Title

Optimisation of waiting at Shake Shack fast food restaurant at Jewel Changi Airport through application of Queuing Theory

Introduction

Earlier this year in April, Shake Shack, an American fast casual restaurant chain based in New York City finally opened its first outlet in Singapore at Jewel Changi Airport. Shake Shack is one of the titans in the fast-food industry, expanding to more than 70 locations worldwide, including Dubai and South Korea.[1] Since day one of business, snaking queues can be seen outside Shake Shack. Due to the overwhelming anticipation from its debut, queuing has become the major hindrance to trying out Shake Shack. According to the Straits Times, the queue saw an estimated average of 200 people through the day with an average waiting time of 1.5 hours.[2] When it first opened, the first customer had to start queuing about 6 hours in advance.[3]

In a fast-paced country such as Singapore, wasting time on queuing has a huge impact on Singaporeans' productivity results in a huge loss with such huge waiting times; The customers could be spending the time on more productive activities such as working or studying. Since waiting time of 10 minutes is reasonable for queuing compared to 1.5 hours, **this mathematical exploration seeks to optimize the number of counters at Shake Shack so that the average waiting time is 10 minutes and below by modelling the waiting times, queue lengths and idle times of counters.**

I will be analyzing the behavior of the queue which is characterized by the arrival process, the service process, the number of servers in the system, the queue discipline, the capacity of the queue, the number of servers in the system, the queueing discipline, the capacity of the queue and the size of the client population. These six parameters are part of a standard system known as Kendall's notation.[4] After that, Chi Square Goodness of Fit Test will be used to test whether the data follows the Poisson distribution, so that queuing theory can be applied to the collected set of data.

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[1] Cna. (2018, October 10). Burger chain Shake Shack to open first Singapore outlet at Jewel Changi Airport. Retrieved October 6, 2019, from https://cnalifestyle.channelnewsasia.com/dining/shake-shack-first-singapore-outlet-at-jewel-changi-airport-10809810.

[2] Loh, D., & Quek, E. (2019, April 20). Long lines for taste of Shake Shack in Singapore. Retrieved October 6, 2019, from https://www.straitstimes.com/lifestyle/food/long-lines-for-taste-of-shake-shack-in-singapore.

[3] Co, C. (2019, April 27). Foodies make a beeline for popular eateries as Jewel Changi Airport opens. Retrieved October 6, 2019, from https://www.channelnewsasia.com/news/singapore/foodies-queue-shake-shack-jewel-changi-airport-11452930.

[4] Teknomo, K. (n.d.). Classification of Queuing Model using Kendal Notation. Retrieved October 6, 2019, from https://people.revoledu.com/kardi/tutorial/Queuing/Kendall-Notation.html.

Rationale

The huge waiting times could be due to the fact that the restaurant could be too understaffed to the overwhelming number of customers, or there are not enough counters to serve the customers. This model can aid Shake Shack in improving system performance by lowering waiting time and queuing time and increasing number of customers that can be served per unit time. The restaurant will then be able to deduce whether to increase the number of counters. With shorter waiting time, the company can profit more with more efficient service.

Methodology

Introduction to the Queuing Theory

Queuing theory is a mathematical study which models the act of waiting in lines.[5] The model allows us to predict the queue lengths and waiting time. The nature of the process of queuing theory is stochastic; it consists of the arrival rates of customers, the length of time taken for each customer to be served and lastly the service rates.

Queuing theory is vital for the firms to achieve a balanced system that serve the customers efficiently without having too much cost to be sustainable. Queuing theory also aids to the development of more efficient price mechanism and staffing solutions.[6]

A queue in this exploration will be defined as a line of people waiting their turn to receive the service. There are three main components of a queuing model, which are:

1. Arrival Process
2. Service Mechanism
3. Queue discipline[7]

Firstly, arrival process analyses the pattern of customers' arrival, for example singly or in groups, the interval time distribution and whether nature of customer population is infinite.[8] Secondly, service

[5] Revolvy, L. L. C. (n.d.). "Queueing theory" on Revolvy.com. Retrieved October 6, 2019, from https://www.revolvy.com/page/Queueing-theory.
[6] Kenton, W. (2019, July 10). Queuing Theory and the Business of Waiting in Line. Retrieved October 6, 2019, from https://www.investopedia.com/terms/q/queuing-theory.asp.

[7] Beasley, J. E. (n.d.). OR-Notes. Retrieved October 6, 2019, from http://people.brunel.ac.uk/~mastjjb/jeb/or/queue.html.
[8] ibid

mechanism analyses whether the servers are in series or in parallel.[9] Lastly, queue discipline is a description of the order of how the customers are served.[10] There are two major categories – First In First Out (FIFO) and Last In First Out (LIFO). The order of queuing discipline is usually FIFO[11] which can maintain the fairness among the customers. For LIFO, last customers are served first.[12] For the case of fast food restaurants, customer who comes first usually are served first. Hence, queues at Shake Shack follows FIFO.

Identifying Peak Hour

I stationed myself outside Shake Shack for 2 hours between 5.30pm and 7.30pm, which is during dinnertime for 2 days to find out when there are the greatest number of customers. Number of customers joining the queue per 20 minutes has been recorded:

Time	1800	1820	1840	1900	1920	1940	2000
No. of customers joining the queue	43	65	86	79	78	64	50

From the observation, the greatest number of customers (229) joined the queue in between 6.40pm and 7.40pm. I will thus be using this time period to be defined as 'peak hour'.

