Rectification

Introduction

Rectification also known as Conversion in the power electronics world. It involves converting some kind of alternating voltage into a DC voltage. The AC voltage source could be the mains supply or it could be the output from a high frequency transformer in a switching power supply.

The basic element of a rectifier is the diode. Low frequency rectifiers typically make use of bridge rectifiers or diodes such one from the 1N400x series. High frequency rectifiers typically make use of fast recovery diodes or perhaps even MOSFETS.

Single phase half wave rectifier.

Figure 1 shows a single phase half controlled rectifier. It is a poor rectifier in many ways. The output voltage has large gaps requiring lots of smoothing to provide a smooth DC output. The input current is unidirectional which will cause a transformer that supplies it (V1) to become magnetically saturated leading a lowering of voltage and an increase in losses in the transformer. This problem can largely be solved by using full wave rectification

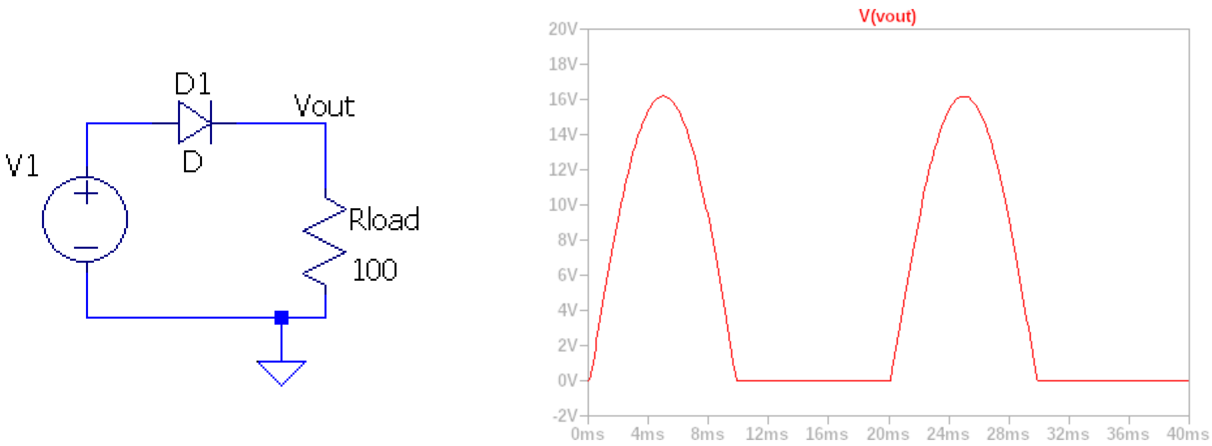


Figure 1: A single phase half wave rectifer

Single phase push-pull rectifier.

Figure 2 shows a transformer with split primary and secondary windings. These windings are not electrically connected (until you connect them). If you are connecting to a 230V mains you connect the primary windings (top) in series with a link as shown. Neutral and Live can then be connected. Mains earth is bonded to the body of the transformer using a lug of some kind.

The secondary side (bottom) is also split and, if joined as shown will produce a 24V rms AC output with a centre tap. This can be used to drive a push pull rectifier as shown in Figure 3

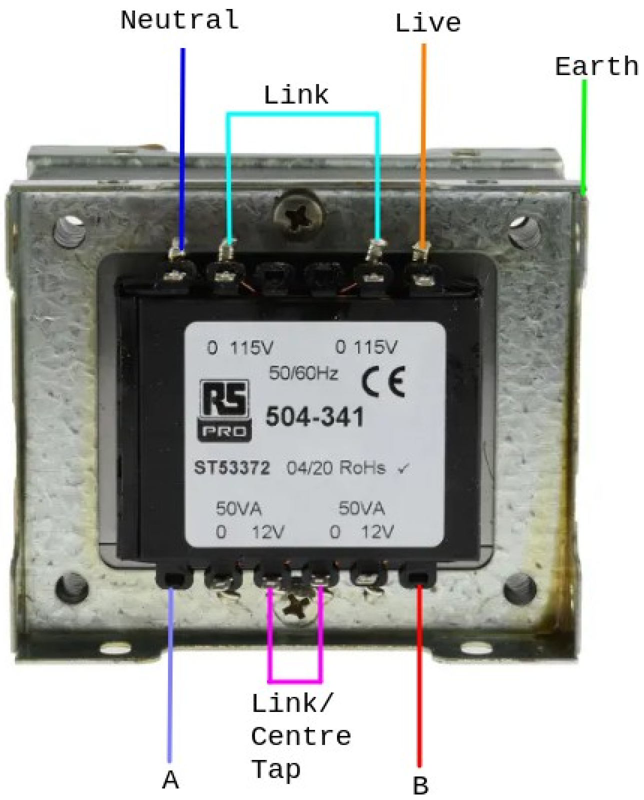


Figure 2: A 100VA transformer

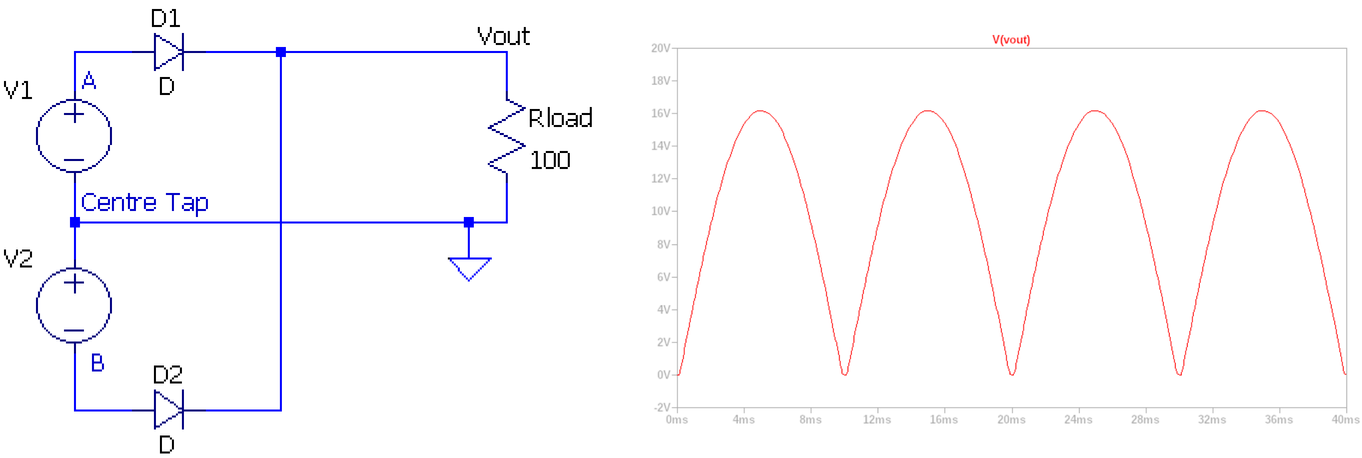


Figure 3: A push pull rectifier.

Note: The output voltage peak is given by $\hat{Vo}=\sqrt{2}.12-0.7$. The "0.7V" figure being the forward voltage drop of a diode.

Single phase full wave rectification with bridge rectifier.

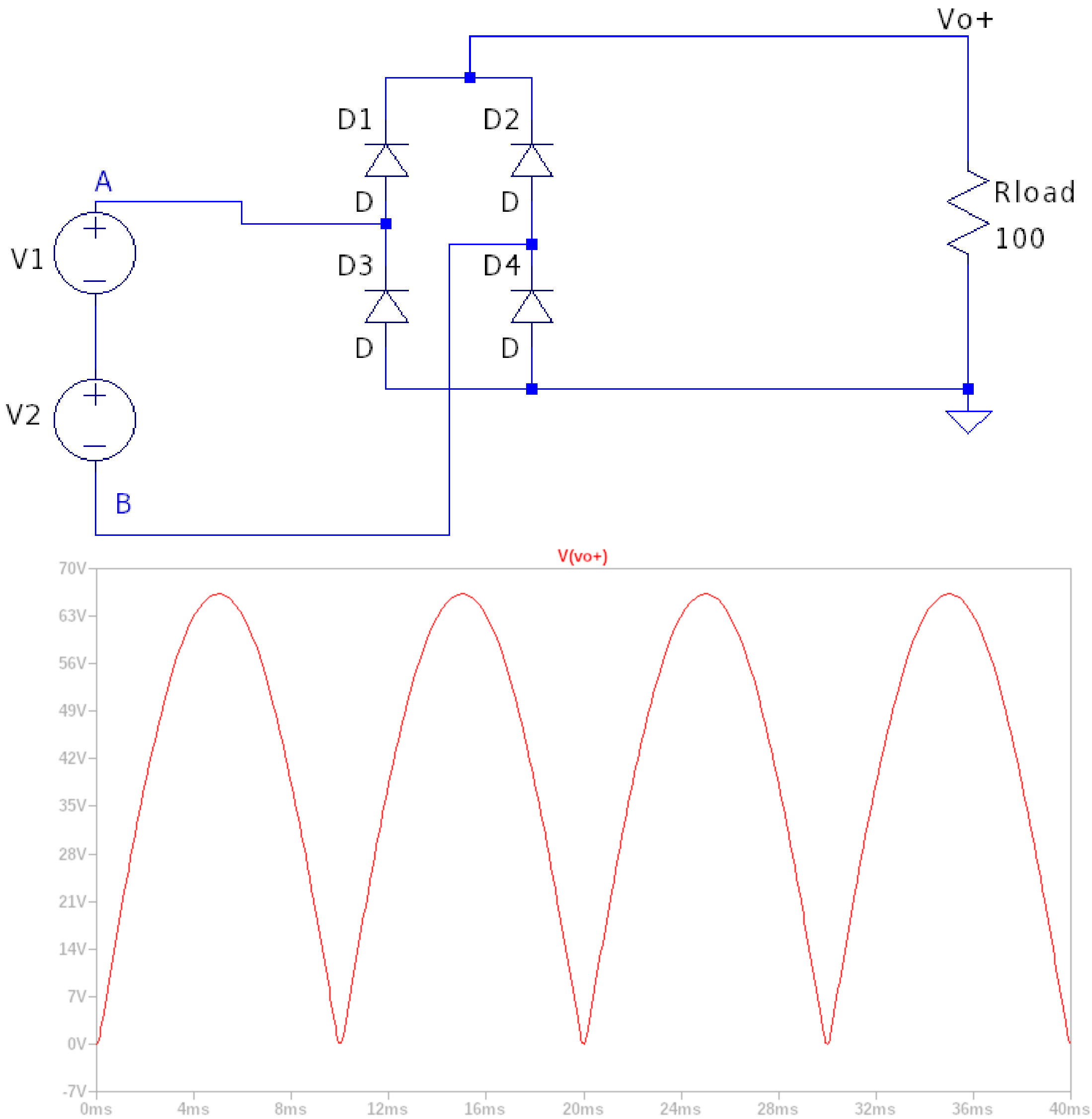
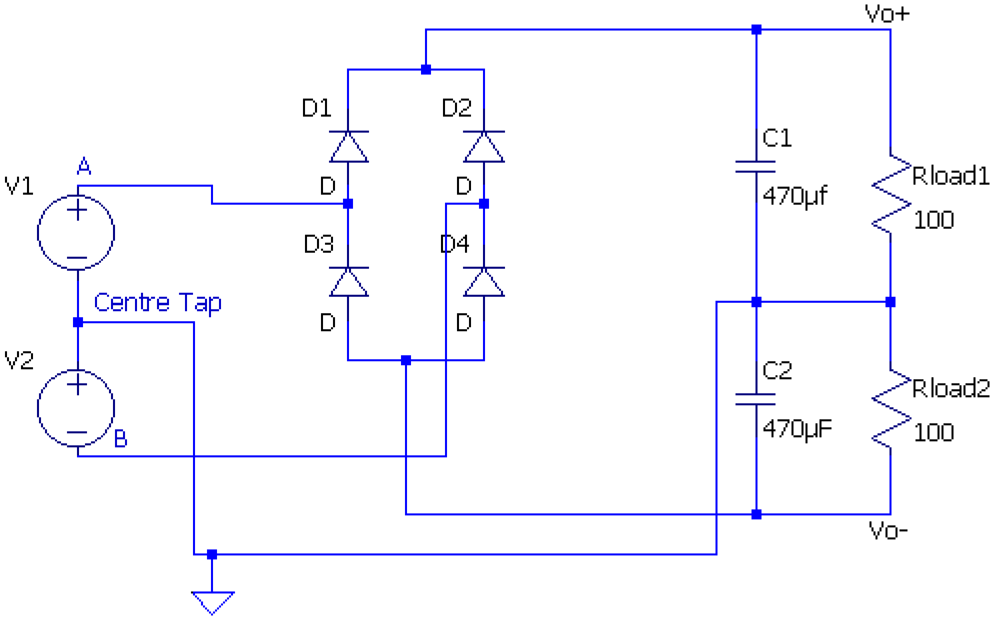


