Homework #2

① ⓐ $\lim_{x \to 1} (4x-1) = 3$

For every $\varepsilon > 0$ there exists a $\delta > 0$ such that if

$0 < |x-1| < \delta$ then $|4x-1-3| < \varepsilon$

$0 < |x-1| < \delta$ then $|4x-4| < \varepsilon$

$$4|x-1| < \varepsilon$$

$$|x-1| < \frac{\varepsilon}{4} = \delta$$

ⓒ $\lim_{x \to 0^-} (e^{\frac{1}{x}}) = 0$

For every $\varepsilon > 0$ there exists a $\delta > 0$ such that if

$0 < 0 - x < \delta \Rightarrow e^{\frac{1}{x}} < \varepsilon$

$0 < -x < \delta \Rightarrow e^{\frac{1}{x}} < \varepsilon$

$0 < -x < \delta \Rightarrow \frac{1}{\ln\varepsilon} < x$

$$\delta = \frac{-1}{\ln\varepsilon}$$

ⓕ $\lim_{x \to 0^+} (\ln x) = -\infty$

for every $\varepsilon > 0$ there exists a $\delta > 0$ such that if

$0 < x - 0 < \delta \Rightarrow \ln x < B$

$0 < x < \delta \Rightarrow x < e^B$

$$\delta = e^B$$

ⓗ $\lim_{x \to -\infty} \left(\frac{1}{\ln x^2}\right) = 0$

for every $\varepsilon > 0$ there exists a $\delta > 0$ such that if

$x < R \Rightarrow \left| \frac{1}{\ln x^2} - 0 \right| < \varepsilon$

$x<R \Rightarrow \frac{1}{\ln x^2} < \varepsilon$

$x<R \Rightarrow \frac{1}{\varepsilon} < \ln x^2 \quad / \ln \sqrt{\ }$

$x<R \Rightarrow \sqrt{e^{\frac{1}{\varepsilon}}} < |x|$

$x<R \Rightarrow \sqrt{e^{\frac{1}{\varepsilon}}} < -x$

$x<R \Rightarrow x < -\sqrt{e^{\frac{1}{\varepsilon}}}$

$R = -\sqrt{e^{\frac{1}{\varepsilon}}}$

(11) $\lim_{x \to 0^+} \left(\frac{1}{1+2^{\frac{1}{x}}} \right) = 0$

For every $\varepsilon > 0$ there exists a $\delta > 0$ such that if

$0 < x < \delta \Rightarrow \left| \frac{1}{1+2^{\frac{1}{x}}} \right| < \varepsilon$

$0 < x < \delta \Rightarrow \frac{1}{1+2^{\frac{1}{x}}} < \varepsilon$

$0 < x < \delta \Rightarrow \frac{1}{\varepsilon} < 1+2^{\frac{1}{x}}$

$0 < x < \delta \Rightarrow \frac{1}{\varepsilon} - 1 < 2^{\frac{1}{x}}$

$0 < x < \delta \Rightarrow \frac{1-\varepsilon}{\varepsilon} < 2^{\frac{1}{x}}$

$0 < x < \delta \Rightarrow \log_2 \left(\frac{1-\varepsilon}{\varepsilon} \right) < \frac{1}{x}$

$0 < x < \delta \Rightarrow x < \frac{1}{\log_2 \left(\frac{1-\varepsilon}{\varepsilon} \right)}$

$\delta = \frac{1}{\log_2 \frac{1-\varepsilon}{\varepsilon}}$

(2) (e) $\lim_{x \to a} \frac{7x^2-2x-5}{x^2-3x+2}$

- $a=1$
- $a=2^+$
- $a=2$
- $a=-1$

- $\lim_{x \to 1} = \frac{0}{0}$ $\frac{(7x+5)(x-1)}{(x-2)(x-1)} = -\frac{12}{1} = -12$

- $\lim_{x \to -1} \frac{7x^2-2x-5}{x^2-3x+2} = \frac{4}{6}$

- $\lim_{x \to 2} \frac{(7x+5)(x-1)}{(x-2)(x-1)} = \frac{19}{0} = \pm\infty$ one sided limit

