EEP 225 : Electric Machines

Lecture 6

AC Machines II

Taha Ahmed

1 Pitch (Chording) Factor

Sometimes the coil span is made less than 180° by angle α which is called the chording angle.

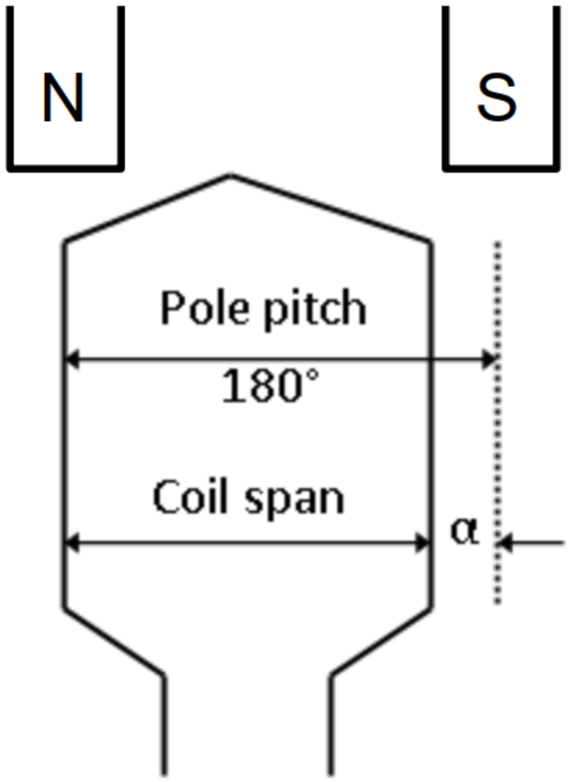


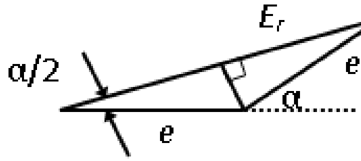
Figure 1: Coil span here is less than 180°

$\boxed{\text{1. Q}}$ — What is the advantages of reducing the coil span angle?

A —

1. To save copper of end connections.
2. To improve the waveform of the generated emf (which supposed to be sinusoidal) by eliminating or reducing the distorting harmonics.

$$K_p = \frac{\text{Voltage of short chorded coil}}{\text{Voltage of full pitch coil}}$$



$$K_p = \frac{2\,e\cos\left(\dfrac{\alpha}{2}\right)}{2\,e}$$

$$= \cos\left(\frac{\alpha}{2}\right)$$

For full-pitch coil (180°), $\alpha = 0$ and then $K_p = 1$

$\boxed{\text{2. Q}}$ — A 3 phase, 16-pole, Y-connected alternator has 144 slots with 10 conductors per slot. The coil span is 150°. Find the phase and line induced emfs if the flux is 30 mWb and the machine runs at 375 rpm.

A —

$$E_{\text{ph}} = 4.44\,f\,\phi\,N_{ph}\,K_w$$
$$f\ (\text{frequency}) = \frac{P\,N_s}{120}$$

P : Total number of field poles.
N_s : Speed of the rotor (synchronous speed) in rpm.
f_s : Frequency of generated emf in Hz.

$$f = \frac{16 \times 375}{120} = 50\,\text{Hz}$$
$$N_{\text{ph}} = \frac{Z}{2 \times \text{number of phases}}$$

