EEP 225 : Electric Machines

Lecture 7
AC Machines III

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1 Voltage Regulation of Synchronous Generator

Voltage regulation is a property of generator not motor, no motor has voltage regulation, but motors have speed regulation[1].

Voltage regulation (V. R.) is the change in terminal voltage when the load is removed.

It is a measure of the voltage drop inside the generator due to its internal impedance not resistance and armature reaction.

it is a measure to the magnitude only, don't consider the angle

$$\text{Voltage regulation} = \frac{|V_{\text{no load}}| - |V_{\text{load}}|}{|V_{\text{load}}|}$$

Since at no load $I_a = 0$, therefore $V_{\text{no load}} = E$

$$\text{Voltage regulation} = \frac{|E| - |V_{\text{load}}|}{|V_{\text{load}}|}$$

In the case of **lagging** power factor (inductive loads): $E > V_t$, so voltage regulation is **always positive**

In the case of **leading** power factor (capacitive loads) : may be

- $E > V_t$, voltage regulation **is positive**
- $E = V_t$, voltage regulation **is zero**
- $E < V_t$, voltage regulation **is negative**

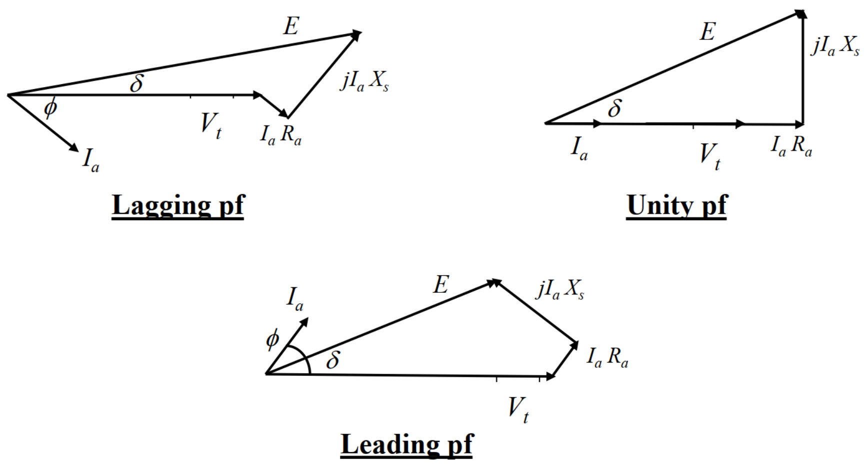


Figure 1

[1] $\frac{\text{speed at no load - speed at load}}{\text{speed at load}}$

it should be kept as small as possible. Practical values of voltage regulation ranging from 5% to 30% according to the power factor and ratings.

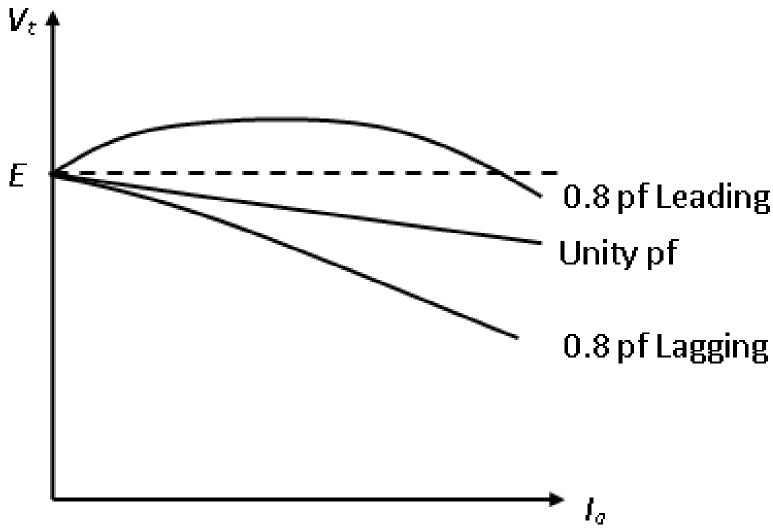


Figure 2: Variation of terminal Voltage versus armature current

2 Measuring Synchronous Generator Model Parameters

Synchronous machines are different from DC machines in parameters, in DC machines the equivalent circuit had R_a (armature resistance) which can be measured easily.

