(1)

$f(x) = x^3 + 3x^2 + 3x + 1$ (1)

$f'(x) = 3x^2 + 6x + 3$

(2)

$f(x) = \frac{1}{x}$

$f(x) = x^{-1}$ $f'(x) = -x^{-2} = \frac{1}{-x^2}$

(3)

$f(x) = \sqrt{x} =$

$f(x) = x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}} =$

$\frac{1}{\frac{1}{2}x^{\frac{1}{2}}} = \frac{1}{2\sqrt{x}}$

(4)

$f(x) = x(x+1)$

$f(x) = x^2 + x$

$f'(x) = 2x + 1$

(5)

$f(x) = \frac{x}{x+1} =$

$\frac{1 \cdot (x+1) - [x \ (x+1)']}{(x+1)^2} = \frac{x+1-x}{(x+1)^2} = \frac{1}{(x+1)^2}$

(6)

$f(x) = \frac{4x^3 - 3x^2}{5x^7 + 1} =$

$\frac{(12x^2 - 6x)(5x^7+1) - [(4x^3 - 3x^2)(35x^6)]}{(5x^7-1)^2} =$

$\frac{60x^9 + 12x^2 - 30x^8 - 6x - [140x^9 - 105x^8]}{(5x^7-1)^2} =$

$\frac{-80x^9 + 75x^8 + 12x^2 - 6x}{(5x^7-1)^2}$

(ב) הוכיחו כי $F(x)=\int_0^x \frac{1}{1+t^2}\,dt+\int_0^{\frac{1}{x}} \frac{1}{1+t^2}\,dt$ קבועה בקטע $(0,\infty)$

(נראה ש- $F'(x)=0$ בקטע $(0,\infty)$

נגדיר: $g:(0,\infty)\to R$ ע"י

$$g(x)=\int_0^x \frac{1}{1+t^2}\,dt$$

מהמשפט היסודי, k גזירה

$$g'(x)=\frac{1}{1+x^2}$$

נגדיר $k:(0,\infty)\to \mathbb{R}$ ע"י $k(x)=g\left(\frac{1}{x}\right)$

אז: $k(x)=\int_0^{\frac{1}{x}} \frac{1}{1+t^2}\,dt$

$$\boxed{H(x)=\int_a^x h(t)\,dt \quad H'(x)=h(x) \Leftarrow}$$

$$k'(x)=g'\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)=\frac{1}{1+\left(\frac{1}{x}\right)^2}\left(-\frac{1}{x^2}\right)=$$

$$\left(g'(x)=\frac{1}{1+x^2}\right) \Bigg| =\frac{x^2}{x^2+1}\left(-\frac{1}{x^2}\right)=-\frac{1}{1+x^2}$$

$$F(x)=\underbrace{\int_0^x \frac{1}{1+t^2}\,dt}_{g(x)}+\underbrace{\int_0^{\frac{1}{x}} \frac{1}{1+t^2}\,dt}_{k(x)=g\left(\frac{1}{x}\right)}$$

$$F'(x)=g'(x)+k'(x)=\frac{1}{1+x^2}-\frac{1}{1+x^2}=0$$

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$$F(x)=\int_0^x \sqrt{4t^2+9}\,dt \qquad ③$$

$$F(2)=\int_2^2 \sqrt{4t^2+9}\,dt=0$$

$$F'(2)=\sqrt{4\cdot 2^2+9}=\sqrt{25}=5$$

$$F''(2)= \quad F''(x)=\frac{8x}{2\sqrt{4x^2+9}}=\frac{4x}{\sqrt{4x^2+9}}$$

$$F''(2)=\frac{8}{\sqrt{4\cdot 2^2+4}}=\frac{8}{5}$$

$$g(x)=\int_2^{x^2} \tan^2 t\,dt \qquad ① \quad ④$$

$$g(x)=f(x^2) \leftarrow f(x)=\int_2^x \tan^2 t\,dt$$ נקח פונ'

$$g'(x)=F'(x^2)\cdot 2x$$

$$f'(x)=f(x)=\tan^2 x$$

$$g'(x)=\tan^2(x^2)\cdot 2x$$

$$\boxed{g'(x)=2x\tan^2(x^2)}$$

$$g(x)=\int_{-3x}^{0} \sin 2t\,dt=-\int_0^{-3x}\sin(2t)\,dt \qquad ②$$

$$g(x)=F(-3x) \longleftarrow F(x)=\int_0^x \sin(2t)\,dt$$ נקח

$$g'(x)=-(F'(-3x)\cdot -3)$$

$$F'(x)=F(x)=\sin(2x)$$

$$g'(x)=-(\sin(-6x)\cdot -3)$$

$$g'(x)=-(-3\sin(-6x))=3\sin(-6x)$$

$$g(x)=\int_{x^3}^{x^5}\sqrt{t}\,dt, \quad g(x)=\int_0^{x^5}\sqrt{t}\,dt-\int_0^{x^3}\sqrt{t}\,dt \qquad ③$$

