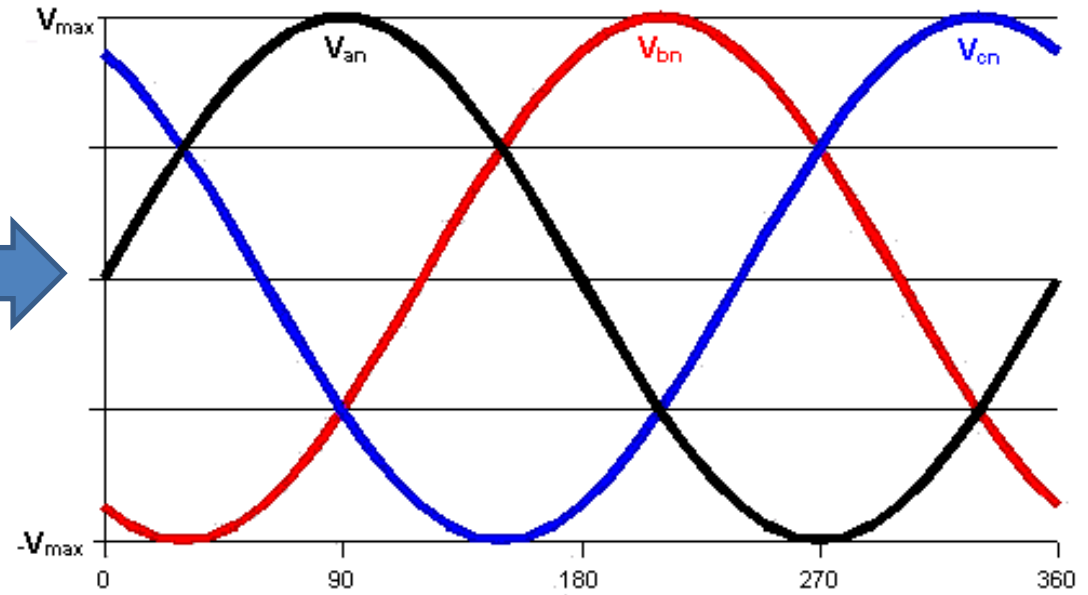
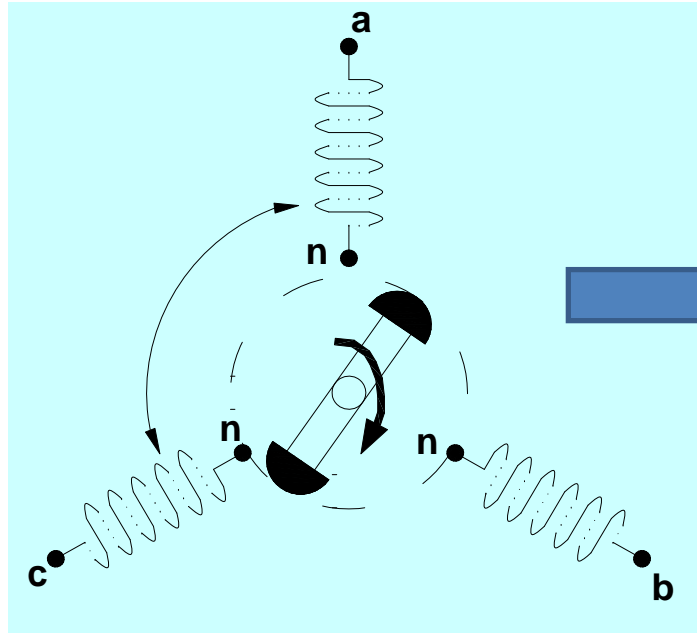


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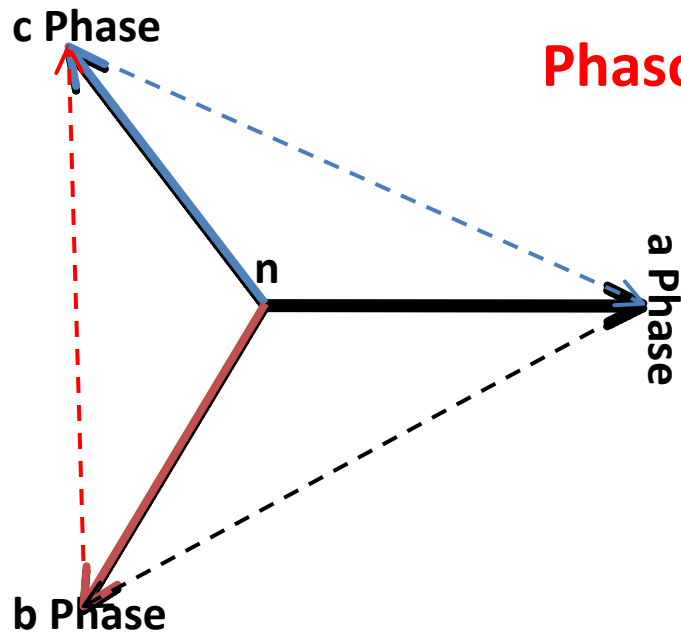
Three Phase Systems

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Sinusoidal voltages are induced in three coils placed 120° out of phase with each other



Phasor Representation

Anticlockwise-abc Sequence

Clockwise-acb Sequence

V_{an} , V_{bn} , V_{cn} --Phase Voltages

V_{ab} , V_{bc} , V_{ca} --Line Voltages

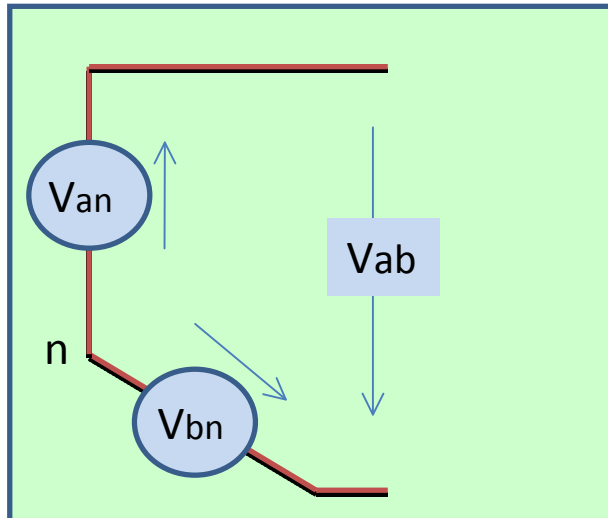
The induced voltages:

$$V_{an} = V \sin \omega t$$

$$V_{bn} = V \sin (\omega t - 120^\circ)$$

$$V_{cn} = V \sin (\omega t - 240^\circ)$$

Writing a KVL equation



Aside: $V_{ab} = V_{an} - V_{bn}$
 $= V\angle 0^\circ - V\angle -120^\circ$
 $= V - V \cdot e^{+j(-120^\circ)}$

Eulers rule:

$$z = x + jy = |z|(\cos\theta + j\sin\theta)$$

$$= |z| e^{j\theta}$$

$$\begin{aligned} \Rightarrow V_{ab} &= V - V\{\cos(-120^\circ) + j\sin(-120^\circ)\} \\ &= V - V\left\{-0.5 - j\frac{\sqrt{3}}{2}\right\} \\ &= V\left[1 - \left\{-0.5 - j\frac{\sqrt{3}}{2}\right\}\right] \\ &= V\left[1 + 0.5 + j\frac{\sqrt{3}}{2}\right] \\ &= V\left[\frac{3}{2} + j\frac{\sqrt{3}}{2}\right] = \sqrt{3}V\left[\frac{\sqrt{3}}{2} + j\frac{1}{2}\right] = \sqrt{3}V\angle 30^\circ \\ \Rightarrow V_{ab} &= \sqrt{3}V\angle 30^\circ \end{aligned}$$