In synchronous machines the equivalent circuit had R_a and X_l and X_{ar}, (remember synchronous reactance $X_s = X_{\text{ar}} + X_l$).

X_l is physical impedance and can be measured unlike X_{ar} which is an imaginary impedance so can't me measured and it changes when the current changes.

Hence, to calculate the parameters we need two tests.

2.1 Open Circuit Test (No Load Test)

- The generator works at the rated speed.
- The terminals are disconnected from all loads.
- The field current is set to zero.
- Increase the field current I_f gradually (in steps) and measure terminal V_t

When terminals are open (no load), $I_a = 0$ so $E = V_t$ (no voltage drop). So it is easy to plot E or V_t vs I_f. It is called **open circuit characteristics (OCC)**

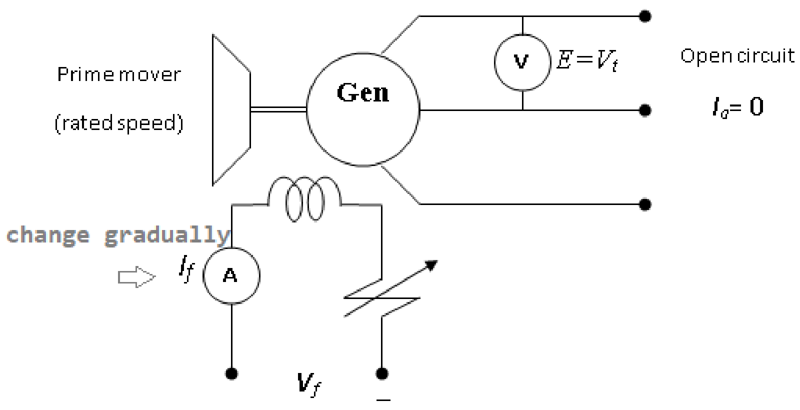


Figure 3: Open circuit test (note we test at constant speed)

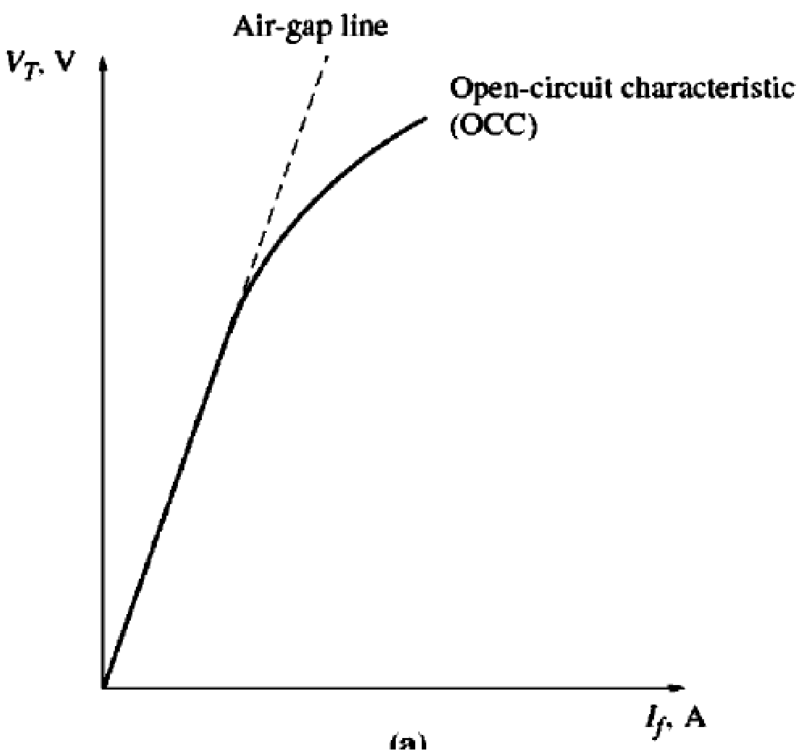


Figure 4: Open circuit characteristics (OCC)

Notice that at first the curve is almost perfectly linear, until some saturation is observed at high field currents. The linear portion of an OCC is called the air-gap line.

