EEP 225 Electric Machines Cheat Sheet [1]
Induction Motor

- Self Starting

$$N_s = 120 \times \frac{f_s}{P} \tag{1}$$

N_s : Speed of rotor
f_s : Frequency of supply
P : Number of poles

Slip:

$$s = \frac{N_s - N_r}{N_r} \qquad N_r = N_s(1-s) \tag{2}$$

$$0 < s \leq 1 \tag{3}$$

practically (0.05 ∽ 0.1)

$$f_r = s f_s \tag{4}$$

f_r : Frequency of the induced current in rotor
f_s : Frequency of supply
s : Slip

Equivalent circuit

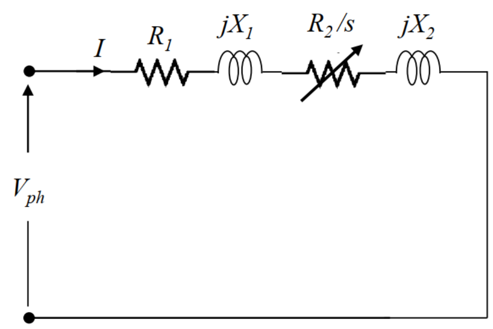

Figure 1: Equivalent Circuit Per Phase.

We divide $\frac{R_2}{s}$ to $\left(R_2 + R_2\frac{1-s}{s}\right)$

Why? to separate R_2 which is a physical resistance and $R_2\frac{1-s}{s}$ on which the developed power is noticed (called dynamic resistance)

[1] Taha Ahmed

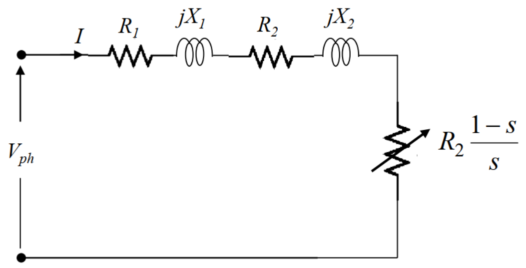

Figure 2: Equivalent Circuit Per Phase.

Power Flow And Losses

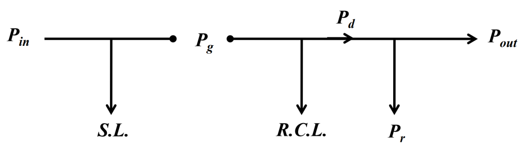

Figure 3: Power flow and losses in induction motor.

input power:

$$P_{\text{in}} = 3V_{\text{ph}}I_{\text{ph}}\cos(\phi) \tag{5}$$

Air gap power = input power − stator losses
it is the power that transfers from stator to rotor.

$$P_g = P_{\text{in}} - S.L. = 3I^2\frac{R_2}{s} \tag{6}$$

Developed power = Air gap power − rotor copper loss

$$R.C.L = 3I^2R_2 \tag{7}$$

$$P_d = P_g - R.C.L = 3I^2R_2\frac{1-s}{s} \tag{8}$$

Output power = developed power − rotational losses

$$P_{\text{out}} = P_d - P_r \tag{9}$$

$$\boxed{P_g : R.C.L : P_d = 1 : s : 1-s} \tag{10}$$

$\boxed{1.\ \text{Q}}$ — A 10 hp, 6-pole, 440 V, 60 Hz, Y-connected, three-phase induction motor is designed to operate at 3% slip on full load. The stator loss is 250 W and the rotational loss is 4% of the power output. Determine :

1. Air gap power.
2. Rotor copper loss R.C.L.
3. Input power.
4. Efficiency.
5. Output torque.
6. Developed torque.

A — How to solve?

