בס"ד

תרגיל גזרת 12

(1) $\lim\limits_{x\to 2} \frac{x^2+x-6}{x^2-4} = "\frac{0}{0}" \text{ (ל.ו.פ)} = \lim\limits_{x\to 2} \frac{2x+1}{2x} = \frac{5}{4}$

(2) $\lim\limits_{x\to 1} \frac{x^3-x^2-x+1}{x^3-2x^2+x} = \frac{1-1-1+1}{1-2+1} = "\frac{0}{0}" \text{ (ל.ו.פ)} = \lim\limits_{x\to 1} \frac{3x^2-2x-1}{3x^2-4x+1} =$

$\frac{3-3}{3-4+1} = "\frac{0}{0}" \text{ (ל.ו.פ)} \ \frac{6x-2}{6x-4} = \frac{6-2}{6-4} = 2$

(3) $\lim\limits_{x\to -1} \frac{\ln(2+x)}{x+1} = \frac{\ln(1)}{0} = "\frac{0}{0}" \text{ (ל.ו.פ)} = \lim\limits_{x\to -1} \frac{\frac{1}{2+x}}{1} = \frac{1}{1} = 1$

(4) $\lim\limits_{x\to 2} \frac{e^x-e^2}{x-2} = \frac{e^2-e^2}{2-2} = "\frac{0}{0}" \text{ (ל.ו.פ)} = \lim\limits_{x\to 2} \frac{e^x}{1} = e^2$

(5) $\lim\limits_{x\to 0} \frac{e^x+e^{-x}-x^2-2}{\sin^2 x-x^2} = \frac{1+1-0-2}{0} = "\frac{0}{0}" \text{ (ל.ו.פ)} = \lim\limits_{x\to 0} \frac{e^x-1e^{-x}-2x}{2\sin x\cos x-2x} =$

$\frac{1-1-0}{0} = "\frac{0}{0}" \text{ (ל.ו.פ)} = \lim\limits_{x\to 0} \frac{e^x+e^{-x}-2}{2(\sin x\cdot -\sin x+\cos x\cdot\cos x)-2} = \frac{1+1-2}{2(0+1)-2} =$

$= "\frac{0}{0}" \text{ (ל.ו.פ)} = \lim \frac{e^x-e^{-x}}{2(-2\sin x\cos x-2\cos x\sin x)} = \frac{1-1}{2(0+0+0)} = "\frac{0}{0}" \text{ (ל.ו.פ)}$

$= \frac{e^x+e^x}{-8(\sin x\sin x-\cos x\cos x)} = -\frac{2}{8} = -\frac{1}{4}$

(6) $\lim\limits_{x\to 0} \frac{8^x-2^x}{x} = \frac{8^0-2^0}{0} = \frac{1-1}{0} = "\frac{0}{0}" \text{ (ל.ו.פ)} = \lim\limits_{x\to 0} \frac{e^{\ln 8^x}-e^{\ln 2^x}}{x}$

$= \lim\limits_{x\to 0} \frac{e^{x\ln 8}-e^{x\ln 2}}{x} = \lim\limits_{x\to 0} \frac{e^{x\ln 8}\cdot\ln 8-e^{x\ln 2}\cdot\ln 2}{1} = \frac{1\cdot\ln 8-1\ln 2}{1}$

$= \ln 8-\ln 2$

(7) $\lim\limits_{x\to 0} \frac{2\tan^{-1}x-x}{2x-\tan^{-1}x} = "\frac{0}{0}" \text{ (ל.ו.פ)} = \frac{\frac{-2\tan^{-2}(x)-1}{\cos^2 x}}{\frac{2+1\tan^{-2}(x)}{\cos^2(x)}} = -\frac{1}{2}$

(8) $\lim\limits_{x\to +\infty} \frac{\ln x}{\sqrt{x}} = "\frac{\infty}{\infty}" \text{ (ל.ו.פ)} = \lim\limits_{x\to +\infty} \frac{\frac{1}{x}}{\frac{1}{2\sqrt{x}}} = "\frac{0}{0}" \text{ (ל.ו.פ)} =$