We did similar test in DC machines to get the magnetization curve, whatever the machine is DC, AC, generator, motor or even a transformer, the test doesn't change. why? because it describes the magnetic properties of the material made by the machine.

2.2 Short Circuit Test

- Adjust the field current I_f to zero
- Short-circuit the terminals of the generator through an ammeter.
- Increase field current I_f and, measure armature I_a.

Plot I_f vs I_a, this is called short circuit characteristic (SCC), and it is straight line why? because it follows ohm's law.

$$\overline{E} = \overline{I_a} \, . \, \overline{Z_s}$$

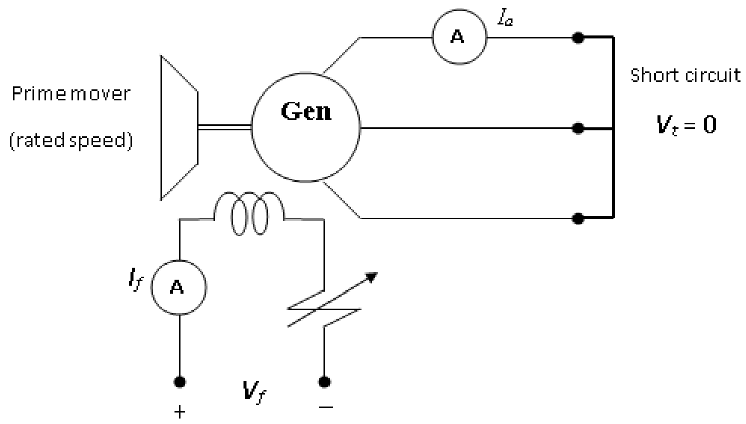


Figure 5: Short circuit test

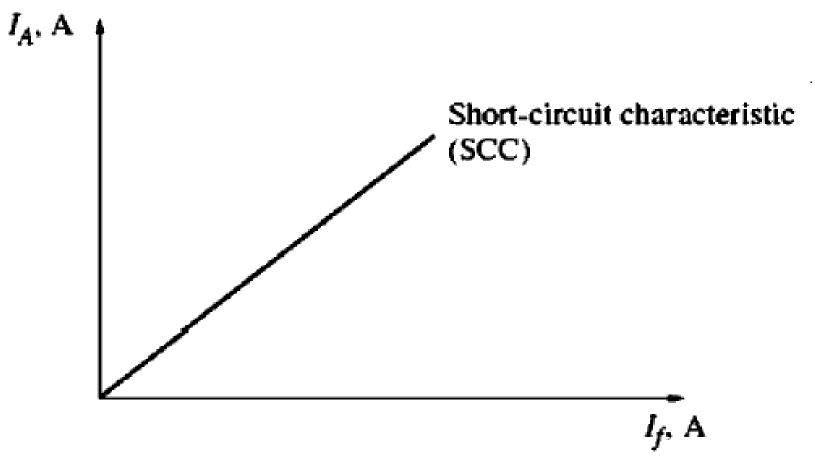
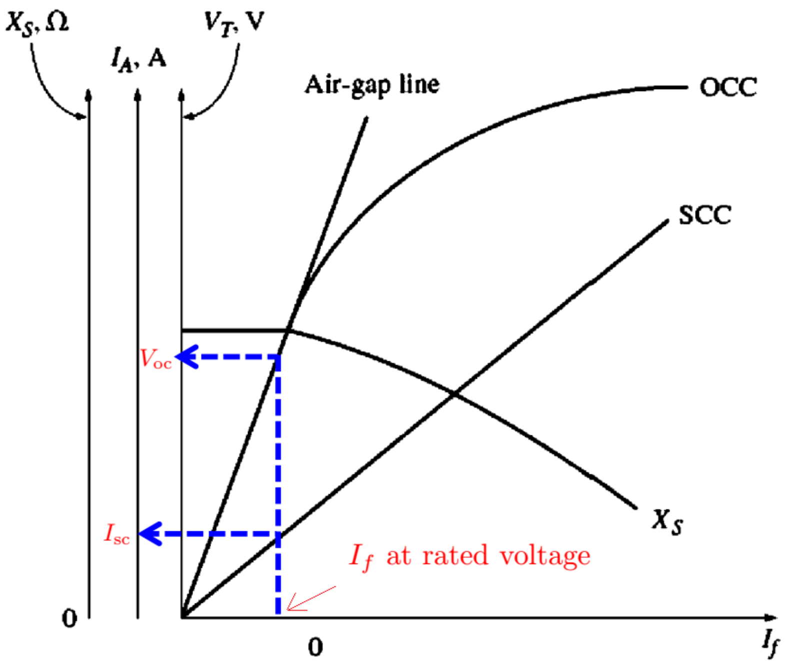


