Probability HW #6

① ⓐ let x = # of customers in 10 minutes $x \sim P(10 \cdot \frac{3}{10}) = P(3)$

$P(x=3) = \frac{e^{-3} 3^3}{3!} = \frac{e^{-3} 27}{6} = \frac{9}{2} e^{-3}$

ⓑ let X = # of customers in 5 minutes, Y = # in 3 min, Z = # in 2 min

$X \sim P(\frac{3}{2})$ $Y \sim P(\frac{9}{10})$ $Z \sim P(\frac{3}{5})$

$P(X=1) \cdot P(Y=1) \cdot P(Z=1) = e^{-\frac{3}{2}} (\frac{3}{2}) \cdot e^{-\frac{9}{10}} (\frac{9}{10}) \cdot e^{-\frac{3}{5}} (\frac{3}{5}) = 0.81 e^{-3}$

ⓒ let x = # of customers in 4 minutes $X \sim P(\frac{6}{5})$

$P(x \geq 3) = 1 - P(x=0) - P(X=1) - P(x=2) = 1 - e^{-\frac{6}{5}} - (\frac{6}{5}) e^{-\frac{6}{5}} - \frac{e^{-\frac{6}{5}} (\frac{6}{5})^2}{2!}$

$= 1 - 2.92 e^{-(\frac{6}{5})}$

⑤ let X_1 = # of calls from city A, $X_1 \sim P(5)$, X_2 = # from B, $X_2 \sim P(3)$

ⓐ $X = X_1 + X_2$, $X \sim P(8)$, $E[X] = V(X) = 8$ $P(X \geq 1) = 1 - P(X=0) = 1 - e^{-8}$

Define Events: A = all calls arrived in the first min B = 5 calls in 2 min

$$P(A|B) = \frac{P_1(X=5) \cdot P_1(X=0)}{P_2(X=5)} = \frac{\frac{e^{-8} 8^5}{5!} e^{-8}}{\frac{e^{-16} 16^5}{5!}} = \frac{8^5}{16^5} = \frac{1}{32}$$

ⓑ let X_1' = # from A in 2 min, $X_1' \sim P(10)$, X_2' = # from B in 2 min, $X_2 \sim P(6)$

$P(X_1' = 10) \cdot P(X_2 \geq 1) = \frac{e^{-10} 10^{10}}{10!} \cdot (1 - e^{-6}) = 0.1247$

ⓒ $X \sim B(10, 0.2)$ $P(X=3) = \binom{10}{3} (0.2)^3 (0.8)^7 = 0.2013$

⑪ ⓐ $X \sim H(100, 3, \frac{4}{100})$ $P(X \geq 1) = 1 - P(X=0) = 1 - \frac{\binom{4}{0}\binom{96}{3}}{\binom{100}{3}} = 0.1163$

$X \sim B(3, 0.04)$ $P(X \geq 1) = 1 - P(X=0) = 1 - \binom{3}{0} (0.04)^0 (0.96)^3 = 0.1153$

ⓑ $X \sim H(20{,}000, 5, \frac{8000}{20000})$ $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$$\frac{\binom{8000}{3}\binom{12000}{2}}{\binom{20000}{5}} + \frac{\binom{8000}{4}\binom{12000}{1}}{\binom{20000}{5}} + \frac{\binom{8000}{5}\binom{12000}{0}}{\binom{20000}{5}} = 0.3174$$

$X \sim B(5, 0.4)$ $P(X \geq 3) = P(X=3) + P(X=4) + P(X=5)$

$\binom{5}{3} (0.4)^3 (0.6)^2 + \binom{5}{4} (0.4)^4 (0.6) + \binom{5}{5} (0.4)^5 = 0.3174$

⑬ $X \sim B(10000, 0.0001)$ $X \sim P(10)$

ⓐ $P(X=8) = e^{-10} \cdot \frac{10^8}{8!} = 0.1126$

ⓑ $P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = e^{-10} + 10 e^{-10} + e^{-10} \cdot \frac{10^2}{2!} = 0.002769$

ⓒ $P(X \neq 6) =$

ⓓ $$P(X=2 | X \leq 3) = \frac{P(X=2)}{P(X \leq 3)} = \frac{e^{-10} \cdot \frac{10^2}{2!}}{(b) + \frac{e^{-10} 10^3}{3!}} = 0.2196$$

$x' = \#$ of stalls 2000 eggs, $x' \sim P(2)$

(16) $x = \#$ of stalls every 1000 eggs, $x \sim P(1)$,

(a) $P(x'=4) = e^{-2}\frac{2^4}{4!}$

(b) $P(x'=1) = 2e^{-2}$

(c) $P(x' \geq 2) = 1 - e^{-2} - 2e^{-2}$

(d) $P(x'=0) = e^{-2}$

(18) $x = \#$ of people recieved own letter

define $X_k = \begin{cases} 1 & \text{person } k \text{ recieved own letter} \quad p = \frac{1}{n} \\ 0 & \text{otherwise} \quad p = 1 - \frac{1}{n} \end{cases}$

$E[X_k] = \frac{1}{n}$ $\quad E(x) = n \cdot \frac{1}{n} = 1$

$x = x_1 + \dots + x_n$ $\quad E[x] = E[x_1] + \dots + E[x_n]$

(19) (a) define $X_k = \begin{cases} 1 & \text{student } k \text{ won } \underline{\text{no}} \text{ letteries} \quad p = \left(\frac{49}{50}\right) \\ 0 & \text{otherwise} \end{cases}$

$E[x_k] = \left(\frac{49}{50}\right)^{100}$ $\quad E[x] = 50 \cdot \left(\frac{49}{50}\right)^{100} = \frac{49^{100}}{50^{100}}$

(c) define $X_k = \begin{cases} 1 & \text{student } k \text{ won exactly 2 times} \quad p = \binom{100}{2}\left(\frac{49}{50}\right)^{98}\left(\frac{1}{50}\right)^2 \\ 0 & \text{otherwise} \end{cases}$

$E[X_k] = \frac{4950}{2500}\left(\frac{49}{50}\right)^{98} = \frac{99}{50}\left(\frac{49}{50}\right)^{98}$

$E[x] = 50 \cdot E[X_k] = 99\left(\frac{49}{50}\right)^{98}$

(e) Define $X_k = \begin{cases} 1 & \text{student } k \text{ won exactly 10 times} \quad p = \binom{100}{10}\left(\frac{49}{50}\right)^{90}\left(\frac{1}{50}\right)^{10} \\ 0 & \text{otherwise} \end{cases}$

$E[XK] = \binom{100}{10}\left(\frac{49}{50}\right)^{90}\left(\frac{1}{50}\right)^{10}$ $\quad E[x] = 50 \cdot E[X_k] = \binom{100}{10}\left(\frac{49}{50}\right)^{80}\left(\frac{1}{50}\right)^{9}$

(22) (b) $x \sim B(5 \cdot 10^5, 7 \cdot 10^{-5})$ $\quad x \sim P(35)$ $\quad x' = 2$ min $\quad x' \sim P(70)$

$P(x'=0) = e^{-70}$