EEP 225 : Electric Machines

Problems on Induction Motor

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$\boxed{1.\ Q}$ — A 120V, 60Hz, 6-poles, Δ-connected, wound-rotor induction motor has a rotor resistance and standstill reactance of 0.15Ω and 0.9Ω per phase respectively. Calculate the starting and maximum torques, and the speed at maximum torque. (Neglect stator impedance). Also find the starting current.
Find the maximum developed power and the speed at which occurs

(Lecture problem)

A — What is standstill reactance X_2?

At standstill, $s = 1$, $f_r = f_s$, note that reactance depend on frequency

$$N_s = 120 \times \frac{f_s}{P} = 120 \times \frac{60}{6} = 1200 \text{ rpm.}$$

$$\omega_s = 2 \times \pi \times \frac{N_s}{f_s} = 125.6 \text{ rad / sec}$$

$$T_{\text{start}} = \frac{3R_2}{\omega_s \times s} \times \frac{V_{\text{ph}}^2}{\left(R_1 + \dfrac{R_2}{s}\right)^2 + (X_1 + X_2)^2}$$

neglecting stator impedance (R_1 and X_1), and substituting with $s = 1$ (Starting slip is always equals 1)

$$T_{\text{start}} = \frac{3R_2}{\omega_s} \times \frac{V_{\text{ph}}^2}{R_2^2 + X_2^2}$$

$$T_{\text{start}} = \frac{3 \times 0.15}{125.6} \times \frac{120^2}{0.15^2 + 0.9^2} = 62 \text{ N.m.}$$

$$s_{mT} = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}}$$

neglecting stator impedance (R_1 and X_1)

$$s_{mT} = \frac{R_2}{X_2} = \frac{0.15}{0.9} = 0.167$$

Rotor speed corresponding to the maximum torque

$$N_{r_{mT}} = N_s(1 - s_{mT}) = 1200(1 - 0.167) = 1000 \text{ rpm}$$

$$T_{\max} = \frac{3R_2}{\omega_s \times s_{mT}} \times \frac{V_{\text{ph}}^2}{\left(R_1 + \dfrac{R_2}{s_{mT}}\right)^2 + (X_1 + X_2)^2}$$

neglecting stator impedance (R_1 and X_1)

$$T_{\max} = \frac{3R_2}{\omega_s \times s_{mT}} \times \frac{V_{\text{ph}}^2}{\left(\dfrac{R_2}{s_{mT}}\right)^2 + X_2^2}$$

$$T_{\max} = \frac{3 \times 0.15}{125.6 \times 0.167} \times \frac{120^2}{\left(\dfrac{0.15}{0.167}\right)^2 + 0.9^2} = 190 \text{ N.m.}$$

$$I_{\text{start}} = \frac{V_{\text{ph}}}{\sqrt{\left(R_1 + \dfrac{R_2}{s}\right)^2 + (X_1 + X_2)^2}}$$

neglecting stator impedance (R_1 and X_1), and substituting with $s = 1$ (Starting slip is always equals 1)

$$I_{\text{start}} = \frac{V_{\text{ph}}}{\sqrt{(R_2)^2 + (X_2)^2}} = \frac{120}{\sqrt{0.15^2 + 0.9^2}} = 131.5 \text{ Amp}$$

R_L (resistance at which maximum developed power occurs)

$$R_L = \sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2}$$

neglecting stator impedance (R_1 and X_1)

$$R_L = \sqrt{R_2^2 + X_2^2} = \sqrt{0.15^2 + 0.9^2} = 0.912\Omega$$

$$s_{mP} = \frac{R_2}{R_2 + R_L} = \frac{0.15}{0.15 + 0.92} = 0.14$$

$N_{r_{mP}}$ (Speed of rotor at maximum developed power)

$$N_{r_{mP}} = N_s(1 - s_{mP}) = 1200(1 - 0.14) = 1032 \text{ rpm}$$

I_{mP} (current at maximum power)

$$I_{mP} = \frac{V_{\text{ph}}}{\sqrt{\left(\frac{R_2}{s_{mP}}\right)^2 + (X_2)^2}} = \frac{120}{\sqrt{\frac{0.15}{0.14}^2 + 0.9^2}} = 85.75 \text{ Amp}$$

$$P_{d_{\max}} = 3I_{mP}^2 R_L = 3 \times 85.75^2 \times 0.912 = 20117.9 \text{ Watt}$$

$\boxed{\text{2. Q}}$ — If if an external resistance equals to the rotor resistance is added to each phase of the rotor, calculate the new values of the motor starting and maximum torques, and speed at maximum torque. Also calculate the starting current.
Find the maximum developed power and the speed at which occurs

A — What will change?

