Machine Learning

A Brief Introduction

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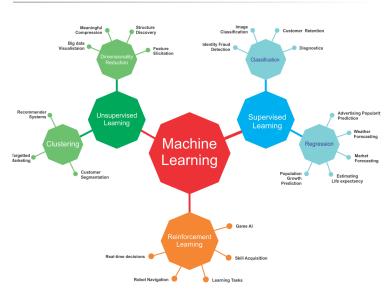
December 2022

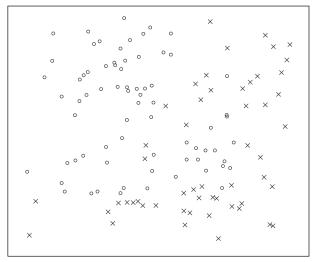
Outline

1. Introduction

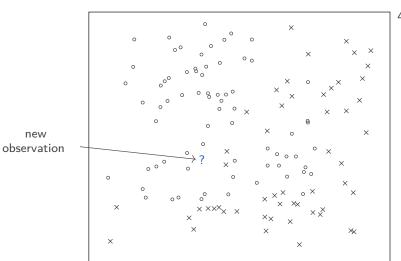
- 2. Supervised Learning
- 3. Decision Trees & Random Forests
- 4. Model Evaluation & Resampling
- 5. Penalized Regression
- 6. Artificial Neural Networks
- 7. Hyperparameter Tuning & Benchmarking
- 8. Discussion

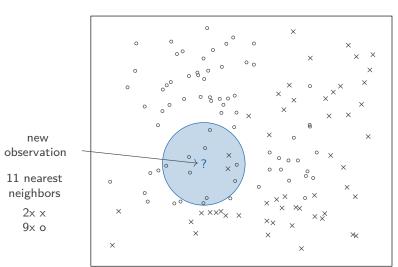
Machine Learning

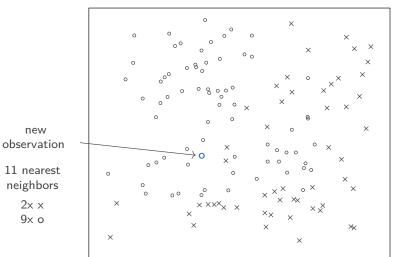




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Example: House Prices

Predict the price for a house in a certain area

	Target y			
square footage of the house	number of bedrooms	swimming pool (yes/no)		house price in US\$
1,180	3	0		221,900
2,570	3	1		538,000
770	2	0		180,000
1,960	4	1		604,000



Example: Length of Hospital Stay

Predict days a patient has to stay in hospital

Features x					Target y
diagnosis category	admission type	gender	age		Length-of-stay in the hospital in days
heart disease	elective	male	75		4.6
injury	emergency	male	22		2.6
psychosis	newborn	female	0		8
pneumonia	urgent	female	67		5.5



Example: Life Insurance

Predict risk category for a life insurance customer

	Target y			
job type	age	smoker		risk group
carpenter	34	1		3
stuntman	25	0		5
student	23	0		1
white-collar worker	39	0		2



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Learn a functional relationship between $\ensuremath{\mathbf{features}}\ x$ and $\ensuremath{\mathbf{target}}\ y$

	Feat	ures x	Target	y
	People in Office (Feature 1) x_1	Salary (Feature 2) x_2	Worked Minute (Target Varia	
	4	4300 € 🗼	2220	
$n=3$ $\left< ight.$	y 12	2700 €	1800	
\downarrow	5	3100 €	1920	_
$x_1^{(2)}$	p =	= 2	$x_2^{(1)}$	$y^{(3)}$

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Use labeled data to learn a model f Use model f to predict target y of new data

	x_1	x_2	Functional Relationship	y
	4	4300 €	n a	2200
Already seen	12	2700 €		1800
Lata	15	3100 €	f	1920
New Data	6	3300 €	no to	???
New Data	5	3100 €		???