$$F(x)=\int_0^x \sqrt{t}\,dt$$ נקח

$$g'(x)=F'(x^5)\cdot 5x^4+F'(x^3)\cdot 3x^2$$

$$F'(x)=\sqrt{x}$$

$$g'(x)=\sqrt{x^5}\cdot 5x^4-\sqrt{x^3}\cdot 3x^2$$

$$\boxed{g'(x)=5x^4\sqrt{x^5}-3x^2\sqrt{x^3}}$$

202

7 5

$$\int_4^0 x\sqrt{1+2x}\,dx = \Big/ \quad y = 1+2x,\quad dy = 2\,dx,\quad dx = \frac{dy}{2} \qquad x = \frac{1}{2}y - \frac{1}{2}$$

$$= \int_9^1 \left(\frac{1}{2}y - \frac{1}{2}\right)\sqrt{y}\ \frac{1}{2}\,dy =$$

$$= \int_9^1 \left(\frac{1}{2}y^{\frac{3}{2}} - \frac{1}{2}y^{\frac{1}{2}}\right)\frac{1}{2}\,dy$$

$$\frac{1}{4}\int_9^1 \left(y^{\frac{3}{2}} - y^{\frac{1}{2}}\right)dy = \frac{1}{4}\int_9^1 \left(y^{\frac{3}{2}} - y^{\frac{1}{2}}\right)dy =$$

$$\frac{1}{4}\left[\left(\frac{2}{5}y^{\frac{5}{2}} - \frac{2}{3}y^{\frac{3}{2}}\right)\right]_9^1$$

$$= \frac{1}{4}\left(\frac{2}{5} - \frac{2}{3}\right) - \frac{1}{4}\left(\frac{2}{5}\cdot 9^{\frac{5}{2}} - \frac{2}{3}\cdot 9^{\frac{3}{2}}\right) =$$

1 הצבנו $\qquad$ 9 הצבנו

$$= \frac{298}{-15}$$

14

$$\int_0^{\frac{\pi}{2}} \cos x\,\sin(\underbrace{\sin x}_{y})\,dx = \qquad y = \sin x,\quad dy = \cos x\,dx$$

$$= \int_0^1 \sin y\,dy = \Big[-\cos y\Big]_0^1 = -\cos 1 - (-\cos 0)$$

$$= -\cos 1 + 1 = 0.459$$

44

$$g(x) = \int_{2x+1}^{x^2-1} \cos t\,dt =$$

$$\int_{2x+1}^{0} \cos t\,dt + \int_0^{x^2-1} \cos t\,dt =$$

$$-\int_0^{2x+1} \cos t\,dt + \int_0^{x^2-1} \cos t\,dt$$

$$F(x) = \int_0^x \cos t\,dt \qquad \text{נגדיר}$$

$$F'(x) = \cos x$$

$$g(x) = -F(2x+1) + F(x^2-1)$$

$$g'(x) = -\cos(2x+1)\cdot 2 + \cos(x^2-1)\,2x$$

$$g(x) = \int_{2x}^{3x^2} \frac{t^2-1}{t^2+1}\,dt =$$ ⑤ ④

$$\int_{0}^{3x^2} \frac{t^2-1}{t^2+1}\,dt + \int_{2x}^{0} \frac{t^2-1}{t^2+1}\,dt =$$

$$\int_{0}^{3x^2} \frac{t^2-1}{t^2+1}\,dt - \int_{0}^{2x} \frac{t^2-1}{t^2+1}\,dt$$