Figure 4: A bridge rectifier

Bridge rectifiers can be made from 4 individual diodes or bought as a module. Figure 4 above shows the output from the bridge rectifier when it is driven by the series connection of the two secondary windings of the transformer in Figure 2. Note the height of the output voltage. If a lower voltage, higher current output is required then the secondary windings could be connected in parallel (0 to 0, 12 to 12 !!).

Figure 5: A bridge rectifier module

Bipolar power supply



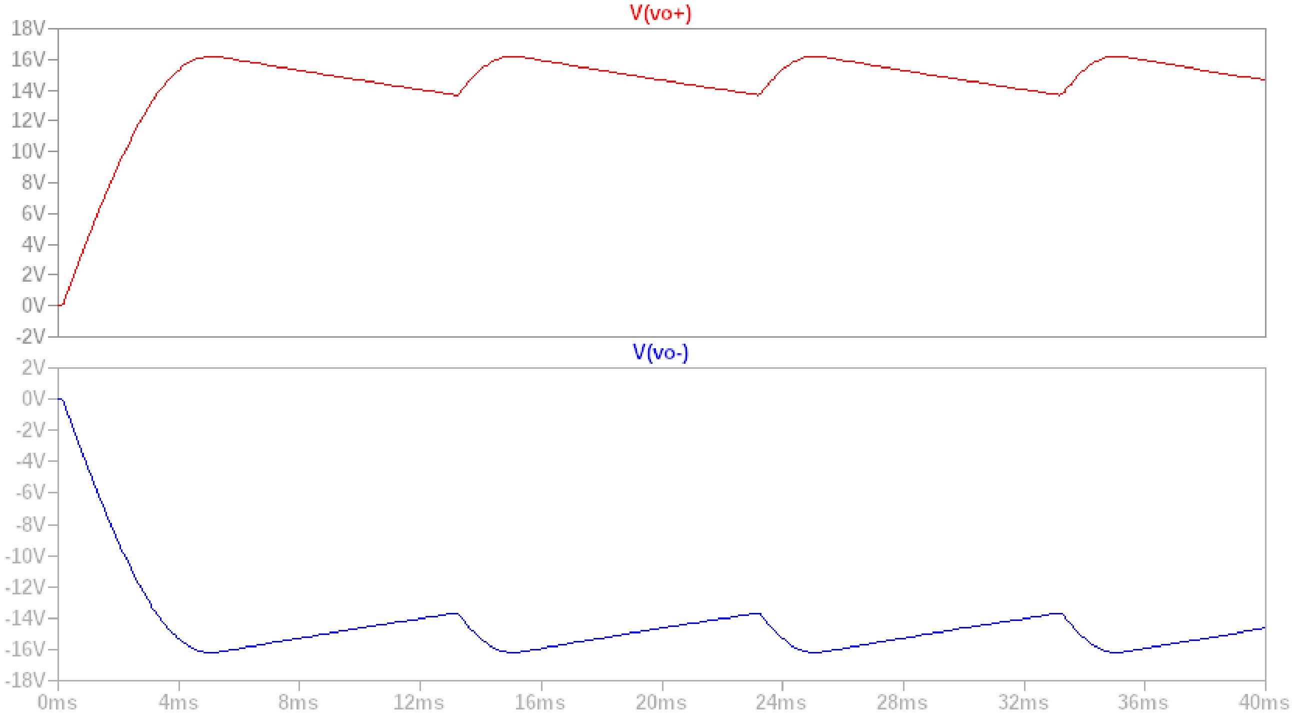


Figure 6: A bipolar DC power supply.

Referring back to the transformer in Figure 2 it is possible to make a bipolar DC power supply using a single bridge rectifier as shown in Figure 6. Note that the output voltage magnitude drops to about 14V. This makes it unsuitable for regulation using linear 15V voltage regulators (e.g. 7815). It would however be suitable for use with 12V linear regulators.

Smoothing rectifier output voltage.

A smoothing or reservoir capacitor can be used to smooth the output voltage from a rectifier as shown in Figure 7. A large electrolytic capacitor can supply current to the load when the incoming voltage falls between peaks. The larger the smoothing capacitor, the smoother the output. There is a serious disadvantage however in using ever larger smoothing capacitors: It leads to large spikes in the input current (I(V1) in Figure 7).

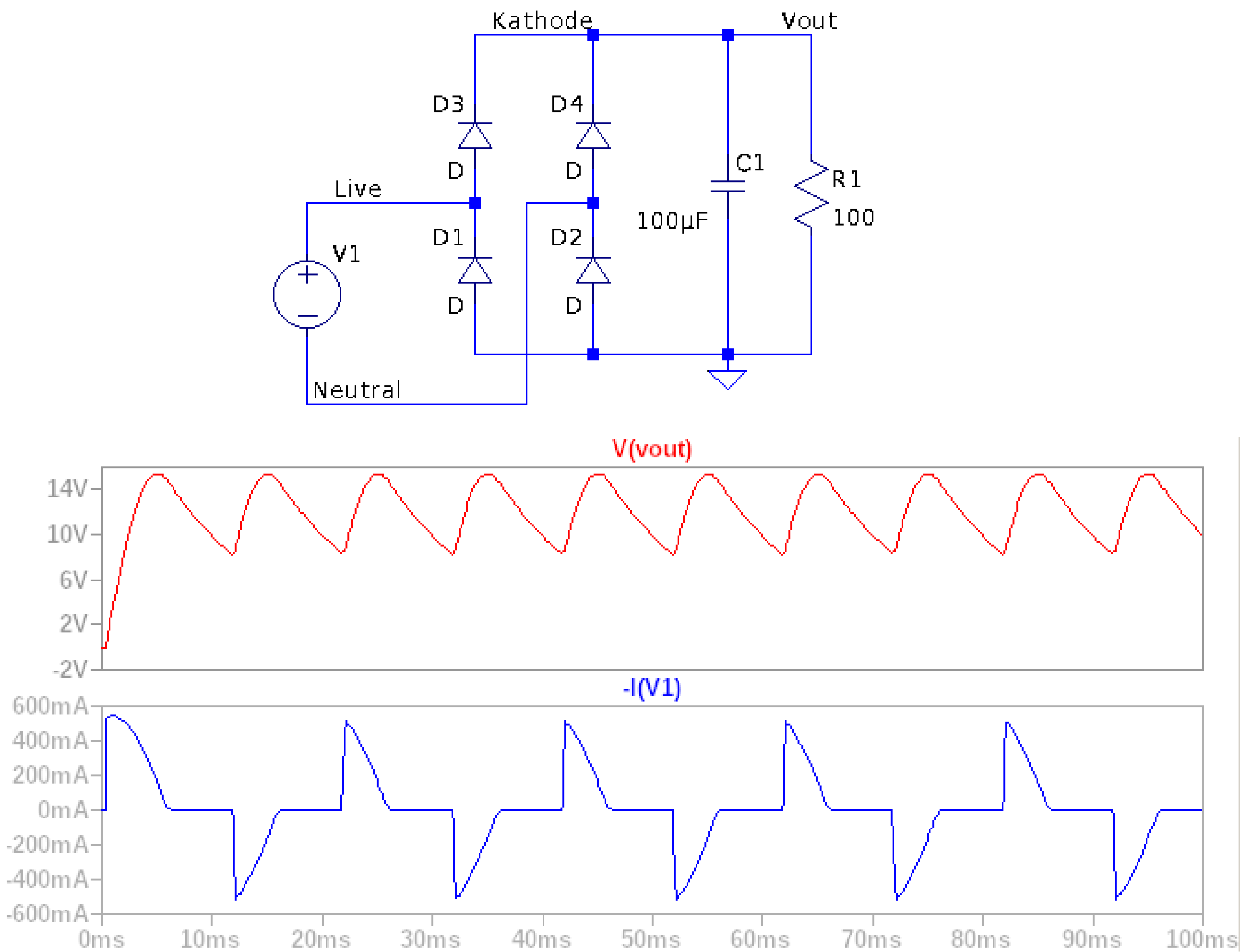


Figure 7: A rectifier with capacitor smoothing

An estimate of a value of C that will achieve a desired output ripple can be obtained relatively easily.

