Mathmatical Logic HW #1

1. (a) not a proposition at all

   (b) a simple proposition

   Formalize: $A \vee B$

   (c) a compound proposition

   Formalize: $A \to (B \wedge C)$

   (d) a simple proposition

   Formalize: $A \leftrightarrow B$

   (e) a compound proposition

   formalize: $A \to (B \wedge C)$

   (f) a simple proposition

   Formalize: $A \leftrightarrow B$

   (g) a simple proposition

   Formalize: $A \leftrightarrow B$

   (h) a simple proposition

   Formalize: $A \wedge B$

   (i) a simple proposition

   (j) not a proposition at all

2.

A	B	C	$\neg C$	$B \wedge \neg C$	$\overbrace{A \to (B \wedge \neg C)}^{p}$	$A \to \neg C$	$\neg B$	$\neg B \vee A$	$\overbrace{(\neg B \vee A) \to (A \to \neg C)}^{q}$
F	F	F	T	F	T	T	T	T	T
F	F	T	F	F	T	T	T	T	T
F	T	F	T	T	T	T	F	F	T
F	T	T	F	F	T	T	F	F	T
T	F	F	T	F	F	T	T	T	T
T	F	T	F	F	F	F	T	T	F
T	T	F	T	T	T	T	F	T	T
T	T	T	F	F	F	F	F	T	F

$p \leftrightarrow q$
T
T
T
T
F
T
T
T

Scanned with CamScanner

2 (a) $g(A)=T$, $g(B)=F$, $g(C)=T$

$$(A\to(B\wedge\neg C))\leftrightarrow((\neg B\vee A)\to(A\to\neg C))$$

$$T \quad F \quad \neg T \qquad \neg F \quad T \quad T \quad \neg T$$

$$(T\to F)\leftrightarrow(T\to F)$$

$$F\leftrightarrow F$$

$$T$$

(b) $g(A)=F$, $g(B)=F$, $g(C)=F$

$$(A\to(B\wedge\neg C))\leftrightarrow((\neg B\vee A)\to(A\to\neg C))$$

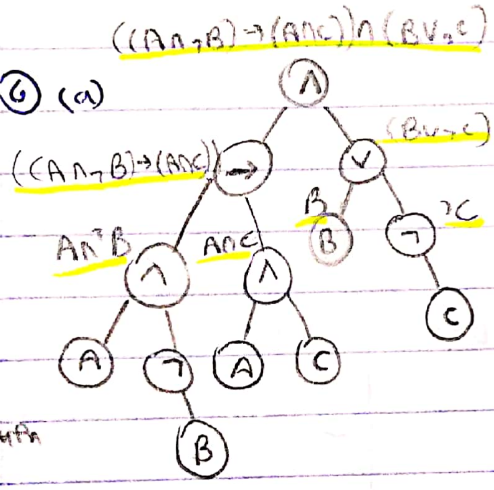
$$F \quad F \quad \neg F \qquad \neg F \quad F \quad F \quad \neg F$$

$$(F\to F)\leftrightarrow(T\to T)$$

$$T\leftrightarrow T$$

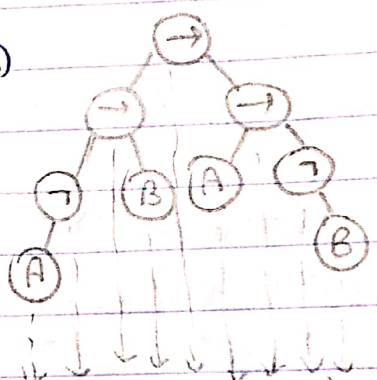
$$T$$

6 (a)



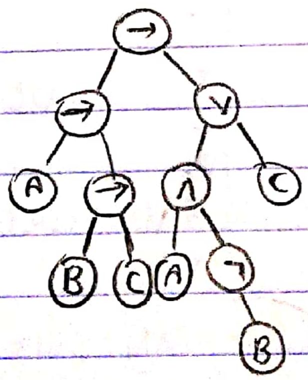
these line correspond with its truth table

(b)

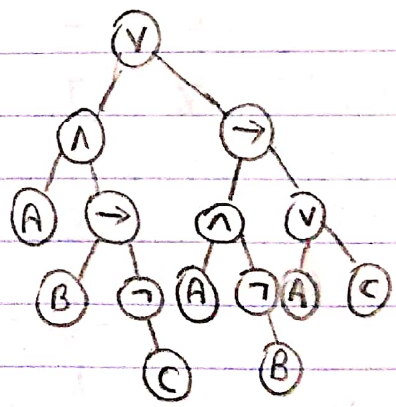


$(A\neg\to B)\to(A\to\neg B)$

(c)



(d)



* to build a tree, start by looking at which symbols is on the most out skirts, then work your way in through the sentence while going down the tree

Scanned with CamScanner

$\overbrace{((A\wedge\neg B)\to(A\wedge C))}^{\alpha}\wedge(B\vee\neg C)$

5 (a)

