מטלה 4
רפאל

אב לינארית 2 (ד)

1 א (עמוד 2 בסיסיים)

$B = \{1, x, x^2, x^3\}$

$C = \{1, 1+x, x^2+x^3, x^2-x^3\}$

נתונה ה"ל $T: R[x]_3 \to R[x]_3$, $T(p(x)) = p(x) + p'(x)$

$$(i)\ \left[\, T[1]_B \;\; T[x]_B \;\; T[x^2]_B \;\; T[x^3]_B \,\right]$$

$T[1]_B:\ T(1) = 1+0 = 1 \qquad 1 = \alpha\cdot 1 + \beta\cdot x + \gamma\cdot x^2 + \delta\cdot x^3$

$\alpha = 1$

$\beta = \gamma = \delta = 0$

קיבלנו: $(1,0,0,0)$

★

$T[x^2]:\ T(v^2) = x^2 + 2x = \alpha\cdot 1 + \beta x + \gamma x^2 + \delta x^3$

$\gamma = 1$

$\beta = 2$

$\alpha = \delta = 0$

קיבלנו: $(0,2,1,0)$

$T[x^3]:\ T(x^3) = x^3 + 3x^2 = \alpha\cdot 1 + \beta x + \gamma x^2 + \delta x^3$

$\delta = 1$

$\gamma = 3$

$\alpha = \beta = 0$

קיבלנו: $(0,0,3,1)$

$$[T]_B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

★ $T[x]_B:\ T(x) = x+1 = \alpha\cdot 1 + \beta x + \gamma x^2 + \delta x^3$

$\alpha = \beta = 1$

$\gamma = \delta = 0 \qquad$ קיבלנו: $(1,1,0,0)$

$$(ii)\ [T]_C^B \left[\, T[1]_B \;\; T[1+x]_B \;\; T[x^2+x^3]_B \;\; T[x^2-x^3] \,\right]$$

$T[1]_B : T(1) = 1+0 = \alpha\cdot 1 + \beta x + \gamma x^2 + \delta x^3$ קיבל

$\alpha = 1$

$\beta = \gamma = \delta = 0$

$(1,0,0,0)$ קיבלנו

$T[1+x]_B : T(1+x) = 1+x+1 = 2+x = \alpha\cdot 1 + \beta x + \gamma x^2 + \delta x^3$

$\alpha = 2$

$\beta = 1$

$\gamma = \delta = 0$

$(2,1,0,0)$ קיבלנו

$T[x^2+x^3] : T(x^2+x^3) = x^2+x^3+2x+3x^2 = x^3+4x^2+2x$

$x^3+4x^2+2x = \alpha\cdot 1 + \beta x + \gamma x^2 + \delta x^3$

$\alpha = 0$

$\beta = 2$

$\gamma = 4$

$\delta = 1$

$(0,2,4,1)$ קיבלנו:

$T[x^2-x^3] : T(x^2-x^3) = x^2-x^3+2x-3x^2 = -x^3-2x^2+2x$

$-x^3-2x^2+2x = \alpha\cdot 1 + \beta x + \gamma x^2 + \delta x^3$

$\alpha = 0$

$\beta = 2$

$\gamma = -2$

$\delta = -1$

$(0,2,-2,-1)$ קיבלנו:

$$[T]^B_C = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$[T]_C \begin{bmatrix} | & | & | & | \\ T[1]_C & T[1+x]_C & T[x^2+x^3]_C & T[x^2-x^3]_C \\ | & | & | & | \end{bmatrix}$$

$T[1]_C : T(1) = 1+0 = \alpha\cdot 1 + \beta(1+x) + \gamma(x^2+x^3) + \delta(x^2-x^3)$

