EEP 225 Electric Machines Cheat Sheet [1]
Synchronous Machines

1 3 Phase Formulas

In Y circuits:

$$V_{\text{line}} = \sqrt{3}V_{\text{phase}} \qquad I_{\text{line}} = I_{\text{phase}} \tag{1}$$

In Δ circuits:

$$V_{\text{line}} = V_{\text{phase}} \qquad I_{\text{line}} = \sqrt{3}I_{\text{phase}} \tag{2}$$

Power:

$$P = \sqrt{3}\, V_{\text{line}}\, I_{\text{line}}\, \cos(\theta) \tag{3}$$

$$P = 3\, V_{\text{phase}}\, I_{\text{phase}}\, \cos(\theta) \tag{4}$$

2 Stator

• Made of thin lamination of highly permeable steel (to reduce eddy current).
• Inside the stator there are slots to accommodate armature conductors.
• Armature conductors are symmetrically arranged to form a balanced polyphase (3 phase) winding.

3 Rotor

The field system is excited by low voltage DC supply (about 220V) through brushes pressed into two slip rings. (Remember that synchronous machines are inverted, the field system rotates unlike DC machines) The rotor has two types:
• **Round pole rotor (Cylindrical rotor)**: used in 2 or 4 poles high speed turbo generator because it is mechanically stable
• **Salient pole rotor:** less mechanically stable, used in low speed applications

4 Brushless Exciter

How to get red of the brushes and slip rings?
Use a brushless exciter → small AC generator, the output of that generator is rectified to DC by a full bridge rectifier (diodes), then it is fed into the field circuit

5 Relation Between Speed and Frequency

$$N_s = 120 \times \frac{f_s}{P} \tag{5}$$

N_s : Speed of the rotor (synchronous speed) in rpm.
f_s Frequency of generated emf in Hz.
P Number of poles.

the rotational speed N_s is **synchronized** with the line frequency f_s. That's why we call it synchronous machines.

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6 Induced EMF

$$E_{\text{ph}} = 4.44 f\, \phi\, N_{\text{ph}}\, K_w \tag{6}$$

$$K_w = K_d \times K_p \tag{7}$$

K_w:Winding factor.
K_d:Distribution factor.
K_p:Pitch factor.

$$N_{\text{ph}} = \frac{Z}{2 \times \text{number of phases}} \tag{8}$$

N_{ph} : number of armature turns per phase.
Z : Number of armature conductors.

$$\frac{E_2}{E_1} = \frac{f_2}{f_1} \times \frac{I_{f\,2}}{I_{f\,1}} \tag{9}$$

$\boxed{\text{1. Q}}$ — Two direct coupled alternators running at 300 rpm having total number of poles of 12 and 16 respectively. Find the frequency of the generated emf of each machine.

A — How to solve?
Since the two machines are direct coupled → $N_1 = N_2$

$$P_1 = 12 \quad P_2 = 16 \quad N_1 = N_2 = 300 \text{ rpm}$$

Use $f = (PN)/120$ from Equation 5 ✓

7 Distribution Factor K_d

$$K_d = \frac{\sin\left(\frac{q\gamma}{2}\right)}{q\frac{\gamma}{2}} \tag{10}$$

$$K_d < 1$$

$$q = S \times \frac{1}{P} \times \frac{1}{n} \tag{11}$$

q : number of slots per pole per phase.

$$\gamma = \frac{180 \times P}{S} \tag{12}$$

γ : slot angle, the displacement between slots

8 Pitch (Chording) Factor

Sometimes the coil span is made less than 180° by angle α which is called the chording angle.
Why? to save copper and to improve the waveform of the emf by reducing the the distorting harmonics

$$K_P = \cos\left(\frac{\alpha}{2}\right) \tag{13}$$

For full-pitch coil (180°), $\alpha = 0$ and then $K_p = 1$

$\boxed{\text{2. Q}}$ — 3 phase, 16-pole, Y-connected alternator has 144 slots with 10 conductors per slot. The coil span is 150°. Find the phase and line induced emfs if the flux is 30 mWb and the machine runs at 375 rpm.

