EEP 225 : Electric Machines

Lecture 10
Induction Motor II

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1 Frequency of The Induced Voltage (or Current) in Rotor Windings f_r

We know that

$$f_s = \frac{PN_s}{120}$$

How can we get f_r? a common mistake to assume that $f_r = \frac{PN_r}{120}$ (wrong), Why? because it doesn't depend on rotor speed N_r, it depends on the relative speed, therefore:

$$f_r = \frac{P(N_s - N_r)}{120}$$

$$f_r = \frac{P(N_s - N_r)}{120} \times \frac{N_s}{N_s}$$

$$\boxed{f_r = sf_s}$$

2 Equivalent Circuit of Three Phase Induction Motor

Remember that the rotor windings are shorted through brushes riding on the slip rings.

What if we removed the brushes? in other words if we removed the short circuit.

1. Motor will not start.
2. Motor will act as a **transformer** Why? there are winding in rotor and stator and there are electromagnetic induction between them.

   In most cases, turns ration in induction motor is 1:1

Therefore, the equivalent circuit of three phase induction motor is similar to the equivalent circuit of the transformer, with two main differences:

1. There are no electrical load on rotor circuit(short circuit)[1]
2. The primary and secondary circuit don't have the same frequency, instead, there are f_s in stator circuit and f_r in rotor circuit.

[1]There are mechanical load, but not considered, there are no electric load e.g. a resistor

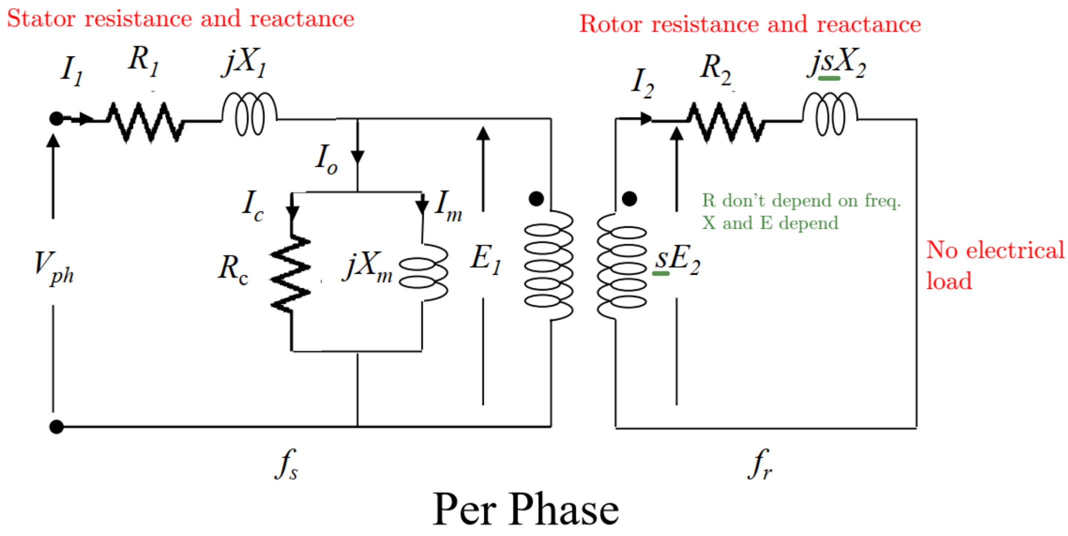

Figure 1

Note: referring is not valid between two circuits with different frequencies.

