

**SCHOOL OF ELECTRICAL AND ELECTRONIC ENGINEERING**

**Bachelor of Engineering (Hons) BE in Elect/Cont/Comm/Comp Eng**  
**Program Code: (DT021A)**  
**<YEAR 4>**

**Name of Module: (Solid-State Electronics, ELTR2603)**

**TU Dublin – Grangegorman**

**Lab 4: FERMI-DIRAC DISTRIBUTION &  
FERMI LEVEL AND CARRIER MOBILITY &  
DIFFUSION**

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**Submission Date:** 06<sup>th</sup> December 2021

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## **1. Part 1 - FERMI-DIRAC DISTRIBUTION & FERMI LEVEL**

### **1.1. Laboratory Objectives**

The purpose of this lab is to learn the following concepts of fermi-dirac distribution and fermi level:

- Fermi level,  $E_F$ , and the occupation probability of quantum states by an electron.
- Temperature effect on electron distribution.

### **1.2. Introduction**

From Schrödinger's wave equation for a free electron confined to a three-dimensional infinite potential well (representing the crystal), the density of states per unit energy and per unit volume is given by

$$g(E) = \frac{4\pi(2m)^{3/2}}{h^3} \sqrt{E} \quad (1)$$

The density of allowed electronic energy states in the conduction band:

$$g_c(E) = \frac{4\pi(2m_n^*)^{3/2}}{h^3} \sqrt{E - E_c} \quad (2)$$

where equation (2) is valid for  $E \geq E_c$ . As the energy of the electron in the conduction band decreases, the number of available quantum states also decreases.

The density of states function in the valence band is given by:

$$g_v(E) = \frac{4\pi(2m_p^*)^{3/2}}{h^3} \sqrt{E_v - E} \quad (3)$$

where equation (3) is valid for  $E \leq E_v$ . The Fermi-Dirac probability function is given by

$$\frac{N(E)}{g(E)} = f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} \quad (4)$$

where  $N(E)$  is the number of particles per unit volume per unit energy and  $g(E)$  is the number of quantum states per unit volume per unit energy.

The Fermi-Dirac distribution gives the probability that an allowed quantum state at the energy  $E$  is occupied by an electron. The parameter  $E_F$  is called the Fermi energy.

When  $E - E_F \gg kT$ , so that the exponential term in the equation (4) is much greater than unity, the Fermi-Dirac distribution function becomes

$$f_F(E) \approx \exp\left[\frac{-(E - E_F)}{kT}\right] \quad (5)$$

Equation (5) is known as the Maxwell-Boltzmann approximation to the Fermi-Dirac distribution function.

## 1.3. Laboratory Procedure

### 1.3.1. Fermi-Dirac Distribution and Fermi level

Running the “Section 1.5.2: Fermi-Dirac Distribution and Fermi Level” simulation and varying the material energy gap, temperature, and doping level to observe the influence on carrier concentration.