A — How to solve?

$$S = 144 \qquad P = 16 \qquad N_s = 375$$
$$Z = 144 \times 10 = 1440 \qquad \alpha = 180^\circ - 150^\circ = 30^\circ \qquad \phi = 0.03$$

$$E_{\text{ph}} = 4.44 \underbrace{f}_{(PN_s)/120} \underbrace{\phi}_{\text{given}} \underbrace{N_{ph}}_{Z/2n} \underbrace{K_d}_{\frac{\sin\left(\frac{q\gamma}{2}\right)}{q\frac{\gamma}{2}}} \times \underbrace{K_p}_{\cos\left(\frac{\alpha}{2}\right)} \checkmark$$

$$E_L = \sqrt{3} \times E_{\text{ph}}$$

9 Effect of Flux Density Harmonics

The generated voltage is highly distorted due to the existing harmonics. The goal is to eliminate mainly 3rd and 5th harmonics. If h is the order of harmonics.

$$\gamma \rightarrow h\gamma \tag{14}$$
$$\alpha \rightarrow h\alpha \tag{15}$$
$$\tag{16}$$

$$K_{dh} = \frac{\sin\left(\frac{q\,h\,\gamma}{2}\right)}{q\,\sin\left(\frac{h\,\gamma}{2}\right)} \tag{17}$$

$$K_{ph} = \cos\left(\frac{h\,\alpha}{2}\right) \tag{18}$$

$$f_{h=3} = 3\,f \tag{19}$$

if it is requires to reduce the 3rd harmonic

$$E_{h=3} = 4.44\,f_{h=3}\,\phi_{h=3}\,N_{ph}\,K_{d\,h=3}\,K_{p\,h=3} \tag{20}$$

The only term that can be zero is the pitch factor K_{p3}

$$K_{p3} = 0$$
$$\cos\left(\frac{3\alpha}{2}\right) = 0$$
$$1.5\,\alpha = 90^\circ$$
$$\alpha = 60^\circ$$

Therefore, in order to eliminate the 3rd harmonic, armature coils have to be chorded by 1/3 of the pole pitch.

$\boxed{\text{3. Q}}$ — For the alternator in $\boxed{\text{2. Q}}$, if the 3rd harmonic flux is 2 mWb. Find the value of the generated emf per phase, and the line-line value.

A — How to solve?

Recalculate K_d and K_p, replace each γ and α with $h\gamma$ and $h\alpha$ where $h = 3$

replace f with hf where $h = 3$

substitute in

$$E_{h=3} = 4.44\,f_{h=3}\,\phi_{h=3}\,N_{ph}\,K_{d\,h=3}\,K_{p\,h=3}$$

How to add RMS voltages (main emf and 3rd harmonic emf)?

$$E_{\text{ph}} = E_1 + E_2 \text{ ✗}$$

$$E_{\text{ph}} = \sqrt{E_1^2 + E_2^2} \text{ ✓✓}$$

10 Equivalent Circuit

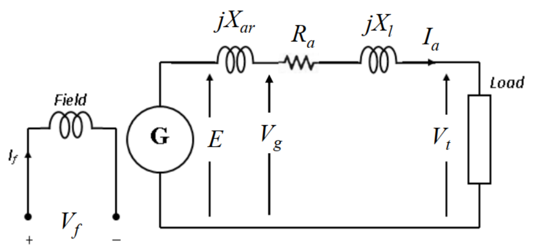

Figure 1: Equivalent circuit of synchronous generator per phase, X_{ar} is the how we represent the armature reaction as voltage drop(it is physical reactance, not real), note that we can't connect the field circuit (DC) with the generator circuit (AC).