because $f_r = sf_s$, we divide the rotor circuit by s

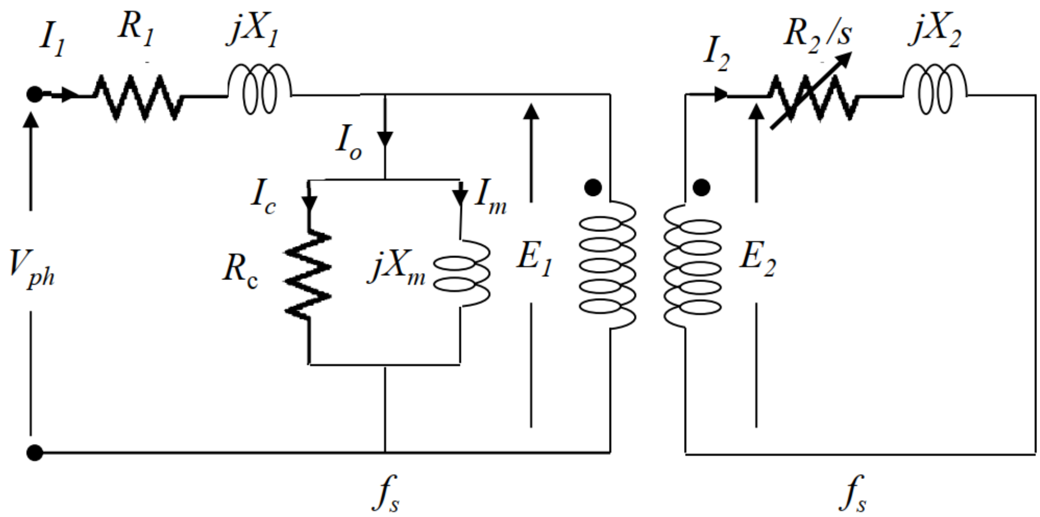

Figure 2

Because turns ratio is 1:1, referring is very simple, no need to multiply or divide.

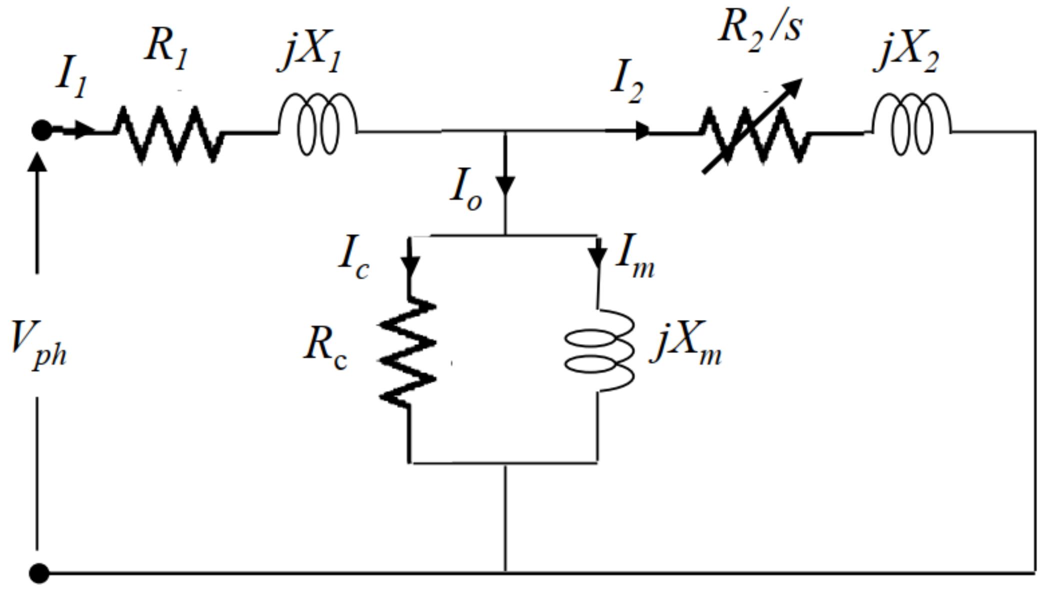

Figure 3

Analysis of this circuit is not easy, we apply two approximations.

