
Lecture 3
DC Machines III

1. Q — A 5 kW, 100 V, 1000 rpm, shunt generator has the following open circuit curve at rated speed:

$I_f(A)$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2
$E_a(V)$	6	16	32	48	63	76	85	94	100	102

The generator has armature and field winding resistances of 0.4Ω and 120Ω respectively. If the generator is driven at 1200 rpm, determine:

1. The no load terminal voltage.
2. The critical field resistance.
3. if you insert a variable resistance to control the field, what is the operating range.
4. The maximum value of the armature current the generator can deliver; determine the terminal voltage in this case. Neglect armature reaction effect.
5. The critical speed.
6. Residual flux.

A —

1. First we find the operating point at no load, which is the intersection of magnetization curve and E_a-I_f line.

The given table is at 1000 rpm, since the generator is driven at 1200 rpm, new table must be created.

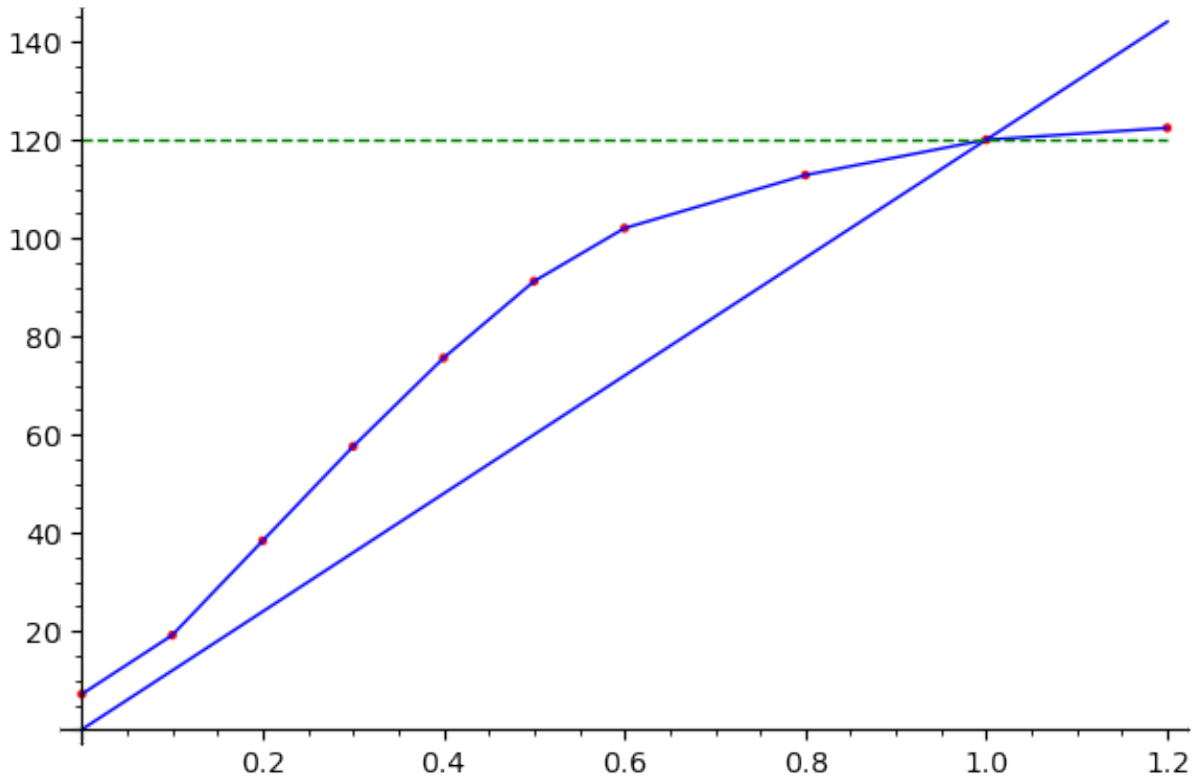
$$E = K\phi N$$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$\therefore E_2 = 1.2E_1$$

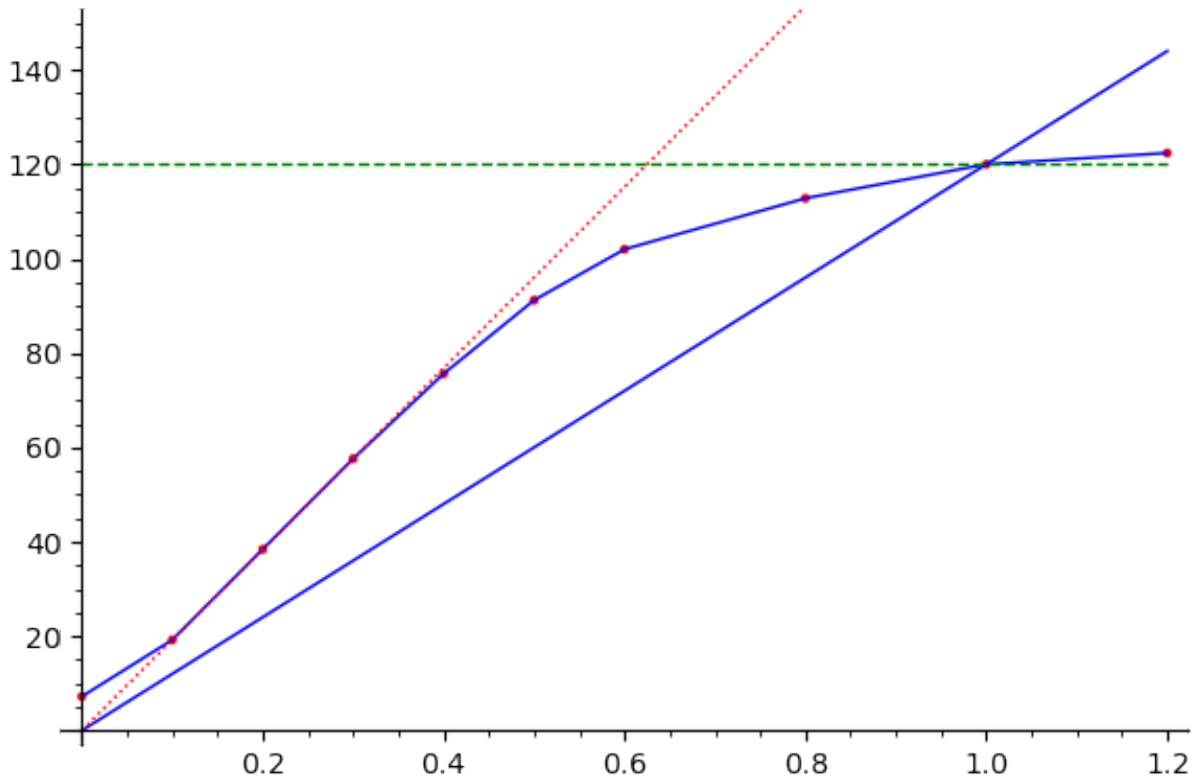
$I_f(A)$	0	0.1	0.2	0.3	0.4	0.5	0.6	0.8	1.0	1.2
$E_{a1}(V)$	6	16	32	48	63	76	85	94	100	102
$E_{a2}(V)$	7.2	19.2	38.4	57.6	75.6	91.2	102	112.8	120	122.4