Substitute with $(R_2 + R_{\text{external}})$

$$T_{\text{start}} = \frac{3(R_2 + R_{\text{external}})}{\omega_s} \times \frac{V_{\text{ph}}^2}{(R_2 + R_{\text{external}})^2 + X_2^2}$$

$$R_{\text{external}} = 0.15$$

$$T_{\text{start}} = \frac{3 \times 0.3}{125.6} \times \frac{120^2}{0.3^2 + 0.9^2} = 114.6 \text{ N.m.}$$

$$s_{mT} = \frac{(R_2 + R_{\text{external}})}{X_2} = \frac{0.3}{0.9} = 0.333$$

$$N_{r_{mT}} = N_s(1 - s_{mT}) = 1200(1 - 0.333) = 800 \text{ rpm}$$

$$T_{\max} = \frac{3(R_2 + R_{\text{external}})}{\omega_s \times s_{mT}} \times \frac{V_{\text{ph}}^2}{\left(\frac{(R_2 + R_{\text{external}})}{s_{mT}}\right)^2 + X_2^2}$$

$$T_{\max} = \frac{3 \times 0.3}{125.6 \times 0.167} \times \frac{120^2}{\left(\frac{0.3}{0.167}\right)^2 + 0.9^2} = 190 \text{ N.m.}$$

Didn't change, because maximum torque doesn't depend on R_2

$$I_{\text{start}} = \frac{V_{\text{ph}}}{\sqrt{(R_2 + R_{\text{external}})^2 + (X_2)^2}} = \frac{120}{\sqrt{0.3^2 + 0.9^2}} = 126.49 \text{ Amp}$$

R_L (resistance at which maximum developed power occurs)

$$R_L = \sqrt{(R_1 + R_2 + R_{\text{external}})^2 + (X_1 + X_2)^2}$$

neglecting stator impedance (R_1 and X_1)

$$R_L = \sqrt{(R_2 + R_{\text{external}})^2 + X_2^2} = \sqrt{0.3^2 + 0.9^2} = 0.948\Omega$$

$$s_{mP} = \frac{(R_2 + R_{\text{external}})}{(R_2 + R_{\text{external}}) + R_L} = \frac{0.3}{0.3 + 0.92} = 0.246$$

$N_{r_{mP}}$ (Speed of rotor at maximum developed power)

$$N_{r_{mP}} = N_s(1 - s_{mP}) = 1200(1 - 0.246) = 904.8 \text{ rpm}$$

I_{mP} (current at maximum power)

$$I_{mP} = \frac{V_{\text{ph}}}{\sqrt{\left(\frac{(R_2 + R_{\text{external}})}{s_{mP}}\right)^2 + (X_2)^2}} = \frac{120}{\sqrt{\frac{0.3}{0.246}^2 + 0.9^2}} = 79.17 \text{ Amp}$$

$$P_{d_{\max}} = 3I_{mP}^2 R_L = 3 \times 79.17^2 \times 0.948 = 17825.8 \text{ Watt}$$

$\boxed{\text{3. Q}}$ — A 380 V, 50 Hz, 4-pole three-phase Y-connected induction motor has the following parameters:
Stator resistance and reactance per phase: 0.8 Ω and 1.5 Ω.
Rotor resistance and reactance per phase: 0.6 Ω and 1.7 Ω.

