(part of) 2 • prove that $\sqrt{2}+\sqrt{3}$ is irrational

Proof: by contradiction

Assume $(\sqrt{2}+\sqrt{3})^2 = \left(\frac{p}{q}\right)^2$ which is a fraction in reduced form

that it is a rational #

$$5+2\sqrt{6} = \frac{p^2}{q^2}$$

$$\sqrt{6} = \frac{\frac{p^2}{q^2}-5}{2} = \frac{p^2}{2q^2} - \frac{5}{2} = \frac{p^2-5q^2}{2q^2}$$

p^2-5q^2 ← integer

$2q^2$ ← natural #

contradiction: $\sqrt{6}$ is irrational

• Prove that $\sqrt{6}$ is irrational

Proof: by contradiction

Assume $\sqrt{6} = \frac{p}{q}$ fraction in reduced form

that it is a rational #

$$6 = \frac{p^2}{q^2}$$

$$6q^2 = p^2$$

p^2 is a multiple of 3

$$2[3\cdot q^2] = p^2$$

p is a multiple of 3

p^2 is even

p is even

lets call $p = 2m$

$$6q^2 = (2m)^2$$

$$6q^2 = 4m^2$$

$$3q^2 = 2m^2$$

$3q^2$ is even

q^2 is even

q is even

n, Another multiple of 2s ... form.

contradiction cuz p & q have common term

1 2

x - rational y - irrational

Proof: assume that $x+y$ is rational

since x is rational, $-x$ is also rational, thus the sum of $x+y$ and $-x$ must be a rational number (since the sum of 2 rational numbers is rational).

Thus $(x+y)+(-x)=y$ is rational, this contradicts our original hypothesis that x is irrational.

Therefore is must not be true that $x+y$ is rational, we have shown that $x+y$ must be irrational.

a) the sum of any 2 rational numbers is rational.

Proof: take 2 rational numbers

$\frac{p}{q}$ and $\frac{m}{n}$ $\quad p, m \in \mathbb{Z}$, $q, n \in \mathbb{N}$

their sum is $\frac{p+m}{q+n}$ $\quad p+m \in \mathbb{Z}$, $q+n \in \mathbb{N}$

approve

2 • $\sqrt{3}$

notebook:

- $\sqrt{2} \cdot \sqrt{3}$
- $\sqrt{6}$

Prove that $\sqrt{3}$ is irrational

Proof by contradiction:

Assume $\sqrt{3} = \frac{p}{q}$ (a rational number) fraction is reduced form

$$3 = \frac{p^2}{q^2}$$

$$3q^2 = p^2$$

p^2 is a multiple of 3, therefore $p = 3m$

$$3q^2 = (3m)^2$$

$$3q^2 = 9m^2$$

$3:$
$$q^2 = 3m^2$$

q^2 is a multiple of 3, therefore $q = 3n$

contradiction: p and q have common term

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• $1+\sqrt{2}$

Proof: by contradiction

assume $1+\sqrt{2}$ is a rational number in reduced form

$1+\sqrt{2} = \frac{p}{q}$

$(1+\sqrt{2})^2$

$(1+\sqrt{2})(1+\sqrt{2})$

$1+2\sqrt{2}+2$

$3+2\sqrt{2} = \frac{p^2}{q^2} \Rightarrow \sqrt{2} = \frac{p^2}{2q^2} - \frac{3}{2}$

~~$3q^2 = p^2$ p^2 is a multiple of 3 $p = 3m$~~

~~$3q^2 = (3m)^2$~~

~~$3q^2 = 9m^2$~~ $/ \div 3$

~~$q^2 = 3m^2$~~ ~~q^2 is a multiple of 3~~

~~contradiction!~~

contradiction: $\sqrt{2}$ is irrational

Proof by contradiction:

Assume $\sqrt{2}$ is a rational number in reduced form

$\sqrt{2} = \frac{p}{q}$

$2 = \frac{p^2}{q^2}$

$2q^2 = p^2$

p^2 is a multiple of 2 $p = 2m$

$2q^2 = (2m)^2$

$2q^2 = 4m^2$ $/ :2$

$q^2 = 2m^2$

q^2 is a multiple of 2

contradiction: p and q have common terms

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③ ⓓ $\lim_{n\to\infty} \frac{1}{\sqrt{2n^2-5}} = 0$