Kendall Notation

Aforementioned components of a queuing model can be represented concisely by Kendall's Notation. His full notation is extended to A/S/c/K/N/D,[13] where **A** is the time between arrivals to the queue, **S** is the service time distribution, **c** is the number of servers, **K** is the capacity of the queue, **N** is the size of the population of jobs to be served, and **D** is the queuing discipline.

[9] ibid

[10] panel, A. links open overlay, & Publisher SummaryThis chapter focuses on queuing theory models. Queuing theory (the theory of waiting lines) is a discipline of operational research. (2008, May 5). 8 Quenung Theory Models. Retrieved October 6, 2019, from https://www.sciencedirect.com/science/article/pii/S0167564808709274.

[11] This is also known as first come first serve

[12] Teknomo, K. (n.d.). Queuing Discipline. Retrieved October 6, 2019, from https://people.revoledu.com/kardi/tutorial/Queuing/Queuing-Discipline.html.

[13] Revolvy, L. L. C. (n.d.). "Kendall's notation" on Revolvy.com. Retrieved October 6, 2019, from https://www.revolvy.com/page/Kendall's-notation.

Assumption: This exploration will use the abbreviated Kendall notation (A/S/c) as the observed queue is D = FIFO and the customer population size and the capacity of the queue are assumed to be infinite. ($K \to \infty$, $N \to \infty$)

M/M/c

The arrival pattern of customers is random in the case of Shake Shack, as customers arrive without any dependence on others' choices. Thus, the arrival process, (A) will be represented as 'M', Markovian or memoryless. Likewise, service time distribution (S) will be random in the case of Shake Shack therefore will also be represented as 'M'. Henceforth, this exploration will focus on the M/M/c process, a single queuing node where there is a single queue wherein there are more servers. This exploration will assume that the service time follows cumulative exponential distribution if the arrival of the customers is verified as a Poisson process.

M/M/c model is also known as Erlang-C model, and it is a multi-server queuing model. It is a stochastic process, which is a sequence of random variables that represent a system changing over time. Its state space is the set {0, 1, 2, 3, …} where the value corresponds to the number of customers in the system, including any currently in service.

Verifying the arrival of the customers as a Poisson Process

During the aforementioned peak hour, I have also recorded the number of customers observed in a minute and its frequency (Refer to Appendix A):

Table 1: Observed frequency of the number of customers joining the queue in a minute

Number of customers observed joining the queue in a minute (x)	Number of such minutes (O)
0	2
1	5
2	7
3	10
4	14
5	9
6	5
7	3
8	3
9	2

To check whether Poisson distribution is followed for this set of data, a **Chi Square Goodness of Fit** test is used. Firstly, average arrival rate is derived as it is required for the calculation the theoretical probability of Poisson distribution.

$$\lambda = \frac{\sum(O \times x)}{60} = \frac{243}{60} = 4.05$$

λ = Average arrival rate
O = number of such minutes
x= number of customers entering the queue per minute

Null hypothesis (H_0) is that the distribution follows the Poisson process (parameter 4.05)
Alternate hypothesis (H_1) is that the distribution does not follow the Poisson process.

With λ = 4.05, the probability of a certain number of customers in a given time period is:

$$P(x) = e^{-4.05} \frac{4.05^x}{x!}$$

Table 2: Probability of Poisson Distribution

Number of customers observed joining the queue in a minute (x)	Number of such minutes (O)	Poisson Distribution Probability
0	2	0.017422375
1	5	0.070560617
2	7	0.14288525
3	10	0.192895088
4	14	0.195306276
5	9	0.158198084
6	5	0.106783706
7	3	0.061782002
8	3	0.031277138
9	2	0.014074712

For Chi-square distribution, observed values below 5 are considered invalid, thus number of customers observed joining the queue in 8 minutes and above has been grouped together, then expected number of such minutes is calculated:

Table 3: Observed and Expected Frequencies

Number of Customers observed joining the queue in a minute (X)	Observed Number of such minutes (O)	Expected Number of such minutes (K)	Poisson Distribution Probability
0	2	1.045342478	0.017422375
1	5	4.233637037	0.070560617
2	7	8.573115001	0.14288525
3	10	11.57370525	0.192895088
4	14	11.71837657	0.195306276
5	9	9.491885019	0.158198084
6	5	6.407022388	0.106783706
7	3	3.706920096	0.061782002
8≤	5	2.721111033	0.045351851

Above observed and expected values will be used to calculate Chi Square (χ^2):

Table 4: Chi Square Calculation

Number of Customers observed joining the queue in a minute (X)	Observed Number of such minutes (O)	Expected Number of such minutes (K)	$\frac{(O - T)^2}{T}$
0	2	1.045342478	0.8718396157
1	5	4.233637037	0.1387252110
2	7	8.573115001	0.2886571341
3	10	11.57370525	0.2139805845
4	14	11.71837657	0.4442428902
5	9	9.491885019	0.0254902869
6	5	6.407022388	0.3089909603
7	3	3.706920096	0.1348116520
8≤	5	2.721111033	1.9085347350

$$\chi^2 = \sum \frac{(O - T)^2}{T} = 4.33527307$$

Degree of Freedom

Degree of freedom are utilized to determine if a certain null hypothesis can be rejected according to the total number of variables and samples within the experiment.[14] The null hypothesis for this exploration is that the distribution follows a Poisson distribution with a parameter of the same value as average arrival rate. The formula to calculate the degree of freedom is as follows:

$$Degree\ of\ Freedom\ =\ Number\ of\ Classes\ -\ 1\ - Number\ of\ Parameters$$

Degree of Freedom for this case will be:

$$DF\ =\ 9\ -\ 1\ -\ 1\ =\ 7$$

Critical value can be found with the degree of freedom and the level of significance.[15] Critical value aids in finding out whether to reject the null hypothesis; if the calculated Chi Square value is bigger than the critical value, the data does not fit into the model, the null hypothesis is rejected. On the other hand, if the Chi square value is smaller than the critical value, the data fits into the model and the null hypothesis does not have to get rejected.[16]

This exploration will use level of significance of 0.05,[17] which is a standard choice. With degree of freedom value 7, the critical value is **14.07** (Refer to Appendix B).