רצף

$$\lim_{x\to\infty^+} \frac{-\frac{1}{x^2}}{\frac{1}{2}\cdot -0.5\frac{1}{x^{\frac{3}{2}}}} = \frac{-\frac{1}{x^2}}{-(\frac{1}{2})^2\cdot\frac{1}{\sqrt{x^3}}} = \frac{\frac{1}{x^2}}{\lim\limits_{x\to\infty^+}\frac{1}{4}\cdot\frac{1}{\sqrt{x^3}}} = \lim_{x\to\infty^+}\frac{\frac{1}{x^2}}{\frac{1}{4\sqrt{x^3}}}$$

$$= \lim_{x\to\infty^+} \frac{1}{x^2}\cdot\frac{4\cdot\sqrt{x^3}}{1} = 4\cdot x^{-0.5} = \frac{4\cdot 1}{\sqrt{x}} = 0$$

(9) $\lim\limits_{x\to\infty^+} \frac{x^4-x^2}{e^x+1} = "\frac{\infty}{\infty}"$ לופי' $= \lim\limits_{x\to\infty^+}\frac{4x^3-2x}{e^x} = "\frac{\infty}{\infty}"$ לופי' $=$

$= \lim\limits_{x\to\infty^+}\frac{12x^2-2}{e^x} = "\frac{\infty}{\infty}"$ לופי' $= \frac{24x}{e^x} = "\frac{\infty}{\infty}"$ לופי' $= \lim\limits_{x\to\infty^+}\frac{24}{e^x} = \frac{24}{\infty} = 0$

(13) $\cancel{\lim\limits_{x\to\infty^+} x^2 e^{-x}}$

$\lim\limits_{x\to 0}(e^x+3x)^{\frac{1}{x}} = \lim\limits_{x\to 0} e^{\ln(e^x+3x)^{\frac{1}{x}}} = \lim\limits_{x\to 0} e^{\frac{1}{x}\ln(e^x+3x)}$

פתרון המעריך

$\lim\limits_{x\to 0}\frac{1}{x}\ln(e^x+3x) = \infty\cdot\ln(1+0) = "\infty\cdot 0" =$

$\lim\limits_{x\to 0}\frac{\ln(e^x+3x)}{x} = \frac{\ln 1}{0} = "\frac{0}{0}"$ לופי' $= \lim\limits_{x\to 0}\frac{\ln(e^x+3x)}{x} =$

$\lim\limits_{x\to 0}\frac{\frac{1}{e^x+3x}\cdot(e^x+3)}{1} = \cancel{\lim\limits_{x\to 0}\frac{\ln(e^x+3x)}{x}}\ \lim\limits_{x\to 0}\frac{e^x+3}{e^x+3x} = \frac{1+3}{1+0}$

$\lim\limits_{x\to 0} e^{\frac{1}{x}\ln(e^x+3x)} = \lim\limits_{x\to 0} e^4 = e^4$ סוף

(11) $\lim\limits_{x\to\infty^+} x^2 e^{-x} = \lim\limits_{x\to\infty^+}\frac{x^2}{\frac{1}{e^{-x}}} = \frac{\infty}{\frac{1}{0}} = "\frac{\infty}{\infty}"$ לופי' $= \lim\limits_{x\to\infty^+}\frac{2x}{-1(e^{-x})^2\cdot e^{-x}\cdot -1}$

$= \lim\limits_{x\to\infty^+}\frac{2x}{\frac{1\cdot e^{-x}}{(e^{-x})^2}} = "\frac{\infty}{\frac{0}{0}}" = \frac{(e^{-x})^2\cdot 2x}{e^{-x}} = e^{-x}\cdot 2x = \lim\limits_{x\to 1}\frac{2x}{\frac{1}{e^{-x}}} = "\frac{\infty}{\infty}"$ לופי'

$= \frac{2}{-1(e^{-x})^2}\cdot e^{-x}\cdot -1 = \frac{\frac{2}{e^{-x}}}{(e^{-x})^2} = \frac{(e^{-x})^2}{e^{-x}}\cdot 2$

$= \lim\limits_{x\to\infty^+} e^{-x}\cdot 2 = 0\cdot 2 = 0$

(14) $\lim_{x\to1} x^{\tan\frac{\pi x}{2}} = \lim_{x\to1} e^{\tan\frac{\pi}{2}x\ln x}$ // 6(c)

