Linear Algebra hw #4

① $T(x,y) = (2x-y, x-3y)$

ⓐ $[T]_E^E$

$T(e_1) = (1,0) = (2,1) \Rightarrow 2e_1 + 1e_2$

$T(e_2) = (0,1) = (-1,-3) \Rightarrow -1e_1 - 3e_2$

$[T]_E^E = \begin{pmatrix} 2 & -1 \\ 1 & -3 \end{pmatrix}$

ⓑ $[T]_C^B$ $\quad B = (b_1, b_2)$ $\quad C = (c_1, c_2)$

$b_1 = (1,1)$ $\quad c_1 = (2,2)$

$b_2 = (2,5)$ $\quad c_2 = (0,1)$

$T(b_1) = T(1,1) \Rightarrow (1,-2) \Rightarrow \alpha c_1 + \beta c_2$

$T(b_2) = T(2,5) \Rightarrow (-1,-13) \Rightarrow \alpha c_1 + \beta c_2$

$\left(\begin{array}{cc|c} 2 & 2 & 1 \\ 0 & 1 & -2 \end{array}\right) \xrightarrow{R_1 = \frac{R_1}{2}} \left(\begin{array}{cc|c} 1 & 1 & \frac{1}{2} \\ 0 & 1 & -2 \end{array}\right) \xrightarrow{R_1 = R_1 - R_2} \left(\begin{array}{cc|c} 1 & 0 & 2.5 \\ 0 & 1 & -2 \end{array}\right)$

$\left(\begin{array}{cc|c} 2 & 2 & -1 \\ 0 & 1 & -13 \end{array}\right) \xrightarrow{R_1 = \frac{R_1}{2}} \left(\begin{array}{cc|c} 1 & 1 & -\frac{1}{2} \\ 0 & 1 & -13 \end{array}\right) \xrightarrow{R_1 = R_1 - R_2}$

$\left(\begin{array}{cc|c} 1 & 0 & 13.5 \\ 0 & 1 & 13 \end{array}\right)$

$2.5(2,2) - 2(0,1) = (5,3)$

$13.5(2,2) + 13(0,1) = (27, 40)$

$a\begin{pmatrix} 2 \\ 2 \end{pmatrix} + b\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 2a \\ 2a+b \end{pmatrix}$

$T(b_1) = \begin{pmatrix} 2a \\ 2a+b \end{pmatrix} \Rightarrow 2a = 1, \quad 2a+b = -2$ $\quad a = \frac{1}{2}$, $b = -3$

$T(b_2) = \begin{pmatrix} 2a \\ 2a+b \end{pmatrix} \Rightarrow 2a = -1, \quad 2a+b = -13$ $\quad a = -\frac{1}{2}$, $b = -12$

$[T]_C^B = \begin{bmatrix} \frac{1}{2} & -3 \\ -\frac{1}{2} & -12 \end{bmatrix}$

ⓒ $[T]_E^B$ $\quad T(b_1) = \begin{pmatrix} a \\ b \end{pmatrix}$ $\quad T(b_2) = \begin{pmatrix} a \\ b \end{pmatrix}$

$[T]_E^B = \begin{bmatrix} 1 & -2 \\ -1 & -13 \end{bmatrix}$

ⓓ $b_1(1,1)$ $\quad b_2(2,5)$

$T(b_1) = \begin{pmatrix} a + 2b \\ a + 5b \end{pmatrix}$ $\qquad T(b_2) = \begin{pmatrix} a + 2b \\ a + 5b \end{pmatrix}$

$a + 2b = 1$ $\qquad a + 2b = -1$

$a + 5b = -2$ $\qquad a + 5b = -13$

$a = 3, \; b = -1$ $\qquad a = 7 \quad b = -4$

$[T]_B^B = \begin{pmatrix} -2 & -1 \\ 7 & -4 \end{pmatrix}$

(e) $[T(v)]_C \qquad v = (3, 16)$

$$T(v) = (2(3) - 16,\ 3 - 3(16)) = (-10, -45)$$

$$\begin{bmatrix} 2 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -10 \\ -45 \end{bmatrix} = \begin{bmatrix} -20 \\ -65 \end{bmatrix}$$

