

Probability HW #10

⑤ a) $x \sim N(75, 6^2)$ $x \sim N(75, 15.6^2)$
 $P(X < 55) = 0.1$ $P(Z < \frac{55-75}{6}) = 0.1$ $\Phi(\frac{-20}{6}) = 0.1$ $1 - \Phi(\frac{20}{6}) = 0.1$
 $\Phi(\frac{20}{6}) = 0.9$ $\frac{20}{6} = 1.282$ $6 = \frac{20}{1.282}$

$P(X > K) = 0.1$ $P(Z > \frac{K-75}{15.6}) = 0.1$ $1 - \Phi(\frac{K-75}{15.6}) = 0.1$ $\Phi(\frac{K-75}{15.6}) = 0.9$
 $\frac{K-75}{15.6} = 1.282$ $K = 1.282 \cdot 15.6 + 75 = 95$

⑥ $P(72 < x < 88) = \frac{12345}{n}$ $P(\frac{72-75}{15.6} < Z < \frac{88-75}{15.6}) = P(0.192 < Z < 0.83) = \frac{12345}{n}$
 $\Phi(0.83) - (1 - \Phi(0.192)) = (0.7967) - 1 + (0.5753) = 0.372 = \frac{12345}{n} \Rightarrow n = 33185.4$

⑦ a) $x \sim N(200, 15^2)$ $P(X > 224) = P(Z > \frac{224-200}{15}) = 1 - \Phi(1.6) = 0.0548$

b) $P(191 < X < 209) = P(\frac{191-200}{15} < Z < \frac{209-200}{15}) = \Phi(0.6) - \Phi(-0.6) = 2 \cdot (0.7257) \cdot 1$
 $= 0.4514$

c) $P(X > 230) = P(Z > \frac{30}{15}) = 1 - \Phi(2) = 1 - 0.9772 = 0.0228$

d) $P(X < K) = 0.25$ $P(Z < \frac{K-200}{15}) = 0.25$ $\Phi(\frac{K-200}{15}) = 0.25$ $\Phi(\frac{300-K}{15}) = 0.75$
 $\frac{300-K}{15} = 0.67$ $300-K = 10.05$ $K = 189.95$

⑨ $x \sim N(10, 2^2)$ $P(X < K) = 0.03$ $P(Z < \frac{K-10}{2}) = 0.03$ $\Phi(\frac{K-10}{2}) = 0.03$
 $\Phi(\frac{10-K}{2}) = 0.97$ $\frac{10-K}{2} = 1.88$ $K = 10 - 2 \cdot 1.88 = 6.24$ years

⑪ a) $x \sim N(1, 6^2)$ $P(1 - (1.3)6 < x < 1 + (1.3)6) = P(-1.3 < Z < 1.3) = \Phi(1.3) - \Phi(-1.3)$
 $2 \cdot (0.9032) - 1 = 0.8064$ prob. of x out of range: $1 - 0.8064 = 0.1936$

b) $P(1 - (0.52)6 < x < 1 + (0.52)6) = P(-0.52 < Z < 0.52) = \Phi(0.52) - \Phi(-0.52)$
 $2 \cdot (0.6985) - 1 = 0.397$

⑫ $x \sim N(160, 6^2)$
 $P(120 < X < 200) = 0.8$ $P(\frac{120-160}{6} < Z < \frac{200-160}{6}) = \Phi(\frac{40}{6}) - \Phi(-\frac{40}{6}) = 0.8$
 $2 \cdot \Phi(\frac{40}{6}) - 1 = 0.8$ $\Phi(\frac{40}{6}) = 0.9$ $\frac{40}{6} = 1.282$ $6 = \frac{40}{1.282} = 31.2$
 $P(X > 170) = P(Z > \frac{170-160}{31.2}) = 1 - \Phi(0.32) = 1 - (0.6255) = 0.3745$