assume $\left| \frac{1}{\sqrt{2n^2-5}} - 0 \right| < \varepsilon$

$\frac{1}{\sqrt{2n^2-5}} < \varepsilon$

$1 < \varepsilon\left(\sqrt{2n^2-5}\right)$

$\frac{1}{\varepsilon} < \sqrt{2n^2-5}$

$\frac{1}{\varepsilon^2} < 2n^2-5$

$\frac{1}{\varepsilon^2} + 5 < 2n^2$

$\frac{1}{2\varepsilon^2} + 2.5 < n^2$

$\sqrt{\frac{1}{2\varepsilon^2} + 2.5} < n$

$= n_0$

ⓔ $\left| \frac{2n+1}{3n+2} - \frac{2}{3} \right| < \varepsilon$

$\frac{3(2n+1) - 2(3n+2)}{3(3n+2)}$

$\frac{6n+3-6n-4}{9n+6} < \varepsilon$

$\left| \frac{-1}{9n+6} \right| < \varepsilon$

$\frac{1}{9n+6} < \varepsilon$

$9n+6 > \frac{1}{\varepsilon}$

$9n > \frac{1}{\varepsilon} - 6$

$n > \boxed{\frac{1-6\varepsilon}{9\varepsilon}}$

$= n_0$

⑥ ⓐ $a_n = (-1)^n$

if $L = 0$ or -1 : the sequence is equal to 1 infinitely many times & 1 is not ε away from the limit.

if $L = 1$: the sequence will be equal to -1 which is not close to the limit.

Method 1:

⑪ ⓐ $\lim_{n\to\infty} \frac{1+2+3+\dots+n}{n^2} = \lim_{n\to\infty} \frac{\frac{n(n+1)}{2}}{n^2} = \frac{n(n+1)}{2n^2} = \frac{n+1}{2n} = \frac{n}{2n} = \frac{1}{2}$

Method 2: because there's infinitely many limits - can't break it up - won't get the right answer

③ ⓖ $\lim_{n\to\infty} 3^{-n} = 0$

$|3^{-n} - 0| < \varepsilon$

$|3^{-n}| < \varepsilon$

$\left|\frac{1}{3^n}\right| < \varepsilon$

$1 < \varepsilon 3^n$

$\frac{1}{\varepsilon} < 3^n$

$\log_3 \frac{1}{\varepsilon} < \log_3 3^n$

$\log_3 \frac{1}{\varepsilon} < n$
$\quad\; \parallel$
$\quad n_0$

ⓗ $\lim_{n\to\infty} \frac{2}{4^{-n}+1} = 2$

$\left|\frac{2}{4^{-n}+1} - 2\right| < \varepsilon$

$\left|\frac{2 - 2(4^{-n}+1)}{4^{-n}+1}\right| < \varepsilon$

$\left|\frac{2 - \frac{2}{4^n} - 2}{4^{-n}+1}\right| < \varepsilon$

$\left|\frac{-2}{4^n(4^{-n}+1)}\right| < \varepsilon$

$\left|\frac{-2}{4^n+1}\right| < \varepsilon$

$\frac{2}{4^n+1} < \varepsilon$

$\frac{2}{\varepsilon} < 4^n + 1$

$\log_4 \frac{2}{\varepsilon} - 1 < \log_4 4^n$

$\log_4 \frac{2}{\varepsilon} - 1 < n$
$\qquad\quad \parallel$
$\qquad\quad n_0$

one side of n needs to be positive therefore will take off the minus

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④ ⓐ $n > N, \quad \left| \frac{2n+1}{3n+2} - \frac{2}{3} \right| < \frac{1}{1000}$

$$\left| \frac{3(2n+1) - 2(3n+2)}{3(3n+2)} \right| < \frac{1}{1000}$$

$$\left| \frac{6n+3-6n-4}{9n+6} \right| < \frac{1}{1000}$$

$$\left| \frac{-1}{9n+6} \right| < \frac{1}{1000}$$

$\frac{1}{9n+6} < \frac{1}{1000}$ ← n is positive

$1 < 9n \cdot 10^{-3} + 6 \cdot 10^{-3}$ $/ \cdot 9n+6$

$1 - 6 \cdot 10^{-3} < 9n \cdot 10^{-3}$ $/ -6 \cdot 10^{-3}$

$\frac{1 - 6 \cdot 10^{-3}}{9 \cdot 10^{-3}} < n \; / \; 9 \cdot 10^{-3}$

$110.44 < n$ →

$n = 115$:

$$\left| \frac{2 \cdot 115 + 1}{3 \cdot 115 + 2} - \frac{2}{3} \right| < \frac{1}{1000}$$