(7) (b) $F = ((1,3,-6), (-4,2,0), (1,0,-1))$

$$M^F_B = ([I(f_1)]_B, [I(f_2)]_B, [I(f_3)]_B) = (f_1, f_2, f_3) = \begin{pmatrix} 1 & -4 & 1 \\ 3 & 2 & 0 \\ -6 & 0 & -1 \end{pmatrix}$$

$$M^B_F = (M^F_B)^{-1} = \begin{pmatrix} 1 & -4 & 1 \\ 3 & 2 & 0 \\ -6 & 0 & -1 \end{pmatrix}^{-1} \Rightarrow \left(\begin{array}{ccc|ccc} 1 & -4 & 1 & 1 & 0 & 0 \\ 3 & 2 & 0 & 0 & 1 & 0 \\ -6 & 0 & -1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 + 6R_1 \end{array} \left(\begin{array}{ccc|ccc} 1 & -4 & 1 & 1 & 0 & 0 \\ 0 & 14 & -3 & -3 & 1 & 0 \\ 0 & -24 & 5 & 6 & 0 & 1 \end{array}\right)$$

$$\begin{array}{l} R_3 \to R_3 + \frac{12}{7}R_2 \\ R_2 \to \frac{1}{14}R_2 \end{array} \left(\begin{array}{ccc|ccc} 1 & -4 & 1 & 1 & 0 & 0 \\ 0 & 1 & -\frac{3}{14} & -\frac{3}{14} & \frac{1}{14} & 0 \\ 0 & 0 & -\frac{1}{7} & \frac{6}{7} & \frac{12}{7} & 1 \end{array}\right) \begin{array}{l} R_1 \to R_1 + 4R_2 \\ R_3 \to -7R_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{7} & \frac{1}{7} & \frac{2}{7} & 0 \\ 0 & 1 & -\frac{3}{14} & -\frac{3}{14} & \frac{1}{14} & 0 \\ 0 & 0 & 1 & -6 & -12 & -7 \end{array}\right) \begin{array}{l} R_1 \to R_1 - \frac{1}{7}R_3 \\ R_2 \to R_2 + \frac{3}{14}R_3 \end{array} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 2 & 1 \\ 0 & 1 & 0 & -1\frac{1}{2} & -\frac{5}{2} & -\frac{3}{2} \\ 0 & 0 & 1 & -6 & -12 & -7 \end{array}\right)$$

$$\Rightarrow M^B_F = \begin{pmatrix} 1 & 2 & 1 \\ -1\frac{1}{2} & -2\frac{1}{2} & -1\frac{1}{2} \\ -6 & -12 & -7 \end{pmatrix}$$

⑤ ⓐ $F_1 = (1,0,0)$ $\quad f_1 = 1$ $\quad P_2(x) = 6x+3 = 3\cdot f_1 + 6\cdot f_2 + 0\cdot f_3$

$F_2 = (0,1,0)$ $\quad f_2 = x$

$F_3 = (0,0,1)$ $\quad f_3 = x^2$

$$[p(x)]_F = 1\cdot f_1 + 6\cdot f_2 + 2\cdot f_3 = \begin{pmatrix}1\\6\\2\end{pmatrix}$$

$$[D]^F_G \cdot [p(x)]_F = [D(p(x))]_G = [p'(x)]_G$$

$$[D]^F_G = ([D(f_1)]_G, [D(f_2)]_G, [D(f_3)]_G) = ([D(1)]_G, [D(x)]_G, [D(x^2)]_G) = ([0]_G, [1]_G, [2x]_G) = ((0,0),(1,0),(0,2)) = \begin{pmatrix}0&1&0\\0&0&2\end{pmatrix}$$

③ ⓐ $[T]^E_E$ $\quad T(e_1) = (1,0) \to (2+i, 0) = 2+i\, e_1$ $\quad \begin{pmatrix}2+i & 0\\ 0 & i\end{pmatrix}$

$T(e_2) = (0,1) \to (0,i) = i\, e_2$

5b $$[D(p(x))]_G = [D]^F_G [p(x)]_F = \begin{pmatrix}0&1&0\\0&0&2\end{pmatrix}\begin{pmatrix}1\\6\\2\end{pmatrix} = \begin{pmatrix}6\\4\end{pmatrix} \to p'x = (1,x)\begin{pmatrix}6\\4\end{pmatrix} = 4x+6$$