N_{ph} : Number of armature turns per phase
Z : Number of armature conductors (= 144 slots × 10 conductors per phase = 1440)

$$N_{\mathrm{ph}} = \frac{1440}{2 \times 3}$$

$$q = \frac{S}{P \times n}$$

$$K_d = \frac{\sin\left(\frac{q\gamma}{2}\right)}{q\,\sin\left(\frac{\gamma}{2}\right)}$$

q : Number of slots per pole per phase

$$q = S \times \frac{1}{P} \times \frac{1}{n} = \frac{144}{16 \times 3} = 3$$

$$\gamma = \frac{180 \times P}{S} = \frac{180^\circ \times 16}{144} = 20^\circ$$

$$K_d = \frac{\sin(30)}{3\sin(10)} = 0.96$$

$$\alpha = 180^\circ - 150^\circ = 30^\circ$$

$$K_p = \cos\left(\frac{\alpha}{2}\right) = \cos(15^\circ) = 0.966$$

$$K_w = K_d \times K_p = 0.96 \times 0.966 = 0.927$$

$$E_{\mathrm{ph}} = 4.44 \times 50 \times 0.03 \times 240 \times 0.927 = 1482V$$

$$E_L = \sqrt{3} \times 1482$$

2 Effect of Flux Density Harmonics

$\boxed{\text{3. Q}}$ — What is Fourier transform?

A —

Any periodic waveform can be decomposed into DC component plus series of harmonics (sine and cosine components)

