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# Lecture 7

## AC Machines III

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# 1 Voltage Regulation of Synchronous Generator

Voltage regulation is a property of **generator** not **motor**, no motor has voltage regulation, but motors have speed regulation<sup>1</sup>.

**Voltage regulation (V. R.)** is the change in terminal voltage when the load is removed.

It is a measure of the voltage drop inside the generator due to its internal impedance **not resistance** and armature reaction.

it is a measure to the magnitude only, don't consider the angle

$$\text{Voltage regulation} = \frac{|V_{\text{no load}} - V_{\text{load}}|}{V_{\text{load}}}$$

Since at no load  $I_a = 0$ , therefore  $V_{\text{no load}} = E$

$$\text{Voltage regulation} = \frac{|E - V_{\text{load}}|}{E}$$

In the case of **lagging** power factor (inductive loads):  $E > V_t$ , so voltage regulation is **always positive**

In the case of **leading** power factor (capacitive loads) : may be

- $E > V_t$ , voltage regulation is **positive**
- $E = V_t$ , voltage regulation is **zero**
- $E < V_t$ , voltage regulation is **negative**

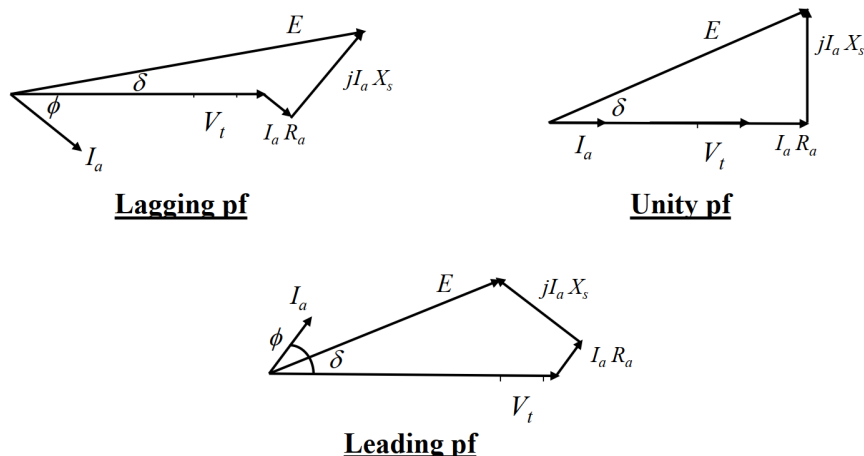


Figure 1

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<sup>1</sup>  $\frac{\text{speed at no load} - \text{speed at load}}{\text{speed at load}}$

it should be kept **as small as possible**. Practical values of voltage regulation ranging from 5% to 30% according to the power factor and ratings.

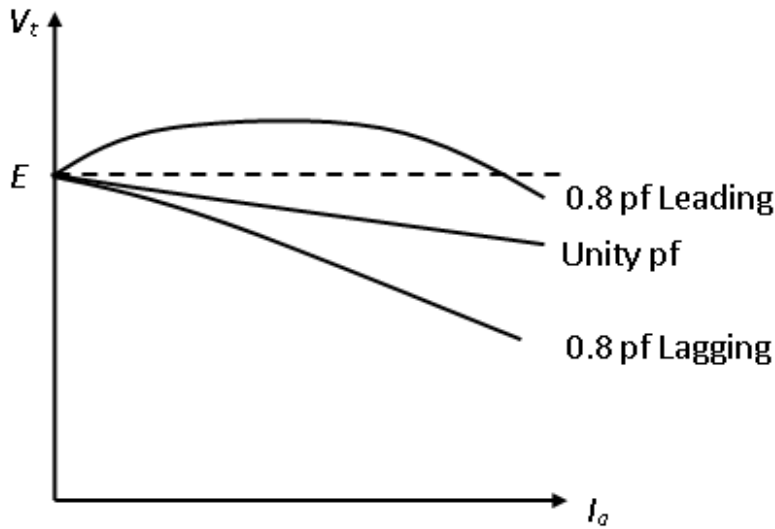


Figure 2: Variation of terminal Voltage versus armature current

## 2 Measuring Synchronous Generator Model Parameters

Synchronous machines are different from DC machines in parameters, in DC machines the equivalent circuit had  $R_a$  (armature resistance) which can be measured easily.