3b $F = (f_1, f_2) = ((6-2i, 12-4i), (0, -6+2i))$, $G = (g_1, g_2) = ((1-i, 1), (1+2i, 1+i))$

$$[T]^F_G = [I]^E_G [T]^F_E = ([I]^G_E)^{-1} \cdot [T]^F_E$$

$$[I]^G_E = ([I(g_1)]_E, [I(g_2)]_E) = (g_1, g_2) = \begin{pmatrix}1-i & 1+2i\\ 1 & 1+i\end{pmatrix} \Rightarrow [I]^E_G = \begin{pmatrix}1-i & 1+2i\\ 1 & 1+i\end{pmatrix}^{-1} = \frac{1}{(1-i)(1+i)-(1+2i)}\begin{pmatrix}1+i & -1-2i\\ -1 & 1-i\end{pmatrix}$$

$$= \frac{1}{1-2i}\begin{pmatrix}1+i & -1-2i\\ -1 & 1-i\end{pmatrix} = \begin{pmatrix}\frac{1+i}{1-2i} & \frac{-1-2i}{1-2i}\\ \frac{-1}{1-2i} & \frac{1-i}{1-2i}\end{pmatrix} = \frac{1}{5}\begin{pmatrix}3i-1 & 3-4i\\ -1-2i & 3+i\end{pmatrix}$$

$$[T]^F_E = ([T(f_1)]_E, [T(f_2)]_E) = (T(f_1), T(f_2)) = (((2+i)(6-2i), i(12-4i)), ((2+i)\cdot 0, i(2i-6)))$$

$$= ((14+2i, 4+12i), (0, -2-6i)) = \begin{pmatrix}14+2i & 0\\ 4+12i & -2-6i\end{pmatrix}$$

$$[T]^F_G = [I]^E_G [T]^F_E = \frac{1}{5}\begin{pmatrix}3i-1 & 3-4i\\ -1-2i & 3+i\end{pmatrix}\begin{pmatrix}14+2i & 0\\ 4+12i & -2-6i\end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix}(3i-1)(14+2i)+(3-4i)(4+12i) & (2+6i)(4i-3)\\ (-1-2i)(14+2i)+(3+i)(4+12i) & -(2+6i)(3+i)\end{pmatrix} = \frac{1}{5}\begin{pmatrix}-20+40i+60+20i & -30-6i\\ -10-30i+40i & -20i\end{pmatrix}$$

$$= \frac{1}{5}\begin{pmatrix}40+60i & -30+10i\\ -10+10i & -20i\end{pmatrix} = \boxed{\begin{pmatrix}8+12i & -6-2i\\ -2+2i & -4i\end{pmatrix}}$$

(3) (1) $B=(1,x,x^2)$. $F=(1+2x^2,\ x+2x^2,\ 5x-12)$

$$M_B^F=([I(f_1)]_B,[I(f_2)]_B,[I(f_3)]_B)=([1+2x^2]_B,[x+2x^2]_B,[5x-12]_B)$$

$$=((1,0,2),(0,1,2),(-12,5,0))=\begin{pmatrix}1&0&-12\\0&1&5\\2&2&0\end{pmatrix}$$

$$M_B^F=(M_B^F)^{-1}=\begin{pmatrix}1&0&-12\\0&1&5\\2&2&0\end{pmatrix}^{-1}\Rightarrow\left(\begin{array}{ccc|ccc}1&0&-12&1&0&0\\0&1&5&0&1&0\\2&2&0&0&0&1\end{array}\right)\xrightarrow{R_3\to R_3-R_1-2R_2}\left(\begin{array}{ccc|ccc}1&0&-12&1&0&0\\0&1&5&0&1&0\\0&0&14&-2&-2&1\end{array}\right)$$

$$\xrightarrow{R_3\to\frac{1}{14}R_3}\left(\begin{array}{ccc|ccc}1&0&-12&1&0&0\\0&1&5&0&1&0\\0&0&1&-\frac{1}{7}&-\frac{1}{7}&\frac{1}{14}\end{array}\right)\xrightarrow[R_2\to R_2-5R_3]{R_1\to R_1+12R_3}\left(\begin{array}{ccc|ccc}1&0&0&-\frac{5}{7}&-\frac{12}{7}&\frac{6}{7}\\0&1&0&\frac{5}{7}&\frac{12}{7}&-\frac{5}{14}\\0&0&1&-\frac{1}{7}&-\frac{1}{7}&\frac{1}{14}\end{array}\right)\Rightarrow M_F^B=\frac{1}{14}\begin{pmatrix}-10&-24&12\\10&24&-5\\-2&-2&1\end{pmatrix}$$

