LA2 hw #7

13 (a) $g_1 = 1$, $g_2 = x+1$, $g_3 = x^2$, $\langle f, g\rangle = \int_{-1}^{1} f(x)\cdot g(x)\,dx$

$f_1 = g_1 = 1$

$f_2 = g_2 - \frac{\langle g_2, f_1\rangle}{\langle f_1, f_1\rangle}\cdot f_1 = x+1 - \frac{\int_{-1}^{1} x+1\,dx}{2}\cdot 1 = x+1 - \frac{1}{2}\left[\frac{1}{2}x^2 + x\right]_{-1}^{1} = x+1-\frac{1}{2}\left(1\frac{1}{2}+\frac{1}{2}\right) = x$

$f_3 = g_3 - \frac{\langle g_3, f_1\rangle}{\langle f_1, f_1\rangle}\cdot f_1 - \frac{\langle g_3, f_2\rangle}{\langle f_2, f_2\rangle}\cdot f_2 = x^2 - \frac{\int_{-1}^{1} x^2\,dx}{2} - \frac{\int_{-1}^{1} x^3\,dx}{\int_{-1}^{1} x^2\,dx}\cdot x = x^2 - \frac{1}{3} \Rightarrow C = \left\{1, x, x^2 - \frac{1}{3}\right\}$

(b) $c_1 = \frac{\langle x^4, 1\rangle}{\|1\|^2} = \frac{\frac{2}{5}}{2} = \frac{1}{5}$, $c_2 = \frac{\langle x^4, x\rangle}{\|x\|^2} = 0$, $c_3 = \frac{\langle x^4, x^2 - \frac{1}{3}\rangle}{\|x^2 - \frac{1}{3}\|^2} = \frac{\left(\frac{1}{7}x^7 - \frac{1}{15}x^5\right]_{-1}^{1}}{\left[\frac{1}{5}x^5 - \frac{2}{9}x^3 + \frac{1}{9}x\right]_{-1}^{1}} = \frac{\frac{2}{7} - \frac{2}{15}}{\frac{2}{5} - \frac{4}{9} + \frac{2}{9}} = \frac{6}{7}$

$\Rightarrow y = P_u(x^4) = \frac{1}{5} + \frac{6}{7}x^2 - \frac{2}{7} = \frac{6}{7}x^2 - \frac{3}{35}$

(c) $\|x^4 - y\| = \|x^4 - \frac{6}{7}\cdot x^2 + \frac{3}{35}\| = \sqrt{\int_{-1}^{1} x^8 - \frac{12}{7}x^6 + \frac{222}{245}x^4 - \frac{36}{245}x^2 + \frac{9}{1225}\,dx}$

$= \sqrt{\left[\frac{1}{9}x^9 - \frac{12}{49}x^7 + \frac{222}{1225}x^5 - \frac{12}{245}x^3 + \frac{9}{1225}x\right]_{-1}^{1}} = \sqrt{\frac{2}{9} - \frac{24}{49} + \frac{444}{1225} - \frac{24}{245} + \frac{18}{1225}} = \frac{8\sqrt{2}}{105} = 0{,}1077$

12 (a) $g_1 = x$, $g_2 = x^2$, $g_3 = x^3$, $\langle f, g\rangle = \int_{-1}^{1} f(x)\cdot g(x)\,dx$

$f_1 = g_1 = x$

$f_2 = g_2 - P_{f_1}(g_2) = g_2 - \frac{\langle g_2, f_1\rangle}{\langle f_1, f_1\rangle}\cdot f_1 = x^2 - 0\cdot f_1 = x^2$

$f_3 = g_3 - \frac{\langle g_3, f_1\rangle}{\langle f_1, f_1\rangle}\cdot f_1 - \frac{\langle g_3, f_2\rangle}{\langle f_2, f_2\rangle}\cdot f_2 = x^3 - \frac{\int_{-1}^{1} x^4\,dx}{\int_{-1}^{1} x^2\,dx}\cdot x - 0\cdot f_2 = x^3 - \frac{\frac{2}{5}}{\frac{2}{3}}x = x^3 - \frac{3}{5}x$

$\Rightarrow C = \left\{x, x^2, x^3 - \frac{3}{5}x\right\}$

(b) $\begin{pmatrix} \langle x^5, x\rangle \\ \langle x^5, x^2\rangle \\ \langle x^5, x^3\rangle \end{pmatrix} = \begin{pmatrix} \langle x, x\rangle & \langle x^2, x\rangle & \langle x^3, x\rangle \\ \langle x, x^2\rangle & \langle x^2, x^2\rangle & \langle x^3, x^2\rangle \\ \langle x, x^3\rangle & \langle x^2, x^3\rangle & \langle x^3, x^3\rangle \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{2}{7} \\ 0 \\ \frac{2}{9} \end{pmatrix} = \begin{pmatrix} \frac{2}{3} & 0 & \frac{2}{5} \\ 0 & \frac{2}{5} & 0 \\ \frac{2}{5} & 0 & \frac{2}{7} \end{pmatrix}\begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \Rightarrow \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} \frac{75}{8} & 0 & -\frac{105}{8} \\ 0 & \frac{5}{2} & 0 \\ -\frac{105}{8} & 0 & \frac{175}{8} \end{pmatrix}$