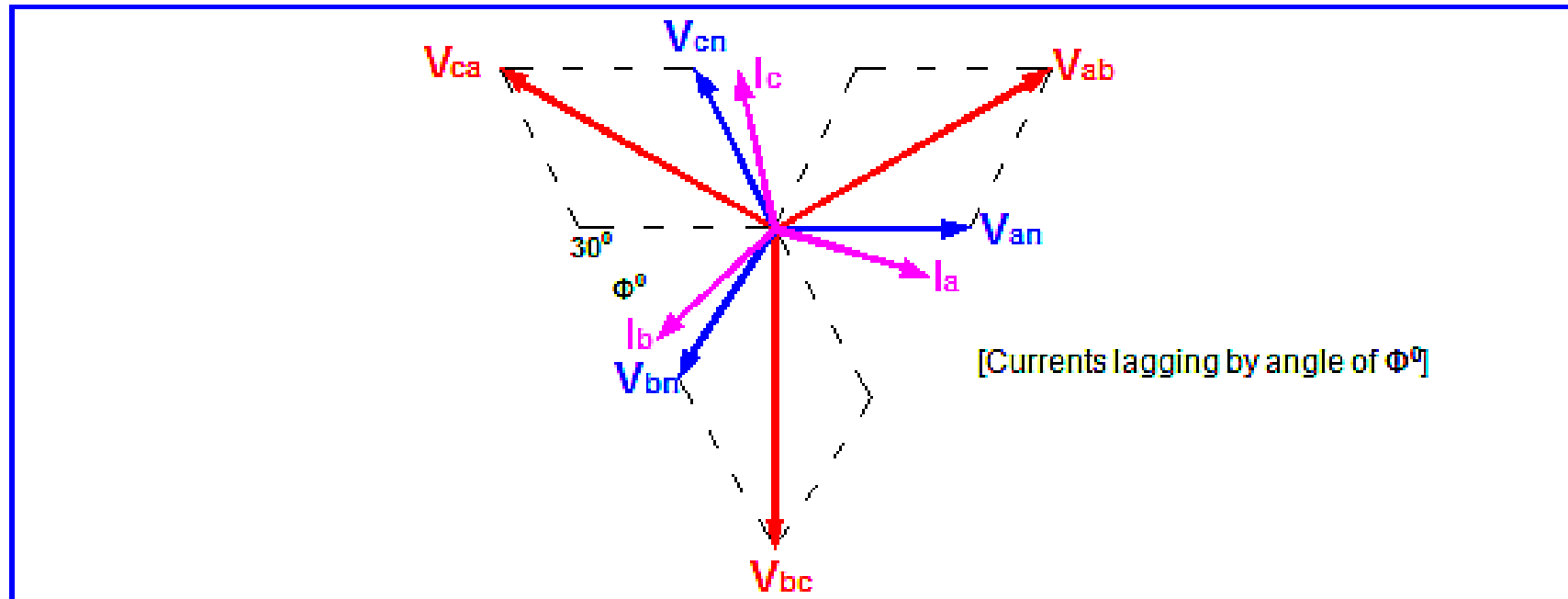
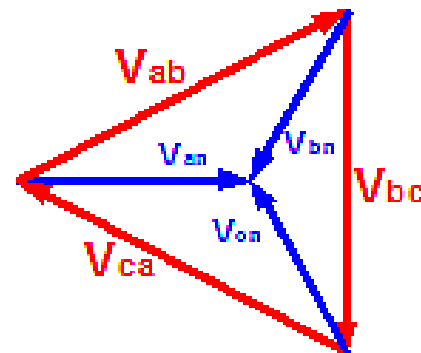


Figure 4: Phasor diagram of a star connected generator



Note: Since balanced line-to-line voltages form a closed triangle, their phasor sum is zero. Also in a balanced system, $\vec{V}_{an} + \vec{V}_{bn} + \vec{V}_{cn} = 0$

Example 1: A balanced star-connected load is supplied from a symmetrical 3 wire, three-phase 400 V supply. The magnitude of the current in each phase is 30 A and lags 30° behind the respective phase voltage. The phase sequence is ABC and the reference phasor is V_{AN}

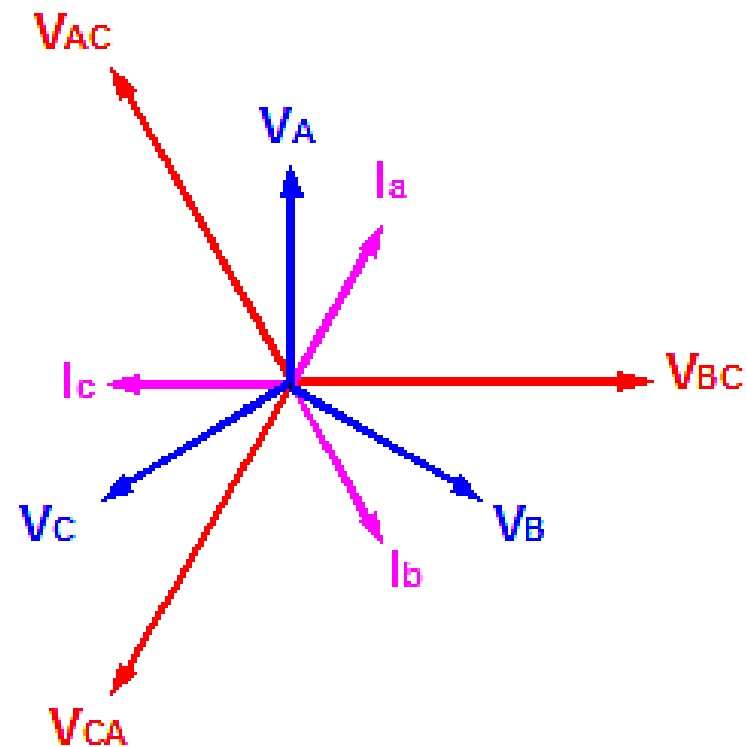
- Calculate the phase voltage magnitude and the active power.
- Draw a phasor diagram showing the phase currents and the line-to-line voltages.
- What are the active and reactive components of the “A” phase current?

Solution:

$$a) \quad |V_{ph}| = \frac{|V_{Line}|}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.9 \quad [V]$$

$$\begin{aligned} P &= V.I.\cos\phi = V.I.\cos 30^\circ \\ &= (230.9).(30).\cos 30^\circ \\ &= 6000 \quad [W/ph] \\ &= (6000).(3) = 18000 \quad [W] \end{aligned}$$

b)



Phasor Diagram
illustrating the phase
currents and line-to-
line voltages

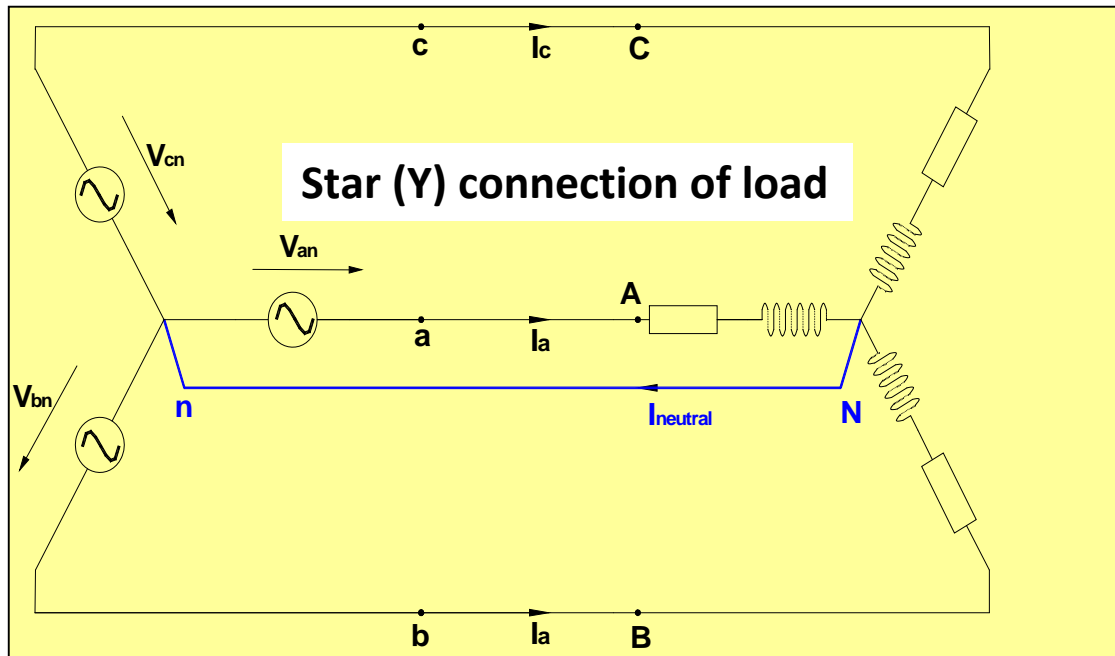
Phasors are not drawn to scale

c)