Seeing as

$$\chi^2\ <\ 14.07$$

Null hypothesis (H_0) is true; the queuing model of Shake Shack follows a Poisson distribution.

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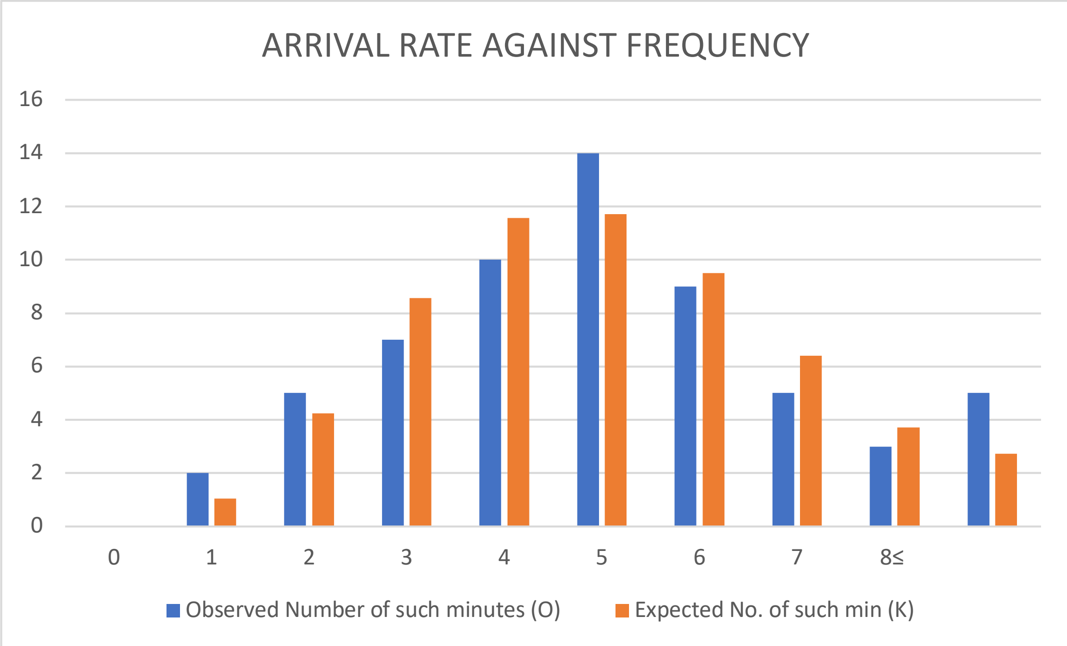
[14] 'Degree of Freedom | Mathematics and Statistics', Encyclopedia Britannica, accessed 8 January 2020, https://www.britannica.com/science/degree-of-freedom-mathematics-and-statistics.
[15] Stat Trek. (n.d.). Retrieved October 9, 2019, from https://stattrek.com/chi-square-test/independence.aspx.
[16] https://mathbench.umd.edu/modules/statistical-tests_chisquare_advanced/page14.htm
[17] Stat Trek. (n.d.). Retrieved October 11, 2019, from https://stattrek.com/chi-square-test/independence.aspx.

Figure 1: Graph of arrival rate of customers against frequency



Analysis of the Service Time

To calculate the average service time, I counted the number of customers leaving the counter during the peak hour, 6.40pm to 7.40pm as mentioned above. It will be assumed that service rate for each counter will be the same and service time for each customer will be constant.

Number of customers who left the counter during peak	21 customers
Mean service rate (μ)	21/60 = 0.35 customers per minute

Finding the optimal number of counters

Utilization of the server refers to the fraction of time when the counter is serving the customers.[18] With the known mean arrival rate (λ) and mean service rate (μ), the utilization of the server (ρ) can now be calculated:

$$\rho = \frac{\lambda}{\mu c} = \frac{4.05}{(0.35)c}$$

c → number of servers

[18] Queue system server utilization. (2018, January 9). Retrieved October 13, 2019, from http://cpntools.org/2018/01/09/queue-system-server-utilization/.