P_{out} is given, but in hp.

$$\text{Watt} = \text{hp} \times 764 \tag{11}$$

P_r is given (4% of P_{out})
Get P_d from P_{out} and P_r
via $\boxed{P_g : R.C.L : P_d = 1 : s : 1-s}$ you can get P_g and $R.C.L$ from P_d ✓ ✓
$P_{\text{in}} = P_g + S.L.$, stator losses are given. ✓

$$\text{efficiency } (\eta) = \frac{P_{\text{out}}}{P_{\text{in}}} \checkmark$$

Output torque (Use output power):

$$T_{\text{out}} = \frac{P_{\text{out}}}{\omega_r} \checkmark \tag{12}$$

Developed torque (Use developed power):

$$T_{\text{d}} = \frac{P_{\text{d}}}{\omega_r} \checkmark \tag{13}$$

$$\omega_r = 2\pi\frac{N_r}{60} \tag{14}$$

DON'T substitute with electricity frequency to get ω_r

get N_r from N_s and N_s from $N_s = 120\frac{f}{P}$

Torque-Slip Relationship

$$T_{\mathrm{d}} = \frac{3R_2}{\omega_s \times s} \times \frac{V_{\mathrm{ph}}^2}{\left(R_1 + \frac{R_2}{s}\right)^2 + (X_1 + X_2)^2} \tag{15}$$

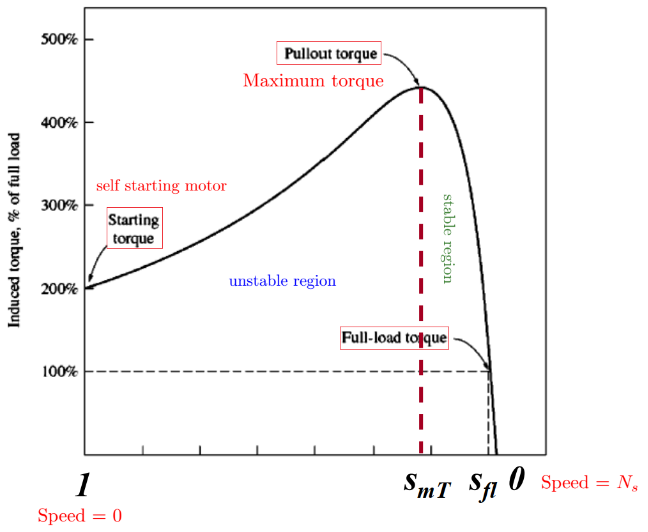

Figure 4: Torque-Slip relationship

How to get s_{mT} (Slip corresponding to the maximum torque)

$$s_{mT} = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}} \tag{16}$$

If stator impedance is small and can be ignored

$$s_{mT} = \frac{R_2}{X_2} \tag{17}$$

$N_{r_{mP}}$ the rotor speed corresponding to maximum torque

$$N_{r_{mT}} = N_s(1 - s_{mT}) \tag{18}$$

To get the maximum torque, substitute by s_{mT} in the general torque equation

$$T_{\max} = \frac{3R_2}{\omega_s \times s_{mT}} \times \frac{V_{\mathrm{ph}}^2}{\left(R_1 + \frac{R_2}{s_{mT}}\right)^2 + (X_1 + X_2)^2} \tag{19}$$

Maximum Developed Power

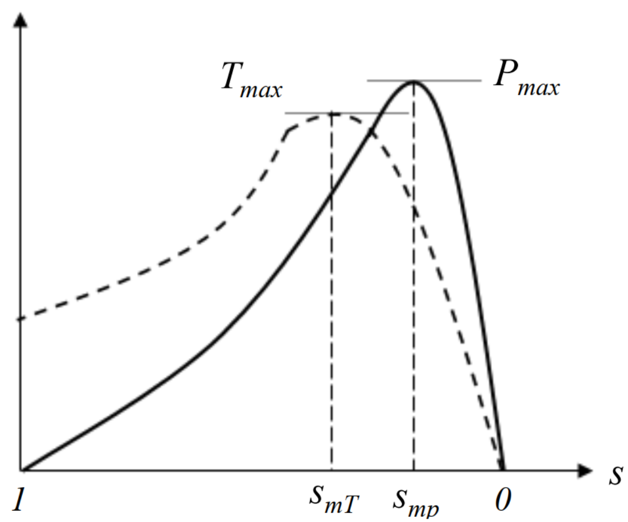

Figure 5: $s_{mT} \neq s_{mP}$

Note : Maximum developed power and maximum developed torque occur at different points, a common mistake is getting maximum torque and use $T = \frac{P}{\omega}$ to get the maximum power, recalculate both from scratch, recalculate s How to get s_{mP} (Slip corresponding to the maximum power)

$$s_{mP} = \frac{R_2}{R_2 + R_L} \tag{20}$$

$$R_L = \sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2} = R_2 \frac{1 - s_{mP}}{s_{mP}} \tag{21}$$

$$N_{r_{mP}} = N_s(1 - s_{mP}) \tag{22}$$

$$P_{d_{\max}} = 3I_{mP}^2 R_L \tag{23}$$

$$P_{d_{\max}} = \frac{3V_{mP}^2}{2(R_1 + R_2 + R_L)} \tag{24}$$

(25)