$$F(x) = \int_0^x \frac{t^2-1}{t^2+1}\,dt$$ נגדיר

$$g'(x) = F'(3x^2)\cdot 6x - F'(2x)\cdot 2$$

$$F'(x) = f(x) = \frac{t^2-1}{t^2+1}$$

$$g'(x) = \frac{9x^4-1}{9x^4+1}\cdot 6x - \frac{4x^2-1}{4x^2+1}\cdot 2$$

$$\boxed{g'(x) = \frac{54x^5-6x}{9x^4+1} - \frac{8x^2-2}{4x^2+1}}$$

$$g(x) = \int_{-x^2}^{2x} \sin 3t^2\,dt =$$ ⑥

$$\int_0^{2x} \sin 3t^2\,dt + \int_{-x^2}^{0} \sin 3t^2\,dt =$$

$$\int_0^{2x} \sin(3t^2)\,dt - \int_0^{-x^2} \sin(3t^2)\,dt$$

$$F(x) = \int_0^x \sin 3t^2\,dt$$ נגדיר

$$g'(x) = F'(2x)\cdot 2 - F'(-x^2)\cdot -2x$$

$$F'(x) = F(x) = \sin 3x^2$$

$$g'(x) = 2\sin 3(2x)^2 + 2x\sin 3(-x^2)^2$$

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7 5

$$\int_1^4 (5x^2 - 8x + 5)\,dx =$$
$$\left[\frac{5x^3}{3} - \frac{8x^2}{2} + 5x + C\right]_1^4 =$$
$$\left[\frac{5x^3}{3} - 4x^2 + 5x\right] =$$
$$\left[\frac{5\cdot 4^3}{3} - 4\cdot 4^2 + 5\cdot 4\right] - \left[\frac{5\cdot 1^3}{3} - 4\cdot 1^2 + 5\cdot 1\right] =$$
$$\left[106\tfrac{2}{3} - 64 + 20\right] - \left[\frac{5}{3} - 4 + 5\right] =$$
$$\left[106\tfrac{2}{3} - 44\right] - \left[1\tfrac{5}{3}\right] = 62\tfrac{2}{3} - 2\tfrac{2}{3} = 60$$

2

$$\int_1^9 \left(x^{\frac{3}{2}} + 2x + 3\right)dx =$$
$$\left[\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^2}{2} + 3x\right]_1^9 = \left[\frac{2}{5}x^{\frac{5}{2}} + x^2 + 3x\right]_1^9 =$$
$$\left[\frac{2}{5}\cdot 9^{\frac{5}{2}} + 9^2 + 3\cdot 9\right] - \left[\frac{2}{5}\cdot 1^{\frac{5}{2}} + 1^2 + 3\cdot 1\right] =$$
$$\left[97.2 + 81 + 27\right] - \left[\frac{2}{5} + 1 + 3\right]$$
$$\left[205.2\right] - \left[4\tfrac{2}{5}\right] = 200.8$$

3

$$\int_4^9 \left(\sqrt{x} + \frac{1}{3\sqrt{x}}\right)dx = \int_4^9 \left(x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}\right)dx =$$
$$\left[\frac{2}{3}x^{\frac{3}{2}} + \frac{3x^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^9 = \left[\frac{2}{3}x^{\frac{3}{2}} + 1.5x^{\frac{1}{2}}\right]_4^9 =$$
$$\left[\frac{2}{3}\cdot 9^{\frac{3}{2}} + 1.5\cdot 9^{\frac{1}{2}}\right] - \left[\frac{2}{3}\cdot 4^{\frac{3}{2}} + 1.5\cdot 4^{\frac{1}{2}}\right] =$$
$$\left[18 + 4.5\right] - \left[5\tfrac{1}{3} + 3\right] =$$
$$\left[22.5\right] - \left[8\tfrac{1}{3}\right] = 14\tfrac{1}{6}$$

4

$$\int_1^4 \frac{5}{x^3}\,dx = \int_1^4 5x^{-3}\,dx = \left[\frac{5x^{-2}}{-2}\right]_1^4 =$$
$$\left[\frac{5\cdot 4^{-2}}{-2}\right] - \left[\frac{5\cdot 1^{-2}}{-2}\right] = \left[-\frac{5}{32}\right] - \left[-2.5\right] = 2\tfrac{11}{32}$$

$$\int_{-1}^{2} (1+3t)\, t^2\, dt = \quad f(x) = 1+3t$$
$$f'(x) = 3$$
$$g'(x) = t^2$$
$$g(x) = \int t^2\, dx = \frac{t^3}{3}$$

⑤

$$\left[(1+3t)\frac{t^3}{3} - \int 3\cdot\frac{t^3}{3}\, dt\right]_{-1}^{2} = \left[\frac{t^3}{3} + t^4 - \frac{t^4}{4}\right]_{-1}^{2} =$$

$$\left[\frac{2^3}{3} + 2^4 - \frac{2^4}{4}\right] - \left[\frac{(-1)^3}{3} + (-1)^4 - \frac{(-1)^4}{4}\right] =$$

$$\left[\frac{8}{3} + 16 - 4\right] - \left[-\frac{1}{3} + 1 - \frac{1}{4}\right] =$$