$$I_a = 30 \angle 30^\circ = 25.98 - j15$$

↙
Active
↘
Reactive

Three Phase Connections



KVL for loop "a-A-N-n-a"

$$\Rightarrow V_{an} - I_a \cdot Z \angle \phi = 0$$

$$\Rightarrow V_{an} = I_a Z \angle \phi$$

$$\Rightarrow I_a = \frac{V_{an}}{Z \angle \phi} = \frac{V \angle 0}{Z \angle \phi} = \frac{V}{Z} \angle -\phi$$

$$I_b = \frac{V \angle -120}{Z \angle \phi} = \frac{V}{Z} \angle -120 - \phi$$

$$I_c = \frac{V \angle +120}{Z \angle \phi} = \frac{V}{Z} \angle +120 - \phi$$

➤ Line Currents are same as phase currents

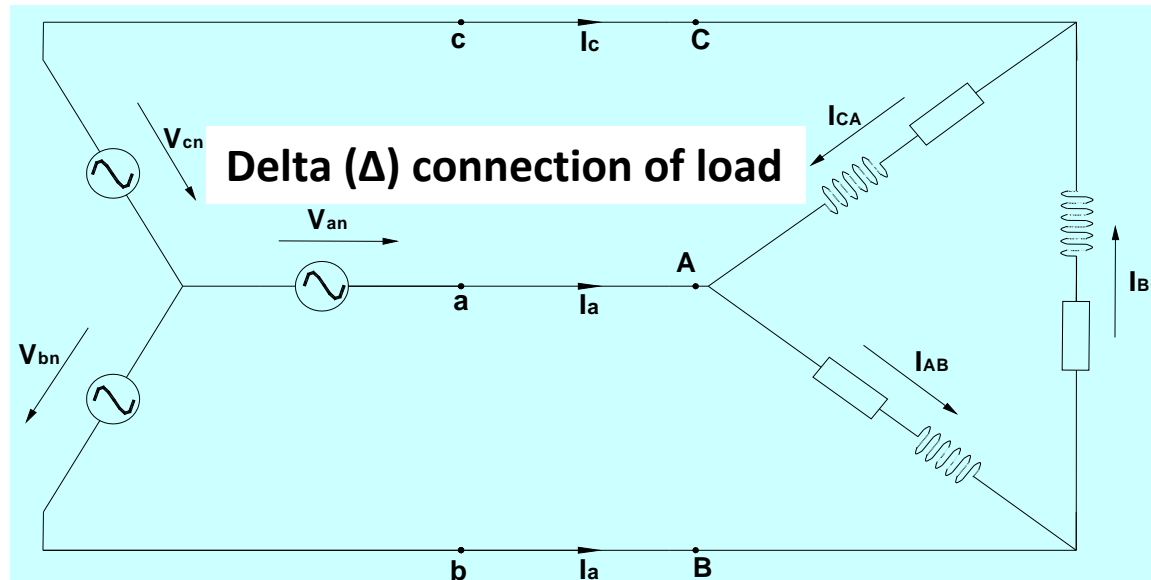
➤ Electrically displaced by 120° , with magnitude V/Z

➤ $I_{neutral} = I_a + I_b + I_c = 0$ for a balanced system

➤ Line voltages lead phase voltages by 30°

➤ Line voltages = $\sqrt{3}$ * Phase voltages

Three Phase Connections Cntd...



Applying KVL to the loop "a-A-B-b-a" \Rightarrow Phase current I_{AB} can be determined.

$$\Rightarrow V_{an} - I_{AB} \cdot Z \angle \phi - V_{bn} = 0$$

$$\Rightarrow V_{an} - V_{bn} = I_{AB} \cdot Z \angle \phi \quad \therefore V_{ab} = I_{AB} \cdot Z \angle \phi$$

$$\Rightarrow \frac{V_{ab}}{Z \angle \phi} = I_{AB} \Rightarrow \frac{\sqrt{3} V \sqrt{30}}{Z \angle \phi} = \frac{\sqrt{3} V \angle 30 - \phi}{Z} = I_{AB}$$

The phase currents in the delta connected load are balanced and displaced from one another by $120^\circ \Rightarrow \text{Balanced}$

Three Phase Connections Cntd...

Determining the line currents: - KCL

$$I_a = I_{AB} - I_{CA}$$

$$\frac{\sqrt{3}V\angle 30 - \phi}{Z} - \frac{\sqrt{3}V\angle 150 - \phi}{Z}$$

$$\frac{\sqrt{3}}{Z} \cdot V \{ \cos(30 - \phi) + j\sin(30 - \phi) - \cos(150 - \phi) - j\sin(150 - \phi) \}$$

Recall:

$$\cos A - \cos B = -2\sin\left[\frac{A+B}{2}\right]\sin\left[\frac{A-B}{2}\right]$$

$$\sin A - \sin B = +2\cos\left[\frac{A+B}{2}\right]\sin\left[\frac{A-B}{2}\right]$$

$$\begin{aligned} \Rightarrow \vec{I}_a &= \frac{\sqrt{3}}{Z} V \left\{ \left(-2\sin\left[\frac{30 - \phi + 150 - \phi}{2}\right] \right) \left(\sin\left[\frac{30 - \phi - 150 + \phi}{2}\right] \right) \right. \\ &\quad \left. + \left(2j\cos\left[\frac{30 - \phi + 150 - \phi}{2}\right] \right) \left(\sin\left[\frac{30 - \phi - 150 + \phi}{2}\right] \right) \right\} \\ &= \frac{\sqrt{3}}{Z} \cdot V \{ -2\sin(90 - \phi)\sin(-60) + 2j\cos(90 - \phi)\sin(-60) \} \end{aligned}$$

Three Phase Connections Cntd...

$$\begin{aligned}
 &= \frac{\sqrt{3}}{Z} V \left\{ -2 \sin(90 - \phi) - \frac{\sqrt{3}}{2} + 2j \cos(90 - \phi) - \frac{\sqrt{3}}{2} \right\} \\
 &= \frac{\sqrt{3}}{Z} V \left\{ \sqrt{3} \sin(90 - \phi) - \sqrt{3} j \cos(90 - \phi) \right\} \\
 &= \frac{\sqrt{3} V}{Z} \left\{ \sqrt{3} \sin(90 - \phi) - \sqrt{3} j \cos(90 - \phi) \right\} \\
 &= \frac{3V}{Z} \{ \cos \phi - j \sin \phi \} \\
 &\Rightarrow \vec{I}_a = \frac{3V}{Z} \angle -\phi
 \end{aligned}$$

Note: $\sin\left(\frac{\pi}{2} - \phi\right) = \cos \phi$
 $\cos\left(\frac{\pi}{2} - \phi\right) = \sin \phi$

Recall: $\vec{I}_{AB} = \sqrt{3} \cdot \frac{V}{Z} \angle 30 - \phi$

$$\Rightarrow I_a = \sqrt{3} I_{AB} \angle -30 \Rightarrow \text{i.e. line current lags phase currents } 30^\circ.$$

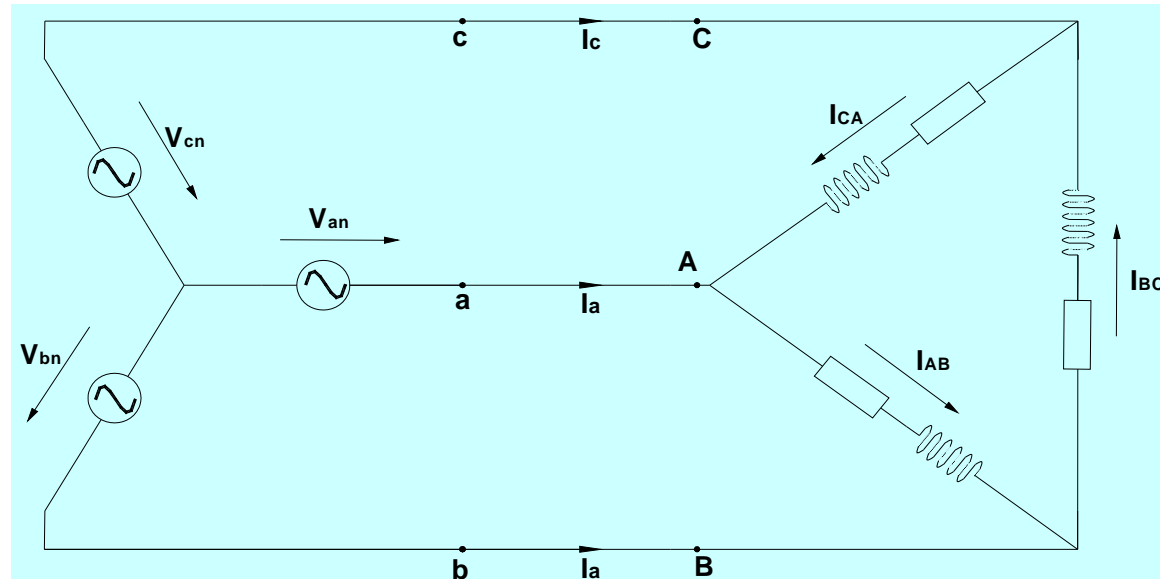
$$\text{i.e. } |I_a| = \sqrt{3} |I_{AB}|$$

Similarly

$$\vec{I}_b = \sqrt{3} \vec{I}_{BC} \angle -30$$

$$\vec{I}_c = \sqrt{3} \vec{I}_{CA} \angle -30$$

Three Phase Connections Cntd...