Little's Law shows the relationship between the average number of customers in the queue and the average waiting time in the queue.[19]

$$L_q = \lambda Wq$$

L_q → Average number of customers in the queue

W_q → Average waiting time in the queue

λ → Average arrival rate

To find the average number of customers in the queue, the probability of having no customers in the system has to be expressed as follows:

$$P(x=0) = \left[\sum_{r=0}^{c-1} \frac{(c\rho)^r}{r!} + \frac{(c\rho)^c}{c!\,(1-\rho)}\right]^{-1}$$

Whereas L_q is:

$$L_q = \frac{P(x=0)(\frac{\lambda}{\mu})^c \rho}{c!\,(1-\rho)^2}$$

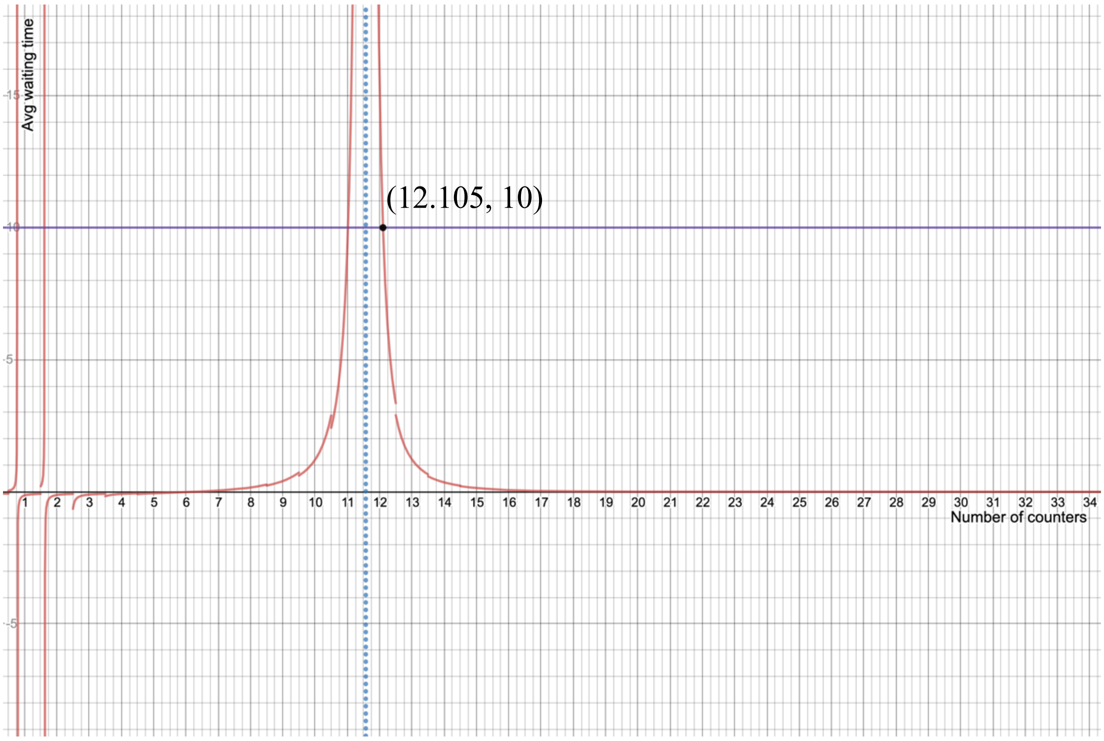
To plot a graph, aforementioned formulas have been combined to plot a graph of average waiting time in the queue against the number of servers:

$$W_q = \frac{L_q}{\lambda} = \frac{P(x=0)(\frac{\lambda}{\mu})^c \rho}{c!\,(1-\rho)^2 \lambda} = \frac{\left[\sum_{r=0}^{c-1} \frac{(c\rho)^r}{r!} + \frac{(c\rho)^c}{c!\,(1-\rho)}\right]^{-1} (\frac{\lambda}{\mu})^c \rho}{c!\,(1-\rho)^2 \lambda}$$

$$= \frac{\left[\sum_{r=0}^{c-1} \frac{(c\frac{4.05}{(0.35)c})^r}{r!} + \frac{(c\frac{4.05}{(0.35)c})^c}{c!\,(1-\frac{4.05}{(0.35)c})}\right]^{-1} (\frac{4.05}{0.35})^c \frac{4.05}{(0.35)c}}{c!\,(1-\frac{4.05}{(0.35)c})^2 4.05}, c \in \mathbb{Z}^+, c \neq \frac{4.05}{0.35}$$

[19] Notes on Little's Law. (2009). Retrieved October 13, 2019, from http://www.columbia.edu/~ks20/stochastic-I/stochastic-I-LL.pdf.

Figure 2: Graph of average waiting time (min) against number of counters



As the number of counters increase, the average waiting time for customers should decrease as the customers can be served at more counters. Thus, the domain of the graph to find the optimal number of counters will be $c > \frac{4.05}{0.35}$.

A line of equation y = 10 has been plotted so that we find the number of counters that will result in the average waiting time of 10 minutes. **Thus, the optimal number of counters at Shake Shack to have average waiting time of 10 minutes and below is 12.**

Evaluation

To verify the effectiveness of having 12 counters, a Monte Carlo simulation was run; it analyses the probability of different results using random set of values generated for modelling uncertainty of a system. This simulation will generate thousands of results, which are all possible outcomes. Mean arrival rate (λ) and mean service rate (μ) derived from the data I have collected were used and the service duration of each customer was randomised to simulate the exact condition of the queue. (Refer to Appendix C)

This simulation was run 10 times and the average individual waiting times were recorded:

Simulation	Average individual waiting time (min)
1	0.221
2	0.508
3	0.410
4	0.247
5	0.123
6	0.227
7	0.263
8	0.478
9	0.416
10	0.323

The average individual waiting time derived from the simulation is 0.322 min. Since it is well below 10 min, having 12 counters indeed is effective in reducing the waiting time for Shake Shack's customer.