In synchronous machines the equivalent circuit had  $R_a$  and  $X_l$  and  $X_{ar}$ , (remember **synchronous reactance**  $X_s = X_{ar} + X_l$ ).

$X_l$  is **physical impedance** and can be measured unlike  $X_{ar}$  which is an **imaginary impedance** so can't be measured and it changes when the current changes.

Hence, to calculate the parameters we need **two tests**.

### 2.1 Open Circuit Test (No Load Test)

- The generator works at the rated speed.
- The terminals are disconnected from all loads.
- The field current is set to zero.
- Increase the field current  $I_f$  gradually (in steps) and measure terminal  $V_t$

When terminals are open (no load),  $I_a = 0$  so  $E = V_t$  (no voltage drop). So it is easy to plot  $E$  or  $V_t$  vs  $I_f$ . It is called **open circuit characteristics (OCC)**

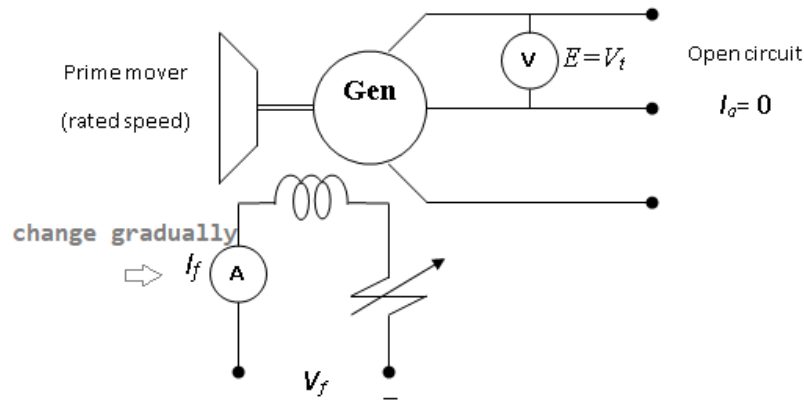


Figure 3: Open circuit test (note we test at constant speed)

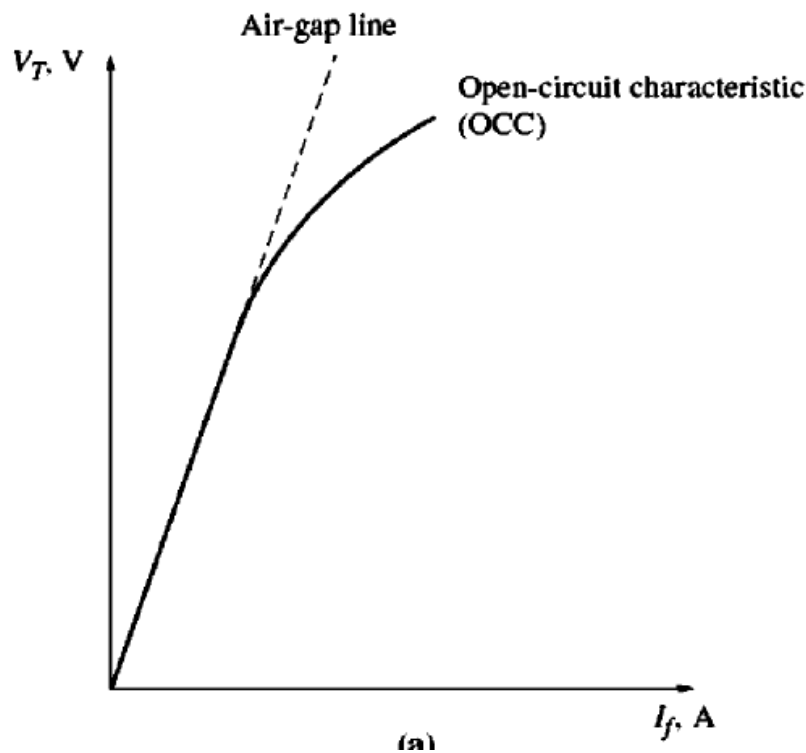


Figure 4: Open circuit characteristics (OCC)

Notice that at first the curve is almost perfectly linear, until some saturation is observed at high field currents. The linear portion of an OCC is called the **air-gap line**.