$\lim_{x\to1} \tan\frac{\pi}{2}x \ln x = "0\cdot\infty" = \lim_{x\to1}\frac{\ln x}{\frac{1}{\tan\frac{\pi}{2}x}} = "\frac{0}{0}"$ (L)

$\lim_{x\to1}\frac{\frac{1}{x}}{\frac{-1\tan^{-2}(\frac{\pi}{2}x)\cdot1}{\cos^2(\frac{\pi}{2}x)}\cdot\frac{\pi}{2}} = \lim_{x\to1}\frac{\frac{1}{x}}{\frac{-\frac{\pi}{2}}{\tan^2(\frac{\pi}{2}x)\cdot\cos^2(\frac{\pi}{2}x)}} = \frac{\frac{1}{x}}{\frac{-\frac{\pi}{2}}{\frac{\sin^2(\frac{\pi}{2}x)\cdot\cancel{\cos^2(\frac{\pi}{2})}}{\cancel{\cos^2(\frac{\pi}{2}x)}}}}$

$= \lim_{x\to1}\frac{\frac{1}{x}}{\frac{-\frac{\pi}{2}}{\sin^2(\frac{\pi}{2}x)}} = \lim_{x\to1}\frac{1}{-\frac{\pi}{2}} = \frac{-2}{\pi}$

$\lim_{x\to1} e^{-\frac{2}{\pi}} = e^{-\frac{2}{\pi}}$

(12) $\lim_{x\to2}\left(\frac{4}{x^2-4}-\frac{1}{x-2}\right) = \frac{4}{0}-\frac{1}{0} = "\infty-\infty"$

$\lim_{x\to2}\left(\frac{4}{(x+2)(x-2)}-\frac{1}{(x-2)}\right) = \lim_{x\to2}\frac{4-1(x+2)}{(x+2)(x-2)}$

$= \lim_{x\to2}\frac{4-x-2}{(x+2)(x-2)} = \lim_{x\to2}\frac{2-x}{(x+2)(x-2)} = \lim_{x\to2}\frac{-\cancel{(x-2)}}{(x+2)\cancel{(x-2)}} = -\frac{1}{4}$

(17) $\lim_{x\to0^+} x^x = \lim_{x\to0^+} e^{x\ln x}$ — $\lim_{x\to0^+} x\ln x = "0\cdot-\infty"$

$\lim_{x\to0^+}\frac{\ln x}{\frac{1}{x}} = \frac{-\infty}{\infty}$ (L) $= \lim_{x\to0^+}\frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to0^+}\frac{-x^2}{x} = \lim_{x\to0^+} -x = 0$

$\lim_{x\to0^+} e^0 = 1$

(16)

$$\lim_{x\to 0^+} \frac{\ln(\tan 2x)}{\ln(\tan 3x)} = \frac{\ln 0}{\ln 0} = \frac{-\infty}{-\infty} \text{ (cond)} =$$

$$\lim_{x\to 0^+} \frac{\frac{1}{\tan(2x)} \cdot \cos^2(2x)\cdot 2}{\frac{1}{\tan(3x)} \cdot \cos^2(3x)\cdot 3} = \frac{\frac{2\cos^2(2x)}{\tan(2x)}}{\frac{3\cos^2(3x)}{\tan(3x)}}$$

$$= \lim_{x\to 0^+} \frac{\frac{2\cos^2(2x)\sin(3x)}{\cos(3x)}}{\frac{\sin(2x)\cos^2(3x)\cdot 3}{\cos(2x)}} \text{ (cond)} =$$

$$\lim_{x\to 0^+} \frac{2\cos^3(2x)\sin(3x)}{3\cos^3(3x)\sin(2x)} = \lim_{x\to 0^+} \text{"}\frac{0}{0}\text{"} \text{ (cond)}$$

$$\lim_{x\to 0^+} \frac{2\cdot 3\cos^2(2x)\cdot -\sin(2x)\cdot 2 + 2\cos^3(2x)\cos(3x)\cdot 3}{-3\cdot 3\cos^2(3x)\sin(3x)\cdot 3 + 3\cos^3(3x)\cos(2x)\cdot 2}$$

$$\lim_{x\to 0^+} \frac{2\cdot 3\cdot 1\cdot 0 + 2\cdot 1\cdot 1\cdot 3}{-3\cdot 1\cdot 0\cdot 3 + 3\cdot 1\cdot 1\cdot 2} = \frac{6}{6} = 1$$

