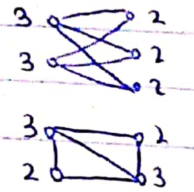
Mathmatical Logic HW 10

(2) (b) 5,4,3,2,2,0 , 3,2,1,1,-1

no, by havel-hakimi algorithm no simple graph can be created with these degrees. (5 must be edges to 5 other nodes with positive degree, but only 4 are > 0)

(d)



yes

3333 22222
222222222
1122222
2222211
~~[illegible]~~
112211
221111
10111
11110
0110
1100
000 by havel Hakimi, graph exists.

(F) 5,4,4,3,2,1
3,3,2,1,0
2,1,0,0
0,-1,0

No, by Havel-Hakimi algorithm no simple graph can be made.

after disconnecting the 2 vertices with longest degree, not enough vertices have positive degree)

(3) For every vertex in one component, there is no connection in G to any vertex in another component (otherwise they would be one component). $\bar{G}$ must therefore have a connection from every vertex in the first component to every vertex in the second component. Since for any vertex in the second component, it is connected to everything in the first and everything in the first is connected to everything in the second. The two components disconnected in G are connected in $\bar{G}$.

(4) <u>counterexample:</u> for any $G(V,E)$, $S = \{\}$ is a proper subset of V. however, there is no neighbor not in S that has a neighbor in S (there are no vertices in S).

<u>Assuming $S \neq \{\}$:</u> Since G is connected, all vertices in V are neighbors of other vertices in V. if you take a subset, all elements in the subset will be neighbors of another element which is in S or $\bar{S}$. Assuming $S \neq V$, which must be true since S is a proper subset,

there must be at least one vertex in S which is a neighbor of an element of $\bar{S}$ (otherwise $G(V,E)$ wouldn't be connected)

5 (a) The number of ways to choose 2 elements from X is $\binom{|X|}{2} = \frac{|X|(|X|-1)}{2} = \frac{20}{2} = 10$

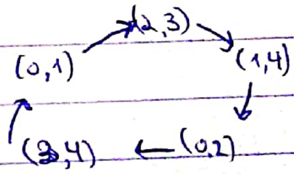
(b) for a given pair u, there are $3(|X|-|u|)$ elements not in u. The possible pairs v such that $u \cap v = \emptyset$ are pairs from $\bar{u}$.

Number of pairs $= \binom{3}{2} = 3$, There are 3 for each of the 10, divided by 2 since each pair u,v goes both ways. Therefore: $(3 \cdot 10)/2 = 15$ edges

(c) since there are $\binom{|X|-|u|}{|v|} = \binom{3}{2} = 3$ possible disjoint pairs V for a given pair u.

(d) its not a tree since its not acyclic.

There are cycles for exp:



6 # of possible directed edges = # of ordered pairs of vertices $= |V| \times |V| = n^2$

subtract n since this includes a loop for each of the n vertices.

$\therefore$ Number of directed edges not including loops $= n^2 - n = n(n-1)$

another proof: The number of possible pairings of 2 vertices (no loops since you choose without replacement) is $\binom{n}{2} = \frac{n(n-1)}{2}$

since the graph is directed, for every unordered set $\{a,b\}$ you have at most 2 edges -- (a,b) and (b,a). Therefore multiply the result by 2 to get $2 \cdot \binom{n}{2} = n \cdot (n-1)$