➤ Both line currents and phase currents in a delta connector balanced load are balanced

➤ The phasor sum of the balanced delta phase currents equals zero ($I_{AB} + I_{BC} + I_{CA} = 0$)

➤ The phasor sum of the line currents is always zero for a delta load even if the load is unbalanced because there is no neutral wire ($I_a + I_b + I_c = 0$)

➤ The line current lags the phase current by 30° for a balanced Delta load

➤ Line Voltages and Phase voltages are same in a delta connection

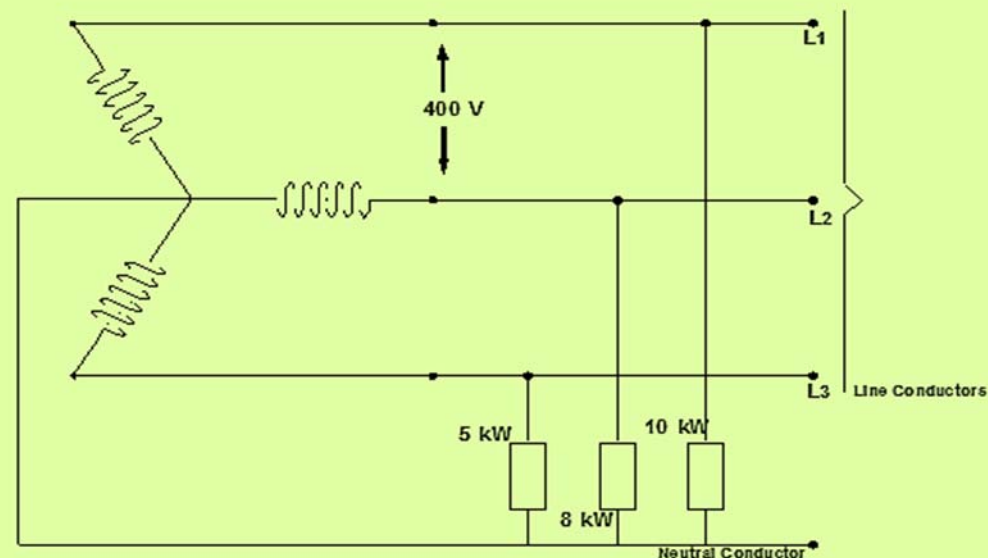
Example 2: In a three-phase four wire system, the line voltage is 400V and non-inductive loads of 10 kW [L_1], 8 kW [L_2] and 5 kW [L_3] are connected between the three lines and the neutral conductor.

Calculate:

- a) the current in each line, and
- b) the current in the neutral conductor

Hughes Electrical Technology [7th Edition]

Solution:



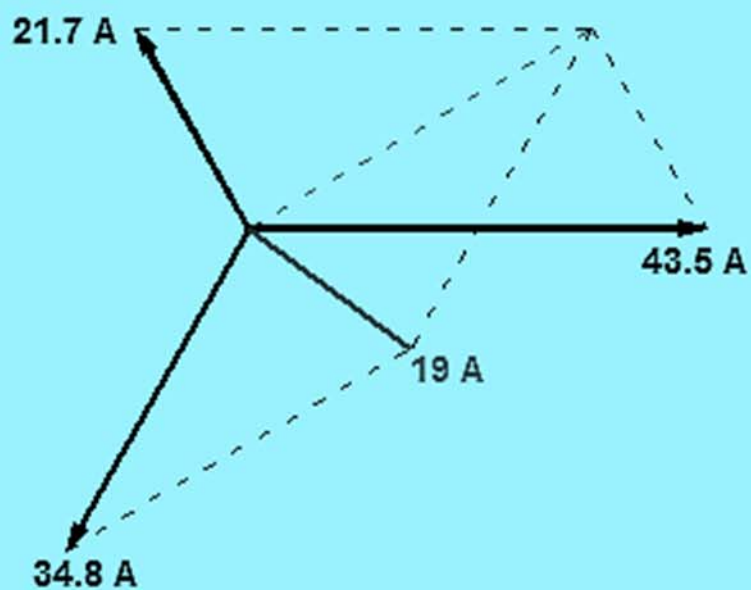
$$V_{ph} = \frac{V_{Line}}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230 \text{ V}$$

$$I_{ph} = I_{Line}$$

$$\Rightarrow |I_{L_1}| = \frac{10000}{230} = 43.5 \text{ A}$$

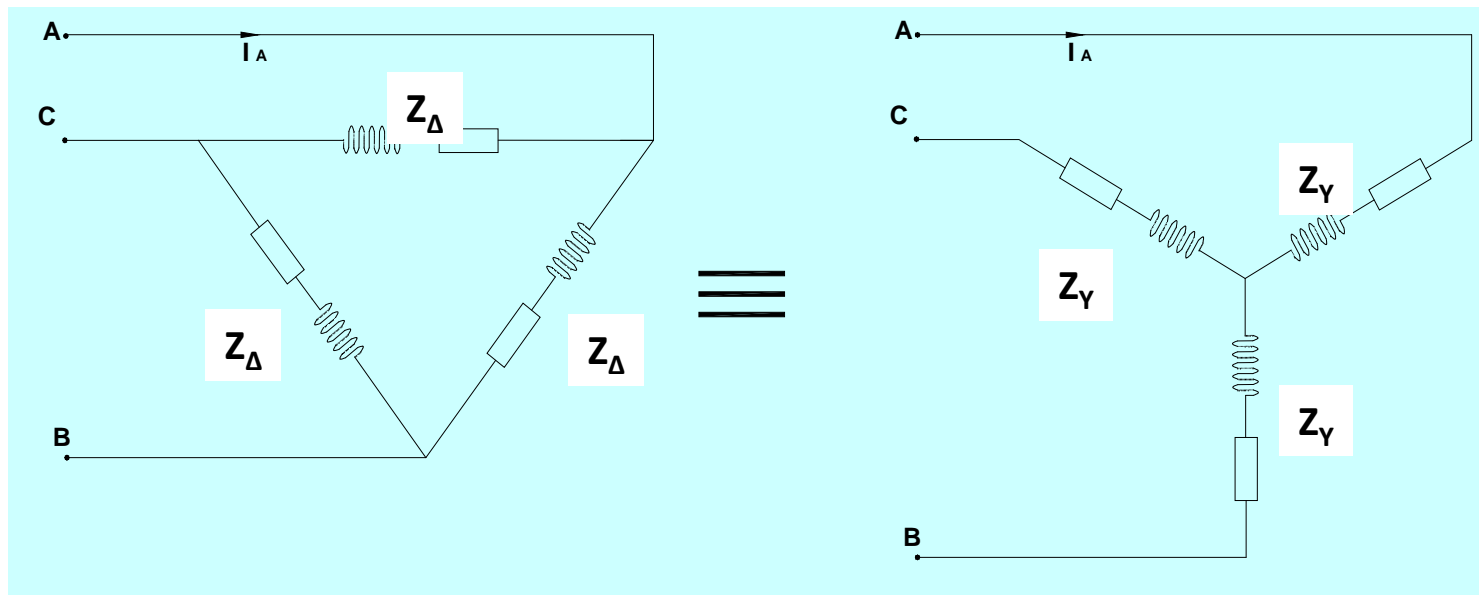
$$|I_{L_2}| = \frac{8000}{230} = 34.8 \text{ A}$$

$$|I_{L_3}| = \frac{5000}{230} = 21.7 \text{ A}$$



The Neutral Current, $I_N = 43.5 \angle 0^\circ + 34.8 \angle -120^\circ + 21.7 \angle -240^\circ$