Figure 6: short circuit characteristic (SCC)

Determination of Synchronous Impedance



for a given field current I_f :

$$Z_s = \frac{V_{\text{open circuit}}}{I_{\text{short circuit}}}$$

If R_a is known or measured :

$$X_s = \sqrt{Z_s^2 - R_a^2}$$

The value of X_s is **constant** before saturation[2], after saturation X_s decreases

Z_s : Synchronous reactance.

X_s : Synchronous impedance.

Z_s : Armature resistance.

For large alternators with ratings higher than 100 kVA.

$$R_a \approx 0 \qquad Z_s \approx X_s$$

[2]Because OCC and SCC are linear

3 Power Flow, Losses and Efficiency

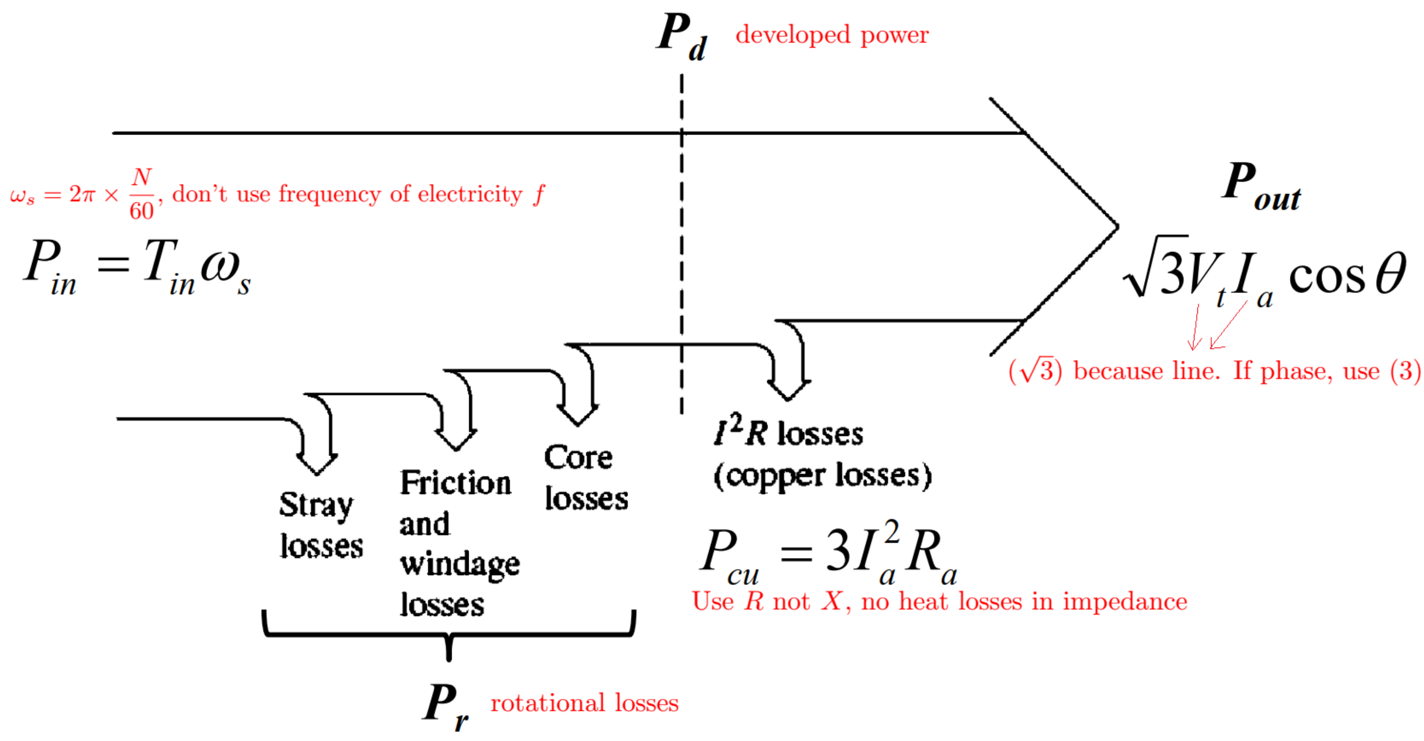


Figure 7

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{r}} + P_{\text{cu}}}$$

$\boxed{\text{1. Q}}$ — A 100 kVA, 3000 V, 50 Hz, 3 phase, Y connected alternator has an effective armature resistance = 0.2 Ω. The field current of 40 A produces short-circuit current of 200 A and an open-circuit emf of 1040 V. Calculate the full-load voltage regulation at 0.8 pf lagging, and 0.8 pf leading.

Note : In any 3 phase problem, all given data are **line values**

A —

$$Z_s = \frac{V_{\text{oc}}}{I_{\text{sc}}} = \frac{1040/\sqrt{3}}{200} = 3\Omega$$

$$X_s = \sqrt{3^2 - 0.2^2} = 2.99\Omega$$