②

$f(x) = e^{-x^2-2x-1}$

$f'(x) = (e^{-x^2-2x-1}) \cdot (-2x-2)$

הפונקציה e^x תמיד חיובית ולכן נשאר לבדוק את
קיצון רק

$-2x-2=0 \Rightarrow -2x=2 \Rightarrow x=-1$

$x=-2 \; ; \; f'(-2)>0$

$x=0 \; ; \; f'(0)<0$

תחום עליה: $(-\infty,-1]$

תחום ירידה: $[1,\infty)$

ערך ה-a הוא: -1

תחום ההגדרה של f^{-1} הוא קבוצת הערכים של f:

$f(\infty,-1] = (\lim_{x\to-\infty} f(x), f(-1)] = (\lim_{x\to-\infty} e^{-x^2-2x-1}, e^{-(-1)^2-2(-1)-1}]$

תחום ההגדרה: הוא $\leftarrow$ $D(f^{-1}) = (0,1]$

$(f^{-1})'(\frac{1}{e}) = \frac{1}{f'(f^{-1}(b))} = \frac{1}{f'(a)}$

נמצא את a $\Leftarrow$ נציב את f ל- $\frac{1}{e}$

$e^{-x^2-2x-1} = \frac{1}{e} \Rightarrow e^{-x^2-2x-1} = e^{-1}$

נשווה מעריכים:

$-x^2-2x-1 = -1 \Rightarrow -x^2-2x=0 \Rightarrow x(x-2)$

$x=0 \qquad x=-2$

$x=0$ אינו בתחום $(-\infty,-1]$ נבחר את $x=-2$

$(f^{-1})'(\frac{1}{e}) = \frac{1}{f'(-2)} = \frac{1}{e^{-(-2)^2-2\cdot-2-1}(-2\cdot-2-2)} =$

$\frac{1}{2e^{-1}} = \frac{1}{\frac{2}{e}} = \frac{e}{2}$

$$F(x) = \ln^2 x - 4\ln x + 3$$

$$F'(x) = 2\ln x \cdot \frac{1}{x} - \frac{4}{x} = \frac{2\ln x - 4}{x}$$

נשווה לאפס:

$$\frac{2\ln x - 4}{x} = 0 \Rightarrow 2\ln x - 4 = 0 \Rightarrow 2\ln x = 4 \Rightarrow \ln x = 2 \Rightarrow e^{\ln(x)} = e^2$$

$$\boxed{x = e^2}$$

$x = 1$: -4 $f'(1) < 0$ תחום ירידה $(0, e^2]$

$x = e^3$: $\frac{2}{e^3}$ $f'(e^3) > 0$ תחום עליה $[e^2, \infty)$

תחום ההגדרה של f הוא $x > 0$, לכן, נבחר את התחום $(0, a]$ כך ש- $e^2 = a$.

התחום הוא $(0, e^2]$

נמצא את הטווח של f:

$$f[e^2, 0) = [\, f(e^2), \lim_{x \to 0^+} f(x) \,) =$$

$$[\ln^2 e^2 - 4\ln e^2 + 3, \lim_{x \to 0^+} \ln^2 x - 4\ln x + 3)$$

$$= [-1, \text{"}(-\infty)^2 - 4\cdot(-\infty) + 3\text{"}) = [-1, \infty)$$

תחום ההגדרה של f^{-1} הוא $D(f^{-1}) = [-1, \infty)$

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))} = \frac{1}{f'(a)}$$

נשווה את f ל-3:

$$\ln^2 x - 4\ln x + 3 = 3 \Rightarrow \ln^2 x - 4\ln x = 0 \Rightarrow \ln x(\ln x - 4) = 0$$

