Linear Algebra #2

① ⓑ ~~[illegible]~~ $T: \mathbb{R}^2 \to \mathbb{R}^2$ so that $T(x_1, x_2) = (2x_1 + |x_2|, -2x_2)$

False, counter example: $\underline{v}_1 = (2,-3)$ $\underline{v}_2 = (2,-4)$ $\underline{v}_1 + \underline{v}_2 = (4,-7)$

$T(\underline{v}_1 + \underline{v}_2) = 2\cdot 4 + |-7|, -2(-7) \Rightarrow 8+7-14 = 1$

ⓓ $T: \mathbb{R}^3 \to \mathbb{R}$ so that $T(x_1, x_2, x_3) = x_2$

True

$\underline{v}_1 = (x_1, x_2, x_3)$

$\underline{v}_2 = (y_1, y_2, y_3)$

add: $T(\underline{v}_1 + \underline{v}_2) = T(x_1 + y_1, x_2 + y_2, x_3 + y_3) = x_2 + y_2 \Rightarrow T(x_2 + y_2) = T(x_2) + T(y_2)$

multi: $T(c\underline{v}) = T(cx_1, cx_2, cx_3) \Rightarrow c(x_1, x_2, x_3) \Rightarrow cT(x_2) \Rightarrow cT(\underline{v})$ ✓

ⓕ $T: \mathbb{C}^2 \to \mathbb{C}^2$ so that $T(x_1, x_2) = (x_1 + ix_2, x_1 - 2ix_2)$ both vectors spaces over $\mathbb{C}$

$V_1 = (z_1, z_2)$ $V_2 = (w_1, w_2)$ $\left(\begin{array}{ll} z_1 = a_1 + bi_1 & z_2 = a_2 + bi_2 \\ w_1 = a_3 + bi & w_2 = a_4 + bi_4 \end{array} \right)$

add: $T(\underline{v}_1 + \underline{v}_2) = [(z_1 + w_1) + i(z_2, w_2), (z_1 + w_1 - 2i(z_2 + w_2))]$

$\Rightarrow [(z_1 + w_1 + iz_2 + iw_2), (z_1 + w_1 - 2iz_2 - 2iw_2)]$

$\Rightarrow (z_1 + iz_2, z_1 - 2iz_2) + (w_1 + iw_2, w_1 - 2iw_2)$

$\Rightarrow T(z_1 + iz_2, z_1 - 2iz_2) + T(w_1 + iw_2, w_1 - 2iw_2) \Rightarrow T(\underline{v}_1) + T(\underline{v}_2)$ ✓

mult: $T(c\underline{V}) = T(cx_1 + cix_2, cx_1 - 2cix_2)$

$\Rightarrow c(x_1 + ix_2, x_1 - 2ix_2) \Rightarrow cT(\underline{v})$ ✓

ⓗ $T: \mathbb{C} \to \mathbb{C}$ so that $T(z) = \bar{z}$ both vectors spaces over $\mathbb{C}$

False, counter exp: $z_1 = 2-8i$ $z_2 = 5-7i$

add: $T(z_1 + z_2) = \overline{2-8i + 5-7i} \Rightarrow \overline{2-8i} + \overline{5-7i}$

$\Rightarrow T(z_1) + T(z_2)$

mult: $\left.\begin{array}{l} T(cz) = \overline{cz} \\ cT(z) = c\cdot\bar{z} \end{array}\right\}$ ~~not equal~~ ← c cannot change to $\bar{c}$ since has imaginary

ⓘ $T: \mathbb{C} \to \mathbb{C}$ so that $T(z) = \bar{z}$ both vector spaces over $\mathbb{R}$

True, check conditions:

mult: $T(cz) = \overline{cz} \Rightarrow \bar{c}\cdot\bar{z} \Rightarrow c\cdot T(z)$ ✓ ← $c = \bar{c}$ since c is a real number

add: $T(\underline{v}_1 + \underline{v}_2)$ $z_1 = 2-8i$ $z_2 = 5-7i$

$= \overline{2-8i + 5-7i} \Rightarrow \overline{2-8i} + \overline{5-7i} \Rightarrow T(z_1) + T(z_2)$