We draw magnetization curve from the table, and current field line from the equation $V_t = R_f I_f$, since $R_f = 120 \Omega$, $V_t = 120 I_f$



As shown in the figure, the intersection point (no load terminal voltage) = 120 V

- To get the critical field resistance, we need to find a tangential line to the magnetization curve at the linear region (region excluding residual volt region and saturated region), since the magnetization curve is linear at point (0.1, 19.2), the critical field resistance = $\frac{19.2}{0.1} = 192 \Omega$

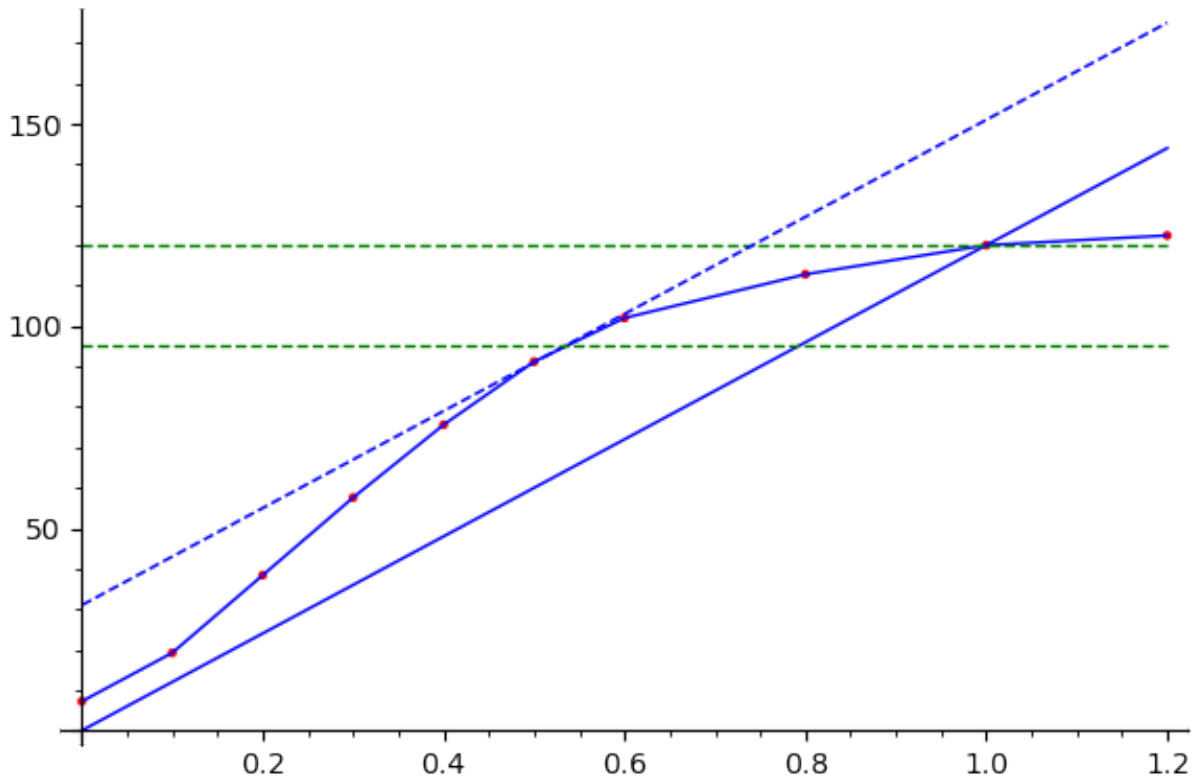


3. Since the maximum resistance allowed is $192 \, \Omega$, field resistance is $120 \, \Omega$, the operating range is $[0, 192 - 120] = [0, 72] \, \Omega$

4. Recall:

$$E_a = V_t + I_a R_a$$

We tune the value of $I_a R_a$ till the line becomes tangential to the magnetization curve without changing its slope.



as shown in the figure

$$I_a R_a = 31V$$

$$I_a = \frac{31}{0.4} = 77.5A$$

intersection point is at E_a (no load voltage at maximum current) = 95 V

$$V_t = E_a - I_a R_a$$

$$V_t = 95 - 31 = 64V$$

5. how to find the critical speed?

start with the linear region of magnetization curve, adjust the slope of this region which is $\frac{\Delta y}{\Delta x}$ to be equal the slope of $E_a - I_f$ line, which is 120

$$n \times \frac{19.2}{0.1} = 120$$

$$n = 0.625$$

$$\therefore \text{critical speed} = 0.625 \times 1200 = 750 \text{ rpm}$$

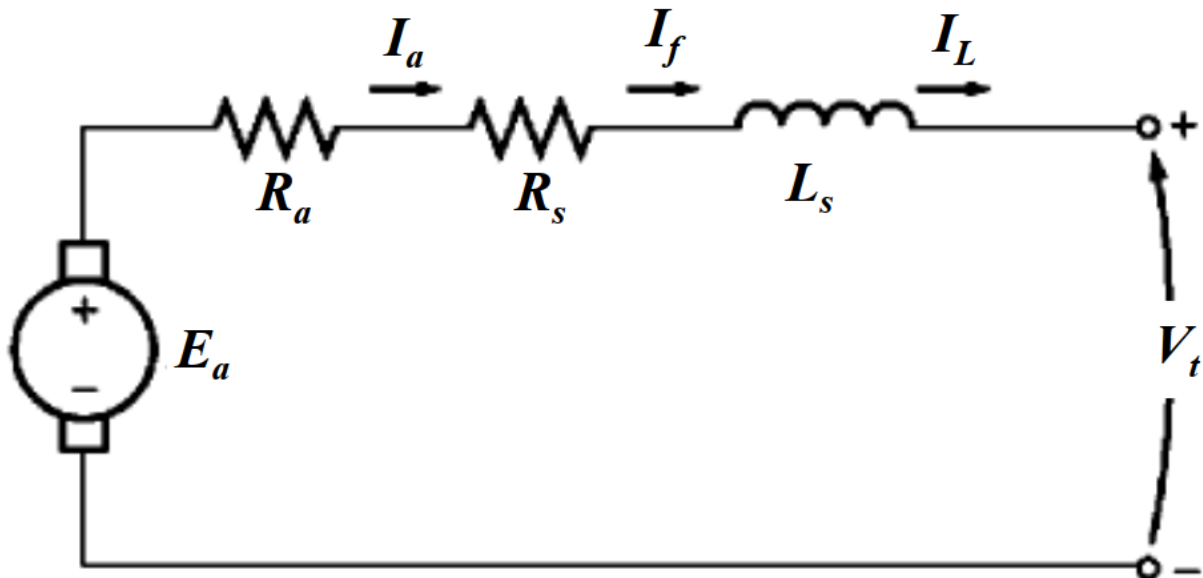
6. Try it yourself.

