30 p

ODE hw#6

1) a) $y''(x) - ay(x) = 0$, $y(0) = y(6) = 0$, $x \in [0,6]$

case 1: $a=0$, $y''=0$, therefore $y(x) = Ax+B$

$0 = y(0) = B \Rightarrow y(x) = Ax$, $0 = y(6) = 6A \Rightarrow A=0$

trivial answer

case 2: $a<0$ $\quad a=-w^2 \; (0<w)$ therefore $r = \pm w$ $\quad y(x) = Ae^{wx} + Be^{-wx}$

$0 = y(0) = A+B \Rightarrow A=-B \Rightarrow y(x) = A(e^{wx} - e^{-wx})$

$0 = y(6) = A(e^{6x} - e^{-6x}) \Rightarrow$ $6w>0$ therefore $e^{6x} - e^{-6x} > 0$ therefore $A=0$ therefore $B=0$

we got again a trivial answer

case 3: $a>0$ $\quad a=w^2$ therefore $r = \pm wi$ $\quad$ solution: $y(x) = A\cos wx + B\sin wx$

$0 = y(0) = A \Rightarrow y(x) = B\sin wx$

$0 = y(6) = B\sin(6w)$, $B \neq 0 \Rightarrow \sin(6w) = 0 \Rightarrow w = \frac{\pi n}{6}$, $n \in \mathbb{N}$

$y(x) = \sin\left(\frac{n\pi x}{6}\right)$ when $n \in \mathbb{N}$

$a = w^2 = \left(\frac{n\pi}{6}\right)^2$

b) $y''(x) + by(x) = 0$, $y'(0) = y'(2) = 0$, $x \in [0,2]$

case 1: $b=0$ therefore $y''=0$ $\quad y(x) = Ax+B$

$0 = y'(0) = A \Rightarrow y(x) = B$

$0 = y'(2) = A = 0 \Rightarrow$ for all $B \Rightarrow$ $y(x) = B$, $x_0 = 0$

case 2:

$b<0$ $\quad b=-w^2$ therefore $r = \pm w$ $\quad y(x) = Ae^{wx} + Be^{-wx}$

$0 = y'(0) = Aw - Bw \Rightarrow A=B \Rightarrow y(x) = A(e^{wx} - e^{-wx})$

$0 = y'(2) = A(e^{2w} - e^{-2w})$, $e^{2w} - e^{-2w} > 0 \Rightarrow A=0 \Rightarrow$ trivial answer

case 3: $b>0$ $\quad b=w^2$ $\quad r = \pm wi$ $\quad y(x) = A\cos wx + B\sin wx$

$0 = y'(0) = -Aw\sin(0) + Bw\cos(0) = B \Rightarrow y(x) = A\cos wx$

$0 = y'(2) = -Aw\sin(2w)$, $A \neq 0 \Rightarrow \sin 2w = 0 \Rightarrow w = \frac{n\pi}{2}$, $n \in \mathbb{N}$

solution: $y(x) = B$ or $y(x) = \cos\left(\frac{n\pi x}{2}\right)$

$b=0$ or $b = w^2 = \left(\frac{n\pi}{2}\right)^2$

$$(f(x), g(x)) = \int_a^b f(x)\cdot\overline{g(x)}\,dx = \int_0^2 \cos\left(\frac{n\pi x}{2}\right)\cdot\cos\left(\frac{m\pi x}{2}\right)dx$$

$$= \frac{1}{2}\int_0^2 2\cos\left(\frac{\frac{(n+m)\pi x}{2} + \frac{(n-m)\pi x}{2}}{2}\right)\cos\left(\frac{\frac{(n+m)\pi x}{2} - \frac{(n-m)\pi x}{2}}{2}\right)dx = \frac{1}{2}\int_0^2 \cos\left(\frac{(n+m)\pi x}{2}\right) + \cos\left(\frac{(n-m)\pi x}{2}\right)dx$$

$$= \frac{1}{2}\left(\frac{2}{(n+m)\pi}\sin\left(\frac{(n+m)\pi x}{2}\right) + \frac{2}{(n-m)\pi}\sin\left(\frac{(n-m)\pi x}{2}\right)\right]_0^2 = \frac{\sin((n+m)\pi)}{(n+m)\pi} + \frac{\sin((n-m)\pi)}{(n-m)\pi} = 0$$

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c) $y''(x) + cy(x) = 0$, $y(0) = 0$, $y(8) + y'(8) = 0$, $x \in [0,8]$

case 1: $c = 0 \Rightarrow y(x) = Ax + B \Rightarrow$ $\quad 0 = y(0) = B \Rightarrow y(x) = Ax$

$y(8) + y'(8) = 0 = 8A + A \Rightarrow A = 0 \Rightarrow$ trivial answer

case 2: $c < 0$ $\quad c = -w^2$ $\quad r = \pm w$ $\quad y(x) = Ae^{wx} + Be^{-wx}$

$y'(x) = Awe^{wx} - Bwe^{-wx}$, $\quad 0 = y(0) = A + B \Rightarrow y(x) = A(e^{wx} - e^{-wx})$, $y'(x) = Aw(e^{wx} + e^{-wx})$

$0 = y(8) + y'(8) = A(e^{8} - e^{-8w}) + Aw(e^{8w} + e^{-8w})$, $A \neq 0 \Rightarrow -w = \frac{e^{8w} - e^{-8w}}{e^{8w} + e^{-8w}} = \tanh(8w)$

function $\tanh(x)$ increase and odd, therefore $w = 0$ and meaning a trivial answer.

case 3: $c > 0$ $\quad c = w^2$ $\quad r = \pm wi$ $\quad y = A\cos wx + B\sin wx$

$0 = y(0) \Rightarrow A = 0 \Rightarrow y = B\sin wx$, $y' = Bw\cos wx$

$y(8) + y'(8) = B\sin(8w) + Bw\cos(8w) = 0$, $B \neq 0 \Rightarrow -w = \tan(8w)$

for all the answers (except 0) we'll get $c = w^2$

$y(x) = B\sin(wx)$