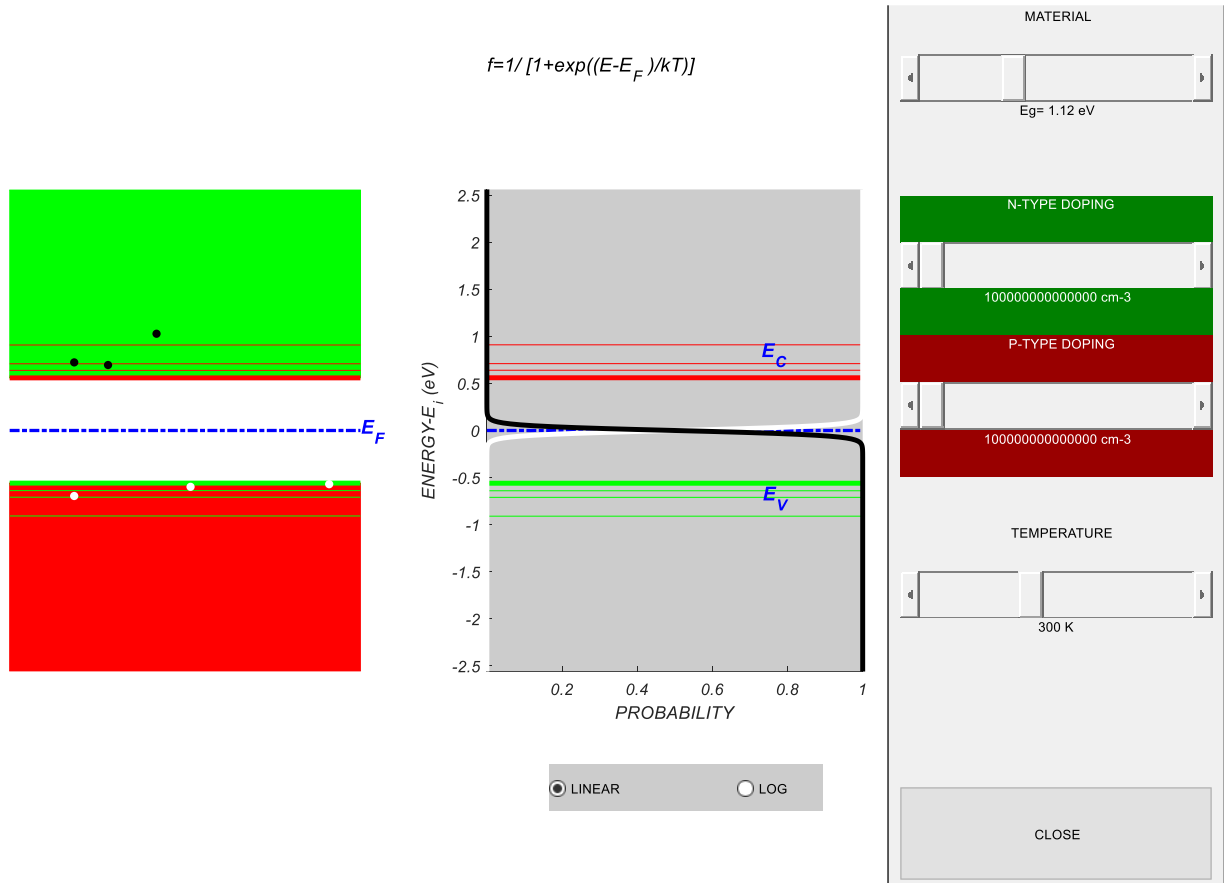


Figure 1 - Fermi-Dirac Distribution and Fermi Level Simulation

### 1.3.2. Material energy gap change

The energy gap ( $E_g$ ) is the gap between the valence and conduction band. When material gap ( $E_g$ ) increases, more energy is required to transfer the electrons from the valence band to the conduction band. Valence band is formed due to the low energy orbitals combined, and the higher energy orbitals combine forms a conduction band.

As the energy gap ( $E_g$ ) gets above 1.4 eV in the simulation, no electrons are transferred because the energy gap got wider meaning that more energy is required to transfer electrons from the valence band to the conduction band.

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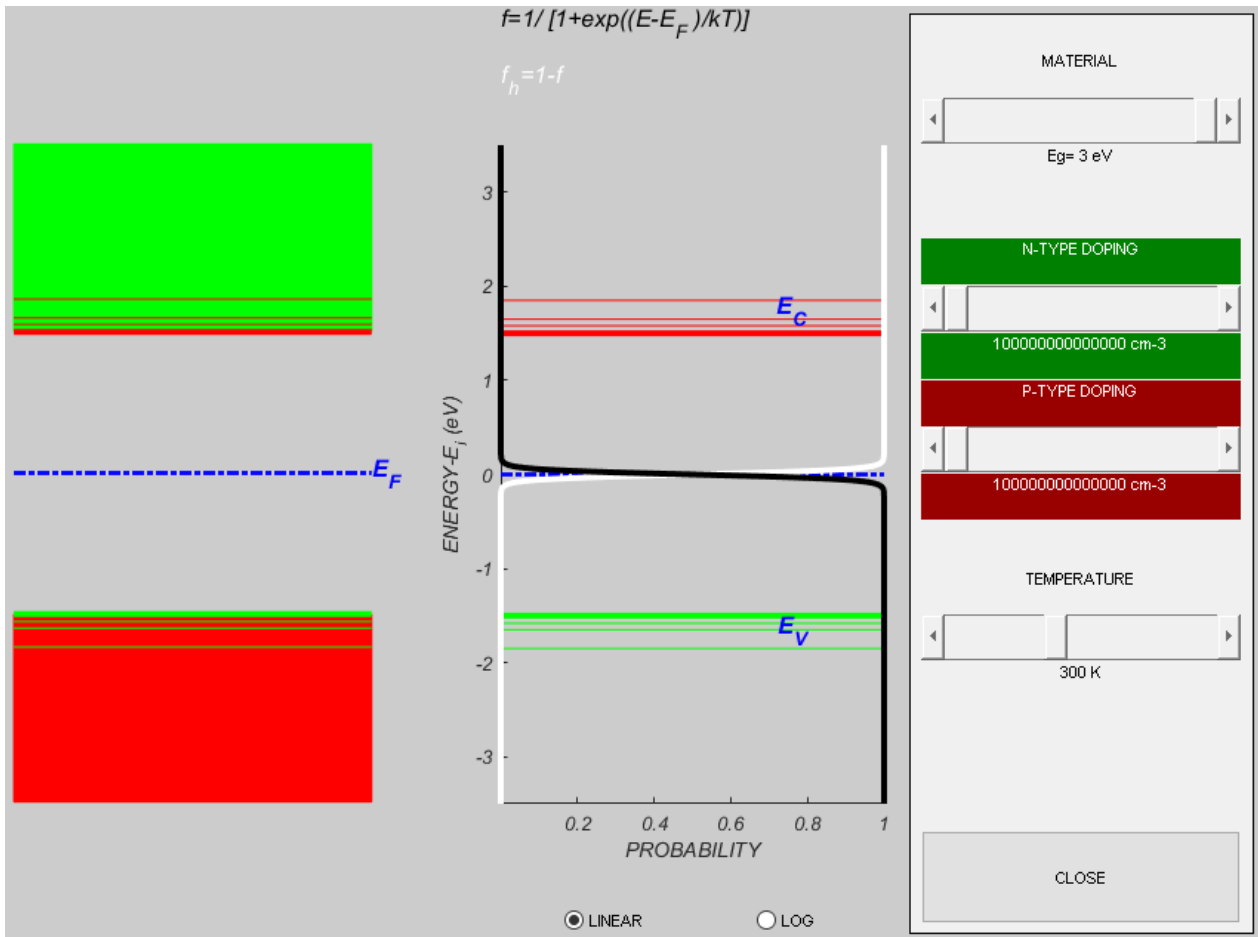