1 Self Excited DC Generator

1.1 Series Generator

In a series generator, the field flux is produced by connecting the field circuit in series with the armature of the generator

No load voltage = residual voltage



$$I_a = I_l = I_f \quad V_t = E_a - I_a(R_a + R_s) \quad E_a = K_a \phi_s \omega_m$$

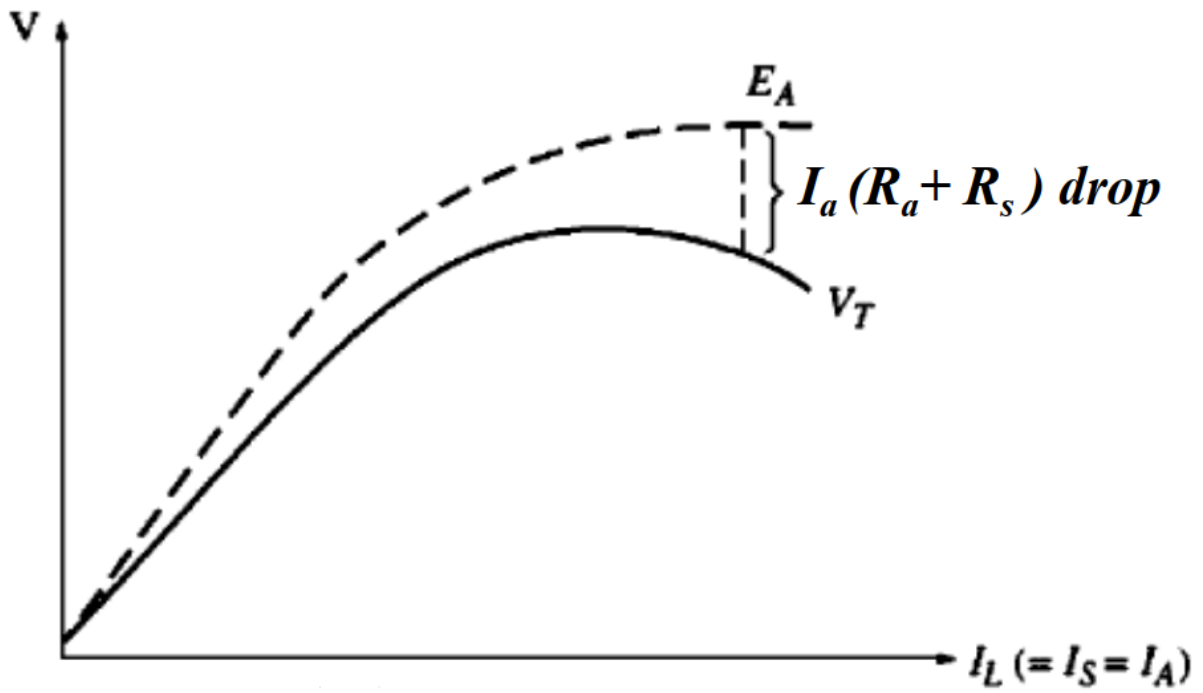
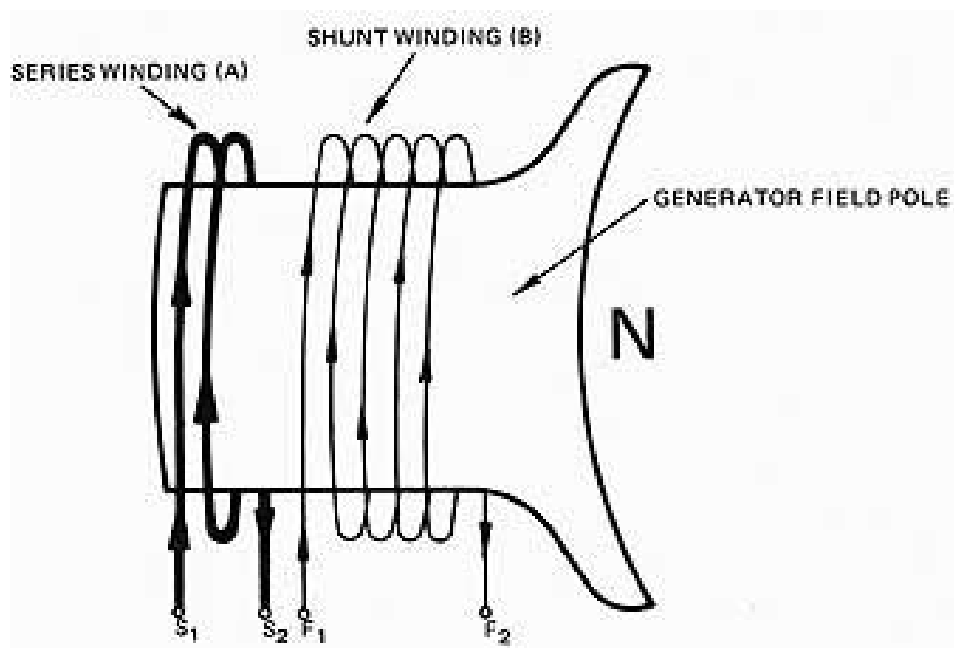