- $\lim_{x \to 2^+} \frac{(7x+5)(x-1)}{(x-2)(x-1)} = \frac{19}{0^+} = +\infty$

(9) $\lim_{x \to a} \frac{x^4-7x^3+2x^2+5x-1}{x^3+2x^2-x-2}$

- $a=2$

$\lim_{x \to 2} \frac{x^4-7x^3+2x^2+5x-1}{x^3+2x^2-x-2} = \frac{-23}{12}$

- $a=1$

$\lim_{x \to 1} \frac{x^4-7x^3+2x^2+5x-1}{x^3+2x^2-x-2} = \frac{0}{0}$

$\frac{(x-1)(x^3-6x^2-4x-1)}{(x-1)(x^2+3x+2)} = \lim_{n \to 1} \frac{x^3-6x^2-4x+1}{x^2+3x+2} = \frac{-8}{6} = \frac{-4}{3}$

- $a=-1$

$\lim_{x \to 1} \frac{x^4-7x^3+2x^2+5x-1}{x^3+2x^2-x-2} = \frac{4}{0} = \infty$

$\lim_{x \to -1^+} \frac{x^4-7x^3+2x^2+5x-1}{x^3+2x^2-x-2} = \frac{-}{-} = +\infty$

$\lim_{x \to -1^-} \frac{x^4-7x^3+2x^2+5x-1}{x^3+2x^2-x-2} = \frac{-}{+} = -\infty$

Type II Infinite discon

- $a=-2$

$\lim_{x \to -2} \frac{x^4-7x^3+2x^2+5x-1}{x^3+2x^2-x-2} = \frac{64}{0} = \infty$

Type II Infinite discon.

$\lim_{x \to -2^+} \frac{x^4-7x^3+2x^2+5x-1}{x^3+2x^2-x-2} = \frac{+}{+} = +\infty$

$\lim_{x \to -2^-} \frac{x^4-7x^3+2x^2+5x-1}{x^3+2x^2-x-2} = \frac{+}{-} = -\infty$

- $a = -\infty$

[illegible]

$$\lim_{x \to -\infty} \frac{x^4 - 7x^3 + 2x^2 + 5x - 1}{x^3 + 2x^2 - x - 2} = \frac{x^4\left(\overbrace{1}^{L=1} - \overbrace{\frac{7}{x}}^{L=0} + \overbrace{\frac{2}{x^2}}^{L=0} + \overbrace{\frac{5}{x^3}}^{L=0} - \overbrace{\frac{1}{x^4}}^{L=0}\right)}{x^3\left(\underbrace{1}_{L=1} + \underbrace{\frac{2}{x}}_{L=0} - \underbrace{\frac{1}{x^2}}_{L=0} - \underbrace{\frac{2}{x^3}}_{L=0}\right)} = \frac{x^4}{x^3} = -\infty$$

(j) $\lim\limits_{x \to 0} \dfrac{\sqrt{x^2+3x+1}-1}{x} = \dfrac{0}{0}$

$$\frac{\sqrt{x^2+3x+1}-1}{x} \cdot \frac{(\sqrt{x^2+3x+1}+1)}{(\sqrt{x^2+3x+1}+1)} = \frac{x^2+3x+1-1}{x\sqrt{x^2+3x+1}+x} =$$

$$\lim_{x \to 0} \frac{x+3}{\sqrt{x^2+3x+1}+1} = \frac{3}{2}$$

(l) $\lim (\sqrt{x^2+3x}+x) \dfrac{(\sqrt{x^2+3x}-x)}{(\sqrt{x^2+3x}-x)} = \dfrac{x^2+3x-x^2}{\sqrt{x^2+3x}-x} = \dfrac{3x}{\sqrt{x^2+3x}-x} =$

$$\frac{\frac{3x}{-x}}{\sqrt{\frac{x^2}{x^2}+\frac{3x}{x^2}}-\frac{x}{-x}} = \frac{-3}{\sqrt{1+\frac{3}{x}}+1} = \frac{-3}{\sqrt{1}+1} = -\frac{3}{2}$$