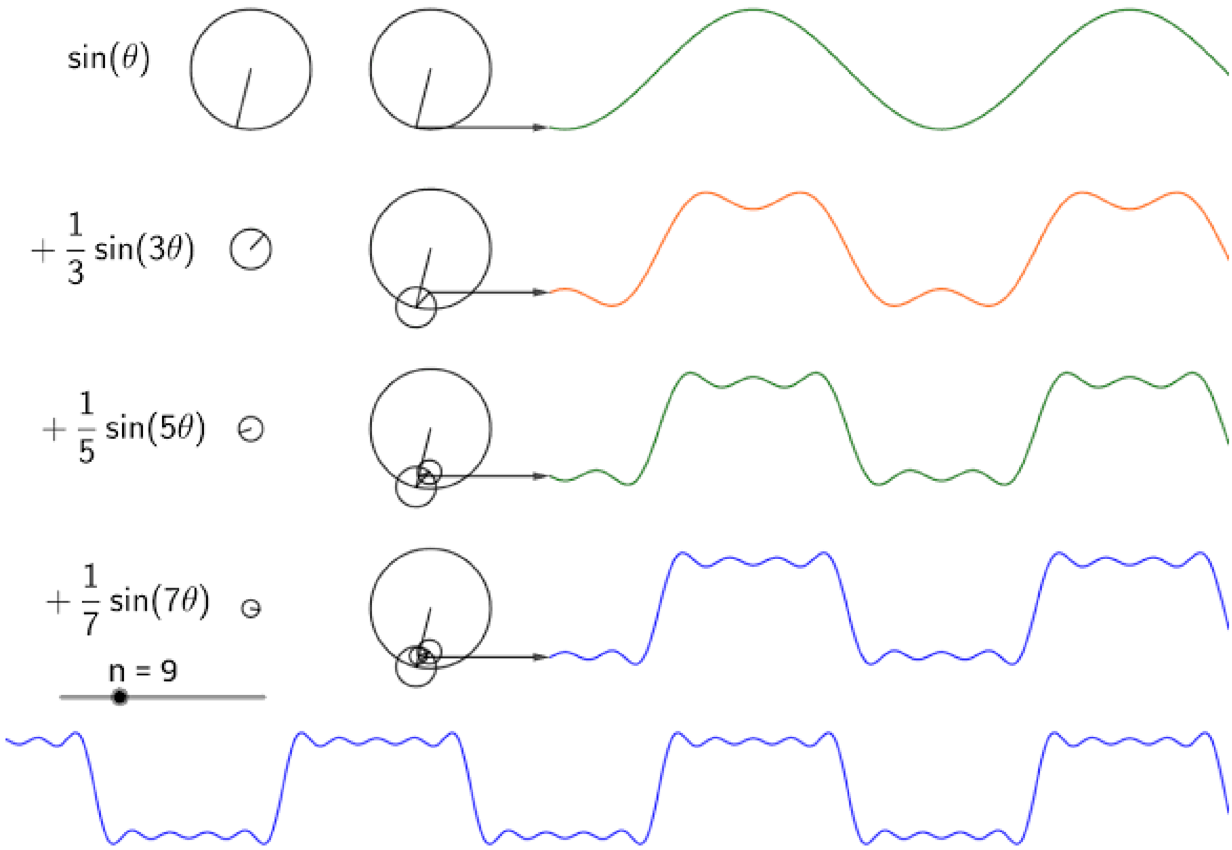


Figure 2: Fourier Transform (note no even harmonics exists,because they cancel each other), also not that the 3rd, 5th and 7th harmonics are the most effective ones

