Homework I did it with Shira Feldman 341321776

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Homework 1 Data Structure

① ⓐ improvedSelection (A, n)

ⓑ

```
{ int temp1, temp2
    for i=0, j=n to i<n/2, j>n/2, by i by 1 j by -1      // n/2
      {    temp1 = A[i]                                  // n/2
           temp2 = A[j]                                  // n/2
           for k = i+1 to j-1 by 1                       // n/2 · (n-i-j)
         { if A[k] < temp1                               // n/2 · (n-i-j)
              temp1 = A[k]
           if A[k] > temp2                               // n/2 · (n-i-j)
              temp2 = A[k]
         }
        A[i] = temp1        // n/2
        A[j] = temp2        // n/2
      }
          return A    // 1
}
```

ⓑ $\frac{n}{2}(n-i-j)+1$

ⓒ $O(n^2)$ — this is the best case and the worst case

ⓓ No, the order of magnitude for selection sort is also n^2

② 2^{48}, $\frac{n}{\log_5 n}$, $(\log n)^{2n}$, $28n+(\log n)^{0.1}$, $n+1000$, $n^{\log n}$, n^4, 2^n

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③ (ii) n^2 (top left)
(i) $n+n$ (top right)
(iii) $n(n+\frac{n}{2})$
(iv) $\log\log(n)$

④ a

⑤ $3n^2+0.5n^4-2\log n=\Omega(n^4)$
disprove   proof by contradiction
assume that $f(n)=\Omega(n^4)$ such that there are constants $c>2$ and $N \ni n_0$
That for every $n \geq n_0$

$$3n^2+0.5n^4-2\log n \geq cn^4$$

$$0.5n^4 \geq cn^4-3n^2+2\log n$$

$$0.5n^4 \geq cn^4-3n^2+2\log n \geq cn^4-3n^2+1 \geq cn^4-n^4 \geq n^4(c-1)+1 > n^4(c-2)$$

That is $0.5n^4 > n^4(c-2)$
and if we chose any $0.5n^4 > n^4(c-2)$ we get a contradiction to the original claim

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① ⓐ $f(n) = \Omega(g(n))$ $\quad$ $f(n) = O(g(n))$

$f(n) = n^{2.5} + n \qquad g(n) = n^{1.4} \log n \qquad f(n) = \Theta(g(n))$

we need to choose c and n_0 such that, for every $n \geq n_0$ $f(n) = \Omega(g(n))$

$$n^{2.5} + n \geq c(n^{1.4} \log n)$$

choose $c = 2$

$$n^{2.5} + n \geq 2n^{1.4} \log n \quad / n^{1.4}$$

$$n^{1.1} - n^{-0.4} \geq 2 \log n$$

$$n^{1.1} - \frac{1}{n^{0.4}} \geq 2 \log n$$

for $n \geq 1$ $\quad 1 + 1 \geq 2 \log 2$

$\Rightarrow 2 \geq 2 \log 2 \Rightarrow 2 \geq 0.6021$

ⓑ $f(n) = \Omega(g(n))$

$f(n) = n^5 \qquad g(n) = 42n^3 + 50n + 17$

$$n^5 \geq c(42n^3 + 50n + 17)$$

choose $c = 1$ $\quad n^5 \geq 1(42n^3 + 50n + 17)$

$$n^5 \geq 42n^3 + 50n + 17 \qquad / : n^3$$

$$n^2 \geq 42 + \frac{50}{n^2} + \frac{17}{n^3}$$

for every $n \geq 7$

$$7^2 \geq 42 + \frac{50}{7^2} + \frac{17}{7^3}$$

$$49 \geq 43 + \frac{17}{343} \quad \checkmark$$

ⓒ $f(n) = \Theta(g(n))$

$f(n) = 5n \qquad g(n) = 4n + 5000$

$c(4n + 500) <= 5n <= c(4n + 5000)$

for all $n > 1$:

$n + 500 <= 5n <= 4n^2 + 5000$

choose: $c_1 = 1 \qquad c_2 = 3$

$4n + 500 <= 5n <= 12n + 15000$

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