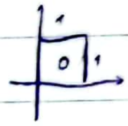
Calc 2 hw #6

(B) (3) $R=[0,1]\times[0,2]$, $\iint_R \sqrt{x+2y}\,dx\,dy = \int_0^2\int_0^1 \sqrt{x+2y}\,dx\,dy$

$= \int_0^2 \left[\frac{2}{3}(x+2y)^{\frac{3}{2}}\right]_0^1 dy = \frac{2}{3}\int_0^2 (1+2y)^{\frac{3}{2}}-(2y)^{\frac{3}{2}}\,dy = \frac{2}{3}\left[\frac{1}{5}(1+2y)^{\frac{5}{2}}-\frac{1}{5}(2y)^{\frac{5}{2}}\right]_0^2$

$= \frac{2}{15}\left(5^{2.5}-4^{2.5}-1\right) = 3.053$

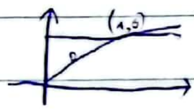
(5) $\int_0^1\int_0^1 xy^2e^{x+y}\,dy\,dx = \int_0^1\int_0^1 xe^x\cdot y^2e^y\,dy\,dx = \int_0^1 xe^x\,dx \cdot \int_0^1 y^2e^y\,dy$



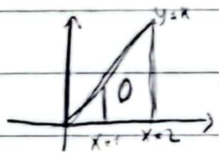
$= \left[xe^x-e^x\right]_0^1 \cdot \left[y^2e^y-2ye^y+2e^y\right]_0^1 = 1\cdot(e-2) = 0.718$

(E) (2) $\int_0^1\int_{\sqrt{x}}^1 \sqrt{1+y^3}\,dy\,dx = \int_0^1\int_0^{y^2}\sqrt{1+y^3}\,dx\,dy = \int_0^1\left[x\sqrt{1+y^3}\right]_0^{y^2}dy = \int_0^1 y^2\sqrt{1+y^3}\,dy$

$= \left[\frac{2}{9}(1+y^3)^{\frac{3}{2}}\right]_0^1 = \frac{2}{9}\left(\sqrt{8}-1\right) = 0.406$



(F) (3) $\int_1^2\int_0^x \frac{1}{(x^2+y^2)^{\frac{3}{2}}}\,dy\,dx \Rightarrow$



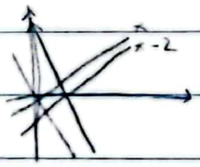
$\Rightarrow$ $x=1 = r\cos\theta \Rightarrow r = \frac{1}{\cos\theta}$
$x=2 = r\cos\theta \Rightarrow r = \frac{2}{\cos\theta}$ $\Rightarrow \int_0^{\frac{\pi}{4}}\int_{\sec\theta}^{2\sec\theta} \frac{1}{r^3}\,r\,dr\,d\theta$

$= \int_0^{\frac{\pi}{4}}\left[-\frac{1}{r}\right]_{2\sec\theta}^{2\sec\theta} d\theta = \int_0^{\frac{\pi}{4}} -\frac{1}{2}\cos\theta+\cos\theta\,d\theta = \frac{1}{2}\int_0^{\frac{\pi}{4}}\cos\theta\,d\theta = \frac{1}{2}\left[\sin\theta\right]_0^{\frac{\pi}{4}} = \frac{\sqrt{2}}{4} = 0.353$

(A) (2) $\int_0^1\int_0^x xe^y\,dy\,dx = \int_0^x\int_0^1 xe^y\,dx\,dy$

false, if you flip x do the first integration to x and then the second you do integration to y, doesn't come out the same

(G) (2) $D=\{(x,y)\mid x-2\le y\le x,\ -2x\le y\le 3-2x\}$, $\iint_D 3x+4y\,dA$

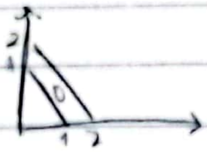


$\Rightarrow$ $u = y-x$, $v = y+2x$ $\Rightarrow$ $x = \frac{v-u}{3}$, $y = \frac{2u+v}{3}$ $\Rightarrow J = \begin{vmatrix}\frac{\partial x}{\partial u} & \frac{\partial y}{\partial u}\\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v}\end{vmatrix} = \begin{vmatrix}-\frac{1}{3} & \frac{2}{3}\\ \frac{1}{3} & \frac{1}{3}\end{vmatrix}$

$\Rightarrow J = -\frac{1}{3} \Rightarrow \iint_D 3x+4y\,dA = \int_0^3\int_{-2}^0\left(v-u+\frac{4}{3}(2u+v)\right)\cdot\left|-\frac{1}{3}\right|du\,dv$