Model 11

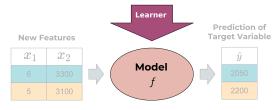
Functional relationship between **features** x and **target** y

Learner (or inducer)

Algorithm for finding model

Train Set

y	x_1	x_2
2200	4	4300
1800	12	2700
1920	15	3100



Example

Learner: Artificial neural network (as a concept)

Model: Actual network with learned weights

Models differ in size and complexity

• Linear model: Coefficients β

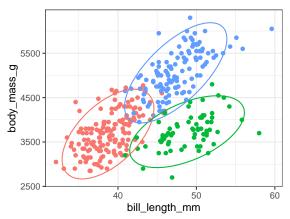
Neural network: Weights for all units in all layers

Decision trees: Many binary splits

k-nearest neighbors: Complete training data

Unsupervised Learning

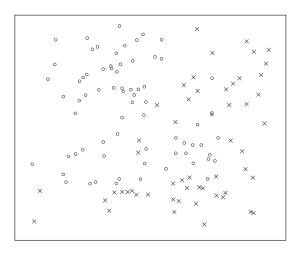
No target y available Search for patterns in the data x, e.g. clustering:

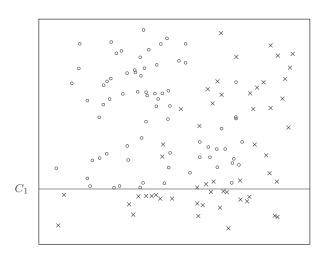


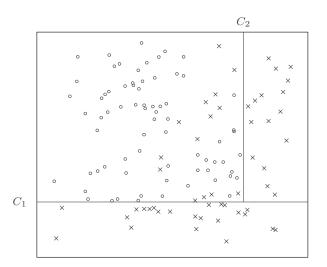
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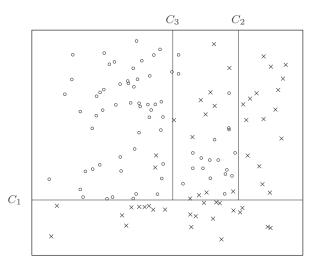




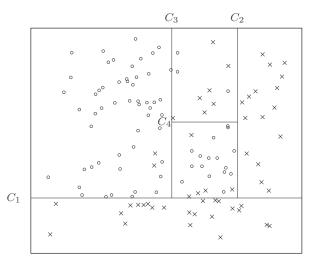




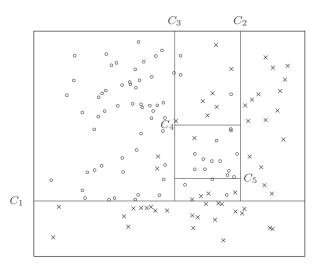


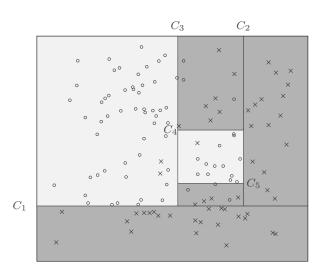


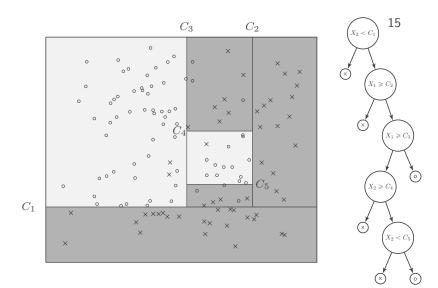


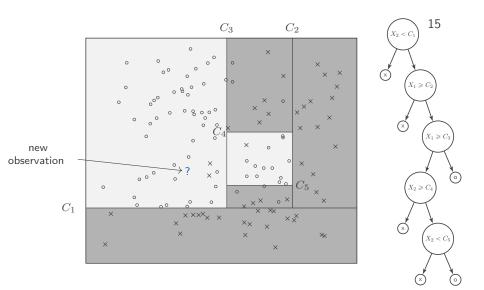


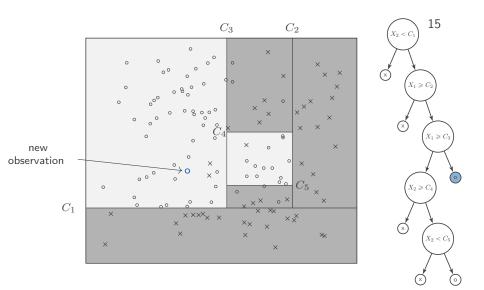












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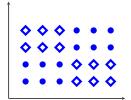
Advantages of decision trees

