Fourier series for output voltages of inverter waveforms.

The Fourier series for a periodic function $v_o(\omega t)$ can be expressed as

$$v_o(\omega t) = a_o + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

For an odd quarter-wave symmetry waveform,

$$a_o = 0 \quad a_n = 0$$

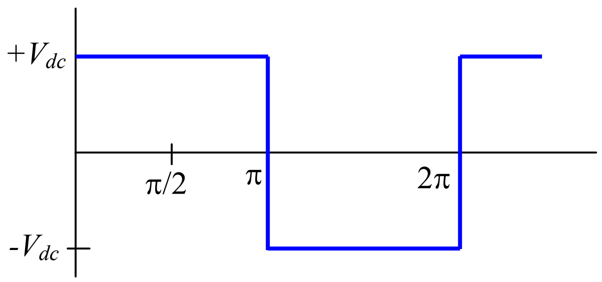
and

$$b_n = \begin{cases} \dfrac{4}{\pi} \displaystyle\int_0^{\frac{\pi}{2}} v_o \sin(n\omega t)\,d(\omega t) & \text{for odd } n \\ 0 & \text{for even } n \end{cases}$$

Therefore, $v_o(\omega t)$ can be written as

$$v_o(\omega t) = \sum_{n=odd}^{\infty} b_n \sin(n\omega t)$$

(i) Square-wave



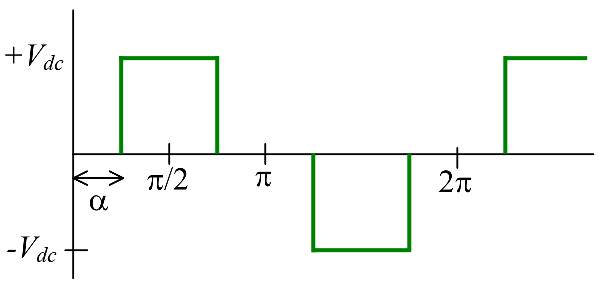
$$b_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} v_o \sin(n\omega t)\,d(\omega t)$$

$$= \frac{4}{\pi}\int_0^{\frac{\pi}{2}} V_{dc} \sin(n\omega t)\,d(\omega t)$$

$$= \frac{4V_{dc}}{n\pi}\left[-con(n\omega t\right]_0^{\frac{\pi}{2}}$$

$$= \frac{4V_{dc}}{n\pi}$$

(ii) Quasi square-wave



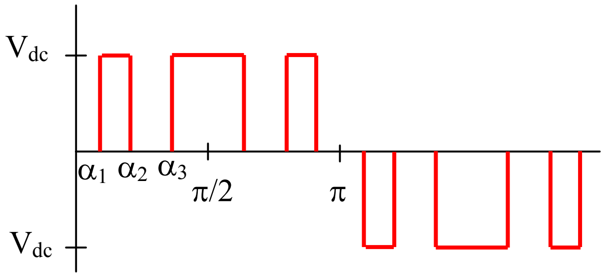
$$b_n = \frac{4}{\pi}\int_\alpha^{\frac{\pi}{2}} v_o \sin(n\omega t)\,d(\omega t)$$

$$= \frac{4}{\pi}\int_\alpha^{\frac{\pi}{2}} V_{dc} \sin(n\omega t)\,d(\omega t)$$

$$= \frac{4V_{dc}}{n\pi}\left[-con(n\omega t\right]_\alpha^{\frac{\pi}{2}}$$

$$= \frac{4V_{dc}}{n\pi}\cos(n\alpha)$$

(iii) Notched waveform (Harmonics Elimination PWM)



$$b_n = \frac{4}{\pi}\int_0^{\frac{\pi}{2}} v_o \sin(n\omega t)d(\omega t)$$

$$= \frac{4}{\pi}\left[\int_{\alpha_1}^{\alpha_2} V_{dc}\sin(n\omega t)d(\omega t) + \int_{\alpha_3}^{\frac{\pi}{2}} V_{dc}\sin(n\omega t)d(\omega t)\right]$$

