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Linear algebra hw 0.5

① ⓐ $2x+y-2z=10$

$3x+2y-2z=1$

$5x+4y+3z=4$

$$\left(\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 3 & 2 & -2 & 1 \\ 5 & 4 & 3 & 4 \end{array}\right)$$

$$|A| = \left|\begin{array}{ccc|c} 2 & 1 & -2 & 10 \\ 3 & 2 & -2 & 1 \\ 5 & 4 & 3 & 4 \end{array}\right| = 2\begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix} - \begin{vmatrix} 3 & -2 \\ 5 & 3 \end{vmatrix} - 2\begin{vmatrix} 3 & 2 \\ 5 & 4 \end{vmatrix} = 28-19-4 = 5 \neq 0$$

the determinent is not 0, therefore there is only 1 solution

$$X = \frac{|A_{1b}|}{|A|} = \frac{\begin{vmatrix} 10 & 1 & -2 \\ 1 & 2 & -2 \\ 4 & 4 & 3 \end{vmatrix}}{5} = \frac{1}{5}(10\cdot 14 - 11 + 2\cdot 4) = \frac{137}{5} = 27.4$$

$$y = \frac{|A_{2b}|}{|A|} = \frac{\begin{vmatrix} 2 & 4 & -2 \\ 3 & 1 & -2 \\ 5 & 10 & 3 \end{vmatrix}}{5} = \frac{1}{5}(2\cdot 11 - 10\cdot 19 - 2\cdot 7) = -36.4 \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} 27.4 \\ -36.4 \\ 4.2 \end{pmatrix}$$

$$Z = \frac{|A_{3b}|}{|A|} = \frac{\begin{vmatrix} 2 & 1 & 10 \\ 3 & 2 & 1 \\ 5 & 4 & 4 \end{vmatrix}}{5} = \frac{1}{5}(2\cdot 4 - 7 + 10\cdot 2) = 4.2$$

② ⓒ $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$

$(-1)^2$

$C_{11} = (-1)^2|A_{11}| = \begin{vmatrix} 3 & 4 \\ 4 & 3 \end{vmatrix} = -7$ $\quad C_{21} = (-1)^3|A_{21}| = \begin{vmatrix} 2 & 3 \\ 4 & 3 \end{vmatrix} = 6$ $\quad C_{31} = (-1)^4|A_{31}| = \begin{vmatrix} 2 & 3 \\ 3 & 4 \end{vmatrix} = -1$

$C_{12} = (-1)^3|A_{12}| = \begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix} = 1$ $\quad C_{22} = (-1)^4|A_{22}| = \begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = 0$ $\quad C_{32} = (-1)^5|A_{32}| = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = -1$

$C_{13} = (-1)^4|A_{13}| = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 1$ $\quad C_{23} = (-1)^5|A_{23}| = \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = -2$ $\quad C_{33} = (-1)^6|A_{33}| = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1$

$$\Rightarrow \begin{bmatrix} -7 & 1 & 1 \\ 6 & 0 & -2 \\ -1 & -1 & 1 \end{bmatrix}$$

Adj (A) = cofactor ^{T}

$$= \begin{bmatrix} -7 & 6 & -1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{vmatrix} = 1\cdot(-7) - 2\cdot(-1) + 3\cdot 1 = -2$$

inverse matrix

$$\begin{bmatrix} \frac{7}{2} & -3 & \frac{-1}{2} \\ \frac{1}{2} & 0 & \frac{-1}{2} \\ \frac{-1}{2} & 1 & \frac{-1}{2} \end{bmatrix}$$

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(3) Let A, B be square matrixes of order n.

Prove : $Adj(AB) = Adj(B)\,Adj(A)$

$$A \cdot adj(A) = det(A) \cdot I$$

$\therefore$ $AB \cdot adj(AB) = det(AB) \cdot I = det(A) \cdot det(B) \cdot I = det(B) \cdot det(A) \cdot I =$

$det(B) \cdot A \cdot adj(A)$

but $det(B) \cdot A \cdot adj(A) = A \cdot det(B) \cdot adj(A)$

$= A \cdot (det(B) \cdot I) \cdot adj(A) = AB \cdot adj(B)\,adj(A)$

$\therefore$ $AB \cdot adj(AB) = AB \cdot adj(B) \cdot adj(A)$

Since A and B are of order n, they are invertible, and AB is invertible, therefore multiplying by $(AB)^{-1}$ on both sides gives

$$adj(AB) = adj(B) \cdot adj(A)$$

(5) Given that $|A| = \pm 1$ and $A^{-1} = adj(A) / |A|$,

$A^{-1} = adj(A)$ or $-adj(A)$

$adj(A)$ is the matrix of cofactors transposed which must have only integer entries because all determinants must be integers (if a, b, c, d are integers, then $ac - bd$ is an integer). all integers multiplied by -1 are still integers, so A^{-1} must contain only integers