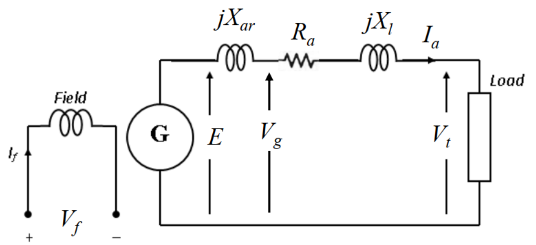

Figure 2: Equivalent circuit of synchronous generator per phase.

$$X_s = X_{\text{ar}} + X_l \tag{21}$$
$$Z_s = R_a + j\,X_s \tag{22}$$

R_a : Armature resistance
X_s : Synchronous reactance
Z_s : Synchronous impedance

$$\overline{E} = \overline{V_t} + \overline{I_a} \cdot \overline{Z_s} \tag{23}$$

11 Phasor Diagram

$$\overline{E} = \overline{V_t} + \overline{I_a} \cdot \overline{Z_s}$$
$$E\angle\delta = V_t\angle 0 + I_a \times Z_s\angle\theta$$
$$E\angle\delta = V_t\angle 0 + I_a \times (R_a + jX_s)$$

δ : Load angle (Power angle).

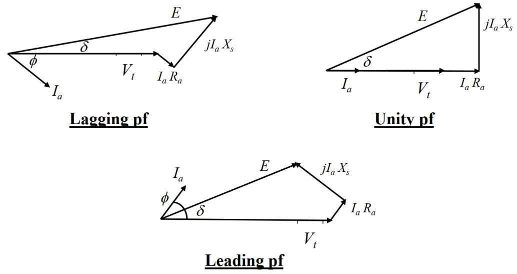

Figure 3: Phasor diagram of synchronous generator

12 Voltage Regulation

$$\text{Voltage regulation} = \frac{|V_{\text{no load}} - V_{\text{load}}|}{V_{\text{load}}} = \frac{|E - V_{\text{load}}|}{V_{\text{load}}}$$

In the case of **lagging** power factor (inductive loads): $E > V_t$, so voltage regulation is **always** positive

In the case of **leading** power factor (capacitive loads) : may be

- $E > V_t$, voltage regulation positive.
- $E = V_t$, voltage regulation **zero**.
- $E < V_t$, voltage regulation **negative**.

Should be kept small (5% ∼ 30%).

13 Measure Synchronous Model Parameters

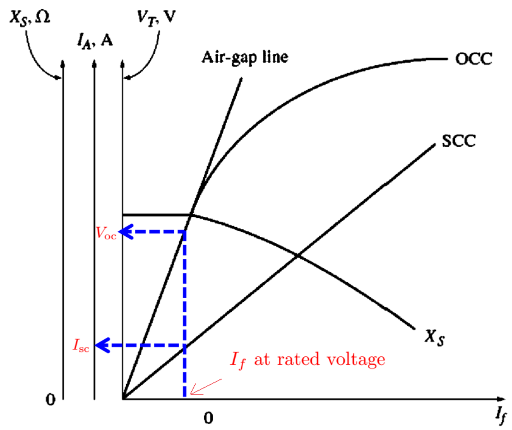

Figure 4: At rated voltage, divide V_{oc} and T_{sc} to get Z_s

Z_s : synchronous reactance

$$Z_s = \frac{V_{\text{open circuit test}}}{I_{\text{short circuit test}}} \tag{24}$$

$$X_s = \sqrt{Z_s^2 + R_a^2} \tag{25}$$

Open circuit test: The generator works at rated speed and disconnected from load, then set field current I_f to zero and increase gradually and measure terminal voltage V_t