(m) $\lim\limits_{x \to -3} \dfrac{\sqrt{x^2+2x+6}-\sqrt{2x^2-9}}{1-\sqrt{x+4}} = \dfrac{0}{0}$ $\quad \dfrac{\sqrt{x^2+2x+6}-\sqrt{2x^2-9}}{1-\sqrt{x+4}} \cdot \dfrac{(\sqrt{x^2+2x+6}+\sqrt{2x^2-9})}{(\sqrt{x^2+2x+6}+\sqrt{2x^2-9})}$

$$\frac{x^2+2x+6-(2x^2-9)}{(1-\sqrt{x+4})(\sqrt{x^2+2x+6}+\sqrt{2x^2-9})} =$$

$$\frac{-x^2+2x-15}{(1-\sqrt{x+4})(\sqrt{x^2+2x+6}+\sqrt{2x^2-9})} =$$

$$\frac{(1+\sqrt{x+4})\ (-x^2+2x-15)}{(1+\sqrt{x+4})(1-\sqrt{x+4})(\sqrt{x^2+2x+6}+\sqrt{2x^2-9})} =$$

$$\frac{(1-\sqrt{x+4})(x+5)(\cancel{-x-3})}{(1-\cancel{x}-4)(\sqrt{x^2+2x+6}+\sqrt{2x^2-9})} = \frac{(1+\sqrt{x+4})(x+5)}{(\sqrt{x^2+2x+6}+\sqrt{2x^2-9})} =$$

$$\boxed{-\frac{8}{3}}$$

notebook

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(P) $\lim_{x\to -\infty} \dfrac{\sqrt{x^2+3x+8}}{2x+3} = \dfrac{\sqrt{\frac{x^2}{x^2}+\frac{3x}{x^2}+\frac{8}{x^2}}}{\frac{2x}{-x}+\frac{3}{x}} = \dfrac{\sqrt{1+0+0}}{-\frac{2}{1}+0} = -\frac{1}{2}$

(5) $\lim_{x\to -\infty} \left(\sqrt{x^2+3x+8}-\sqrt{x^2+1}\right) = \left(\sqrt{x^2+3x+8}-\sqrt{x^2+1}\right)\left(\dfrac{\sqrt{x^2+3x+8}+\sqrt{x^2+1}}{\sqrt{x^2+3x+8}+\sqrt{x^2+1}}\right)$

$\dfrac{x^2+3x+8-(x^2+1)}{\sqrt{x^2+3x+8}+\sqrt{x^2+1}} = \dfrac{3x+7}{\sqrt{x^2+3x+8}+\sqrt{x^2+1}} =$

$\dfrac{\frac{3x}{-x}+\frac{7}{-x}}{\sqrt{\frac{x^2}{x^2}+\frac{3x}{x^2}+\frac{8}{x^2}}+\sqrt{\frac{x^2}{x^2}+\frac{1}{x^2}}} = \dfrac{-3+\frac{7}{-x} \leftarrow L=0}{\sqrt{1+\frac{3}{x}+\frac{8}{x^2}}+\sqrt{1+\frac{1}{x^2}}} = \dfrac{-3}{\sqrt{1}+\sqrt{1}} = \dfrac{-3}{2}$

$L=0 \quad L=0 \quad L=0 \quad L=1 \quad L=0$

(4) notebook

(2) $f(x) = \begin{cases} 2x^3-5x^2-\frac{3}{x} & x<3 \\ 4 & x=3 \\ \frac{x+5}{(x-4)^2} & x>3 \end{cases}$

- $\lim_{x\to 0} 2x^3-5x^2-\frac{3}{x} = \dfrac{2x^4-5x^3-3}{x} = \dfrac{-3}{0}$

$\lim_{x\to 0^+} f(x) = \dfrac{2x^4-5x^3-3}{x} = \dfrac{-}{+} = -\infty$

$\lim_{x\to 0^-} f(x) = \dfrac{2x^4-5x^3-3}{x} = \dfrac{-}{-} = +\infty$

} Type III Infinite discon.

- $\lim_{x\to 2} 2x^3-5x^2-\frac{3}{x} = -5.5$

- $\lim_{x\to 3^-} f(x) = 2x^3-5x^2-\frac{3}{x} = \boxed{8}$ $\quad \lim_{x\to 3^+} f(x) = \dfrac{x+5}{(x-4)^2} = \dfrac{8}{1} = \boxed{8}$ $\quad \lim_{x\to 3} f(x) = 8$

explanation next pg.

