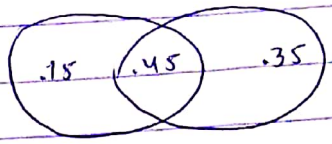
Probability HW #3

(3) $P(S) = .6 \quad P(H) = .8 \quad P(S \cup H) = .95 \quad P(S \cap H) = .6 + .8 - .95 = .45$

(a) $P(S|H) = \frac{P(S \cap H)}{P(H)} = \frac{.45}{.8} = .5625$



(b) $P(H|S) = \frac{P(H \cap S)}{P(S)} = \frac{.45}{.6} = .75$

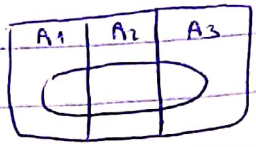
(c) $P(S) \cdot P(H) = .6 \cdot .8 = .48 \neq .45 = P(S \cap H) \Rightarrow$ dependent

(6) Let B = item produced is defective

Let A_1 = item produced by machine A

Let A_2 = item produced by machine B

Let A_3 = item produced by machine C



(a) $P(B) = P(A_1) \cdot P(B|A_1) + P(A_2) \cdot P(B|A_2) + P(A_3) \cdot P(B|A_3)$

$= 0.1 \cdot 0.4 + 0.6 \cdot 0.3 + 0.3 \cdot 0.25 = 0.295$

(b) $P(A_1|B) = \frac{P(A_1) \cdot P(B|A_1)}{P(B)} = \frac{0.1 \cdot 0.4}{0.295} = \frac{8}{59}$

$P(A_2|B) = \frac{P(A_2) \cdot P(B|A_2)}{P(B)} = \frac{0.6 \cdot 0.3}{0.295} = \frac{36}{59}$

$P(A_3|B) = \frac{P(A_3) \cdot P(B|A_3)}{P(B)} = \frac{0.3 \cdot 0.25}{0.295} = \frac{18}{59}$

(10) Let B = process is successfully executed

Let A_1, A_2, A_3 = methods A, B, C were used

(a) $P(B) = (\frac{1}{3})(0.8) + (\frac{1}{3})(0.6) + (\frac{1}{3})(0.4) = 0.6 = \frac{3}{5}$

(b) $P(A_1|B) = \frac{P(A_1 \cap B)}{P(B)} = \frac{P(A_1) \cdot P(B|A_1)}{P(B)} = \frac{(1/3)(0.8)}{0.6} = \frac{4}{9}$

(12) Let B = chosen spare part is found defective

Let A_1 = part is defective, A_2 = part is non-defective

(a) $P(B) = (0.02)(0.999) + (0.98)(0.01) = 0.02998$

(b) $P(A_1|B) = \frac{(0.02)(0.999)}{0.02998} = 0.67092$

(13) Let B = number of hits

(a) $P(B=0) = (0.9)^{12} = 0.2824 \quad P(B \geq 1) = 1 - P(B=0) = 1 - 0.2824 = 0.7175$

(b) $P(B=1) = (0.9)^{11} \cdot (0.1) \cdot 12 = 0.3768$

(c) Let part (a) = A, part (b) = B

$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(B)}{P(A)} = \frac{0.3768}{0.7175} = 0.5249$

(16) (b) $P(A \cap B) = P(A) \cdot P(B) \Rightarrow \frac{1}{6} = P(A) \cdot P(B) \Rightarrow P(B) = \frac{1}{6P(A)}$

$P(A \cap B) = P(A) + P(B) - P(A \cup B) \Rightarrow \frac{1}{6} + \frac{2}{3} = P(A) + P(B) \Rightarrow P(B) = \frac{5}{6} - P(A)$

$\frac{1}{6P(A)} = \frac{5}{6} - P(A) \Rightarrow 1 = 5P(A) - 6P(A)^2 \Rightarrow 6P(A)^2 - 5P(A) + 1 = 0$

$(6P(A) - 2)(6P(A) - \frac{1}{2}) = 0 \Rightarrow P(A) = \frac{1}{3}, \frac{1}{2} \quad P(B) = \frac{1}{3}, \frac{1}{2}$

(d) $P(A \cap B) = P(A)P(B) = 0.4x \quad P(A \cup B) = 0.4 + x - 0.4x \Rightarrow 0.7 = 0.4 + 0.6x$

$0.3 = 0.6x \Rightarrow x = 0.5$

(17) (b) $$\frac{P(A|A \cup B)}{P(B|A \cup B)} = \frac{\frac{P(A \cap (A \cup B))}{P(A \cup B)}}{\frac{P(B \cap (A \cup B))}{P(A \cup B)}} = \frac{\frac{P(A)}{P(A \cup B)}}{\frac{P(B)}{P(A \cup B)}} = \frac{P(A)}{P(B)}$$

(18) (c) $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) = 1 - (P(A) + P(B) - P(A \cap B)) = 1 - P(A) - P(B) + P(A)P(B)$

$P(\bar{A} \cap \bar{B}) = P(\bar{A})P(\bar{B}) = (1 - P(A))(1 - P(B)) = 1 - P(B) - P(A) + P(A)P(B)$

(21) (b) $P(\text{no error in a minute}) = 1 - 7 \cdot 10^{-5} = 0.9993$

$P(\text{no error in 2 minutes}) = (0.9993)^{5 \cdot 10^5 \cdot 2} = 3.9657 \times 10^{-31}$