$x = 1$ $\qquad$ $x = e^4$

e^4 לא בתחום $(0, e^2]$ נבחר $x = 1$

$$(f^{-1})'(3) = \frac{1}{f'(1)} = \frac{1}{\frac{2\ln 1 - 4}{1}} = \boxed{-\frac{1}{4}}$$

4) 1) $f(x) = (x^2-4x+3)(x-1)$

$f'(x) = (2x-4)(x+1) + x^2 - 4x + 3 =$

$2x^2 + 2x - 4x - 4 + x^2 - 4x + 3 =$

$3x^2 - 6x - 1$

2) $f(x) = \sqrt[8]{(\cancel{8}x^4 - 4x^2)} = (x^4 - 4x^2)^{\frac{1}{8}}$

$f'(x) = \frac{1}{8}(x^4 - 4x^2)^{-\frac{7}{8}} \cdot 4x^3 - 8x =$

$\frac{1}{8} \cdot \sqrt[8]{(x^4-4x^2)^7} \cdot 4x^3 - 8x =$

$$\frac{4(x^3-2x)}{8\sqrt[8]{(x^4-4x^2)^7}} = \frac{(x^3-2x)}{2\sqrt[8]{(x^4-4x^2)^7}}$$

3) $f(x) = \left(\frac{x}{x^2+1}\right)^3 = \frac{x^3}{(x^2+1)^3} = \frac{3x^2(x^2+1)^3 - x^3\,3(x^2+1)^2 \cdot 2x}{((x+1)^3)^2}$

$$= \frac{3x^2(x^2+1)^3 - 6x^4(x^2+1)^2}{(x+1)^6} =$$

$$\frac{(x^2+1)^2\,(3x^2(x^2+1) - 6x^4)}{(x+1)^6}$$

4) $f(x) = \sqrt[4]{\left(\frac{2x-5}{5x+2}\right)} = \left(\frac{2x-5}{5x+2}\right)^{\frac{1}{4}}$

$f'(x) = \frac{1}{4}\left(\frac{2x-5}{5x+2}\right)^{-\frac{3}{4}} \cdot (2x - 5(5) + 2(5x+2)) =$

$$\frac{1}{4\sqrt[4]{\left(\frac{2x-5}{5x+2}\right)^3}} \cdot (2x - 25 + 10x + 4) = \frac{12x - 21}{4\sqrt[4]{\left(\frac{2x-5}{5x+2}\right)^3}}$$

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⑤ $f(x) = \left(x + \frac{1}{x}\right)\left(x^2 - \frac{1}{x^2}\right)$

$$f'(x) = \left(x + \frac{1}{x}\right) \cdot \left(2x + \frac{2x}{x^3}\right) - \left(1 - \frac{1}{x^2}\right)\left(x^2 - \frac{1}{x^2}\right)$$

$$= \left(\frac{x^2+1}{x}\right)\left(\frac{2x^4+2x}{x^3}\right) - \left(\frac{x^2-1}{x^2}\right)\left(\frac{x^4-1}{x^2}\right)$$

$$= \frac{(x^2+1)(2x^4+2x)}{x^4} - \frac{(x^2-1)(x^4-1)}{x^4}$$

$$= \frac{(x^2+1)(2x^4+2x) - (x^2-1)(x^4-1)}{x^4}$$

(5) (6) $F(x) = 4\cos x + 2\sin x$

$f'(x) = -4\sin x + 2\cos x = 2(-2\sin x + \cos x)$

(7) $f'(x) = 2\cdot 2\sin x\cdot\cos x = 2\cdot(\sin x\cdot\cos x) = 2\sin 2x$

(8) $F'(x) = 2 + \frac{\cos x\cdot x - \sin x}{x^2} = 2 + \frac{x\cos x - \sin x}{x^2}$

(9) $f'(x) = \frac{\cos x(x^2+\sin x) - ((2x-\cos x)\sin x)}{(x^2+\sin x)^2}$

$= \frac{x^2\cos x + \cos x\sin x - 2x\sin x + \cos x\sin x}{(x^2+\sin x)^2}$

$= \frac{x^2\cos x + 2\cos x\sin x - 2\sin x}{(x^2+\sin x)^2}$

$\left(= \frac{x^2\cos x + \sin 2x - \sin 2x}{(x^2+\sin x)^2}\right)$

(10) $f'(x) = 4\cos^3 x\cdot -\sin x - 4x = -4(\cos^3 x\sin x + x)$

(11) $F'(x) = \left(f'\ \frac{1}{\tan 6x}\right)' = \left((\tan 6x)^{-1}\right)' = -1\tan 6x^{-2}\cdot\frac{1}{\cos^2 6x}\cdot 6$