(2) $F=((2+i,5i),(-4i,3-2i))$

$$M_B^F=([I(f_1)]_B,[I(f_2)]_B)=(f_1,f_2)=\begin{pmatrix}2+i&-4i\\5i&3-2i\end{pmatrix}$$

$$M_F^B=(M_B^F)^{-1}=\frac{1}{(2+i)(3-2i)+4i\cdot 5i}\begin{pmatrix}3-2i&4i\\-5i&2+i\end{pmatrix}=\frac{1}{-12-i}\begin{pmatrix}3-2i&4i\\-5i&2+i\end{pmatrix}=\begin{pmatrix}\frac{2i-3}{12+i}&\frac{-4i}{12+i}\\\frac{5i}{12+i}&\frac{-2-i}{12+i}\end{pmatrix}$$

$$=\frac{1}{145}\begin{pmatrix}(2i-3)(12-i)&-4i(12-i)\\5i(12-i)&(2+i)(i-12)\end{pmatrix}=\frac{1}{145}\begin{pmatrix}29i-34&-4-48i\\5+60i&-25-10i\end{pmatrix}$$

(3) $F=\left(\begin{pmatrix}1&-3\\0&0\end{pmatrix},\begin{pmatrix}-4&2\\0&2\end{pmatrix},\begin{pmatrix}1&0\\1&0\end{pmatrix},\begin{pmatrix}0&0\\2&1\end{pmatrix}\right)$

$$M_B^F=((1,-3,0,0),(-4,2,0,2),(1,0,1,0),(0,0,2,1))=\begin{pmatrix}1&-4&1&0\\-3&2&0&0\\0&0&1&2\\0&2&0&1\end{pmatrix}$$

$$M_F^B=(M_B^F)^{-1}\Rightarrow\left(\begin{array}{cccc|cccc}1&-4&1&0&1&0&0&0\\-3&2&0&0&0&1&0&0\\0&0&1&2&0&0&1&0\\0&2&0&1&0&0&0&1\end{array}\right)\xrightarrow{R_2\to R_2+3R_1}\left(\begin{array}{cccc|cccc}1&-4&1&0&1&0&0&0\\0&-10&3&0&3&1&0&0\\0&0&1&2&0&0&1&0\\0&2&0&1&0&0&0&1\end{array}\right)$$

$$\xrightarrow[R_1\to R_1+2R_4]{R_2\to\frac{1}{-10}R_2}\left(\begin{array}{cccc|cccc}1&0&1&2&1&0&0&2\\0&1&-\frac{3}{10}&0&-\frac{3}{10}&-\frac{1}{10}&0&0\\0&0&1&2&0&0&1&0\\0&2&0&1&0&0&0&1\end{array}\right)\xrightarrow[R_4\to R_4-2R_2]{R_1\to R_1-R_3}\left(\begin{array}{cccc|cccc}1&0&0&0&1&0&-1&2\\0&1&-\frac{3}{10}&0&-\frac{3}{10}&\frac{1}{10}&0&0\\0&0&1&2&0&0&1&0\\0&0&\frac{3}{5}&1&\frac{3}{5}&\frac{1}{5}&0&1\end{array}\right)\xrightarrow[R_2\to R_2+\frac{3}{10}R_3]{R_4\to R_4-\frac{3}{5}R_3}$$

$$\left(\begin{array}{cccc|cccc}1&0&0&0&1&0&-1&2\\0&1&0&\frac{3}{5}&-\frac{3}{10}&-\frac{1}{10}&\frac{3}{10}&0\\0&0&1&2&0&0&1&0\\0&0&0&-\frac{1}{5}&\frac{3}{5}&\frac{1}{5}&-\frac{3}{5}&1\end{array}\right)\xrightarrow[\substack{R_3\to R_3+10R_4\\R_2\to R_2+3R_4}]{R_4\to-5R_4}\left(\begin{array}{cccc|cccc}1&0&0&0&1&0&-1&2\\0&1&0&0&1.5&\frac{1}{2}&-1\frac{1}{2}&3\\0&0&1&0&6&2&-5&10\\0&0&0&1&-3&-1&3&-5\end{array}\right)\Rightarrow M_F^B=\begin{pmatrix}1&0&-1&2\\1.5&\frac{1}{2}&-1\frac{1}{2}&3\\6&2&-5&10\\-3&-1&3&-5\end{pmatrix}$$