- $\lim_{x \to 4} f(x) = \frac{x+5}{(x-4)^2} = \frac{9}{"0"} = +\infty$

$\lim_{x \to 4^+} \frac{x+5}{(x-4)^2} = \frac{+}{+} = +\infty$

$\lim_{x \to 4^-} \frac{x+5}{(x-4)^2} = \frac{+}{+} = +\infty$

- $\lim_{x \to \pm\infty} f(x) = \frac{x+5}{(x-4)^2} = \frac{\infty}{\infty} = \frac{x+5}{x^2-8x+16} \begin{matrix} 1:x \\ 1:x^2 \end{matrix} = \frac{\frac{x}{x}+\frac{5}{x}}{\frac{x^2}{x^2}-\frac{8x}{x^2}+\frac{16}{x^2}} = \frac{x(\overset{L=1}{1}+\overset{L=0}{\frac{5}{x}})}{x^2(\underset{L=0}{1}-\underset{}{\frac{8}{x}}+\underset{L=0}{\frac{16}{x^2}})} =$

$\frac{x}{x^2} = \frac{1}{x} = 0$

- $\lim_{x \to -\infty} f(x) = 2x^3 - 5x - \frac{3}{x} = x^3(\underset{L=2}{2} - \underset{L=0}{\frac{5}{x}} - \underset{L=0}{\frac{3}{x^4}}) = 2x^3 = -\infty$

$2-0-0=2$

$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$ — $\lim_{x\to 0} \frac{\sin x}{x} = 1$

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③ ⓐ $\lim_{x\to 0} \frac{\sin 3x}{\sin 5x}$

$\frac{\sin 3x}{\sin 5x} \cdot \frac{3x}{3x} \cdot \frac{5x}{5x} = \frac{3x}{5x} = \frac{3}{5}$

$=1$

$=1$

ⓒ $\lim_{x\to 0} \frac{\tan 4x \cos 5x}{\sin 3x} = \frac{\sin 4x}{\cos 4x} \cdot \frac{\cos 5x}{\sin 3x} \quad /\cdot\frac{3x}{3x} \quad /\cdot\frac{4x}{4x} = \frac{\cos 5x \cdot 4x}{\cos 4x \cdot 3x} = \frac{\cos 5x}{\cos 4x} \cdot \frac{4}{3} =$

$\lim_{x\to 0} \frac{\cos 5x}{\cos 4x} \cdot \frac{4}{3} = \frac{4}{3}$

ⓖ $\lim_{x\to 0} \frac{1-\cos 10x}{\sin^2 3x} = \quad /\cdot(10x)^2 \quad \frac{1-\cos 10x \cdot (10x)^2}{\sin^2 3x \cdot (10x)^2} = \frac{1}{2} \cdot \frac{(10x)^2}{\sin^2 3x} \quad /\cdot(3x)^2 =$

$=\frac{1}{2}$

$\frac{1}{2} \cdot \frac{(10x)^2 \cdot 3x^2}{\sin^2 3x \, (3x)^2} = \frac{1}{2} \cdot \frac{(10x)^2}{(3x)^2} = \frac{50x^2}{9x^2} = \frac{50}{9}$

$=1$

ⓙ $\lim_{x\to 0} \frac{x^2 \cdot \sin\frac{1}{x}}{\sin x} = \frac{x \cdot x \sin\frac{1}{x}}{\sin x} = x \cdot \sin\frac{1}{x} = 0$

$=1$ vanishing function $\cdot$ bounded function = vanishing

① $\lim\limits_{x \to \frac{\pi}{2}} \dfrac{\cos 3x}{\left(\frac{\pi}{2} - x\right)} = \dfrac{0}{0}$ $\qquad t = x - \frac{\pi}{2}$

$\qquad x = t + \frac{\pi}{2}$

$\lim\limits_{t \to 0} \dfrac{\cos 3\left(t + \frac{\pi}{2}\right)}{t} = \dfrac{\cos\left(3t + \frac{3\pi}{2}\right)}{t}$

(We will use the identity: $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$)

$\Downarrow$

$= \dfrac{\cos 3t \cos\frac{3\pi}{2} - \sin 3t \sin\frac{3\pi}{2}}{t}$

$\lim\limits_{t \to 0} = (-1) \cdot \dfrac{\sin 3t}{t} = -1 \cdot \dfrac{\sin 3t}{3t} \cdot 3 = \boxed{-3}$

