## linear algebra HW #1

② ⓑ $z^2 = -10 + 20i$

$i = \sqrt{-1}$

$i^2 = -1$

$(a+bi)(a+bi) = -10 + 20i$

$a^2 + 2abi + b^2 i^2 = -10 + 20i$

$a^2 + 2abi - b^2 = -10 + 20i$

$a^2 - b^2 = -10 \rightarrow a^2 - \left(\frac{10}{a}\right)^2 = -10 \rightarrow a^2 - \frac{100}{a^2} = -10$

$2abi = 20i$

$2ab = 20 \rightarrow b = \frac{10}{a}$

$\boxed{t = a^2}$

$t^2 - 100 = -10t$

$t^2 + 10t - 100 = 0$

$t_{1,2} = \frac{-10 \sqrt{10^2 - 4 \cdot 1 \cdot -100}}{-2}$

$t_{1,2} = \frac{-10 \sqrt{500}}{-2}$

$t_1 = \frac{-10 + \sqrt{500}}{-2} = -6.1803$

$t_2 = \frac{-10 \pm \sqrt{500}}{-2} = 16.180$

$a^2 = 16.180$

$a = \pm\sqrt{16.1803}$

$b = \frac{10}{\pm\sqrt{16.1803}}$

ⓓ $z^2 + |z|^2 = 2 - 4i \qquad z = a + bi$

$|z| = r = \sqrt{a^2 + b^2}$

$a^2 + 2abi + b^2 i^2 + \left(\sqrt{a^2 + b^2}\right)^2 = 2 - 4i$

$a^2 + 2abi + b^2 i^2 + a^2 + b^2 = 2 - 4i$

$2a^2 + 2abi + b^2 i^2 + b^2 = 2 - 4i$

$2a^2 + 2abi - \cancel{b^2} + \cancel{b^2} = 2 - 4i$

$2a^2 = 2$

$a = \pm 1$

$2abi = -4i$

$2ab = -4$

$b = \pm 2$

$z = 1 - 2i$ or $z = -1 + 2i$

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sop

3 a $\overline{z_1}+\overline{z_2}=\overline{z_1+z_2}$

$z_1=a+bi \Rightarrow \bar{z}_1=a-bi$

$z_2=a+bi \Rightarrow \bar{z}_2=a-bi$

$\overline{z_1}+\overline{z_2}=a_1+b_1i+a_2+b_2i \Rightarrow \boxed{a_1-b_1i+a_2-b_2i}$ equal

$\overline{z_1+z_2}=(a_1+b_1i)+(a_2+b_2i) \Rightarrow (a_1+a_2)-(b_1i+b_2i) \Rightarrow \boxed{a_1-b_1i+a_2-b_2i}$

d $\bar{\bar{z}}_1=z_1$

$z_1=\boxed{a_1+b_1i} \Rightarrow \bar{z}_1=a_1-b_1i \Rightarrow \bar{\bar{z}}_1=\boxed{a_1+b_1i}$

therefore $\bar{\bar{z}}_1=z_1$

they are both equal to $a+bi$

$r=|z|$

$r=\sqrt{a^2+b^2}$

cartesian = $z=a^2+b^2$

or could be $z=r(\cos(\ )+i\sin(\ ))$

4 b $1-i$

$r=\sqrt{1^2+(-1)^2}$

$r=\sqrt{1+1}=2 \Rightarrow r=\sqrt{2}$

$z=\sqrt{2}(\cos\theta+i\sin\theta)$

$\theta=\arctan\left(\frac{-1}{1}\right) \Rightarrow -45 \Rightarrow x=\frac{45\pi}{180} \Rightarrow \frac{-\pi}{4}$

$z=\sqrt{2}\left(\cos\frac{-\pi}{4}+i\sin\frac{-\pi}{4}\right)$

d $4-4i$

$r=\sqrt{4^2+(-4)^2}=\sqrt{32}$

$z=\sqrt{20}(\cos\theta+i\sin\theta)$

$\theta=\arctan\left(\frac{-4}{4}\right) \Rightarrow -45 \Rightarrow x=\frac{45\pi}{180} \Rightarrow \frac{-\pi}{4}$

$z=\sqrt{32}\left(\cos-\frac{\pi}{4}+i\sin-\frac{\pi}{4}\right)$

5 c $\left(\frac{1+2i}{-2+i}\right)^{2048}$

polar representation $re^{i\theta}$

$\left(\frac{(1+2i)(i-2)}{(i-2)(i+2)}\right)^{2048}$

$\left(\frac{i+2-2+4i}{-1+2i-2i-4}\right)^{2048}$

$\left(\frac{5i}{-5}\right)^{2048}$

$(i)^{2048}\cdot(-1)^{2048} \Rightarrow 1\cdot1=1$

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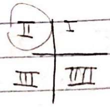
$z_K = \sqrt[n]{r} \cdot e^{\frac{\theta + 2\pi K}{n}}$ $\quad K = 0, \ldots, n-1$

$a = r \sin\theta$




$x+yi$

6 b $z^3 = -2+2i$

$z = \sqrt[3]{-2+2i}$

$r = \sqrt{(-2)^2 + 2^2} = \sqrt{8}$ $\qquad -2 = \sqrt{8} \cdot \cos\theta$

$2 = \sqrt{8} \cdot \cos\theta$

$z^3 = \sqrt{8}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

$\sqrt[n]{r(\cos\theta + i\sin\theta)} = \sqrt[n]{r}\left(\cos\left(\frac{\theta}{n} + \frac{2\pi K}{n}\right) + i\sin\left(\frac{\theta}{n} + \frac{2\pi K}{n}\right)\right)$