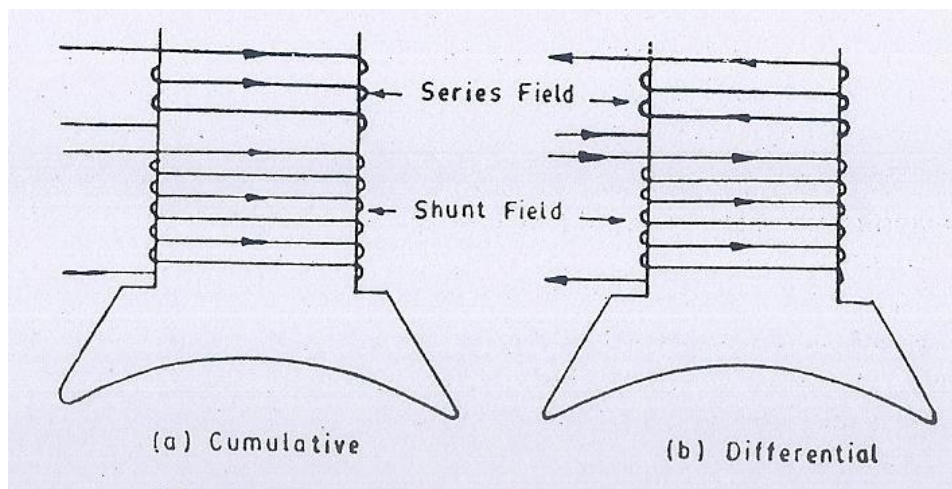
Figure 1: Terminal characteristics of series motor

1.1.1 Shunt and Series Field Winding



Shunt Field Winding	Series Field Winding
<ol style="list-style-type: none"> 1. Large number of turns. 2. Thin wire with high resistance (100s of Ohms). 3. Carry small field current. 4. $AT_{sh} = N_{sh}I_f$ <p> AT : Ampere turn¹ N : Number of turns I : Current </p>	<ol style="list-style-type: none"> 1. Small number of turns. 2. Thick wire with low resistance (0.1 ~ 1 Ohms). 3. Carry Large field current. 4. $AT_s = N_sI_a$ <p> AT : Ampere turn N : Number of turns I : Current </p>

1.1.2 Cumulative and Differential Field Flux



- Cumulative field flux : when series and shunt flux are in the same direction

$$\phi_{\text{net}} = \phi_{sh} + \phi_s$$

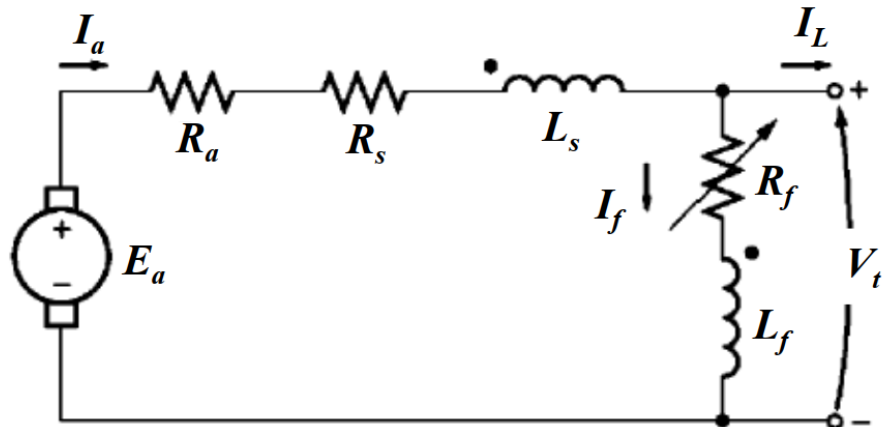
- Differential field flux : when series and shunt flux are in opposite directions.

$$\phi_{\text{net}} = \phi_{sh} - \phi_s$$

¹Magnetomotive force (mmf) is a quantity appearing in the equation for the magnetic flux in a magnetic circuit, often called as ohm's law for magnetic circuits, the SI unit of mmf is ampere, the same as the unit of current (analogously the units of emf and voltage is volt), Informally and frequently (this unit is stated as the ampere-turn to avoid confusion with the current).

1.2 Compound Generator

1.2.1 Long Shunt

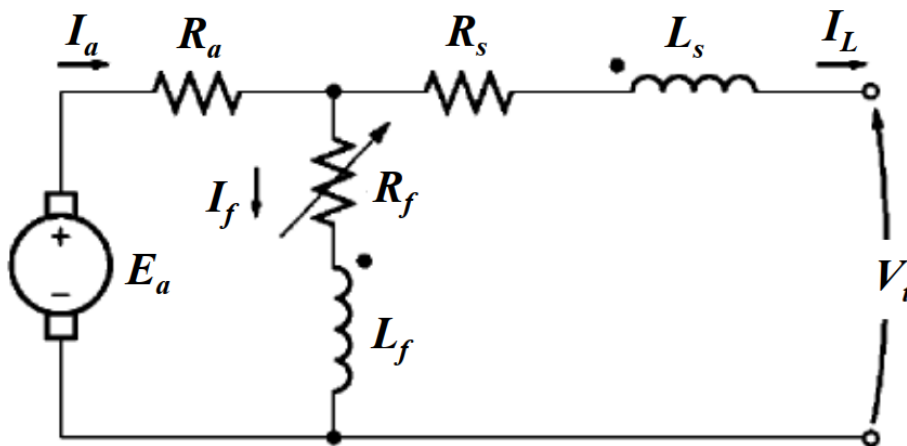


$$I_a = I_L + I_f \quad V_t = I_f R_f$$

$$E_a = V_t + I_a(R_a + R_s) \quad E_a = K_a(\phi_{sh} \pm \phi_s)\omega_m$$

- In a cumulatively compounded generator, both a shunt and a series field are in the same direction, and their effects are additive.
- In a differentially compounded generator, both a shunt and a series field are in opposite directions, and their effects are subtractive.