⓪ $\lim\limits_{x \to \infty} \dfrac{2x + 5\sin x}{5x - 2} = \dfrac{2x}{5x-2} + \underbrace{\dfrac{5\sin x}{5x-2}}_{L=0} \Big\backslash \cdot \dfrac{x}{x} = \dfrac{2x}{5x-2} + \dfrac{5x}{5x-2} \cdot 0 = \dfrac{2x}{5x-2} \Big\backslash \cdot x =$

$\dfrac{\frac{2x}{x}}{\frac{5x}{x} - \frac{2}{x}} = \dfrac{2}{5 - \frac{2}{x}} = \dfrac{2}{5}$

Ⓟ $\lim\limits_{x \to 0} \dfrac{2x + 5\sin x}{5x - 2} = \dfrac{2x}{5x-2} + \underbrace{\dfrac{5(\sin x}{5x-2}}_{1} \Big\backslash \cdot \dfrac{x}{x} = \dfrac{2x}{5x-2} + \dfrac{5x}{5x-2} = \dfrac{7x}{5x-2} \Big\backslash \cdot x =$

$\dfrac{\frac{7x}{x}}{\frac{5x}{x} - \frac{2}{x}} = \dfrac{7}{5 - \underbrace{\frac{2}{x}}_{\infty}} = \dfrac{7}{5 - \infty} = 0$

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(r) $\lim_{x\to\pi} \frac{\sin 3x}{\sin 4x} = \frac{0}{0}$ 　　　 $x = \pi + t$ 　　 207

$\lim_{x\to t} \frac{\sin 3(\pi+t)}{\sin 4(\pi+t)} = \lim_{x\to t} \frac{\sin(3\pi+3t)}{\sin(4\pi+4t)}$

$= \lim_{x\to t} \frac{-\sin 3t}{\sin 4t} = \frac{-\sin 3t}{3t} \cdot \frac{3t}{4t} \cdot \frac{4t}{\sin 4t} = \frac{3}{4}$

$\uparrow$ -1 　 $\uparrow$ 3m 　 $\uparrow$ 1

(t) $\lim_{x\to 0} \frac{\tan x - \sin x}{\sin^3 x} \backslash \cdot \frac{x^3}{x^3} = \frac{\tan x - \sin x}{x^3} = \frac{\frac{\sin x}{\cos x} - \sin x}{x^3} = \frac{\sin x - \cos x \sin x}{x^3} =$

$\frac{\frac{\sin x}{\cos x} - \sin x}{\sin^3 x} = \frac{\frac{\sin x - \cos x \sin x}{\cos x}}{\sin^3 x} = \frac{\sin x(1-\cos x)}{\sin^3 x \cos x} =$

$\frac{1-\cos x}{\sin^2 x \cos x} \cdot \frac{x^2}{x^2} \; \lim_{x\to 0} \frac{1-\cos x}{\cos x \, x^2} = \frac{1}{2\cos x} = \frac{1}{2}$

$L = \frac{1}{2}$ 　　 $L = 1$

(4) $\lim_{x\to 0} \tan 4x \cot 7x = \frac{\sin 4x}{\cos 4x} \cdot \frac{\cos 7x}{\sin 7x} \cdot \frac{4x}{4x} \cdot \frac{7x}{7x} = \frac{4x}{\cos 4x} \cdot \frac{\cos 7x}{7x} = \frac{4\cos 7x}{7\cos 4x} \boxed{\frac{4}{7}}$

(4) (b) $\lim_{x\to 0} \left(x\left[\frac{7}{x}\right]\right)$ 　　　 (d) $\lim_{x\to 0} \left(x\sqrt{3-2\cos\frac{1}{x}}\right)$

given: $\frac{7}{x} - 1 \le \left[\frac{7}{x}\right] \le \frac{7}{x}$ 　 $/ \cdot x$