$z = \sqrt[3]{8}\left(\cos\left(\frac{\pi}{4} + \frac{2\pi K}{3}\right) + i\sin\left(\frac{\pi}{4} + \frac{2\pi K}{3}\right)\right)$

$= \sqrt{2}\left(\cos\left(\frac{3\pi + 8\pi K}{12}\right) + i\sin\left(\frac{3\pi + 8\pi K}{12}\right)\right)$

$K=0: \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right) = \boxed{1+i}$

$K=1: \sqrt{2}\left(\cos\left(\frac{11\pi}{12}\right) + i\sin\left(\frac{11\pi}{12}\right)\right) = \boxed{(-1-\sqrt{3}) + (\sqrt{3}-1)i}$

$K=2: \sqrt{2}\left(\cos\left(\frac{19\pi}{12}\right) + i\sin\left(\frac{19\pi}{12}\right)\right) = \boxed{(\sqrt{3}-1) + (-1-\sqrt{3})i}$

9 a $z = \left(\frac{1+i}{1-i}\right)^{19} \Rightarrow \frac{(1+i)(1+i)}{(1-i)(1+i)} \Rightarrow \frac{1+2i+i^2}{1-i^2} = \left(\frac{2i}{2}\right)^{19} = 1 \cdot i^{19} = i$ $\quad \boxed{\text{True}}$

b $z^5 = 2-2i \Rightarrow r = \sqrt{2^2 + (-2)^2} \Rightarrow r = \sqrt{8}$ $\quad n=5$

$\theta = -\frac{\pi}{4}$ $\quad z = 2^{\frac{3}{10}}\left(\cos\left(\frac{-\pi}{20} + \frac{2\pi K}{5}\right) + i\sin\left(-\frac{\pi}{20} + \frac{2\pi K}{5}\right)\right)$ $\quad = \theta$

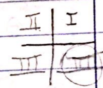
$K=0: 2^{\frac{3}{10}}\left(\cos\left(\frac{-\pi}{20}\right) + i\sin\left(\frac{-\pi}{20}\right)\right)$

$K=1: 2^{\frac{3}{10}}\left(\cos\left(\frac{7\pi}{20}\right) + i\sin\left(\frac{7\pi}{20}\right)\right)$

$K=2: 2^{\frac{3}{10}}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

$K=3: 2^{\frac{3}{10}}\left(\cos\left(\frac{23\pi}{20}\right) + i\sin\left(\frac{23\pi}{20}\right)\right)$

$K=4: 2^{\frac{3}{10}}\left(\cos\left(\frac{31\pi}{20}\right) + i\sin\left(\frac{31\pi}{20}\right)\right)$ $\quad \therefore \boxed{\text{True}}$




10 a given $u = \sqrt{3}(1+i)$, calculate $u^4$.

$u^4 = \sqrt{3}^4 (1+i)^4 = 9\left((1+i)^2\right)^2 = 9(1+i+i-1)^2 = 9(2i)^2 = 9 \cdot -4 = -36$

c 1 $z^5 - 1 = 0$

$r=1, \theta=0$ $\quad z = \cos\left(\frac{2\pi K}{5}\right) + i\sin\left(\frac{2\pi K}{5}\right)$

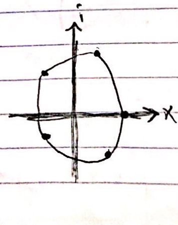
~~$K=0: \cos(0) + i\sin(0)$~~ $= 1$

$K=1: \cos\left(\frac{2\pi}{5}\right) + i\sin\left(\frac{2\pi}{5}\right)$

$K=2: \cos\left(\frac{4\pi}{5}\right) + i\sin\left(\frac{4\pi}{5}\right)$

$K=3: \cos\left(\frac{6\pi}{5}\right) + i\sin\left(\frac{6\pi}{5}\right)$

$K=4: \cos\left(\frac{8\pi}{5}\right) + i\sin\left(\frac{8\pi}{5}\right)$



$\widetilde{\text{pop}}$

(14) (b) When $\mathbb{C}^2$ is a vector space over $\mathbb{R}$, it has dimension 4.

Therefore any basis must have 4 vectors, the simplest being

$\{(1,0),(i,0),(0,1),(0,i)\}$

(c) $\{((1-i),(1+i)),(1+i,2-i)\}$ is linearly independent.

since $(1-i)i = 1+i$ but $(1+i)i = i-1 \neq 2-1$

$\therefore$ They are not multiples of each other

(16) a.b.c.

$$\left[\begin{array}{cc|c} c & (1+i) & c \\ (1-i) & -2c & 2-2c \end{array}\right] \xrightarrow[c\neq 0]{R_1 = \frac{R_1}{c}} \left[\begin{array}{cc|c} 1 & \frac{1+i}{c} & 1 \\ 1-i & -2c & 2-2c \end{array}\right] \xrightarrow{R_2 = R_2 - (1-i)R_1}$$

$$\left[\begin{array}{cc|c} 1 & \frac{1+i}{c} & 1 \\ 0 & (-2c - \frac{2}{c} & (-2c+1i) \end{array}\right] \xrightarrow[c \neq \pm i]{R_2 = \frac{R_2}{-2c-\frac{2}{c}}} \left[\begin{array}{cc|c} 1 & \frac{1+i}{c} & 1 \\ 0 & 1 & \frac{-2c+1+i}{-2c-\frac{2}{c}} \end{array}\right]$$

$c = \pm i$ : No solutions.

For all $c \neq \pm i$, there is a unique solution.

There are no values that have infinitely many solutions.

The equations are linearly independent.