

SvdImage Manual

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1 Description

In an abstracted form, images are matrices of color values. Therefore, we can perform linear algebra operations on images just as we do on general matrices. Specifically, when we perform a Singular Value Decomposition (SVD) on an image, we can represent it as the product of 3 matrices: the left-singular vectors, a diagonal matrix of the decreasing singular values and the right-singular vectors.

$$A = USV^T \tag{1}$$

Performing an SVD gets interesting for images when we truncate the factor matrices. Truncation in this sense is the removal of a certain number of trailing columns of U and V and trailing columns and rows of S . By multiplying the matrix back into A after truncating, we get back a similar image to the original. Depending on the amount of truncation, we can keep a “good” representation of the image in less space!

SvdImage is a program and developer library that performs a truncated SVD. It is written in the Ruby programming language and provides a command line interface.

2 Features

- Command Line Interface
- Grayscale, RGB, and CMYK channel processing

- Level of truncation can be “automatic” or user-specified
- Supports most image file formats
- Codec provides targeted file format (.svd)
- Ruby Gem distribution
- Can be used a library

3 Compression

The main application of `SvdImage` is to compress images. However, this does not come for free. We are indeed losing information about the original image. This happens by reducing the rank our image to some value. For example, an image `dog.jpg` may have 200 linearly independent rows and columns. If we run `SvdImage` on this image with a k value of 150, the output will be an image that looks similar to `dog.jpg`, but is actually now comprised of 150 linearly independent rows and columns. In other words, 50 rows and columns will be the linear combination of the other 150 rows and columns.

Another caveat is that you may not always notice compression in file size. This is dependent on a few factors (image content, colorspace, file format). Not each file format stores truncated SVDs in a way optimal to this operation. I can’t imagine a file format that would be able to understand if some rows or columns are linearly dependent.

However, this is not to say you will never see compression! Personally, most times, I have.

4 Sigma Ratio Threshold

`SvdImage` comes with the option to use the `-a/--auto-k` flags. These tell `SvdImage` that you would like to try to find a good truncation automatically. This truncation is unique to the image’s singular values and the sigma threshold. First, we set our sigma threshold equal to some value. By default, `SvdImage` uses 0.2, but you may also specify this yourself by providing an argument to the `-a/--auto-k` flags. Then, starting with a $k = 1$, we iterate through all k until:

$$\text{sigma_threshold} \geq \frac{\sum_{n=k+1}^t \sqrt{\sigma_n^2}}{\sum_{n=1}^t \sqrt{\sigma_n^2}} \quad (2)$$

Intuitively, you may think of this as leaving out $\text{sigma_threshold}\%$ of the image's information, but this is not exactly correct.

5 Question

The project poses a question:

To get a good easily recognizable image, do you need to have σ_{k+1} small compared to 255, or just small compared to σ_1 , or is there some other criterion of smallness that is even more relevant? Try this compression with portraits of faces. How small can you make k , and still keep the portrait recognizable? In other words (roughly), what is the rank of a human face?

I believe I have answered this question with my explanation of the sigma ratio threshold. A good k is not dependent on 255 or just being small, but it is instead dependent on the complexity of the image. k must be chosen

6 Requirements

6.1 System

- Ruby 1.9.2
- GNU Scientific Library 1.15
- ImageMagick 6.7.6-0
- RubyGems 1.8.17

6.2 Ruby Gems

- rake
- bundler

Bundler will be used to easily download and install other gems.

7 Installation

Once all requirements have been met, you can install `SvdImage` . If you are unable to get these requirements or perform the following installation, please contact me!

```
%> tar -xzf svdimage-0.1.0.tar.gz
      Extract the archive contents.
```

```
%> cd svdimage-0.1.0/
      Change into the SvdImage directory.
```

```
%> rake install
      Install SvdImage to your system.
```

And that's it! `SvdImage` should be in your `$PATH` and you can start using it anywhere

8 Usage

```
Usage: svdimage INPUT_IMAGE OUTPUT_IMAGE [OPTIONS]
  -c, --colorspace COLORSPACE    Defines the colorspace of output-image.
                                  Must be "rgb", "gray", or "cmyk".
                                  Defaults to "rgb".
  -k RANK                        Truncates the SVD to rank RANK. May not be
                                  used with -a/--auto-k
  -a, --auto-k [SIGMA_THRESHOLD] Truncates the SVD to a rank determined by
                                  the unique singular values of input-file.
                                  Compares the sum of the square roots of
                                  the squares of the singular values to
                                  SIGMA_THRESHOLD, which may be provided as
                                  an argument or defaults to 0.2.
                                  May not be used with -k

  -h, --help                    Show this message
  --version                     Show version
```

9 Examples

For the following examples, you must have

```
%> svdimage in.jpg out.jpg -k 100
    Truncate in.jpg to have a rank of 100 and write it out as out.jpg.
    Ranks must be  $1 \leq k < \min(\text{image\_height}, \text{image\_width})$ .
```

```
%> svdimage format1.png format2.gif -k 100
    Like previous, but notice how you can use file formats interchangeably.
```

```
%> svdimage in.jpg out.jpg -a
    Truncate in.jpg to a “reasonable” rank. This algorithm attempts to
    remove unneeded ranks, but what ranks you consider unneeded may
    differ from me!
```

```
%> svdimage in.jpg out.jpg -a 0.05
    Like previous, but with a specification of the sigma ratio. See the
    respective section for more information on how to use this.
```

```
%> svdimage in.jpg out.svd -k 20
    Perform a truncation, but output to .svd format, a format designed to
    store SVDs.
```

```
%> svdimage color.jpg bw.jpg -k 43 -c gray
    Decompose to grayscale. The default colorspace is rgb.
```

10 License

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