Delta star conversion (Δ -Y) for balanced loads



For the Δ -load :

$$I_A = \sqrt{3} I_{AB} \angle -30^\circ = \frac{\sqrt{3} \cdot E_{AB} \angle -30^\circ}{Z_{\Delta}}$$

For the Y -load :

$$I_A = \frac{E_{AN}}{Z_y} = \frac{E_{AB} \angle -30^\circ}{\sqrt{3} Z_y}$$

Comparing two equations I_A will be same if

$$Z_y = \frac{Z_{\Delta}}{3}$$

Similarly

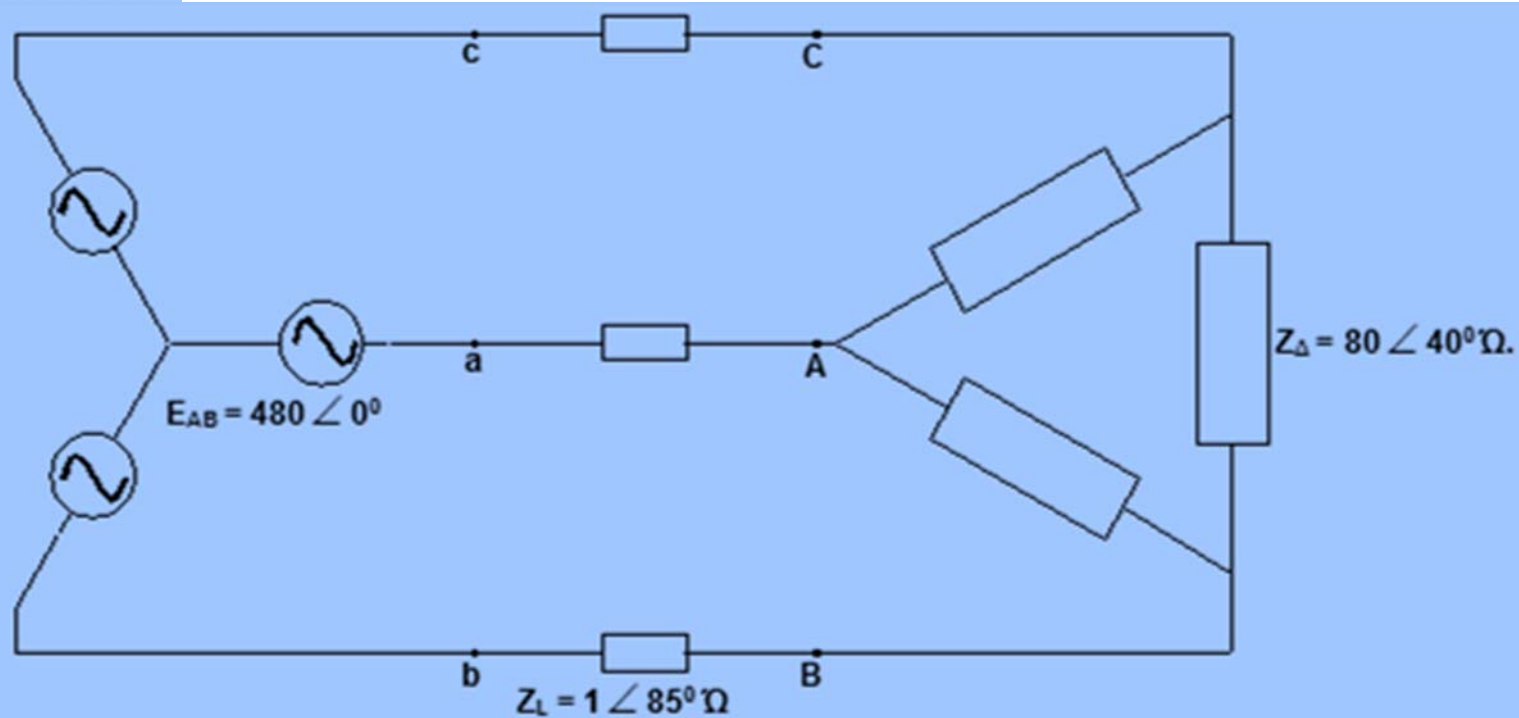
$$Z_{\Delta} = 3Z_y$$

Example 3: A balanced star connected positive sequence voltage source with $E_{AB} = 480 \angle 0^\circ$ is applied to a balanced delta connected load with $Z_\Delta = 30 \angle 40^\circ \Omega$. The line impedance between the source and the load is $Z_L = 1 \angle 85^\circ \Omega$ per phase.

Calculate:

- The line currents
- The voltage at the load terminals

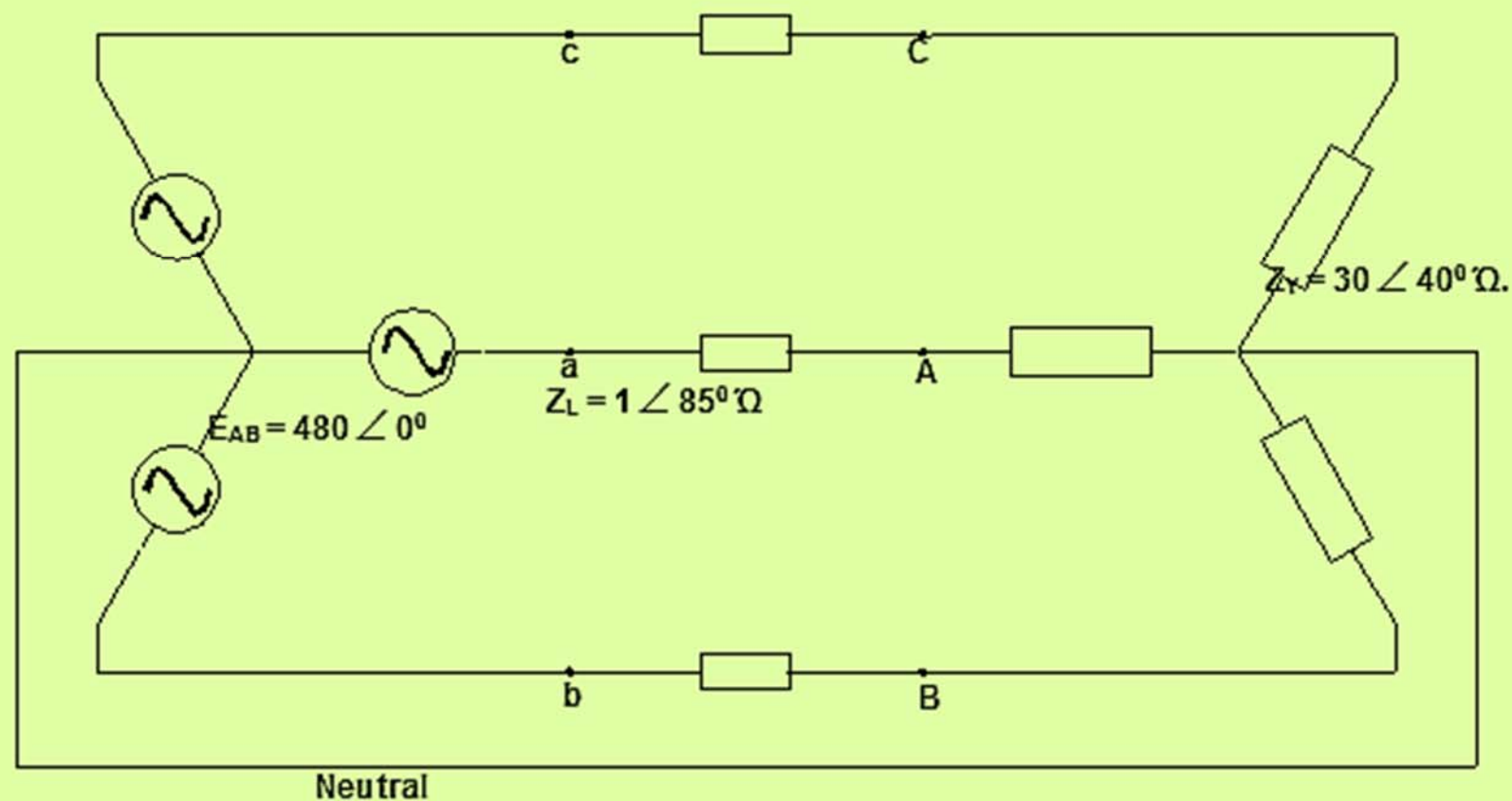
Solution:



Converting the Delta Impedance to the Star equivalent:

$$Z_{\Delta} \rightarrow Z_Y \therefore Z_Y = \frac{Z_{\Delta}}{3} = \frac{30}{3} \angle 40^{\circ} = 10 \angle 40^{\circ}$$

Therefore, this transposes to:



$$I_A = \frac{E_{AN}}{Z_{Line} = Z_Y} = \frac{\frac{480}{\sqrt{3}} \angle -30^\circ}{1 \angle 85^\circ + 10 \angle 40^\circ}$$

Recall, in a star-connected system,
 V_{Line} leads V_{Phase} by 30°

$$\therefore I_A = 25.83 \angle -73.78^\circ [A]$$

$$I_B = 25.83 \angle -193.78^\circ [A]$$

$$I_C = 25.83 \angle 46.22^\circ [A]$$

Note: The magnitude of each current is the same and their respective phase displacement is 120° .

The Delta load currents are:

$$I_{AB} = \frac{I_a}{\sqrt{3}} \angle 30^\circ = \frac{25.83}{\sqrt{3}} \angle (-73.78 + 30)^\circ = 14.91 \angle -43.78^\circ [A]$$

$$I_{AB} = 14.91 \angle -163.78^\circ [A]$$

$$I_{AB} = 14.91 \angle +76.22^\circ [A]$$

Recall, in a delta-connected system, I_{Line} lags I_{Phase} by 30°

The voltages at the load terminals:

$$E_{AB} = Z_{\Delta} I_{AB} = (30 \angle 40^\circ) (14.91 \angle -43.78^\circ) = 447.3 \angle -3.78^\circ$$

$$E_{BC} = 447.3 \angle -123.78^\circ$$

$$E_{CA} = 447.3 \angle 116.22^\circ$$

Instantaneous Power

For a single phase circuit instantaneous power can be given as a product of instantaneous voltage and current values

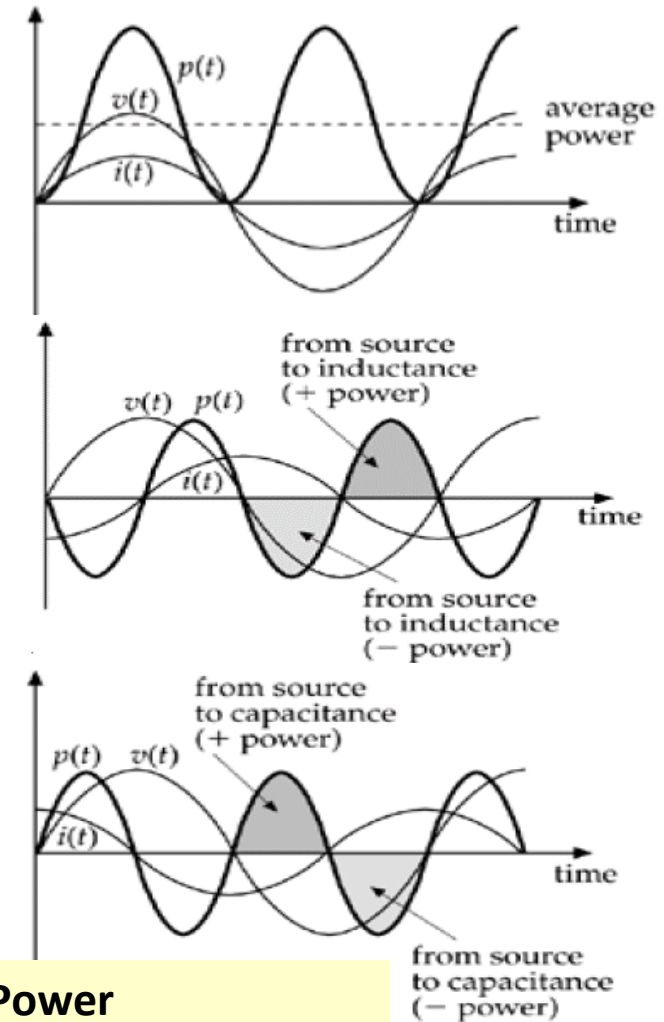
$$\begin{aligned}
 p(t) &= v(t) \cdot i(t) = V_m I_m \sin \omega t \sin(\omega t - \phi) \\
 &= V_m I_m \sin \omega t \{ \sin \omega t \cos \phi - \cos \omega t \sin \phi \} \\
 &= V_m I_m \underbrace{\sin^2 \omega t \cos \phi}_{\frac{1}{2}\{1 - \cos 2\omega t\}} - \underbrace{\sin \omega t \cos \omega t \sin \phi}_{\frac{1}{2} \sin 2\omega t} \\
 \Rightarrow & V_m I_m \left[\frac{1}{2} \{1 - \cos 2\omega t\} \cos \phi - \frac{1}{2} \{ \sin 2\omega t \} \sin \phi \right] \\
 &= \frac{V_m I_m}{2} \{ (1 - \cos 2_{\omega t}) \cos \phi - (\sin 2_{\omega t}) \sin \phi \} \\
 &= \frac{\sqrt{2} V \cdot \sqrt{2} I}{2} \{ (1 - \cos 2\omega t) \cos \phi - (\sin 2\omega t) \sin \phi \} \\
 \Rightarrow & p(t) = V I [(1 - \cos 2_{\omega t}) \cos \phi - (\sin 2_{\omega t}) \sin \phi]
 \end{aligned}$$

Active Power

- Average power to the load
- Pulsates at twice the supply frequency

Reactive Power

- Power exchanges between load and source, average is zero
- Pulsates at twice the supply frequency



Instantaneous Power Contd...

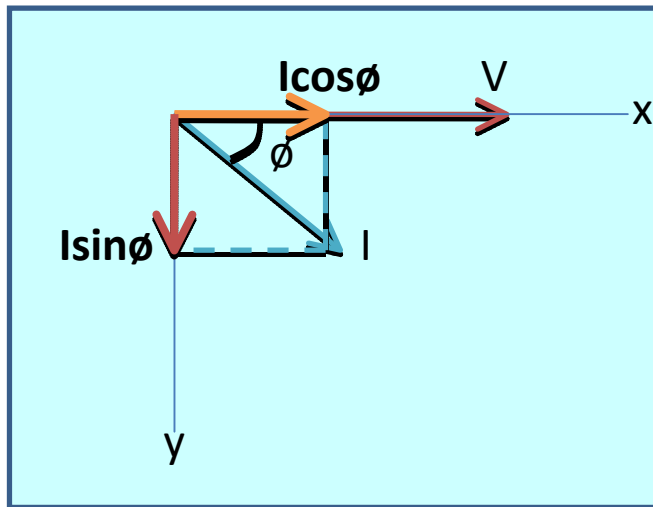
Similarly If equations are solved for a three phase balanced circuit scenario:

$$\begin{aligned} p_{3-\phi(t)} &= V_m \sin \omega t . I_m \sin(\omega t - \phi) + V_m \sin(\omega t - 120) . I_m \sin(\omega t - \phi - 120) \\ &\quad + V_m \sin(\omega t + 120) I_m \sin(\omega t - \phi + 120) \\ &= \frac{V_m . I_m}{2} \left\{ \cos \phi - \cos(2\omega t - \phi) + \cos \phi - \cos(2\omega t - \phi - 240) + \cos \phi - \cos(2\omega t - \phi - 120^\circ) \right\} \\ \text{Since } &[\cos(2\omega t - \phi) + \cos(2\omega t - \phi - 240) + \cos(2\omega t - \phi - 120^\circ)] = 0 \\ \Rightarrow p(t) &= \frac{V_m . I_m}{2} . 3 \cos \phi \\ \Rightarrow \sqrt{3} . V_{Line} . I_{Line} \cos \phi \end{aligned}$$

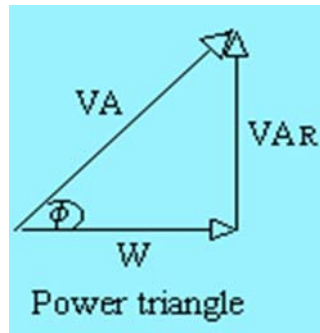
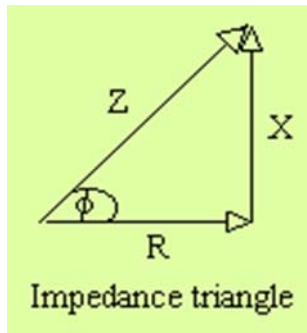
- The total power = 3 x times the active power in each phase *for balanced loads*.
- For both Star and Delta connections, the power is described by: $\sqrt{3} . V_{Line} . I_{Line} \cos \phi$
- The three phase power does not pulsate as in the single phase case

Power Factor

If some angle ϕ exists between the current and voltage then the current can be split into two components along Voltage axis.