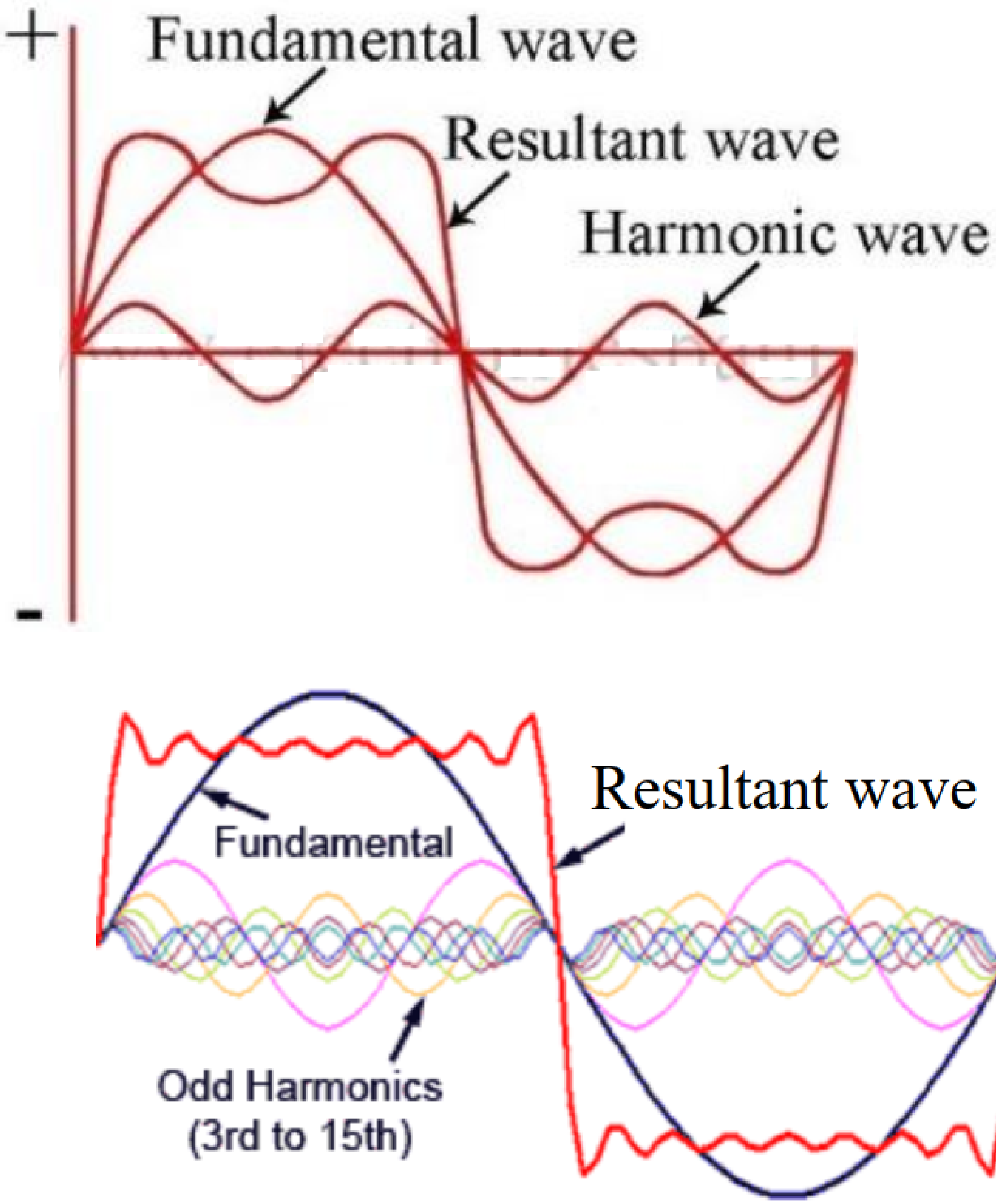


Figure 3: Fourier Transform

$\boxed{\text{4. Q}}$ — Explain the effect of flux density harmonics

A —

The actual flux density distribution is not sinusoidal as it has been presumed when deriving the emf equation, the generated voltage is highly distorted due to the existing harmonics.

The disadvantages of harmonics are distorting the output voltage, increase machine losses, and decreases the efficiency.

Our target to eliminates the higher order harmonics, mainly the 3rd harmonic or the 5th harmonic.

If γ and α are the slot angle and the pitch angle for the fundamental flux wave, then their values for different harmonics are $h\,\gamma$ and $h\,\alpha$, where h is the order of harmonics.

$$K_{dh} = \frac{\sin\left(\dfrac{q\,h\,\gamma}{2}\right)}{q\,\sin\left(\dfrac{h\,\gamma}{2}\right)}$$

$$K_{ph} = \cos\left(\frac{h\,\alpha}{2}\right)$$

$$f_{h=3} = 3\,f$$

It is requires to reduce the 3rd harmonic

$$E_{h=3} = 4.44\,f_{h=3}\,\phi_{h=3}\,N_{ph}\,K_{dh=3}\,K_{ph=3}$$

The only term that can be zero is the pitch factor K_{p3}

$$\begin{aligned} K_{p3} &= 0 \\ \cos\left(\frac{3\alpha}{2}\right) &= 0 \\ 1.5\,\alpha &= 90^\circ \\ \alpha &= 60^\circ \end{aligned}$$

Therefore, in order to eliminate the 3rd harmonic, armature coils have to be chorded by 1/3 of the pole pitch.

$\boxed{\text{5. Q}}$ — For the alternator in $\boxed{\text{2. Q}}$, if the 3rd harmonic flux is 2 mWb. Find the value of the generated emf per phase, and the line-line value.

A —

$$K_{dh=3} = \frac{\sin\left(\frac{q\,3\,\gamma}{2}\right)}{q\,\sin\left(\frac{3\,\gamma}{2}\right)} = \frac{\sin(90)}{3\sin(30)} = 0.667$$

$$K_{ph=3} = \cos\left(\frac{3\,\alpha}{2}\right) = \cos(45) = 0.707$$

$$K_{ph=3} = K_{dh=3} \times K_{ph=3} = 0.667 \times 0.707 = 0.471$$

$$f_{h=3} = 3f = 150\,\text{Hz}$$

$$E_{h=3} = 4.44\,f_{h=3}\,\phi_{h=3}\,N_{ph}\,K_{wh=3} = 4.44 \times 150 \times 2 \times 10^{-3} \times 240 \times 0.471 = 150\,V$$

$$E_{\text{ph}} = \sqrt{E_1^2 + E_3^2} = \sqrt{1483^2 + 150^2} = 1490\,V \qquad \text{(That is how we add RMS voltages)}$$

$$E_{\text{line}} = \sqrt{3} \times 1482 = 2567\,V$$

Note that in both Y and Δ connections, the 3rd harmonic and its multiples cancel out at the line terminals because they are co-phased.[1]

Midterm exams will cover the subjects discussed till this point

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3 Synchronous Generator on Load

If the alternator is run unloaded, then the flux in the air gap will be produced by the field system only.

