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① ⓓ $f(x) = \sqrt{3-4x}$

$f(x) = \sqrt{3-4x}$

$$\frac{f(x+h) - f(x)}{h} = \frac{\sqrt{3-4(x+h)} - \sqrt{3-4x}}{h} \cdot \frac{\sqrt{3-4(x+h)} + \sqrt{3-4x}}{\sqrt{3-4(x+h)} + \sqrt{3-4x}} =$$

$$\frac{3-4(x+h) - (3-4x)}{h\left(\sqrt{3-4(x+h)} + \sqrt{3-4x}\right)} = \frac{\cancel{3}-\cancel{4x}-4\cancel{h}-\cancel{3}+\cancel{4x}}{\cancel{h}\left(\sqrt{3-4(x+h)} + \sqrt{3-4x}\right)} = \lim_{h\to 0} \frac{-4}{\sqrt{3-4(x+h)} + \sqrt{3-4x}} =$$

$$\frac{-4}{2\sqrt{3-4x}} = \frac{-2}{\sqrt{3-4x}}, \quad x < \frac{3}{4}$$

Calculus #4

(1) (b) $f(x) = \frac{3}{2x-5}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{3}{2(x+h)-5} - \frac{3}{2x-5}}{h} =$$

$$\lim_{h\to 0} \frac{\frac{3(2x-5)-3(2(x+h)-5)}{(2(x+h)-5)(2x-5)}}{h} = \frac{\frac{6x-\cancel{15}-6(x+h)+\cancel{15}}{(2(x+h)-5)(2x-5)}}{h} = \frac{\frac{6x-6(x+h)}{(2(x+h)-5)(2x-5)}}{h} =$$

$$\frac{6x-6(x+h)}{h((2(x+h)-5)(2x-5))} = \frac{\cancel{6x}-\cancel{6x}-6h}{h(2x+2h-5)(2x-5)} =$$

$$\frac{-6\cancel{h}}{\cancel{h}(2x+2h-5)(2x-5)} = \lim_{h\to 0} \frac{-6}{(2x+2h-5)(2x-5)} = \frac{-6}{(2x-5)^2}$$

(2) (14) $f(x) = \sin^5 3x$

$$f'(x) = 5(\sin(3x))^4 \cdot \cos(3x) \cdot 3 = 15(\sin 3x)^4 \cos(3x)$$

(19) $f(x) = \sqrt{\cos(\cot x)}$

$$(\cos(\cot x))^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}((\cos(\cot x))^{-\frac{1}{2}} \cdot -\sin(\cot x) \cdot \frac{-1}{\sin^2 x} =$$

$$\frac{1}{2}(\cos(\cot x))^{-\frac{1}{2}} \cdot \frac{\sin(\cot x)}{\sin^2 x}$$

(24) $f(x) = \arcsin \frac{\sqrt{x^2-1}}{x}$

$$f'(x) = \frac{1}{\sqrt{1-\frac{x^2+1}{x^2}}} \cdot \frac{\frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot 2x : x - (1 \cdot (x^2-1)^{\frac{1}{2}})}{x^2}$$

$$\frac{1}{\sqrt{\frac{x^2-x^2+1}{x^2}}} \cdot \frac{\frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot 2x^2 - (x^2-1)^{\frac{1}{2}}}{x^2} = \frac{\frac{1}{2}(x^2-1)^{-\frac{1}{2}} \cdot 2x^2 - (x^2-1)^{\frac{1}{2}}}{x^2\sqrt{\frac{1}{x^2}}} = \frac{x^2}{x} = \boxed{x}$$

$x \neq 0$

(23) $f(x) = \begin{cases} x^2 - 3x & x \in [1, +\infty) \\ x^3 - 2x^2 - 1 & x \in (-2, 1) \\ -2x^2 + 12x + 9 & x \in (-\infty, -2] \end{cases}$

$f'(x) = \quad 2x - 3$

$3x^2 - 4x$

$-4x + 12$

$x = 1^+ : \lim_{x \to 1^+} x^2 - 3x = -2$

$x = 1^- : \lim_{x \to 1^-} x^3 - 2x^2 - 1 = -2$

function is differentiable at $x = 1$

$x = -2^+ : \lim_{x \to -2^+} x^3 - 2x^2 - 1 = -17$

$\lim_{x \to -2^-} -2x^2 + 12x + 9 = -23$

function is not differentiable at $x = -2$

$f'(x) = \begin{cases} 2x - 3 & x \in [1, +\infty) \\ 3x^2 - 4x & x \in (-2, 1) \\ -4x + 12 & x \in (-\infty, -2] \end{cases}$