- Useful power delivered by the in-phase component of current, $P = V.I \cos \phi$ Watts
- Power exchanged between source and load is delivered by the quadrature component of the current, $Q = V.I \sin \phi$ VAr
- The product of current and voltage is apparent power , $S = V.I$
- Ratio of useful power (real power) to apparent power is called power factor (p.f.), $p.f. = P/S = \cos \phi$
- Increased VAr has following disadvantages:
 - ✓ Burdens the supply with higher current
 - ✓ Increases the transmission loss
 - ✓ Reduces efficiency
 - ✓ Transformers, cables, protection devices have to be overrated to accommodate higher current



$$\cos \phi = \frac{R}{Z} = \frac{W}{VA}$$

Example 4: A three-phase, 400V, star-connected motor has an output of 50kW, with an efficiency of 90% and a power factor of 0.85.

Calculate

- The line current.
- If the motor windings were connected in mesh, what would be the current flowing through the windings of the motor

Solution

a) Efficiency = $\frac{\text{Output Power [W]}}{\text{Input Power [W]}}$

$$\eta = \frac{\text{Output Power [W]}}{\sqrt{3} \cdot V_{\text{Line}} \cdot I_{\text{Line}} \cdot \cos\phi [W]}$$

$$0.90 = \frac{20 \times 10^3}{\sqrt{3} \cdot 400 \cdot I_{\text{Line}} \cdot 0.85}$$

$$\Rightarrow I_{\text{Line}} = 37.74 [A]$$

b) For a Mesh/Delta connected winding,

$$I_{ph} = \frac{I_{\text{Line}}}{\sqrt{3}} \therefore I_{\text{Mtr-Winding}} = \frac{37.74}{\sqrt{3}} = 21.79 [A]$$

Example 5:

- a) Find the complex power ($S = \text{power} + j \text{ Vars}$) when $V = (200 - j150)$ and $I = (-2.8 - j6.3)$
b) Find the current if $S = (1200 - j600)$ and $(V = -120 + j300)$.

Solution:

$$\begin{aligned} \text{a)} \quad S &= V \cdot I^* = (200 - j150)(-2.8 + j6.3) \text{ [VA]} \\ &= (385 + j1680) \text{ [VA]} \\ &= (P + jQ) \end{aligned}$$

Therefore: $P = 385 \text{ [W]}$
 $Q = 1680 \text{ [VAR]}$

- b) Find the current if $S = 1200 - j600$
 $V = -120 + j300$

$$\begin{aligned} S &= V \cdot I^* \\ \Rightarrow I^* &= \frac{S}{V} = \frac{(1200 - j600)}{(-120 + j300)} \\ \Rightarrow I^* &= -3.1 - j2.758 \\ \Rightarrow I &= -3.1 + j2.758 \text{ [A]} \end{aligned}$$

Example 7: Two 3ph generators supply a 3Ø load although separate 3Ø lines. The load absorbs 30kw @0.8pf lag. The line impedances:

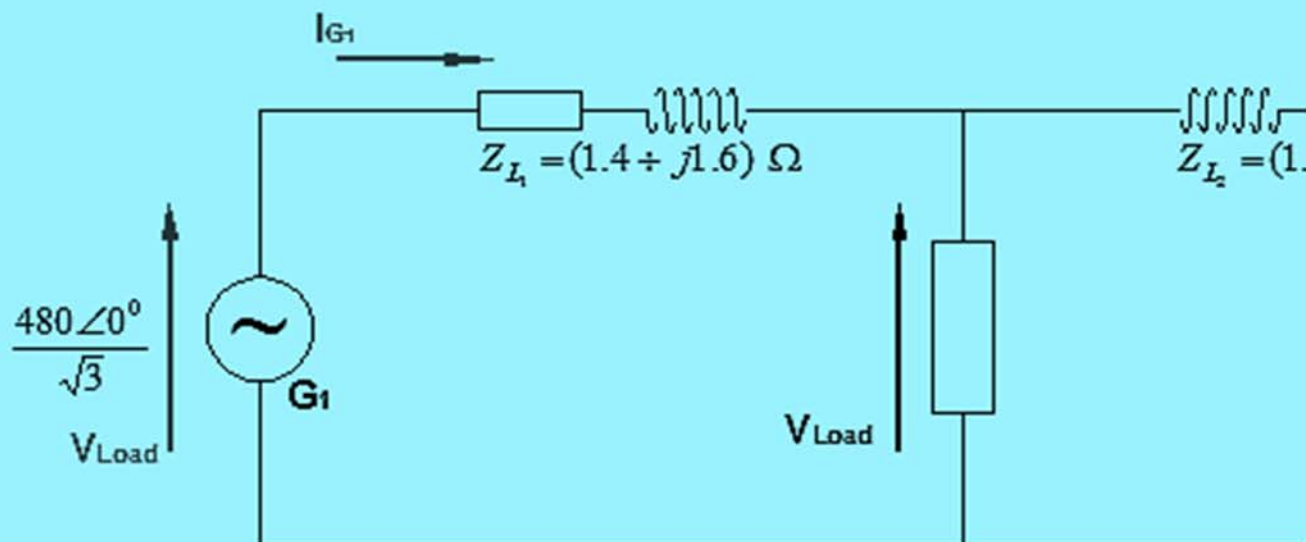
- $Z_{L-G_1} = (1.4 + j1.6)\Omega/ph$
- $Z_{L-G_2} = (1.8 + j1)\Omega/ph$

If generator G_1 supplies 15kw @0.8pF lag with a terminal voltage of 480V ($|V|=460$, {Line], determine

- a) The voltage at the load terminals
- b) The voltage at terminals G_2
- c) The real and reactive power supplied by G_2

Solution:

- a) Assume balanced operation and utilise an equivalent per phase approach



$$\vec{I}_{G1} = \frac{P_{\phi_1}}{\sqrt{3} \cdot V_L \cdot \cos \phi} = \frac{15 \times 10^3}{\sqrt{3} \cdot 460.08} \angle 0^\circ - \cos^{-1} 0.8 = 23.53 \angle -36.87^\circ \quad [A]$$

$$\begin{aligned} \vec{V}_{Load} &= \vec{V}_{G1} - \vec{Z}_{Load} \vec{I}_{G1} = \frac{460}{\sqrt{3}} \angle 0^\circ - [(1.4 + j1.6) \cdot (23.53 \angle -36.87^\circ)] [V] \\ &= 265.6 \angle 0^\circ - 50.08 \angle 11.94^\circ = 216.7 - j10.35 \quad [V] \end{aligned}$$

$$\begin{aligned} \Rightarrow \vec{V}_{Load} &= 216.9 \angle -2.73^\circ & [V_{L-N}] \\ &= 375.7 & [V_{L-L}] \end{aligned}$$

$$\begin{aligned}
 \text{b) } \vec{I}_{Load} &= \frac{30 \times 10^3}{\sqrt{3} \cdot 375 \cdot 7.08} \angle -2.73 - (\cos - 0.8) = 57.63 \angle -39.6 \quad [A] \\
 \Rightarrow \vec{I}_{G_2} &= \vec{I}_{Load} - \vec{I}_{G_1} = (57.63 \angle -39.6) - (23.57 \angle -36.87) \quad [A] \\
 &= 34.14 \angle -41.49 \quad [A]
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow \vec{V}_{G_2} &= \vec{V}_{Load} + \vec{Z}_{Load} \cdot \vec{I}_{G_2} \\
 &= (216.9 \angle -2.73) + ((0.8 + j1.0) 34.14 \angle -41.49^\circ) \quad [V] \\
 &= 259.7 \angle -0.63^\circ \quad [V_{L-N}]
 \end{aligned}$$

