$$③ \; ⓑ \quad A = \begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}, \quad X_1(t) = \begin{pmatrix} e^{2t}\sin t \\ -e^{2t}\cos t \end{pmatrix}, \quad X_2(t) = \begin{pmatrix} e^{2t}\cos t \\ e^{2t}\sin t \end{pmatrix}$$

$$\cancel{\begin{matrix} 2e^{2t}\sin t \\ e^{2t}\cos t \end{matrix}} \qquad X_1 \begin{pmatrix} 2e^{2t}\sin t & e^{2t}\cos t \\ e^{2t}\sin t & -2e^{2t}\cos t \end{pmatrix}$$

$$X_2 \begin{pmatrix} 2e^{2t}\cos t & -e^{2t}\sin t \\ e^{2t}\cos t & 2e^{2t}\sin t \end{pmatrix}$$

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7 b $A = \begin{pmatrix} 6 & 4 \\ 1 & 3 \end{pmatrix}$, $\vec{f}(t) = \begin{pmatrix} e^t \\ 2e^t \end{pmatrix}$

$$\begin{pmatrix} 6-r & 4 \\ 1 & 3-r \end{pmatrix}$$

$$(6-r)\cdot(3-r) - 4$$

$$18 - 6r - 3r + r^2 - 4$$

$$r^2 - 9r + 14$$

$r_1 = 2$ eigenvalue

$r_2 = 7$

eigenvectors $r_1 = 2$ $\begin{pmatrix} 6-2 & 4 \\ 1 & 3-2 \end{pmatrix} = \begin{pmatrix} 4 & 4 \\ 1 & 1 \end{pmatrix}$

$R_1 = R_1 - 4R_2$

$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_1} \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$ $x + y = 0$

$y = t$

$x + t = 0$

$x = -t$

$t\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t}$

$r_2 = 7$ $\begin{pmatrix} 6-7 & 4 \\ 1 & 3-7 \end{pmatrix} = \begin{pmatrix} -1 & 4 \\ 1 & -4 \end{pmatrix} \xrightarrow{R_2 = R_2 + R_1}$

$\begin{pmatrix} -1 & 4 \\ 0 & 0 \end{pmatrix}$ $-x + 4y = 0$

$y = t$

$-x + 4t = 0$

$4t = x$

$t\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 4 \\ 1 \end{pmatrix} e^{7t}$

general solution $X_{gh} = c_1 \begin{pmatrix} 4 \\ 1 \end{pmatrix} e^{7t} + c_2 \begin{pmatrix} -1 \\ 1 \end{pmatrix} e^{2t}$

$$X_c = \begin{pmatrix} 4e^{7t} & -e^{2t} \\ e^{7t} & e^{2t} \end{pmatrix}$$

formula for particular solution $\rightarrow X_{pn} = X_c \cdot \int X_c^{-1} f(t)\, dt$ *

$X_c^{-1} = \dfrac{1}{(4e^{7t}\cdot e^{2t}) - (-e^{2t}\cdot e^{7t})} = \dfrac{1}{4e^{9t} + e^{9t}} \cdot \begin{pmatrix} e^{2t} & e^{2t} \\ -e^{7t} & 4e^{7t} \end{pmatrix}$

switch main diagnol, the rest of number, add minus

$$\frac{1}{5e^{9t}} \cdot \begin{pmatrix} e^{2t} & e^{2t} \\ -e^{7t} & 4e^{7t} \end{pmatrix} = \begin{pmatrix} \frac{1}{5}e^{-7t} & \frac{1}{5}e^{-7t} \\ -\frac{1}{5}e^{-2t} & \frac{4}{5}e^{-2t} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{5}e^{-7t} & \frac{1}{5}e^{-7t} \\ \frac{-1}{5}e^{-2t} & \frac{4}{5}e^{-2t} \end{pmatrix} \begin{pmatrix} e^{t} \\ 2e^{t} \end{pmatrix} = \begin{pmatrix} e^t \cdot \frac{1}{5}e^{-7t} & 2e^t \cdot \frac{1}{5}e^{-7t} \\ e^t \cdot -\frac{1}{5}e^{-2t} & 2e^t \cdot \frac{4}{5}e^{-2t} \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{5}e^{-6t} + \frac{2}{5}e^{-6t} \\ -\frac{1}{5}e^{-t} + \frac{8}{5}e^{-t} \end{pmatrix} = \begin{pmatrix} \frac{3}{5}e^{-6t} \\ \frac{7}{5}e^{-t} \end{pmatrix} \overset{\text{integral}}{\Longrightarrow} \begin{pmatrix} \frac{\frac{3}{5}}{-6} e^{-6t} \\ \frac{\frac{7}{5}}{-1} e^{-t} \end{pmatrix} \Rightarrow \begin{pmatrix} \frac{3}{-30}e^{-6t} \\ \frac{7}{-5}e^{-t} \end{pmatrix}$$

applying particular solution formula

$$X_{pn} = \overset{X_c}{\begin{pmatrix} 4e^{7t} & -e^{2t} \\ e^{7t} & e^{2t} \end{pmatrix}} \cdot \begin{pmatrix} -\frac{1}{10}e^{-6t} \\ \frac{7}{-5}e^{-t} \end{pmatrix}$$

