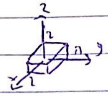
Calc 2 HW #7

(A) (4) $I = \int_0^{\pi}\int_0^{2}\int_0^{\sqrt{4-z^2}} z\sin y \, dx\,dz\,dy \Rightarrow$

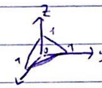


$\Rightarrow I \int_0^{\pi}\int_0^{2}\int_0^{\sqrt{4-z^2}} z\sin y \, dz\,dx\,dy$

$= \int_0^{\pi}\int_0^{2}\left[\frac{1}{2}z^2\sin y\right]_0^{\sqrt{4-x^2}} dx\,dy = \int_0^{\pi}\int_0^{2}(2-\frac{1}{2}x^2)\sin y \, dx\,dy = \int_0^{\pi}\sin y\,dy \cdot \int_0^{2} 2-\frac{1}{2}x^2\,dx$

$= \left[-\cos y\right]_0^{\pi} \cdot \left[2x - \frac{1}{6}x^3\right]_0^2 = 2\cdot 2\frac{2}{3} = 5\frac{1}{3}$

(7) $I = \iiint_B z\,dx\,dy\,dz$, $x=y=z=0$, $x+y+z=1 \Rightarrow$



$I = \int_0^{1}\int_0^{1-z}\int_0^{1-y-z} z\,dx\,dy\,dz = \int_0^{1}\int_0^{1-z}\left[zx\right]_0^{1-y-z} dy\,dz = \int_0^{1}\int_0^{1-z} z-yz-z^2\,dy\,dz$

$= \int_0^{1}\left[(z-z^2)y - \frac{1}{2}zy^2\right]_0^{1-z} dz = \int_0^{1} z(1-z)^2 - \frac{1}{2}z(1-z)^2\,dz = \frac{1}{2}\int_0^{1} z^3-2z^2+z\,dz$

$= \frac{1}{2}\left[\frac{1}{4}z^4 - \frac{2}{3}z^3 + \frac{1}{2}z^2\right]_0^1 = \frac{1}{2}\left(\frac{1}{4}-\frac{2}{3}+\frac{1}{2}\right) = \frac{1}{24}$

(B)

Cylindrical coordinates	Spherical coordinates
$x = r\cos\theta$	$x = r\cos\theta\sin\varphi$
$y = r\sin\theta$	$y = r\sin\theta\sin\varphi$
$z = z$	$z = r\cos\varphi$

(2) $-2 = r\cos\theta \Rightarrow r = \frac{-2}{\cos\theta}$

$1 = r\sin\theta = -2\tan\theta \Rightarrow \theta = \tan^{-1}(-\frac{1}{2})$

$\Rightarrow \theta = 2.68 \Rightarrow r = -\frac{2}{\cos(2.68)} = \sqrt{5}$

we added π because the pt is 2 not 4, $z = -3$

$1 = r\sin(2.68)\sin(\varphi) \Rightarrow r = \frac{\sqrt{5}}{\sin(\varphi)}$

$-3 = r\cos(\varphi) = \sqrt{5}\cot(\varphi)$

$\Rightarrow \varphi = \cot^{-1}\left(-\frac{3}{\sqrt{5}}\right) = 2.5 \Rightarrow r = \sqrt{14}$

cylindrical coordinates: $(r,\theta,z) = (\sqrt{5}, 2.68, -3)$

Spherical coordinates: $(r,\theta,\varphi) = (\sqrt{14}, 2.68, 2.5)$

(4) cylindrical coordinates | Spherical coordinates

$x = 3\cos 2\pi \sin\frac{\pi}{2} = 3 \Longrightarrow x = r\cos\theta \Rightarrow 3 = r$

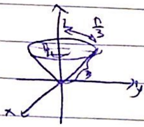
$y = 3\sin 2\pi \sin\frac{\pi}{2} = 0$

$z = 3\cos\frac{\pi}{2} = 0$

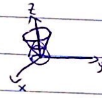
cartesian coordinates: $(x,y,z) = (3,0,0)$

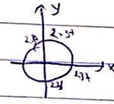
cylindrical coordinates: $(r,\theta,z) = (3,2\pi,0)$

Ⓓ ① $0 \le \varphi \le \frac{\pi}{3}$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, $\rho \le 3$



Ⓔ ② $I = \iiint_B z\,dx\,dy\,dz$, $z = 8 - x^2 - y^2$, $z = \sqrt{x^2+y^2} \Rightarrow$