$= \frac{-6}{\cos^2 6x\cdot\tan^2 6x} = \frac{-6}{\cancel{\cos^2 6x}\cdot\frac{\sin^2 6x}{\cancel{\cos^2 6x}}} = \frac{-6}{\sin^2 6x}$

(12) $F'(x) = -\sin(x^2)\cdot 2x = -2x\sin(x^2)$

(13) $F'(x) = \frac{1}{\cos^2 x}\cdot\sec x + \tan x\cdot\tan x\sec x = \frac{\sec x}{\cos x} + \tan^2 x\sec x$

(14) $f'(x) = \dfrac{\cos x(1-\sin x) - (1+\sin x)(-\cos x)}{(1-\sin x)^2} =$

$\dfrac{\cos x - \cos x \sin x - (-\cos x - \sin x \cos x)}{(1-\sin x)^2}$

$\dfrac{\cos x - \cancel{\cos x \sin x} + \cos x + \cancel{\cos x \sin x}}{(1-\sin x)^2} = \dfrac{2\cos x}{(1-\sin x)^2}$

(15) $f(x) = \csc^2(x^2) = \dfrac{1}{\sin^2(2x)}$ $\quad f'(x) = \sin^{-2}(2x)$

$f'(x) = -2(\sin^{-3}(2x)) \cdot \cos(2x) \cdot 2 = \dfrac{-4\cos(2x)}{\sin^3(2x)}$

(16) $f'(x) = \dfrac{1}{2\sqrt{\cot x}} \cdot -\operatorname{cosec}^2(x) + 2 \cdot 2.7x =$

$\dfrac{-\operatorname{cosec}^2 x}{2\sqrt{\cot(x)}} + 2 \cdot 2.7x$

⑥

1) $f'(x) = 3(1+x)^2$

2) $f'(x) = \frac{1}{2}\left(x+\frac{1}{x}\right)^{-0.5} \cdot \left(1-\frac{1}{x^2}\right) =$

$$\frac{1 \cdot \left(1-\frac{1}{x^2}\right)}{2\sqrt{x+\frac{1}{x}}} = \frac{1-\frac{1}{x^2}}{2\sqrt{x+\frac{1}{x}}}$$

3) $f'(x) = 4(2x^2+x)^3 \cdot (4x+1)$

4) $(x^2+a^2)^{\frac{1}{2}} = \frac{1}{2}(x^2+a^2)^{-\frac{1}{2}} \cdot 2x = \frac{1 \cdot \cancel{2}x}{\cancel{2}\sqrt{x^2+a^2}} = \frac{x}{\sqrt{x^2+a^2}}$

5) $f(x) = -\frac{1}{\sqrt{x^2+4}} = -(x^2+4)^{-0.5}$

$f'(x) = 0.5(x^2+4)^{-1.5} \cdot 2x = \frac{1}{\cancel{2}\sqrt{(x^2+4)^3}} \cdot \cancel{2}x = \frac{x}{\sqrt{(x^2+4)^3}}$

6) $f'(x) = \frac{1}{\cos^2\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\cos^2\sqrt{x}}$

7) $f'(x) = 4\tan^3(x^3) \cdot \frac{1}{\cos^2(x^3)} \cdot 3x^2 = \frac{12x^2\tan^3(x^3)}{\cos^2(x^3)}$

8) $f'(x) = \frac{1}{2\sqrt{3x-\sin^2(4x)}} \cdot 3 - 2\sin(4x) \cdot \cos(4x) \cdot 4$

$$= \frac{3+8\sin(4x)\cos(4x)}{2\sqrt{3x-\sin^2(4x)}}$$

9) $f'(x) = 12\left(1+\sin^3(x^5)\right)^{11} \cdot 3\sin^2(x^5) \cdot \cos(x^5) \cdot 5x^4$

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