$= \frac{1}{9}\int_0^3\int_{-2}^0 5u+7v\,du\,dv = \frac{1}{9}\int_0^3\left[\frac{5}{2}u^2+7uv\right]_{-2}^0 dv = \frac{1}{9}\int_0^3 -10+14v\,dv = \frac{1}{9}\left(7v^2-10v\right]_0^3$

$= 7-3\frac{1}{3} = 3\frac{2}{3}$

③ 

$\iint_0 \cos\frac{y-x}{y+x}\,dA \Rightarrow 1\le x+y\le 2$

$0\le \frac{y}{x}$

$\Rightarrow u = y+x$, $v=\frac{y}{x}$ $\Rightarrow x=\frac{u}{v+1}$, $y=\frac{vu}{v+1}$ $\Rightarrow J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v}\end{vmatrix} = \begin{vmatrix}\frac{1}{v+1} & \frac{v}{v+1}\\ \frac{-u}{(v+1)^2} & \frac{u}{(v+1)^2}\end{vmatrix}$

$= \frac{u}{(v+1)^3} + \frac{vu}{(v+1)^3} = \frac{u}{(v+1)^2} \Rightarrow \iint_0 \cos\frac{y-x}{y+x}\,dA = \lim_{n\to\infty}\left(\int_0^n\int_1^2 \cos\left(\frac{v-1}{v+1}\right)\frac{1}{(v+1)^2}\cdot u\,du\,dv\right)$

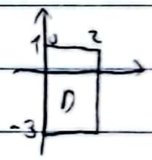
$=\lim_{n\to\infty}\left(\int_1^2 u\,du\cdot\int_0^n \cos\left(\frac{v-1}{v+1}\right)\frac{1}{(v+1)^2}\,dv\right) = \frac{3}{2}\lim_{n\to\infty}\int_0^n \cos\left(\frac{v-1}{v+1}\right)\cdot\frac{1}{(v+1)^2}\,dv$

$=\frac{3}{4}\cdot\lim_{n\to\infty}\left[\sin\left(\frac{v-1}{v+1}\right)\right]_0^n = \frac{3}{4}\left(\sin(1)-\sin(-1)\right) = 1.262$

Ⓗ ② $f(x,y)=x^3+y^2+2$, $R=\{(x,y)\mid 0\le x\le 2, -3\le y\le 1\}$

$V=\int_0^2\int_3^1 x^2+y^2+2\,dy\,dx = \int_0^2\left[x^2y+\frac13y^3+2y\right]_{-3}^1dx = \int_0^2\left(x^3+2\frac13+3x^3+9+6\right)dx$

$=\int_0^2\left(4x^3+17\frac13\right)dx = \left[x^4+17\frac13x\right]_0^2 = 16+34\frac23 = 50\frac23$



Ⓙ ② $\iint_D \rho(x,y)\,dy\,dx = \int_1^2\int_1^3 x^3y^2\,dy\,dx = \int_1^2\left[\frac13x^3y^3\right]_1^3dx = \int_1^2\left(8\frac23x^3\right)dx$

$=\left[2\frac16x^4\right]_1^2 = 2\frac16\cdot16-2\frac16=32.5$

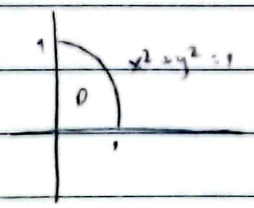
Ⓐ ⑥ false, $-1\le x\le1$ $-\sqrt{1-x^2}\le y\le\sqrt{1-x^2}$

the circle bound and the integral bound aren't going to create the same type of circle since the integral will give a $\frac12$ circle and, unlike $x^2+y^2\le4$, which would be a whole circle

Ⓐ ④ true, $\int_0^1\int_0^1 f(x)\cdot f(y)\,dy\,dx \Rightarrow \int_0^1 f(x)F(y)\Big|_0^1 dx \Rightarrow \int_0 f(x)\left(F(1)-F(0)\right)dx \Rightarrow \left(F(1)-F(0)\right)F(x)\Big|_0$

$\Rightarrow [F(1)-F(0)][F(1)-F(0)] \Rightarrow [F(1)-F(0)]^2 \Rightarrow \left(F(z)\Big|_0^1\right)^2 \Rightarrow \left(\int f(z)\,dz\right)^2$

Ⓒ ③ $f(x,y)=xy$



$0\le x\le1$

$0\le y\le\sqrt{1-x^2}$

$$\int_0^1\int_0^{\sqrt{1-x^2}} xy\,dy\,dx \Rightarrow \int_0^1 \frac{xy^2}{2}\Big|_0^{\sqrt{1-x^2}} dx \Rightarrow \int_0^1 \frac{x\sqrt{1-x^2}^2}{2} - 0\,dx$$

bounds of substitution:

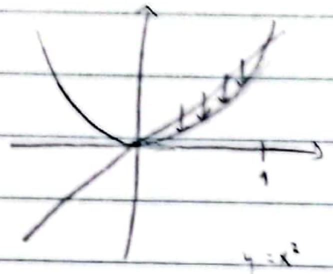
$x=0 \quad t=1$

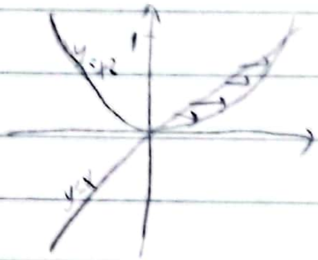
$x=1 \quad t=0$

$$\Rightarrow \begin{array}{l} t=1-x^2 \\ dt=-2x\,dx \end{array} : \frac{1}{-2}\int_0^1 \frac{2x\sqrt{1-x^2}^2}{2}\,dx \Rightarrow -\frac{1}{2}\int_1^0 \frac{dt\sqrt{t}^2}{2}$$

$$\Rightarrow -\frac{1}{2}\int_1^0 \frac{t}{2}^{\frac{1}{2}}\,dt \Rightarrow -\frac{1}{2}\,\frac{t^{\frac{3}{2}}}{3}\Big|_1^0 \Rightarrow 0^{\circ}-\left(-\frac{1}{2}\right)\cdot\frac{1^{\frac{3}{2}}}{3} \Rightarrow \frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$$

D 7 $\displaystyle\int_0^1\int_{x^2}^{x} f(x,y)\,dy\,dx$ $\qquad\displaystyle\int_0^1\int_{\sqrt{y}}^{y} f(x,y)\,dy\,dx$





when $x=0,1$

$x=x^2$

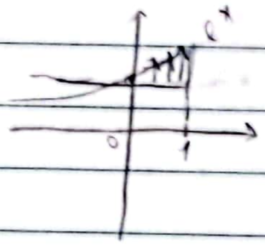
$y=x^2$

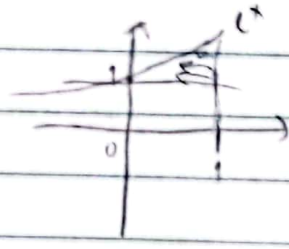
$y=x$

$0\le x\le 1$

$x\le y\le x^2$

6 $\displaystyle\int_0^1\int_1^{e^x} f(x,y)\,dy\,dx$ $\qquad\displaystyle\int_1^e\int_{\ln y}^{1} f(x,y)\,dx\,dy$





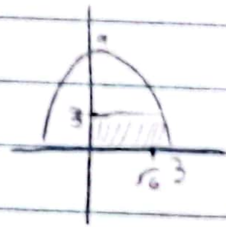
$y=e^x$

$x=\ln y$

$0\le x\le 1$ $\qquad \ln y\le x\le 1$

$1\le y\le e^x$ $\qquad 1\le y\le e$

2 $\displaystyle\int_0^3\int_0^{\sqrt{9-y}} f(x,y)\,dx\,dy$ $\qquad \displaystyle\int_{\sqrt{6}}^{3}\int_0^{9-x^2} f(x,y)\,dy\,dx+\int_0^{\sqrt{6}}\int_0^{3} f(x,y)\,dy\,dx$



$0\le y\le 3$

$0\le x\le\sqrt{9-y}$

$y=9-x^2$

if: $y=0$ $x=3$

if: $y=3$ $x=\sqrt{6}$

E 4 $\displaystyle\int_1^4\int_0^{\sqrt{y}} e^{\frac{x}{\sqrt{y}}}\,dx\,dy = \frac{e^{\frac{x}{\sqrt{y}}}}{\frac{1}{\sqrt{y}}} = \left[\sqrt{2}e^{\frac{x}{\sqrt{2}}}\right]_0^{\sqrt{y}} \sqrt{y}\,e^{\frac{\sqrt{y}}{\sqrt{y}}} - \sqrt{y}\,e^{0}$

$\displaystyle\int_1^4 \sqrt{y}e-\sqrt{y}=(e-1)\sqrt{y} \rightarrow \int\sqrt{y}-\int y^{\frac{1}{2}}=(e-1)\left[y^{\frac{3}{2}}\,\frac{2}{3}\right]_1^4 = \left(8\cdot\frac{2}{3}-1\cdot\frac{2}{3}\right)(e-1)=5(e-1)$