First approximation:

Voltage drop on stator is very low such that supply voltage V_{ph} is the same on magnetizing branch. (we can move the magnetizing branch as shown in fig 4)

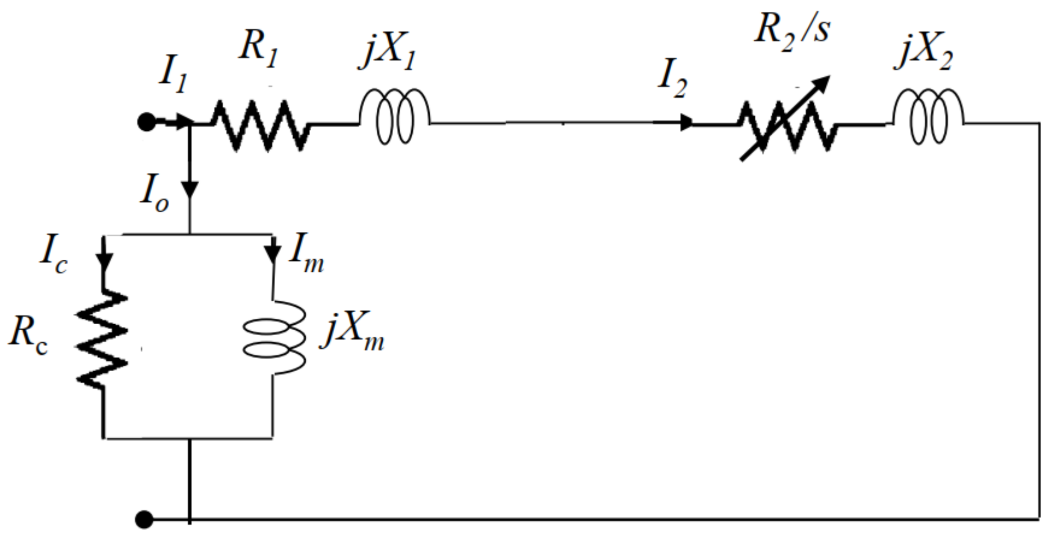

Figure 4

Second approximation: I_0 is very small relative to I_1 (nearly 5%) (magnetizing branch can be ignored)

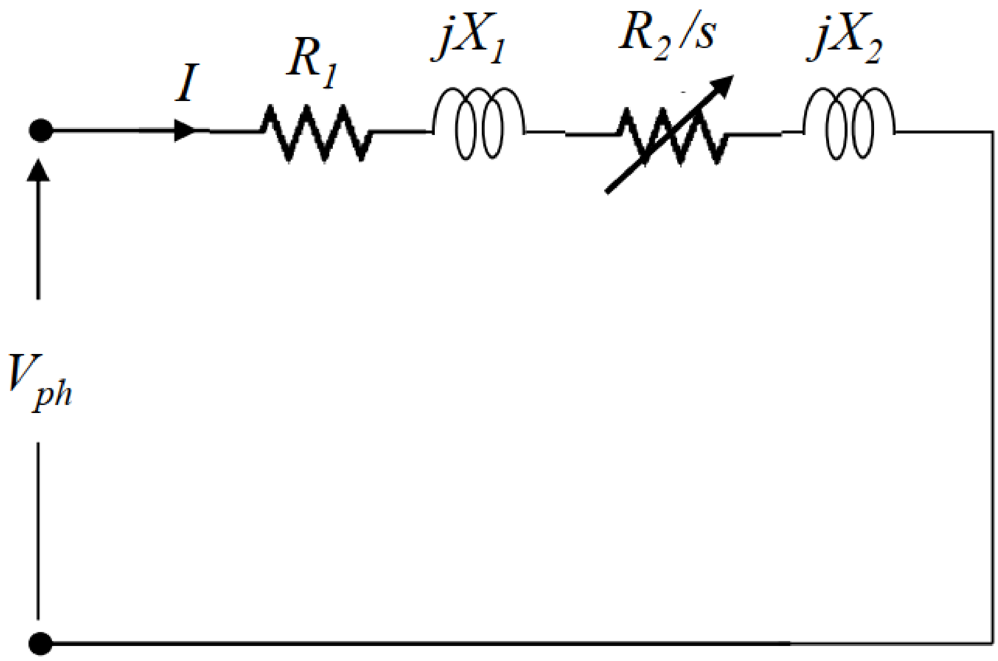

Figure 5: Equivalent Circuit Per Phase.

