Mathmatical Logic #2

① ⓑ $x \to x$ is a tautology, since $\to$ is only false when the left operand is true but the right operand is false. Since in $x \to x$ both operands are the same, it can not occur that the left operand be true and the right one be false.

Therefore $(x \to x) \to x = T \to x$. However, $T \to x$ is only true when x is true, so $(x \to x) \to x = x$. Therefore:

$$(((x \to x) \to x) \to x) \to x = (x \to x) \to x] = x$$

Which is neither a tautology or a contradiction

ⓓ as the truth table below shows, when A is false and B and C are true, the proposition is false, otherwise its true. Therefore this proposition is neither a tautology or a contradiction

A	B	C	$((A \vee \neg B) \to (A \vee (B \wedge C))) \to (C \to A)$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	T
T	T	T	T

② ⓐ Simply an atomic proposition. Its not a contradiction, even though we know that factually its incorrect.

ⓑ This is neither a tautology ~~...~~ nor a ~~...~~ contradiction, since although we know that in the world we live in we need clouds for there to be rain, regarding mathmatical logic, we can in theory assign a truth value to "Its rainy" and a false value to "its cloudy", thus giving our proposition a value of falsehood in that case.

ⓒ This is neither a tautology nor a contradiction. Let R be the atomic proposition "my name is Ron". The former proposition is equivalent to $R \to \neg R$. If R takes the value 'True', then $\neg R$ take the value 'False', in which case $R \to \neg R = T \to F = F$. Proving that the proposition in question is not a tautology. However, if R takes the value 'False', then $R \to \neg B = F \to T$, which evaluates to 'true', so the proposition is also not a contradiction.

③ ⓑ $(A \to B) \land (C \to B) \Leftrightarrow (A \lor C) \to B$

Proof: $(A \to B) \land (C \to B)$

$= (\neg(A \land \neg B)) \land (\neg(C \land \neg B))$

$= (\neg A \lor B) \land (\neg C \lor B)$

$= (\neg A \lor \neg C) \lor B$

$= \neg(A \lor C) \lor B$

$= \neg((A \lor C) \land \neg B)$

$= (A \lor C) \to B$

④ ⓑ Let α = "a is larger than b", β = "a is larger than 0", γ = "b is larger than 0"

δ = "b is equal to 0". We want to determine if

$(\alpha \land (\gamma \lor \delta)) \to \beta \Leftrightarrow (\alpha \land \neg\beta) \to (\neg\gamma \land \neg\delta)$

$(\alpha \land (\gamma \lor \delta)) \to \beta$

$= \neg(\alpha \land (\gamma \lor \delta) \land \neg\beta)$

$= \neg\alpha \lor \neg(\gamma \lor \delta) \lor \beta$

$= \neg\alpha \lor \beta \lor (\neg\gamma \land \neg\delta)$

$= \neg(\alpha \land \neg\beta) \lor (\neg\gamma \land \neg\delta)$

$= \neg((\alpha \land \neg\beta) \land \neg(\neg\gamma \land \neg\delta))$

$= (\alpha \land \neg\beta) \to (\neg\gamma \land \neg\delta)$

⑤ ⓑ $(X \lor Y) \land (X \lor \neg Y)$

$= X \lor (Y \land \neg Y)$

$= X \lor F$

$= X$

ⓓ $(X \land Y \land Z) \lor (X \land Y \land \neg Z) \lor (X \land \neg Y)$

$= X \land ((Y \land Z) \lor (Y \land \neg Z) \lor \neg Y)$

$= X \land ((Y \land (Z \lor \neg Z)) \lor \neg Y)$

$= X \land ((Y \land T) \lor \neg Y)$

$= X \land (Y \lor \neg Y)$

$= X \land T$

$= X$

⑥ ⓑ Let P be false, and R and S be true, $P \vee R$ is true because R is true, $P \to S$ is true because P is false. $R \to S$ is true because both R and S are true. This is an example where all 3 given proposition hold, yet P is false. Therefore the given implication don't hold.

ⓓ Let R be False, and P and S be true. $P \vee R$ is true because P is true, $R \to S$ is true because R is false. $P \to S$ is true because both P and S are true. This is an example where all 3 given propositions hold, yet $R \vee \neg S$ is false. Therefore the given implications do not hold.

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