Figure 2 - Material Energy gap ( $E_g$ ) increased

### 1.3.3. n-, p- type doping levels increase

When the n-type doping is increased the Fermi energy level increases.

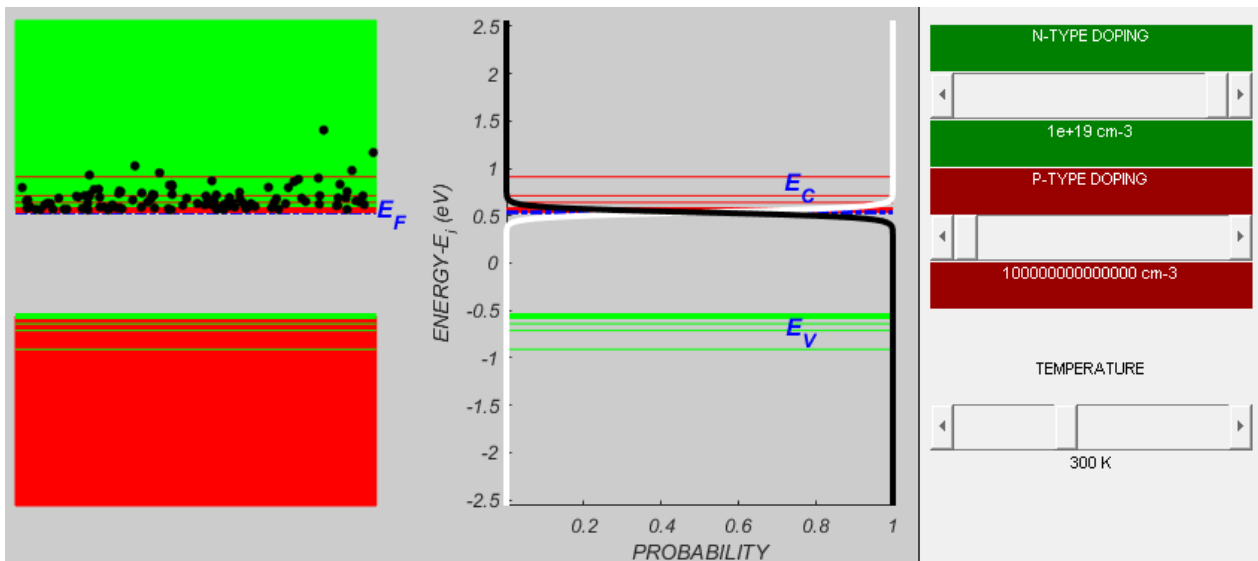


Figure 3 - N-type doping increased

When p-type doping is increased the Fermi energy level decreases.