(24) $f(x) = \dfrac{|x^2 - 1|}{x - 1}$ $\qquad x^2 - 1 \to$ differentiable: $x = 1 \quad x = -1$

$f'(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1} & x < -1 \\ \dfrac{-x^2 + 1}{x - 1} & -1 < x < 1 \\ \dfrac{x^2 - 1}{x - 1} & x > 1 \end{cases}$

$x = 1^+ ; \lim_{x \to 1^+} \dfrac{x^2 - 1}{x - 1} = \dfrac{0}{0} = \dfrac{x(x - \cancel{1})}{x - \cancel{1}} = x = 1$

$x = 1^- : \lim_{x \to 1^-} \dfrac{-x^2 + 1}{x - 1} = \dfrac{0}{0} = \dfrac{-x(x - 1)}{x - 1} = -x = -1$

function isn't differentiable at $1 = x$

$x = -1^+ : \lim_{x \to -1^+} \dfrac{-x^2 + 1}{x - 1} = 0$

$x = -1^- : \lim_{x \to -1^-} \dfrac{x^2 - 1}{x - 1} = 0$

function is differentiable at $-1 = x$

3'07

$$f'(x) = \begin{cases} 1 & x < -1 \\ -1 & -1 < x < 1 \\ 1 & 1 < x \end{cases}$$

$\frac{x^2-1}{x-1}$ $f'(x) = \frac{2x(x-1) - 1(x^2-1)}{(x-1)^2} = \frac{2x^2-2x-x^2+1}{(x-1)^2} = \frac{x^2-2x+1}{(x-1)^2} = 1$

$\frac{-x^2+1}{x-1}$ $f'(x) = \frac{-2x(x-1) - 1(-x^2+1)}{(x-1)^2} = \frac{-2x^2+2x+x^2-1}{(x-1)^2} = \frac{-x^2+2x-1}{(x-1)^2} = -1$

(30) $f(x) = \ln(1+|x|) = \begin{cases} \ln(1-x) & x < -1 \\ \ln(1+x), & x \geq -1 \end{cases}$

$1+|x| > 0$

$|x| > -1$

$= \begin{cases} \frac{-1}{1-x} & x < -1 \\ \frac{1}{1+x} & x > -1 \end{cases}$

Domain: -1 does not exist, therefore its not differentiable.

(35) $f(x) = (3x^2-2x)^{5x+3}$

$f(x) = e^{5x+3 \cdot \ln(3x^2-2x)}$

$f'(x) = e^{5x+3 \cdot \ln(3x^2-2x)} \cdot 5\ln(3x^2-2x) + (5x+3) \cdot \frac{1}{3x^2-2x} \cdot (6x-2)$

$e^{5x+3 \cdot \ln(3x^2-2x)} \cdot 5\ln(3x^2-2x) \cdot \left(\frac{(6x-2)\cdot(5x+3)}{3x^2-2x}\right)$

(37) $f(x) = (\ln x)^{\arctan x}$

$f(x) = e^{\arctan x \ln x \ln x}$

$f'(x) = e^{\arctan x \ln x \ln x} \cdot \frac{1}{1+x^2} \cdot \ln x \ln x + \arctan x \cdot \frac{1}{\ln x} \cdot \frac{1}{x} =$

$e^{\arctan x \ln x \ln x} \cdot \frac{\ln x \ln x}{1+x^2} + \frac{\arctan x}{x \ln x}$

② 33 $f(x) = \dfrac{\sqrt{x} \cdot \cos x}{\sqrt[3]{x^2-3x} \cdot \ln x}$ using e & $\ln$

$f(x) = e^{\ln(\ \ )}$

$f'(x) = \underbrace{e^{\ln(\ \ \ \ )}}_{f(x)} \cdot$ derivative of inside

- $\ln AB = \ln A + \ln B$
- $\ln \frac{A}{B} = \ln A - \ln B$
- $\ln A^p = p \ln A$

using this here

$\ln\left(\dfrac{\sqrt{x}\cos x}{\sqrt[5]{x^2-3x} \cdot \ln x}\right) = \underbrace{\left[\ln \sqrt{x} + \cos x\right]}_{\text{num}} - \underbrace{\left[\ln \sqrt[5]{x^2-3x} + \ln(\ln x)\right]}_{\text{denom}}$

$\left(\ln(\ \ \ \ )\right)' = \left(\frac{1}{2}\ln x + \ln \cos x - \frac{1}{5}\ln(x^2-3x) - \ln(\ln x)\right)'$

$= \frac{1}{2x} + \frac{1}{\cos x} \cdot (-\sin x) - \frac{1}{5(x^2-3x)} \cdot (2x-3) - \frac{1}{\ln x} \cdot \frac{1}{x}$