We did similar test in DC machines to get the magnetization curve, whatever the machine is DC, AC, generator, motor or even a transformer, the test doesn't change. **why?** because it describes the magnetic properties of the material made by the machine.

## 2.2 Short Circuit Test

- Adjust the field current  $I_f$  to zero
- Short-circuit the terminals of the generator through an ammeter.
- Increase field current  $I_f$  and, measure armature  $I_a$ .

Plot  $I_f$  vs  $I_a$ , this is called **short circuit characteristic (SCC)**, and it is straight line **why?** because it follows ohm's law.

$$\overline{E} = \overline{I_a} \cdot \overline{Z_s}$$

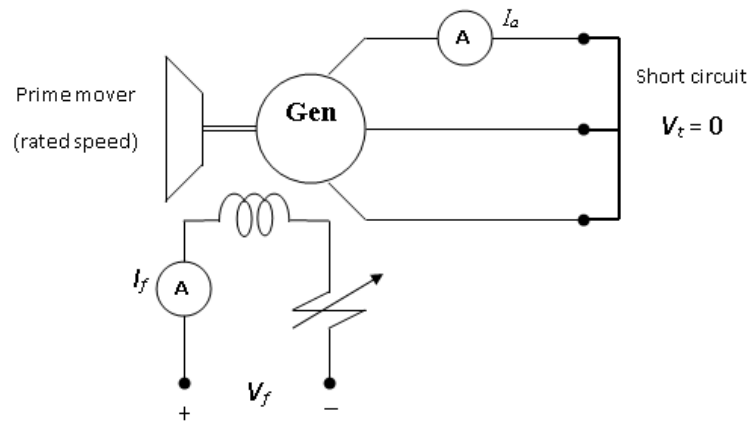


Figure 5: Short circuit test

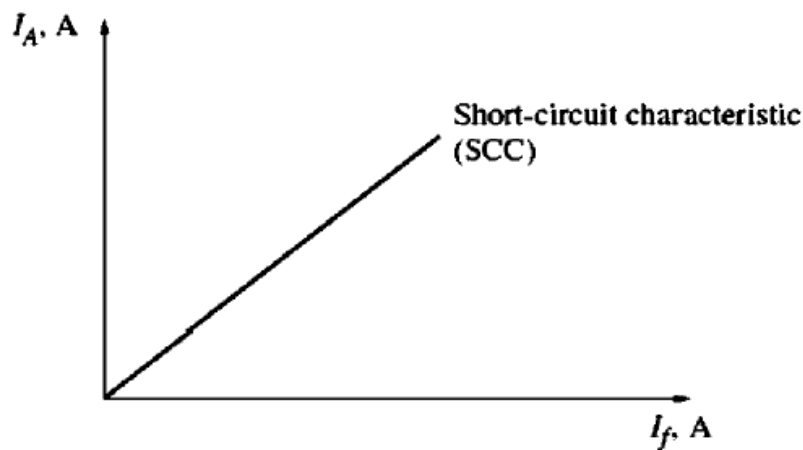
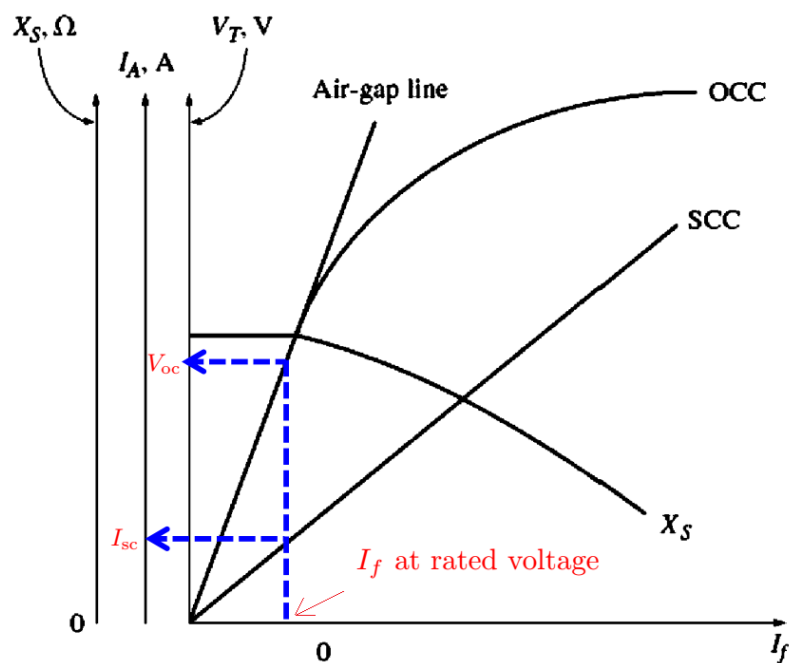