We divide $\dfrac{R_2}{s}$ to $\left(R_2 + R_2\dfrac{1-s}{s}\right)$

Why? to separate R_2 which is a physical resistance and $R_2\dfrac{1-s}{s}$ on which the developed power is noticed (called dynamic resistance)

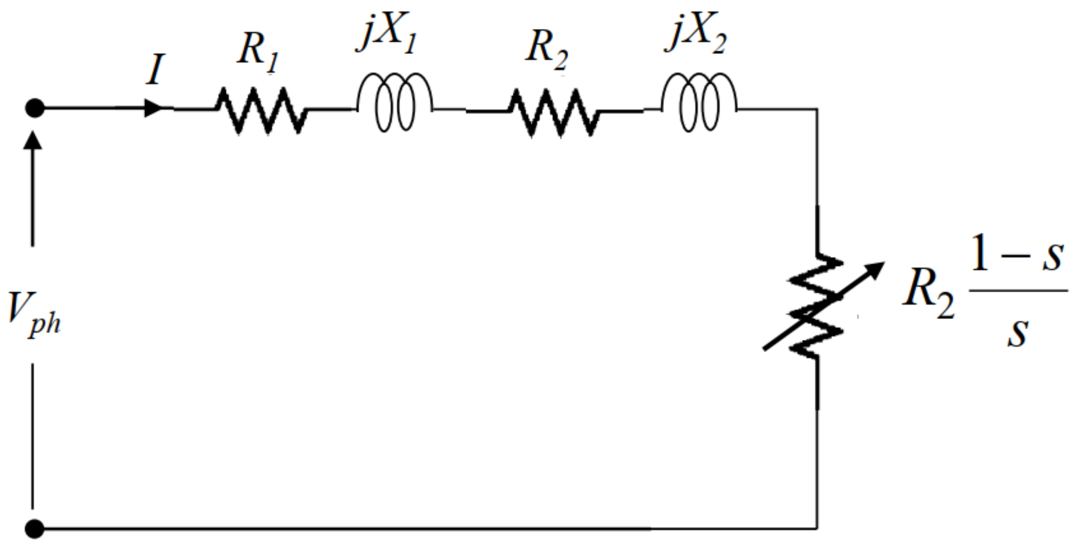

Figure 6: Equivalent Circuit Per Phase.

3 Power Flow and Losses

From ohm's law:

$$\overline{I_{\text{ph}}} = \frac{V_{\text{ph}}}{\left(R_1 + \frac{R_2}{s}\right) + j(X_1 + X_2)} = |I_{\text{ph}}|\underline{/\phi}$$

Note the power factor in 3 phase induction motor is always lagging.

We know that input power:

$$P_{\text{in}} = 3V_{\text{ph}}I_{\text{ph}}\cos(\phi)$$

Air gap power = input power − stator losses

it is the power that transfers from stator to rotor.

$$P_g = P_{\text{in}} - S.L. = 3I^2\frac{R_2}{s}$$

Developed power = Air gap power - rotor copper loss

$$R.C.L = 3I^2R_2$$

$$P_d = P_g - R.C.L = 3I^2R_2\frac{1-s}{s}$$

Output power = developed power - rotational losses

$$P_{\text{out}} = P_d - P_r$$

$$\boxed{P_g : R.C.L : P_d = 1 : s : 1-s}$$

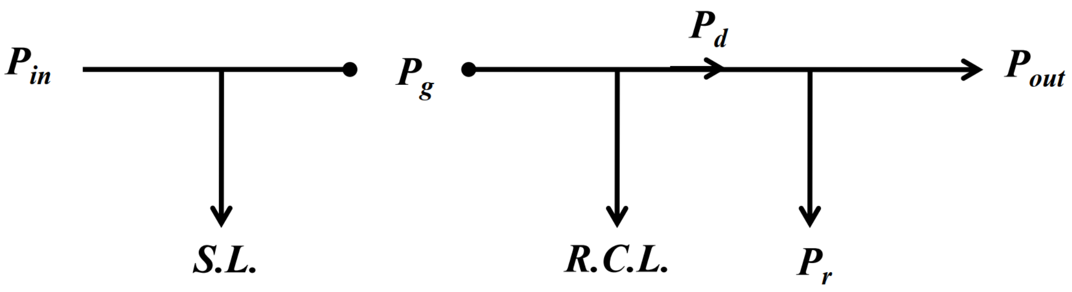

Figure 7: Power flow and losses in induction motor.