1) Assume that the current drawn from the capacitor when it supplies the load is given by:

$$i_C = \frac{\hat{V}}{R_{load}}$$

This exaggerates the actual current flow a little over the course of a full cycle.

2) Assume that the capacitor discharges for the full interval between peaks (i.e. for a half cycle).

Using these assumptions we can say:

$$\Delta Q = \frac{\hat{V}}{R}\frac{T}{2}$$

This represent an exaggerated measure of the charge removed from the capacitor when it is alone supplies the load. This leads to a fall in capacitor voltage given by:

$$\Delta V_c = \frac{\Delta Q}{C}$$

In a typical scenario, ΔV_c would be a design target. Apart from C, the remainder of the variables are known so C can be calculated.

Example:

A full wave 50Hz rectifier has a peak output voltage of 17V and drives a 200Ohm load. Calculate a suitable value for an output smoothing capacitor such that the output ripple voltage is at most 1V (peak to peak).

$$\Delta Q = \frac{\hat{V}}{R}\frac{T}{2}$$

$$\Delta Q = \frac{17}{200}.10\,x\,10^{-3}$$

$$\Delta Q = 0.00085\,Coulombs$$

$$\Delta V_c = \frac{\Delta Q}{C}$$

$$C = \frac{\Delta Q}{\Delta V_c}$$

$$C = \frac{0.00085}{1}$$

$$C = 850\,\mu F$$

Simulating this in LTSpice reveals the actual ripple voltage to be 0.8V approx.

Given the effect on the input current of capacitor smoothing alone it is not recommended to add ever larger capacitors to a rectifier output in an effort to reduce ripple. A combination of an L-C filter could be used or alternatively a smaller smoothing capacitor and an active linear or switching voltage regulator. In the latter case, the lowest output voltage should not go below the minimum input voltage for the active voltage regulator.

Single phase half Controlled Rectification.

Figure 8 shows a full wave rectifier with two of the diodes replaced by thyristors. Thyristors only turn on when they are forward biased and a pulse of current is sent in to their gates. The idea behind a controlled rectifier is that you can regulate the output voltage by allowing a controlled portion of each half cycle to be rectified.

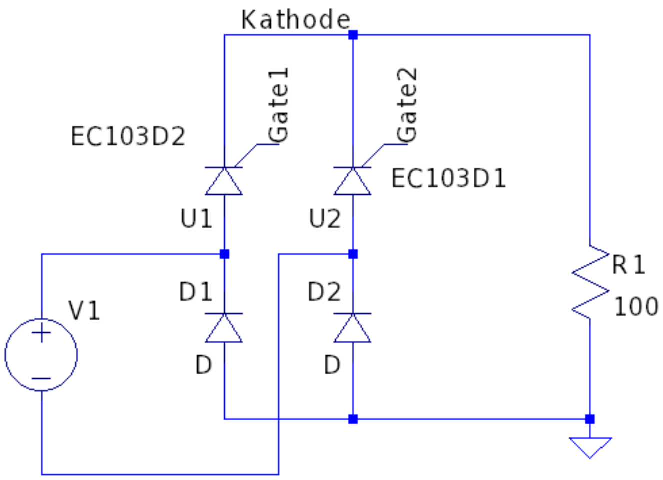


Figure 8 : A half controlled single phase rectifier

Figure 9 shows the input voltage (red) and the output voltage (blue). As you can see, only part of each half cycle is permitted to pass through to the load. The turning on of each of the thyristors has been delayed by approx 100 degrees in this case. This delay is commonly called the firing angle α.

The effect of changing α is to alter the average output voltage and hence the power to the load. Typically these sorts of rectifiers are used to drive loads that can tolerate a large voltage ripple like this e.g. DC motors.

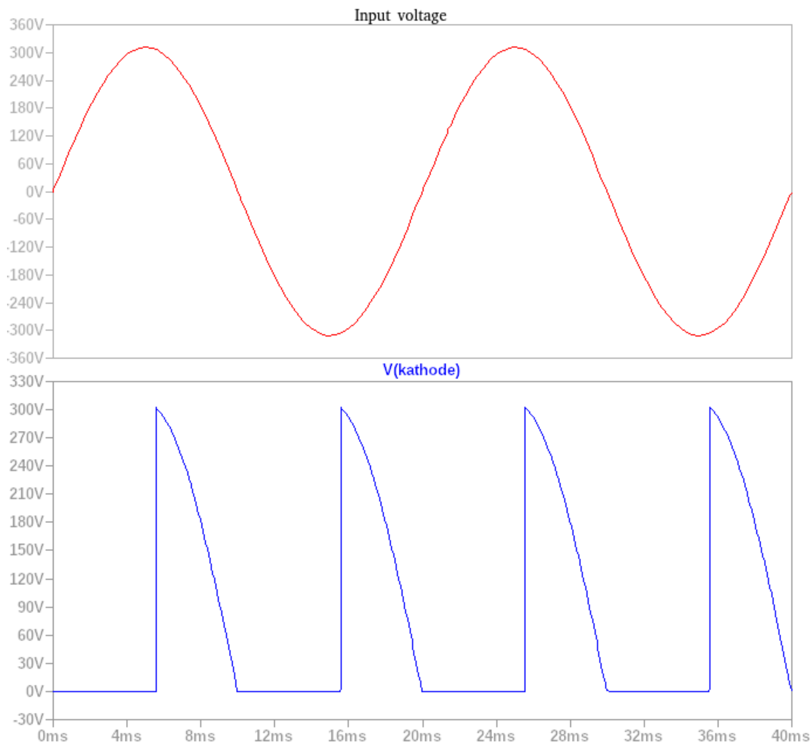


Figure 9: Input and output voltages for a single phase controlled rectifer

If all diodes were replaced by thyristors you would have a fully controlled rectifier. Such rectifiers are capable of controlling power flow in both directions i.e. they can invert a DC supply back to AC.

Three phase rectifiers.

Three phase rectifiers can be supplied from a star or delta connection.

This rectifier is said to be a six-pulse rectifier as the output voltage has six small peaks for each full cycle of the mains input. The overlap between rectified phases leads to an output voltage with very little ripple as shown in Figure 11.

By using a mix of star and delta connections from a suitable transformer it is possible to construct a 12 pulse rectifier which has even smaller output ripple. This makes it suitable for use in many applications without large filter components.

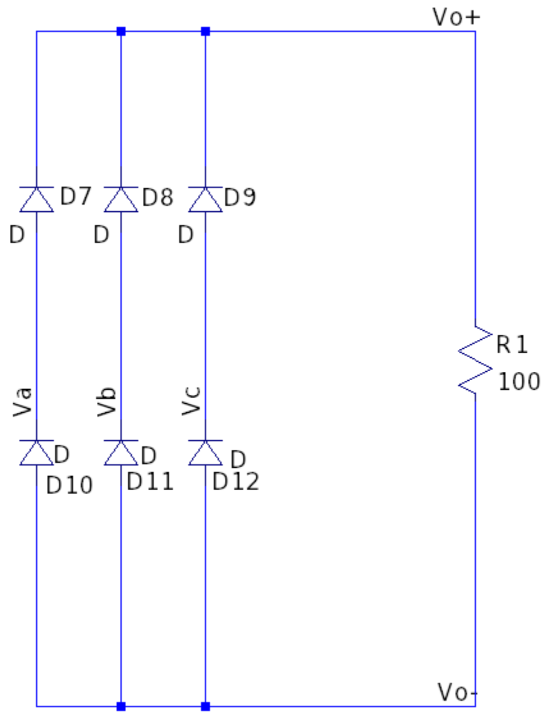


Figure 10: 3 phase rectifier.