Figure 6: short circuit characteristic (SCC)

## Determination of Synchronous Impedance



for a given field current  $I_f$  :

$$Z_s = \frac{V_{\text{open circuit}}}{I_{\text{short circuit}}}$$

If  $R_a$  is known or measured :

$$X_s = \sqrt{Z_s^2 - R_a^2}$$

The value of  $X_s$  is **constant** before saturation<sup>2</sup>, after saturation  $X_s$  decreases

$Z_s$  : Synchronous reactance.

$X_s$  : Synchronous impedance.

$Z_s$  : Armature resistance.

For large alternators with ratings higher than 100 kVA.

$$R_a \approx 0 \quad Z_s \approx X_s$$

<sup>2</sup>Because OCC and SCC are linear

### 3 Power Flow, Losses and Efficiency

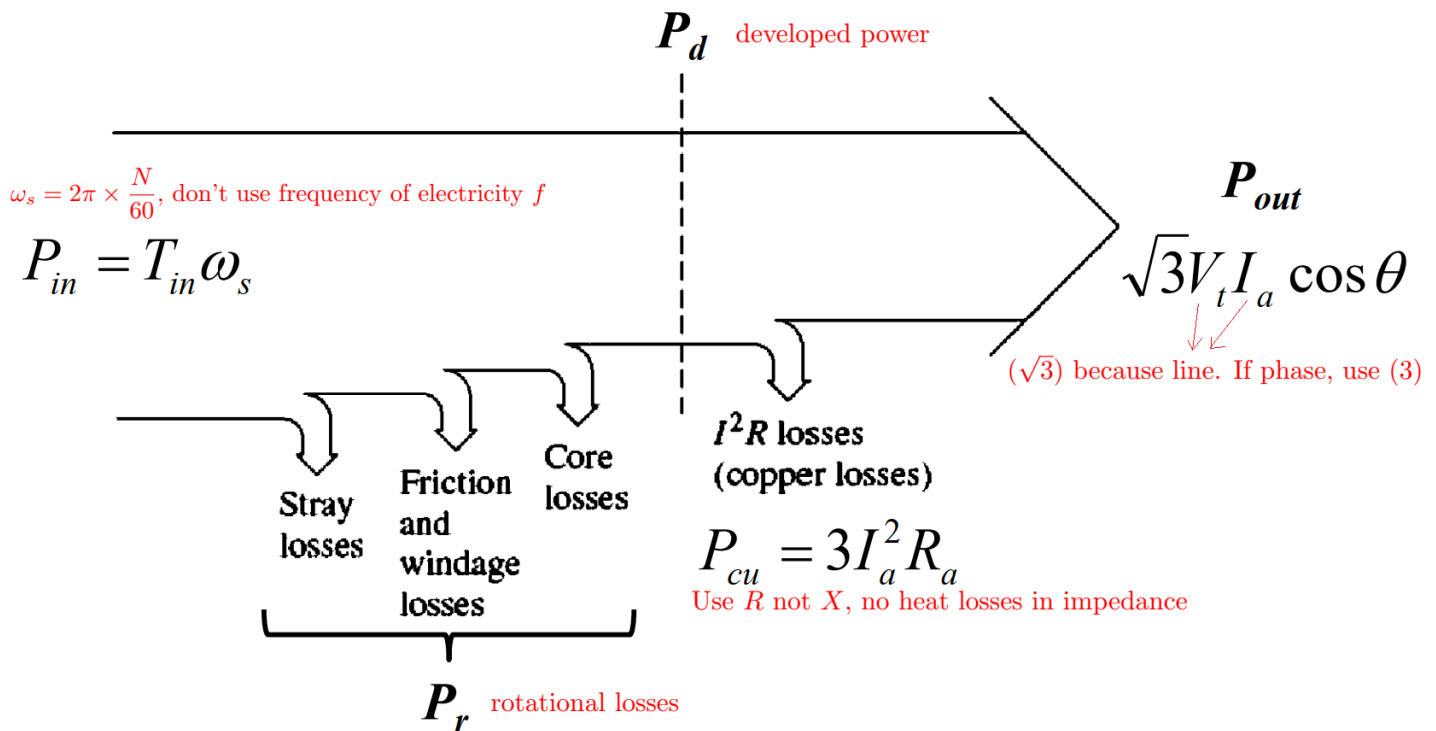


Figure 7

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_r + P_{cu}}$$

1. Q — A 100 kVA, 3000 V, 50 Hz, 3 phase, Y connected alternator has an effective armature resistance =  $0.2 \Omega$ . The field current of 40 A produces short-circuit current of 200 A and an open-circuit emf of 1040 V. Calculate the full-load voltage regulation at 0.8 pf lagging, and 0.8 pf leading.

Note : In any 3 phase problem, all given data are **line values**

A —

$$Z_s = \frac{V_{oc}}{I_{sc}} = \frac{1040/\sqrt{3}}{200} = 3\Omega$$

$$X_s = \sqrt{3^2 - 0.2^2} = 2.99\Omega$$

$$P = \sqrt{3}V_t I_a \cos(\theta)$$