$$\left[12\tfrac{8}{3}\right] - \left[\frac{5}{12}\right] = 14{,}25$$

$$\int_{-2}^{1} (2t^2-1)^2\, dt = \int_{-2}^{1} (4t^4 - 4t^2 + 1)\, dt =$$

⑥

$$\left[\frac{4t^5}{5} - \frac{4t^3}{3} + x + c\right]_{-2}^{1} =$$

$$\left[\frac{4\cdot 1^5}{5} - \frac{4\cdot 1^3}{3} + 1\right] - \left[\frac{4\cdot(-2)^5}{5} - \frac{4\cdot(-2)^3}{3} + (-2)\right]$$

$$\left[\frac{4}{5} - \frac{4}{3} + 1\right] - \left[-\frac{128}{5} + \frac{32}{3} - 2\right]$$

$$\left[\frac{7}{15}\right] - \left[-16\tfrac{14}{15}\right] = 17{,}4$$

$$\int_{-2}^{2} \frac{1}{x^2+4}\, dx = \frac{1}{4}\int_{-2}^{2} \frac{1}{1+\left(\frac{x}{2}\right)^2} = \Big/ \; u = \frac{x}{2} \quad dx = 2du$$
$$du = \frac{1}{2}dx$$

⑧

$$= \frac{1}{4}\cdot\int_{-1}^{1} \frac{1}{1-u^2}\, 2du = \frac{1}{2}\int_{-1}^{1} \frac{1}{1+u^2}\, du = \frac{1}{2}\left[\arctan u\right]_{-1}^{1} =$$

$$\frac{1}{2}\left((\arctan 1) - \frac{1}{2}(\arctan -1)\right) = \frac{1}{4}\pi$$

$$\int_{1}^{4} \frac{1}{\sqrt{x}\,(\sqrt{x}+1)^2}\, dx = \quad u = \sqrt{x}+1 \qquad dx = 2\sqrt{x}\, du$$
$$du = \frac{1}{2\sqrt{x}}$$

⑩

$$\int_{1}^{4} \frac{1}{\sqrt{x}\cdot u^2}\, 2\sqrt{x}\, du = 2\int_{2}^{3} \frac{1}{u^2}\, du = 2\left[\frac{u^{-1}}{-1}\right]_{2}^{3}$$

$$2\left[\frac{3^{-1}}{-1}\right] - 2\left[\frac{2^{-1}}{-1}\right] = \frac{1}{3}$$

307

(11)

$$\int_0^{\pi} x^2 \cos x \, dx$$

$f(x) = x^2 \qquad g'(x) = \cos(x)$

$f'(x) = 2x \qquad g(x) = \sin(x)$

<u>אינטגרציה בחלקים</u>

$x^2 \sin x - 2\int x \sin x \, dx$

$f(x) = x \qquad g'(x) = \sin(x)$

$f'(x) = 1 \qquad g(x) = -\cos(x)$

$-x\cos x + \int \cos x = -x\cos(x) + \sin(x)$ <u>אינטגרציה בחלקים:</u>

<u>חיבור תשובות!</u>

$$\left[x^2 \sin x + 2x\cos x - 2\sin x\right]_0^{\pi}$$

$$\left(\pi^2 \sin(\pi) + 2\pi\cos(\pi) - 2\sin(\pi)\right) - 0 = \boxed{-2\pi}$$

(12)

$$\int_0^3 x\sqrt{9-x^2}\,dx = \Big/ \begin{array}{l} u = 9-x^2 \\ du = -2x\,dx \\ dx = \frac{du}{-2x} \end{array}$$

$$-\frac{1}{2}\int_0^3 x\sqrt{u}\,\frac{du}{x} = -\frac{1}{2}\int_9^0 \sqrt{u}\,du = -\frac{1}{2}\cdot\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_9^0$$

$$= -\frac{1}{3}\cdot\left(0-\sqrt{9}^{\,3}\right) = \boxed{9}$$

(13)

$$\int_{-1}^{2} x^2\sqrt{x^3+1}\,dx = \Big/ \begin{array}{l} u = x^3+1 \\ du = 3x^2\,dx \\ dx = \frac{du}{3x^2} \end{array}$$

$$\frac{1}{3}\int_1^2 \cancel{x^2}\sqrt{u}\,\frac{du}{\cancel{x^2}} = \frac{1}{3}\int_0^9 \sqrt{u}\,du = \frac{1}{3}\cdot\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^9 =$$

$$\frac{2}{9}\left[\sqrt{u^3}\right]_0^9 = \boxed{6}$$

(15)

$$\int_{-3}^{0} (2x+6)^4\,dx = \Big/ \begin{array}{l} u = 2x+6 \\ du = 2\,dx \end{array} \quad dx = \frac{du}{2}$$

$$\frac{1}{2}\int_0^6 u^4\,du = \frac{1}{2}\cdot\left[\frac{u^5}{5}\right]_0^6 = \frac{1}{2}\left(\frac{6^5}{5}\right) - \frac{1}{2}\left(\frac{0^5}{5} = \frac{1}{10}\cdot 6^5 = \boxed{\frac{3888}{5}}\right.$$