However, from the perspective of management, there were instances of having more than 2 counters available despite the overwhelming number of customers during the peak hour. Thus, whilst having 12 counters may be optimal theoretically, it will not be practical as having free counters indicate cost inefficiency. Other renowned fast-food restaurants such as McDonald's usually have between 2 and 4 counters and soon enough, the demand for Shake Shack will decrease and the number of customers will be around other famous restaurants' average number of customers, concluding the necessity of having 12 counters to suit the high demand Shake Shack currently has.

Conclusion

This exploration has successfully found the solution to the aim, which was to find the optimal number of counters at Shake Shack, which is 12 counters. However, there are various limitations that make this number only theoretically possible. The cost of building new counters have not been taken into account of, and the increase in the number of counters will be pointless as some point if Shake Shack is understaffed.

Furthermore, there were many assumptions to this exploration which leads to uncertainties when applied to practical use; in reality, there is a chance that the queue will not follow the Poisson distribution, and likewise, the service rate might not be exponentially distributed. Thus, there is no distribution that can exactly depict how the service and arrival processes are distributed. The knowledge from this queuing model can be useful and important for management of Shake Shack, but due to its limitations, they should only be used as a guide to analyze the impact of increasing the number of customers and much larger sample pool size should be used jointly with other relevant information such as the variance in the cost of production when variables such as the number of counters are changed.

Instead of increasing the number of counters, self-checkout machines can be installed. In fact, many fast food restaurants such as McDonald's have self-checkout machines to serve most customers at given time. Furthermore, Shake Shack does not have to hire more employees which is cost-saving.

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17. Teknomo, K. (n.d.). Queuing Discipline. Retrieved October 6, 2019, from https://people.revoledu.com/kardi/tutorial/Queuing/Queuing-Discipline.html.

Appendix

Appendix A

Time	Number of custoemrs joining the queue	Time	Number of custoemrs joining the queue
1840	2	1910	4
1841	4	1911	3
1842	5	1912	1
1843	8	1913	4
1844	5	1914	1
1845	3	1915	4
1846	9	1916	4
1847	3	1917	7
1848	5	1918	4
1849	3	1919	1
1850	8	1920	4
1851	2	1921	4
1852	4	1922	5
1853	3	1923	2
1854	5	1924	1
1855	3	1925	7
1856	5	1926	6
1857	5	1927	3
1858	3	1928	2
1859	1	1929	4
1900	2	1930	4
1901	7	1931	4
1902	6	1932	5
1903	2	1933	0
1904	2	1934	6
1905	3	1935	4
1906	3	1936	5
1907	9	1937	6
1908	8	1938	0
1909	4	1939	6

Appendix B

Percentage Points of the Chi-Square Distribution

Degrees of Freedom	Probability of a larger value of x^2								
	0.99	**0.95**	**0.90**	**0.75**	**0.50**	**0.25**	**0.10**	**0.05**	**0.01**
1	0.000	0.004	0.016	0.102	0.455	1.32	2.71	3.84	6.63
2	0.020	0.103	0.211	0.575	1.386	2.77	4.61	5.99	9.21
3	0.115	0.352	0.584	1.212	2.366	4.11	6.25	7.81	11.34
4	0.297	0.711	1.064	1.923	3.357	5.39	7.78	9.49	13.28
5	0.554	1.145	1.610	2.675	4.351	6.63	9.24	11.07	15.09
6	0.872	1.635	2.204	3.455	5.348	7.84	10.64	12.59	16.81
7	1.239	2.167	2.833	4.255	6.346	9.04	12.02	14.07	18.48
8	1.647	2.733	3.490	5.071	7.344	10.22	13.36	15.51	20.09
9	2.088	3.325	4.168	5.899	8.343	11.39	14.68	16.92	21.67
10	2.558	3.940	4.865	6.737	9.342	12.55	15.99	18.31	23.21
11	3.053	4.575	5.578	7.584	10.341	13.70	17.28	19.68	24.72
12	3.571	5.226	6.304	8.438	11.340	14.85	18.55	21.03	26.22
13	4.107	5.892	7.042	9.299	12.340	15.98	19.81	22.36	27.69
14	4.660	6.571	7.790	10.165	13.339	17.12	21.06	23.68	29.14
15	5.229	7.261	8.547	11.037	14.339	18.25	22.31	25.00	30.58
16	5.812	7.962	9.312	11.912	15.338	19.37	23.54	26.30	32.00
17	6.408	8.672	10.085	12.792	16.338	20.49	24.77	27.59	33.41
18	7.015	9.390	10.865	13.675	17.338	21.60	25.99	28.87	34.80
19	7.633	10.117	11.651	14.562	18.338	22.72	27.20	30.14	36.19
20	8.260	10.851	12.443	15.452	19.337	23.83	28.41	31.41	37.57
22	9.542	12.338	14.041	17.240	21.337	26.04	30.81	33.92	40.29
24	10.856	13.848	15.659	19.037	23.337	28.24	33.20	36.42	42.98
26	12.198	15.379	17.292	20.843	25.336	30.43	35.56	38.89	45.64
28	13.565	16.928	18.939	22.657	27.336	32.62	37.92	41.34	48.28
30	14.953	18.493	20.599	24.478	29.336	34.80	40.26	43.77	50.89
40	22.164	26.509	29.051	33.660	39.335	45.62	51.80	55.76	63.69
50	27.707	34.764	37.689	42.942	49.335	56.33	63.17	67.50	76.15
60	37.485	43.188	46.459	52.294	59.335	66.98	74.40	79.08	88.38