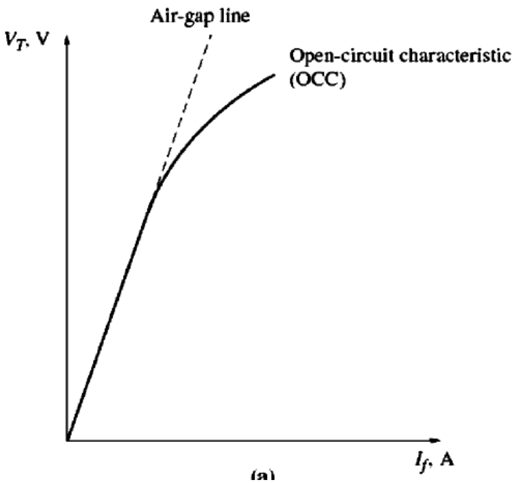

Figure 5: Open circuit characteristics (OCC), at no load $I_a = 0$ so $E = V_t$

Short circuit test: Disconnected the generator from load, then set field current I_f to zero and increase gradually and measure armature current I_a.

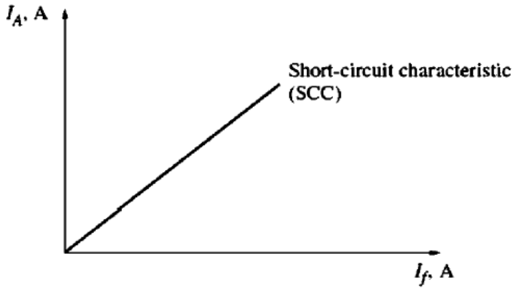

Figure 6: Short circuit characteristics (OCC), plot I_f vs I_a

For large generators with ratings higher than 100 kVA.

$$R_a \approx 0 \qquad Z_s \approx X_s$$

14 Power Flow, Losses and Efficiency

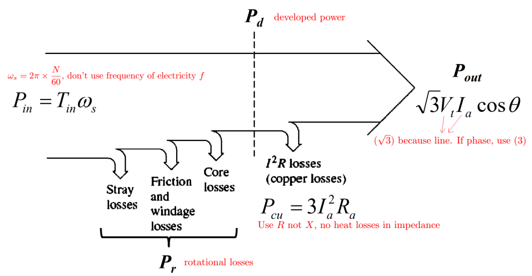

Figure 7

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{r}} + P_{\text{cu}}} \tag{26}$$

$\boxed{\text{4. Q}}$ — A 100 kVA, 3000 V, 50 Hz, 3 phase, Y connected alternator has an effective armature resistance = 0.2 Ω. The field current of 40 A produces short-circuit current of 200 A and an open-circuit emf of 1040 V. Calculate the full-load voltage regulation at 0.8 pf lagging, and 0.8 pf leading.

A — How to solve?

Note : In any 3 phase problem, all given data are **line values**

You have $V_{\text{open circuit}}$ and $I_{\text{short circuit}}$

$$Z_s \frac{V_{\text{open circuit}}}{I_{\text{short circuit}}}$$

calculate $|I|$ from

$$P = \sqrt{3} V_t I_a \cos(\theta)$$

Calculate $\underline{/\theta}$ of the current from

$$\theta = \cos^{-1}(\text{pf})$$

If lagging pf → negative θ.
If leading pf → positive θ.