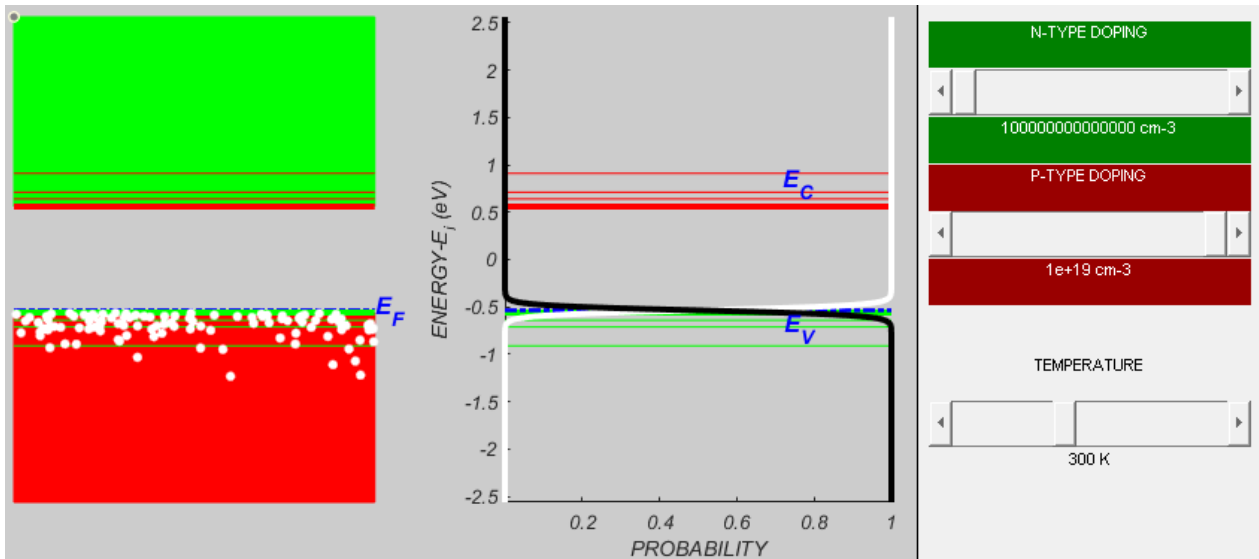


Figure 4 - P-type doping increased

## 1.3.4. Temperature change

As temperature increases, electrons from the lower energy levels are moving to higher energy levels because they are getting more energy due to the increase in temperature and therefore, gets excited to higher energy levels.