$$\therefore I_{a(\text{full load})} = \frac{100000}{\sqrt{3} \times 3000} = 19.2/\underline{\pm 36.7^\circ} A.$$

Why this angle?  $\cos^{-1}(0.8) = 36.7^\circ$ .

Why positive and negative sign?

If 0.8 **lagging**  $\rightarrow$  **negative**.

If 0.8 **leading**  $\rightarrow$  **positive**.

$$V_{\text{terminal - phsae}} = \frac{3000}{\sqrt{3}} = 1732.05V$$

Remember from last lecture:

$$E/\underline{\delta} = V_t/\underline{0} + I_a \times Z_s/\underline{\theta}$$

if 0.8 **lagging**:

$$\therefore \overline{E} = 1732.05/\underline{0} + 19.2/\underline{-36.7^\circ} \times (0.2 + j2.99) = 1769.5/\underline{1.4^\circ}$$

$$\therefore \text{Voltage regulation} = \frac{1769.5 - 1732.05}{1732.05} = 2.16\%$$

if 0.8 **leading**:

$$\therefore \overline{E} = 1732.05/\underline{0} + 19.2/\underline{36.7^\circ} \times (0.2 + j2.99) = 1700.7/\underline{1.4^\circ}$$

$$\therefore \text{Voltage regulation} = \frac{1700.7 - 1732.05}{1732.05} = -1.83\%$$

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**2. Q** — A 1000 kVA, 6600 V, 3-phase, Y-connected alternator has an armature resistance of  $0.5\Omega$  and a synchronous reactance of  $10\Omega$  per phase. Calculate the generated voltage, load angle, voltage regulation, and efficiency at full load at 0.8 pf lagging. Rotational loss is 10kW.