$0.96 \cdot 10^{-3} < \frac{1}{1000}$

$n = 150$:

$$\left| \frac{2 \cdot 150 + 1}{3 \cdot 150 + 2} - \frac{2}{3} \right| < \frac{1}{1000}$$

$0.737 \cdot 10^{-3} < \frac{1}{1000}$

⑦ ⓑ $\lim_{n \to \infty} 2^n = +\infty$

Proof: assume $2^n > B$

$\log_2 2^n > \log_2 B$

$n > \log_2 B$
$\quad \| $
$\quad n_0$

ⓓ $\lim_{n \to \infty} \log_2 n = +\infty$

$\log_2 n > B$

$2^{\log_2 n} > 2^B$

$n > 2^B$
$\quad \|$
$\quad n_0$

9 b

1 $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = +\infty$

$a_n = \frac{1}{n}$

$b_n = \frac{1}{n^2}$

2 $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = 4$

$a_n = \frac{4}{n}$

$b_n = \frac{1}{n}$

3 $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = 0$

$a_n = \frac{1}{n}$, $\frac{1}{n^2}$

$b_n = 0.5^n$, $\frac{1}{n}$

4 $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = -\infty$

$a_n = -\frac{1}{n}$

$b_n = \frac{1}{n^2}$

5 $\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right)$ does not exist

$a_n = \frac{\sin n}{n}$

$b_n = \frac{1}{n}$

(10)

(c) $$\lim_{n\to\infty} \frac{5n^2+n+3}{10000-2n^2} = \frac{\frac{5n^2}{n^2}+\frac{n}{n^2}+\frac{3}{n^2}}{\frac{-2n^2}{n^2}+\frac{10000}{n^2}} = \frac{\overbrace{5}^{\text{limit}=5}+\overbrace{\frac{1}{n}}^{\text{limit}=0}+\overbrace{\frac{3}{n^2}}^{\text{limit}=0}}{\underbrace{-2}_{\text{limit}=-2}+\underbrace{\frac{10000}{n^2}}_{\text{limit}=0}} = -\frac{5}{2}$$

(e) $$\lim_{n\to\infty} \frac{2n+3}{2n^2-n-1} = \frac{\frac{2n}{n^2}+\frac{3}{n^2}}{\frac{2n^2}{n^2}-\frac{n}{n^2}-\frac{1}{n^2}} = \frac{\overbrace{\frac{2}{n}}^{\text{limit}=0}+\overbrace{\frac{3}{n^2}}^{\text{limit}=0}}{\underbrace{2}_{\text{limit}=2}-\underbrace{\frac{1}{n}}_{\text{limit}=0}-\underbrace{\frac{1}{n^2}}_{\text{limit}=0}} = \frac{0}{2} = 0$$

(f) $$\lim_{n\to\infty} \frac{-3n^3-5n^2+8n+1}{5-2n^2} = \frac{\frac{-3n^3}{n^2}-\frac{5n^2}{n^2}+\frac{8n}{n^2}+\frac{1}{n^2}}{\frac{-2n^2}{n^2}+\frac{5}{n^2}} =$$

$$\frac{\overbrace{-3n}^{\text{limit}=-\infty}-\overbrace{5}^{\text{limit}=5}+\overbrace{\frac{8}{n}}^{\text{limit}=0}+\overbrace{\frac{1}{n^2}}^{\text{limit}=0}}{\underbrace{-2}_{\text{limit}=-2}+\underbrace{\frac{5}{n^2}}_{\text{limit}=0}} = \frac{-\infty}{-2} = \infty$$

(i) $$\lim_{n\to\infty} \frac{4^n+3^n}{5^n-2^n} = \frac{\frac{4^n}{5^n}+\frac{3^n}{5^n}}{\frac{5^n}{5^n}-\frac{2^n}{5^n}} = \frac{\overbrace{\left(\frac{4}{5}\right)^n}^{\text{limit}=0}+\overbrace{\left(\frac{3}{5}\right)^n}^{\text{limit}=0}}{\underbrace{1}_{\text{limit}=1}-\underbrace{\left(\frac{2}{5}\right)^n}_{\text{limit}=0}} = 0$$