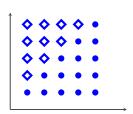
- Procedure intuitive
- Small trees simple to interpret
- Intrinsic variable selection
- Simple handling of outliers
- Fast training
- Usually better prediction performance than kNN

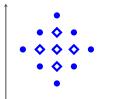
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Disadvantages of decision trees

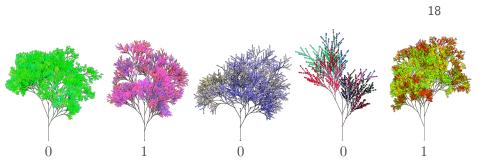
- Trees unstable
- Pruning can be computationally intensive
- Usually worse prediction performance than random forests (covered later) and boosted trees
- Problematic data sets

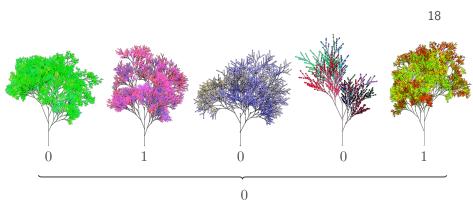




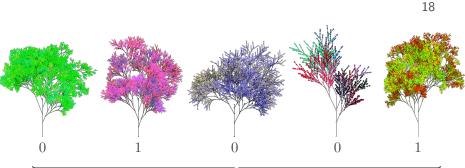








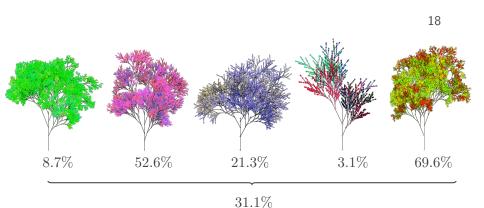
Classification: majority vote over all trees



$$0.4 < 0.5 \Rightarrow 0$$

Classification: majority vote over all trees Identical to average over all trees, cut point 0.5

Breiman 2001 Mach Learn 45:5-32 • Malley et al. 2012 Methods Inf Med 51:74-81 • luc.devroye.org



Probability estimation: Average over all trees

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Two components of randomization

- Data manipulation in rows: bootstrapping / subsampling
- Data manipulation in columns: feature subsampling

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Bootstrap aggregating (bagging)

- Ensemble = committee of experts
- Single weak learner = single committee member
- Ensemble decision = committee decision

Fundamental idea of bagging (bootstrap aggregating)

Any machine can be used as base learner, e.g. kNN or tree

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Bootstrapping

- Sampling with replacement
- Original sample size n, resampled sample size n
- On average $\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n\approx 0.632\approx 2/3$ resampled

Subsampling

- Sampling without replacement
- Original sample size n, resampled sample size < n
- Standard: resampling of 0.632n

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Feature subsampling

At a node consider only subset of features

- Trees vary
- "Experts" differ in their opinion
- Reduce correlation between trees

Number of features considered at split

 $\mathtt{mtry} = \sqrt{d}$, $\ln d$ or $d/3 \to \mathsf{Tuning}$ possible (later)

Random Forests

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Random forest algorithm

For each tree

- 1. Draw bootstrap sample with replacement
- 2. Grow tree
 - a) Use random subset of variables (mtry) at each node
 - b) Stop if minimum node size reached
- 3. Determine proportion of '1' in each terminal node

New subject

- 1. Drop down subject in each single tree
- 2. Store proportion from all trees
- 3. Average proportion of '1's over all trees

Random Forests

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Advantages of random forests

- As with trees: Procedure intuitive, intrinsic variable selection, simple handling of outliers, fast training
- Work well with high dimensional data
- Work well without (or with only a little) tuning
- Usually better prediction performance than a single tree

Random Forests

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Disadvantages of random forests

- Not simple to interpret
- Sometimes worse prediction performance than well tuned boosted trees
- Bad prediction performance on image, text and speech data

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How goood is a prediction model?

Compare true target y with predicted target \hat{y}

Examples

- How many patients correctly diagnosed?
- How many emails correctly detected as ham or spam?
- How close is the predicted price of a house to the true value?
- How close is the length of hospitalization to the true value?