1.2.2 Short Shunt



$$\begin{aligned}
I_a &= I_L + I_f & V_t &= I_f R_f - I_L R_s \\
E_a &= V_t + I_a R_a + I_L R_s & E_a &= K_a (\phi_{sh} \pm \phi_s) \omega_m
\end{aligned}$$

2. Q — At constant I_a , the output voltage is greater in long shunt or short shunt?

A —

In long shunt:

$$V_t = E_a - I_a(R_a + R_s)$$

In short shunt

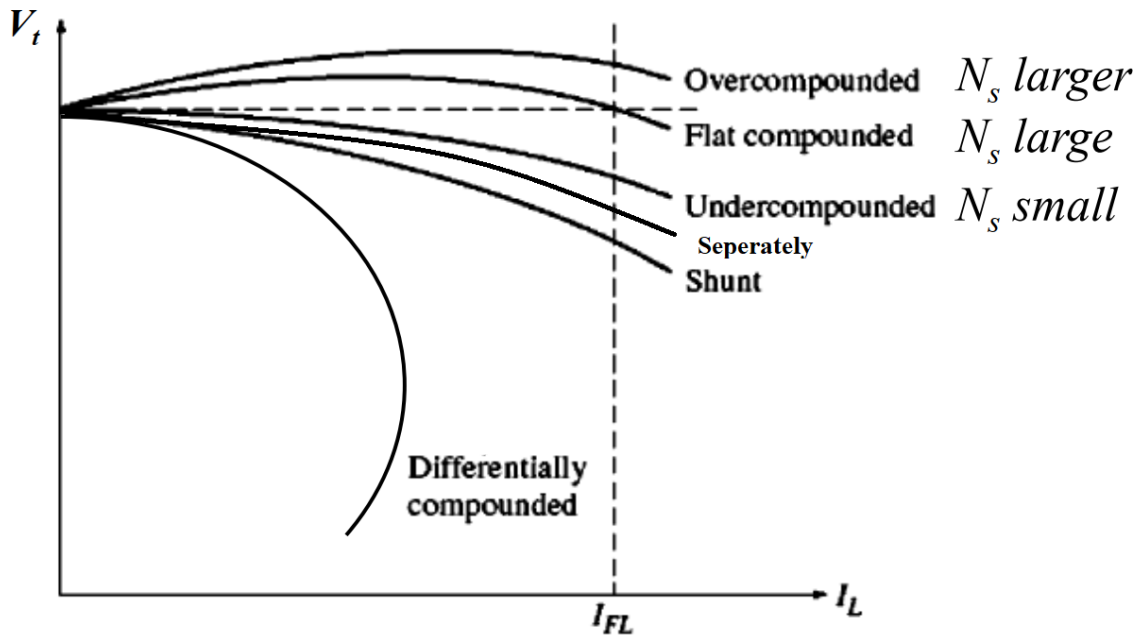
$$V_t = E_a - (I_a R_a + I_L R_s)$$

since

$$I_a(R_a + R_s) > I_a R_a + I_L R_s$$

V_t in short shunt is greater than V_t in long shunt .

1.2.3 Terminal Characteristics of Compound Generators



$$AT_{eff} = AT_{sh} \pm AT_s$$

$$N_f I_{f(eff)} = N_f I_f \pm N_s I_a$$

$$I_{f(eff)} = I_f \pm \frac{N_s}{N_f} I_a$$

2 Voltage Regulation

- DC generators are compared by their voltages, power ratings, efficiency, and voltage regulation.
- **Voltage regulation** : It is the change in terminal voltage when the load is switched off
- Voltage regulation is a measure of internal voltage drop inside generator due to internal resistance and armature reaction
- Smaller voltage regulation means better generator
-

$$VR = \frac{V_{\text{no load}} - V_{\text{full load}}}{V_{\text{full load}}}$$

$$VR = \frac{E_a - V_t}{V_t}$$

3 Power Flow, Losses and Efficiency

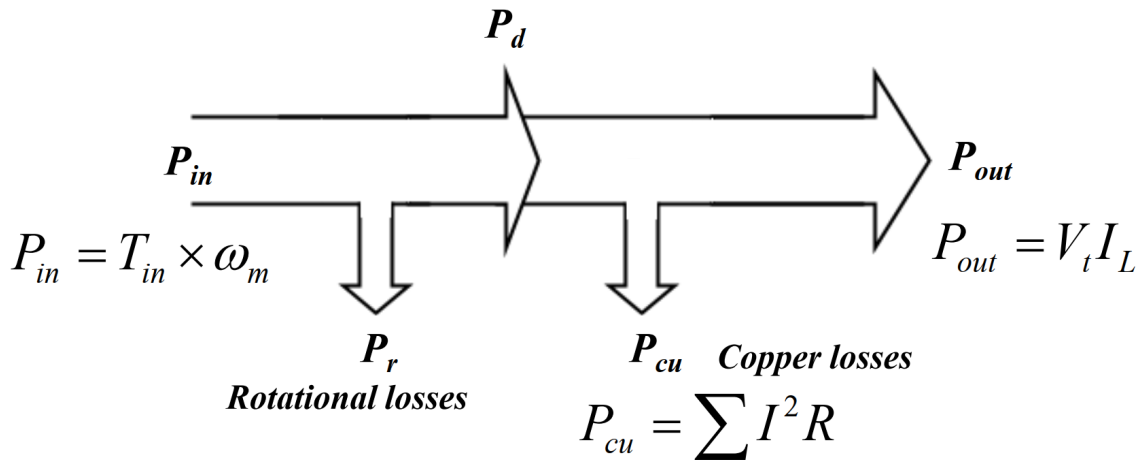


Figure 2: Power Flow

$$P_{in} = T_{in} \times \omega_m$$

P_{in} : Input power (Watt)

T_{in} : Input torque (N.m)

ω_m : Radial speed (rad/sec)

$$P_d = E_a I_a = P_{in} - P_r$$

P_d : Developed power

P_r : Rotational losses

$$P_{cu} = \sum I^2 R$$

P_{cu} : Copper losses

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_r + P_{cu}}$$

In practice, the efficiency for real generators is around 85% - 90%

3. Q — At what operating point we obtain the maximum efficiency?

A — We obtain the maximum efficiency when Rotational losses = Copper losses, this happens at certain load

4 Applications of DC Generators

1. Separately Excited Generations²

- Lighting systems.
- Power supply.
- Battery charges.

2. Series Excited Generations³

- Lighting systems.

²Why these applications? cuz we want constant volt and variable current, as all loads are connected in parallel

³Same as separately excited

- Power supply.
- Battery charges.