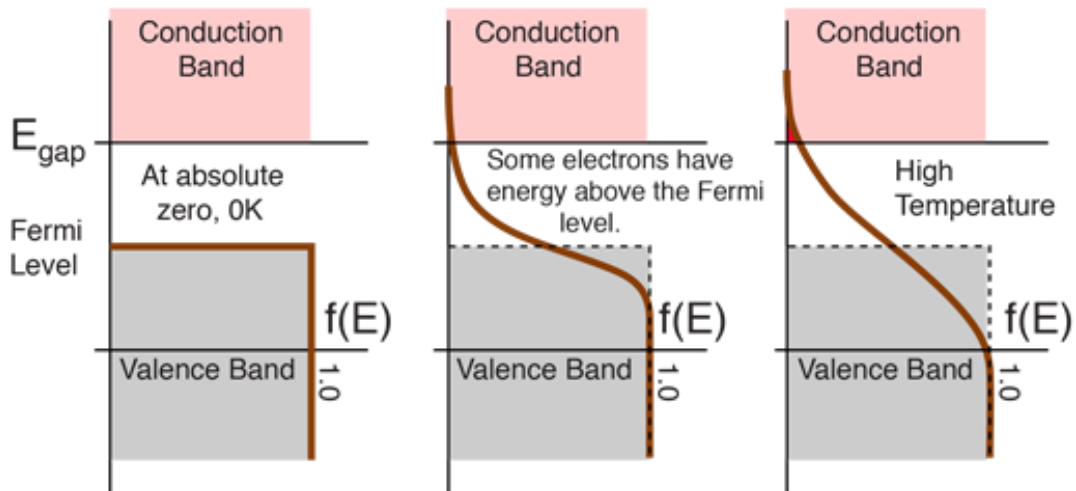


Figure 5 - Effects of a Fermi level due to temperature

### 1.3.5. Conclusion

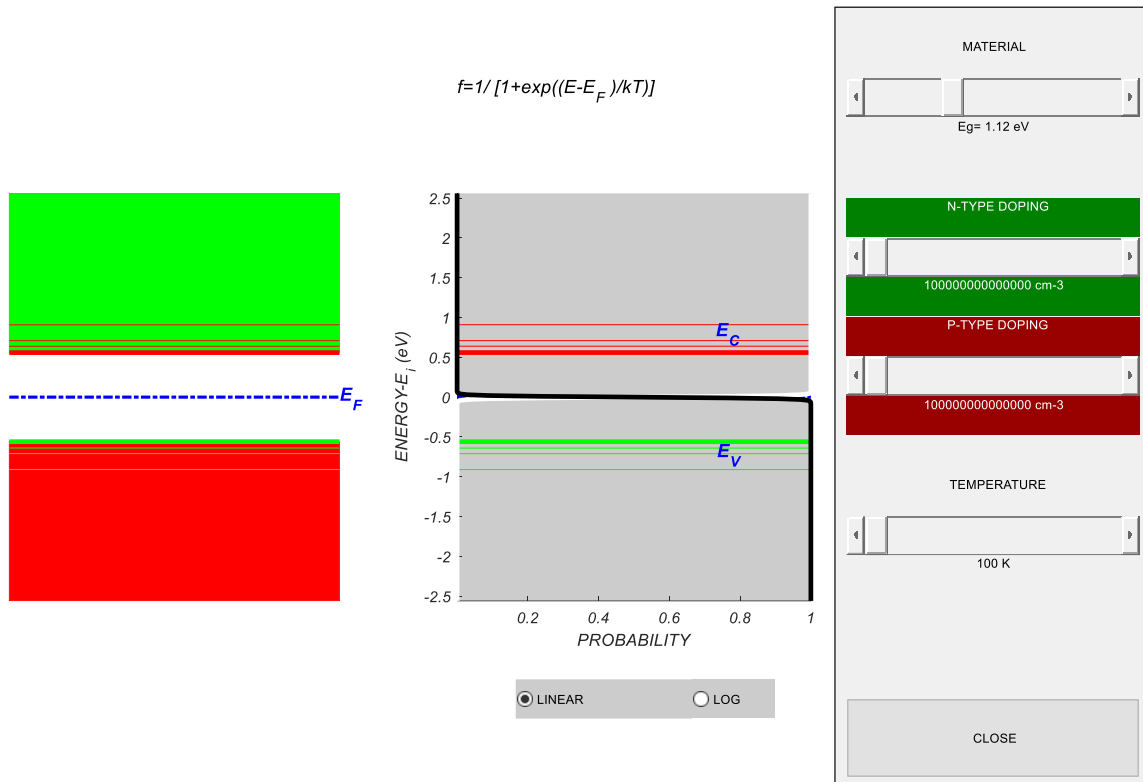


Figure 6 - Temperature at 0 K

The highest occupied energy level at zero temperature (0K) is called as Fermi level. Where, no electrons can be above the Fermi level at 0K, since none have energy above the Fermi level and there are no available energy states in band gap.

At higher temperature ( $T > 0K$ ), the probability of occupancy for the energy levels below the Fermi level may not be 1 and the probability of occupancy for the energy levels above the Fermi level may not be 0.

The distribution of electrons over a range of allowed energy levels at thermal equilibrium is given by Fermi-Dirac Distribution function

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}$$

Where  $f(E)$  is the probability of occupancy for energy level  $E$ ,  $E_F$  is Fermi energy,  $T$  is temperature in Kelvin (K) and  $k = 1.38 \times 10^{-23} J/K$  which is  $k = 8.625 \times 10^{-5} eV/K$  in electron volt / Kelvin.

The probability of occupancy can be used to find the number of electrons when we know the density of states by multiplying density of states, that's the number of energy levels per unit volume.