$f'(x) = \left[\dfrac{\sqrt{x}\cos x}{\sqrt[5]{x^2-3x} \cdot \ln x}\right]\left[\frac{1}{2x} - \frac{\sin x}{\cos x} - \frac{2x-3}{5x^2-15x} - \frac{1}{x\ln x}\right]$

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③ ⓑ $f(x) = \dfrac{(3x^2 - 5x + 2)^5}{(4-3x)^7}$ $x \neq \dfrac{4}{3}$

$$f'(x) = \frac{(4-3x)^7 \cdot 5(3x^2-5x+2)^4 \cdot (6x-5) - ((3x^2-5x+2)^5 \cdot 7(4-3x)^6 \cdot -3}{((4-3x)^7)^2}$$

$$f'(x) = \frac{(4-3x)^7(3x^2-5x+2)^4 \cdot (30x-25) - (3x^2-5x+2)^5 \cdot -21(4-3x)^6}{(4-3x)^{14}}$$

$$f'(x) = \frac{(4-3x)^6(3x^2-5x+2)^4[(4-3x)(30x-25) - (3x^2-5x+2)-21]}{(4-3x)^{14}}$$

$$f'(x) = \frac{(3x^2-5x+2)^4[120x-100-90x^2+75x-(-63x^2+105x-42)]}{(4-3x)^8}$$

$$f'(x) = \frac{(3x^2-5x+2)^4[195x-90x^2-100+63x^2-105x+42]}{(4-3x)^8}$$

$$f'(x) = \frac{(3x^2-5x+2)^4[-27x^2+90x-58]}{(4-3x)^8}$$

ⓓ $f(x) = (2x-3)\sqrt{4x-x^2}$

$$f'(x) = 2 \cdot \sqrt{4x-x^2} + (2x-3)\underbrace{\frac{1}{2}(4x-x^2)^{-\frac{1}{2}}}_{\frac{1}{2\sqrt{4x-x^2}}}(-2x+4)$$

$$f'(x) = 2 \cdot \sqrt{4x-x^2} + \frac{(2x-3)(-2x+4)}{2\sqrt{4x-x^2}} = \frac{4(4x-x^2)+(2x-3)(-2x+4)}{2\sqrt{4x-x^2}} =$$

$$\frac{16x-4x^2+(-4x^2+8x+6x-12)}{2\sqrt{4x-x^2}} = \frac{-8x^2+30x-12}{2\sqrt{4-x^2}} = \frac{2(-4x^2+15x-6)}{2\sqrt{4-x^2}}$$

③ ⓕ $f(x) = \tan x - \frac{1}{3}\tan^3 x + \frac{1}{5}\tan^5 x$

$f'(x) = 1 + \tan^2 x - \tan^2 x\,(1 + \tan^2 x) + \tan^4 x\,(1 + \tan^2 x)$

$f'(x) = (1 + \tan^2 x)(1 - \tan^2 x + \tan^4 x)$

$f'(x) = 1 - \cancel{\tan^2 x} + \tan^4 x + \tan^2 x - \tan^4 x + \tan^6 x = 1 + \tan^6 x$

ⓘ $f(x) = x\ln\left(x + \sqrt{1+x^2}\right) - \sqrt{1+x^2}$

$f'(x) = \ln\left(x + \sqrt{1+x^2}\right) + x \cdot \frac{1}{x + \sqrt{1+x^2}} \cdot \left(1 + \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x\right) - \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x$

$= \ln\left(x + \sqrt{1+x^2}\right) + x\,\frac{\left(1 + \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x\right)}{x + \sqrt{1+x^2}} - x(1+x^2)^{-\frac{1}{2}}$

$= \ln\left(x + \sqrt{1+x^2}\right) + x\,\frac{\left(1 + \frac{x}{\sqrt{1+x^2}}\right)}{x + \sqrt{1+x^2}} - x(1+x^2)^{-\frac{1}{2}}$

$\ln\left(x + \sqrt{1+x^2}\right) + x\left(\frac{\cancel{\sqrt{1+x^2} + x}}{\cancel{\left(x + \sqrt{1+x^2}\right)}\left(\sqrt{1+x^2}\right)}\right) - \frac{x}{\sqrt{1+x^2}}$

$\ln\left(x + \sqrt{1+x^2}\right) + \left(\cancel{\frac{x}{\sqrt{1+x^2}}}\right) - \cancel{\frac{x}{\sqrt{1+x^2}}} = \ln\left(x + \sqrt{1+x^2}\right)$

ⓙ $f(x) = \arcsin(1 + x^2)$

$f'(x) = \frac{2x}{\sqrt{1-(1+x^2)^2}} = \frac{2x}{\sqrt{1-(1+2x^2+x^4)}} = \frac{2x}{\sqrt{2x^2 - x^4}}$