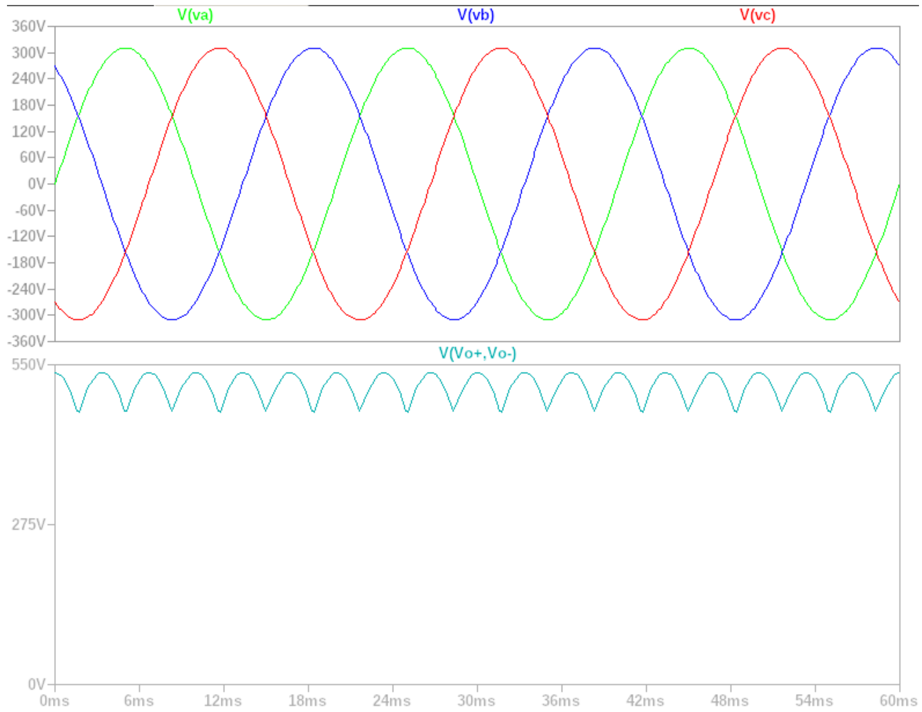


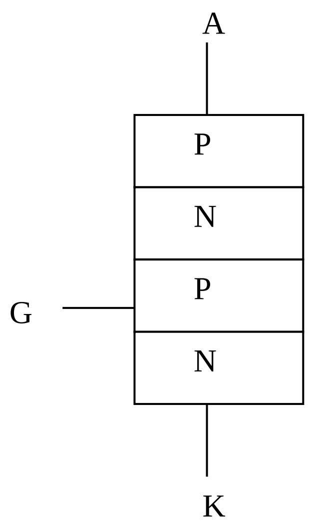
Figure 11: Input and output voltage for a 3 phase rectifier

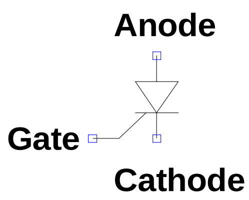
The following notes deal provide a mathematical analysis of rectifiers and were produced by Kevin Gaughan.

DT021-3 Thyristors and Thyristor Controlled Rectifiers

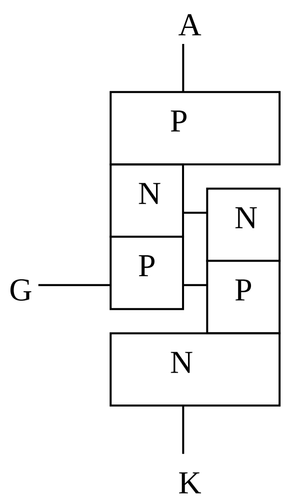
1. **The Thyristor also known as a Silicon Controlled Rectifer (SCR)**

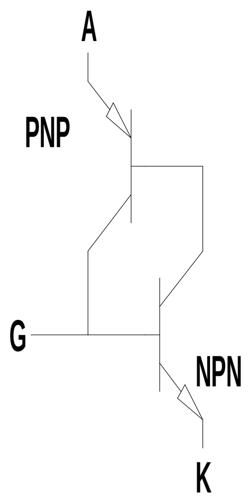
Construction:





Two Transistor Equivalent Model:





Explanation of Two Transistor Model:
If the pnp transistor is initially off and no current is provided at the gate then the pnp transistor has no base current and is off. This in turn means that the PNP transistor had no base current and also remains off. The thryistor remains in its non conducting state.

On the other hand if we introduce a small current into the gate this will turn on the NPN transistor. Providing there is external voltage applied between Anode and Cathode then current flow into the base of the PNP and through the npn. This will turn on the pnp which in turn will supply base current to the NPN **even after the external base current has been removed.** The thyristor latches on It will remain on until the external voltage is removed at which time it will revert to its non-conducting state.

In practise once the gate current is removed the device will only remain on providing the the anode current has exceeded a certain minimum value known as the **Latching current**. Once the latching current has been achieved the device will remain on until the anode current falls below a lower current level called the **holding current**.
Note that $I_L > I_H$

Note: Latching current and holding current scale with current rating of the thyristor so it is important to correctly size a thyristor for an application. If you use a thyristor with too generous a current rating the circuit current may not build up to the latching current during the gate pulse and the thyristor may not latch on.

Note that once a standard thyristor has started conducting it cannot be turned off via the gate. It behaves like a diode until such time as the anode current falls below the holding current and the device reverts to its blocking state.

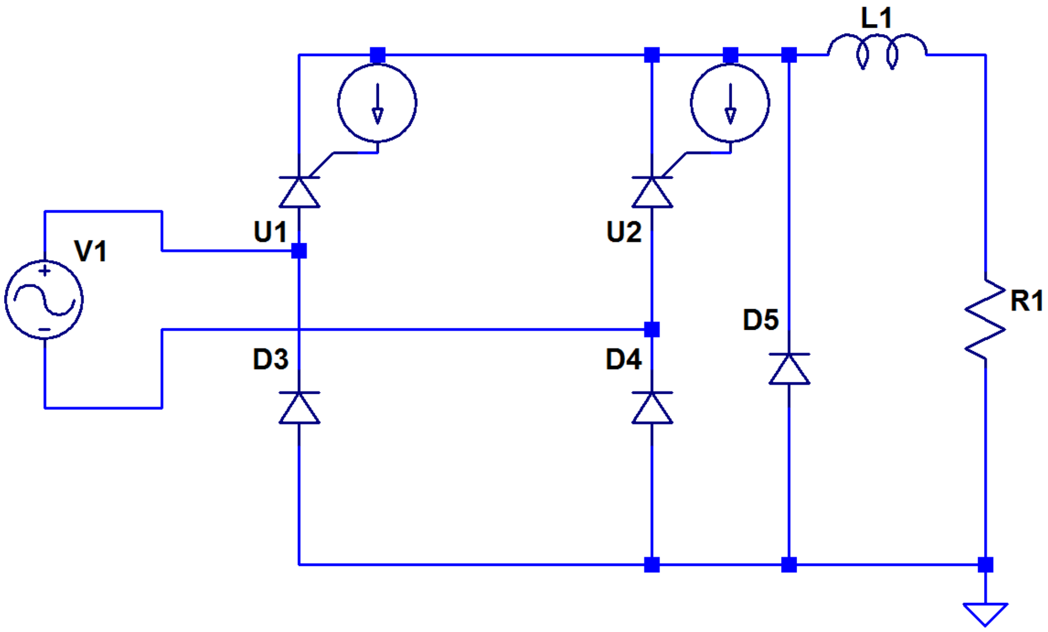
Applications of Thyristors
Thyristors are extremely robust semi-conductors and a wide range of models are available up to very high levels of voltage and current. Thyristor converters have been implemented at power levels of up to 100's MW for example in the DC link between the power grids of Ireland and Scotland. A version of the thyristor called a TRIAC is also widely used in low cost low power AC control systems (for example in dimmer switches for incandescent light bulbs.

Thyristor Derivatives:

Triac: Effectively two back to back thyristors in one package with a single gate. Widely used for AC power control.
GTO: A thyristor which can be forced to turn off by pulling current out of the gate.
IGCT: Based on an GTO with integrated gate drive circuit. Optimised for fast turn on and off.
MCT: A hybrid of Mosfet and Thyristor. The thyristor may be turned on or off by application of a appropriate voltage signal to the gate. Allows high speed switching.

Mathematical analysis of a half Controlled Single Phase Bridge Rectifier (Semiconverter)

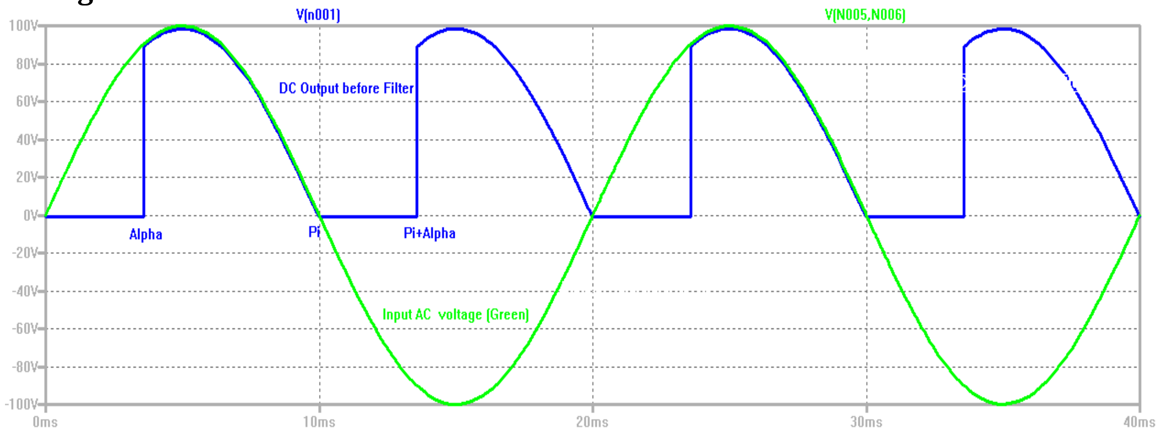
Circuit Diagram:



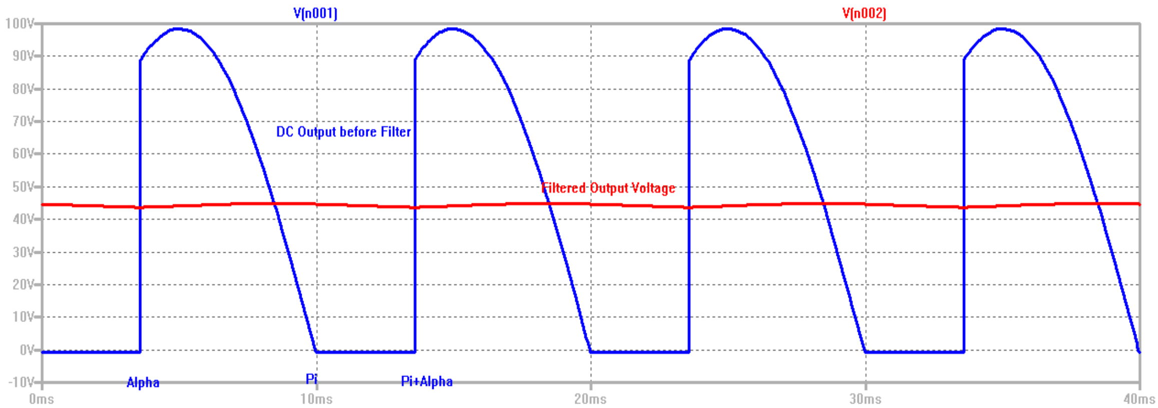
Description: A full wave bridge rectifier in which the top two diodes are replaced by thyristors. The turn on of each thyristor is delayed by a time α (radians) from the time that a diode in that position would naturally conduct. By delaying conduction in this way the average output voltage may be controlled. An inductor is included to smooth the output ripple. We assume that the corner frequency of the filter provided by L and R is much lower than the frequency of ripple ($2xf_{ac}$) so that the ripple is greatly attenuated and the output current in R is almost pure DC. For good filtering: $\dfrac{1}{2\pi \frac{L}{R}} << 2 f_{ac}$

During the delay time (α) the inductor current requires a freewheel path and this is provided by D5. In fact the circuit can work without a freewheel diode because the current can also freewheel through a leg diode and associated thyristor (for example D3 and U1). However a separate freewheel diode reduces losses because the voltage drop across the freewheel path is lower.

