Mathmatical Logic HW # 8

① ⓑ $\neg(\exists x(P(x)\wedge Q(x)) \rightarrow \forall x(P(x)\rightarrow Q(x)))$

$\neg(\neg\exists x(P(x)\wedge Q(x)) \vee \forall x(P(x)\rightarrow Q(x)))$

$\exists x(P(x)\wedge Q(x)) \wedge \neg\forall x(P(x)\rightarrow Q(x))$

$\exists x(P(x)\wedge Q(x)) \wedge \exists x(\neg P(x)\rightarrow Q(x)))$

$\exists x(P(x)\wedge Q(x)) \wedge \exists x(P(x)\wedge \neg Q(x))$

② ⓑ $\forall x(P(x)\vee Q(x)), \forall x(\neg P(x)) \vdash \exists x(Q(x))$

1	$\forall x(P(x)\vee Q(x))$	P
2	$\forall x(\neg P(x))$	P
3	$P(y)\vee Q(y)$	R5, 1
4	$\neg P(y)$	R5, 2
5	$Q(y)$	I6, 3, 4
6	$\exists x Q(x)$	R7

ⓓ $\neg\forall x(P(x)\wedge Q(x)), \forall x(P(x)) \vdash \neg\forall x(Q(x))$

1	$\neg\forall x(P(x)\wedge Q(x))$	P
2	$\forall x(P(x))$	P
3	$\exists x(\neg(P(x)\wedge Q(x)))$	E24, 1
4	$\neg(P(y)\wedge Q(y))$	R6
5	$\neg P(y)\vee\neg Q(y)$	E17, 4
6	$P(y)$	R5, 2
7	$\neg Q(y)$	I6, 5, 6
8	$\exists x(\neg Q(x))$	R7, 7
9	$\neg\forall x(Q(x))$	E24, 8

ⓕ $\forall x(P(x)\rightarrow(Q(y)\wedge R(x))), \exists x P(x) \vdash Q(y)\wedge\exists x(P(x)\wedge R(x))$

1	$\forall x(P(x)\rightarrow(Q(y)\wedge R(x)))$	P
2	$\exists x P(x)$	P
3	$P(z)\rightarrow(Q(y)\wedge R(z))$	R5, 1
4	$P(z)$	R6, 2
5	$(Q(y)\wedge R(z))$	I11, 3, 4
6	$R(z)$	I4, 5
7	$Q(y)$	I3, 5
8	$P(z)\wedge R(z)$	I5, 4, 6
9	$\exists x(P(x)\wedge R(x))$	R7, 8
10	$Q(y)\wedge\exists x(P(x)\wedge R(x))$	I5, 7, 9

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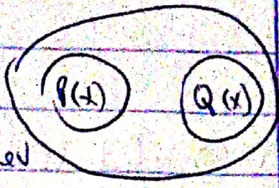
③ ⓑ $\{\exists x P(x), \exists x Q(x)\} \Rightarrow \exists x (P(x) \wedge Q(x))$



False, counter example - $P(x)$ = x studies at Machon Lev
universe - students $Q(x)$ = x studies at Machon Tal

There exists a student who studies at Machon Lev ($\exists x P(x)$)

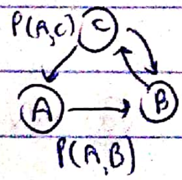
There exists a student who studies at Machon Tal ($\exists x Q(x)$)

it is not true that there exists a student at Machon Lev and Machon Tal
($\exists x (P(x) \wedge Q(x))$ is not true.

ⓓ $\forall x (P(x) \rightarrow Q(x)) \Rightarrow (\forall x P(x) \rightarrow \forall x Q(x))$

1	$\forall x (P(x) \rightarrow Q(x))$	P
2	$\forall x (P(x)) \rightarrow \forall x (Q(x))$	I_{20},1

④ $\forall x \exists y P(x,y) \Rightarrow \exists y \forall x P(x,y)$



P(B,C)
P(C,B)
so one in this universe knows A.

False. universe - people $P(x,y)$ = x knows y

$\forall x \exists y P(x,y)$ = Every person knows at least one person

$\exists x \forall x P(x,y)$ = There is someone who is known by everyone

it is not true that if every person knows someone, then everyone knows someone.

The proof is incorrect since in step (4), the universal quantifier is introduced but z,w are free in a step obtained by R6.