⑭ a) $x \sim N(160, 6^2)$ $P(120 < X < 200) = P(\frac{120-160}{6} < Z < \frac{200-160}{6}) = 0.8$
 $2 \cdot \Phi(\frac{40}{6}) - 1$ $6 = 31.2$

$$1. P(X < 200) = P(Z < \frac{200 - 160}{31.2}) = \Phi(1.282) = 0.9$$

$$2. P(X > 120) = P(Z > \frac{120 - 160}{31.2}) = 1 - \Phi(-1.282) = 0.9$$

$$3. P(120 < X < 200 | X < 200) = \frac{P(120 < X < 200)}{P(X < 200)} = \frac{0.8}{0.9} = \frac{8}{9}$$

$$4. P(120 < X < 200 | X > 200) = \frac{0}{9} = 0$$

$$5. P(X > 170 | 120 < X < 200) = \frac{P(170 < X < 200)}{P(120 < X < 200)} = \frac{0.2745}{0.8} = 0.343$$

$$P(170 < X < 200) = P(\frac{170 - 160}{31.2} < Z < \frac{200 - 160}{31.2}) = \Phi(1.282) - \Phi(0.32) = 0.9 - 0.6255$$

$$① \text{ let } Y = X_1 - X_2 \quad Y \sim N(320.2; 31.2^2) \quad P(240 < Y < 400)$$

$$P(\frac{240 - 320.2}{31.2\sqrt{2}} < Z < \frac{400 - 320.2}{31.2\sqrt{2}}) = \Phi(1.81) - \Phi(-1.81) = 2 \cdot (0.9649) - 1 = 0.9298$$

$$② P(320 - 6 < Y < 320 + 6) = 0.8 \quad P(-\frac{6}{31.2\sqrt{2}} < Z < \frac{6}{31.2\sqrt{2}}) = 0.8 \quad 2 \cdot \Phi(\frac{6}{31.2\sqrt{2}}) - 1 = 0.8$$

$$\Phi(\frac{6}{31.2\sqrt{2}}) = 0.9 \quad \frac{6}{31.2\sqrt{2}} = 1.282 \quad c = 56.566$$

$$③ \text{ let } Y' = X_1 - X_2 \quad Y' \sim N(0.2; 31.2^2) \quad P(-40 < Y' < 40)$$

$$P(\frac{-40}{31.2\sqrt{2}} < Z < \frac{40}{31.2\sqrt{2}}) = 2 \cdot \Phi(0.9065) - 1 = 2 \cdot (0.8159) - 1 = 0.6318$$

$$④ \text{ a) } P(X < 20) = P(Z < \frac{20 - \mu}{\sigma}) = \frac{250}{400} \quad \Phi(\frac{20 - \mu}{\sigma}) = 0.625 \quad \frac{20 - \mu}{\sigma} = 0.32$$

$$P(X > 12) = P(Z > \frac{12 - \mu}{\sigma}) = \frac{380}{400} \quad 1 - \Phi(\frac{12 - \mu}{\sigma}) = 0.95 \quad \Phi(\frac{12 - \mu}{\sigma}) = 0.05$$

$$\Phi(\frac{\mu - 12}{\sigma}) = 0.95 \quad \frac{\mu - 12}{\sigma} = 1.645$$

$$[20 - \mu = 0.32\sigma] \quad [1 - 12 = 1.645\sigma] \Rightarrow \sigma = 1.9658 \quad \sigma = 4.071$$

$$\mu - 12 = 1.645(4.071) \quad \mu = 18.697$$

$$X \sim N(18.697, 4.071^2) \quad E[X] = 18.697 \quad V(X) = 16.57 \quad \sigma(X) = 4.071$$

$$\text{b) } P(X > 15) = P(Z > \frac{15 - 18.697}{4.071}) = 1 - \Phi(-0.908) = \Phi(0.908) = 0.8186$$

$$\text{c) } P(X < 18 | X > 15) = \frac{P(15 < X < 18)}{P(X > 15)} = \frac{0.2511}{0.8186} = 0.3067$$

$$P(15 < X < 18) = P(\frac{15 - 18.697}{4.071} < Z < \frac{18 - 18.697}{4.071}) = \Phi(-0.171) - \Phi(-0.908)$$

$$[1 - \Phi(0.171)] - [1 - \Phi(0.91)] = \Phi(0.91) - \Phi(0.171) = (0.8186) - (0.5675) = 0.2511$$