Voltage Waveforms



The plot above shows the Input Voltage sinewave (Green) and the unfiltered DC output measured across D5 (Blue). The firing angle Alpha in this case is 63.5°. The plot below shows the effec tof the LR filter. The red line is the filtered output voltage which is a smooth DC voltage equal to the average of the voltage across D5.

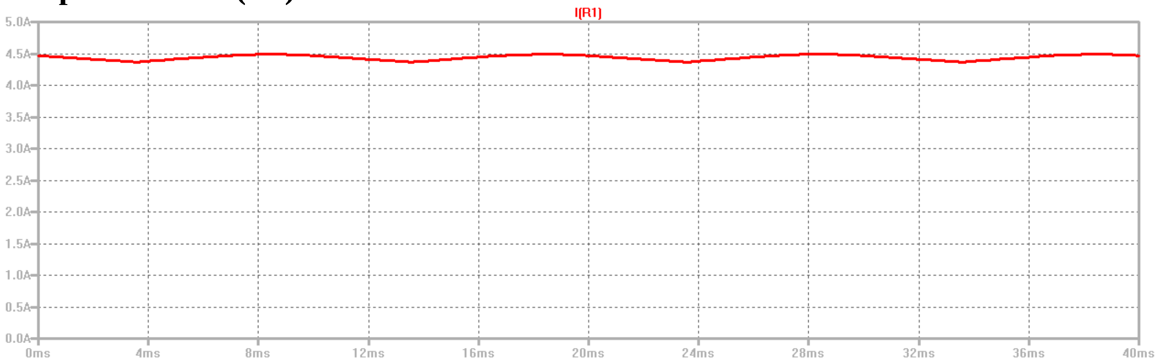


If the firing angle (α) is zero then the output is identical to a diode bridge rectifier. As the firing angle is increased the proportion of the rectified wave that is let through diminishes and the average output voltage falls.

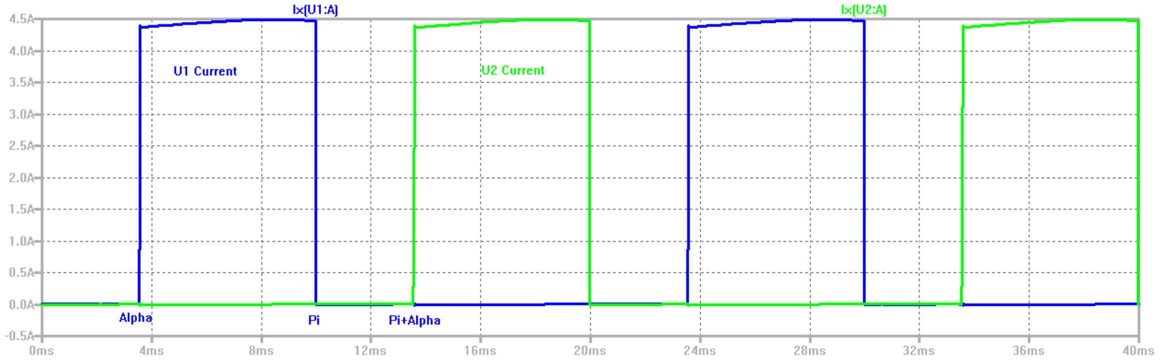
Notice that the region for which alpha makes sense is: $0 \leq \alpha \leq \pi$ radians.

Current Waveforms

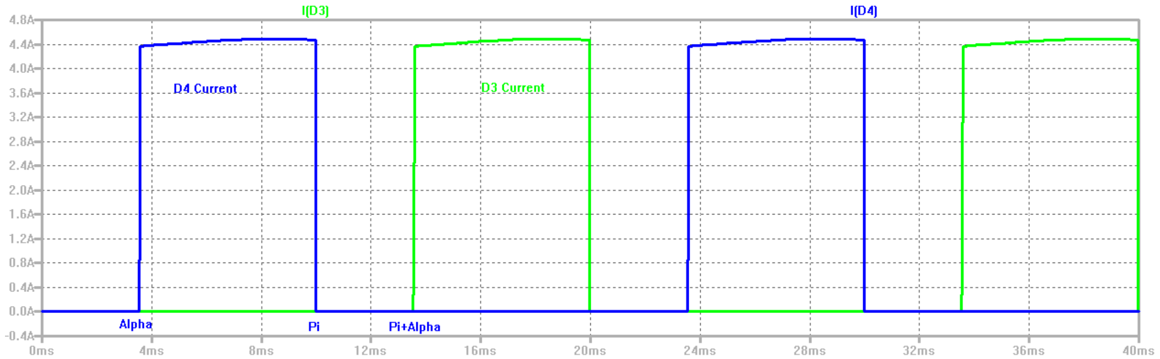
Outpur Current (R1)



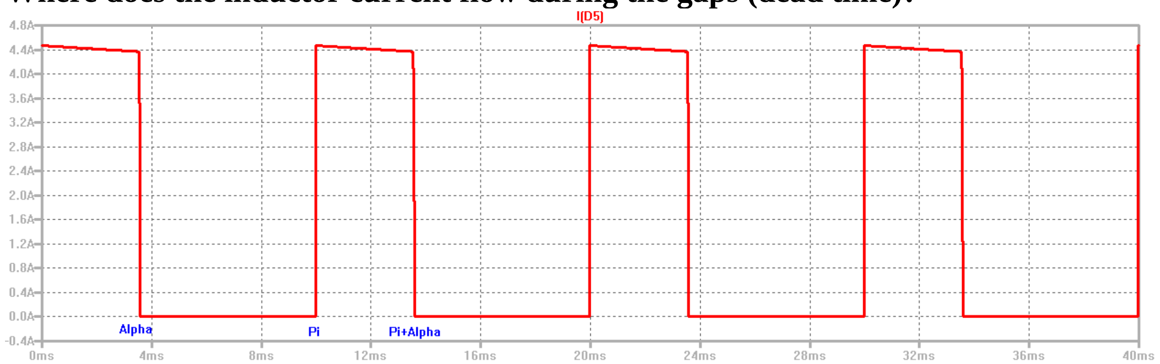
U1 anode current and U2 Anode Current



D3 and D4 current

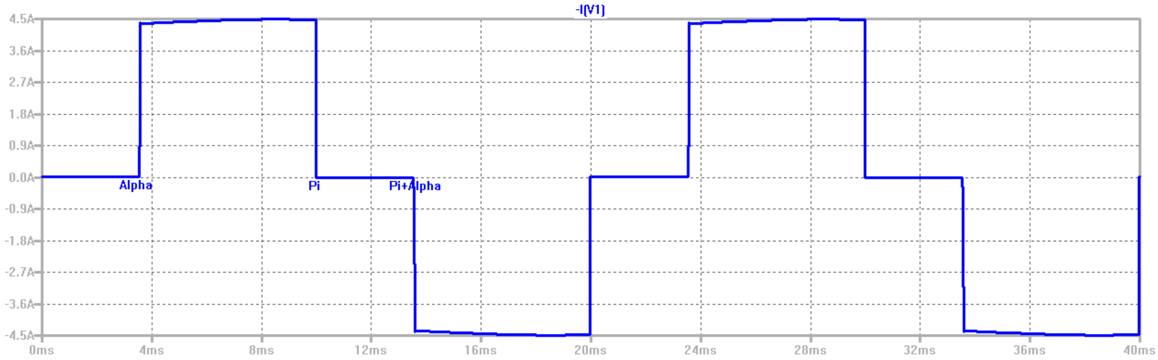


Where does the inductor current flow during the gaps (dead time)?



Notice that the output current is quite smooth with low ripple. During the positive half cycle the output current flows through U1 and D4. During the negative half cycle it flows through U2 and D3. During the dead times (between 0 and α and between π and π+α the inductor causes the current to freewheels through D5).

AC Line Current



From Kirchoff's first law we can see that the AC input current equals U1 current minus D3 current. This is a square wave with dead time in the regions 0 to α and π to π+α.

Analysis
Average output Voltage:
The output voltage V_o can be obtained by calculating the average of the voltage across D5:

$$V_o = \frac{\int_{\alpha}^{\pi} V_m \sin(\omega t).d\omega t}{\pi} = V_m \left| -\cos(\omega t) \right|_{\alpha}^{\pi} = \frac{V_m(1+\cos(\alpha))}{\pi}$$

Where
V_o=Average DC output voltage
V_m=Peak of AC input sinewave voltage
α=firing angle (0< π radians)

Notice that at α=0 the output voltage is $2V_m/\pi$ which is just the average value of a full wave rectified sinewave and is the same output voltage eyou would get from a four diode bridge rectifier with LR filter.
As α increases the voltage falls off along a cosine curve falling to zero at α= π.
Values of α greater than π radians do not make sense because the thyristors are reversed biased after π radians and will not conduct even if they get a gate pulse.