Effect of Varying Rotor Resistance

We want to change rotor resistance from R_2 to $R_2 + R_{\mathrm{external}}$

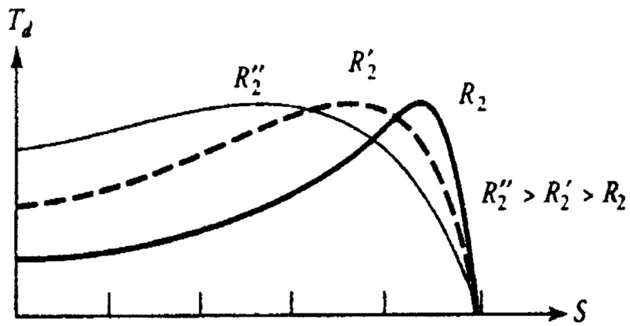

Figure 6: Effect of varying rotor resistance

Maximum torque does not depend on R_2
When R_2 increases ↑, starting torque increases ↑

$$R_{\mathrm{external}} = \sqrt{R_1^2 + (X_1 + X_2)^2} - R_2 \tag{26}$$

Note that this method can be used only in wound rotor, not the squirrel cage rotor.

$\boxed{\text{2. Q}}$ — A 120 V, 60 Hz, 6-poles, Δ-connected, wound-rotor induction motor has a rotor resistance and standstill reactance of 0.15 Ω and 0.9 Ω per phase respectively. Calculate the starting and maximum torques, and the speed at maximum torque. (Neglect stator impedance). Also find the starting current.
Find the maximum developed power and the speed at which occurs

A — How to solve

$$T = \frac{3R_2}{\omega_s \times s} \times \frac{V_{\mathrm{ph}}^2}{\left(R_1 + \frac{R_2}{s}\right)^2 + (X_1 + X_2)^2}$$

Neglect stator impedance (R_1 and X_1)

$$T = \frac{3R_2}{\omega_s \times s} \times \frac{V_{\mathrm{ph}}^2}{R_2^2 + X_2^2}$$

Note that the only thing changes here is s.
Substitute with s twice

1. $s_{\mathrm{start}} = 1 \rightarrow$ to get starting torque ✓
2. $s_{mT} \rightarrow$ to get starting torque ✓

Approximate s_{mT}

$$s_{mT} = \frac{R_2}{X_2}$$

Rotor speed at any torque (N_s) doesn't change for the same source

$$N_{r\text{ at any point}} = N_s(1 - s_{\text{at that point}})\ ✓$$

Current? → ohm's law from Figure 1 of Figure 2 (and don't forget to approximate) ✓

Maximum developed power ? $\rightarrow$ from equations 20 calculate s_{mP}, then calculate R_L and I_{mP} (use ohms law on equivalent circuit) to calculate $3I_{mP}^2R_L$ ✓

$\boxed{\text{3. Q}}$ — If if an external resistance equals to the rotor resistance is added to each phase of the rotor, calculate the new values of the motor starting and maximum torques, and speed at maximum torque. Also calculate the starting current.
Find the maximum developed power and the speed at which occurs

A — How to solve?
Same steps, but whenever you see R_2, replace it with $R_2 + R_{\text{external}}$ and solve ✓
Note : Maximum torque shouldn't change if you change R_2. See Figure 6.

Property	**Squirrel-cage rotor**	**Wound rotor**
Maintenance	Almost no maintenance required	Requires periodic maintenance
Efficiency	High (no coils, just bars)	Low (because of winding)
Advantages	• Cheaper and more robust. • Higher power factor. • The risk of sparks is eliminated by the absence of slip rings and brushes.	• Higher starting torque. • Lower starting current. • Means of varying speed by use of external rotor resistance.

Advantages

- Simple and rugged (strong) construction
- Low cost, high reliability
- High efficiency (85% - 90%)
- Good lagging pf (0.8 - 0.85)
- Requires minimum maintenance