$\downarrow$

f isn't differentiable anywhere in $\mathbb{R}$

④ ⓒ $f(x) = \begin{cases} x \cdot \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$

ⓐ $\lim_{x \to 0} x \cdot \sin\frac{1}{x} = 0$

ⓑ $\lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \frac{\cancel{x}\sin(\frac{1}{x})}{\cancel{x}} = \sin(\frac{1}{x}) = f'(0) \rightarrow$ no limit

ⓒ $\lim_{x \to 0^+} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^+} \sin\frac{1}{x} =$ no limit

$\lim_{x \to 0^-} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0^-} \sin\frac{1}{x} =$ no limit

f is not differentiable from right or left.

ⓓ $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

ⓐ $\lim_{x \to 0} x^2 \sin\frac{1}{x} = 0$

ⓑ $f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \to 0} \frac{x^2 \sin\frac{1}{x}}{x} = x\sin\frac{1}{x} = 0 \leftarrow$ f differentiable

$f'(0) = 0$

$x \neq 0 \quad f'(x) = 2x \cdot \sin\frac{1}{x} - \cos\frac{1}{x} \cdot x^2 \cdot \frac{1}{x^2} = 2x \cdot \sin\frac{1}{x} - \cos\frac{1}{x}$

$\lim_{x \to 0} 2x \cdot \sin\frac{1}{x} - \cos\frac{1}{x} =$ no limits

ⓔ $f(x) = \begin{cases} x^3 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$

ⓐ $\lim_{x \to 0} x^3 \sin\frac{1}{x} = 0$

ⓑ $f'(0) = \lim_{x \to 0} \frac{f(x) - f(0)}{x} = x^2 \sin\frac{1}{x} = 0$

$f'(0) = 0$

ⓒ $x \neq 0 \quad f'(x) = 3x^2 \sin\frac{1}{x} - \cos\frac{1}{x} \cdot x$

$\lim_{x \to 0} 3x^2 \cdot \sin\frac{1}{x} - \cos\frac{1}{x} \cdot x = 0$

⑤ ⓐ $f(x) = x^2 - 3x$

$g(x) = |x|$

$h(x) = |x^2 - 3x|$

$x^2 - 3x = x(x-3) \longrightarrow$ differentiable ; $x=0 \quad x=3$

$$h(x) = x^2 - 3x = \begin{cases} x^2 - 3x & x \leq 0 \\ -x^2 + 3x & 0 < x < 3 \\ x^2 - 3x & x \geq 3 \end{cases}$$

$$h'(x) = \begin{cases} 2x - 3 & x \leq 0 \\ -2x + 3 & 0 < x < 3 \\ 2x - 3 & x \geq 3 \end{cases}$$

$\lim_{x \to 3^+} 2x - 3 = 3$

$\lim_{x \to 3^-} -2x + 3 = -3$

$3 \neq -3$

so $h(x)$ is not differentiable at $x=0$

$\lim_{x \to 0^+} 2x - 3 = -3$

$\lim_{x \to 0^-} -2x + 3 = 3$

$3 \neq -3$

so $h(x)$ is not differentiable at $x=3$

ⓒ $f(x) = x^3 - 6x^2 + 9x$

$g(x) = |x|$

$h(x) = |x^3 - 6x^2 + 9x|$

$x^3 - 6x^2 + 9x = x(x^2 - 6x + 9)$

$x(x-3)(x-3) \rightarrow x=0 \quad x=3$

$$h(x) = x^3 - 6x^2 + 9x = \begin{cases} -x^3 + 6x^2 - 9x & x \leq 0 \\ x^3 - 6x^2 + 9x & 0 < x < 3 \\ x^3 - 6x^2 + 9x & x \geq 3 \end{cases}$$

$-x^3 + 6x^2 - 9x \quad x \leq 0$

$x^3 - 6x^2 + 9x \quad x > 0$

$$h'(x) = \begin{cases} -3x^2 + 12x - 9 & x \leq 0 \\ 3x^2 - 12x + 9 & x > 0 \end{cases}$$