$\alpha+\beta = 1 \Rightarrow \alpha = 1$

$\beta = 0$

המשך

$$\gamma+\delta=0 \qquad \gamma-\delta=0 \Rightarrow \delta=\gamma=0$$

קיבלנו: $(1,0,0,0)$

$T[1+x]_C$: $T(1+x)=1+x+1=2+x$

$$2+x=\alpha\cdot 1+\beta(1+x)+\gamma(x^2+x^3)+\delta(x^2-x^3)$$

$$\alpha+\beta=2 \Rightarrow \alpha=1$$

$$\beta=1$$

$$\gamma+\delta=0 \qquad \gamma-\delta=0 \Rightarrow \gamma=\delta=0$$

קיבלנו: $(1,1,0,0)$

$T[x^2+x^3]$: $T(x^2+x^3)=x^2+x^3+2x+3x^2=x^3+4x^2+2x$

$$x^3+4x^2+2x=\alpha\cdot 1+\beta(1+x)+\gamma(x^2+x^3)+\delta(x^2-x^3)$$

$$\alpha+\beta=\Rightarrow \alpha=-2$$

$$\beta=2$$

$$\gamma+\delta=4 \Rightarrow 1+2\delta=4 \qquad \delta=\frac{3}{2}$$

$$\gamma-\delta=1 \Rightarrow \gamma=1+\delta \qquad \gamma=-\frac{1}{2}$$

קיבלנו: $(-2,2,\ 3/2,\ -1/2)$

$T[x^2-x^3]$: $T(x^2-x^3)=x^2-x^3+2x-3x^2=-x^3-2x^2+2x$

$$-x^3-2x^2+2x=\alpha\cdot 1+\beta(1+x)+\gamma(x^2+x^3)+\delta(x^2-x^3)$$

$$\alpha+\beta=0 \Rightarrow \alpha=-2$$

$$\beta=2$$

$$\gamma+\delta=-2 \Rightarrow -1+2\delta=-2 \Rightarrow \delta=-1/2$$

$$\gamma-\delta=-1 \Rightarrow \gamma=-1+\delta \Rightarrow \gamma=-3/2$$

קיבלנו: $(-2,2,-1/2,-3/2)$

$$[T]_C=\begin{bmatrix}1&1&-2&-2\\0&1&2&2\\0&0&3/2&-1/2\\0&0&-1/2&-3/2\end{bmatrix}$$

(ב) נמצא 2 בסיסים:

$$C=\left\{\begin{bmatrix}1&1\\0&0\end{bmatrix}\begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}1&0\\1&0\end{bmatrix}\begin{bmatrix}0&1\\0&1\end{bmatrix}\right\}$$

$$B = \left\{ \begin{bmatrix}1&0\\0&0\end{bmatrix} \begin{bmatrix}0&1\\0&0\end{bmatrix} \begin{bmatrix}0&0\\1&0\end{bmatrix} \begin{bmatrix}0&0\\1&0\end{bmatrix} \right\}$$

$$\left[T\left[\begin{pmatrix}1\\0\\0\\0\end{pmatrix}\right]_B \; T\left[\begin{pmatrix}0\\1\\0\\0\end{pmatrix}\right]_B \; T\left[\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\right]_B \; T\left[\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right]_B \right]$$

$T[(1,0,0,0)]_B = T(1000) = (1,0,0,0) \quad \alpha=1 \quad \beta=\gamma=\delta=0$

$(1,0,0,0)$ קבלנו:

$T[(0,1,0,0)]_B = T(0100) = (0,0,1,0) \quad \alpha=\beta=\delta=0 \quad \gamma=1$

$(0,0,1,0)$ קבלנו:

$T[(0,0,1,0)]_B = T(0010) = (0,1,0,0) \quad \alpha=\gamma=\delta=0 \quad \beta=1$

$(0,1,0,0)$ קבלנו:

$T[(0,0,0,1)]_B = T(0001) = (0,0,0,1) \quad \alpha=\beta=\gamma=0 \quad \delta=1$

$(0,0,0,1)$ קבלנו:

$$[T]_B = \begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}$$

$$\left[T\left[\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\right]_B \; T\left[\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\right]_B \; T\left[\begin{pmatrix}1\\0\\1\\0\end{pmatrix}\right]_B \; T\left[\begin{pmatrix}0\\1\\0\\1\end{pmatrix}\right]_B \right]$$