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Dichotomous (binary) outcome

- Proportion of correct classifications (PC); also accuracy: $\widehat{PC} = \frac{1}{n} \sum_{i=1}^{n} \mathbf{1}_{y_i = \hat{y}_i}$
- Sensitivity, specificity, ROC, AUC: $\hat{\mathbb{P}}(y=1\mid x)$
- Brier score (BS), i.e., MSE of probability estimates; also probability score (PS): $\widehat{BS} = \frac{1}{n} \sum_{i=1}^{n} \left(y_i \hat{\mathbb{P}} \left(y_i = 1 \mid x_i \right) \right)^2$

Multicategory outcome

- Proportion of correct classifications (PC)
- Averaged class-wise PC
- ROC, AUC: several extensions

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Continuous outcome

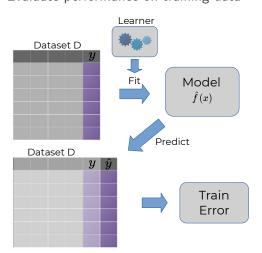
- MSE: $\widehat{MSE} = \frac{1}{n} \sum_{i=1}^{n} (y_i \hat{y}_i)^2$
- MAE: $\widehat{MAE} = \frac{1}{n} \sum_{i=1}^{n} |y_i \hat{y}_i|$
- RMSE: $\widehat{RMSE} = \sqrt{\widehat{MSE}}$
- Explained variance: $\hat{R}^2 = \frac{1 \bar{M}S\bar{E}}{\widehat{\mathbb{V}ar}(y)}$

Survival outcome

- Time-dependent Brier Score
- Integrated Brier score
- C-Index

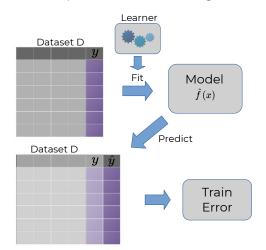
Training error

Evaluate performance on training data



Training error

Evaluate performance on training data

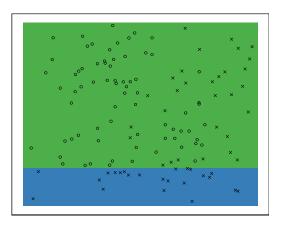


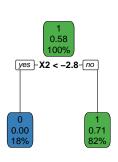
Problem: Overfitting

Overfitting

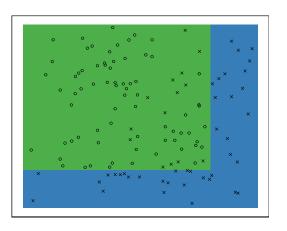
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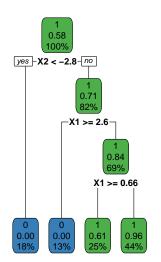
Overfitting 31



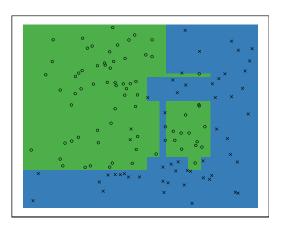


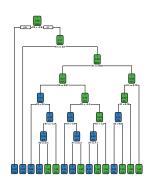
Overfitting 31



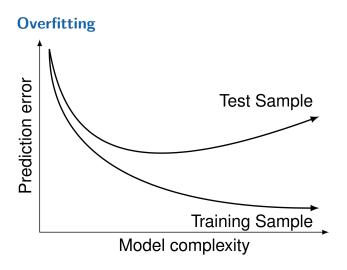


Overfitting 31



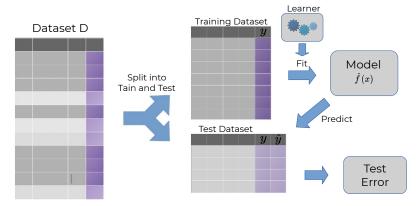






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Test error



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Training and test error

- Training error heavily biased
- Test error (almost) unbiased but variance unknown

Resampling

- Repeated training/test splits (subsampling)
- Cross validation
- Repeated cross validation
- Bootstrap

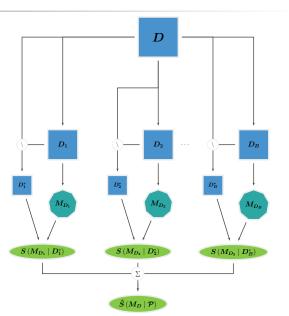
Hyperparameters

Most (all?) learners have hyperparameters, e.g.:

- k-nearest neighbors: Number of neighbors k, distance weighting, etc.
- Decision trees: Tree depth, splitting criterion, etc.
- Neural networks: Number and size of layers, activation function, regularization, etc.