$8 - x^2 - y^2 = \sqrt{x^2+y^2} \Rightarrow 8 - r^2 = r$

$\Rightarrow r^2 + r - 8 = 0$ $\quad r = \frac{-1 \pm \sqrt{1+32}}{2} = 2.37$ radius has to be positive

integral $\Rightarrow I = \int_0^{2.37}\int_0^{2\pi}\int_r^{8-r^2} z\,r\,dz\,d\theta\,dr = \int_0^{2.37}\int_0^{2\pi} \left[\frac{1}{2}z^2 r\right]_r^{8-r^2} d\theta\,dr = \int_0^{2.37}\int_0^{2\pi} \frac{1}{2}(8-r^2)^2 r - \frac{1}{2}r^3\,d\theta\,dr$

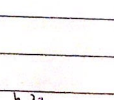
$= \frac{1}{2}\int_0^{2\pi} d\theta \int_0^{2.37} r^5 - 17r^3 + 64r\,dr = \pi\left[\frac{1}{6}r^6 - \frac{17}{4}r^4 + 32r^2\right]_0^{2.37} = \pi \cdot 75.19 = 236.22$

Ⓕ ③ $D = \left\{(x,y,z) \;\middle|\; \begin{matrix} 0 \le 2x - 3y + z \le 5 \\ 1 \le x + 2y \le 4 \\ -3 \le x - z \le 6 \end{matrix}\right\} \Rightarrow \begin{matrix} t = 2x - 3y + z \\ u = x + 2y \\ v = x - z \end{matrix}$

$\Rightarrow J = \dfrac{1}{\begin{vmatrix} \frac{\partial t}{\partial x} & \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial t}{\partial y} & \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \\ \frac{\partial t}{\partial z} & \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} 2 & 1 & 1 \\ -3 & 2 & 0 \\ 1 & 0 & -1 \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} 1 & 1 \\ 2 & 0 \end{vmatrix} - \begin{vmatrix} 2 & 1 \\ -3 & 2 \end{vmatrix}} = \dfrac{1}{-2-4-3} = -\dfrac{1}{9}$

$\Rightarrow V = \int_0^5\int_1^4\int_{-3}^6 \left|-\frac{1}{9}\right| dv\,du\,dt = \frac{1}{9}\int_0^5 dt \int_1^4 du \int_{-3}^6 dv = \frac{1}{9} \cdot 5 \cdot 3 \cdot 9 = 15$

Ⓖ ③ $\sqrt{x^2+y^2} \le z \le 4 - \sqrt{x^2+y^2} \Rightarrow$



$V = \int_0^2\int_0^{2\pi}\int_r^{4-r} r\,dz\,d\theta\,dr = \int_0^2\int_0^{2\pi} [rz]_r^{4-r} d\theta\,dr = \int_0^2\int_0^{2\pi} r(4-r) - r^2\,d\theta\,dr = \int_0^{2\pi} d\theta \int_0^2 4r - 2r^2\,dr =$

$2\pi \cdot \left[2r^2 - \frac{2}{3}r^3\right]_0^2 = 2\pi\left(8 - \frac{16}{3}\right) = \frac{16\pi}{3}$

⑤

$\Rightarrow V = \int_0^h\int_0^{2\pi}\int_0^{\frac{a}{h}z} r\,dr\,d\theta\,dz = \int_0^h\int_0^{2\pi} \left[\frac{1}{2}r^2\right]_0^{\frac{a}{h}z} d\theta\,dz$

$= \frac{a^2}{2h^2}\int_0^{2\pi} d\theta \int_0^h z^2\,dz = \frac{a^2\pi}{h^2}\left[\frac{1}{3}z^3\right]_0^h = \frac{a^2 h\pi}{3}$

Scanned with CamScanner

$$⑥\ \left\{(x,y,z) \,\middle|\, \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}\leqslant 1\right\} \Rightarrow \begin{array}{l} t=\frac{x}{a} \\ u=\frac{y}{b} \\ v=\frac{z}{c}\end{array} \Rightarrow J = \frac{1}{\begin{vmatrix} \frac{\partial t}{\partial x} & \frac{\partial u}{\partial x} & \frac{\partial v}{\partial x} \\ \frac{\partial t}{\partial y} & \frac{\partial u}{\partial y} & \frac{\partial v}{\partial y} \\ \frac{\partial t}{\partial z} & \frac{\partial u}{\partial z} & \frac{\partial v}{\partial z} \end{vmatrix}} = \frac{1}{\begin{vmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{vmatrix}}$$

$\Rightarrow J = abc \Rightarrow V = \iiint_B |abc|\, dt\,du\,dv \;\Big|\; B = \{t,u,v \mid t^2+u^2+v^2\leqslant 1\} \Rightarrow V =$ circle volume that radius is $1 \cdot abc$

$$= \boxed{V = \frac{4\pi abc}{3}}$$