## LAB 4: FERMI-DIRAC DISTRIBUTION. CARRIER MOBILITY & DIFFUSION

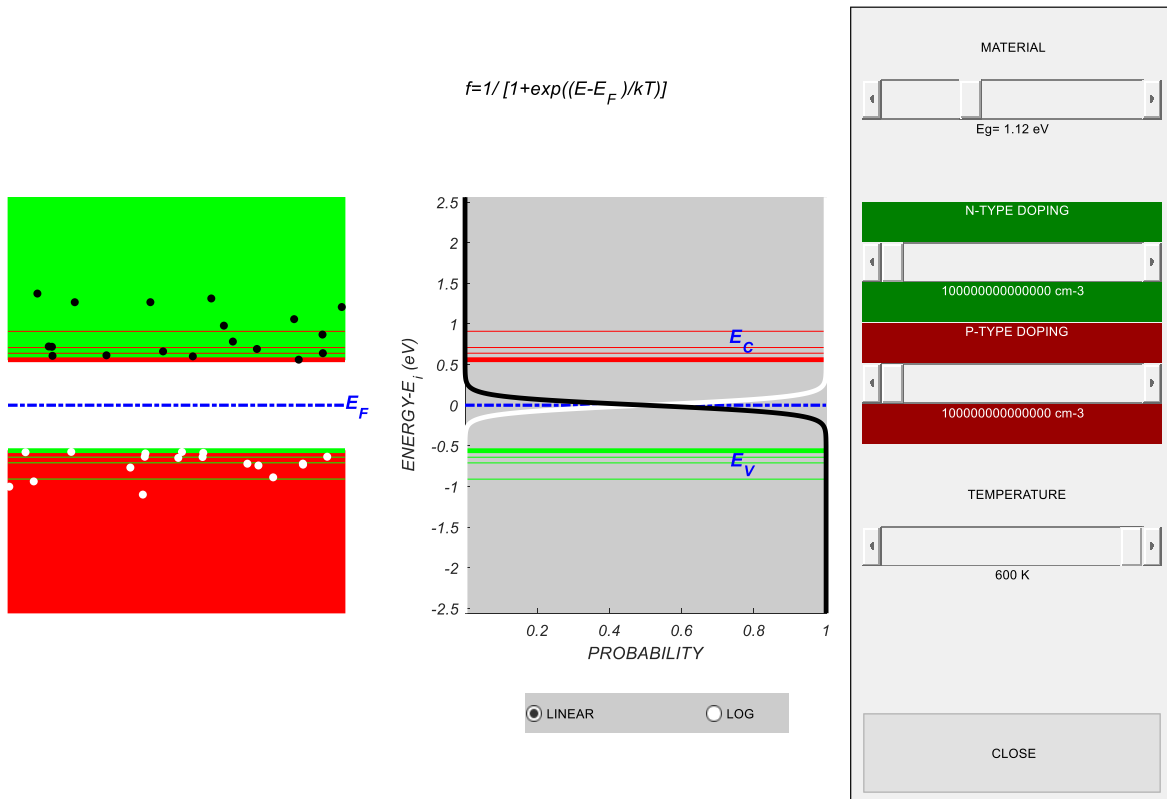


Figure 7 - High temperature

At high temperatures, some electrons can reach the conduction band and contribute to electric current.



## 2. Part 2 - CARRIER MOBILITY AND DIFFUSION. HAYNES-SHOCKLEY EXPERIMENT

### 2.1. Objectives

The aim of this experiment is to learn the concepts of carrier drift and diffusion, experimental technique to determine mobility and diffusion coefficients for holes in a semiconductor.

### 2.2. Introduction

Drift and diffusion are two separate mechanisms. This can be demonstrated by the Haynes-Shockley experiment, illustrated in Figure 1. To monitor the process of drift and diffusion, a pulse of minority carriers is generated in a very narrow region of a semiconductor bar for a very short time interval.

In the case shown in Fig.1 this is achieved by a flash of light that illuminates a narrow region at the end of an n-type silicon bar at time  $t = 0$ . The light generates both electrons and holes, however, the direction of the electric field applied is such that the electrons are quickly collected by the positive contact, while the holes must travel through the silicon bar to be collected by the negatively biased contact.

The drift velocity is  $v_d = \mu_p E$ , and if the diffusion did not exist, a short current pulse would be detected by the ammeter as the holes are collected by the negative contact after the time interval equal to  $\frac{L}{v_d}$ . However, as the holes diffuse in either direction the hole distribution widens with time. As illustrated in the Fig. 1. The holes distribution can be expressed by the Gauss distribution:

$$P = P_{max} e^{-\frac{(x-x_{max})^2}{4D_p t}} \quad (6)$$

The position of the distribution peak  $x - x_{max}$  shifts as the holes drift along the electric field:  $x_{max} = v_d t$ . Both the mobility and diffusion coefficient for the holes can be extracted from the measured time dependence of the current. Figure 1c illustrates that the maximum of the current coincides with the arrival of the peak of the hole's distribution at the negative contact. The drift velocity can be calculated from the measured time between the current maximum and the flash of light as  $v_d = \frac{L}{t_{max}}$ .

The mobility is then calculated as:

$$\mu_p = \frac{v_d}{E} = \frac{L^2}{t_{max} V} \quad (7)$$

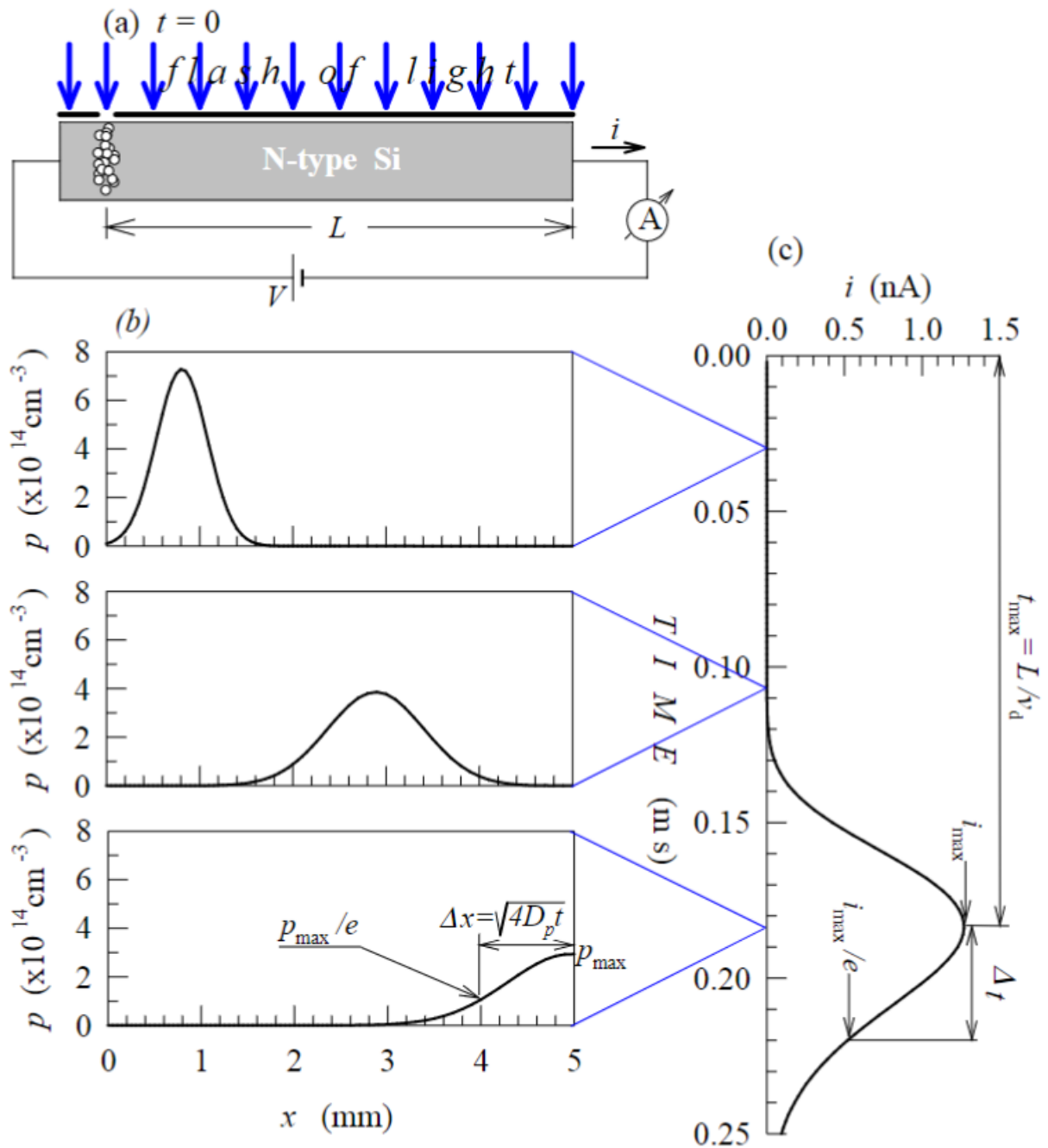


Figure 8 - Haynes-Shockley experiment: (a) a flash of light is used to generate a narrow pulse of minority carriers (holes); (b) the motion of the hole peak illustrates the hole drift, while the widening of the hole distribution illustrates the hole diffusion, and (c) the measured current at the end of the semiconductor that can be used to calculate  $\mu_p$  and  $D_p$

The diffusion coefficient can be determined from the width of the current pulse. Equation (6) shows that  $P = \frac{P_{\max}}{e}$  for  $(x - x_{\max})^2 = 4D_p t$ , that is  $\Delta x = x - x_{\max} = \sqrt{4D_p t}$

The current that corresponds to this distribution is  $\frac{i_{\max}}{e}$ . As it takes time of  $\Delta t = \frac{\Delta x}{v_d}$  for the holes to travel the distance of  $\Delta x$  :

$$\Delta t = \frac{\Delta x}{v_d} = \frac{\sqrt{4D_P(t_{max} + \Delta t)}}{v_d} \quad (8)$$

Using  $v_d = L/t_{max}$ , the diffusion coefficient can be expressed in terms of the measured values of  $t_{max}$  and  $\Delta t$ .

$$D_P = \frac{\Delta t^2 \left(\frac{L}{t_{max}}\right)^2}{4(t_{max} + \Delta t)} \quad (9)$$

### 2.3. Procedure

The MATLAB Animations “Section 1.4.4: Haynes-Shockley Experiment” was run and observing the processes of drift and diffusion separately. When running Haynes-Shockley experiment, it takes 20.57 seconds from the beginning of the experiment to the end of the experiment by using a stopwatch to measure the time.