ⓚ $T: \mathbb{R}^3 \to \mathbb{R}$ so that $T(x_1, x_2, x_3) = x_1 + x_2 + \pi x_3$ both vector spaces over $\mathbb{R}$

True, check conditions

add: $T(V_1+V_2) = T(x_1+y_1, x_2+y_2, x_3+y_3) =$

$x_1+y_1+x_2+y_2+\pi(x_3+y_3)$

$= x_1+y_1+x_2+y_2+\pi x_3+\pi y_3$

$(x_1+x_2+\pi x_3)+(y_1+y_2+\pi y_3) = T(V_1)+T(V_2)$ ✓

mult: $T(cV) = T(cx_1, cx_2, cx_3) \Rightarrow cx_1+cx_2+c\pi x_3$

$\Rightarrow c(x_1+x_2+\pi x_3) \Rightarrow cT(v)$ ✓

② ⓒ $T: c^0[-\pi,\pi] \to \mathbb{R}$ so that $T(f) = f(0)$

add: $T((f+g)(x)) \Rightarrow (f+g)\,0 \Rightarrow f(0)+g(0) \Rightarrow T(f)+T(g)$ ✓

mult: $T((cf)(x)) \Rightarrow cf(0) \Rightarrow cf(0) \Rightarrow cT(f)$

③ ⓐ $T: \mathbb{R}^2 \to \mathbb{R}^2$ so that $T(x_1,x_2) = (x_2,x_1)$

and line $L_1: \{(x,y) \mid x-y=0\}$ $L_2: \{(x,y) \mid x-3y=0\}$

L_1: image is $\{(x,y): x-y=0\}$

L_2: image is $\{(x,y): y+3x=0\}$

④ ⓑ False, counter exp: let T be the transformation $T(x_1,x_2) = x_1+x_2$

if we map: $\{(1,0),(10,1)\}$ using T

we get $\{T(1,0), T(10,1)\} = \{1,11\}$

$\{(1,0),(10,1)\}$ is linearly independent

but $\{1,11\}$ is dependant since 11 is a multiple of 1.

⑥ ⓐ let the cartesian representation of ImP be the following

$ax'+by'+cz' = K$

We must now find a,b,c for some constant K that satisfies this equation $(x',y',z') = T(x,y,z)$

By substituting equation 2 into 1, we obtain this equation.

$a(2x-11y-7z)+b(-5y-5z)+c(4x-5y+3z) = K$

now we can rearrange equation 3 into the following form.

$(2a+4c)x+(-11a-5b-5c)y+(-7a-5b+3c)z = K$

Next, we are able to choose any three triples (x,y,z) which lie on the plane P, which will when substituted into equation 4, give us a system of 3 equations with 3 unknowns. The points we shall use are $(0,0,1)$, $(1,2,1)$ and $(\frac{1}{2},0,0)$: giving the following 3 equations.

$$-7a-5b+3c=k$$

$$-27a-15b-11c=k$$

$$a+2c=k$$

by forming a matrix and reducing it to echelon form, we obtain the following.

$$\begin{bmatrix} 1 & 0 & 0 & 2k \\ 0 & 1 & 0 & -\frac{33}{10}k \\ 0 & 0 & 1 & -\frac{1}{2}k \end{bmatrix} \Longrightarrow \begin{cases} a = 2k \\ b = -\frac{33}{10}k \\ c = -\frac{1}{2}k \end{cases}$$

Finally, by selecting any arbitrary k, let us say $k=10$, we can have a cartesian representation for our transformed plane.

$$20x' - 33y' - 5z' = 10$$

(b) ~~given~~ ~~t~~

The point $(1,3,2) \in L$ transforms to $T(1,3,2) = (21,-25,-5)$.

The point $(1,5,3)$, also in L (when $t=1$), transforms into $T(1,5,3) = (36,-40,-12)$.

With these two points which we know to be on the image of L, we can determine the direction vector of $\operatorname{Im}L$ as the vector between the 2 pts.

That is, $(36-21, -40-25, -12-5) = (15,-65,-17)$.

which brings us to the parametric representation of $\operatorname{Im}L$.

$$\operatorname{Im}L = \{(21,-25,-5) + t(15,-65,-17) : t \in \mathbb{R}\}$$

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