$$V_{\text{phase}} = \frac{V_{\text{line}}}{\sqrt{3}}$$

substitute in

$$E\underline{/\delta} = V_t\underline{/0} + I_a \times Z_s\underline{/\theta}$$

Voltage regulation

$$\text{Voltage regulation} = \frac{|E - V_{\text{load}}|}{V_{\text{load}}}$$

15 Power Angle Relationship and Maximum Power

Consider large synchronous, R_a can be ignored so

$$\overline{E} = \overline{V_t} + \overline{I_a} \times j\,\overline{X_s}$$

From the corresponding phasor diagram

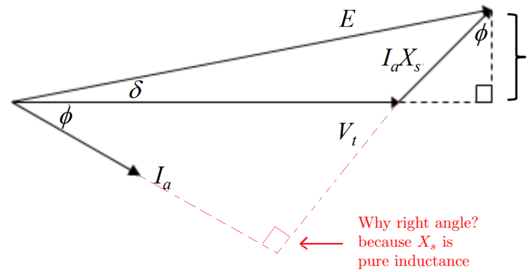

Figure 8

$$P_{\text{out}} = \frac{3EV_t}{X_s}\sin(\delta) \tag{27}$$

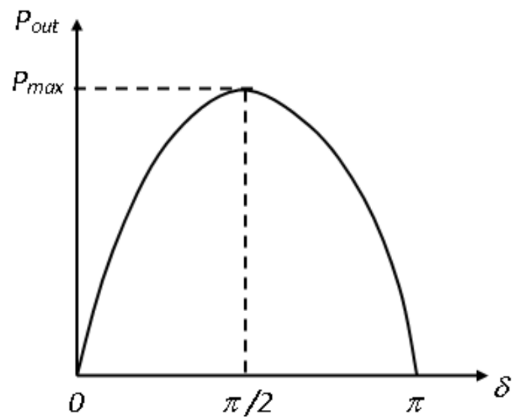

Figure 9: Plot of the swing equation

The **maximum output power** occurs at $\delta = \frac{\pi}{2}$ and is given by :

$$P_{\text{out},\delta=0.5\pi} = 3\frac{|E||V_t|}{X_s} \tag{28}$$

$\boxed{\text{5. Q}}$ — A 1000 V, 120 kVA, Δ connected, three-phase, synchronous generator has a synchronous reactance per phase of 5Ω and a neglected armature resistance. It supplies the rated load at 0.9 pf lagging. Determine

- The generated voltage.
- The power angle.
- The power at generated voltage.
- The maximum developed power and the corresponding power angle.

A — How to solve?

calculate $|I_a|$ from

$$P = \sqrt{3}\, V_{\text{line}}\, I_{\text{line}}\, \cos(\theta)$$

Calculate $\angle\theta$ of the current from

$$\theta = \cos^{-1}(\text{pf})$$

with negative sign because power factor is lagging.

Calculate the E from

$$E\angle\delta = V_t\angle 0 + I_a \times Z_s\angle\theta$$

Calculate the power from

$$P = 3\frac{|E||V_t|}{X_s}\sin(\delta)$$

Substitute with $\delta = 90°$ to get the maximum power.

16 Electrical Load Diagram

Check lecture 8 notes.

17 Synchronous Motor Equivalent circuit

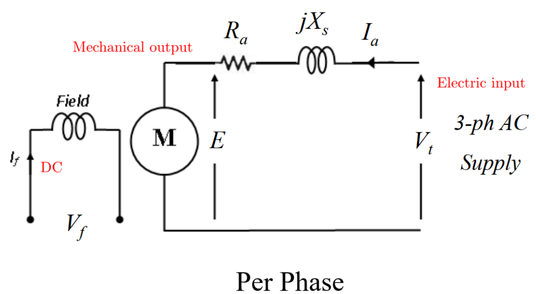

Figure 10: Equivalent circuit of synchronous motor

$$\begin{aligned}
\overline{E} &= \overline{V_t} - \overline{I_a} \times \overline{Z_s} \\
E\angle\delta &= V_t\angle 0 - I_a\angle\phi \times Z_s\angle\theta \\
E\angle\delta &= V_t\angle 0 - I_a\angle\phi \times (R_a + jX_s)
\end{aligned} \tag{29}$$

(Notice that we use negative sign in motor, positive sign in generator)

18 Phasor Diagram of Synchronous Motors

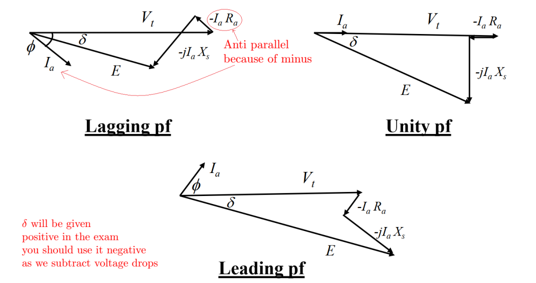

Figure 11: Phasor diagram of synchronous motors

Synchronous motors are the **only** motors in which power factor can be controlled (by controlling excitation E), all other motors have lagging (inductive) power factor.

