Probability HW#7

(3) (a)

K	P(X=K)
1	$0.2+0.25+0.05=0.5$
2	$0.3+0.05+0.15=0.5$

k	P(Y=k)
1	$0.2+0.3=0.5$
2	$0.25+0.05=0.3$
3	$0.05+0.15=0.2$

k	P(XY=K)
1	0.2
2	$0.3+0.25=0.55$
3	0.05
4	0.05
5	
6	0.15

(b) $E[X]=1\cdot0.5+2\cdot0.5=1.5$ $\quad E[Y]=1\cdot0.5+2\cdot0.3+3\cdot0.2=1.7$

$E[XY]=1\cdot0.2+2\cdot0.55+3\cdot0.05+4\cdot0.05+6\cdot0.15=0.2+1.1+0.15+0.2+0.9=2.55$

$Cov(X,Y)=E[XY]-E[X]E[Y]=2.55-(1.7)(1.5)=2.55-2.55=0$

(c) although uncorrelated since $Cov(X,Y)=0$, X, Y <u>are</u> dependent. proof: probability table is <u>not</u> a mult table

E.g $P(X=1)\cdot P(Y=1)=0.5\cdot0.5=0.25\neq0.2=P(X=1,Y=1)$

(4) (a)

X\Y	1	2	3	PX
2	0.15	0.09	0.06	0.3
3	0.15	0.09	0.06	0.3
4	0.2	0.12	0.08	0.4
PY	0.5	0.3	0.2	

(b) $E[X]=2\cdot0.3+3\cdot0.3+4\cdot0.4=0.6+0.9+1.6=3.1$ $\quad E[X^2]=4\cdot0.3+9\cdot0.3+16\cdot0.4$

$=1.2+2.7+6.4=10.3$ $\quad V(X)=E[X^2]-(E(X))^2=10.3-(3.1)^2=10.3-9.61=0.69$

$E[Y]=1\cdot0.5+2\cdot0.3+3\cdot0.2=0.5+0.6+0.6=1.7$ $\quad E[Y^2]=1\cdot0.5+4\cdot0.3+9\cdot0.2$

$=0.5+1.2+1.8=3.5$ $\quad V(Y)=E[Y^2]-E[Y]^2=3.5-(1.7)^2=3.5-2.89=0.61$

(c) since the prob. table is a mult. table X, Y are indep. Hence $cov(X,Y)=\rho(X,Y)=0$

(6) (a)

X_1\X_2	1	2	3	4	Px
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
2	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
3	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
4	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
P_{X^2}	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	

(b)

X\Y	1	2	3	4	PX
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
2	0	$\frac{2}{16}$	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
3	0	0	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{4}$
4	0	0	0	$\frac{4}{16}$	$\frac{1}{4}$
Py	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{5}{16}$	$\frac{7}{16}$	

(c) $P(Y=2\mid X=2)=\dfrac{P(Y=2,X=2)}{P(X=2)}=\dfrac{\frac{2}{16}}{\frac{4}{16}}=\frac{1}{2}$

⑦ ⓓ $E[X] = 1\cdot\frac{1}{4} + 2\cdot\frac{1}{4} + 3\cdot\frac{1}{4} + 4\cdot\frac{1}{4} = \frac{10}{4} = \frac{5}{2}$ $\quad E[X^2] = \frac{1}{4}(1+4+9+16) = \frac{30}{4} = \frac{15}{2}$

$V(X) = \frac{15}{2} - \left(\frac{5}{2}\right)^2 = \frac{30}{4} - \frac{25}{4} = \frac{5}{4}$

$E[Y] = 1\cdot\frac{1}{16} + 2\cdot\frac{3}{16} + 3\cdot\frac{5}{16} + 4\cdot\frac{7}{16} = \frac{1}{16}(1+6+15+28) = \frac{50}{16} = \frac{25}{8}$

$E[Y^2] = 1\cdot\frac{1}{16} + 4\cdot\frac{3}{16} + 9\cdot\frac{5}{16} + 16\cdot\frac{7}{16} = \frac{1}{16}(1+12+45+112) = \frac{170}{16} = \frac{85}{8}$

$V(Y) = \frac{85}{8} - \left(\frac{85}{8}\right)^2 = \frac{680-625}{64} = \frac{55}{64}$

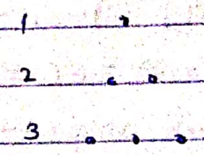
$E[XY] = \frac{1}{16}(1+2+3+4+6+8+12) + 4\cdot\frac{2}{16} + 9\cdot\frac{3}{16} + 16\cdot\frac{4}{16} = \frac{1}{16}(36+8+27+64) = \frac{135}{16}$