$x\left(\frac{7}{x}-1\right) \le x\left[\frac{7}{x}\right] \le \frac{7x}{x}$

$\lim_{x\to 0}$ 　 $7 - x \le x\left[\frac{7}{x}\right] \le 7$

$\downarrow$ $L=7$ 　 $\downarrow$ $L=7$

therefore $L=7$

$-1 \le \cos\frac{1}{x} \le 1$ 　 $/ \cdot -2$

$2 \ge -2\cos\frac{1}{x} \ge -2$ 　 $/ +3$

$5 \ge -2\cos\frac{1}{x} + 3 \ge 1$ 　 $/ \sqrt{\ }$

$\sqrt{5} \ge \sqrt{-2\cos\frac{1}{x}+3} \ge 1$ 　 $/ \cdot x$

$x\sqrt{5} \ge x\sqrt{-2\cos\frac{1}{x}+3} \ge x$

$\downarrow$ $L=0$ 　 $\downarrow$ $L=0$

therefore $L=0$

⑤ ⓑ $\lim_{x\to 0} (1-2x)^{\frac{3}{4x}}$

$\lim_{x\to 0} \left(\underbrace{\left((1-2x)^{-\frac{1}{2x}}\right)}_{e}\right)^{\frac{3}{-2}} = e^{-\frac{3}{2}}$

ⓔ $\lim_{x\to -\infty} \left(1-\frac{4}{2x-1}\right)^{6x-3} =$

$\left(1-\frac{4}{2x-1}\right)^{3(2x-1)} = \lim_{x\to -\infty} \underbrace{\left(\left(1-\frac{4}{2x-1}\right)^{\frac{(2x-1)}{-4}}\right)^{-12}}_{e^{-12}} = e^{-12}$

Prove that

⑨ ① $\lim_{x\to\pm\infty} \left(1+\frac{1}{x}\right)^{x+1} = e$ & ② $\lim_{x\to\pm\infty} \left(1-\frac{1}{x}\right)^{x+1} = e^{-1}$

① $\lim_{x\to\pm\infty} \left(1+\frac{1}{x}\right)^{x+1} = \underbrace{\left(1+\frac{1}{x}\right)^{x}}_{e} \cdot \left(1+\frac{1}{x}\right)^{1} = e\cdot 1 = e$

$L=1 \quad L=0$

② $\lim_{x\to\pm\infty} \left(1-\frac{1}{x}\right)^{x+1} = \left(1-\frac{1}{x}\right)^{x} \cdot \left(1-\frac{1}{x}\right)^{1}$

$L=1 \quad L=0$

$\left(\underbrace{\left(\left(1+\frac{1}{-x}\right)^{-x}\right)}_{e^{-1}}^{\frac{-1}{x}}\right)^{x} \longrightarrow e^{-1}\cdot 1 = e^{-1}$

ⓙ $\lim_{x\to+\infty} \left(\frac{2x+5}{2x+1}\right)^{2x-3} = \left(\frac{2x+1+4}{2x+1}\right)^{2x-3} = \left(1+\frac{4}{2x+1}\right)^{2x-3} =$

$\left(\underbrace{\left(\left(1+\frac{4}{2x+1}\right)^{\frac{2x+1}{4}}\right)}_{e}^{\frac{4}{2x+1}}\right)^{2x-3}$

$e = \lim_{x\to+\infty} e^{\frac{8x-12}{2x+1}} = \lim_{x\to+\infty} \frac{8x-12}{2x+1} /\div x = \frac{8-\overset{L=0}{\frac{12}{x}}}{2+\underset{L=0}{\frac{1}{x}}} = \frac{8}{2} = e^{4}$

ⓜ $\lim_{x\to+\infty} \left(2+\frac{1}{x}\right)^{x} = 2^{x}\left(1+\frac{1}{2x}\right)^{x} = 2^{x}\underbrace{\left(\left(1+\frac{1}{2x}\right)^{2x}\right)}_{e}^{\frac{1}{2}} = 2^{x}e^{\frac{1}{2}} =$

$\lim_{x\to+\infty} \underset{\infty}{2^{x}}\cdot e^{\frac{1}{2}} = \infty$

⑨ $\lim_{x\to+\infty}\left(\frac{2x-3}{x-4}\right)^{3x-5} = \frac{(2x-8)+11}{x-4} = \frac{2(x-4)+11}{x-4} =$

$\left(2\cdot 1+\frac{11}{x-4}\right)^{3x-5} = 2\left(1+\frac{11}{2(x-4)}\right)^{3x-5} =$

$\left(2\left(1+\frac{11}{2x-8}\right)^{3x-5}\right) = 2\left(1+\frac{1}{\frac{2x-8}{11}}\right)^{3x-5} :$