Is there poles in induction motor?

Yes, but not similar to synchronous or DC machines, in synchronous or DC machines the poles are real, physical poles, in induction motor, poles depends on the way of winding.

In examples: the efficiency η of induction motor is around 85%.

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1. Q — A 10 hp, 6-pole, 440 V, 60 Hz, Y-connected, three-phase induction motor is designed to operate at 3% slip on full load. The stator loss is 250 W and the rotational loss is 4% of the power output. Determine :

1. Air gap power.
2. Rotor copper loss R.C.L.
3. Input power
4. Efficiency.
5. Output torque.
6. Developed torque

A —

$$P_{\text{out}} = 10\text{hp} \times 746 = 7460W$$

$$P_r = 0.04 \times 7460 = 298.4W$$

$$P_d = P_{\text{out}} + P_r = 7460 + 298.4 = 7758.4W$$

(1)

$$P_g = \frac{P_d}{1-s} = \frac{7758.4}{1-0.03} = 7998.3W$$

(2)

$$R.C.L. = s \times P_g = 0.03 \times 7998.3 = 239.9W$$

or

$$R.C.L. = P_g - P_d = 7998.3 - 7758.4 = 239.9W$$

(3)

$$P_{\text{in}} = P_g + \text{stator losses S.L.} = 7998.3 + 250 = 8248.3W$$

(4)

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{7460}{8248.3} = 90.44\%$$

(5)

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_r}$$

$$\omega_r = 2\pi\frac{N_r}{60}$$

$$N_r = N_s(1-s)$$

$$N_s = 120 \times \frac{f}{P} = 120 \times \frac{60}{6} = 1200 \text{ rpm}$$

$$\therefore N_r = 1200(1-0.03) = 1164 \text{ rpm}$$

$$\therefore \omega_r = 2\pi\frac{1164}{60} = 121.83 \text{ rad/sec}$$

$$\therefore T_{\text{out}} = \frac{7460}{121.83} = 61.23 \text{ N.m.}$$

(6)

$$T_{\text{d}} = \frac{P_{\text{d}}}{\omega_r} = \frac{7758.4}{121.83} = 63.68 \text{ N.m.}$$

4 Torque-Slip Relationship

We know

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_r}$$

Therefore :

$$T_{\text{d}} = \frac{P_{\text{d}}}{\omega_r} = \frac{P_{\text{g}}(1-s)}{\omega_s(1-s)} = \frac{P_{\text{g}}}{\omega_s}$$

Why we did that? because ω_s is constant, unlike ω_r

Remember:

$$P_g = 3I^2\frac{R_2}{s} = 3R_s \times \frac{1}{s} \times \underbrace{\frac{V_{\text{ph}}^2}{\left(R_1 + \frac{R_2}{s}\right)^2 + (X_1+X_2)^2}}_{I^2}$$

$$\therefore T_{\text{d}} = \frac{3R_2}{\omega_s \times s} \times \frac{V_{\text{ph}}^2}{\left(R_1 + \frac{R_2}{s}\right)^2 + (X_1+X_2)^2}$$

Everything in this equation is constant, except (s)

Note that (s) varies from 1 to 0, not from 0 to 1, because start occur first.