$$P = \sqrt{3}V_t I_a \cos(\theta)$$

$$\therefore I_{a(\text{full load})} = \frac{100000}{\sqrt{3} \times 3000} = 19.2\underline{/\pm 36.7^\circ} A.$$

Why this angle? $\cos^{-1}(0.8) = 36.7^\circ$.

Why positive and negative sign?

If 0.8 lagging $\rightarrow$ negative.

If 0.8 leading $\rightarrow$ positive.

$$V_{\text{terminal - phsae}} = \frac{3000}{\sqrt{3}} = 1732.05V$$

Remember from last lecture:

$$E\underline{/\delta} = V_t\underline{/0} + I_a \times Z_s\underline{/\theta}$$

if 0.8 lagging:

$$\therefore \overline{E} = 1732.05\underline{/0} + 19.2\underline{/-36.7^\circ} \times (0.2 + j2.99) = 1769.5\underline{/1.4^\circ}$$

$$\therefore \text{Voltage regulation} = \frac{1769.5 - 1732.05}{1732.05} = 2.16\%$$

if 0.8 leading:

$$\therefore \overline{E} = 1732.05\underline{/0} + 19.2\underline{/36.7^\circ} \times (0.2 + j2.99) = 1700.7\underline{/1.4^\circ}$$

$$\therefore \text{Voltage regulation} = \frac{1700.7 - 1732.05}{1732.05} = -1.83\%$$

$\boxed{\text{2. Q}}$ — A 1000 kVA, 6600 V, 3-phase, Y-connected alternator has an armature resistance of 0.5Ω and a synchronous reactance of 10Ω per phase. Calculate the generated voltage, load angle, voltage regulation, and efficiency at full load at 0.8 pf lagging. Rotational loss is 10kW.