Taken from: https://i.stack.imgur.com/PbEqv.jpg

Appendix C

Cust No.	Inter-Arrival Time	Actual Arrival Time	Service Durati	Servers available	Start Time	End Time	Waiting Time
1	0.000	0.000	2.066	12	0.000	2.066	0.000
2	0.115	0.115	0.002	11	0.115	0.117	0.000
3	0.092	0.207	3.283	11	0.207	3.490	0.000
4	0.212	0.419	0.552	10	0.419	0.971	0.000
5	0.193	0.613	0.093	9	0.613	0.706	0.000
6	0.677	1.290	0.705	10	1.290	1.995	0.000
7	0.126	1.416	0.193	9	1.416	1.609	0.000
8	0.342	1.758	2.430	9	1.758	4.188	0.000
9	0.040	1.798	0.266	8	1.798	2.065	0.000
10	0.053	1.851	12.253	7	1.851	14.105	0.000
11	0.719	2.571	3.445	9	2.571	6.016	0.000
12	0.004	2.575	0.335	8	2.575	2.911	0.000
13	0.389	2.964	1.409	8	2.964	4.373	0.000
14	0.623	3.587	0.597	8	3.587	4.184	0.000
15	0.050	3.638	1.504	7	3.638	5.142	0.000
16	0.062	3.700	0.758	6	3.700	4.458	0.000
17	0.302	4.002	3.860	5	4.002	7.862	0.000
18	0.052	4.055	1.922	4	4.055	5.976	0.000
19	0.318	4.373	3.508	5	4.373	7.881	0.000
20	0.167	4.540	0.548	6	4.540	5.089	0.000
21	0.018	4.558	2.398	5	4.558	6.956	0.000
22	0.387	4.945	5.989	4	4.945	10.934	0.000
23	0.557	5.502	2.043	5	5.502	7.545	0.000
24	0.085	5.588	5.406	4	5.588	10.994	0.000
25	0.003	5.591	2.832	3	5.591	8.423	0.000
26	0.190	5.781	1.674	2	5.781	7.455	0.000
27	0.287	6.068	2.601	3	6.068	8.670	0.000
28	0.302	6.371	0.973	2	6.371	7.344	0.000
29	0.130	6.501	5.478	1	6.501	11.979	0.000
30	0.085	6.586	6.967	0	6.956	13.553	0.370
31	0.007	6.593	1.878	0	7.344	8.471	0.751
32	0.016	6.609	5.823	0	7.455	12.432	0.845
33	0.029	6.639	3.541	0	7.545	10.180	0.907
34	0.224	6.862	0.963	0	7.862	7.825	1.000
35	0.206	7.068	3.657	0	7.862	10.725	0.793
36	0.054	7.122	0.815	0	7.881	7.937	0.759
37	0.121	7.243	2.651	0	7.937	9.895	0.693
38	0.243	7.486	1.191	0	8.423	8.677	0.937
39	0.259	7.746	0.458	0	8.471	8.203	0.726
40	0.685	8.431	1.115	0	8.471	9.546	0.041
41	0.608	9.039	4.686	2	9.039	13.725	0.000
42	0.013	9.052	3.898	1	9.052	12.950	0.000
43	0.098	9.150	1.400	0	9.546	10.550	0.395
44	0.196	9.347	1.007	0	9.895	10.354	0.548
45	0.039	9.386	10.683	0	10.180	20.069	0.794
46	0.297	9.683	13.661	0	10.354	23.344	0.670
47	0.153	9.837	5.541	0	10.550	15.377	0.713
48	0.101	9.937	1.513	0	10.725	11.450	0.788
49	0.072	10.009	6.411	0	10.934	16.419	0.925
50	0.343	10.352	0.240	0	10.994	10.592	0.642
51	0.189	10.541	6.031	0	10.994	16.572	0.453
52	0.190	10.731	0.791	0	11.450	11.522	0.719
53	0.511	11.242	2.749	0	11.522	13.991	0.280
54	0.082	11.324	0.019	0	11.979	11.343	0.655
55	0.515	11.839	1.951	0	11.979	13.790	0.140
56	0.008	11.847	5.645	0	12.432	17.492	0.585
57	0.165	12.011	1.167	0	12.950	13.179	0.939
58	0.040	12.051	1.882	0	13.179	13.933	1.127
59	0.175	12.226	6.166	0	13.553	18.392	1.327
60	0.204	12.430	2.777	0	13.725	15.207	1.295
61	0.030	12.460	0.093	0	13.790	12.553	1.330
62	0.269	12.729	0.387	0	13.790	13.116	1.061
63	0.110	12.839	5.164	0	13.790	18.003	0.951
64	0.273	13.111	4.202	0	13.933	17.314	0.822
65	0.237	13.349	0.439	0	13.991	13.787	0.643
66	0.020	13.369	9.341	0	13.991	22.710	0.623
67	0.162	13.531	11.129	0	14.105	24.660	0.574
68	0.023	13.554	1.725	0	15.207	15.278	1.654
69	0.055	13.608	10.755	0	15.278	24.363	1.670
70	0.021	13.630	4.905	0	15.377	18.534	1.748
71	0.305	13.934	1.331	0	16.419	15.265	2.485
72	0.893	14.828	5.897	0	16.419	20.725	1.592
73	0.394	15.222	1.094	0	16.572	16.316	1.350
74	0.615	15.837	1.221	0	16.572	17.058	0.735
75	0.072	15.909	2.622	0	17.058	18.531	1.150
76	0.037	15.945	4.468	0	17.314	20.413	1.369
77	0.185	16.130	2.679	0	17.492	18.808	1.362
78	0.059	16.189	0.617	0	18.003	16.806	1.814
79	0.005	16.194	0.191	0	18.003	16.385	1.809
80	0.013	16.206	0.013	0	18.003	16.219	1.796
81	0.047	16.253	0.030	0	18.003	16.283	1.750
82	0.375	16.628	3.886	0	18.003	20.513	1.375
83	0.369	16.997	1.996	0	18.392	18.993	1.395
84	0.033	17.030	1.584	0	18.531	18.614	1.501
85	0.953	17.983	0.319	0	18.534	18.302	0.551
86	0.009	17.992	3.144	0	18.534	21.137	0.542
87	0.098	18.090	6.918	0	18.614	25.008	0.524