A —

$$P = \sqrt{3}V_t I_a \cos(\theta)$$



$$\therefore I_{a(\text{full load})} = \frac{1000000}{\sqrt{3} \times 6600} = 87.5 \angle -36.7^\circ A.$$

Why this angle?  $\cos^{-1}(0.8) = 36.8^\circ$ .

Why positive and negative sign?

If 0.8 **lagging**  $\rightarrow$  **negative**.

$$V_{\text{terminal - phsae}} = \frac{6600}{\sqrt{3}} = 3810V$$

Remember from last lecture:

$$E \angle \delta = V_t \angle 0 + I_a \times Z_s \angle \theta$$

$$\therefore \overline{E} = 3810 \angle 0 + 87.5 \angle -36.8^\circ \times (0.5 + j10) = 4421 \angle 8.8^\circ$$

$$\therefore \text{Voltage regulation} = \frac{4421 - 3810}{3810} = 16\%$$

$$P_{\text{cu}} = 3I_a^2 R_a = 3 \times 87.5^2 \times 0.5 = 11484 \text{ Watt}$$

$$\eta = \frac{800000}{800000 + 10000 + 11484} = 97.4\%$$

Note that  $R_a \ll X_s$  which is a **good design** for the machine **why?** to **limit the current** and **limit the losses**.

## 4 Power-Angle Relationship

Consider the case of **large synchronous machines with ratings greater than 100 kVA**, as **armature resistance is so small and may be neglected** for simplicity. Therefore :