$\lim_{x \to 0^+} 3x^2 - 12x + 9 = 9$

$\lim_{x \to 0^-} -3x^2 + 12x - 9 = -9$

$9 \neq -9$ so $h(x)$ is not differentiable at $x=0$

differentiable at $x=3$

(a) (a) f invertible in the neighborhood of $x_0 = 3$

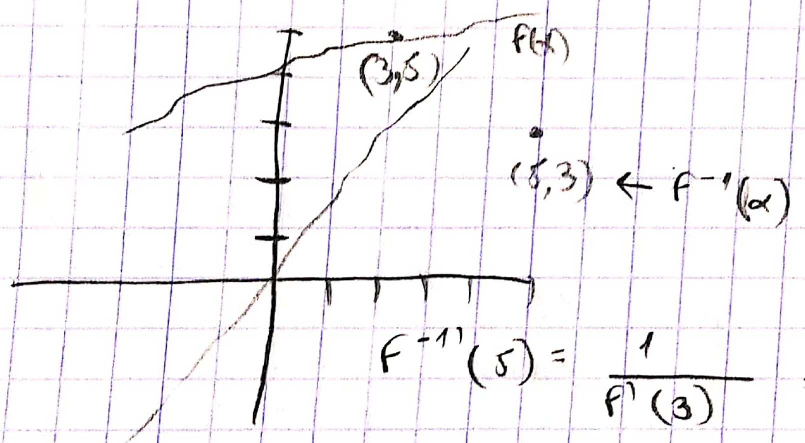
$f(3) = 5$

$f'(3) = 2$

$(f^{-1})'(5)$?

$$\boxed{f'(x_0) = \frac{1}{f'^{-1}(y_0)}}$$

$$\boxed{(f^{-1})'(x_0) = \frac{1}{f'(y_0)}}$$



$$f^{-1\prime}(5) = \frac{1}{f'(3)} = \frac{1}{2}$$

$$\left(f^{-1\prime}(3) = \frac{1}{f'(5)} \right)$$

⑥ (b) $f(x) = \frac{3}{2x-4}$

① first way to find deriv. of inverse: find inverse & derive

$$y = \frac{3}{2x-4} \Rightarrow x = \frac{3}{2y-4} \Rightarrow 2y-4 = \frac{3}{x} \Rightarrow y = \frac{\frac{3}{x}+4}{2} = \frac{3+4x}{2x} = \frac{3}{2x}+2$$

$$y' = \frac{-3}{2x^2}$$

② use rule $f^{-1'}(y_0) = \frac{1}{f'(x_0)}$ $\quad f'(x) = -3(2x-4)^{-2}(2) = \frac{-6}{(2x-4)^2}$

$$f^{-1'}(y_0) = \frac{(2x_0-4)^2}{6}$$

(c) $f(x) = x+\ln x$ $\quad (f^{-1})'(e+1)$

<u>Domain:</u> $x>0$

- x & $\ln x$ are continuous function in their domain so $f(x)$ is continuous.
- <u>monotonic</u>: $f'(x) = 1+\frac{1}{x}$ never be zero in our domain.

$1+\frac{1}{x} > 0$ so the function is always increasing

so $f(x)$ is invertible $\quad$ ($x = y+\ln y$ can't find the inverse directly)

$$(x_0, y_0) = (x_0, e+1) = (e, e+1)$$

$f(x) = x_0+\ln x_0 = e+1$

$x_0 = e$

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)} = \frac{1}{f'(e)}$$

$f'(x) = 1+\frac{1}{x}$

$f'(e) = 1+\frac{1}{e}$

$$= \frac{1}{1+\frac{1}{e}} = \frac{e}{e+1}$$

7 b $y = x + \arctan(xy^2)$

$$y' = 1 + \frac{y^2 + x2y \cdot y'}{1 + (xy^2)^2}$$

$$y'(1 + (x^2y^4)) = y^2 + x2yy' + 1 + x^2y^4$$

$$y'(1 + x^2y^4) - x2yy' = y^2 + 1 + x^2y^4$$

$$y'(1 + x^2y^4 - x2y) = y^2 + 1 + x^2y^4$$

$$y' = \frac{y^2 + 1 + x^2y^4}{1 + x^2y^4 - x2y}$$ for $1 + x^2y^4 - x2y \neq 0$

7 d $((x+2y)^3)' - (3x^2 \ln y) + (x)' = (y^2)' - (3e^{(x+2)y})'$

$\Leftrightarrow (3(x+2y)^2 - (6x \ln y + 3x^2 (\frac{y'}{y})) + 1 = 2yy' - e^{(x+2)y} + 3(y + (x+2)y')(e^{(x+2)y})$

$\Leftrightarrow 3(x+2y)^2 - 6x \ln y - 3x^2 (\frac{y'}{y}) + 1 = 2yy' - e^{(x+2)y} + 3(y + (x+2)y')(e^{(x+2)y})$

$\Leftrightarrow -3x^2(\frac{y'}{y}) - 2yy' - 3(y + (x+2)y')(e^{(x+2)y}) = 6x \ln y - 3(x+2y)^2 - 1 - e^{(x+2)y}$