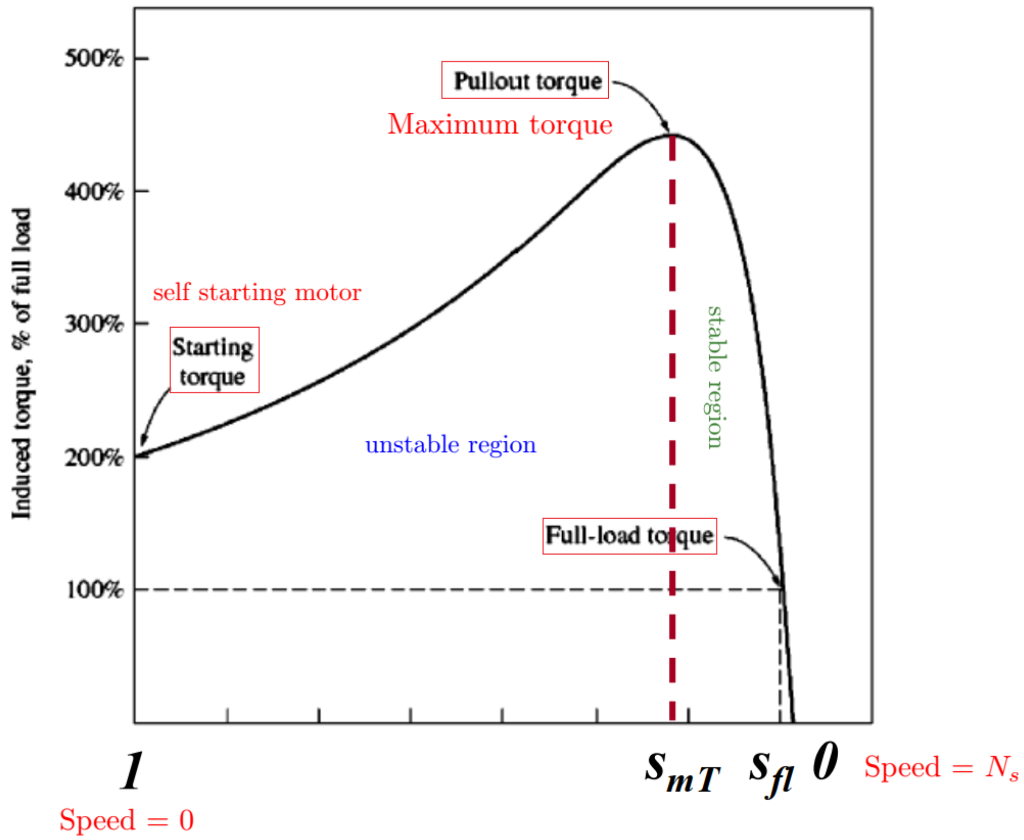

Figure 8: Torque-Slip relationship - To get stable operation in any motor, $\frac{dT}{d\omega}$ = negative, when one increases the other decreases.

s starting = 1.

s at full load will be given.

What about s_{mT} ?

We want to find s_{mT} (slip corresponding to maximum torque)

It can be obtained by differentiation, but there are easier way.

The maximum possible torque occurs when the air-gap power P_g is maximum.

Using maximum power transfer theorem, the maximum power transfer to the load resistor $\dfrac{R_2}{s}$ occurs when the magnitude of that resistance = the magnitude of the impedance seen by the source.

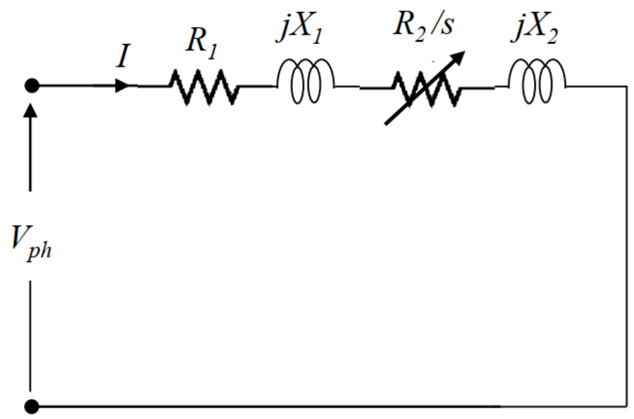

Figure 9

$$\left|R_1 + \frac{R_2}{s_{mT}}\right| = |j(X_1 + X_2)|$$

$$\therefore \frac{R_2}{s_{mT}} = |R_1 + j(X_1 + X_2)| = \sqrt{R_1^2 + (X_1 + X_2)^2}$$

$$s_{mT} = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}$$

In some examples, stator impedance is small and can be ignored, then $s_{mT} = \frac{R_2}{X_2}$

$N_{r_{mt}}$ is the rotor speed corresponding to maximum torque, and is the minimum speed motor operate at.

$$N_{r_{mT}} = N_s(1 - s_{mT})$$

To get the maximum torque, substitute by s_{mT} in the general torque equation

$$T_{\max} = \frac{3R_2}{\omega_s \times s_{mT}} \times \frac{V_{\mathrm{ph}}^2}{\left(R_1 + \frac{R_2}{s_{mT}}\right)^2 + (X_1 + X_2)^2}$$

5 Maximum Developed Power

The maximum developed power occurs when the power consumed in the $R_2\dfrac{1-s}{s}$ resistor is maximum

It can be obtained by differentiation, but there are easier way.