88	0.012	18.102	1.583	0	18.808	19.685	0.706
89	0.120	18.222	0.651	0	18.993	18.873	0.771
90	0.024	18.246	1.683	0	18.993	19.929	0.747
91	0.574	18.820	1.179	0	19.685	19.999	0.865
92	0.346	19.166	1.887	0	19.929	21.053	0.763
93	0.176	19.342	4.347	0	19.999	23.689	0.658
94	0.445	19.786	1.229	0	20.069	21.016	0.283
95	0.359	20.145	2.223	0	20.413	22.368	0.268
96	0.490	20.635	0.508	1	20.635	21.143	0.000
97	0.063	20.698	3.720	0	20.725	24.418	0.026
98	0.453	21.151	0.769	4	21.151	21.919	0.000
99	0.019	21.170	0.726	3	21.170	21.895	0.000
100	0.114	21.284	0.394	2	21.284	21.678	0.000
101	0.101	21.385	2.998	1	21.385	24.383	0.000
102	0.362	21.747	1.019	1	21.747	22.766	0.000
103	0.119	21.867	1.336	0	21.895	23.203	0.029
104	0.101	21.967	0.451	1	21.967	22.419	0.000
105	0.036	22.003	7.306	0	22.368	29.309	0.365
106	0.024	22.027	0.067	0	22.419	22.094	0.392
107	0.193	22.220	8.659	0	22.419	30.879	0.199
108	0.255	22.475	1.656	0	22.710	24.131	0.235
109	0.710	23.185	1.058	1	23.185	24.242	0.000
110	0.281	23.466	2.386	2	23.466	25.852	0.000
111	0.037	23.503	4.401	1	23.503	27.904	0.000
112	0.002	23.505	3.583	0	23.689	27.088	0.184
113	0.017	23.522	2.464	0	24.131	25.986	0.609
114	0.109	23.631	3.634	0	24.242	27.265	0.611
115	0.182	23.813	10.183	0	24.363	33.996	0.550
116	0.737	24.550	1.977	2	24.550	26.527	0.000
117	0.286	24.836	3.548	2	24.836	28.383	0.000
118	0.020	24.856	0.663	1	24.856	25.518	0.000
119	0.211	25.066	2.477	1	25.066	27.544	0.000
120	0.064	25.131	0.638	0	25.518	25.769	0.388
121	0.602	25.732	0.233	0	25.769	25.965	0.037
122	0.512	26.245	0.301	3	26.245	26.546	0.000
123	0.327	26.572	0.858	4	26.572	27.430	0.000
124	0.398	26.970	4.653	3	26.970	31.623	0.000
125	0.140	27.111	1.418	3	27.111	28.528	0.000
126	0.060	27.171	0.031	2	27.171	27.201	0.000
127	0.305	27.476	1.095	4	27.476	28.571	0.000
128	0.188	27.664	0.135	4	27.664	27.799	0.000
129	0.064	27.728	2.564	3	27.728	30.292	0.000
130	0.473	28.201	0.267	4	28.201	28.467	0.000
131	0.052	28.253	2.577	3	28.253	30.830	0.000
132	0.240	28.492	1.819	4	28.492	30.312	0.000
133	0.275	28.767	1.755	5	28.767	30.523	0.000
134	0.091	28.859	4.499	4	28.859	33.358	0.000
135	0.035	28.894	0.313	3	28.894	29.207	0.000
136	0.538	29.432	5.758	4	29.432	35.190	0.000
137	0.736	30.168	2.435	3	30.168	32.602	0.000
138	0.095	30.263	6.188	2	30.263	36.450	0.000
139	0.122	30.385	0.128	3	30.385	30.513	0.000
140	0.095	30.480	0.903	2	30.480	31.382	0.000
141	0.106	30.585	2.162	3	30.585	32.747	0.000
142	0.200	30.785	2.769	2	30.785	33.554	0.000
143	0.019	30.805	0.060	1	30.805	30.865	0.000
144	0.190	30.995	1.177	3	30.995	32.172	0.000
145	0.314	31.310	1.134	2	31.310	32.443	0.000
146	0.121	31.431	1.225	2	31.431	32.656	0.000
147	0.005	31.436	0.232	1	31.436	31.668	0.000
148	0.145	31.581	2.073	0	31.623	33.654	0.042
149	0.633	32.214	0.923	2	32.214	33.137	0.000
150	0.038	32.252	2.093	1	32.252	34.345	0.000
151	0.001	32.253	2.068	0	32.443	34.321	0.190
152	0.251	32.505	0.786	0	32.602	33.290	0.098
153	0.244	32.749	12.659	2	32.749	45.408	0.000
154	0.421	33.170	2.258	2	33.170	35.427	0.000
155	0.309	33.478	1.357	3	33.478	34.836	0.000
156	0.108	33.587	0.991	3	33.587	34.578	0.000
157	0.023	33.610	3.292	2	33.610	36.902	0.000
158	0.186	33.796	4.226	2	33.796	38.023	0.000
159	0.095	33.891	5.992	1	33.891	39.883	0.000
160	0.025	33.916	1.517	0	33.996	35.433	0.080
161	0.192	34.108	3.571	0	34.321	37.679	0.213
162	0.090	34.198	11.641	0	34.345	45.839	0.147
163	0.125	34.323	0.872	0	34.578	35.195	0.254
164	0.244	34.568	0.299	0	34.836	34.867	0.268
165	0.091	34.658	2.119	0	34.867	36.778	0.208
166	0.693	35.351	0.480	2	35.351	35.832	0.000
167	0.068	35.419	5.567	1	35.419	40.986	0.000
168	0.325	35.743	2.860	2	35.743	38.603	0.000
169	0.221	35.965	3.971	2	35.965	39.936	0.000
170	0.176	36.141	2.100	1	36.141	38.240	0.000
171	0.218	36.359	9.227	0	36.450	45.586	0.091
172	0.063	36.422	2.631	0	36.778	39.053	0.356
173	0.188	36.610	0.790	0	36.902	37.400	0.292