$\Leftrightarrow \frac{-3x^2 y' - 2y^2y' - 3ye^{xy+2y} - 3ye^{xy+2y} xy' - 6ye^{xy+2y} y'}{y} = 6x \ln y - 3(x+2y)^2 - 1 - e^{(x+2)y}$

$\Leftrightarrow y'(-3x^2 - 2y^2 - 3e^{xy+2y} - 3ye^{xy+2y} x - 6ye^{xy+2y}) = 6xy \ln y - 3y(x+2y)^2 - y - ye^{(x+2)y}$

$\Leftrightarrow y' = \frac{y(6x \ln y - 3(x+2y)^2 - 1 - e^{(x+2)y})}{-3x^2 - 2y^2 - 3e^{xy+2y} - 3ye^{xy+2y} x - 6ye^{xy+2y}}$

at the pt $(-2, 1)$

$y' = \frac{-1-1}{-17} = \frac{2}{17} \Rightarrow$ slope $= \frac{2}{17}$

$\Rightarrow y - 1 = \frac{2}{17}(x+2) \Leftrightarrow y = \frac{2}{17}x + \frac{21}{17}$

9 b $y' = \ln x + 1$

$\ln x + 1 = -1$

$\ln x = -2$

$x = e^{-2}$

$\Downarrow$

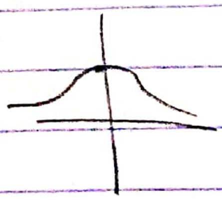
$y = e^{-2} \ln(e^{-2}) = -2e^{-2}$

$y + 2e^{-2} = 1(x - e^{-2})$

$y = x - 3e^{-2}$

Finding the slope of the line

d $y = e^{-|x|}$



$$y = \begin{cases} e^{-x} & x > 0 \\ e^{x} & x < 0 \end{cases}$$

$$y' = \begin{cases} -e^{-x} & x > 0 \\ e^{x} & x < 0 \end{cases}$$

$f'(0^+) = -1$

$f'(0^-) = 1$

$\alpha = \arctan(-1) = -45$

$\beta = \arctan(1) = 45$

$\gamma = 45° - (-45°) = 90°$

(f) $y=2 \quad f'(x)=\frac{-c}{x^2}$

$$\begin{cases} 2x+3=\frac{c}{x} \\ 2=\frac{-c}{x^2} \end{cases} \Rightarrow \begin{cases} c=2x^2+3x \\ c=-2x^2 \end{cases}$$

$$-2x^2=2x^2+3x$$

$$4x^2+3x=0$$

$$4x=-3$$

$x\neq 0$ therefore $x=-\frac{3}{4}$

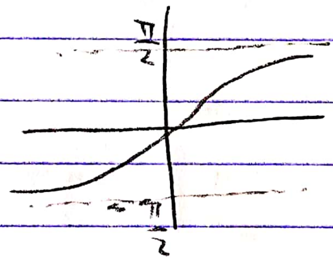
$$\Downarrow$$

$$c=-2\cdot\left(\frac{3}{4}\right)^2=-\frac{9}{8}$$

(11) $f(x)=\arctan\left(\frac{1+x}{1-x}\right) \quad g(x)=\arctan x$

(a) $1-x\neq 0 \Rightarrow x\neq 1 \Rightarrow \mathbb{R}/\{1\}$

graph of $g(x)$



(b) $\left(\frac{1+x}{1-x}\right)'=\frac{1\cdot(1-x)-(-1)\cdot(1+x)}{(1-x)^2}=\frac{1-x+1+x}{(1-x)^2}=\frac{2}{(1-x)^2}$

$$f'(x)=\frac{\frac{2}{(1-x)^2}}{1+\left(\frac{1+x}{1-x}\right)^2}=\frac{2}{(1-x)^2\cdot\left[1+\left(\frac{1+x}{1-x}\right)^2\right]}=\frac{2}{\frac{(1-x)^2+(1+x)^2}{(1-x)^2}\cdot(1-x)^2}=$$

$$\frac{2}{(x^2-2x+1)+(x^2+2x+1)}=\frac{2}{2+2x^2}=\frac{1}{1+x^2}$$

$$g'(x)=\frac{1}{1+x^2}$$

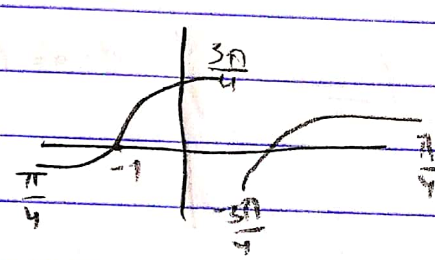
2c3

(c) $\arctan\left(\frac{1+x}{1-x}\right) = \arctan(x) + \frac{\pi}{4}$: $(-\infty, 1)$