$\lim_{x\to+\infty} 2^{3x-5}\left(\left(\underbrace{\left(1+\frac{1}{\frac{2x-8}{11}}\right)^{\frac{2x-8}{11}}}_{e}\right)^{\frac{11}{2x-8}}\right)^{3x-5} = 2^{3x-5}\cdot e^{\frac{11(3x-5)}{2x-8}} =$

$\lim_{x\to+\infty} 2^{\infty}\cdot e^{\frac{33x-55}{2x-8}} /:x = 2^{\infty}\cdot e^{\frac{33}{2}} = \infty$

⑩ $\lim_{x\to+\infty}\left(\frac{2x+1}{3x-2}\right)^{4x-3} = \frac{2x+1}{3x-2} /:x \quad \frac{2+\overset{L=0}{\frac{1}{x}}}{3-\underset{L=0}{\frac{2}{x}}} = \frac{2}{3} \qquad 0<\frac{2}{3}<1$

$\lim_{x\to+\infty}\left(\frac{2x+1}{3x-2}\right)^{4x-3} = 0$

⑪ $\lim_{x\to0^-}\left(\frac{2x+3}{x+4}\right)^{\frac{3}{x}} = \left(\frac{(2x+8)-5}{x+4}\right)^{\frac{3}{x}} = \left(2-\frac{5}{x+4}\right)^{\frac{3}{x}} = \left(2\left(1-\frac{5}{2(x+4)}\right)\right)^{\frac{3}{x}} =$

$2^{\frac{3}{x}}\left(1-\frac{1}{\frac{2(x+4)}{5}}\right)^{\frac{3}{x}} = 2^{\frac{3}{x}}\left(\underbrace{\left(1-\frac{1}{\frac{2x+8}{5}}\right)^{\frac{2x+8}{5}}}_{e}\right)^{\frac{5}{2x+8}\cdot\frac{3}{x}} = 2^{\frac{3}{x}}\cdot e^{\frac{15}{2x^2+8x}} =$

$\lim_{x\to0^-} 2^{\frac{3}{x}}\cdot\frac{\frac{15}{x^2}}{e^{2+\frac{8}{x}}} = \infty$

⑫ $\lim_{x\to1^+} 2^{\frac{1}{1-x}} \leftarrow\infty = \lim_{x\to1^-} 2^{\infty} = \infty$

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(4) **example 1:** $F(x) = 1 + \frac{x}{1}$ $\quad g(x) = \frac{1}{x}$

$$\lim_{x \to 0^+} F(x) = 1 \quad , \quad \lim_{x \to 0^+} g(x) = +\infty$$

$$\lim (F(x))^{g(x)} = \left(1 + \frac{x}{1}\right)^{\frac{1}{x}} = e$$

example 2: $F(x) = 1 + \frac{3x}{1}$ $\quad g(x) = \frac{1}{x}$

$$\lim_{x \to 0^+} F(x) = 1 \qquad \lim_{x \to 0^+} g(x) = +\infty$$

$$\lim (F(x))^{g(x)} = \left(1 + \frac{3x}{1}\right)^{\frac{1}{x}} = \Bigg[\underbrace{\left(1 + \frac{3x}{1}\right)^{\frac{1}{3x}}}_{e}\Bigg]^3 = e^3$$

example 3: $F(x) = 1 + \frac{4x}{1}$ $\quad g(x) = \frac{1}{x}$

$$\lim_{x \to 0^+} F(x) = 1 \qquad \lim_{x \to 0^+} g(x) = +\infty$$

$$\lim_{x \to 0^+} (F(x))^{g(x)} = \left(1 + \frac{4x}{1}\right)^{\frac{1}{x}} = \Bigg[\underbrace{\left(1 + \frac{4x}{1}\right)^{\frac{1}{4x}}}_{e}\Bigg]^4 = e^4$$

(6) (c) $\lim_{x \to a} \frac{\sin x - \sin a}{x - a} = \cos a$

$$\lim_{x \to a} \frac{\sin x - \sin a}{x - a}$$

$$\lim_{x \to a} \frac{2\sin\left(\frac{x-a}{2}\right) \cdot \cos\left(\frac{x+a}{2}\right)}{x - a}$$

$t = x - a$ $\qquad$ $\to 3)$

$$\lim_{t \to 0} = 2 \cdot \frac{\sin \frac{t}{2}}{\frac{t}{2}} \cdot \frac{1}{2} \cdot \cos\left(\frac{t+2a}{2}\right) = \lim_{t \to 0} \cos\left(\frac{2a}{2}\right) = \cos a$$

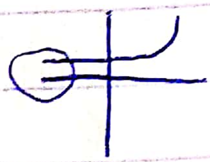
5.?