Therefore, synchronous motors can be over excited $E \uparrow\uparrow$ for power factor correction.

19 Power Flow, Losses and Efficiency

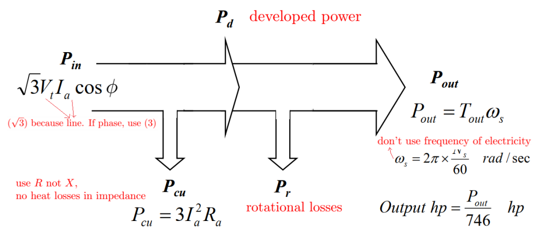

Figure 12: Power flow in synchronous motor

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{P_{\text{in}} - P_{\text{cu}} - P_{\text{r}}}{P_{\text{in}}} \tag{30}$$

We neglect armature resistance R_a, so the copper loss becomes zero, therefore developed power equals input power $P_d = P_{\text{in}}$.

So, power angle relationship of synchronous generator will be valid for the synchronous motor

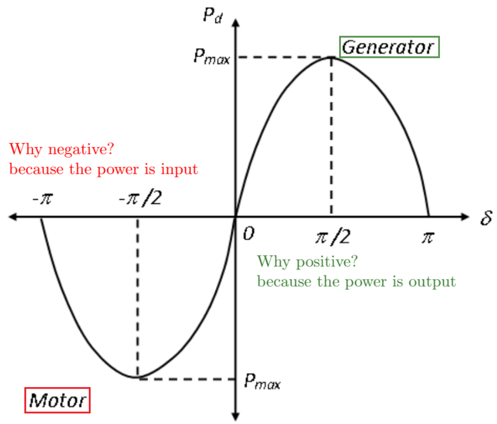

Figure 13: Complete power angle relationship for both synchronous motor and generator

The **maximum (developed = input) power** occurs at $\delta = \frac{\pi}{2}$ and is given by :

$$P_{\text{d max}} = 3\frac{|E||V_t|}{X_s} \tag{31}$$

To get the **maximum developed torque** occurs at $\delta = \frac{\pi}{2}$ and is given by:

$$T_{\text{d max}} = \frac{P_{\text{d max}}}{\omega_s} = 3\frac{|E||V_t|}{\omega_s X_s} \tag{32}$$

20 Synchronous Motor Torque-Speed Characteristics

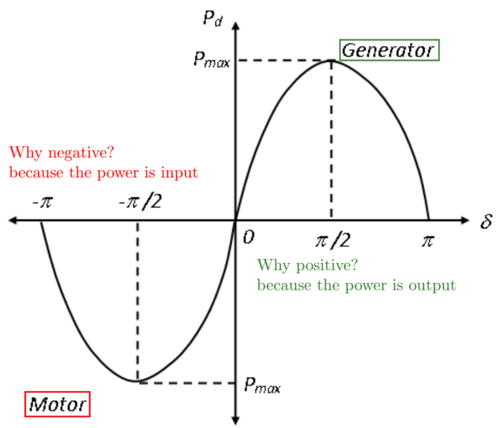

Figure 14: Synchronous motor torque-speed characteristics

Why it is constant?
Speed of the motor is synchronized (remember why it is called synchronous) with the supply frequency regardless of the load (remember Equation 5)

21 Speed Regulation

Analogous to voltage regulation in generators, in motors, speed regulation is the change in speed when the load changes

$$\text{Speed regulation} = \frac{\text{speed at no load - speed at full load}}{\text{speed at full load}} = \frac{N_{\text{nl}} - N_{\text{fl}}}{N_{\text{fl}}} \tag{33}$$

In synchronous motor, speed at no load = speed at full load, so speed regulation is **zero**.
In other motors, speed at no load > speed at full load, so speed regulation is positive.
In differential motor (unstable DC motor), speed at no load < speed at full, so speed regulation is negative.