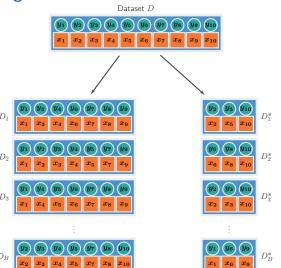
Hyperparameter tuning

- Optimize (tune) the hyperparameters
- Do not tune and evaluate on same data
- → 3-fold split into training, validation, test
- → Nested resampling



- Estimate performance on independent data
- Used for
 - Performance estimation
 - Hyperparameter tuning
 - Model selection
- Resampling based performance estimation
 - 1. Split dataset in several (smaller) datasets D_b
 - 2. On each dataset D_b :
 - 2.1 Train learner
 - 2.2 Estimate performance on $D_b^* = D \backslash D_b$
 - 3. Aggregate performance estimates

Subsampling

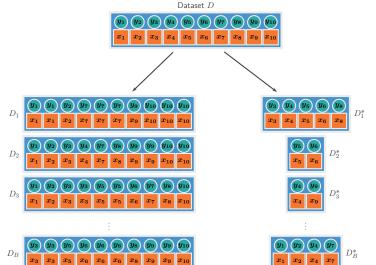


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Subsampling

- Sample B training datasets D_b from D without replacement, usually $n_b = \frac{2}{3}n$
- Use $D_b^* = D \backslash D_b$ as test datasets
- D_b and D_b^* disjunct
- D_1 and D_2 not disjunct
- D_1^* and D_2^* not disjunct
- Performance estimator biased
- No optimal B, usually 100 < B < 1000
- Special case with B=1: Single train/test split (holdout)





Bootstrapping

- Sample B training datasets D_b from D with replacement, usually $n_b=n$
- Use $D_b^* = D \backslash D_b$ as test datasets
- D_b and D_b^* disjunct
- D_1 and D_2 not disjunct
- D_1^* and D_2^* not disjunct
- Performance estimator biased
- Adaptive weighting to reduce bias (.632+ bootstrap)
- Small variance (large B)
- No optimal B, usually 100 < B < 1000

Cross validation (CV)

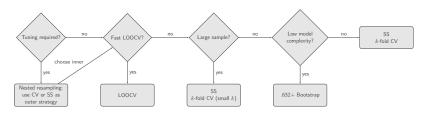
Dataset D D_3^* D_3^* D_2^* D_5

Cross validation (CV)

- Split D in B test datasets D_b^*
- Use $D_b = D \backslash D_b^*$ as training datasets
- D_b and D_b^* disjunct
- D_1 and D_2 not disjunct
- D_1^* and D_2^* disjunct
- Special case with B=n: Leave-one-out CV (LOOCV)
 - Small bias, high variance
 - Long runtime
- No optimal B, usually B=5,10
 - Slightly more bias than LOOCV, but lower variance
 - Lowest B of all resampling methods \rightarrow fast computation

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How to choose the resampling method?



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Generalized linear model

$$g(\mathbb{E}(Y)) = \beta_0 + \beta_1 \cdot X_1 + \ldots + \beta_p \cdot X_p$$

= $X\beta$

g: Link function

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Generalized linear model

$$g(\mathbb{E}(Y)) = \beta_0 + \beta_1 \cdot X_1 + \ldots + \beta_p \cdot X_p$$

= $X\beta$

g: Link function

Linear model

$$\mathbb{E}(Y) = X\beta$$

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Ordinary least squares

Minimize squared differences

$$L_{OLS} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$
$$= \|y - X\beta\|_2^2$$
$$= (y - X\beta)'(y - X\beta)$$

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Ordinary least squares

Minimize squared differences

$$L_{OLS} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

= $||y - X\beta||_2^2$
= $(y - X\beta)'(y - X\beta)$

Solution:

$$\beta_{\mathsf{OLS}} = \left(X'X \right)^{-1} X'y$$

Ridge regression

Penalize large parameter estimates (L2 regularization)

$$L_{\text{Ridge}} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^{m} \beta_j^2$$
$$= \|y - X\beta\|_2^2 + \lambda \|\beta\|_2^2$$

