Probability Hw#2

① (d) $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C) \cup (A \cap B \cap C)$

$= (A \cap B) \cup (A \cap C) \cup (B \cap C)$

(g) $(\bar{A} \cap \bar{B} \cap \bar{C}) \cup (\bar{A} \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap \bar{C})$

$= (\bar{A} \cap \bar{B}) \cup (\bar{A} \cap \bar{C}) \cup (\bar{B} \cap \bar{C})$

(i) $(A \cap B \cap \bar{C}) \cup (A \cap \bar{B} \cap C) \cup (\bar{A} \cap B \cap C)$

② (a) $|\Omega| = 32 = (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2)$

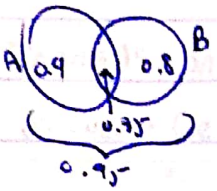
only gate 1 dysfunctional $\bar{T_1} \cap T_2 \cap T_3 \cap T_4 \cap T_5$

only gate 5 dysfunctional $T_1 \cap T_2 \cap T_3 \cap T_4 \cap \bar{T_5}$

⑥ $P(A) = 0.9$

$P(B) = 0.8$

$P(A \cup B) = 0.95$



(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.9 + 0.8 - 0.95 = 0.75$

(b) $P(\overline{A \cap B}) = 1 - 0.75 = 0.25$

(c) $P(A \cup B) - P(A \cap B) = 0.95 - 0.75 = 0.2$

(d) $P(A) - P(A \cap B) = 0.9 - 0.75 = 0.15$

⑧ $P(A) = 0.2 \qquad P(B) = 0.3$

(a) yes. $P(A \cup B) = 0.4$

$P(A \cap B) = 0.2 + 0.3 - 0.4 = 0.1$

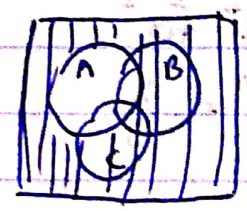
$P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.1$

(b) no

(c) $P(A) + P(B) = 0.5$

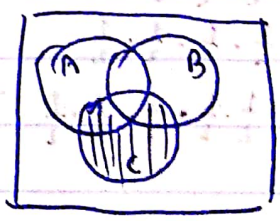
(d) $P(B) = 0.3$

⑨ (d)



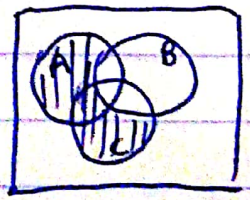
$\bar{A}$

(h)



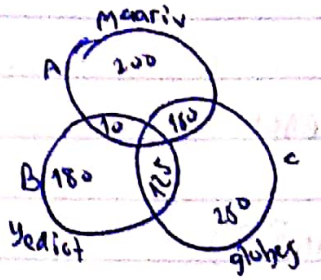
$(\bar{A} \cup \bar{B}) \cap C$

(f)



$A \cup C$

(10)



$P(A) = \frac{200}{500}$ $P(A \cap B) = \frac{70}{500}$

$P(B) = \frac{180}{500}$ $P(A \cap C) = \frac{130}{500}$

$P(C) = \frac{250}{500}$ $P(B \cap C) = \frac{125}{500}$

$P(A \cap B \cap C) = \frac{50}{500}$

(b) $P(A \cap B \cap \bar{C}) + P(B \cap C \cap \bar{A}) + P(A \cap C \cap \bar{B}) = \frac{(70-50) + (125-50) + (130-50)}{500} = \frac{195}{500} = \frac{39}{100}$

(d) $P(A) - (P(A \cap B \cap \bar{C})) - (P(A \cap C))$

$\frac{200}{500} - \frac{(70-50)}{500} - \frac{130}{500} = \frac{30}{500} = \frac{3}{50}$

(e) $1 - (P(A) + P(B) - P(B \cap C) - P(A \cap C \cap \bar{B}) + P(C) - P(A \cap B))$

$= 1 - \frac{200 + 180 - 125 - (130-50) + 250 - 70}{500} = 1 - \frac{335}{500} = \frac{165}{500} = \frac{33}{100}$

(12)

1 = 10%
2 = 10%
3 = 10%
4 = 20%
5 = 20%
6 = 30%

(a) result at least 3

$0.1 + 0.2 + 0.2 + 0.3 = 0.8$

(b) at most 2

$0.1 + 0.1 = 0.2$

(c) larger than 4

$0.2 + 0.3 = 0.5$

(13) y = # of times flipped before getting same side 2x in succession

$P(y=1) = 0$

$P(y=2) = \frac{1}{2}$

$P(y=3) = \frac{1}{2} \cdot \frac{1}{2}$

$P(y=4) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$P(y=5) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$P(y=6) = \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

(a) before 6th toss

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \frac{15}{16}$

(b) $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \frac{1}{128} + \dots$

$= \sum_{n=1}^{\infty} 2^{1-2n} = \frac{2}{3}$