3. Series Generators

- Boosters in DC distributed systems in railway service.⁴

4. Compound Generators

(a) Cumulative Compound

- Lamp loads.
- Heavy power service such as electric railways

(b) Differential Compound

- Arc welding⁵

4. Q — A 5.5 kW, 220 V DC shunt generator has a field resistance of 110 W, an armature resistance of 0.5 W, and rotational losses of 100 W. Find at rated load⁶

1. The armature current.
2. The generated voltage.
3. The voltage regulation.

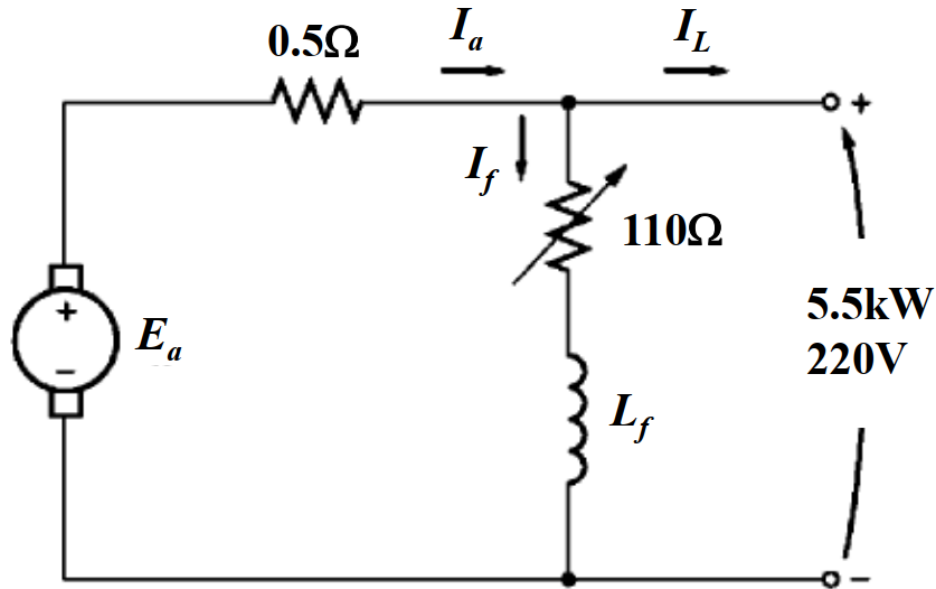
A —

what is 5.5 kW and 220 V ? they are the rated value (full load values), $V_t = 220V$

⁴Nowadays, power electronics (Solid state boosters) are used instead.

⁵Because we want high current low voltage

⁶What if we want to solve at (50% of rated value?), the power will be halved (2.25 KW), but the voltage will remain the same



$$1. I_a = I_L + I_f$$

$$I_L = \frac{P_{\text{out}}}{V_t} = \frac{5500}{220} = 25A$$

$$I_f = \frac{V_t}{R_f} = \frac{220}{110} = 2A$$

$$I_a = 25 + 2 = 27A$$

$$2. E_a = V_t + I_a R_a = 220 + 27 \times 0.5 = 233.5V$$

$$3. VR = \frac{233.5 - 220}{220} = 6.13\%$$

$$4. \eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_R + P_{\text{cu}}}$$

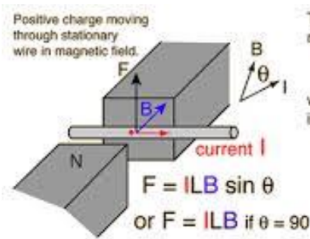
$$P_{\text{cu}} = \sum I^2 R = 27^2 \times 0.5 + 2^2 \times 110 = 804.5W$$

$$\eta = \frac{5500}{5500 + 100 + 804.5} = 85.87\%$$

5 DC Motors

5.1 Principle of Operation

- Remember : A current-carrying wire in the presence of a magnetic field has a force induced on it. (This is the basis of **motor action**)



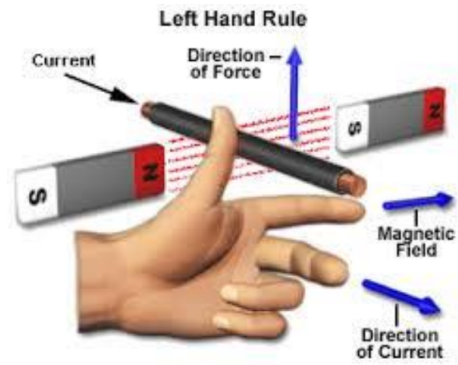
This relationship arises from the basic magnetic force:

$$F = qvB \sin \theta$$

which for a charge q traveling length L in a wire can be written

$$F = q \frac{L}{t} B \sin \theta$$

$$F = \frac{q}{t} LB \sin \theta$$

$$F = ILB \sin \theta$$


- Remember: Fleming left hand rule from figure
- Remember:

$$F = BIL \sin(\theta)$$

F = force

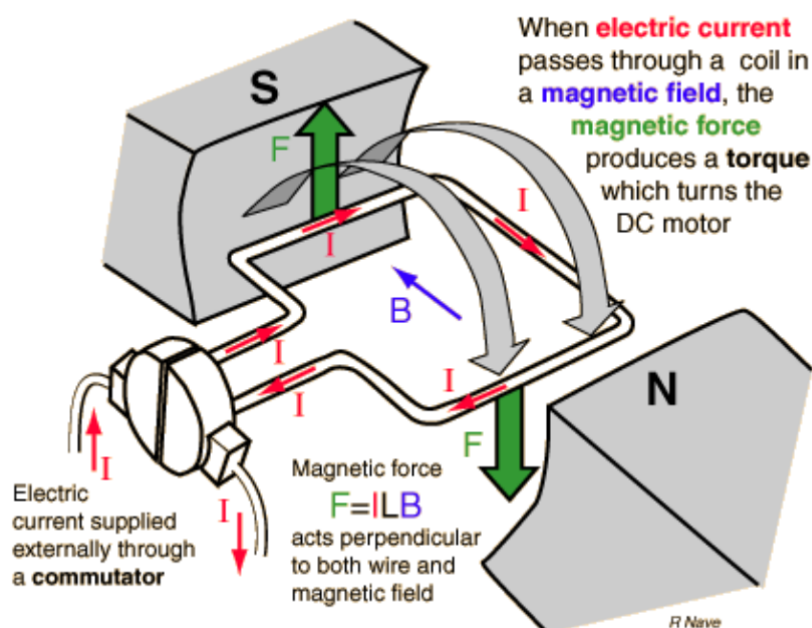
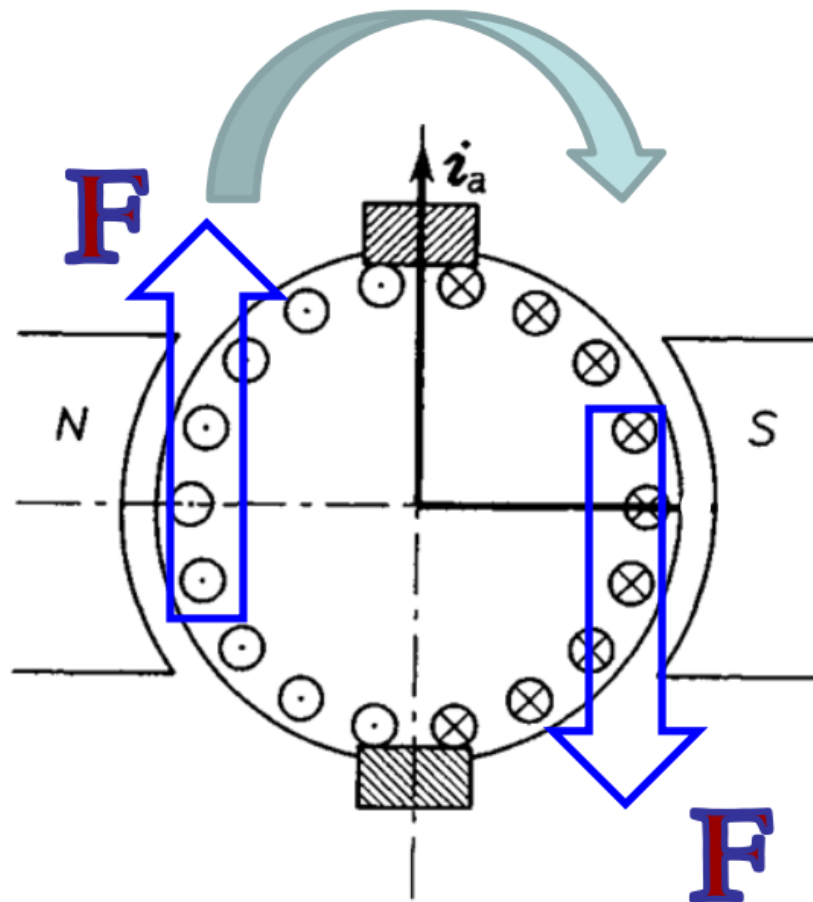
B = magnetic flux density