$\arctan\left(\frac{1+x}{1-x}\right) = \arctan(x) - \frac{3\pi}{4}$: $(1, \infty)$

(d) $\arctan\left(\frac{1+x}{1-x}\right)$ has a zero denominator at 1 and there is a type I discontinuity (jump)

(e)



12) $f(x) = \arctan\left(\frac{x}{1+\sqrt{1-x^2}}\right)$

$\left\{ \text{connect this to } g(x) = \arcsin x \ (-1,1) \quad g'(x) = \frac{1}{\sqrt{1-x^2}} \right\}$

$$\boxed{\left(\sqrt{1-x^2}\right)^2 = 1-x^2}$$

$$f'(x) = \frac{1}{1+\left(\frac{x}{1+\sqrt{1-x^2}}\right)^2} \cdot \left[\frac{(1+\sqrt{1-x^2})(1) - x \cdot \left(\frac{1}{2\sqrt{1-x^2}} \cdot -2x\right)}{(1+\sqrt{1-x^2})^2}\right]$$

$\left\{ g'(x) = \frac{1}{\sqrt{1-x^2}} \right.$

$\left\{ \begin{array}{l} h(x) = \arctan x \\ h'(x) = \frac{1}{1+x^2} \end{array} \right.$

$$= \frac{1}{1+\frac{x^2}{(1+\sqrt{1-x^2})^2}} \cdot \frac{(1+\sqrt{1-x^2}) + \frac{x^2}{\sqrt{1-x^2}}}{(1+\sqrt{1-x^2})^2} = \frac{1+\sqrt{1-x^2} + \frac{x^2}{\sqrt{1-x^2}}}{(1+\sqrt{1-x^2})^2 + x^2} \cdot \left(\frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\right)$$

207

$$\frac{\sqrt{1-x^2} + 1 - \cancel{x^2} + \cancel{x^2}}{\sqrt{1-x^2}\left(\left[1+2\sqrt{1-x^2}+1-x^2\right]+x^2\right)} = \frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}+2(1-x^2)+\sqrt{1-x^2}} = \frac{\sqrt{1-x^2}+1}{2(1-x^2+\sqrt{1-x^2})}$$

$$= \frac{\cancel{\sqrt{1-x^2}+1}}{2\sqrt{1-x^2}\left(\cancel{\sqrt{1-x^2}+1}\right)} = \frac{1}{2\sqrt{1-x^2}} = \text{our derivative}$$

$g(x) = \arcsin x$

$g'(x) = \frac{1}{\sqrt{1-x^2}}$

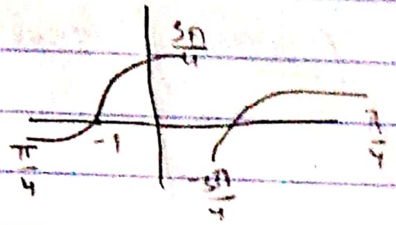
$f(x) = \arctan\left(\frac{x^2}{1+\sqrt{1-x^2}}\right) = \frac{1}{2}\arcsin x \ (+k)$ $\qquad (cf(x))' = cf'(x)$

(c) $\arctan\left(\frac{1+x}{1-x}\right) = \arctan(x) + \frac{\pi}{4}$: $(-\infty, 1)$

$\arctan\left(\frac{1+x}{1-x}\right) = \arctan(x) - \frac{3\pi}{4}$: $(1, \infty)$

(d) $\arctan\left(\frac{1+x}{1-x}\right)$ has a zero denominator at 1 and there is a type I discontinuity (jump)

(e)



(14) (c) $\lim_{x\to\infty} \frac{(\ln x)^2}{\sqrt[3]{x}} = \lim_{x\to\infty} \frac{2(\ln x)\cdot\frac{1}{x}}{\frac{1}{3}x^{-\frac{2}{3}}} = \lim_{x\to\infty} \frac{2\ln x}{\frac{x^{\frac{1}{3}}}{3}} = \lim_{x\to\infty} \frac{6\ln x}{\frac{x}{?}^{\frac{1}{3}}} =$

$\lim_{x\to\infty} \frac{6\cdot\frac{1}{x}}{\frac{1}{3}x^{-\frac{2}{3}}} = \lim_{x\to\infty} \frac{6}{x^{\frac{1}{3}}} = 0$

(f) $\lim_{x\to 0^+} (\sqrt{x}\cdot\ln x) = \sqrt{x\ln^2 x} = \sqrt{\lim_{x\to 0^+} x\ln^2 x} = \sqrt{\lim_{x\to 0^+} \frac{\ln^2 x}{\frac{1}{x}}} = \sqrt{\frac{2\ln x\cdot\frac{1}{x}}{\frac{1}{x^2}}}$

