Linear Algebra HW #15

① ⓐ False - if $z=-1$, $Re(z)$ is not >0.

ⓑ False - if $z=1$, $Im(z)$ equals 0.

ⓒ True, the n^{th} root of both sides leaves $x=\pm 1$ for real solutions if n is even.

ⓓ True, X can be 1 but not -1 if n is odd, because -1 to an odd power is -1.

② All solutions for roots are separated by an angle of $\frac{2\pi}{n}$

By De Moivre's theorym $z=\cos\left(\frac{2\pi k}{n}\right)+i\sin\left(\frac{2\pi k}{n}\right)$, for $k\in\mathbb{N}<n-1$

Since the angles are all good, connecting the segments leaves a regular polygon.

③ $x^m=1$ $\quad m=c\cdot n$

using De moivre's therum:

$z=\cos\left(\frac{2\pi k}{n}\right)+i\sin\left(\frac{2\pi k}{n}\right)$, $k\in 0,1,\ldots m-1$

$$\frac{2\pi k}{m}=\frac{2\pi k}{c\cdot n}=\frac{2\pi\left(\frac{k}{c}\right)}{n}$$

For all values where $\frac{k}{c}\in 0,1,\cdots,n-1,$

$k\in,0,1\ldots,m-1$

since m is $c\cdot n$.

Therefore if z is a root of order n, its also a root of order m.