$cov(X,Y) = \frac{135}{16} - \frac{5}{2}\cdot\frac{25}{8} = \frac{135-125}{16} = \frac{5}{8}$ $\quad \rho(X,Y) = \frac{\frac{5}{8}}{\sqrt{\frac{5}{4}}\sqrt{\frac{55}{64}}} = \frac{2\sqrt{11}}{11} = 0.603$

– strong positive correlation

⑧ ⓐ

$X \backslash Y$	1	2	3	P_X
1	$\frac{1}{6}$	0	0	$\frac{1}{6}$
2	$\frac{1}{6}$	$\frac{1}{6}$	0	$\frac{2}{6}$
3	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{3}{6}$
P_Y	$\frac{3}{6}$	$\frac{2}{6}$	$\frac{1}{6}$	



ⓑ X, Y are clearly dependent because 0 exists. Hence, table is not a mult. table

E.g. $P(X=1)\cdot P(Y=1) = \frac{3}{6}\cdot\frac{1}{6} = \frac{1}{12} \neq \frac{1}{6} = P(X=1, Y=1)$

ⓒ $E[X] = 1\cdot\frac{1}{6} + 2\cdot\frac{2}{6} + 3\cdot\frac{3}{6} = \frac{14}{6} = \frac{7}{3}$ $\quad E[X^2] = 1\cdot\frac{1}{6} + 4\cdot\frac{2}{6} + 9\cdot\frac{3}{6} = \frac{36}{6} = 6$

$V(X) = 6 - \left(\frac{7}{3}\right)^2 = \frac{54-49}{9} = \frac{5}{9}$

$E[Y] = 1\cdot\frac{3}{6} + 2\cdot\frac{2}{6} + 3\cdot\frac{1}{6} = \frac{10}{6} = \frac{5}{3}$ $\quad E[Y^2] = 1\cdot\frac{3}{6} + 4\cdot\frac{2}{6} + 9\cdot\frac{1}{6} = \frac{20}{6} = \frac{10}{3}$

$V(Y) = \frac{10}{3} - \left(\frac{5}{3}\right)^2 = \frac{30-25}{9} = \frac{5}{9}$

$E[XY] = \frac{1}{6}(1+2+4+3+6+9) = \frac{25}{6}$ $\quad cov(X,Y) = \frac{25}{6} - \frac{7}{3}\cdot\frac{5}{3} = \frac{75-70}{18} = \frac{5}{18}$

$\rho(X,Y) = \frac{\frac{5}{18}}{\frac{5}{9}} = \frac{1}{2}$ – moderate positive correlation

⑪ ⓑ

$X_1 \backslash X_2$	2	3	P_{X_1}
2	0.4	0.3	0.7
3	0.3	0	0.3
P_{X_2}	0.7	0.3	

k	$P(X_1+X_2=k)$
4	0.4
5	0.3+0.3=0.6
6	0

k	$P(2X_1=k)$
4	0.7
6	0.3

k	$P(X_1^2=k)$
4	0.7
9	0.3

k	$P(X_1+X_2=k)$
4	0.4
6	0.6
9	0

$E[X_1 + X_2] = 4 \cdot 0.4 + 5 \cdot 0.6 = 4.6 \quad E[(X_1+X_2)^2] = 16 \cdot 0.4 + 25 \cdot 0.6 = 6.4 + 15 = 21.4$

$V(X_1 + X_2) = 21.4 - (4.6)^2 = 21.4 - 21.16 = 0.24$

$E[2X_1] = 4 \cdot 0.7 + 6 \cdot 0.3 = 4.6 \quad E[(2X_1)^2] = 16 \cdot 0.7 + 36 \cdot 0.3 = 11.2 + 10.8 = 22$

$V(2X_1) = 22 - (4.6)^2 = 22 - 21.16 = 0.84$

$E[X_1 \cdot X_2] = 4 \cdot 0.4 + 6 \cdot 0.6 = 5.2 \quad E[(X_1 \cdot X_2)^2] = 16 \cdot 0.4 + 36 \cdot 0.6 = 6.4 + 21.6 = 28$

$V(X_1 \cdot X_2) = 28 - (5.2)^2 = 28 - 27.04 = 0.96$

$E[X_1^2] = 4 \cdot 0.7 + 9 \cdot 0.3 = 5.5 \quad E[(X_1^2)^2] = 16 \cdot 0.7 + 81 \cdot 0.3 = 11.2 + 24.3 = 35.5$

$V(X_1^2) = 35.5 - (5.5)^2 = 35.5 - 30.25 = 5.25$