[1]If Y, the 3rd harmonic and its multiples are grounded. If Δ, they create circulating current I_c, can be calculated using the volt and the impedance of the 3rd harmonic, note that impedance depends on the frequency of the waveform

Loading the alternator (connecting a load to armature turns), will cause a flow of current in armature windings. This current will be responsible of flow of another flux component.

Therefore the air gap flux will be the resultant of:

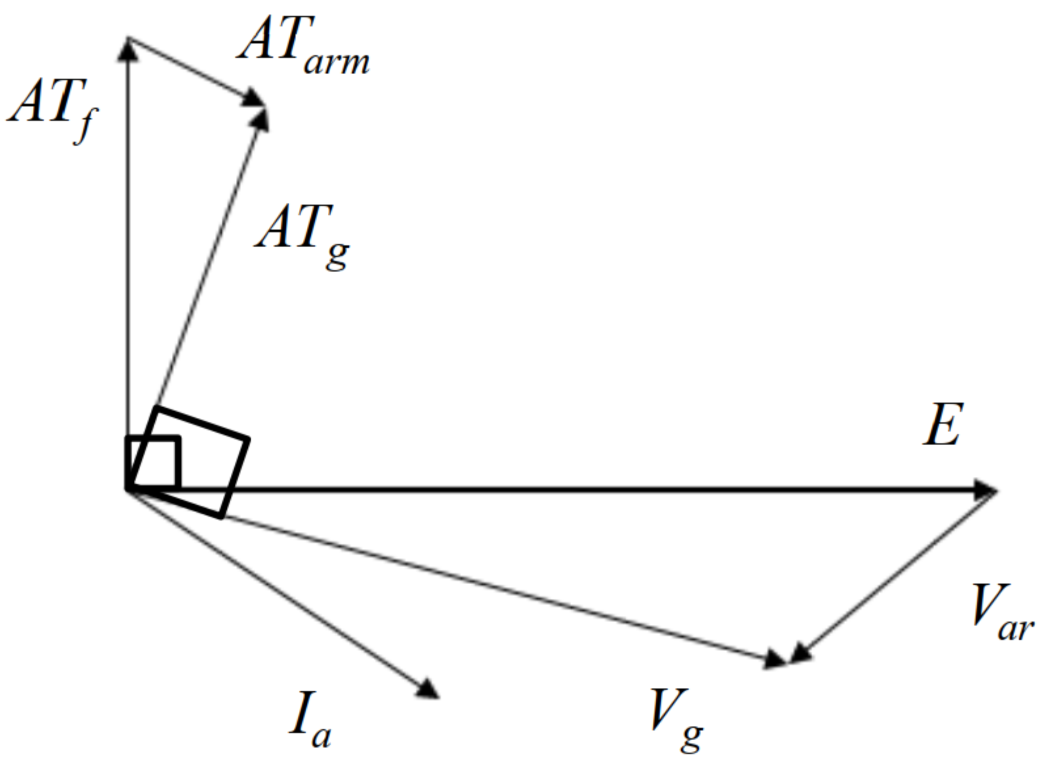
1. Field ampere-turn(AT_f)
2. Armature ampere-turn (AT_{armature})

The effect of armature ampere-turns on the field ampere-turns is called **(Armature Reaction)**

$$\overline{AT_g} = \overline{AT_f} + \overline{AT_{\text{armature}}}$$

4 Equivalent Circuit of Synchronous Generator

Each ampere-turns produces a voltage which lags[2] its corresponding ampere-turns by 90°



[2]Because Faraday's law: emf $= -N\frac{d\phi}{dt}$, differentiation causes this lag

$$
\begin{aligned}
\overline{AT_g} &= \overline{AT_f} + \overline{AT_{\text{armature}}} \\
\overline{E} + \overline{V}_{ar} &= \overline{V}_g \\
\overline{E} &= \overline{V}_g + \underbrace{I_a X_{ar}}_{\text{Voltage drop}}
\end{aligned}
$$