22 V-Curves of Synchronous Motors

The relationship between the field current I_f and armature current I_a for different values of load power

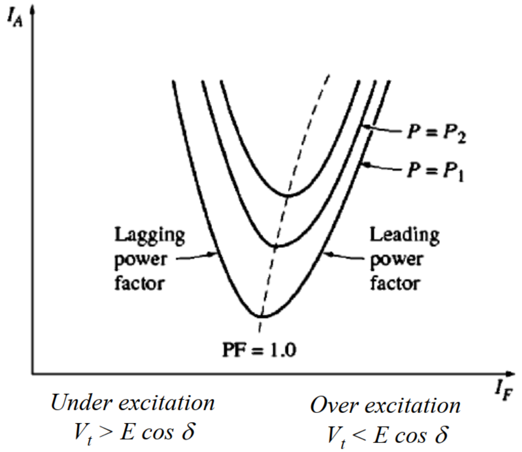

Figure 15: V-Curves of Synchronous Motors

From any phasor diagram, how to determine if it is leading, lagging or unity power factor?
if $E\cos(\delta) < V_t \rightarrow$ lagging power factor
if $E\cos(\delta) = V_t \rightarrow$ unity power factor
if $E\cos(\delta) > V_t \rightarrow$ leading power factor
which means that in synchronous motor, power factor can be controlled by controlling E, which is controlled by controlling I_f.

23 Synchronous Motors As Capacitor

Synchronous motors can act as capacitors if it operates at leading power factor region, motor then is called **synchronous condenser** or **synchronous capacitor**
Used for **power factor correction**

24 Starting of Synchronous Motor

Synchronous Motors are **not self starting motor**
To start the motor, manufacture the motor with **damped bars**, energize it with 3 phase supply without energizing the field circuit. This will make the motor start as squirrel cage induction motor, when the speed reaches near the synchronous speed, energize the field circuit and it will reach the synchronous speed.
If there are no damped bars, we need **auxiliary motor**(another motor to start it)

$\boxed{\text{6. Q}}$ — A 230 V, 60 Hz, 4-pole, Y-connected synchronous motor has an armature resistance and synchronous reactance of 0.5Ω and 5Ω per phase respectively. If the motor takes an input power of 7 kW at 0.707 pf leading and the rotational loss is 100 W. Determine:
1. The generated emf and the power angle.
2. The output hp (horse power), and the output torque.
3. The efficiency.

A — How to solve?
Given line values, calculate the phase value
get I_a from

$$P = 3\, V_{\text{ph}}\, I_{\text{ph}}\, \cos(\theta) = 3\, V_{\text{ph}}\, I_{\text{ph}} \times \text{pf}$$

Get $|E|$ and $\angle\delta$ (power angle) from

$$E\angle\delta = V_t\angle 0 - I_a\angle\phi \times (R_a + jX_s)$$

$$\text{output power} = \text{input power} - \text{rotational losses} - \underbrace{\text{copper losses}}_{3I_a^2 R_a}$$

$$\text{hp}\frac{\text{Watt}}{hp}$$

$$T_{\text{out}} = \frac{P_{\text{our}}}{\omega_s}$$

$$\omega_s = 2\pi\frac{N_s}{60} \qquad N_s = 120 \times \frac{f}{P}$$

$$\text{Efficiency} = \frac{P_{\text{out}}}{P_{\text{in}}}$$

25 Advantages

• The output current are led from fixed terminals on the stator to the load without need to pass it through brushes.
• It is easier to insulate stationary armature winding for high AC voltages
• Slip rings and brushes are transferred to the low-voltage, low-power DC field circuit, which can be easily insulated.

26 Disadvantages

• Synchronous motor's cost is higher than the cost of other motors at the same rating.
• Synchronous motor requires two supplies (DC and AC)