EEP 225 : Electric Machines

Problem on Synchronous Machines

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I solved it on my own, so mistakes are probable

$\boxed{1.\ \text{Q}}$ — A three phase, Y connected 50 Hz, 1000 rpm synchronous generator has 15 slots per pole and 6 conductors per slot. The armature coils are short chorded by 3 slots. The flux per pole has a fundamental component of 100 mWb. The flux density distribution has 20% third harmonic and 10% fifth harmonic, while higher order harmonics may be neglected.

Calculate the rms value of the resultant phase and line voltages of the induced emf

Final exam 2011 - 2012

A —

$$E_{\text{ph}} = 4.44 f\,\phi\,N_{\text{ph}}\,K_w$$

$$f = \frac{PN}{120}$$

$$\therefore P = \frac{50 \times 120}{1000} = 6 \text{ poles}$$

Given that there are 15 slots per pole

$$\therefore \text{Number of slots } (S) = 15 \times 6 = 90$$

$$\text{Number of conductors} = 6 \times 90 = 540$$

$$\therefore N_{\text{ph}} = \frac{Z}{2 \times \text{number of phases}} = \frac{540}{2 \times 3} = 90$$

$$q = S \times \frac{1}{P} \times \frac{1}{n} = 90 \times \frac{1}{6} \times \frac{1}{3} = 5$$

$$\gamma = \frac{180 \times P}{S} = \frac{180 \times 6}{90} = 12^\circ$$

$$\therefore K_d = \frac{\sin\left(\frac{q\gamma}{2}\right)}{q\sin\left(\frac{\gamma}{2}\right)} = \frac{\sin\left(\frac{5\times 12}{2}\right)}{5\sin\left(\frac{12}{2}\right)} = 0.9566$$

$$\alpha = \frac{180^\circ \times P \times \text{short chorded}}{S} = \frac{180^\circ \times 6 \times 3}{90} = 36^\circ$$

$$K_p = \cos\left(\frac{\alpha}{2}\right) = \cos\left(\frac{36^\circ}{2}\right) = 0.951$$

$$E_{\text{ph}} = 4.44 \times 50 \times 100 \times 10^{-3} \times 90 \times 0.9566 \times 0.951 = 1817.6 \text{ V}$$

For the 3rd harmonic

$$\phi = 100 \text{ mWb} \times 0.2 = 20 \text{ mWb}$$

$$f = 3 \times 50 = 150$$

$$K_d = \therefore K_d = \frac{\sin\left(\frac{qh\gamma}{2}\right)}{q\sin\left(\frac{h\gamma}{2}\right)} = \frac{\sin\left(\frac{5\times 3\times 12}{2}\right)}{5\sin\left(\frac{12\times 3}{2}\right)} = 0.6472$$

$$K_p = \cos\left(\frac{h\alpha}{2}\right) = \cos\left(\frac{3\times 36^\circ}{2}\right) = 0.587$$

$$E_{\text{ph}} = 4.44 \times 150 \times 20 \times 10^{-3} \times 90 \times 0.6472 \times 0.587 = 455.43 \text{ V}$$

For the 5th harmonic

$$\phi = 100 \text{ mWb} \times 0.1 = 10 \text{ mWb}$$

$$f = 5 \times 50 = 250$$

$$K_d = \therefore K_d = \frac{\sin\left(\dfrac{qh\gamma}{2}\right)}{q\sin\left(\dfrac{h\gamma}{2}\right)} = \frac{\sin\left(\dfrac{5\times 5\times 12}{2}\right)}{5\sin\left(\dfrac{12\times 5}{2}\right)} = 0.2$$

$$K_p = \cos\left(\frac{h\alpha}{2}\right) = \cos\left(\frac{5\times 36^\circ}{2}\right) = 0$$

$$E_{\text{ph}} = 4.44\times 250\times 10\times 10^{-3}\times 6\times 0.2\times 0 = 0\text{ V}$$

$$E_{\text{ph}} = \sqrt{E_1^2 + E_3^2 + E_5^2} = \sqrt{1817.6^2 + 455.43^2} = 1873.2\text{ Volt}$$

Line voltages:

$$E_1 = \sqrt{3}\times 1817.6 = 3148.1\text{ Volt}$$

$$E_3 = \sqrt{3}\times 455.4 = 788.7\text{ Volt}$$

$$E_5 = 0$$

Note that in both Y and Δ connections **3rd harmonics and its multiples cancels out at the line terminals**

$$E_L = \sqrt{E_1^2 + E_5^2} = \sqrt{3148.1^2} = 3148.1\text{ Volt}$$