ODE HW#2

(2) $\frac{dy}{dx} = (y-2)(y-3)y = y^3 - 5y^2 + 6y$

$f_y = 3y^2 - 10y + 6$

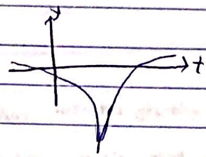
continuity throughout the plane for each rectangle that has the points $(0,1)$ $(6,4)$

therefore exist 1 solution

(6) (a) • $\frac{dy}{dt} = \frac{1}{t-2} \Rightarrow \int dy = \int \frac{dt}{t-2} \Rightarrow y = \ln|t-2| + c \Rightarrow 1 = \ln|9-2| + c$

$\Rightarrow y = \ln|t-2| + 1 - \ln 7$

• $t-2 \neq 0 \Rightarrow t \neq 2$ •



(b) • $\frac{dy}{dt} = \frac{t}{y} \Rightarrow \int y\,dy = \int t\,dt \Rightarrow \frac{1}{2}y^2 = \frac{1}{2}t^2 + c_1 \Rightarrow y^2 = t^2 + 2c_1$

$\Rightarrow y = \sqrt{|t^2 + c|} \Rightarrow y(1) = 3 \Rightarrow 3 = \sqrt{|c+1|} \Rightarrow 9 = c+1 \Rightarrow c = 8 \Rightarrow y = \sqrt{t^2+8}$

• domain: $t \in \mathbb{R}$

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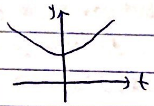
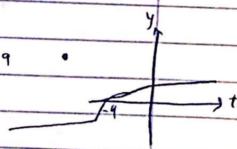


(c) • $\frac{dy}{dt} = \frac{1}{(y+2)^2} \Rightarrow \int (y+2)^2 dy = \int dt \Rightarrow \frac{1}{3}(y+2)^3 = t + c$

$y(0) = 1 \Rightarrow \frac{1}{3}(2+1)^3 = 0 + c \Rightarrow c = 9 \Rightarrow \frac{1}{3}(y+2)^3 = t + 9$

$\Rightarrow (y+2)^3 = 3t + 27 \Rightarrow y = \sqrt[3]{3t+27} - 2$

• $(y+2)^2 \neq 0 \Rightarrow (3t+27)^{\frac{2}{3}} \neq 0 \Rightarrow 3t + 27 \neq 0 \Rightarrow t \neq -9$ •



(7) $y \cdot \frac{dy}{dx} = x \Rightarrow \int y\,dy = \int x\,dx \Rightarrow \frac{1}{2}y^2 = \frac{1}{2}x^2 + c$

$y(0) = 0 \Rightarrow \frac{1}{2}(0)^2 = \frac{1}{2}(0)^2 + c \Rightarrow c = 0 \Rightarrow \frac{1}{2}y^2 = \frac{1}{2}x^2 \Rightarrow y^2 = x^2 \Rightarrow y = \pm x$

we allegedly got 2 solutions therefore contradicts the EUT, although the EUT isn't valid here since we would have to divide y by the original equation in order for it to be $y'(x) = f(x,y)$, then the function won't be continuous in any rectangle, therefore theres no contradiction in the fact that theres 2 solutions.

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⑧ $y^{\frac{2}{3}} \cdot \frac{dy}{dx} = x \Rightarrow \int y^{\frac{2}{3}} dy = \int x\, dx \Rightarrow \frac{3}{5} y^{\frac{5}{3}} = \frac{1}{2}x^2 + c$

$y(0) = 0 \Rightarrow \frac{3}{5}(0)^{\frac{5}{3}} = \frac{1}{2}(0)^2 + c \Rightarrow c = 0 \Rightarrow y^{\frac{5}{3}} = \frac{5}{6}x^2 \Rightarrow y = \left(\frac{5}{6}\right)^{\frac{3}{5}} \cdot x^{\frac{6}{5}}$

a solution is unique when it doesn't include whats not continous. The initial value is $x(0) = 0$ so it doesn't necessarily have to have many solutions, therefore its unique

⑩ $x^2 y' = y^2 \Rightarrow x^2 \cdot \frac{dy}{dx} = y^2 \Rightarrow \int \frac{1}{y^2} dy = \int \frac{1}{x^2} dx \Rightarrow -\frac{1}{y} = -\frac{1}{x} + c_1 = \frac{c_1 x - 1}{x}$

$\Rightarrow y = \frac{x}{1 - cx} \Rightarrow y = \frac{x}{1 + cx} \Rightarrow y(0) = 0 \Rightarrow 0 = \frac{0}{1}$

there are endless solutions

$x = 2 \quad y = 0$

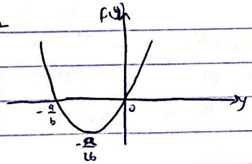
$\frac{2}{1+2c} = 0 \Rightarrow 0 = 2$ no solution to the equation. with the starting conditions. $y(2) = 0$

but we will notice that $y = 0$ is an equilibrium solution that meets the condition. we will move the equation to the next form → $y' = \frac{y^2}{x^2}$, we will get that every point that is not in the form of $(0, y_0)$, exists an open rectangle that contains the point with the function $\frac{y^2}{x^2}$ and is derived by y continuity as a function of 2 variables. we can therefore determine that for the starting conditions: $y(2) = 0$, $y(1) = 8$ exist a unique solution that maintains the equation and meets the conditions.

$x = 1 \quad y = 8 \quad : \quad \frac{1}{1+c} = 8 \Rightarrow \frac{1}{8} = 1 + c \Rightarrow c = -\frac{7}{8} \Rightarrow \boxed{y = \frac{x}{1 - \frac{7}{8}x}}$

The picture does not constitute a contradiction to the existing law and the units. because each solution is unique for different starting positions.