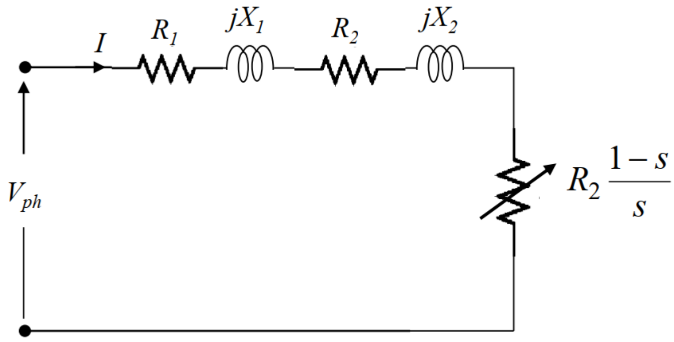

Figure 10

the maximum power transfer to the load resistor occurs when the magnitude of that resistance = the magnitude of the impedance (R_l) seen by the source.

$$R_L = \sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2} = R_2 \frac{1 - s_{mP}}{s_{mP}}$$

s_{mP} is the slip corresponding to maximum power.

$$\therefore s_{mP} = \frac{R_2}{R_2 + R_L}$$

$$N_{r_{mP}} = N_s(1 - s_{mP})$$

$$P_{d_{\max}} = 3I_{mP}^2 R_L$$

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$$P_{d_{\max}} = \frac{3V_{mP}^2}{2(R_1 + R_2 + R_L)}$$