Therefore, armature reaction can be represented by a voltage drop on an inductive reactance X_{ar}[3]. However, the terminal voltage Vt of the machine appears after considering voltage drop across armature resistance R_a and leakage reactance X_l

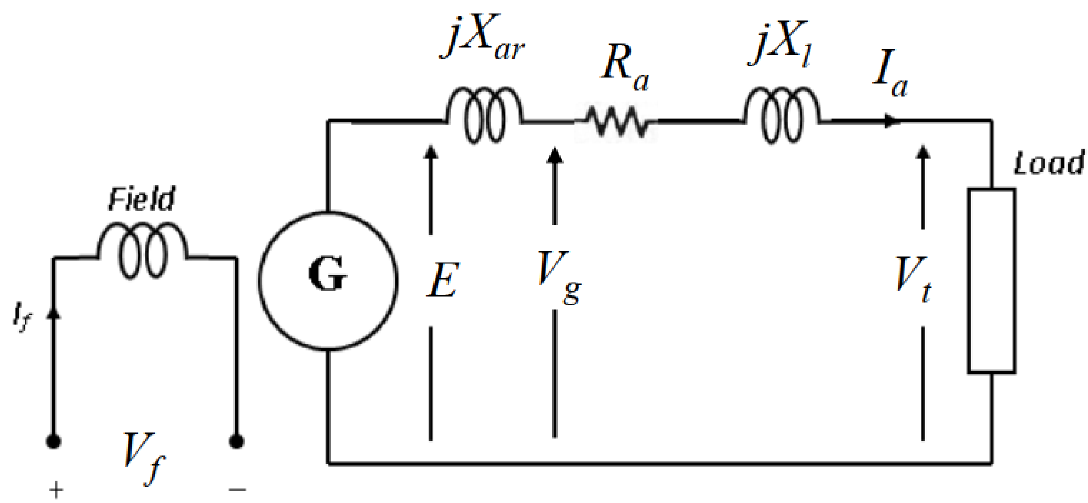


Figure 4: Equivalent circuit of synchronous generator per phase, note that we can't connect the field circuit (DC) with the generator circuit (AC)