Ridge regression

Penalize large parameter estimates (L2 regularization)

$$L_{\text{Ridge}} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^{m} \beta_j^2$$
$$= \|y - X\beta\|_2^2 + \lambda \|\beta\|_2^2$$

Solution:

$$\beta_{\mathsf{Ridge}} = \left(X'X + \lambda I \right)^{-1} X'y$$

Ridge regression

Penalize large parameter estimates (L2 regularization)

$$L_{\text{Ridge}} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^{m} \beta_j^2$$
$$= \|y - X\beta\|_2^2 + \lambda \|\beta\|_2^2$$

Solution:

$$\beta_{\mathsf{Ridge}} = \left(X' X + \lambda I \right)^{-1} X' y$$

Shrink parameter estimates towards zero

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How to find best λ ?

Minimize L_{Ridge} in cross validation

 \rightarrow Hyperparameter tuning

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LASSO: Least absolute shrinkage and selection operator

Penalize large parameter estimates (L1 regularization)

$$L_{\text{LASSO}} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^{m} |\beta_j|$$
$$= ||y - X\beta||_2^2 + \lambda ||\beta||_1$$

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LASSO: Least absolute shrinkage and selection operator

Penalize large parameter estimates (L1 regularization)

$$L_{\text{LASSO}} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^{m} |\beta_j|$$
$$= ||y - X\beta||_2^2 + \lambda ||\beta||_1$$

No closed-form solution

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LASSO: Least absolute shrinkage and selection operator

Penalize large parameter estimates (L1 regularization)

$$L_{\text{LASSO}} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \lambda \sum_{j=1}^{m} |\beta_j|$$
$$= ||y - X\beta||_2^2 + \lambda ||\beta||_1$$

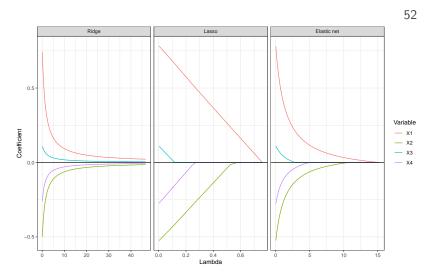
No closed-form solution

Shrink parameter estimates to (exactly) zero

Elastic net: Combination of Ridge and LASSO

L1 and L2 regularization

$$L_{\text{Elnet}} = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \lambda_1 \sum_{j=1}^{m} |\beta_j| + \lambda_2 \sum_{j=1}^{m} \beta_j^2$$
$$= \|y - X\beta\|_2^2 + \lambda_1 \|\beta\|_1 + \lambda_2 \|\beta\|_2^2$$



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Advantages of penalized regression

- Reduces overfitting
- Avoid multicolinarity issues of (non-penalized) regression models
 - → Work well with high-dimensional data
- Same general concept of (non-penalized) regression models
 - → Interpretable model
- Better prediction performance than non-penalized regression (less variance)
- Implicit variable selection (LASSO)

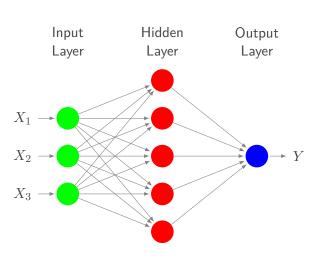
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Disadvantages of penalized regression

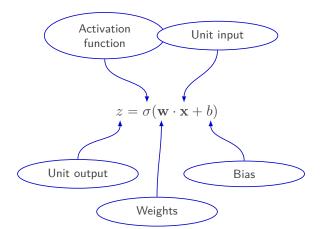
- Biased parameter estimates
- Cannot use statistical inference methods used in non-penalized regression
- Interactions and non-linear effects have to be explicitly specified
- Often worse prediction performance than (other) machine learning algorithms

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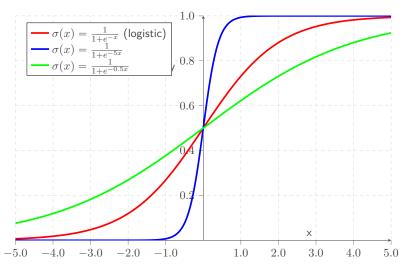
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For each hidden unit:

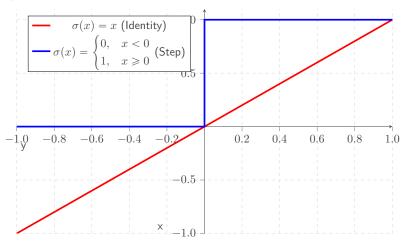


Activation function: Sigmoid



Special cases

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Identity: Linear model

Step: Perceptron

How to fit a neural network?