7 b If $\lim_{x \to -\infty} f(x) = l < 0$ & $\lim g(x) = -\infty$

ex: $f(x) = e^x - 1$ $\quad \lim_{x \to -\infty} e^x = 0 \; (\Rightarrow \frac{1}{e^x})$

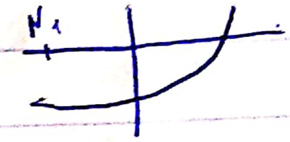
$g(x) = x$

$\lim_{x \to -\infty} x = -\infty$ $\quad \downarrow$



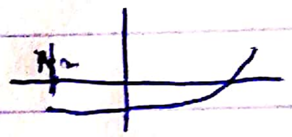
$\lim_{x \to \infty} x(e^x - 1) = \infty$ $\quad \lim_{x \to -\infty} e^x - 1 = -1$

$\lim_{x \to -\infty} f(x) = l < 0$



$\lim_{x \to -\infty} g(x) = -\infty$

Prove: $\lim_{x \to -\infty} f(x)g(x) = \infty$



$f(x)$ for every $\varepsilon > 0$ $\exists$ $N < 0$ s.t if $x < N$, $|f(x) - L| < \varepsilon$

$g(x)$ for every $M < 0$ $\exists$ $N_2 < 0$ s.t if $x < N_2$ $g(x) < M$ take

$N = \min(N_1, N_2)$ then $\lim_{x \to -\infty} f(x)g(x) = \infty$

Need: definition lim of $f(x)\,g(x)$

8) given: $\lim_{x \to 0} \frac{\sin x}{x} = 1$

a) $\lim_{x \to \pi} \frac{\sin(x-\pi)}{x-\pi} = \frac{0}{0}$

$t = x - \pi$ $\quad \lim_{t \to 0} \frac{\sin t}{t} = 1$

b) $\lim_{x \to 0} \frac{\sin 7x}{7x} = 1$

$t = 7x$ $\quad \lim_{t \to 0} \frac{\sin t}{t} = 1$

5) i) $\lim_{x \to \infty} \left(\frac{x+4}{x+5}\right)^x = \lim_{x \to \infty} \left(\frac{x+5-1}{x+5}\right)^x =$

$\lim_{x \to \infty} \left(1 + \left(\frac{-1}{x+5}\right)\right)^x = \lim_{x \to \infty} \left(\left(\left(1 + \frac{1}{-(x+5)}\right)^x\right)^{-x-5}\right)^{\frac{1}{-x-5}} =$

$\lim_{x \to \infty} \left(\underbrace{\left(1 + \frac{1}{-x-5}\right)^{-x-5}}_{e}\right)^{\frac{x}{-x-5}} =$

$\lim_{x \to \infty} e^{\frac{x}{-x-5}} \to e^{\lim_{x \to \infty} \frac{x}{-x-5}}$ $\quad /:x$

$\lim \frac{x}{-x-5} = \frac{\frac{x}{x}}{-\frac{x}{x} - \frac{5}{x}} = \frac{1}{-1-\frac{5}{x}} = -1 \quad = e^{-1}$

$\frac{5}{x} \to L = 0$

6 a $\lim_{x\to a} \frac{x^n - a^n}{x-a} = na^{n-1}$ ($= \frac{0}{0}$ plug in)

$x^n - a^n = (x-a)(x^{n-1} + x^{n-2}a + \dots + xa^{n-2} + a^{n-1})$

$\lim_{x\to a} \frac{\cancel{(x-a)}(x^{n-1} + x^{n-2}a + \dots + xa^{n-2} + a^{n-1})}{\cancel{(x-a)}}$

$= \underbrace{a^{n-1} + a^{n-1} + \dots + a^{n-1} + a^{n-1}}_{n \text{ terms}} = \boxed{na^{n-1}}$