1. Calculate the starting current, and the starting torque.
2. Determine the maximum developed torque, the speed at which it occurs, and the corresponding developed power.
3. Determine the maximum developed power, the speed at which it occurs, and the corresponding developed torque.
4. Sketch the variations of both the developed torque and developed power versus speed, then insert the values obtained from i, ii, and iii on the graph.
5. If if an external resistance equals to the rotor resistance is added to each phase of the rotor, calculate the new values of the motor starting and maximum torques, and speed at maximum torque. Also calculate the starting current. Find the maximum developed power and the speed at which occurs

(Take Home Exam 2019-2020)
(I solved it on my own, so mistakes are probable)

A —

$$R_1 = 0.8\Omega \qquad X_1 = 1.5\Omega \qquad R_2 = 0.6\Omega \qquad X_2 = 1.7\Omega$$

$$V_{\text{ph}} = \frac{380}{\sqrt{3}} \approx 220 \text{ Volt}$$

1. Starting Current (note s at starting $= 1$)

$$I_{\text{start}} = \frac{V_{\text{ph}}}{\sqrt{\left(R_1 + \frac{R_2}{s}\right)^2 + (X_1 + X_2)^2}}$$

$$I_{\text{start}} = \frac{220}{\sqrt{(0.8+0.6)^2+(1.5+1.7)^2}} = 62.98A$$

2. Determine the maximum developed torque, the speed at which it occurs, and the corresponding developed power.

$$s_{mT} = \frac{R_2}{\sqrt{R_1^2+(X_1+X_2)^2}} = 0.1819$$

$$N_s = 120 \times \frac{f_s}{P} = 120 \times \frac{50}{4} = 1500 \text{ rpm.}$$

$$\omega_s = 2 \times \pi \times \frac{N_s}{60} = 2 \times \pi \times \frac{1500}{60} = 157 \text{ rad / sec}$$

$$T_{\max} = \frac{3R_2}{\omega_s \times s_{mT}} \times \frac{V_{\text{ph}}^2}{\left(R_1 + \dfrac{R_2}{s_{mT}}\right)^2 + (X_1+X_2)^2}$$

$$T_{\max} = \frac{3 \times 0.6}{157 \times 0.1819} \times \frac{220^2}{\left(0.8 + \dfrac{0.6}{0.1819}\right)^2 + (1.5+1.7)^2} = 112.8 \text{ N.m.}$$

$$N_{r_{mT}} = N_s(1-s_{mT}) = 1500 \times (1-0.1819) = 1227.15 \text{ rad/sec}$$

Corresponding developed power

$$T_{d_{\max}} = \frac{P_d}{\omega_r}$$

$$\omega_r = 2\times\pi\times\frac{N_r}{60} = 2\times\pi\times\frac{N_s(1-s_{mT})}{60} = 2\times\pi\times\frac{1500(1-0.1819)}{60} = 128.44 \text{ rad/sec}$$

$$\therefore P_d = 112.8 \times 128.44 = 14488.032 \text{ Watt}$$

3. Determine the maximum developed power, the speed at which it occurs, and the corresponding developed torque.

R_L (resistance at which maximum developed power occurs)

$$R_L = \sqrt{(R_1 + R_2)^2 + (X_1 + X_2)^2} = \sqrt{(0.8 + 0.6)^2 + (1.5 + 1.7)^2} = 3.5\Omega$$

$$s_{mP} = \frac{R_2}{R_2 + R_L} = \frac{0.6}{0.6 + 3.5} = 0.1463$$

$N_{r_{mP}}$ (Speed of rotor at maximum developed power)

$$N_{r_{mP}} = N_s(1 - s_{mP}) = 1500(1 - 0.1463) = 1280.5 \text{ rpm}$$

I_{mP} (current at maximum power)

$$I_{mP} = \frac{V_{\text{ph}}}{\sqrt{\left(R_1 + \frac{R_2}{s_{mP}}\right)^2 + (X_1 + X_2)^2}} = \frac{220}{\sqrt{\left(0.8 + \frac{0.6}{0.1463}\right)^2 + (1.5 + 1.7)^2}} = 37.58 \text{ A}$$

$$P_{d_{\max}} = 3I_{mP}^2 R_L = 3 \times 37.58^2 \times 3.5 = 14828.7 \text{ Watt}$$

Corresponding torque

$$\omega_r = 2\times\pi\times\frac{N_r}{60} = 2\times\pi\times\frac{N_s(1 - s_{mT})}{60} = 2\times\pi\times\frac{1500(1 - 0.1463)}{60} = 134 \text{ rad/sec}$$

$$T = \frac{P}{\omega_r} = \frac{14828.7}{134} = 110.66$$

4. Sketch the variations of both the developed torque and developed power versus speed, then insert the values obtained from 1, 2, and 3 on the graph.