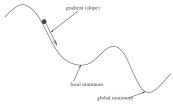
Loss function: Error as function of network weights, e.g,

$$L(W) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

Aim: Find weights that minimize error

Gradient descent

Adjust weights in direction with steepest descent

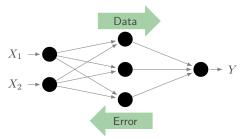


How to fit a neural network?

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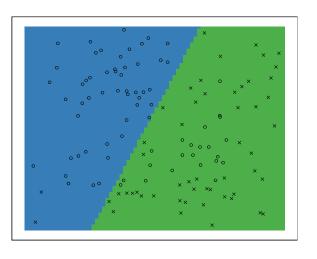
Backpropagation

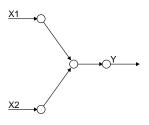
- 1. Initialize weights randomly
- 2. For k iterations repeat
 - a) Compute error function
 - b) Adjust weights in output layer
 - c) Propagate error backwards through network and adjust weights



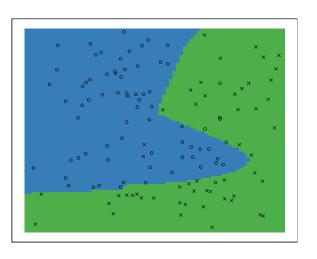
Do neural networks overfit?

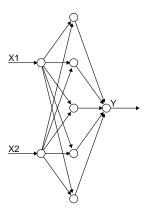
Do neural networks overfit?



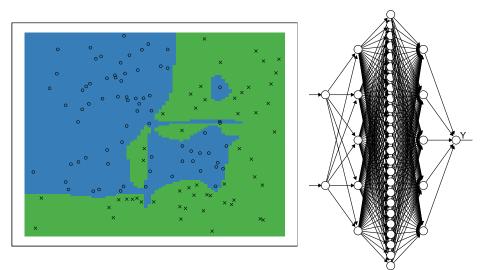


Do neural networks overfit?





Do neural networks overfit?



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L2 regularization

$$L(\mathcal{W}) = \frac{1}{n} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 + \lambda \sum_{\mathbf{w} \in \mathcal{W}} \mathbf{w}^2$$

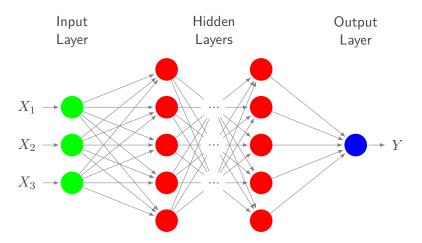
Alternative: Dropout

Temporarily remove units while fitting

Other alternative: Early stopping



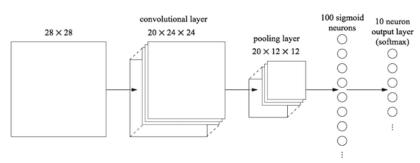
What is (not) deep learning?



What is deep learning?

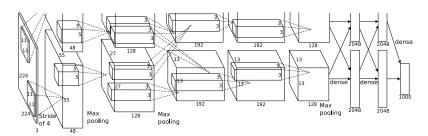
Idea: Build hierarchy of concepts (representations)

Convolutional neural networks



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Example



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Advantages of neural networks

- Can fit any complicated function almost perfectly
- Learns representations
- Online learning possible
- Prediction very fast (matrix multiplication)
- Can be parallelized (GPU computing)

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Disadvantages of neural networks

- Prone to overfitting
- Difficult to design good networks
- Interpretation very difficult (black box)
- Learning can be slow
- Statistical properties not well studied

Outline

- 1. Introduction
- 2. Supervised Learning
- 3. Decision Trees & Random Forests
- 4. Model Evaluation & Resampling
- 5. Penalized Regression
- 6. Artificial Neural Networks
- 7. Hyperparameter Tuning & Benchmarking
- 8. Discussion

...