15) a) $B = ((1,1),(1,0))$, $C = ((3,1),(0,2))$

$[I]^C_B = ([I(3,1)]_B, [I(0,2)]_B) = ([(3,1)]_B, [(0,2)]_B) = ((1,2),(2,-2)) = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}$

b) $[I]^B_C = ([I]^C_B)^{-1} = \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix}^{-1} = \frac{1}{-2-4} \begin{pmatrix} -2 & -2 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{6} \end{pmatrix}$

c) $[I]^B_C [I]^C_B = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{6} \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} \frac{1}{3}+\frac{2}{3} & \frac{2}{3}-\frac{2}{3} \\ \frac{1}{3}-\frac{2}{6} & \frac{2}{3}+\frac{2}{6} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

d) $v = (2,3) = 3(1,1) - (1,0) \Rightarrow [v]_B = (3,-1)$

$[v]_C = [I(v)]_C = [I]^B_C [v]_B = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{6} \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{2}{3} \\ \frac{7}{6} \end{pmatrix} = (\frac{2}{3}, \frac{7}{6})$

16) a) $T(1,0,3) = (1,2,0) = T(e_1) + 3T(e_3) \Rightarrow T(e_1) = (1,2,0) - 3T(e_3)$

$T(1,1,0) = (0,1,0) = T(e_1) + T(e_2) \Rightarrow T(e_2) = (0,1,0) - (1,2,0) + 3T(e_3)$

$T(1,-1,1) = (0,0,0) = T(e_1) - T(e_2) + T(e_3) = (1,2,0) - 3T(e_3) + (1,1,0) - 3T(e_3) + T(e_3)$

$\Rightarrow 5T(e_3) = (2,3,0) \Rightarrow \boxed{T(e_3) = (\frac{2}{5}, \frac{3}{5}, 0)} \Rightarrow \boxed{T(e_2) = (\frac{1}{5}, \frac{4}{5}, 0)} \Rightarrow \boxed{T(e_1) = (-\frac{1}{5}, \frac{1}{5}, 0)}$

$\Rightarrow T(x,y,z) = xT(e_1) + yT(e_2) + zT(e_3) = \boxed{\frac{1}{5}(-x+y+2z, x+4y+3z, 0)}$

b) $T = S$

$v = t_1(1,0,3) + t_2(1,1,0) + t_3(1,-1,1)$

$T(v) = T(t_1(1,0,3) + t_2(1,1,0) + t_3(1,-1,1)) = t_1 T(1,0,3) + t_2 T(1,1,0) + t_3 T(1,-1,1)$

$= t_1(1,2,0) + t_2(0,1,0) + t_3(0,0,0) = t_1 S(1,0,3) + t_2 S(1,1,0) + t_3 S(1,-1,1)$

$= S(t_1(1,0,3) + t_2(1,1,0) + t_3(1,-1,1)) = S(v) \Rightarrow T = S$

c) $[T]^C_C = [T] = (T(e_1), T(e_2), T(e_3)) = \frac{1}{5}\begin{pmatrix} -1 & 1 & 2 \\ 1 & 4 & 3 \\ 0 & 0 & 0 \end{pmatrix}$

17) $T(A) = A^T$, $B = \left(\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right)$, $C = \left(\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right)$

b) $[T]^B_B = \left(\left[T\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\right]_B, \left[T\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right]_B, \left[T\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right]_B, \left[T\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right]_B\right)$

$= \left(\left[\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}\right]_B, \left[\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right]_B, \left[\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right]_B, \left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right]_B\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

c) $M^C_B = \left(\left[\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}\right]_B, \left[\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right]_B, \left[\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right]_B, \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right]_B\right) = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 \end{pmatrix}$

d) $[T]^C_C = \left(\left[\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix}\right]_C, \left[\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right]_C, \left[\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right]_C, \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right]_C\right) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

e) $\left[T\begin{pmatrix}2&1\\2&4\end{pmatrix}\right]_C = [T]_C^C\left[\begin{pmatrix}2&1\\2&4\end{pmatrix}\right]_C = \begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}2\\2\\1\\2\end{pmatrix} = \begin{pmatrix}2\\1\\2\\2\end{pmatrix}$

$\left[\begin{pmatrix}2&1\\2&4\end{pmatrix}^T\right]_C = \begin{pmatrix}2\\1\\2\\2\end{pmatrix}$