$$\overline{E} = \overline{V}_t + \overline{I}_a \times j \overline{X}_s$$

From the corresponding phasor diagram

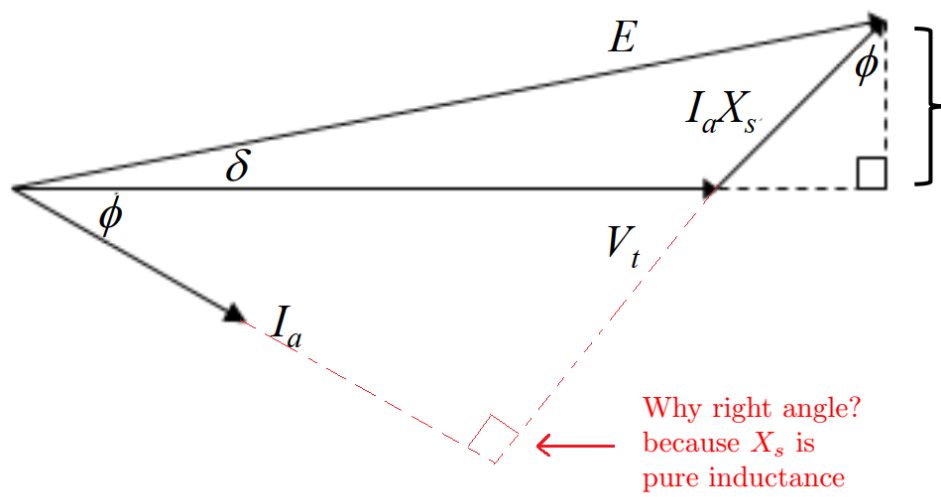


Figure 8

$$I_a X_s \cos(\phi) = E \sin(\delta)$$

$$I_a \cos(\phi) = \frac{E \sin(\delta)}{X_s}$$

$$P_{\text{out}} = 3V_t I_a \cos(\phi)$$

$$P_{\text{out}} = \frac{3EV_t}{X_s} \sin(\delta)$$

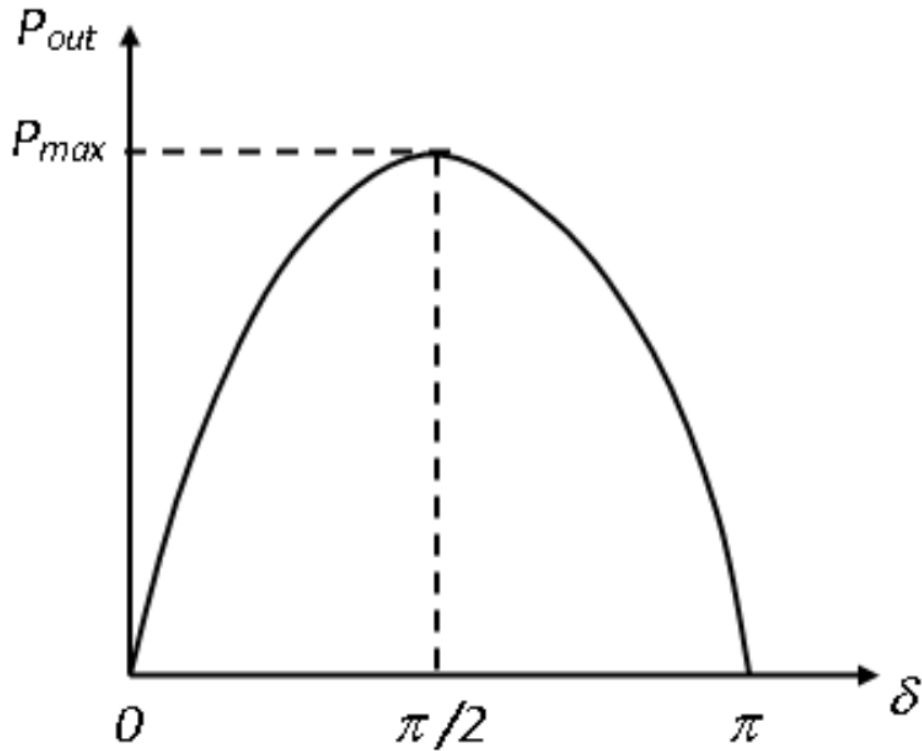


Figure 9: Plot of the **swing equation** we proved

The last equation indicates that the **output power** is function of  $\sin(\delta)$ , power has its **maximum value** at  $\delta = \frac{\pi}{2}$  or  $90^\circ$  and **zero value** at  $\delta = 0$  and  $\delta = \pi$  or  $180^\circ$ . The range  $0 < \delta < \frac{\pi}{2}$  is the **stable range**, while the range  $\frac{\pi}{2} < \delta < \pi$  is **unstable**.

The **maximum output power** occurs at  $\delta = \frac{\pi}{2}$  and is given by :

$$P_{\text{out}, \delta=0.5\pi} = 3 \frac{|E||V_t|}{X_s}$$

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**3. Q** — A 1000 V, 120 kVA,  $\Delta$  connected, three-phase, synchronous generator has a synchronous reactance per phase of  $5\Omega$  and a neglected armature resistance. It supplies the rated load at 0.9 pf lagging. Determine

- The generated voltage.
- The power angle.
- The power at generated voltage.
- The maximum developed power and the corresponding power angle.

A —

$$P_{\text{out}} = 3V_{\text{phase}}I_{\text{phase}}\cos(\phi)$$

$$I_{\text{phase}} = \frac{120000}{\sqrt{3} \times 1000} = 69.28$$

$$I_{\text{line}} = \sqrt{3} \times I_{\text{phase}} = \sqrt{3} \times 69.28 = 120$$

$$I_{a(\text{full load})} = 40\angle -25.8^\circ$$

Why the angle?

$$\cos^{-1}(0.9) = 25.8^\circ.$$

Why **negative** ?  $\rightarrow$  **lagging**.

(a), (b), (c) :

Remember from last lecture:

$$\underline{E/\delta} = \underline{V_t/0} + I_a \times \underline{Z_s/\theta}$$

$$\therefore \underline{\bar{E}} = 1000\angle 0 + 40\angle -25.8^\circ \times (j5) = 1102\angle 9.4^\circ$$

$$P = 3 \frac{|E||V_t|}{X_s} \sin(\delta) = \frac{3 \times 1102 \times 1000}{5} \sin(9.4^\circ) = 108\text{kW}$$

(d) :

$$\text{maximum output power} = 3 \frac{|E||V_t|}{X_s} = \frac{3 \times 1102 \times 1000}{5} = 661.2\text{kW}$$