A —

$$P = \sqrt{3} V_t I_a \cos(\theta)$$

$$\therefore I_{a(\text{full load})} = \frac{1000000}{\sqrt{3} \times 6600} = 87.5\underline{/-36.7^\circ} A.$$

Why this angle? $\cos^{-1}(0.8) = 36.8^\circ$.

Why positive and negative sign?

If 0.8 lagging $\rightarrow$ negative.

$$V_{\text{terminal - phsae}} = \frac{6600}{\sqrt{3}} = 3810V$$

Remember from last lecture:

$$E\underline{/\delta} = V_t\underline{/0} + I_a \times Z_s\underline{/\theta}$$

$$\therefore \overline{E} = 3810\underline{/0} + 87.5\underline{/-36.8^\circ} \times (0.5 + j10) = 4421\underline{/8.8^\circ}$$

$$\therefore \text{Voltage regulation} = \frac{4421 - 3810}{3810} = 16\%$$

$$P_{\text{cu}} = 3I_a^2 R_a = 3 \times 87.5^2 \times 0.5 = 11484\text{Watt}$$

$$\eta = \frac{800000}{800000 + 10000 + 11484} = 97.4\%$$

Note that $R_a << X_s$ which is a good design for the machine why? to limit the current and limit the losses.

4 Power-Angle Relationship

Consider the case of large synchronous machines with ratings greater than 100 kVA, as armature resistance is so small and may be neglected for simplicity. Therefore :

