Linear Algebra 2 HW#3

①ⓑ not a linear transformation.

ⓓ $\mathbb{R}^3 \to \mathbb{R}$ dimensions are 3×1

standard base $(0,1,0)$

since $(0,1,0) \times \begin{pmatrix} x \\ y \\ z \end{pmatrix} = y$

ⓖ not linear transformation

②ⓐ $T(a,b,c) + T(d,e,f) = (a-2b,\ 2a+b-c) + (d-2e,\ 2d+e-f)$

$(a-2b+d-2e,\ 2a+b-c+2d+e-f)$

$= T(a+d,\ b+e,\ c+f) = (a+d-2(b+e),\ 2(a+d)+b+e-(c+f))$

$= (a-2b+d-2e,\ 2a+b-c+2d+e-f)$

$kT(a,b,c) = T(ka,kb,kc)$

$k(a-2b,\ 2a+b-c) = (ka-2kb,\ 2ka+kb-kc)$

$(ka-2kb,\ 2ka+kb-kc) = (ka-2kb,\ 2ka+kb-kc)$

ⓑ $T(x,y,z) = (1,3)$

$x-2y=1$ $\quad (1,0,-1) \quad (0,-\frac{1}{2},-\frac{7}{2})$

$2x+y-z=3$ $\quad (1,0,-1)+t(1,\frac{1}{2},\frac{5}{2})$

ⓒ let $x=1,\ y=\frac{1}{2}$

$x-2y=0 \quad 2x+y-z=0$

$z=2x+y=\frac{5}{2}$

$(1,\frac{1}{2},\frac{5}{2})$ is a basis

ⓓ $(1,0,0),\ (0,0,1)$

basis $\{(1,2),(0,-1)\}$

④ⓐ No (basis spans $\mathbb{R}^2$ but its not invertible)

ⓑ homogenius matrix row reduced

$$\begin{bmatrix} 1 & 0 & \frac{5}{4} & \frac{7}{4} \\ 0 & 1 & \frac{7}{4} & \frac{9}{4} \end{bmatrix} \qquad x = -\frac{5}{4}x_3 - \frac{7}{4}x_4 \qquad y = -\frac{7}{4}x_3 - \frac{9}{4}x_4$$

$x_3=1,\ x_4=0 \qquad x_3=0,\ x_4=1$

Basis $\{(-\frac{5}{4},-\frac{7}{4},1,0),\ (-\frac{7}{4},-\frac{9}{4},0,1)\}$

ⓒ yes (basis span $\mathbb{R}^2$)

⑤ⓑ $$\begin{bmatrix} i & 2i \\ 2 & i \end{bmatrix}^{-1} = \frac{1}{-(2\cdot 2i)+i\cdot i}\begin{pmatrix} i & -2i \\ -2 & i \end{pmatrix} = \left(\frac{4}{17}+\frac{i}{17}\right)\begin{pmatrix} -1 & 2 \\ -2i & -1 \end{pmatrix}$$

(8) (a) $cT(a+bi) = T(c(a+bi))$

$T(a_1+b_1i) + T(a_2+b_2i) = T(a_1+b_1i + a_2+b_2i)$

$cT(a+bi) = c\begin{pmatrix} a & b \\ -b & a \end{pmatrix} = \begin{pmatrix} ca & cb \\ -cb & ca \end{pmatrix} = T(ca+cbi) = T(c(a+bi))$

$T(a_1+b_1i) + T(a_2+b_2i) = \begin{pmatrix} a_1 & b \\ -b_1 & a_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ -b_2 & a_2 \end{pmatrix} = \begin{pmatrix} (a_1+a_2) & (b_1+b_2) \\ -(b_1+b_2) & (a_1+a_2) \end{pmatrix}$

$= T((a_1+a_2) + (b_1+b_2)i = T(a_1+b_1i + a_2+b_2i)$

(b) $T(zw) = T(z)T(w)$ $\qquad z = a+bi,\ w = a_2+b_2i$

$T(z)T(w) = \begin{pmatrix} a_1 & b_1 \\ -b_1 & a_1 \end{pmatrix}\begin{pmatrix} a_2 & b_2 \\ -b_2 & a_2 \end{pmatrix} = \begin{pmatrix} a_1a_2-b_1b_2 & a_1b_2+b_1a_2 \\ -(a_1b_2+b_1a_2) & a_1a_2-b_1b_2 \end{pmatrix}$

$= T((a_1a_2-b_1b_2) + (a_1b_2+b_1a_2)i)$

$= T(a_1(a_2+b_2i) + b_1(a-b_2+a_2i)i$

$= T(a_1w + b_1(a_2i - b_2)) = T(a_1w + b_1i(a_2 - \frac{b_2}{i}))$

$= T(a_1w + b_1iw) = T((a_1+b_1i)w) = T(zw)$

(10) (b) $[T] = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\quad a+2b = 1,\ a-b = 2,\ a-7b = 8$ $\quad c+2d = 1,\ c-d = 1,\ c-7d = -5$

$a = \frac{5}{3},\ b = -\frac{1}{3}$ $\qquad c = -\frac{1}{3}\quad d = \frac{2}{3}$

$\frac{5}{3} + \frac{7}{3} \neq 8 \Rightarrow$ no linear transformation

(d) false because 2 constraints are not enough 4 are needed for a basis.

(b) $s,t \in \mathbb{R},\ T(2,2,1) + s + tT(1,4,0) = (0,3,1)$

Only true if $T(1,1,1) = T(1,4,0) = (0,0,0)$

$T(2,2,1) = (0,3,1)$

vectors are independent & form a basis of unique

linear transformation.

(11) (a) yes $\qquad$ (c) no

(b) yes $\qquad$ (d) 2

(13) (a) $a = -1$

(b) $\{ s(6,1,-2) + t(6,2,-1) : s,t \in \mathbb{R} \}$

(c) $\operatorname{Ker} T = 0$