$$\Rightarrow |\vec{V}_{G_2}| = \sqrt{3}(259.7) = 449.8 \quad [V.]$$

c) Complex Power

$$\begin{aligned}
 S_{G2} &= 3 \cdot \vec{V}_{G_2} \cdot \vec{I}_{G_2}^* \\
 &= 3(259.7 \angle -0.63^\circ)(34.14 \angle 41.49^\circ) \\
 &= 26.6 \times 10^3 \angle 40.86^\circ = (20.12 + j17.4) \times 10^3
 \end{aligned}$$

$$\Rightarrow P_{G2} = 20.120 \quad [kW]$$

$$Q_{G2} = 17.4 \quad [kVar]$$

Advantages of three phase systems

- A 3-phase system is a dual voltage system. This means that light domestic loads can be from a voltage lower (i.e. V_{ph}) than a commercial or industrial load (V_{line})
- Reduced capital and operating costs of transmissions and distribution - as well as better voltage regulations.
- Total instantaneous power delivered to a system is a constant, not a time dependent variable
- For a given frame surge a 3-phase machine will give an output 50% higher than that of a single-phase m/c of the case frame
- Three Phase supply generates rotational magnetic field to help induction machines to self start unlike a single phase supply case

Example 8: An inductive load draws a current of 10A at a power factor of 0.7 when connected to a 100V, 500Hz supply.

Calculate:

- (a) The apparent power, active power and reactive power drawn by the load, and
- (b) The shunt capacitance that must be connected to the load so that the overall power factor across the load terminals will be unity.

Solution

a) Apparent Power, $S = |V||I| = 10 \times 100 = 1000$ [VA]

Active Power, $P = |V||I|. \cos \phi = 10 \times 100 \times 0.7 = 700$ [W]

Reactive Power, $Q = |V||I| \sin \phi = 10 \times 100 \times \sin \phi = 1000 \times \sin \phi$

$$\phi = \cos^{-1} 0.7 = 45.57^\circ$$

$$\therefore Q = 10 \times 100 \times \sin[45.57^\circ] = 714.1 \quad [\text{VAr}]$$

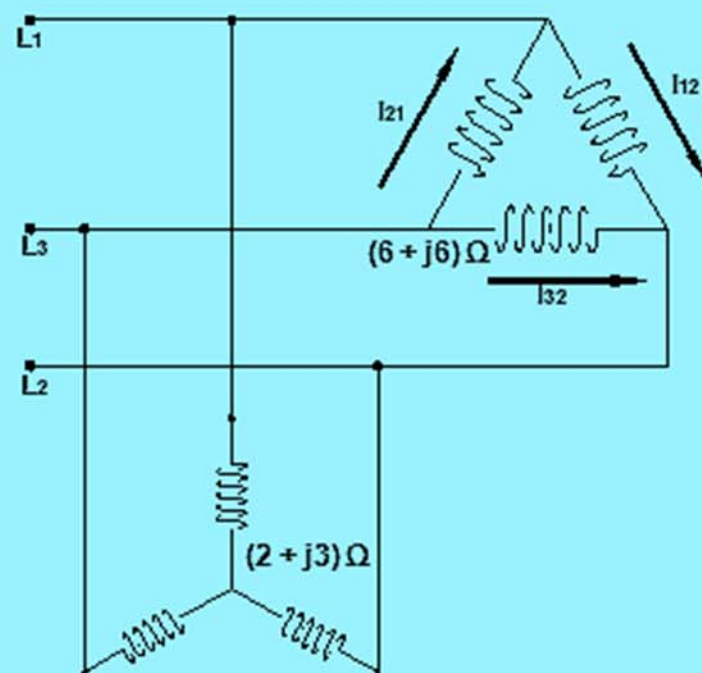
b) $S_c = V.I^* = V \cdot \left[\frac{V}{\frac{1}{j\omega C}} \right]^* = V^2 \cdot (-j2\pi f.C)$ [VA]

$$\Rightarrow 714.1 = 100^2 \cdot (-j2\pi \cdot 50 \cdot C)$$

$$\Rightarrow C = \frac{714.1}{(100^2) \cdot (2\pi \cdot 50)} = 227.3 \quad [\mu F]$$

Example 9: A balanced star-connected load of $(2 + j3)\Omega$ per phase and a balanced delta-connected load of $(6 + j6)\Omega$ per phase are connected in parallel to a three-phase, 400V, 50 Hz supply. Calculate the phase current in each load, the total current in each supply line, the total power supplied and the overall power factor.

Solution



Delta Connected Load:

Essentially V_{Line}

$$I_{12} = \frac{V_{ph} \angle 30^\circ}{Z \angle \Phi^\circ} = \frac{400 \angle 30^\circ}{(6 + j6)} = \frac{400 \angle 30^\circ}{8.485 \angle 45^\circ} = \frac{400 \angle -15^\circ}{8.485} = 47.16 \angle -15^\circ \text{ [A]}$$

$$I_{23} = \frac{V_{ph} \angle -90^\circ}{8.485 \angle 45^\circ} = \frac{400 \angle -135^\circ}{8.485} = 47.16 \angle -135^\circ \text{ [A]}$$

$$I_{31} = 47.16 \angle (-210 - 45)^\circ = 47.16 \angle -255^\circ \text{ [A]}$$

Converting these currents into Line Currents:

$$I_1 = \sqrt{3} \times [47.16 \angle (-15 - 30)^\circ] = 81.68 \angle -45^\circ [A]$$

$$I_1 = \sqrt{3} \times [47.16 \angle (-135 - 30)^\circ] = 81.68 \angle -165^\circ [A]$$

$$I_1 = \sqrt{3} \times [47.16 \angle (-255 - 30)^\circ] = 81.68 \angle -285^\circ [A]$$

Star Connected Load:

$$I_1 = \frac{V_{ph} \angle 0^\circ}{Z \angle \Phi^\circ} = \frac{230 \angle 0^\circ}{(2 + j3)} = \frac{230 \angle 0^\circ}{3.61 \angle 56.31^\circ} = \frac{230 \angle -56.31^\circ}{8.485} = 63.88 \angle -56.31^\circ [A]$$

$$I_2 = 63.88 \angle (-120 - 56.31)^\circ = 63.88 \angle (-176.31)^\circ [A]$$

$$I_3 = 63.88 \angle (-240 - 56.31)^\circ = 63.88 \angle (-296.31)^\circ [A]$$

Total Line Currents:

$$I_{1\text{-Total}} = 81.68 \angle -45^\circ + 63.88 \angle -56.31^\circ = 144.86 \angle -49.96^\circ \quad [\text{A}]$$

$$I_{2\text{-Total}} = 81.68 \angle -165^\circ + 63.88 \angle -176.31^\circ = 144.86 \angle -169.96^\circ \quad [\text{A}]$$

$$I_{3\text{-Total}} = 81.68 \angle -285^\circ + 63.88 \angle -296.31^\circ = 144.86 \angle -289.96^\circ \quad [\text{A}]$$

Power: $S = V.I^*$
 $= [P + jQ]$

$$S_1 = (230.9)(144.86 \angle -49.16) = (21.52 + j25.61) \times 10^3$$

$$S_2 = (230.9 \angle -120^\circ)(144.86 \angle -169.16) = (21.52 + j25.61) \times 10^3$$

$$S_3 = (230.9 \angle -240^\circ)(144.86 \angle -289.16) = (21.52 + j25.61) \times 10^3$$

Therefore: $S = [P \quad + \quad jQ]$

$L_1 = [21.518$	$+$	$j25.61] \times 10^3$
$L_2 = [21.518$	$+$	$j25.61] \times 10^3$
$L_3 = [21.518$	$+$	$j25.61] \times 10^3$
<u>64.55 kW</u>		<u>j76.83 kVAr</u>

$$S_{\text{Total}} = 100.25 \angle -49.96^\circ$$

Power Factor: $\cos \Phi = \cos -49.96^\circ = 0.64 \text{ [Lag]}$