$T[(1,1,0,0)]_B = T(1100) = (1,0,1,0) \quad \alpha=\gamma=1 \quad \beta=\delta=0$

$(1,0,1,0)$ קבלנו:

$T[(0,0,1,0)]_B = T(0010) = (0,1,0,0) \quad \alpha=\gamma=\delta=0 \quad \beta=1$

$(0,1,0,0)$ קבלנו:

$T[(1,0,1,0)]_B = T(1010) = (1,1,0,0) \quad \alpha=\beta=1 \quad \gamma=\delta=0$

$(1,1,0,0)$ קבלנו:

$T[(0,1,0,1)]_B = T(0101) = (0,0,1,1) \quad \alpha=\beta=0 \quad \gamma=\delta=1$

$(0,0,1,1)$ קבלנו:

$$[T]_C^B = \begin{bmatrix}1&0&1&0\\0&1&1&0\\1&0&0&1\\0&0&0&1\end{bmatrix}$$

$$[T]_c = \left[T\left[\begin{pmatrix}1\\1\\0\\0\end{pmatrix}\right]_c \; T\left[\begin{pmatrix}0\\0\\1\\0\end{pmatrix}\right]_c \; T\left[\begin{pmatrix}1\\0\\1\\0\end{pmatrix}\right]_c \; T\left[\begin{pmatrix}0\\1\\0\\1\end{pmatrix}\right]_c \right]$$

$T[(1,1,0,0)]_c = T(1,1,0,0) = (1,0,1,0)$ $\quad \alpha=\beta=\delta=0 \quad \gamma=1$

קיבלנו: $(0,0,1,0)$

$T[(0,0,1,0)]_c = T(0,0,1,0) = (0,1,0,0)$ $\quad \alpha=\beta=1 \quad \gamma=-1 \quad \delta=0$

קיבלנו: $(1,1,-1,0)$

$T[(1,0,1,0)]_c = T(1,0,1,0) = (1,1,0,0)$ $\quad \alpha=1 \quad \beta=\gamma=\delta=0$

קיבלנו: $(1,0,0,0)$

$T[(0,1,0,1)]_c = T(0,1,0,1) = (0,0,1,1)$ $\quad \gamma=\delta=1 \quad \alpha=-1 \quad \beta=0$

קיבלנו: $(-1,0,1,1)$

$$[T]_c = \begin{bmatrix} 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$T(P(x)) = (P(0), P(1), P(-1))$ $\qquad T: \mathbb{R}[x]_2 \to \mathbb{R}^{(3)}$

$B = \{1, x, x^2\}$ $\qquad C = \{(1,0,0)\,(0,1,0)\,(0,0,1)\}$

$$[T]_B^c = \left(T[1]_c \quad T[x]_c \quad T[x^2]_c \right)$$

$T[1]_c = T[1] = (1,1,1)$

$\alpha(1,0,0) + \beta(0,1,0) + \gamma(0,0,1)$

$(1,1,1)$ קיבלנו $\quad \alpha=\beta=\gamma=1$

$T[x]_c = T[x] = (0,1,-1) =$

$\alpha(1,0,0) + \beta(0,1,0) + \gamma(0,0,1)$

$(0,1,-1)$ קיבלנו $\quad \alpha=0 \quad \beta=1 \quad \gamma=-1$

$T[x^2]_c = T[x^2] = (0,1,1)$

$\alpha(1,0,0) + \beta(0,1,0) + \gamma(0,0,1)$

$(0,1,1)$ קיבלנו $\quad \alpha=0, \beta=1, \gamma=1$

$$[T]_B^C = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$$

(3) יהי $P(x)=a+bx+cx^2 \in \mathbb{R}[x]_2$

$L: T[(P(x))]_C : T(P(x))=(a, a+b+c, a-b+c) = T[(P(x))]_C$

$\alpha(1,0,0)+\beta(0,1,0)+\gamma(0,0,1)$

$\alpha=a \quad \beta=a+b+c \quad \gamma=a-b+c$

$R: [T]^C_B \, [P(x)]_B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} [P(x)]_B$