$= \sqrt{\lim_{x\to 0} -2x\ln x} = \sqrt{\lim_{x\to 0} \frac{-2\ln x}{\frac{1}{x}}} = \sqrt{\lim_{x\to 0} \frac{\frac{-2}{x}}{\frac{-1}{x^2}}} = \sqrt{\lim_{x\to 0} -2x} = \sqrt{-2(0)} = 0$

(h) $\lim_{x\to 0}\left(\frac{x+\sin x}{2x-3\sin x}\right) = \frac{0}{0} \overset{H}{=} \lim_{x\to 0} \frac{1+\cos x}{2-3\cos x} = \frac{2}{-1} = -2$

(i) $\lim_{x\to+\infty} \frac{x+\sin x}{2x-3\sin x} = \lim_{x\to+\infty} \frac{1+\frac{\sin x}{x}}{2+\frac{3\sin x}{x}} = \frac{1}{2}$

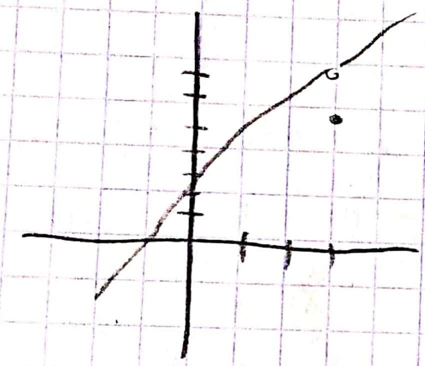
(o) $\lim_{x\to 0^+} (1+\sin x)^{\ln x} = \lim_{x\to 0^+} \left((1+\sin x)^{\frac{1}{\sin x}}\right)^{\sin x\cdot\ln x} = e^{\lim_{x\to 0^+} \sin x\ln x} = e^{\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{\sin x}}}$

$= e^{\frac{\infty}{\infty}} = e^{\lim_{x\to 0^+} \frac{\sin x\tan x}{-x}} = e^{\lim_{x\to 0^+} -\tan x} = e^0 = 1$

(p) $\lim_{x\to 0^+} (1-\sin x)^{\frac{1}{x^2}} = \lim_{x\to 0^+} \left((1-\sin x)^{\frac{1}{-\sin x}}\right)^{\frac{-\sin x}{x^2}} = e^{\lim_{x\to 0} \frac{-\sin x}{x^2}} \overset{H}{=}$

$e^{\lim_{x\to 0^+} \frac{-\cos x}{2x}} = e^{-\infty} = \frac{1}{e^\infty} = \frac{1}{\infty} = 0$

15 a



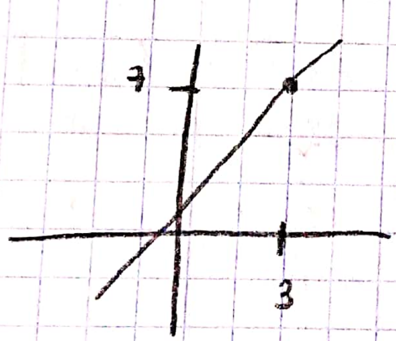
← NOT CONTINOUS at x=3

it does not have a derivative at x=3

differentiable $\Rightarrow$ continous

not continous $\Rightarrow$ not differentiable

b



$f(x) = 2x+1 \quad \forall x$

$f'(x) = 2 \quad \forall x$

d) False

$$f(x) = \begin{cases} x^2 - 4x & x \geq 3 \\ 6 - 3x & x < 3 \end{cases}$$

function defined in parts at $x=3$ we use the definition of derivative

$$f'(x) = \begin{cases} 2x - 4 & x > 3 \\ -3 & x < 3 \end{cases}$$

$$\lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3} \stackrel{?}{=} \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$$

L.H Derivative | R.H Derivative

$$\lim_{x \to 3^-} \frac{6-3x - (-3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{9-3x}{x-3} = \frac{3(3-x)}{x-3} = \boxed{-3}$$

$$= \lim_{x \to 3^+} \frac{x^2 - 4x - (-3)}{x-3} = \lim_{x \to 3^+} \frac{x^2 - 4x + 3}{x-3} = \frac{(x-3)(x-1)}{x-3} = \boxed{2}$$

NOT EQUAL
not TRUE

(d) false $f'(x)=2$ for all $x \in \mathbb{R}$

(e) false $f'(x)$ $\begin{cases} 2x-6 & x>2 \\ -2 & x<2 \end{cases}$ undefined at $x=2$

(f) true $f'(x)=-2$ $f'(2)^{+}=2(2-6)=-2$ $f'(2)=-2$