173	0.188	36.610	0.790	0	36.902	37.400	0.292
174	0.457	37.067	1.635	0	37.400	38.702	0.333
175	0.047	37.114	2.855	0	37.679	39.969	0.565
176	0.097	37.211	1.816	0	38.023	39.027	0.812
177	0.252	37.463	1.873	0	38.240	39.336	0.777
178	0.086	37.549	12.968	0	38.603	50.517	1.054
179	0.038	37.587	7.874	0	38.702	45.461	1.115
180	1.005	38.592	0.170	0	39.027	38.762	0.435
181	0.445	39.037	1.109	1	39.037	40.146	0.000
182	0.005	39.042	0.571	0	39.053	39.613	0.011
183	0.130	39.172	0.141	0	39.336	39.313	0.164
184	0.599	39.771	6.894	2	39.771	46.665	0.000
185	0.080	39.851	0.601	1	39.851	40.452	0.000
186	0.369	40.220	2.479	4	40.220	42.699	0.000
187	0.192	40.411	7.038	3	40.411	47.449	0.000
188	0.314	40.725	7.951	3	40.725	48.676	0.000
189	0.504	41.229	4.179	3	41.229	45.409	0.000
190	0.022	41.252	1.241	2	41.252	42.493	0.000
191	0.201	41.453	0.050	1	41.453	41.503	0.000
192	0.048	41.500	0.979	0	41.503	42.479	0.003
193	0.163	41.663	1.396	0	42.479	43.059	0.816
194	0.153	41.817	0.765	0	42.493	42.582	0.676
195	0.295	42.111	3.253	0	42.582	45.364	0.470
196	0.118	42.230	3.130	0	42.699	45.360	0.469
197	0.055	42.285	1.757	0	43.059	44.041	0.775
198	0.456	42.741	1.843	0	44.041	44.584	1.301
199	0.016	42.757	1.167	0	44.584	43.924	1.827
200	0.433	43.190	10.405	0	44.584	53.594	1.394
201	0.384	43.574	0.448	0	45.360	44.022	1.786
202	0.005	43.579	0.147	0	45.360	43.726	1.781
203	0.285	43.863	1.483	0	45.360	45.347	1.497
204	0.497	44.361	0.255	0	45.360	44.615	0.999
205	0.237	44.598	7.615	0	45.360	52.213	0.762
206	0.746	45.343	5.063	0	45.364	50.406	0.021
207	0.096	45.440	8.178	2	45.440	53.618	0.000
208	0.427	45.867	1.153	4	45.867	47.020	0.000
209	0.605	46.472	3.954	3	46.472	50.426	0.000
210	0.003	46.475	0.951	2	46.475	47.426	0.000
211	0.135	46.610	8.415	1	46.610	55.025	0.000
212	0.197	46.807	0.166	1	46.807	46.973	0.000
213	0.271	47.079	4.570	2	47.079	51.649	0.000
214	0.221	47.300	5.765	1	47.300	53.065	0.000
215	0.021	47.320	0.880	0	47.426	48.200	0.106
216	0.360	47.680	1.487	1	47.680	49.167	0.000
217	0.174	47.854	1.417	0	48.200	49.271	0.347
218	0.048	47.901	0.522	0	48.676	48.423	0.775
219	0.435	48.337	3.800	0	48.676	52.136	0.340
220	0.088	48.425	1.997	0	49.167	50.422	0.742
221	0.304	48.729	0.385	0	49.271	49.114	0.542
222	0.246	48.975	4.249	0	49.271	53.224	0.296
223	0.051	49.026	0.113	0	50.406	49.139	1.381
224	0.178	49.203	0.431	0	50.406	49.635	1.203
225	0.015	49.218	0.108	0	50.406	49.327	1.188
226	0.431	49.650	1.335	0	50.406	50.984	0.757
227	0.008	49.657	1.286	0	50.422	50.943	0.764
228	0.131	49.788	0.750	0	50.426	50.538	0.638
229	0.044	49.832	1.343	0	50.517	51.176	0.685