$[P(x)]_B = a+bx+cx^2 = \alpha\cdot 1+\beta x+\gamma x^2$

$\alpha=a$

$\beta=b$

$\gamma=c$

$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = (a, a+b+c, a-b+c)$

קיבלנו אם ש- $R=L$

(ב) בשאלה 1 סעיף ב' קיבלנו את המטריצה המייצגת את ההעתקה $[T]^C_B$ כעת נחפש את המטריצה ההפיכה

$([T]^C_B)^{-1} = [T^{-1}]^C_B$

$[T]^C_B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix}$

נדרג את המטריצה ע"מ למצוא את המטריצה ההפיכה:

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 1 \end{array}\right] \begin{matrix} R_2-R_1 \\ R_3-R_1 \end{matrix} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & -1 & 1 & -1 & 0 & 1 \end{array}\right] R_3+R_2$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & -2 & 1 & 1 \end{array}\right] R_3=\frac{1}{2}R_3 \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1/2 & 1/2 \end{array}\right] R_2-R_3$

$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & -1/2 \\ 0 & 0 & 1 & -1 & 1/2 & 1/2 \end{array}\right]$

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קיבלנו:

$$[T^{-1}]^{C}_{B} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ -1 & 1/2 & 1/2 \end{bmatrix}$$

כעת נחשב את ההעתקה

מכיוון שבסעיף 2 שאלה 1 ההעתקה הייתה $T: R[x]_2 \to R^{(3)}$

כאן ההעתקה ההפוכה תהיה $T: R^{(3)} \to R[x]_2$

נחשב אותה:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ -1 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} a \\ 1/2(b-c) \\ -a+1/2(b+c) \end{bmatrix}$$

3) נתונה ה"ל $T: R[x]_2 \to R[x]_2$ $\quad T(P(x)) = P(x)+P(x+2)$

הראו כי T ההעתקה הפיכה ומצאו $T^{-1}(a+bx+cx^2)$

נשתמש בבסיס הסטנדרטי $B = \{1, x, x^2\}$

נחשב $[T]^{B}_{B}$ ולכן $[[T]^{B}_{B}]^{-1} = [T^{-1}]^{B}_{B}$

ולכן לכל $P(x)$

$$[T^{-1}P(x)]^{B}_{B} = [T^{-1}]^{B}_{B}[P(x)]_B$$

$$[T]^{B}_{B} = \begin{bmatrix} [T(1)]_B & [T(x)]_B & [T(x^2)]_B \end{bmatrix}$$

$[T(1)]_B$: $T(1) = 1+1 = 2 = \alpha \cdot 1 + \beta x + \gamma x^2$

$\alpha = 2$

$\beta = \gamma = 0$ קיבלנו: $(2,0,0)$

$[T(x)]_B$: $T(x) = x+x+2 = 2x+2 = \alpha \cdot 1 + \beta x + \gamma x^2$

$\alpha = \beta = 2$

$\gamma = 0$ קיבלנו: $(2,2,0)$

$[T(x^2)]_B$: $T(x^2) = x^2+(x+2)^2 = x^2+x^2+4x+4$

$= 2x^2+4x+4 = \alpha \cdot 1 + \beta x + \gamma x^2$

$\alpha = \beta = 4$

$\gamma = 2$ קיבלנו: $(4,4,2)$

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דרך

קבלנו דטרמ׳ :

$$[T]_B^B = \begin{bmatrix} 2 & 2 & 4 \\ 0 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix} \quad \left[[T]_B^B\right]^{-1}$$ נהפך מטריצה כדי לקבל את

$$\left[\begin{array}{ccc|ccc} 2 & 2 & 4 & 1 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 = \frac{1}{2}R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1/2 & 0 & 0 \\ 0 & 2 & 4 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] R_2 = \frac{1}{2}R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 2 & 1/2 & 0 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 - R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \frac{1}{2}R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 1 & 2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right] \xrightarrow{R_2 - 2R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1/2 & -1/2 & 0 \\ 0 & 1 & 0 & 0 & 1/2 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1/2 \end{array}\right]$$

