

Mathematical logic hw #6

- ① (b) well formed \downarrow proposition x
 first x -bounded second x -free
- (d) well formed \downarrow proposition \downarrow
 first x -bounded second x -bounded
- (f) well formed \downarrow proposition \downarrow
 x -bounded
- (h) well formed \downarrow proposition x
 x -bounded second x -free
- (j) well formed \downarrow proposition \downarrow
 x -bounded
- (i) well formed x proposition x (not WFF)
 second y -free

③ Universe: $\{10, 15, 20, 25, 55, 100\}$

$Q(x)$: x divisible by 5

$P(x)$: x is a number whose units place is 5

(B) $x=15$: $F \rightarrow F$: True (D) $F \rightarrow T$: True (E) $\forall x (T \rightarrow T)$: True

$x=35$: $F \rightarrow T$: True

$x=55$: $F \rightarrow T$: True

(H) $x=13$

(J) $\exists x (T \wedge T)$: True (I) not a WFF

$x=35$

$x=55$

(4) (a) 3

(b) $\exists x \forall y (P(x) \leftrightarrow \exists z R(y, z))$

Universe $\{a, b\}$

$\exists x \forall y (P(x) \leftrightarrow (R(y, a) \vee R(y, b)))$

$\exists x ((P(x) \leftrightarrow (R(a, a) \vee R(a, b))) \wedge (P(x) \leftrightarrow (R(b, a) \vee R(b, b))))$

$(P(a) \leftrightarrow (R(a, a) \vee R(a, b))) \wedge (P(a) \leftrightarrow (R(b, a) \vee R(b, b))) \vee (P(b) \leftrightarrow (R(a, a) \vee R(a, b))) \wedge (P(b) \leftrightarrow (R(b, a) \vee R(b, b)))$

(6) (2) true

(3) (b) true, if $R(a, b) = "a \text{ is min in } b" \text{ in } \emptyset$

④ true, if $R(a, b) = "a \text{ is the square root of } b"$, in \mathbb{R}^+

⑤ $\forall x \forall y ((p(x) \wedge p(y) \wedge R(x, y)) \rightarrow \exists z (R(x, z) \wedge R(z, y)))$ Universe: \mathbb{N}

- for any two even numbers where the first is less than the 2nd, there is a number between them