Mathmatical Logic HW#9

① $\varphi_1 : \forall x \exists y (R(x,y) \vee R(y,x))$

$\varphi_2 : \forall x (\exists y (R(x,y)) \vee \exists y (R(y,x)))$

$\varphi_3 : (\forall x \exists y R(x,y)) \vee (\forall x \exists y R(y,x))$

$\varphi_1 \Leftrightarrow \varphi_2$ (by E26)

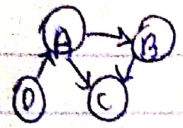
$\varphi_3 \Rightarrow \varphi_2$ (by I19)

$\varphi_3 \Rightarrow \varphi_1$ (since $(\varphi_2 \Leftrightarrow \varphi_1)$

$\varphi_1 \not\Rightarrow \varphi_3$ counter example: $R(x,y)$ = "x knows y"

φ_1 = Every person knows someone or is known by someone

φ_3 - Every person knows someone or every person is known by someone



A knows B
B knows C
A knows C
D knows A

In this model, φ_1 is true ($R(A,B)$, $R(B,C)$, $R(D,A)$)
but φ_3 is not true.

(D is not known by anyone, C doesn't know anyone)

$\varphi_2 \not\Rightarrow \varphi_3$ φ_2 is true in the above model (just as φ_1 is), but still, φ_3 is not true.

② $\exists x (P(x) \wedge Q(x)) \to \forall x (P(x) \to Q(x))$

$\neg \exists x (P(x) \wedge Q(x)) \vee \forall x (P(x) \to Q(x))$

$\forall x (\neg(P(x) \wedge Q(x))) \vee \forall x (P(x) \to Q(x))$

$\forall x ((\neg P(x) \vee \neg Q(x)) \vee (P(x) \to Q(x))$

PNF

③ $\alpha = \exists x (P(x) \to Q(x))$

$\beta = \forall x (P(x)) \to \exists x (Q(x))$

b, $\beta \Rightarrow A$

$\vdash \forall x P(x) \to \exists x Q(x) \Rightarrow \exists x (P(x) \to Q(x))$

True

proof $\longrightarrow$

ⓑ

1	$\forall x P(x) \to \exists x Q(x)$	R9 supposition to prove $\exists (P(x) \to Q(x))$
2	$\neg \exists x (P(x) \to Q(x))$	R_4 supposition
3	$\forall x \neg (P(x) \to Q(x))$	E26, 2
4	$\neg(P(y) \to Q(y))$	R8, 3
5	$P(y) \wedge \neg Q(y)$	E2, E16, 4
6	$P(y)$	I_3, 5
7	$\neg Q(y)$	I_1, 5
8	$\forall x P(x)$	R8, 6
9	$\exists x Q(x)$	I_{11}, 1
10	$\forall x \neg Q(y)$	R8, 7
11	$\neg \exists x Q(x)$	E25, 10
12	$\exists x (P(x) \to Q(x))$	R_4 contradiction 9, 11, (1,2)

Scanned with CamScanner

4) $(\exists x P(x) \to \forall x Q(x)) \Rightarrow \forall x (P(x) \to Q(x))$

a)

1	$\exists x P(x) \to \forall x Q(x)$	R_3 supposition (need to prove $\forall x (P(x) \to Q(x))$)
2	$\neg \forall x (P(x) \to Q(x))$	R_4 supposition (RAA)
3	$\exists x \neg (P(x) \to Q(x))$	E_{24}, 2
4	$\neg (P(y) \to Q(y))$	R_6, 3
5	$P(y) \land \neg Q(y)$	E_{20}, E_{16}, 4
6	$P(y)$	I_3, 5
7	$\exists x (P(x))$	R_7, 6
8	$\forall x Q(x)$	I_{11}, 1
9	$\neg Q(y)$	I_4, 5
10	$Q(y)$	R_5, 8
11	$\forall x (P(x) \to Q(x))$	R_4 contradiction 9, 10 (2-10)
12	$\exists x P(x) \to \forall x Q(x) \Rightarrow \forall x (P(x) \to Q(x))$	deduction 1-11

b) $\forall x (P(x) \to Q(x)) \not\Rightarrow (\exists x P(x) \to \forall x Q(x))$

$P(x)$ = less than 5

$Q(x)$ = less than 10

$\forall x (P(x) \to Q(x))$ – all numbers less than 5 are less than 10

$\exists x P(x) \to \forall x Q(x)$ is <u>False</u>

There exists numbers less than 5 but not all numbers are less than 10.