$\cdot \begin{pmatrix} \frac{2}{7} \\ 0 \\ \frac{2}{9} \end{pmatrix} = \begin{pmatrix} -\frac{5}{21} \\ 0 \\ \frac{10}{9} \end{pmatrix} \Rightarrow y = P_U(x^5) = -\frac{5}{21}x + \frac{10}{9}x^3$

(c) $\|v - y\| = \|x^5 + \frac{5}{21}x - \frac{10}{9}x^3\| = \sqrt{\int_{-1}^{1} x^{10} + \frac{25}{441}x^2 + \frac{100}{81}x^6 + \frac{10}{21}x^6 - \frac{20}{9}x^8 - \frac{100}{129}x^4\,dx}$

$= \sqrt{\left(\frac{1}{11}x^{11} - \frac{20}{81}x^9 + \frac{1}{7}\left(\frac{100}{81} + \frac{10}{21}\right)x^7 - \frac{20}{189}x^5 + \frac{25}{1323}x^3\right]_{-1}^{1}} = \sqrt{\frac{2}{11} - \frac{40}{81} + \frac{2}{7}\left(\frac{100}{81} + \frac{10}{21}\right) - \frac{40}{188} + \frac{50}{1323}} = 0{,}054$

4 (a) (3) $g_1 = 1$, $g_2 = x$, $g_3 = x^2$, $\langle p, q\rangle = \int_{-1}^{1} p(x)\cdot q(x)\,dx$, $V = C[-1, 1]$

$$\begin{pmatrix} \langle 2x^3, g_1\rangle \\ \langle 2x^3, g_2\rangle \\ \langle 2x^3, g_3\rangle \end{pmatrix} = \begin{pmatrix} \langle g_1,g_1\rangle & \langle g_1,g_2\rangle & \langle g_1,g_3\rangle \\ \langle g_2,g_1\rangle & \langle g_2,g_2\rangle & \langle g_2,g_3\rangle \\ \langle g_3,g_1\rangle & \langle g_3,g_2\rangle & \langle g_3,g_3\rangle \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ \frac{4}{5} \\ 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & \frac{2}{5} \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 2 & 0 & \frac{2}{3} \\ 0 & \frac{2}{3} & 0 \\ \frac{2}{3} & 0 & \frac{2}{5} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ \frac{4}{5} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{6}{5} \\ 0 \end{pmatrix} \Rightarrow y = P_{R_2[x]}(2x^3) = c_1 + c_2 x + c_3 x^2 = \frac{6}{5}x$$

(b) (3) $\|v-y\| = \|2x^3 - \frac{6}{5}x\| = \sqrt{\int_{-1}^{1} 4x^6 - \frac{24}{5}x^4 + \frac{36}{25}x^2\,dx} = \sqrt{\left[\frac{4}{7}x^7 - \frac{24}{25}x^5 + \frac{12}{25}x^3\right]_{-1}^{1}}$

$= \sqrt{\frac{4}{7}\cdot 2 - \frac{24}{25}\cdot 2 + \frac{12}{25}\cdot 2} = \sqrt{\frac{2\cdot 16}{7\cdot 25}} = \frac{4}{5}\sqrt{\frac{2}{7}} = 0.427$

(2) (a) $V = Sp\{1, \sin 2x, \cos 2x\}$, $\langle f,g\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)g(x)\,dx$

(1) set is in base V, in the area V, check if its orthogonal:

$\langle 1, \sin 2x\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin 2x\,dx = -\frac{1}{2\pi}\left[\cos 2x\right]_{-\pi}^{\pi} = -\frac{1}{2\pi}\cdot(\cos 2\pi - \cos -2\pi) = 0$

$\langle 1, \cos 2x\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos 2x\,dx = \frac{1}{2\pi}\left[\sin 2x\right]_{-\pi}^{\pi} = \frac{1}{2\pi}\cdot(\sin 2\pi - \sin -2\pi) = 0$

$\langle \sin 2x, \cos 2x\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} \sin 2x \cos 2x\,dx = \frac{-1}{8\pi}\left[\cos 4x\right]_{-\pi}^{\pi} = \frac{-1}{8\pi}(\cos 8\pi - \cos -8\pi) = 0$

(1) in order for it to be an orthogonal set, we need all the inner products to be 1

we will check:

$\langle 1,1\rangle = \frac{1}{\pi}\int_{-\pi}^{\pi} dx = \frac{1}{\pi}\cdot 2\pi = 2 \neq 1 \Rightarrow$ not orthogonal

(3) (a) $g_1 = (1,-1,0)$, $g_2 = (1,3,1)$, $g_3 = (0,1,2)$, $V = \mathbb{R}^3$

try to find a orthogonal basis like $C = \{f_1, f_2, f_3\}$ such that:

$Sp\{f_1\} = Sp\{g_1\}$, $Sp\{f_1,f_2\} = Sp\{g_1,g_2\}$, $Sp\{f_1,f_2,f_3\} = Sp\{g_1,g_2,g_3\}$

step 1: $f_1 = g_1$

Step 2: $f_2 = \Delta$, $g_2 = P_{f_1}(g_2) + \Delta$, $\Delta\cdot f_1 = 0$

$\Rightarrow f_2 = \Delta = g_2 - P_{f_1}(g_2) = g_2 - \frac{g_2\cdot f_1}{f_1^2}\cdot f_1 = (1,3,1) + \frac{2}{2}(1,-1,0) = (2,2,1)$

Step 3: $f_3 = \Delta$, $g_3 = P_{Sp\{f_1,f_2\}}(g_3) + \Delta$

$\Rightarrow f_3 = \Delta = g_3 - P_{Sp\{f_1,f_2\}}(g_3) = g_3 - \left(\frac{g_3\cdot f_1}{f_1^2}\cdot f_1 + \frac{g_3 f_2}{f_2^2}\cdot f_2\right)$

$= (0,1,2) + \frac{1}{2}(1,-1,0) - \frac{4}{9}(2,2,1) = \left(-\frac{7}{18}, -\frac{7}{18}, \frac{14}{9}\right)$

$\Rightarrow C = \left\{(1,-1,0), (2,2,1), \left(-\frac{7}{18}, -\frac{7}{18}, \frac{14}{9}\right)\right\}$

$c = \left\{\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right), \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right), \left(-\frac{1}{3\sqrt{2}}, -\frac{1}{3\sqrt{2}}, \frac{2\sqrt{2}}{3}\right)\right\}$

$\text{ⓒ}\ g_0=1,\ g_1=3x,\ g_2=x^2-2,\ g_3=x^3,\ \langle p(x),q(x)\rangle=\int_{-1}^{1}p(x)q(x)\,dx,\ V=\mathbb{R}_3(x)$

step 1: $f_0=g_0=1$

step 2: $f_1=g_1-P_{f_0}(g_1)=g_1-\frac{\langle g_1,f_0\rangle}{\langle f_0,f_0\rangle}\cdot f_0=3x-\frac{\int_{-1}^{1}3x\,dx}{\int_{-1}^{1}dx}\cdot 1=3x$

step 3: $f_2=g_2-\frac{\langle g_2,f_0\rangle}{\langle f_0,f_0\rangle}\cdot f_0-\frac{\langle g_2,f_1\rangle}{\langle f_1,f_1\rangle}\cdot f_1=x^2-2-\frac{1}{2}\int_{-1}^{1}x^2-2\,dx-\frac{\int_{-1}^{1}3x^3-6x\,dx}{\int_{-1}^{1}9x^2dx}\cdot 3x$

$=x^2-2-\frac{1}{2}\left[\frac{1}{3}x^3-2x\right]_{-1}^{1}=x^2-2-\frac{1}{6}+1=\frac{1}{6}+1=x^2-\frac{1}{3}$

step 4: $f_3=g_3-\frac{\langle g_3,f_0\rangle}{\langle f_0,f_0\rangle}\cdot f_0-\frac{\langle g_3,f_1\rangle}{\langle f_1,f_1\rangle}\cdot f_1-\frac{\langle g_3,f_2\rangle}{\langle f_2,f_2\rangle}\cdot f_2$

$=x^3-\frac{\int_{-1}^{1}x^3dx}{2}-\frac{\int_{-1}^{1}3x^4dx}{\int_{-1}^{1}9x^2dx}\cdot 3x-\frac{\int_{-1}^{1}x^5-\frac{1}{3}x^3\,dx}{\int_{-1}^{1}x^4-\frac{2}{3}x^2+\frac{1}{9}dx}\cdot\left(x^2-\frac{1}{3}\right)=x^3-\frac{\left[\frac{3}{5}x^5\right]_{-1}^{1}}{\left[3x^3\right]_{-1}^{1}}\cdot 3x$

$=x^3-3x\cdot\frac{\frac{6}{5}}{6}=x^3-\frac{3}{5}x\Rightarrow C=\left\{1,3x,x^2-\frac{1}{3},x^2-\frac{3}{5}x\right\}$

① ⓐ $x=\begin{pmatrix}0\\2\\0\end{pmatrix},\ g=\begin{pmatrix}0\\1\\3\end{pmatrix}$ The projection of x onto g is tg when $t=\frac{\langle x,g\rangle}{\|g\|^2}$

$t=\frac{(0,2,0)\cdot(0,1,3)}{0^2+1^2+3^2}=\frac{1}{5}\Rightarrow y=tg=\frac{1}{5}\cdot(0,1,3)=\left(0,\frac{1}{5},\frac{3}{5}\right)$

② ⓐ $g_1=(1,0,2),\ g_2=(3,0,1),\ x=(1,0,-1)$

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