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j) $\lim_{n\to\infty} (3n^2+5n+1) = n^2\left(3+\frac{5n}{n^2}+\frac{1}{n^2}\right) =$

$$\underbrace{n^2}_{\text{limit}=\infty}\left(\underbrace{3}_{\text{limit}=3}+\underbrace{\frac{5}{n}}_{\text{limit}=0}+\underbrace{\frac{1}{n^2}}_{\text{limit}=0}\right) = \infty$$

l) $\lim_{n\to\infty} (-3n^2+5n+1) = \underbrace{n^2}_{\text{limit}=\infty}\left(\underbrace{-3}_{\text{limit}=-3}+\underbrace{\frac{5}{n}}_{\text{limit}=0}+\underbrace{\frac{1}{n^2}}_{\text{limit}=0}\right) = \infty \cdot -3 = -\infty$

n) $\lim_{n\to\infty} \left(\sqrt{n^2+1000}-n\right)$

$$\lim_{n\to\infty} \left(\sqrt{n^2+1000}-n\right)\frac{\left(\sqrt{n^2+1000}+n\right)}{\sqrt{n^2+1000}+n}$$

$$\frac{\cancel{n^2}+1000-\cancel{n^2}}{\sqrt{n^2+1000}+n} = \frac{1000}{\sqrt{n^2+1000}+n} \quad /:\sqrt{n^2}$$

$$\frac{\frac{1000}{n}}{\sqrt{\frac{n^2}{n^2}+\frac{1000}{n^2}}+\frac{n}{n}} = \frac{1000}{n\left(1+\sqrt{\frac{1000}{n^2}}+1\right)} = \frac{1000}{\underbrace{n}_{\text{limit}=0}\left(\underbrace{2}_{\text{limit}=2}+\underbrace{\frac{1000}{n}}_{\text{limit}=0}\right)} = 0$$

p) $\lim_{n\to\infty} \left(\sqrt{3n^2+200n+3}-\sqrt{3n^2-100n-7}\right)$

$$\lim_{n\to\infty} \left(\sqrt{3n^2+200+3}-\sqrt{3n^2-100n-7}\right)\frac{\left(\sqrt{3n^2+200n+3}+\sqrt{3n^2-100n-7}\right)}{\left(\sqrt{3n^2+200n+3}+\sqrt{3n^2-100n-7}\right)}$$

6 d $a_n = 2n-1$ prove by contradiction:

assume $|2n-1-L| < \varepsilon$

$$L-\varepsilon < 2n-1 < \varepsilon + L$$

as n get bigger, many of the terms won't be between these values.

$$(3n^2+200n+3)-(3n^2-100n-7)$$

$$\cancel{3n^2}+200n+3-\cancel{3n^2}+100n+7$$

$$\frac{300n+10}{\left(\sqrt{3n^2+200n+3}+\sqrt{3n^2-100n-7}\right)} \quad / : \sqrt{n^2}$$

$$\frac{\frac{300n}{n}+\frac{10}{n}}{\sqrt{\frac{3n^2}{n^2}+\frac{200n}{n^2}+\frac{3}{n^2}}+\sqrt{\frac{3n^2}{n^2}-\frac{100n}{n^2}-\frac{7}{n^2}}} =$$

limit = 300 ↘ $300 + \frac{10}{n}$ ↙ limit = 0

$$\frac{300+\frac{10}{n}}{\sqrt{3+\frac{200}{n}+\frac{3}{n^2}}+\sqrt{3-\frac{100}{n}-\frac{7}{n^2}}} = \frac{300+0}{\sqrt{3}+\sqrt{3}} = \frac{300}{2\sqrt{3}} = \frac{150}{\sqrt{3}}$$

L=3 L=0 L=0 L=3 L=0 L=0

5 $$\lim_{n\to\infty} \frac{n+\sqrt{n}+\sqrt[4]{n}}{n+\sqrt[3]{n}+\sqrt[5]{n}} = \frac{n+n^{\frac{1}{2}}+n^{\frac{1}{4}}}{n+n^{\frac{1}{3}}+n^{\frac{1}{5}}} \; / : n = \frac{\frac{n}{n}+\frac{n^{\frac{1}{2}}}{n}+\frac{n^{\frac{1}{4}}}{n}}{\frac{n}{n}+\frac{n^{\frac{1}{3}}}{n}+\frac{n^{\frac{1}{5}}}{n}} = \frac{1}{1} = 1$$

L=1 L=0 L=0

L=1 L=0 L=0

12 a $a_1 = 3$

$a_{n+1} = \sqrt{4a_n + 5}$.