קבלנו

$$\left[[T]_B^B\right]^{-1} = [T^{-1}]_B^B = \begin{bmatrix} -1/2 & -1/2 & 0 \\ 0 & 1/2 & 1 \\ 0 & 0 & 1/2 \end{bmatrix}$$

לכן (משפט)

$$[T^{-1}\, a+bx+cx^2]_B^B = [T^{-1}]_B^B\, [P(x)]_B$$

$$[T^{-1}(a+bx+cx^2)]_B^{B} = \begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & 1/2 & 1 \\ 0 & 0 & 1/2 \end{bmatrix} [a+bx+cx^2]_B$$

$$\begin{bmatrix} 1/2 & -1/2 & 0 \\ 0 & 1/2 & 1 \\ 0 & 0 & 1/2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{array}{c} \begin{bmatrix} \frac{1}{2}(a-b) \\ \frac{b}{2}+c \\ c \end{bmatrix} \end{array} \begin{array}{c} 1 \\ x \\ x^2 \end{array}$$

$$T^{-1}(a+bx+cx^2) = \frac{1}{2}(a-b) + \left(\frac{b}{2}+c\right)x + cx^2$$

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(6)

$B' = \{1, 1+x, 1+x+x^2\}$ $\qquad B = \{1, x, x^2\}$ $\qquad R[x]_2$

$[T]_{B'} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ $\qquad$ -1 إذا $T: R[x]_2 \to R[x]_2$

أوجد $[T]_B$

$$[T]_{B'} = \begin{bmatrix} [T(1)]_{B'} & [T(1+x)]_{B'} & [T(1+x+x^2)]_{B'} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

$[T(1)]_{B'} = (1,1,1) \Rightarrow 1(1) + 1(1+x) + 1(1+x+x^2) = 3+2x+x^2$

$[T(x+1)]_{B'} = (1,1,-1) \Rightarrow 1(1) + 1(1+x) - 1(1+x+x^2) = 1-x^2$

$[T(1+x+x^2)]_{B'} = (1,-1,1) \Rightarrow 1(1) - 1(1+x) + 1(1+x+x^2) = 1+x^2$

$$[T]_B = \begin{bmatrix} [T(1)]_B & [T(x)]_B & [T(x^2)]_B \end{bmatrix}$$

$T(1) = 3+2x+x^2$

$T(x+1) = 1-x^2$

$T(1+x+x^2) = 1+x^2$

$T(1) = 3x+2x+x^2 = \alpha \cdot 1 + \beta x + \gamma x^2 \to \alpha = 3 \quad \beta = 2 \quad \gamma = 1$

$[T(1)]_B = (3,2,1)$ $\qquad$ إذن:

$T(x) = T(1+x-1) = T(1+x) - T(1) = 1-x^2 - (3+2x+x^2)$

$-2x^2 - 2x - 2 = \alpha \cdot 1 + \beta x + \gamma x^2 \to \alpha = \beta = \gamma = -2$

$[T(x)]_B = (-2,-2,-2)$ $\qquad$ إذن:

$T(x^2) = T(1+x+x^2-1-x) = T(1+x+x^2) - T(1+x) =$

$1+x^2-(1-x^2) = 2x^2 = \alpha \cdot 1 + \beta x + \gamma x^2 \to \alpha = \beta = 0$

