Calc 2 H3

(L) (2) because $\lim_{\substack{(x,y)\to(0,0)\\(y=0)}} \frac{x^2-y^2}{x^2+y^2} = 1$, we know

$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{x^2+y^2} = 1$

False $\quad y=x: \lim_{(x,x)\to(0,0)} \frac{x^2-x^2}{x^2+y^2} = 0$

the limits are different.

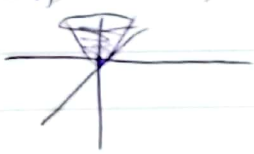
Therefore $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ DNE

(3) (a) F

(b) F

(c) T

(H) (a) $f(x,y) = \ln\sqrt{x^2+y^2}$



(G) (1) b (3) a (5) d

(2) c (4) e

(D) (a) iii (c) v (e) iv

(b) ii (d) i

(C) (1) F (3) c (5) b

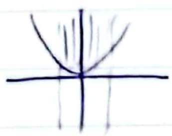
(2) e (4) a (6) d

(E) (2) $f(x,y) = \frac{\sqrt{y-x^2}}{1-x^2}$

$y-x^2 \geq 0$

$y \geq x^2$

$1-x^2 \neq 0 \rightarrow x \neq 1, -1$



(6) $f(x,y) = \ln(9-x^2-9y^2)$

$9-x^2-9y^2 > 0 \rightarrow \frac{x^2}{9} + \frac{y^2}{1} < 1$

(K) ③ $f(x,y) = \begin{cases} \dfrac{\sin(x+y)}{\sin(3x+3y)} & (x,y) \neq (0,0) \\ a & (x,y) = (0,0) \end{cases}$

along $y = x$: $\lim\limits_{(x,x)\to(0,0)} \dfrac{\sin(2x)}{\sin(6x)} = \dfrac{1}{3}$

continuous at $(0,0)$ when $a = \frac{1}{3}$

(J) ② $f(x,y) = \begin{cases} \dfrac{x^4 - y^2}{x^4 + y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

$y = mx^2$: $\lim\limits_{(x,mx^2)\to(0,0)} \dfrac{x^4 - m^2x^4}{x^4 + m^2x^4} = \dfrac{1-m^2}{1+m^2}$ ← $m = 1$ $L = 0$ ← $m = 0$ $L = 1$

this is not continuous when $(x,y) = (0,0)$

(I) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^3 + 2x^2 + xy^2 + 2y^2}{x^2 + y^2} = \lim\limits_{(x,y)\to(0,0)} \dfrac{(x^2+y^2)(x+2)}{x^2+y^2} = 2$