RMS Input Current:
It will be useful also to calculate the rms value of the ac input current. Since the positive and negative half cycles are mirror images of one another we need only calculate the rms over π radians. Neglecting the small ripple on the output current:

$$I_s = \sqrt{\frac{\int_\alpha^\pi I_o^2 . d\omega t}{\pi}} = I_o \sqrt{\frac{\int_\alpha^\pi 1 . d\omega t}{\pi}} = I_o \sqrt{\frac{\left|\omega t\right|_\alpha^\pi}{\pi}} = I_o \sqrt{\frac{(\pi - \alpha)}{\pi}}$$

Remember of course that $I_o = \frac{V_o}{R_1}$

Where
I_s= rms input ac current
I_o=average dc output current

Rectification Ratio (rectification efficency)
With an appropriately sized inductor the output ripple can be reduced to a negligible value giving a rectification ratio as close to 100% as required.

Utilisation Factor:

$$Utilisation\,Factor = \frac{DC\,Output\ power}{InputVA} = \frac{V_o I_o}{V_s I_s} = \frac{\frac{V_m(1+\cos(\alpha))}{\pi}.I_o}{\frac{V_m}{\sqrt{2}} I_o \sqrt{\frac{(\pi - \alpha)}{\pi}}} = \frac{\sqrt{2}(1+\cos(\alpha))}{\sqrt{\pi(\pi - \alpha)}}$$

AC Power Factor

For a single phase system with non sinusoidal input currents the input power factor is defined as:

$$PowerFactor = \frac{AC\ Input\ power}{Input\,VA}$$

From energy balance we can say that *AC Input power = DC output Power + AC Output Power + Losses*

So if we assume that the L/R time constant is large enough to filter out any output voltage ripple and also that losses in the Thyristors and diodes are negligible (assume that the forward voltage drop is a small fraction of the output voltage) then we can approximate:

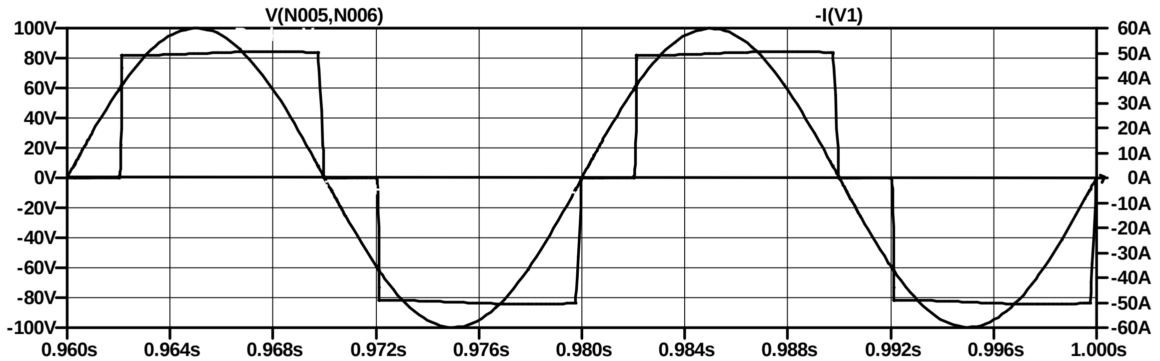
AC input power = DC Output Power

In which case the *Power Factor = Utilisation Factor =* $\frac{\sqrt{2}(1+\cos(\alpha))}{\sqrt{\pi(\pi - \alpha)}}$

Notice that when α=0 Power Factor =0.9
When α=π/2 Power Factor = 0.637
As α approaches π the power factor falls to $\lim_{\alpha \to \pi} \frac{\sqrt{2}(1+\cos(\alpha))}{\sqrt{\pi(\pi - \alpha)}}$ =0

Comment on power Factor
Most utilities impose a sever penalty on commercial customer who have a poor power factor. In Ireland penalties are imposed for power factors below 0.95. A major disadvantage of the thyristor controlled rectifier is that its power factor becomes very poor at low output voltages.

Why does the power factor degrade?



1. Phase shift – By observation of the current waveform we can see that the fundamental of the current waveform is phase shifted by an angle α/2 .This phase shift contributes to the reduction in power factor
2. In systems with non sinusoidal currents the harmonics also contribute to the reduction in power factor.

More Complete Definition of power Factor

Previously you learned that power factor = cos(ϕ)

A more complete definition of power factor is

$$Power\ Factor = \frac{\mathrm{Re}\,al\ Power}{Apparent\ \ power(VA)}$$

In a system with sinusoidal voltage and current this become the familiar form

$$Power\ Factor = \frac{\mathrm{Re}\,al\ Power}{Apparent\ \ power(VA)} = \frac{V_s I_s \cos(\varphi)}{V_s I_s} = \cos(\phi)$$

If the current waveform is no sinusoidal however and has harmonics it turns out that only the fundamental component of the current (I_1) gives real power. The harmonics just contribute to poor power factor.

$$Power\ Factor = \frac{\mathrm{Re}\,al\ Power}{Apparent\ power(VA)} = \frac{V_s I_1 \cos(\varphi_1)}{V_s I_s} = \frac{I_1}{I_s}\cos(\phi_1)$$

Exercise:

A half controlled thyristor rectifier with LR filter is required to supply a 150V / 10Amp resistor load. The AC input voltage is 230V rms at 50Hz. Losses in the rectifier may be assumed to be negligible.

1. Sketch the circuit diagram of the rectifier required.
2. Recommend a value of inductance such that the filter cut off frequency is 5% of the output's ripple frequency. (Answer 477mH)
3. Calculate the firing angle (α) required. (63° / 1.1 radians)
4. Sketch the voltage waveform across the freewheel diode.
5. Sketch the input current waveform.
6. Calculate the rms input current. (8.05A)
7. Calculate the power factor of this rectifier. (0.81)
8. Calculate the fundamental component of the ac supply current. (7.6A)

More Jargon:

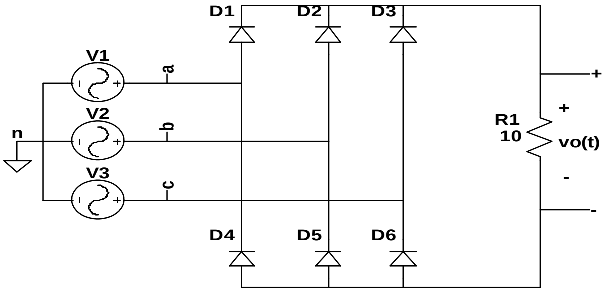
Some further terms you may come across in relation to thyristors and rectifiers:

Phase Control: Thyristor circuits where the thyristor firing angle is determined by the phase of the ac supply.

Line Commutated: Thyristor circuits where the thyristors are left to conduct until the natural part of the cycle where a diode in the same position would turn off.

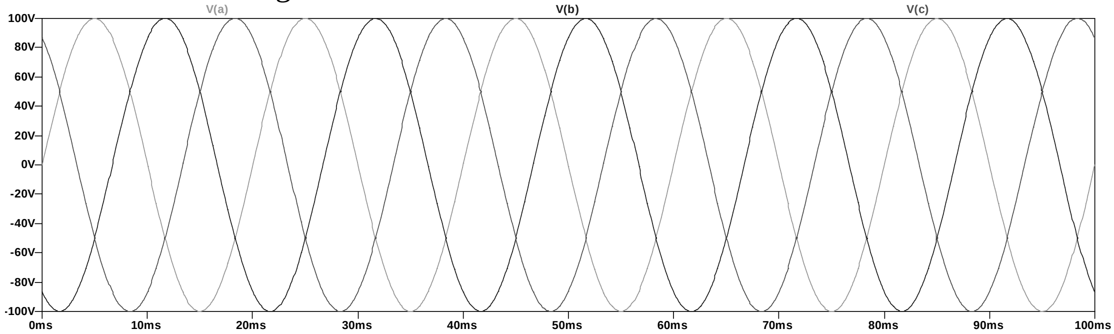
Force Commutated: Thyristor circuits where some means is used to turn of fthe thyristor at a point other than the natural part of the cycle.

Mathematical analysis of a 3 Phase Rectifier with Resistive Load

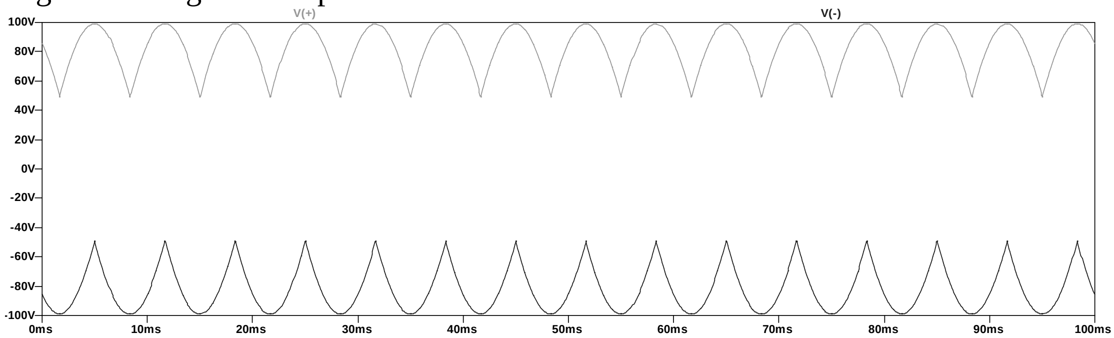


Voltage Waveforms:

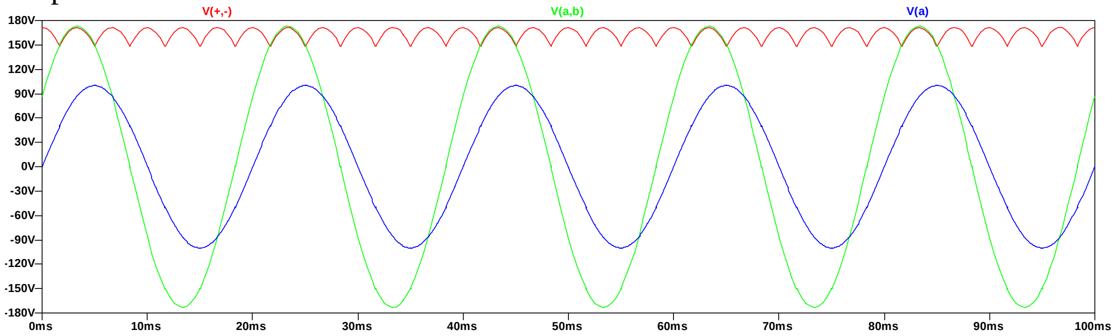
Line to Neutral Voltages:



D1,D2,D3 select the highest positive voltage for Output + while D4,D5,D6, select the lowest negative voltage for Output - :

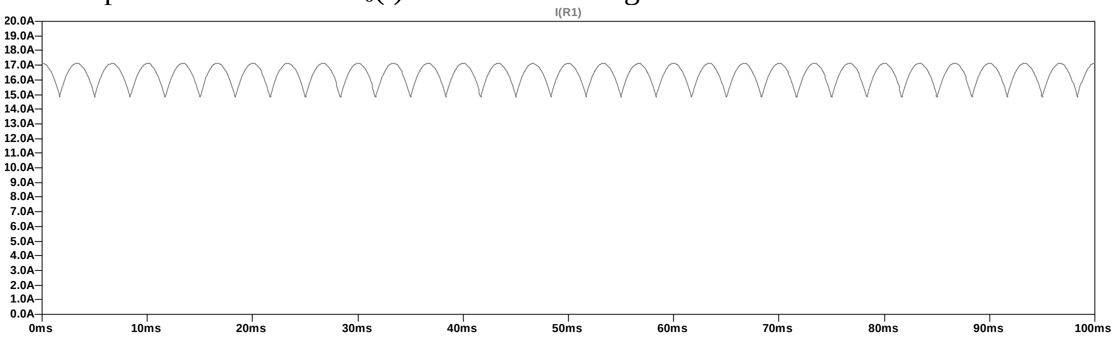


The output voltage is the difference between these two: Please compare the output voltage with eth line to neutral voltage Van and the line to line voltage Vab, The ouput voltage rides along the peaks of the line to line voltages. The output voltage has six ripple pulses per cycle. This is often called a six pulse rectifier.

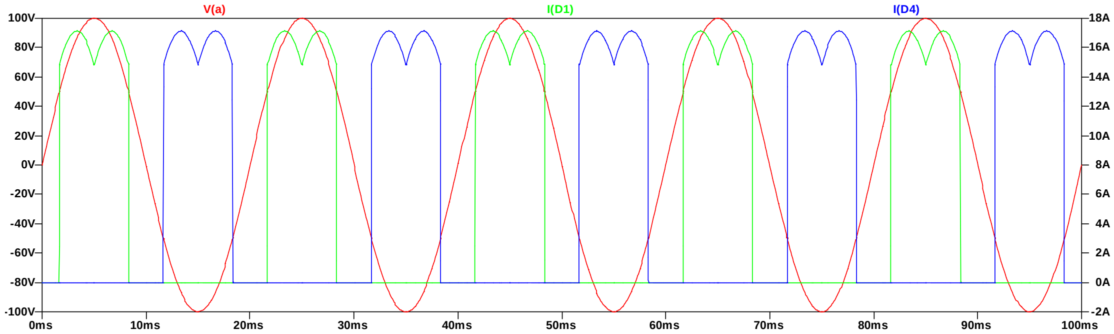


Current Waveforms:

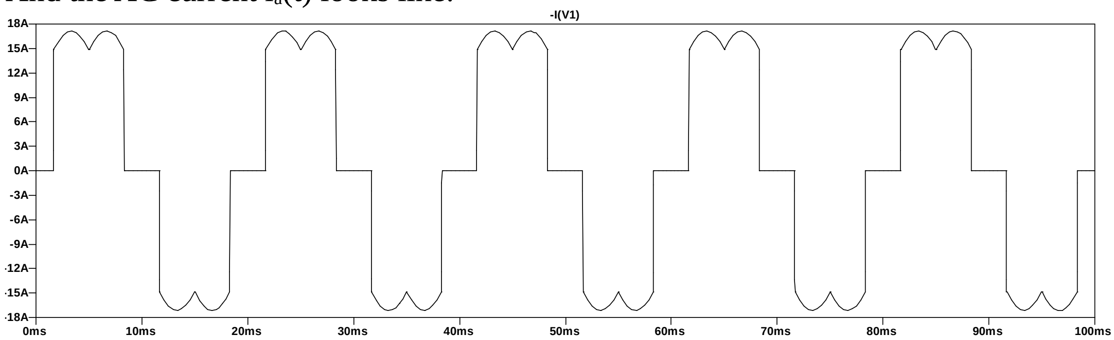
The output current follows $i_o(t)$ follows the voltage across the resistor:



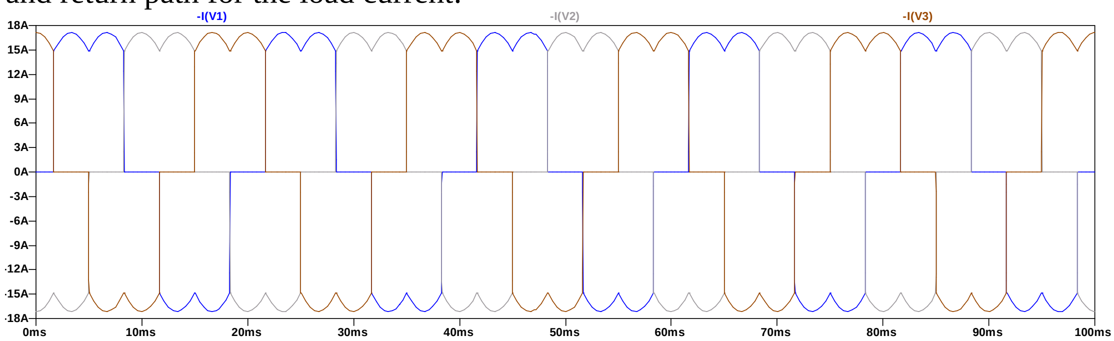
When Van is the highest phase voltage D1 conducts and the load current flows out of phase a supplies the load current. When Van is the lowest phase voltage D4 conducts and the load current flows back into Phase a. It is important to note taht only one top diode conducts at a time and only one bottom diode conducts at a time. The following plot shows the currents in D1 and D4 overlayed onto the Van waveform.



And the AC current $i_a(t)$ looks like:



For reference notice how the three ac currents I_a, I_b and I_c line up to provide a continuous outward and return path for the load current:



Analysis of 3 phase rectifier with resistive load:

Notation: Let V_m = Peak line to neutral voltage

Useful Integration formula: $\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \cos^2(\omega t)\,d\omega t = \frac{\pi}{6} + \frac{\sqrt{3}}{4}$

1. **Peak Output Voltage** = peak of the line of line voltage = $\sqrt{3} \times V_m$

2. **Average Value of the output voltage** may be got by averaging over a single output pulse and using the fact that the output voltage follows a line to line voltage waveform for each pulse.

$$V_o = \frac{\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sqrt{3} \times V_m \cos(\omega t)\,d\omega t}{\frac{2\pi}{6}} = \frac{3\sqrt{3}}{\pi} Vm = 1.654 V_m$$

3. Similarly we can calculate the **rms output voltage** by integrating over a single pulse:

$$V_{o(rms)} = \sqrt{\frac{\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \left(\sqrt{3} \times V_m \cos(\omega t)\right)^2 d\omega t}{\frac{2\pi}{6}}} = V_m \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}} = 1.655 V_m$$

4. Since the output current for a resistive load is just $v_o(t)/R$ we can now calculate the **rectification ratio (efficiency)**

$$\mathrm{Re}\,ctification\,Ratio = \frac{V_o I_o}{V_{o(rms)} I_{o(rms)}} = \frac{V_o^2 / R}{V_{o(rms)}^2 / R} =$$

$$\frac{\left(\frac{3\sqrt{3}}{\pi} Vm\right)^2}{\left(V_m \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}}\right)^2} = \frac{\frac{27}{\pi^2}}{\left(\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}\right)} = 0.998 \ or \ 99.8\%$$

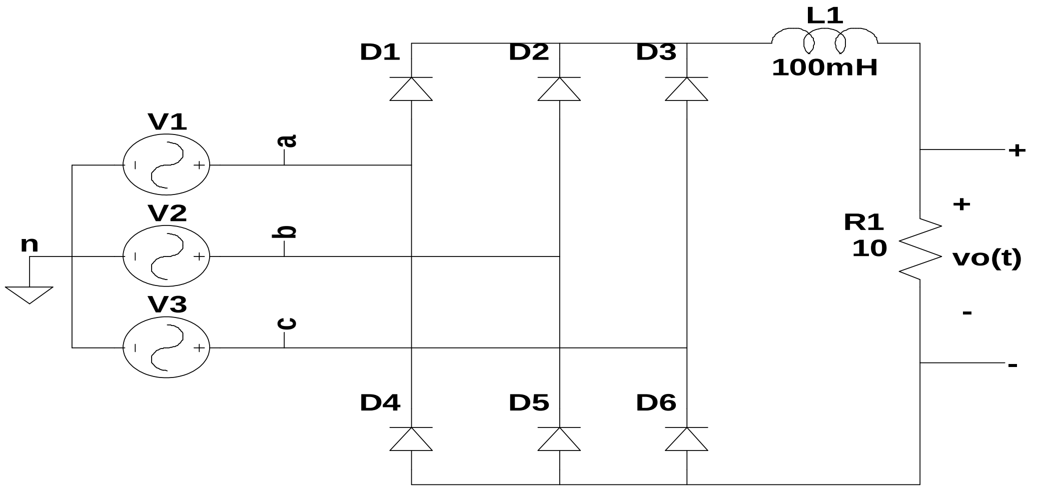
5. In order to calculate the **ripple factor** we must extract the ac component from V_o using the relationship: $V_{o(ac)} = \sqrt{V_{o(rms)}^2 - V_o^2}$

The Ripple factor = $\frac{V_{o(ac)}}{V_o} = \frac{\sqrt{V_{o(rms)}^2 - V_o^2}}{V_o} = \sqrt{\frac{V_{o(rms)}^2}{V_o^2} - 1} = \sqrt{\frac{1}{0.998} - 1} = 0.04\, or\, 4\%$