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Hyperparameters

Learners have hyperparameters, e.g.:

- Number of nearest neighbors k
- Depth of a tree
- Number of features to consider in each split of a random forest (mtry)
- Architecture of neural network

Most learners have several hyperparameters

Have to be jointly optimized

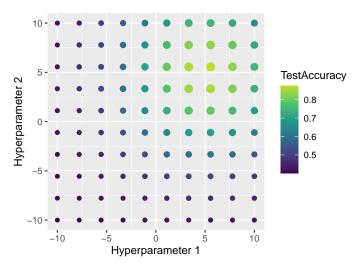
Search entire parameter space

- All possible combinations
- Grid search
- Randomly select combinations
- Model-based optimization

Use resampling

- Evaluate each parameter combination on all resampling iterations/folds
- Choose parameter maximizing aggregated performance measure

Grid search 72



Grid search

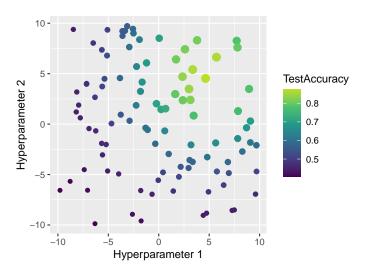
Advantages

- Easy to implement
- All parameter types possible
- · Easily parallelized

Disadvantages

- Computationally intensive
- Inefficient: Searches large irrelevant areas
- Arbitrary: Which values / discretization?

Random search



Random search

Advantages

- Same as grid search: Easy to implement, all parameter types possible, trivial parallelization
- Easy to adjust to computational budget
- No discretization
- Superior performance compared to grid search

Disadvantages

- Computationally intensive
- Inefficient: Searches large irrelevant areas

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Model-based optimization

Surrogate model

Learn relationship between hyperparameters and prediction performance

Algorithm

- 1. Pick initial configuration (e.g. random)
- 2. Learn surrogate model
- 3. Predict new configuration with surrogate model
- 4. Repeat steps 2 and 3

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Model-based optimization

Advantages

- All parameter types possible
- Efficient: Focus on promising areas
- Superior performance compared to grid and random search

Disadvantages

- Computationally intensive
- Non-trivial parallelization
- Harder to implement

Benchmarking

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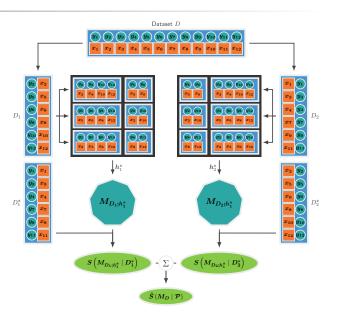
How can performance be compared?

Be fair!

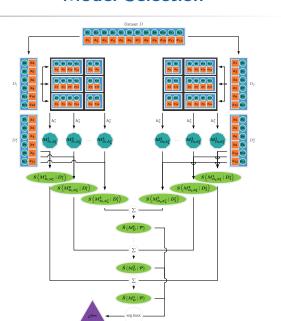
- Compare all learners and models on same data
- Tune parameters of all learners
- Don't overfit
- Don't publish over-optimistic results

Never learn, tune or evaluate on same data!

Nested Resampling



Model Selection



Benchmarking

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How to build a final model?

- 1. Select best learner with nested resampling
- 2. Find optimal hyperparameters of best learner with resampling
- 3. Train best learner with optimal hyperparameters on full data

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Discussion

Is there a single best learner?

No!

Learner recommendations

- Typically RF ≈ Boosting > Tree > kNN
- RF robust, easy to tune and fast
- Boosting often slightly better than RF on tabular data (when properly tuned)
- Support vectot machine (SVM) good alternative for binary classification with numerical features (when properly tuned)
- Image, text and speech data → Deep Learning
- Consider ensembles, e.g. stacking → SuperLearner

Discussion

Tuning recommendations

- Never use default parameter settings
- Tune hyperparameters
- Tune hyperparameters jointly
- Parameter tuning simple and straightforward for
 - kNN
 - Decision trees
 - Boosting
 - Random forests
- Parameter tuning complex and not straightforward for
 - SVM: hyperparameters depend on kernel
 - ANN: tuning of architecture
- Use adequate resampling strategy
- Gold standard: nested cross validation