$[T(x^2)]_B = (0,0,2)$ $\qquad$ إذن: $\qquad \gamma = 2$

$[T]_B = \begin{bmatrix} 3 & -2 & 0 \\ 2 & -2 & 0 \\ 1 & -2 & 2 \end{bmatrix}$ $\qquad$ إذن:

$$\left(\left[T\begin{pmatrix}1\\0\\0\end{pmatrix}\right]_B \left[T\begin{pmatrix}0\\-1\\1\end{pmatrix}\right]_B \left[T\begin{pmatrix}0\\1\\1\end{pmatrix}\right]_B \right) = \begin{pmatrix}1&2&0\\0&1&2\\0&-1&1\end{pmatrix}$$ ⑤

$$\left[T\begin{pmatrix}1\\0\\0\end{pmatrix}\right]_B = 1(1,0,0) + 0(0,-1,1) + 1(0,1,1) = (1,1,1)$$

$$\left[T\begin{pmatrix}0\\-1\\1\end{pmatrix}\right]_B = 2\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + 1\begin{pmatrix}0\\-1\\1\end{pmatrix} + -1\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}2\\-2\\0\end{pmatrix}$$

$$\left[T\begin{pmatrix}0\\1\\1\end{pmatrix}\right]_B = 0\cdot\begin{pmatrix}1\\0\\0\end{pmatrix} + 2\begin{pmatrix}0\\-1\\1\end{pmatrix} + 1\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\-1\\3\end{pmatrix}$$

$$T(a,b,c) = T\left[\alpha_1(1,0,0) + \alpha_2(0,-1,1) + \alpha_3(0,1,1)\right]$$

$$(a,b,c) = \alpha_1(1,0,0) + \alpha_2(0,-1,1) + \alpha_3(0,1,1)$$

$$a = \alpha_1$$
$$b = -\alpha_2 + \alpha_3 \Rightarrow \alpha_3 = b + \alpha_2$$
$$c = \alpha_2 + \alpha_3 \quad c = \alpha_2 + b + \alpha_2 \Rightarrow c - b = 2\alpha_2 \Rightarrow \frac{c-b}{2} = \alpha_2$$

$$\alpha_3 = b + \alpha_2 \Rightarrow \alpha_3 = b + \frac{c-b}{2} \Rightarrow \alpha_3 = \frac{2b + c - b}{2} \Rightarrow \alpha_3 = \frac{b+c}{2}$$

$$\left[T(a,b,c)\right] = T\left[a\begin{pmatrix}1\\0\\0\end{pmatrix} + \frac{c-b}{2}\begin{pmatrix}0\\-1\\1\end{pmatrix} + \frac{b+c}{2}\begin{pmatrix}0\\1\\1\end{pmatrix}\right] =$$

$$aT\begin{pmatrix}1\\0\\0\end{pmatrix} + \frac{c-b}{2}T\begin{pmatrix}0\\-1\\1\end{pmatrix} + \frac{b+c}{2}T\begin{pmatrix}0\\1\\1\end{pmatrix} =$$

$$a\begin{pmatrix}1\\2\\1\end{pmatrix} + \frac{c-b}{2}\begin{pmatrix}2\\-2\\0\end{pmatrix} + \frac{b+c}{2}\begin{pmatrix}0\\-1\\3\end{pmatrix} =$$

$$\begin{pmatrix}a\\a\\a\end{pmatrix} + \begin{pmatrix}\frac{2(c-b)}{2}\\ \frac{-2(c-b)}{2}\\ 0\end{pmatrix} + \begin{pmatrix}0\\ -\frac{(b+c)}{2}\\ \frac{3(b+c)}{2}\end{pmatrix} =$$

$$\begin{pmatrix}a\\a\\a\end{pmatrix} + \begin{pmatrix}c-b\\b-c\\0\end{pmatrix} + \begin{pmatrix}0\\ \frac{-b-c}{2}\\ \frac{3b+3c}{2}\end{pmatrix} = \begin{pmatrix}a+c-b\\ a+b-c\frac{-b-c}{2}\\ a\frac{3b+3c}{2}\end{pmatrix} =$$

$$\begin{pmatrix}a+c-b\\ \frac{2a+2b-2c-b-c}{2}\\ \frac{2a+3b+3c}{2}\end{pmatrix} = \begin{pmatrix}a+c-b\\ \frac{2a+b-3c}{2}\\ \frac{2a+3b+3c}{2}\end{pmatrix} \qquad T\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}a+c-b\\ \frac{2a+b-3c}{2}\\ \frac{2a+3b+3c}{2}\end{pmatrix}$$