Calc2 hw#8

A ② ⓐ $f(x,y) = x+y$, $c: \begin{cases} x(t) = t \\ y(t) = 2t \end{cases}$ $0 \le t \le 1 \Rightarrow \begin{cases} x'(t) = 1 \\ y'(t) = 2 \end{cases}$

$$\int_c f(x,y)\,ds = \int_0^1 f(t,2t)\sqrt{1^2+2^2}\,dt = \sqrt{5}\int_0^1 t+2t\,dt = 3\sqrt{5}\int_0^1 t\,dt = \frac{3\sqrt{5}}{2}$$

ⓑ $c: \begin{cases} x(t) = 1-t \\ y(t) = 2-2t \end{cases}$ $0 \le t \le 1 \Rightarrow \begin{cases} x'(t) = -1 \\ y'(t) = -2 \end{cases}$

$$\int_c f(x,y)\,ds = \int_0^1 (1-t+2-2t)\sqrt{(-1)^2+(-2)^2}\,dt = \int_0^1 (3-3t)\sqrt{5}\,dt = 3\sqrt{5}\left[t - \tfrac{1}{2}t^2\right]_0^1$$

$$= 3\sqrt{5}\cdot\left(1-\tfrac{1}{2}\right) = \frac{3\sqrt{5}}{2}$$

B ③ $F(x,y,z) = -yi + xj$, $c: \begin{cases} x(t) = \cos t \\ y(t) = -\sin t \\ z = 0 \end{cases}$ $0 \le t \le 2\pi \Rightarrow \begin{cases} x'(t) = -\sin t \\ y'(t) = -\cos t \\ z'(t) = 0 \end{cases}$

$$\Rightarrow \int_c \vec{F}\cdot d\vec{r} = \int_0^{2\pi}\left(P\cdot x'(t) + Q\cdot y'(t) + R\cdot z'(t)\right)dt = \int_0^{2\pi}\left(\sin(t)\cdot(-\sin t) + \cos(t)\cdot(-\cos t)\right)dt$$

$$= \int_0^{2\pi} -1\cdot dt = -2\pi$$

ⓒ ③ $c: \begin{cases} x(t) = \cos^3\theta \\ y(t) = \sin^3\theta \\ z(t) = \theta \end{cases}$ $0 \le \theta \le \frac{7\pi}{2} \Rightarrow \begin{cases} x'(\theta) = -3\cos^2\theta\sin\theta \\ y'(\theta) = 3\sin^2\theta\cos\theta \\ z'(\theta) = 1 \end{cases}$

$$\Rightarrow \int_C \sin(z)\,dx + \cos(z)\,dy + (xy)^{\frac{1}{3}}\,dz = \int_0^{\frac{7\pi}{2}}\left(\sin\theta\frac{dx}{d\theta} + \cos\theta\frac{dy}{d\theta} + \frac{1}{2}\sin 2\theta\frac{dz}{d\theta}\right)d\theta$$

$$= \int_0^{\frac{7\pi}{2}}\left(-3\cos^2\theta\sin^2\theta + 3\sin^2\theta\cos^2\theta + \tfrac{1}{2}\sin(2\theta)\right)d\theta = \frac{1}{2}\int_0^{\frac{7\pi}{2}}\sin(2\theta)\,d\theta = -\frac{1}{4}\left[\cos(2\theta)\right]_0^{\frac{7\pi}{2}}$$

$$= -\frac{1}{4}(-1-1) = \frac{1}{2}$$

④ $c: \begin{cases} x(t) = 1+3t \\ y(t) = 1+t \\ z(t) = 1 \end{cases}$ $0 \le t \le 1 \Rightarrow \begin{cases} x'(t) = 3 \\ y'(t) = 1 \\ z'(t) = 0 \end{cases}$

$$\Rightarrow \int_c 2xyz\,dx + x^2z\,dy + x^2y\,dz = \int_0^1 \left(6(1+3t)(1+t) + (1+3t)^2\right)dt = \int_0^1\left(18t^2+24t+6+1+6t+9t^2\right)dt$$

$$= \int_0^1 (27t^2+30t+7)\,dt = \left[9t^2+15t^2+7t\right]_0^1 = 9+15+7 = 31$$

① ② $f(x,y) = xy^4$, $c: \begin{cases} x(t) = 4\cos t \\ y(t) = 4\sin t \end{cases} \quad 0 \leq t \leq \frac{\pi}{2} \Rightarrow \begin{cases} x'(t) = 4\sin t \\ y'(t) = 4\cos t \end{cases}$

$\Rightarrow m = \int_c f(x,y)\,ds = \int_0^{\frac{\pi}{2}} \left(4\cos(t)4^4\sin^4(t)\sqrt{(-4\sin t)^2 + (4\cos t)^2}\right)dt$

$= 4^6\int_0^{\frac{\pi}{2}} \cos(t)\sin^4(t)\,dt = \frac{4^6}{5}\left[\sin^5 t\right]_0^{\frac{\pi}{2}} = \frac{4^6}{5} = 819.2$

③ ⓑ $c: \begin{cases} x(t) = 2\ln(t) \\ y(t) = 2t \\ z(t) = \frac{1}{2}t^2 \end{cases} \quad 1 \leq t \leq 3 \Rightarrow \begin{cases} x'(t) = \frac{2}{t} \\ y'(t) = 2 \\ z'(t) = t \end{cases}$

$\Rightarrow L = \int_1^3 \|\vec{r}\,'(t)\|\,dt = \int_1^3 \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\,dt = \int_1^3 \sqrt{\frac{4}{t^2} + 4 + t^2}\,dt$

$= \int_1^3 \sqrt{\left(t + \frac{2}{t}\right)^2}\,dt = \int_1^3 \left(t + \frac{2}{t}\right)dt = \left[\frac{1}{2}t^2 + 2\ln(t)\right]_1^3 = \frac{9}{2} + 2\ln 3 - \frac{1}{2} = 4 + 2\ln 3$

④ ⓐ $F(x,y,z) = (y^2z, xz^2, 2xyz)$

$c: \begin{cases} x = t \\ y = t \\ z = t \end{cases} \quad 0 \leq t \leq 4 \Rightarrow \begin{cases} x'(t) = 1 \\ y'(t) = 1 \\ z'(t) = 1 \end{cases}$

$W = \int_0^4 (t^3 \cdot 1 + t^3 \cdot 1 + 2t^3 \cdot 1)\,dt = \int_0^4 4t^3\,dt = [t^4]_0^4 = 256$

ⓑ $c: \begin{cases} x = t^2 \\ y = 2t \\ z = \frac{1}{2}t^3 \end{cases} \Rightarrow \begin{cases} x'(t) = 2t \\ y'(t) = 2 \\ z'(t) = \frac{3}{2}t^2 \end{cases} \Rightarrow W = \int_0^2 \left((2t)^2 \cdot \frac{1}{2}t^3 \cdot 2t + t^2 \cdot \left(\frac{1}{2}t^3\right)^2 \cdot 2 + 2 \cdot t^2 \cdot 2t \cdot \frac{1}{2}t^3 \cdot \frac{3}{2}t^2\right)dt$

$= \int_0^2 \left(4t^6 + \frac{1}{2}t^8 + 3t^8\right)dt = \left(\frac{7}{18}t^9 + \frac{4}{7}t^7\right]_0^2 = 2^7\left(\frac{7}{18} + \frac{1}{7}\right) = 272.253$

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