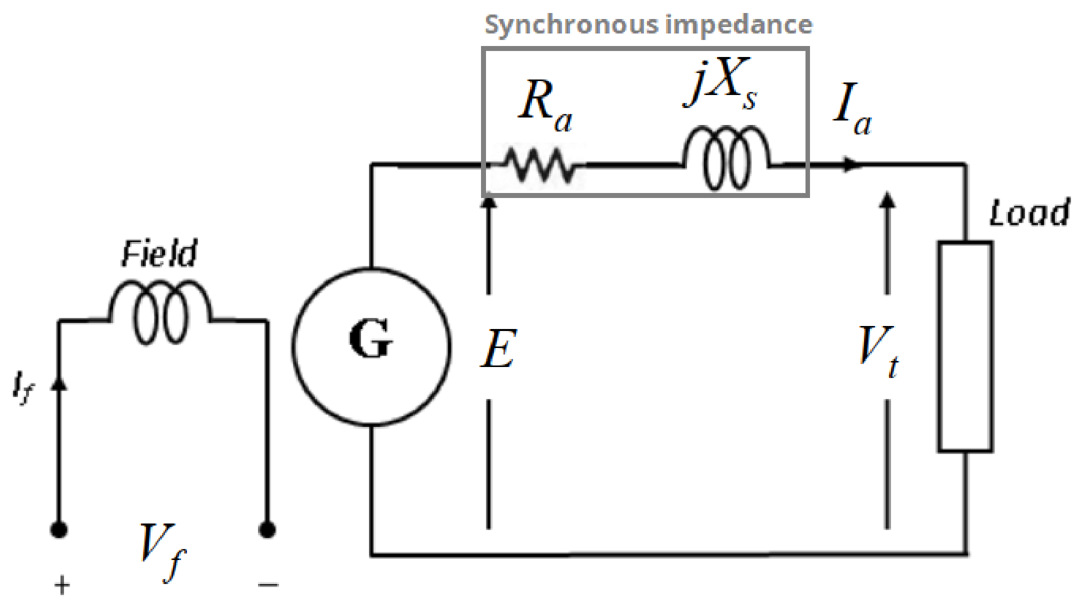


Figure 5: Equivalent circuit of synchronous generator per phase

[3]Imaginary reactance, not physical

$$X_s = X_{\text{ar}} + X_l$$

$$Z_s = R_a + j\,X_s$$

R_a : Armature resistance
X_s : Synchronous reactance
Z_s : Synchronous impedance

$$\overline{E} = \overline{V_t} + \overline{I_a} \cdot \overline{Z_s}$$

5 Phasor Diagram of Synchronous Generator

What is the reference ? Load voltage is the reference since it helps us to calculate the power factor (cosine the angle between load voltage and load current)

$$\overline{E} = \overline{V_t} + \overline{I_a} \cdot \overline{Z_s}$$
$$E\underline{/\delta} = V_t\underline{/0} + I_a \times Z_s\underline{/\theta}$$
$$E\underline{/\delta} = V_t\underline{/0} + I_a \times (R_a + jX_s)$$

δ : Load angle (Power angle).

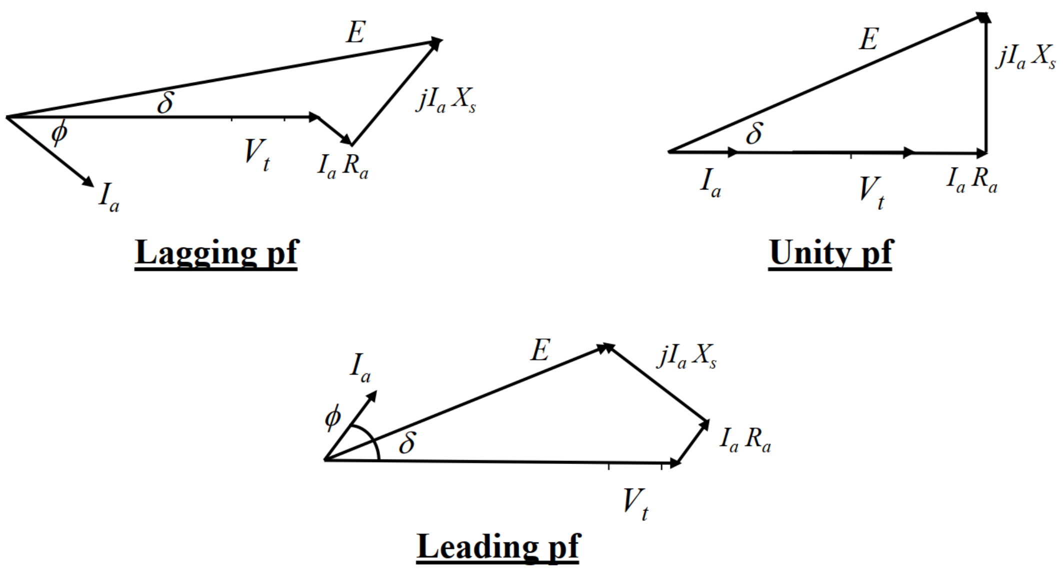


Figure 6

Note that in lagging power factor and unity power factor V terminal is always less than E, in leading power factor V_t may be less than, greater than or equal E.[4]

[4]Counter intuitively, if V_t is less than E, the current will flow from E to V_t, because only in DC currents flows

In lagging power factor, leading power factor and unity power factor, load angle (γ) is always positive

from higher voltage to lower voltage, in AC it not depends on the magnityde of the currents only, also depends on the phase