$a_1 = 3$, $a_2 = \sqrt{4\cdot 3+5} = 4.12$, $a_3 = \sqrt{4\cdot 4.12+5} = 4.63$, $a_4 = \sqrt{4\cdot 4.63+5} = 4.85$

$a_5 = \sqrt{4\cdot 4.85+5} = 4.94$

12 (a_n)

1) $a_1 = 3$

$a_{n+1} = \sqrt{4a_n+5}$

Prove that a_n is bounded and monotonic

bounded: 1 Prove that a_n is bounded or that $3 \leq a_n \leq 5$

basic step: $a_1 = 3 \Rightarrow 3 \leq a_1 \leq 5$

induction step: assume $3 \leq a_n \leq 5$

reminder: $a_{n+1} = \sqrt{4a_n+5}$

$$3 \leq a_n \leq 5$$

$$12 \leq 4a_n \leq 20$$

$$17 \leq 4a_n+5 \leq 25$$

$$\sqrt{17} \leq \sqrt{4a_n+5} \leq \sqrt{25} = 5$$

$$\sqrt{17} \leq a_{n+1} \leq 5$$

done

1 **monotonic:** to prove $a_{n+1} > a_n$

basic case: $a_2 > a_1$

$\sqrt{17} > 3$ ✓

induction step: assume $a_n > a_{n-1}$

to prove: $a_{n+1} > a_n$

$$\sqrt{4a_n+5} > a_n \Rightarrow 4a_n+5 > a_n^2$$

$$a_n^2 - 4a_n - 5 < 0$$

$$t^2 - 4t - 5 < 0$$

$$(t-5)(t+1) < 0$$

this is true when $-1 < t < 5$ or $-1 < a_n < 5$ ✓

2 $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} = L$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \sqrt{4 \cdot a_n + 5}$$

$$L = \sqrt{4L+5}$$

$$L^2 = 4L+5$$

$$L^2 - 4L - 5 = 0$$

$$(L-5)(L+1) = 0$$

$L = 5$

$\cancel{L = -1}$

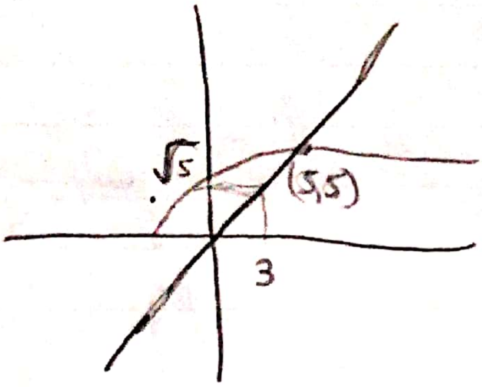
(b) prove that all the terms in the sequence are positive.

basic step: $a_1 = 3 > 0$

Induction: assume $a_n > 0$

$a_{n+1} = \sqrt{a_n + 5} > \sqrt{5}$

(f) $f(x) = \sqrt{4x+5}$



even if the sequence goes down

it's still converge

* change a_1 to be 7

$a_1 = 7$

$\lim a_n = \lim a_{n+1} = L$

$a_{n+1} = \sqrt{4a_n + 5}$

$L = \sqrt{4L+5}$

$L^2 = 4L + 5$

$L^2 - 4L - 5 = 0$

$L_1 = 5 \quad L_2 = -1$

$\lim (a_n | a_1 = 7) = 5$

(6) (d) $a_n = 2n - 1$ does not converge

Proof by contradiction

assume $|2n - 1 - L| < \varepsilon$

$-\varepsilon < 2n - 1 - L < \varepsilon$

$-\varepsilon + L + 1 < 2n < \varepsilon + 1 + L$

$\frac{-\varepsilon + L + 1}{2} < n < \frac{\varepsilon + 1 + L}{2}$

because n is between 2

little values, which is impossible. contradiction.

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(14) (a) Recursive Definition: $a_1 = \sqrt{6}$

$a_{n+1} = \sqrt{a_n + 6}$

(b) prove that a_n is bounded & that $\sqrt{6} \le a_n \le \sqrt{6+\sqrt{6}}$

$2 \le a_n \le 3$

basic step: $a_1 = \sqrt{6} \Rightarrow \sqrt{6} \le a_1 \le \sqrt{6+\sqrt{6}}$

Induction step: assume $\sqrt{6} \le a_n \le \sqrt{6+\sqrt{6}}$

reminder : $a_{n+1} = \sqrt{a_n + \sqrt{6}}$

$2 \le a_n \le 3$

$8 \le a_n + 6 \le 9$

$\sqrt{8} \le \sqrt{a_n+6} \le \sqrt{9}$

$\sqrt{8} \le a_{n+1} \le 3$

monotonic: to prove $a_{n+1} > a_n$

basic case: $a_2 > a_1$

$\sqrt{6+\sqrt{6}} > \sqrt{6}$

induction step: assume $a_n > a_{n-1}$

to prove: $a_{n+1} > a_n$

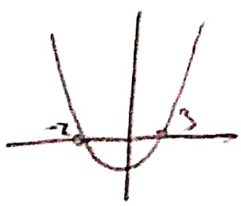
$\sqrt{a_n + 6} > a_n \Rightarrow a_n + 6 > a_n^2$