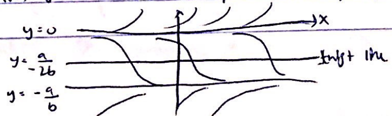
⑭ ⓐ $0 < a, b \quad \frac{dy}{dx} = ay + by^2$



f(y) is a positive parabula that its point on the y axis is $y = 0$ $y = -\frac{a}{b}$

the equilibrium solutions are point f(y) points on the y axis $\Rightarrow$ $y \equiv 0$

$y \equiv -\frac{a}{b}$



explanation: for $\forall$ every $y>0$, we will see that $f(y)$ and $f'(y)$ are positive, therefore $y''(x)>0 \Rightarrow y(x)$ convex

for every $-\frac{a}{b}<y<0$ $f(y)$ is negative, therefore $y(x)$ is decreasing.

for every $-\frac{a}{2b}<y<0$ $f'(y)f(y)$ are negative, therefore $y(x)$ concave.

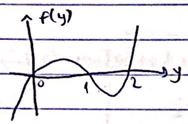
~~for every $-\frac{a}{b}<y<-\frac{a}{2b}$ $f'(y)f(y)$ are positive, therefore the fts convex.~~

For $y=-\frac{a}{2b}$ we will get a twist line

for $y<-\frac{a}{b}$ we will get a positive derivative therefore the function is increasing. the second derivative is negative and therefore the function is concave.

the solution $y\equiv 0$ isn't stable, $y\equiv -\frac{a}{b}$ isn't stable

(b) $\frac{dy}{dx}=y(y-1)(y-2) \Rightarrow$



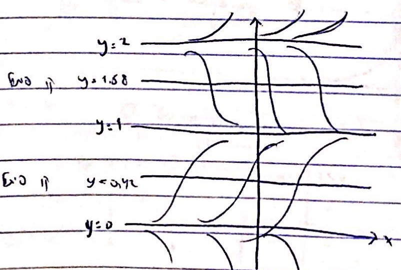
$f(y)$ is 0 for $y\equiv 0$, $y\equiv 1$, $y\equiv 2$ therefore these are the equilibrium solution.

extreme points: $f'(y)=1(y-1)(y-2)+y(1(y-2)+(y-1)1)$

$= y^2-3y+2+2y^2-3y = 3y^2-6y+2 \Rightarrow y_{1,2} = \frac{6\pm\sqrt{36-24}}{6} = 1\pm\sqrt{1-\frac{2}{3}} = 1.58,\ 0.42$

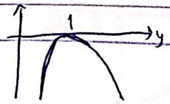
	y	$y<0$	$0<y<0.42$	$0.42<y<1$	$1<y<1.58$	$1.58<y<2$	$2<y$
incline $f(y)$	y'	−	+	+	−	−	+
concave/convex $f(y)f'(y)$	y''	−	+	−	+	−	+

$y(x)$ ⇓



$\Rightarrow$ the solution $y\equiv 0$ isn't stable,
the solution $y\equiv 1$ is stable.
solution $y\equiv 2$ isn't stable.

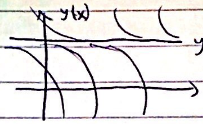
(c) $k>0$ $\frac{dy}{dx}=-k(y-1)^2 \Rightarrow$



$f(y)$ is 0 for $y=1$ therefore this is the only equilibrium solution

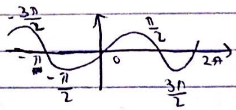
y	$y<1$	$1<y$	
$f(y)=y'$	$-$	$-$	incline
$f(y)f'(y)=y''$	$-$	$+$	convex/concave

$\Rightarrow$



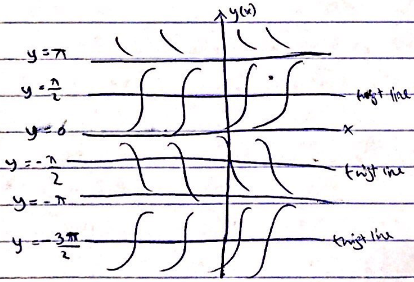
the solution is stable

(j) $\frac{dy}{dx} = \sin y \Rightarrow$



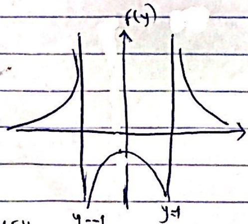
$f(y)$ is 0 for every function of the form $y=\pi k$, $k \in \mathbb{Z}$, therefore for every y in this for is a solution.

y	$y\in(-\pi,\frac{\pi}{2})+2\pi k$	$y\in(-\frac{\pi}{2},0)+2\pi k$	$y\in(0,\frac{\pi}{2})+2\pi k$	$y\in(\frac{\pi}{2},\pi)+2\pi k$	
$f(y)=y'$	$-$	$-$	$+$	$+$	incline
$f(y)\cdot f'(y)=y''$	$+$	$-$	$+$	$-$	concave/convex



all the solutions of the form $y=\pi+2\pi k$ are stable
and all the solutions of the form $y=2\pi k$ are not stable.

(k) $\frac{dy}{dx} = \frac{1}{y^2-1} = \frac{1}{(y-1)(y+1)} \Rightarrow$



$F(y)$ is never 0 so therefore theres no solution to the equation.

y	$y<-1$	$-1<y<0$	$0<y<1$	$1<y$
$f(y)=y'$	$+$	$-$	$-$	$+$
$f(y)f'(y)=y''$	$+$	$-$	$+$	$-$