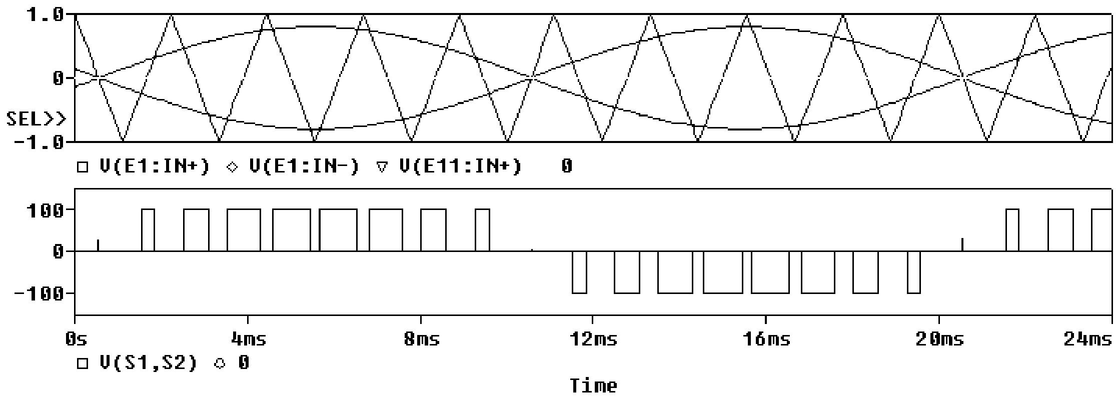
$$= \frac{4V_{dc}}{n\pi}\left\{\left[-con(n\omega t\right]_{\alpha_1}^{\alpha_2} + \left[-con(n\omega t\right]_{\alpha_3}^{\frac{\pi}{2}}\right\}$$

$$= \frac{4V_{dc}}{n\pi}\cos(n\omega t)\Big|_{\alpha_2,\frac{\pi}{2}}^{\alpha_1,\alpha_3}$$

$$= \frac{4V_{dc}}{n\pi}\left[\cos(n\alpha_1) + \cos(n\alpha_3) - \cos(n\alpha_2) - \cos(n\frac{\pi}{2})\right]$$

$$= \frac{4V_{dc}}{n\pi}\left[\cos(n\alpha_1) + \cos(n\alpha_3) - \cos(n\alpha_2)\right]$$

(iv) Sinusoidal PWM (unipolar and bipolar)



Unipolar SPWM waveform as an example

$$v_o(\omega t) = M_a V_{dc}\sin(\omega t) + \frac{4V_{dc}}{\pi}\sum_{n=1}^{\infty}\frac{\sin\left[\frac{n\pi}{2} + (\frac{k}{2})\cos(\omega t)\right]}{n}$$
$$= M_a V_{dc}\sin(\omega t) + \text{Bessel Function for harmonic terms}$$

The tables are required to resolve for the Bessel function for harmonic terms. The harmonics in the inverter output appear as sidebands, centered around the switching frequency, that is, around mf, $2m_f$, $3m_f$ and so on. This general pattern hold true for all values of ma in the range 0 – 1 and m_f>9. The unipolar SPWM switching scheme has the advantage of "effectively" doubling the switching frequency as far as the output harmonics are concerned, compared to the bipolar SPWM switching scheme. Because of that the harmonics in the inverter output of unipolar SPWM are centered around $2m_f$, $4m_f$, $6m_f$ and so on.

TABLE 8.3 NORMALIZED FOURIER COEFFICIENTS V_n/V_{dc} FOR BIPOLAR SPWM

	$M_a = 1$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
n = 1	1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10
$n = m_f$	0.60	0.71	0.82	0.92	1.01	1.08	1.15	1.20	1.24	1.27
$n = m_f \pm 1$	0.32	0.27	0.22	0.17	0.13	0.09	0.06	0.03	0.02	0.00

Table 8.3 shows the first harmonic frequencies in the output spectrum at and around m_f for the bipolar SPWM switching scheme. The harmonics at and around $2m_f$, $3m_f$, $4m_f$ and so on are not indicated.

TABLE 8.5 NORMALIZED FOURIER COEFFICIENTS V_n/V_{dc} FOR UNIPOLAR SPWM

	$M_a = 1$	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
n = 1	1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10
$n = 2m_f \pm 1$	0.18	0.25	0.31	035	0.37	0.36	0.33	0.27	0.19	0.10
$n = 2m_f \pm 3$	0.21	0.18	0.14	0.10	0.07	0.04	0.02	0.01	0.00	0.00

Table 8.5 shows the first harmonic frequencies in the output spectrum at and around $2m_f$ for the uipolar SPWM switching scheme. The harmonics at and around $4m_f$, $6m_f$, $8m_f$ and so on are not indicated.

Table 8.3 and 8.5 can be used to predict the THD for the ouput current of the inverter connected to RL load. Higher order harmonics are assumed to contribute little power and effect, so they can be neglected. To evaluate the THD for the output voltage of the inverter, higher order harmonics should be taken into account.