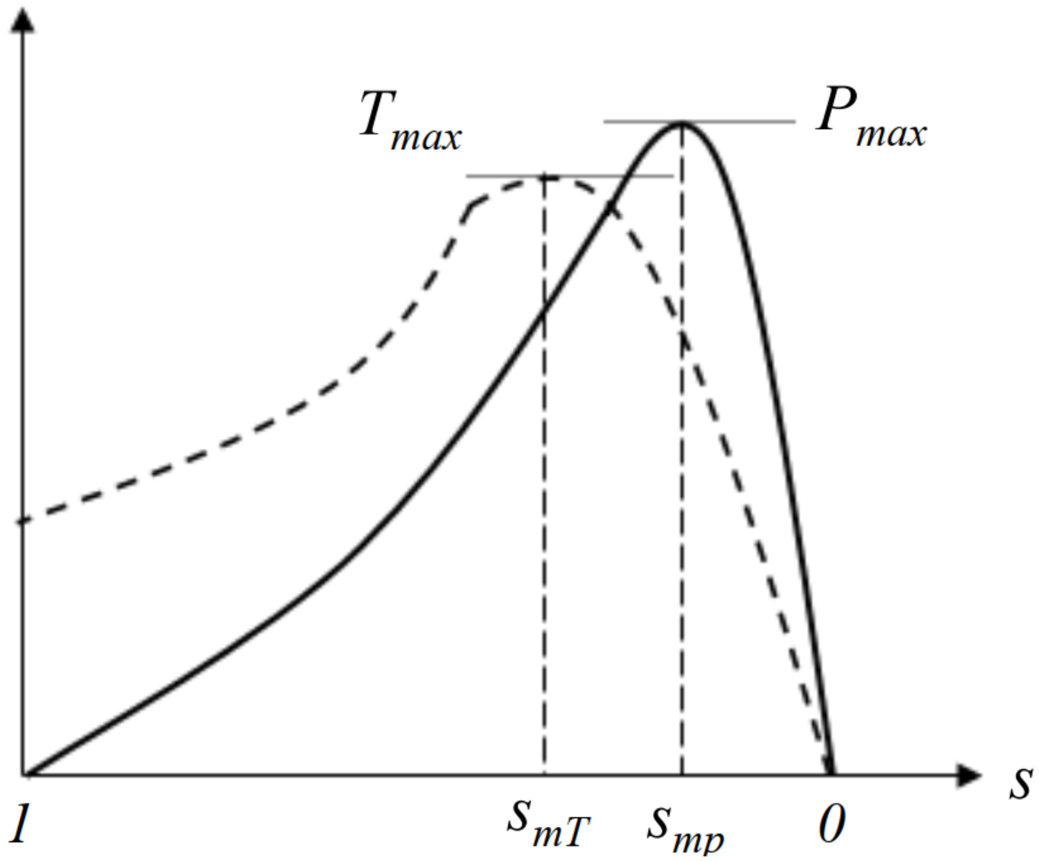

Figure 11: Maximum power and maximum torque occur at two different points

Note that:

$$s_{mT} > s_{mP}$$

6 Effect of Varying Rotor Resistance

$$\because T_{\mathrm{d}} = \frac{3R_2}{\omega_s \times s} \times \frac{V_{\mathrm{ph}}^2}{\left(R_1 + \frac{R_2}{s}\right)^2 + (X_1 + X_2)^2}$$

We want to change rotor resistance, substitute:

$$R_2 = R_2 + R_{\mathrm{ext}}$$

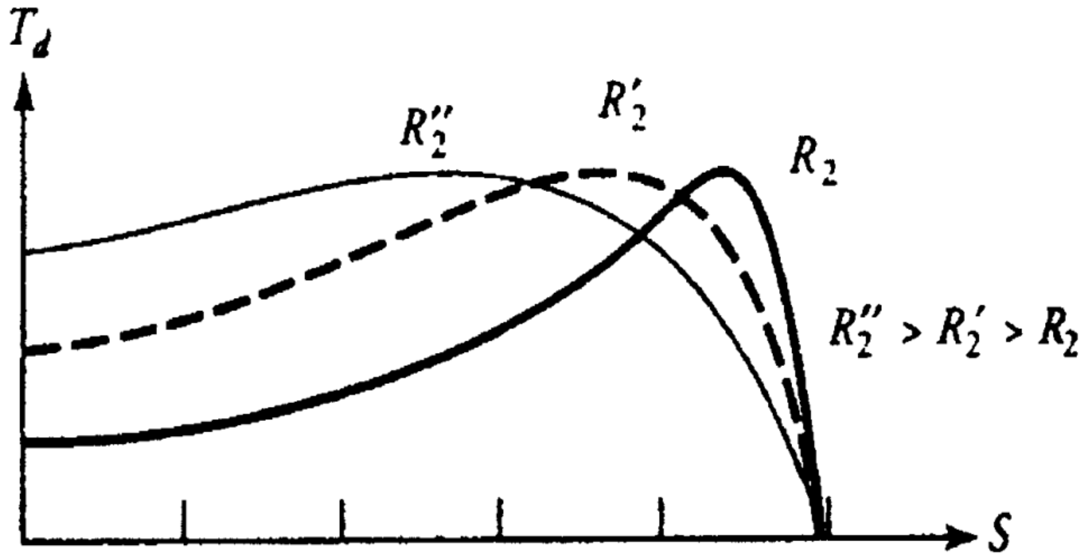

Figure 12: Effect of varying rotor resistance

Note that the maximum torque doesn't depend on R_2

When R_2 increases $\uparrow$, starting torque increases $\uparrow$, which is good, to be able to start, motor torque must be greater than load torque + the friction

At start:

$$s_{mT} = 1$$

We want to get R_{external} to obtain Maximum torque at start:

$$s_{mT} = \frac{R_2 + R_{\text{ext}}}{\sqrt{R_1^2 + (X_1 + X_2)^2}} = 1$$

$$R_{\text{ext}} = \sqrt{R_1^2 + (X_1 + X_2)^2} - R_2$$

Note that this method can be used only in wound rotor, not the squirrel cage rotor.