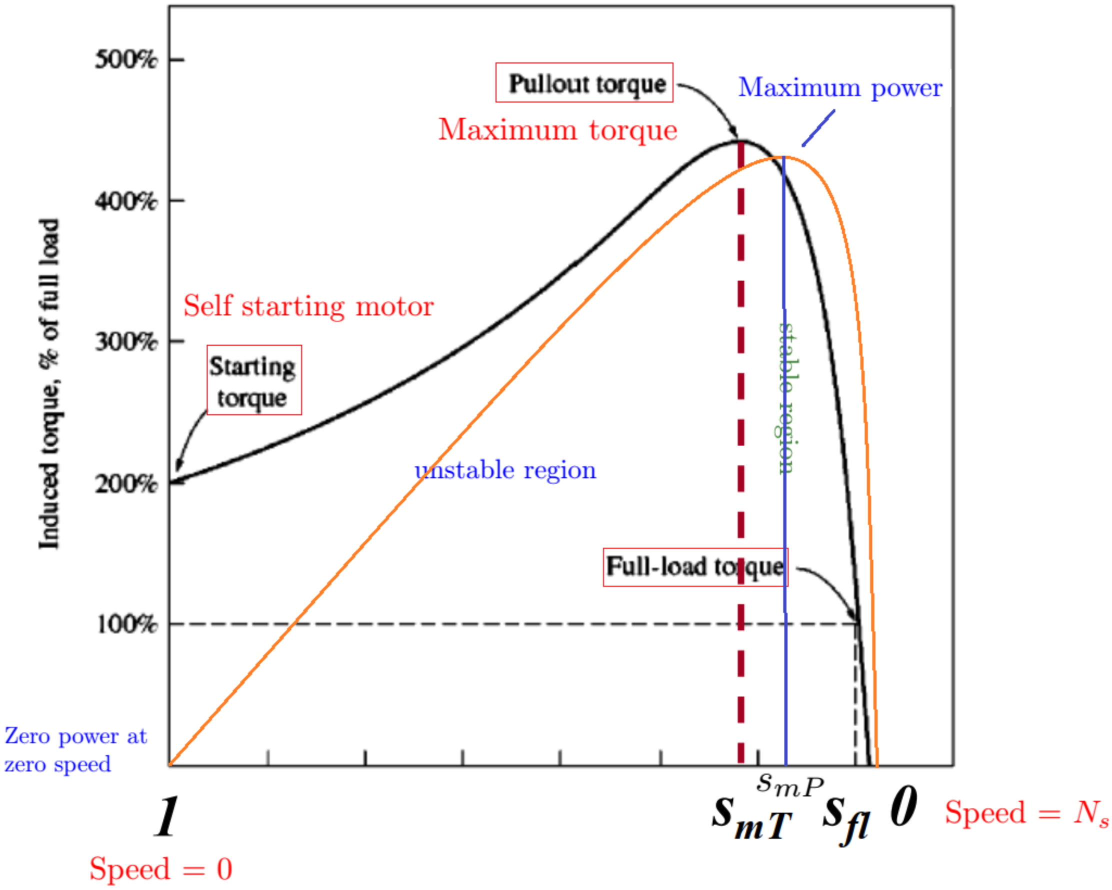

Figure 1

Insert the values obtained from 1, 2, and 3 on the graph on your own

5. If if an external resistance equals to the rotor resistance is added to each phase of the rotor, calculate the new values of the motor starting and maximum torques, and speed at maximum torque. Also calculate the starting current. Find the maximum developed power and the speed at which occurs

   Same steps, but whenever you see R_2, replace it with $R_2 + R_{\text{external}}$ and solve

   Note : Maximum torque shouldn't change if you change R_2.