$0 > a_n^2 - a_n - 6 \qquad a_n = t$

$t^2 - t - 6$

$\frac{1 \pm 5}{2} \rightarrow t_1 = 3 \qquad a_n = 3$

$\rightarrow t_2 = -2 \qquad a_n = -2$



this is true when $-2 < a_n < 3$

עמ 7

(c) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} = L$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \sqrt{a_n + \sqrt{6}}$$

$$L = \sqrt{a_n + \sqrt{6}}$$

$$L^2 = L + 6$$

$$L^2 - L - 6 = 0$$

$$\frac{1 \pm 5}{2} \to L = 3$$

$$\to \cancel{L = -2}$$

because the sequence is bounded, the sequence will never get to -2

$$\lim_{n\to\infty} a_n = 3$$

(17) (c) $\left(\frac{1}{6}\right)^n < \frac{n}{6n} \leq \frac{2n-3}{5n+4} \leq \left(\frac{2n+3\sin n}{5n+4\cos n}\right)^n \leq \frac{2n+3}{5n-4} \leq \frac{2n+n}{5n-n} \leq \frac{3n}{4n} = \left(\frac{3}{4}\right)^n$ $\quad n \geq 4$

$0 < a < 1$

And because

$\lim_{n\to\infty} \left(\frac{3}{4}\right)^n = 0$

$\lim_{n\to\infty} \left(\frac{1}{6}\right)^n = 0$

therefore because of the sandwhich theorem $\lim_{n\to\infty} \left(\frac{2n+3\sin n}{5n+4\cos n}\right)^n = 0$

(a) $\lim_{n\to\infty} \left(\frac{3n+4}{7n+6}\right)^n$ for every $n > 6$

$$\frac{3}{8} = \frac{3n}{8n} \leq \frac{3n}{7n+n} \leq \left(\frac{3n+4}{7n+6}\right)^n \leq \frac{3n+n}{7n+6} \leq \frac{4n}{7n} = \frac{4}{7}$$

for every n that is a natural number. $f(x) = x^n$ increases in the area $[0, +\infty]$

therefore
$$\left(\frac{3}{8}\right)^n < \left(\frac{3n+4}{7n+6}\right)^n < \left(\frac{4}{7}\right)^n$$

and because $\lim_{n\to\infty} \left(\frac{3}{8}\right)^n = 0$

$$\lim_{n\to\infty} \left(\frac{4}{7}\right)^n = 0$$

therefore because of the sandwhich theorem

$$\lim_{n\to\infty} \left(\frac{3n+4}{7n+6}\right)^n = 0$$

18) $1\frac{2}{3} \leq \frac{5n}{3n} \leq \frac{5n}{2n+n} \leq \left(\frac{5n+7}{2n+4}\right)^n \leq \frac{5n+n}{2n+4} \leq \frac{6n}{2n} = 3$

for every n that is a natural number, $f(x)=x^n$ increasing in the area $[0,+\infty)$

therefore

$$\left(1\frac{2}{3}\right)^n \leq \left(\frac{5n+7}{2n+4}\right)^n \leq (3)^n$$

and because $\lim_{n\to\infty} \left(1\frac{2}{3}\right)^n = +\infty$

$\lim_{n\to\infty} (3)^n = +\infty$

therefore because of the sandwich theorem

$$\lim_{n\to\infty} \left(\frac{5n+7}{2n+4}\right)^n = +\infty$$

19) c) Incorrect

we need to prove that both a_n and b_n do not need to be converging sequences in order $a_n \cdot b_n$ to be a converging sequence

contradicting exp: $a_n = n$

$b_n = \frac{1}{n}$

$a_n \cdot b_n = 1$

f) Correct exp: if $a_n = \frac{(-1)^n}{n}$ → vanishing sequence

But $\frac{1}{a_n} = \frac{1}{\frac{(-1)^n}{n}} = \frac{n}{(-1)^n}$ which diverges to ∞ or $-\infty$

h) Incorrect:

contradicting exp: $a_n = \frac{1}{n}$

$\lim \frac{1}{n} = 0$