6. In order to calculate the **transformer utilisation factor** we need to calculate the rms supply current. The easiest way to do this is to notice that ac input current waveform is comprised of four pulses of width 2π/6 over a single cycle of width 2π (remember that positive and negative current contributes equally to the rms).

$$RMS\,Ac\,input\,current = I_s = \sqrt{\frac{4 \times \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \left(\frac{\sqrt{3} \times V_m \cos(\omega t)}{R} \right)^2 d\omega t}{2\pi}} = \frac{V_m}{R}\sqrt{1 + \frac{3\sqrt{3}}{2\pi}} = 1.352\frac{V_m}{R}$$

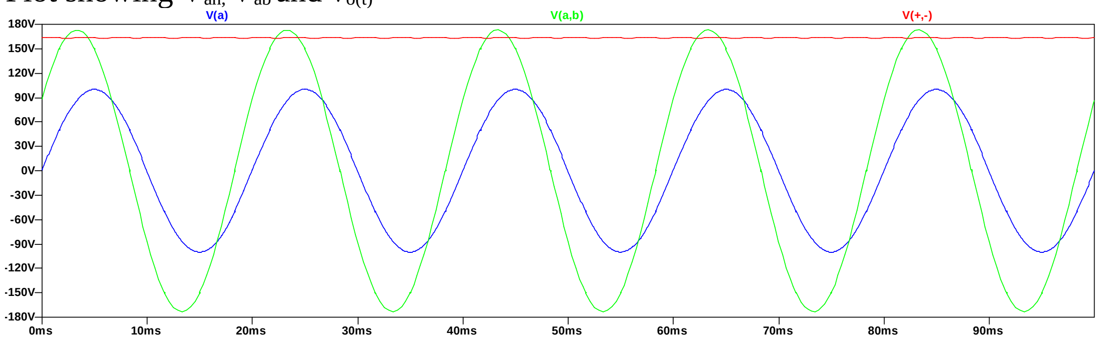
Three Phase Rectifier with highly inductive load.



The ripple frequency in a six pulse rectifier is 6xf where f is the mains frequency. The cut off frequency of an LR filter is $\frac{1}{2\pi \frac{L}{R}}$. If this cut off frequency is much lower than the ripple frequency i.e. if $6f >> \frac{1}{2\pi \frac{L}{R}}$ then the ripple will be almost entirely eliminated leaving a smooth DC load current. The output voltage becomes equal to its average DC value.

Voltage Waveforms:

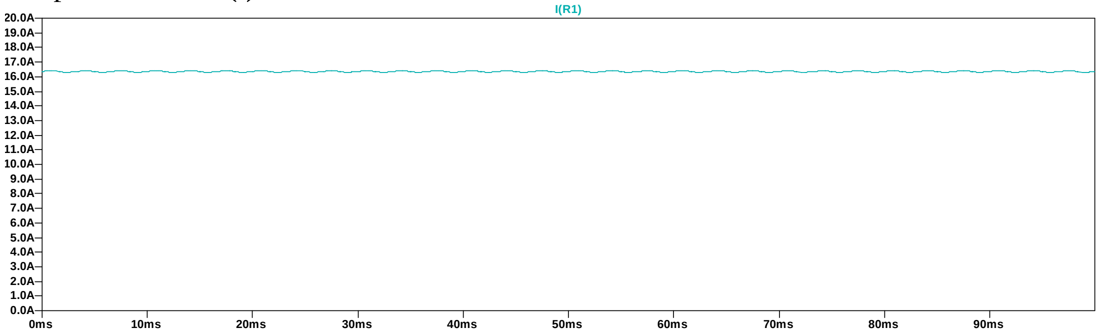
Plot showing V_{an}, V_{ab} and $v_{o(t)}$



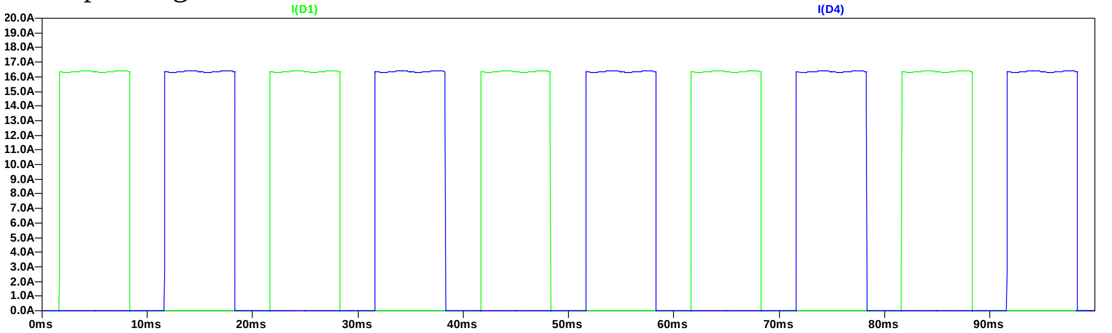
The smooth DC output voltage results in a pure DC output current so the diode currents become square pulses and the input ac current is also "squared up".

Current Waveforms with Inductive Load:

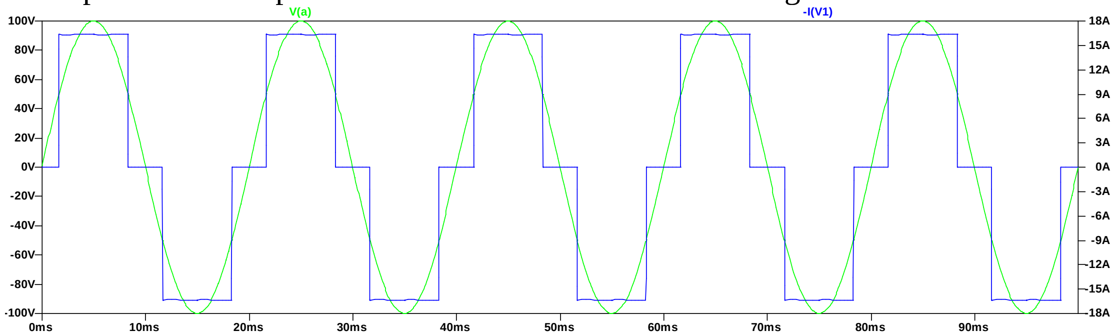
Output Current $i_o(t)$



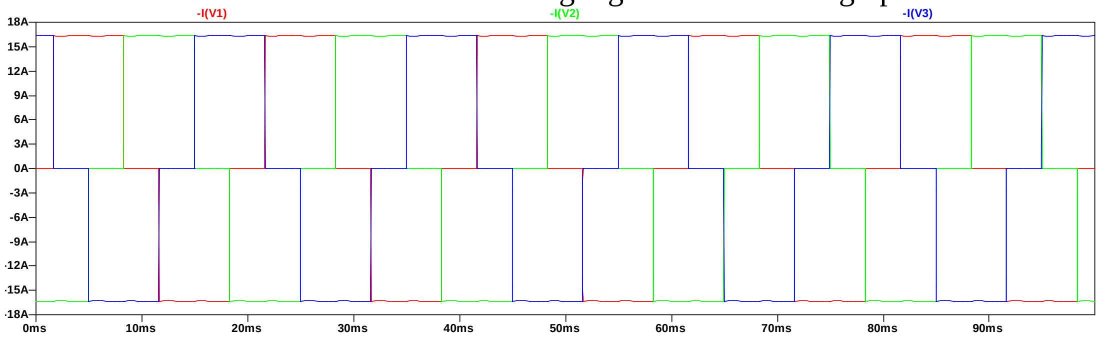
Diode Current Waveforms for D1 and D4. Notice how much squarer these are than the corresponding waveforms for a resistive load:



AC input current for phase a shown with line to neutral voltage for reference:



And all three ac currents are shown meshing together in the next graph:



Analysis of 3 phase rectifier with highly inductive load:

Notation: Let V_m = Peak line to neutral voltage

1. **Peak Output Voltage** = average output voltage = the average of the unfiltered output voltage of the rectifier = $1.654V_m$ from before

2. **Average Value of the output voltage** is the same as before (the inductore attenuates ripple but does not affect average output voltage) $V_o = \dfrac{\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sqrt{3} \times V_m \cos(\omega t) d\omega t}{\frac{2\pi}{6}} = \dfrac{3\sqrt{3}}{\pi} Vm = 1.654V_m$

3. The **rms output voltage** is trivial in this case because there is no ripple

$V_{o(rms)} = V_o = 1.654V_m$

4. Since the output current for a resistive load is just $v_o(t)/R$ we can now calculate the **rectification ratio (efficiency)**

$$\text{Re}\,ctification\,Ratio = \frac{V_o I_o}{V_{o(rms)} I_{o(rms)}} = \frac{V_o^2 / R}{V_{o(rms)}^2 / R} = 100\%$$

5. Since the ripple is vanishingly small $V_{o(ac)} \approx 0 \Rightarrow \dfrac{V_{o(ac)}}{V_o} \approx 0\%$

6. In order to calculate the **transformer utilisation factor** we need to calculate the rms supply current. The easiest way to do this is to integrate over a half cycle. In each half cycle there is a square pulse of current of width $2\pi/3$ and amplitude = I_o = $1.654V_m/R$

$$RMS Ac\,input\,current = I_s = \sqrt{\frac{\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \left(\frac{1.654V_m}{R}\right)^2 d\omega t}{\pi}} = \frac{1.654V_m}{R}\sqrt{\frac{2}{3}} = \frac{1.35V_m}{R}$$

Transformer Utilisation Factor = $\dfrac{DC\ output\ power}{InputVA} = \dfrac{V_o I_o}{3xV_s I_s} = \dfrac{(1.654V_m)(1.654V_m / R)}{3(\frac{V_m}{\sqrt{2}})(1.35\frac{V_m}{R})} = 0.955$

Conclusion: *The three phase bridge rectifier with resistive load has significantly lower output voltage ripple and significantly better transformer utilisation than an unfiltered single phase bridge rectifier. The addition of a sufficiently large filtering inductor can result in negligible output voltage ripple but does not substantially improve transformer utilisation over the unfiltered three phase rectifier,*