A B C	$\neg C$	$\underbrace{B\vee\neg C}_{q}$	$\neg B$	$A\wedge\neg B$	$A\wedge C$	$\underbrace{(A\wedge\neg B)\to(A\wedge C)}_{p}$	$p\wedge q$
F F F	T	T	T	F	F	T	T
F F T	F	F	T	F	F	T	F
F T F	T	T	F	F	F	T	T
F T T	F	T	F	F	F	T	T
T F F	T	T	T	T	F	F	F
T F T	F	F	T	T	T	T	F
T T F	T	T	F	F	F	T	T
T T T	F	T	F	F	T	T	T

$J(A)=J(B)=J(C)=F$ the truth value of α under J, is the value in the row of the truth table corresponding to J

(b) $(\neg A\to B)\to(A\to\neg B)$

A B	$\neg A$	$\neg B$	$\underbrace{\neg A\to B}_{p}$	$\underbrace{A\to\neg B}_{q}$	$p\to q$
F F	T	T	F	T	T
F T	T	F	T	T	T
T F	F	T	T	T	T
T T	F	F	T	F	F

(c) $(A\to(B\to C))\to((A\wedge\neg B)\vee C)$

A B C	$\neg B$	$A\wedge\neg B$	$\underbrace{(A\wedge\neg B)\vee C}_{q}$	$B\to C$	$\underbrace{A\to(B\to C)}_{p}$	$p\to q$
F F F	T	F	F	T	T	F
F F T	T	F	T	T	T	T
F T F	F	F	F	F	T	F
F T T	F	F	T	T	T	T
T F F	T	T	T	T	T	T
T F T	T	T	T	T	T	T
T T F	F	F	F	F	F	T
T T T	F	F	T	T	T	T

(d) $(A\wedge(B\to\neg C))\vee((A\wedge\neg B)\to(A\vee C))$

A B C	$\neg C$	$B\to\neg C$	$\underbrace{A\wedge(B\to\neg C)}_{p}$	$\neg B$	$A\wedge\neg B$	$A\vee C$	$\underbrace{(A\wedge\neg B)\to(A\vee C)}_{q}$	$p\vee q$
F F F	T	T	F	T	F	F	T	T
F F T	F	T	F	T	F	T	T	T
F T F	T	T	F	F	F	F	T	T
F T T	F	F	F	F	F	T	T	T
T F F	T	T	T	T	T	T	T	T
T F T	F	T	T	T	T	T	T	T
T T F	T	T	T	F	F	T	T	T
T T T	F	F	F	F	F	T	T	T

Scanned with CamScanner

7 (a) $J(A) = J(B) = J(C) = F$

a $((F \wedge \neg F) \rightarrow (F \wedge F)) \wedge (F \vee \neg F)$

$(F \rightarrow F) \wedge T$

T

b $(\neg F \rightarrow F) \rightarrow (F \rightarrow \neg F)$

$F \rightarrow T$

T

c $(F \rightarrow (F \rightarrow F)) \rightarrow ((F \wedge \neg F) \vee F)$

$(F \rightarrow T) \rightarrow (F \vee F)$

$T \rightarrow F$

F

d $(F \wedge (F \rightarrow \neg F)) \vee ((F \wedge \neg F) \rightarrow (F \vee F))$

$(F \wedge T) \vee (F \rightarrow F)$

$F \vee T$

T

(b)

	satisfies	doesn't satisfy
a	$J(A) = J(B) = J(C) = F$	$J(A) = J(B) = F$, $J(C) = T$
b	$J(A) = J(B) = F$	$J(A) = J(B) = T$
c	$J(A) = J(B) = F$, $J(C) = T$	$J(A) = J(B) = J(C) = F$
d	$J(A) = J(B) = J(C) = F$	no such interpretation exists

(by truth tables)

(c) $J(A) = T$, $J(B) = T$, $J(C) = F$ (Proven in truth tables)

(d) no, because for truth table (d) everything satisfies

Scanned with CamScanner

8 (a) Let N be a proposition.

Let L be a literal, P and Q be propositions, and @ be a binary logical operator

N can be made up of the following blocks:

N For any beginning of N, the number of left parentheses is larger than or equal to the number of right parentheses.

L True. Before the literal there are no left or right parentheses, and after there are none either.

¬P True, only if it is true for P, since there are no left or right parentheses outside of P.

P@Q True, only if it is true for both P and Q, since there are no parentheses outside of P and Q

(P) True, only if it is true for P, since the only left parentheses outside of P is the first character in N, and the only right one outside of P is the last character in N.

So since N is always true only if its sub propositions are true, and all sub propositions are made up of literals or other sub propositions, and it is always true for literals, therefore it is true for any proposition.

(b) Let N be a proposition. Let N_l be the number of left parentheses and N_r be the number of right parentheses in N.

Let L be a literal, P and Q be propositions, and @ be a binary logical operator.

N can be made up of the following blocks:

N	N_l	N_r
L	0	0
¬P	P_l	P_r
P@Q	P_l+Q_l	P_r+Q_r
(P)	P_l+1	P_r+1