$$\overline{E} = \overline{V_t} + \overline{I_a} \times j\,\overline{X_s}$$

From the corresponding phasor diagram

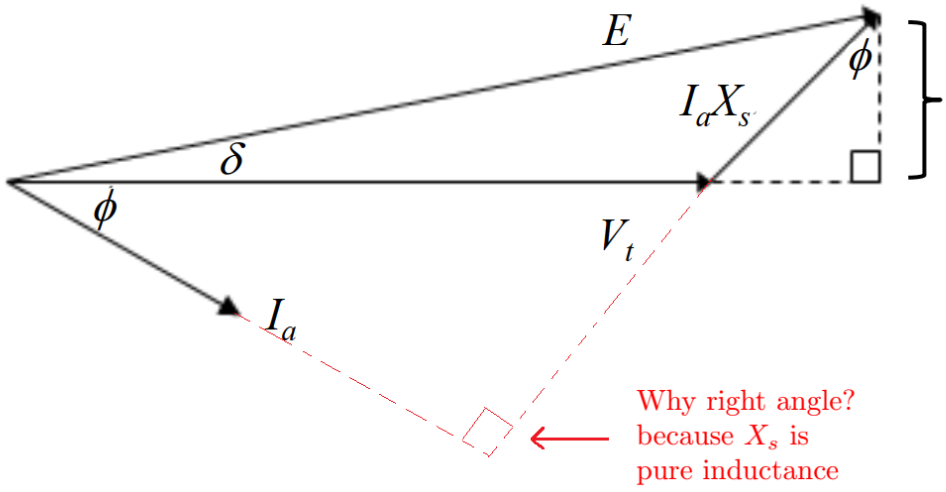


Figure 8

$$I_a X_s \cos(\phi) = E \sin(\delta)$$

$$I_a \cos(\phi) = \frac{E \sin(\delta)}{X_s}$$

$$P_{\text{out}} = 3 V_t I_a \cos(\phi)$$

$$\boxed{P_{\text{out}} = \frac{3 E V_t}{X_s} \sin(\delta)}$$

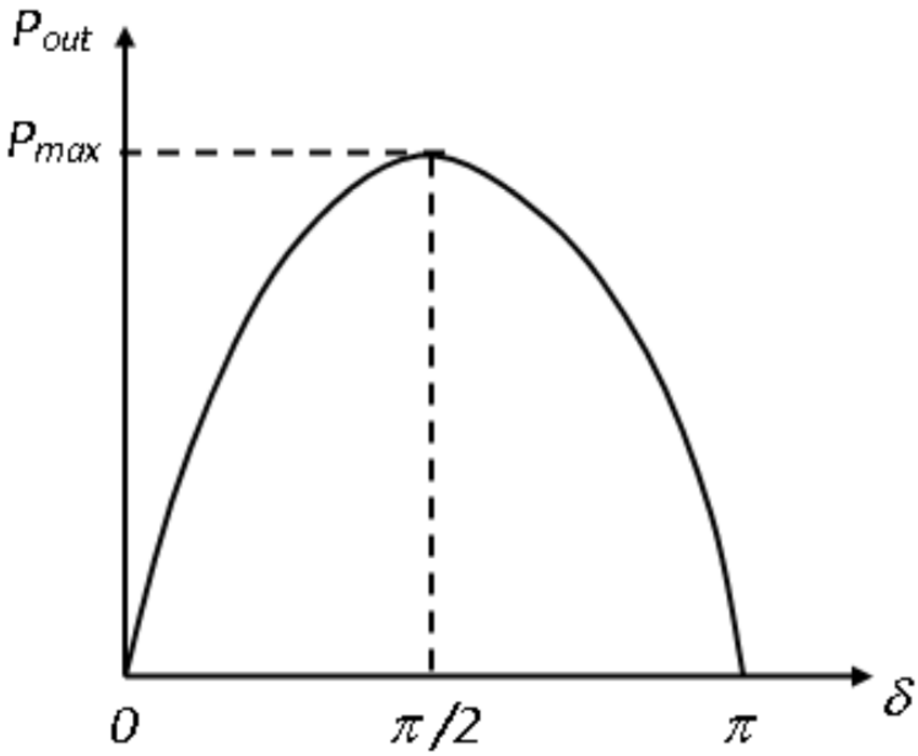


Figure 9: Plot of the **swing equation** we proved

The last equation indicates that the output power is function of $\sin(\delta)$, power has its **maximum value** at $\delta = \frac{\pi}{2}$ or $90°$ and **zero value** at $\delta = 0$ and $\delta = \pi$ or $180°$

The range $0 < \delta < \frac{\pi}{2}$ is the stable range, while the range $\frac{\pi}{2} < \delta < \pi$ is unstable.

The **maximum output power** occurs at $\delta = \frac{\pi}{2}$ and is given by :

$$\boxed{P_{\text{out},\delta=0.5\pi} = 3\frac{|E||V_t|}{X_s}}$$

$\boxed{\text{3. Q}}$ — A 1000 V, 120 kVA, Δ connected, three-phase, synchronous generator has a synchronous reactance per phase of 5Ω and a neglected armature resistance. It supplies the rated load at 0.9 pf lagging. Determine

(a) The generated voltage.

(b) The power angle.

(c) The power at generated voltage.

(d) The maximum developed power and the corresponding power angle.

A —

$$P_{\text{out}} = 3V_{\text{phase}} I_{\text{phase}} \cos(\phi)$$

$$I_{\text{phase}} = \frac{120000}{\sqrt{3} \times 1000} = 69.28$$

$$I_{\text{line}} = \sqrt{3} \times I_{\text{phase}} = \sqrt{3} \times 69.28 = 40$$

$$I_{a(\text{full load})} = 40\underline{/-25.8^\circ}$$

Why the angle?

$\cos^{-1}(0.9) = 25.8^\circ$.

Why negative ? $\rightarrow$ lagging.

(a), (b), (c) :

Remember from last lecture:

$$E\underline{/\delta} = V_t\underline{/0} + I_a \times Z_s\underline{/\theta}$$

$$\therefore \overline{E} = 1000\underline{/0} + 40\underline{/-25.8^\circ} \times (j5) = 1102\underline{/9.4^\circ}$$

$$P = 3\frac{|E||V_t|}{X_s}\sin(\delta) = \frac{3 \times 1102 \times 1000}{5}\sin(9.4^\circ) = 108\text{kW}$$

(d) :

$$\text{maximum output power} = 3\frac{|E||V_t|}{X_s} = \frac{3 \times 1102 \times 1000}{5} = 661.2\text{kW}$$