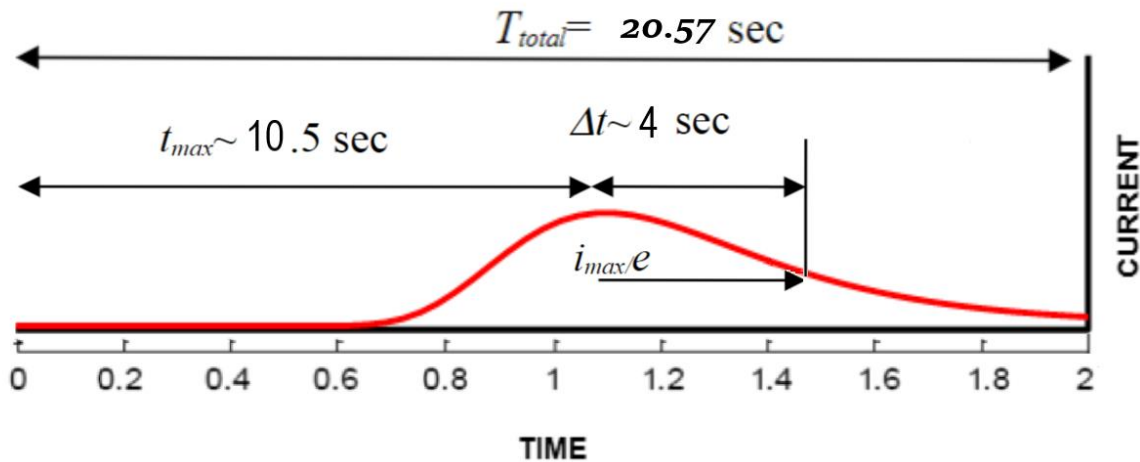


Figure 9 - Current vs. Time graph

#### 2.3.1. Calculation of $\mu_P$ and $D_P$

Calculating  $\mu_P$  and  $D_P$  using equations 7 and 9 after finding  $t_{max}$  and  $\Delta t$ .

Mobility of the minority carriers is calculate using the equation

$$\mu_P = \frac{v_d}{E} = \frac{L^2}{t_{max}V}$$

Where  $v_d$  is the drift velocity of holes,  $v_d = \mu E$ , so, mobility of charge carrier is  $\mu = \frac{q\tau}{m^*}$ .

$E$  is the electric field. So, we calculate mobility of minority carries when  $L = 50$  cm and  $V = 0.3$ v are given.

$$\mu_P = \frac{50^2}{10.5 \times 0.3}$$

$$\mu_P = 793.65 \text{ cm}^2\text{V}^{-1}\text{S}^{-1}$$

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Now, calculating the diffusion constant using the following equation

$$D_P = \frac{\Delta t^2 \left( \frac{L}{t_{\max}} \right)^2}{4(t_{\max} + \Delta t)}$$

Where  $\Delta$  is the width of half maximum of the peak,  $t$  is the time for a light pulse to travel along the bar.

$$D_P = \frac{4^2 \left( \frac{50}{10.5} \right)^2}{4(10.5 + 4)}$$
$$D_P = 6.25 \frac{\text{cm}^2}{\text{s}}$$

### 2.3.2. Einstein relation

Assuming room temperature  $T = 300\text{K}$ , where  $K$  = Boltzmann Constant and  $e$  = charge on one electron.

$$\frac{D_P}{\mu_P} = \frac{KT}{e}$$
$$\frac{D_P}{\mu_P} = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}}$$

Calculating the diffusion coefficient

$$D_P = 0.025875 \text{ V} \left( \frac{793.65 \text{ cm}^2}{\text{V} \cdot \text{s}} \right)$$
$$D_P = 0.025875 \left( 793.65 \frac{\text{cm}^2}{\text{s}} \right)$$
$$D_P = 20.54 \frac{\text{cm}^2}{\text{s}}$$

Calculating Mobility

$$\mu_P = \frac{20.54 \text{ cm}^2}{0.025875 \text{ V} \cdot \text{s}}$$
$$\mu_P = 793.816 \frac{\text{cm}^2}{\text{Vs}}$$

Diffusion coefficient is approximately 40 times smaller in magnitude than the mobility at room temperature.

$$\frac{\mu_P}{D_P} \approx 40$$
$$\frac{793.816}{20.54} = 38.64 \approx 40$$

The mobility of charges in a semiconductor is equal to the magnitude of the charge times the diffusion coefficient divided by the product of the Boltzmann constant and the absolute temperature.

### 2.3.3. Conclusion

The Haynes-Shockley was an experiment that demonstrated that diffusion of minority carries in a semiconductor could end up in a current. In the experiment, a piece of semiconductor gets a pulse of holes, induced by voltage or a short laser pulse.

As the time progresses, the holes spread out by diffusion and move due to the electric field, and their motion is monitored somewhere down the bar.

## 3. References

[1] Nave, R., 2022. *Fermi level and Fermi function*. [online] Hyperphysics.phy-astr.gsu.edu. Available at: <<http://hyperphysics.phy-astr.gsu.edu/hbase/Solids/Fermi.html>> [Accessed 19 November 2021].