I = current

L = length of the conductor

θ = angle between B and I

Torque



Note : DC motor is a self starting motor, most of other motors are not.

6 Classification of DC Machines

There are four major types of DC motors, classified according to the manner in which their field flux is produced (i.e. according to the connection of the field winding with armature).

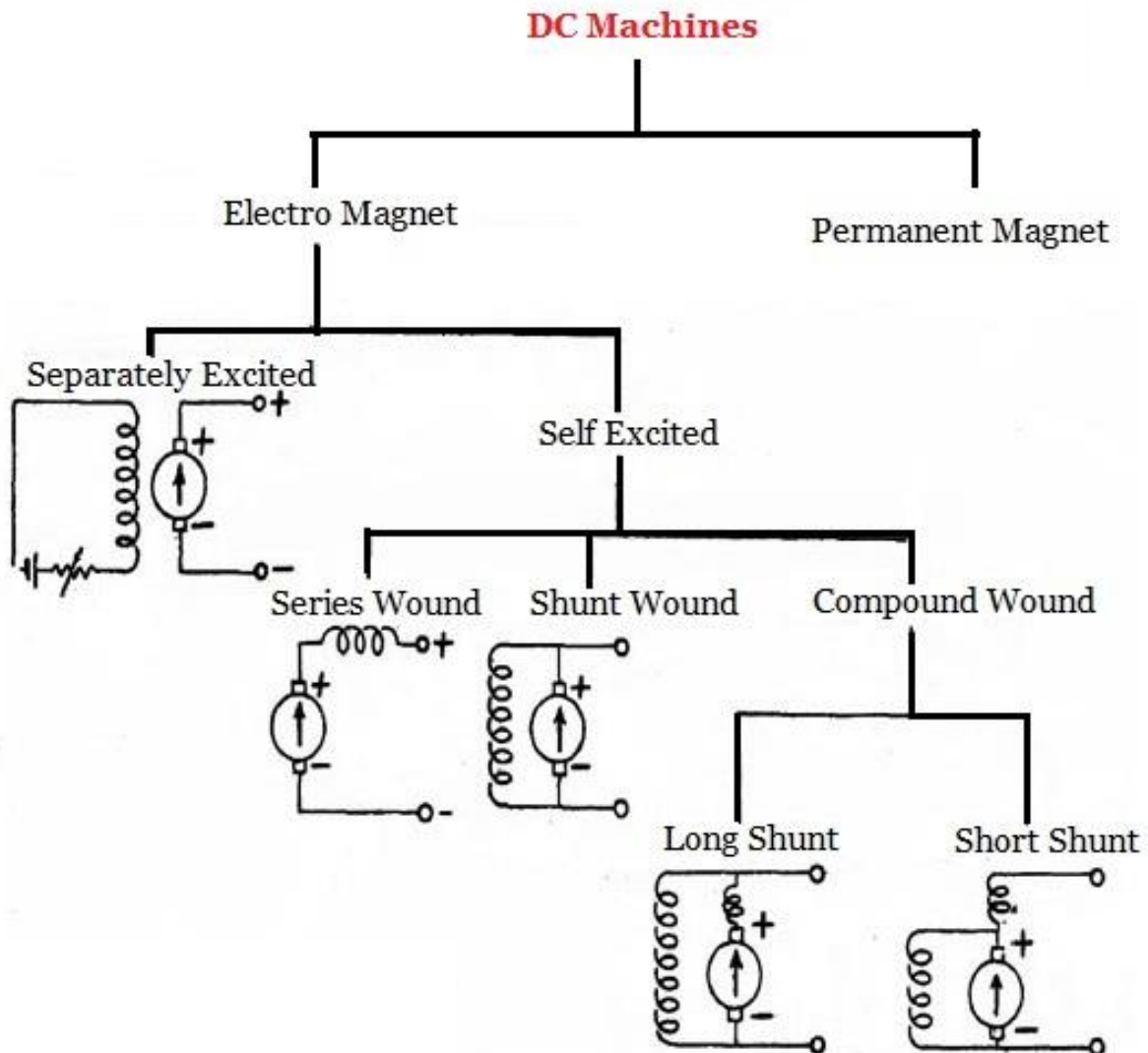


Figure 3: Classification of DC Machines

6.1 Separately Excited DC Motor

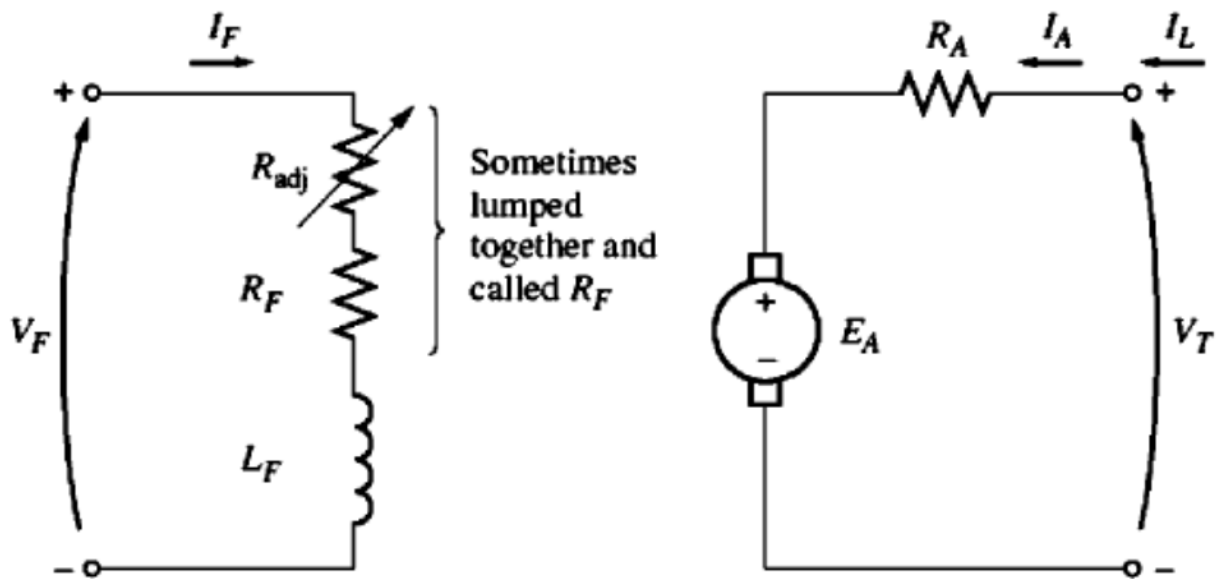


Figure 4: Separately excited DC motor model

Apply KVL

$$V_t = E_a + I_a R_a$$

$$V_f = I_f R_f \quad I_a = I_L \quad V_t = k_a \phi \omega_m + I_a R_a \quad E_a = k_a \phi \omega_m$$

6.2 Self Excited DC Motor

6.2.1 Shunt motor

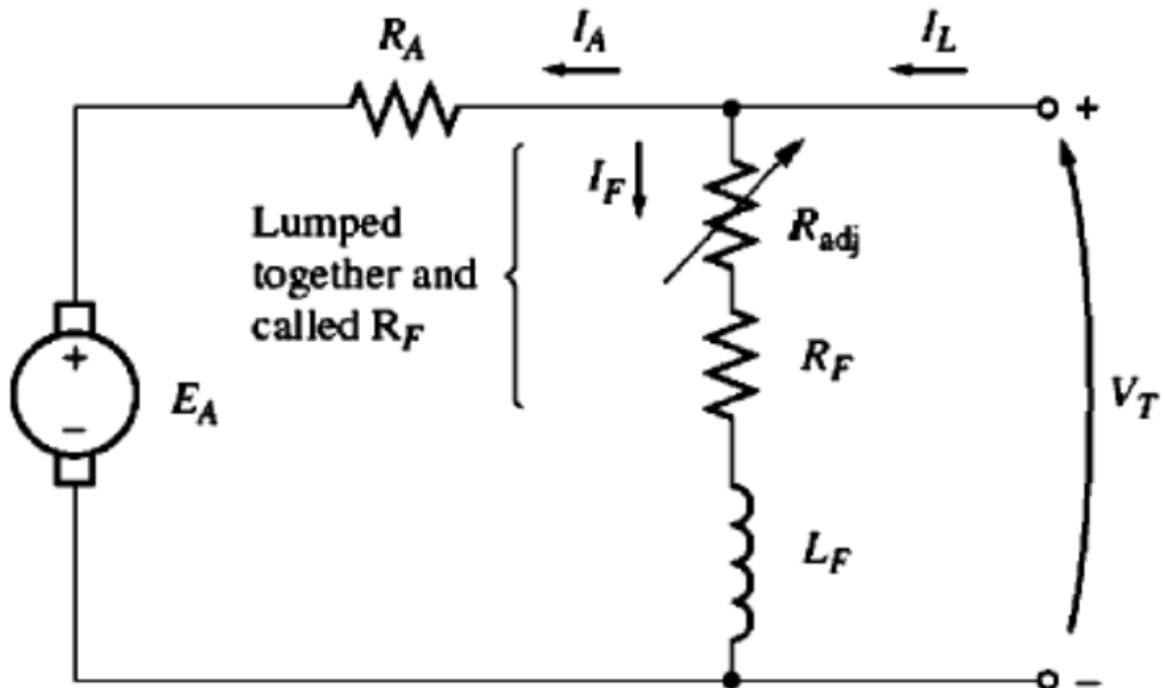


Figure 5: DC shunt motor model

Apply KVL

$$V_t = E_a + I_a R_a$$

$$V_f = I_f R_f \quad I_a = I_L - I_f \quad V_t = k_a \phi \omega_m + I_a R_a \quad E_a = k_a \phi \omega_m$$

7 Appendix - Plotting with SageMath

This appendix is **out of the scope** of this course

In final exam. all graphs for sure will be drawn in hand, since readers are sophomore students, it is expected for them to have experience in drawing graphs by hand with no help.

SageMath is an open source python based software for mathematical computations, that supports research and teaching in algebra, geometry, number theory, cryptography,

numerical computation, and related areas. The overall goal of Sage is to create a viable, free, open-source alternative to Maple, Mathematica, Magma, and MATLAB. SageMath is used in this note to plot graphs.

Software like that helps the student to spend more time thinking beyond math and avoid the cumbersome process of drawing or solving math by hand.

Here is the Sage code used in this note:

```
I_f = [0,0.1,0.2,0.3,0.4,0.5,0.6,0.8,1,1.2]
E_2 = [7.2,19.2,38.4,57.6,75.6,91.2,102,112.8,120,122.4]
data = []
for i in range (len(I_f)):
    data.append((i_f[i], e_2[i]))

# We created data, which is a list of tuples,
# each tuple represents a point to be plotted

f(x) = 120*x

p1 = list_plot(data, color = "red")

p2 = list_plot(data, plotjoined = True)

p3 = plot(f, (x,0,1.2))

p4 = plot(120, (x,0,1.2) , color="green", linestyle = "--")

myplot = p1 + p2 + p3 + p4

myplot
```