16

$$\int_1^2 \frac{x^2}{(x^3+1)^2}\,dx \quad \left(\begin{array}{l} u = x^3+1 \\ du = 3x^2\,dx \end{array} \right. \quad dx = \frac{du}{3x^2}$$

$$\frac{1}{3}\int_1^2 \frac{\cancel{x^2}}{u^2}\,\frac{du}{\cancel{x^2}} = \frac{1}{3}\int_2^9 u^{-2}\,du = \frac{1}{3}\left[\frac{u^{-1}}{-1}\right]_2^9 =$$

$$-\frac{1}{3}\left[\frac{1}{u}\right]_2^9 = -\frac{1}{3}\left(\frac{1}{9}\right) + \frac{1}{3}\cdot\frac{1}{2} = \boxed{\frac{7}{54}}$$

9

$$\int_0^{\frac{\pi}{3}} x^2 \sin 3x\,dx =$$

$f(x) = x^2$ $\qquad g'(x) = \sin(3x)$

$f'(x) = 2x$ $\qquad g(x) = \int \sin(3x)\,dx =$

$u = 3x$

$du = 3dx \qquad dx = \frac{du}{3}$

$$\int \sin(u)\frac{du}{3} = \frac{1}{3}\int \sin u = -\frac{1}{3}\cos(3x)$$

$$\left[x^2\left(-\frac{1}{3}\cos(3x)\right)\right]_0^{\frac{\pi}{3}} - \int_0^{\frac{\pi}{3}}\left(2x \cdot -\frac{1}{3}\cos(3x)\right)dx =$$

$$\left[x^2\left(-\frac{1}{3}\cos(3x)\right)\right]_0^{\frac{\pi}{3}} + \frac{1}{3}\underbrace{\int_0^{\frac{\pi}{3}}\left(2x\cos(3x)\right)dx}_{} =$$

(כמו מקודם אינטגרציה בחלקים)

$f(x) = 2x$ $\qquad g'(x) = \cos(3x)$

$f'(x) = 2$ $\qquad g(x) = \int \cos(3x)\,dx =$ $\quad u = 3x$

$du = 3\,dx$

$dx = \frac{du}{3}$

$$\frac{1}{3}\int \cos(u)\,du = \frac{1}{3}\sin(3x)$$

נציב חזרה לנוסחה:

$$\left[x^2\left(-\frac{1}{3}\cos(3x)\right)\right]_0^{\frac{\pi}{3}} + \frac{1}{3}\left[2x\left(\frac{1}{3}\sin(3x)\right)\right]_0^{\frac{\pi}{3}} - \int 2\cdot\frac{1}{3}\sin(3x)\,dx =$$

$$\left[x^2\left(-\frac{1}{3}\cos(3x)\right)\right]_0^{\frac{\pi}{3}} + \frac{1}{3}\left(\left[2x\left(\frac{1}{3}\sin(3x)\right)\right]_0^{\frac{\pi}{3}} - \frac{2}{3}\int \sin(3x)\,dx\right) =$$

$$\left[x^2\left(-\frac{1}{3}\cos(3x)\right)\right]_0^{\frac{\pi}{3}} + \frac{1}{3}\left(\left[2x\left(\frac{1}{3}\sin(3x)\right)\right]_0^{\frac{\pi}{3}} - \frac{2}{3}\left[-\frac{1}{3}\cos(3x)\right]_0^{\frac{\pi}{3}}\right) =$$

$$\left(\left(\frac{\pi^2}{9}\left(-\frac{1}{3}\cdot -1\right) - (0)\right) + \frac{1}{3}\left(\left(\frac{2\pi}{3}\left(\frac{1}{3}\cdot 0\right)\right) - 0 - \frac{2}{3}\left(\left(-\frac{1}{3}\cdot -1\right) - \left(-\frac{1}{3}\cdot 1\right)\right)\right)\right) =$$

$$\frac{\pi^2}{27} + \frac{1}{3}\left(-\frac{2}{3}\left(\frac{1}{3}+\frac{1}{3}\right)\right) = \frac{\pi^2}{27} - \frac{4}{27} = \frac{\pi^2 - 4}{27} = 0.21739$$