$$\begin{pmatrix} -\frac{1}{10}e^{-6t} \cdot 4e^{7t} & \frac{7}{-5}e^{-t} \cdot -e^{2t} \\ -\frac{1}{10}e^{-6t} \cdot e^{7t} & -\frac{7}{5}e^{-t} \cdot e^{2t} \end{pmatrix} = \begin{matrix} -\frac{4}{10}e^t + \frac{7}{5}e^t \\ -\frac{1}{10}e^t + -\frac{7}{5}e^t \end{matrix} \; /2$$

$$\begin{matrix} -\frac{4}{10}e^t + \frac{14}{10}e^t \\ -\frac{1}{10}e^t + -\frac{14}{10}e^t \end{matrix} = \begin{matrix} \cancel{}\, e^t \\ \frac{15}{10}\cancel{}\, e^t \end{matrix}$$

$$\vec{X}(t) = \vec{x}^h(t) + \vec{x}^p(t) = c_1\binom{-1}{1}e^{2t} + c_2\binom{4}{1}e^{7t} + \binom{1}{1\frac{1}{2}}e^t$$

$$⑦ \; (f) \quad A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad \vec{F}(t) = \begin{pmatrix} -1+\tan^2 t \\ \tan t \end{pmatrix}$$

eigenvalue $\begin{pmatrix} -r & 1 \\ -1 & -r \end{pmatrix}$ $\quad r^2 - (-1) = r^2 + 1$

$r = \pm 1$

eigenvectors $r_1 = i$ $\begin{pmatrix} -i & 1 \\ -1 & -i \end{pmatrix} \xRightarrow{R_2 : R_2 + iR_1} \begin{pmatrix} -i & 1 \\ 0 & 0 \end{pmatrix}$ $\quad -ix + y = 0$

$x = t$

$-it + y = 0$

$y = it$ $\quad t\begin{pmatrix} 1 \\ i \end{pmatrix}$

$$\Rightarrow \vec{a} + \vec{b}i = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} i \Rightarrow \begin{matrix} \vec{x}^1 t = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \sin t + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cos t = \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} \\ \vec{x}^2 t = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \cos t - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \sin t = \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} \end{matrix}$$

general solution: $\vec{x}(t) = c_1 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} + c_2 \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix}$ $\quad \begin{pmatrix} \sin t & \cos t \\ \cos t & -\sin t \end{pmatrix}$

find a non homogeneous solution

$$X_c^{-1} = \frac{1}{(\sin t \cdot -\sin t) - (\cos t \cdot \cos t)} = \frac{1}{-\sin^2 t - \cos^2 t} \cdot \begin{pmatrix} -\sin t & -\cos t \\ -\cos t & \sin t \end{pmatrix}$$

$$\begin{pmatrix} \frac{-\sin t}{-\sin^2 t - \cos^2 t} & \frac{-\cos t}{-\sin t^2 - \cos^2 t} \\ \frac{-\cos t}{\sin^2 t - \cos^2 t} & \frac{\sin t}{-\sin t^2 - \cos t} \end{pmatrix}$$

$-\sin^2 t - \cos^2 t = -1$

$$\begin{pmatrix} \sin t & \cos t \\ \cos t & -\sin t \end{pmatrix} \begin{pmatrix} -1 + \tan^2 t \\ \tan t \end{pmatrix}$$

$$\begin{pmatrix} \sin t(-1+\tan^2 t) + \cos t \tan t \\ \cos t(-1+\tan^2 t) + -\sin t \tan t \end{pmatrix}$$

$-\sin t + \sin t \tan^2 t + \cos t \tan t \Rightarrow -\sin t + \underbrace{\sin t \cdot \frac{\sin^2 t}{\cos^2 t}}_{\sin t \cdot \tan^2 t} + \cancel{\cos t} \frac{\sin t}{\cancel{\cos t}}$

$-\cos t + \cos t \tan^2 t - \sin t \tan t$

$$\Downarrow$$

$$\underbrace{-\cos t + \cancel{\cos t}}_{-\cos t} \frac{\sin^2 t}{\cancel{\cos t}} - \sin t \frac{\sin t}{\cos t}$$

$$\begin{pmatrix} \sin t \cdot \tan^2 t \\ -\cos t \end{pmatrix} \xrightarrow{\text{integral}} \quad \sin t \cdot \tan^2 t \Rightarrow \sin t(\sec^2 t - 1)$$

$$\tan^2 t = \sec^2 t - 1$$

$$\sec^2 t = \frac{1}{\cos^2 t}$$

$$\sin\left(\frac{1}{\cos^2 t} - 1\right) \qquad u = \cos t$$

$$du = -\sin t$$

$$-du = \sin t$$

$$\int -\left(\frac{1}{u^2} - 1\right) du$$

$$\int -\frac{1}{u^2} + 1 \, du$$

$$\frac{1}{u} + u \Rightarrow \cancel{\frac{1}{\cos(x)} + \sec(x)} \quad \cos t + \frac{1}{\cos t}$$

$$\begin{pmatrix} \cos t + \frac{1}{\cos t} \\ -\sin t \end{pmatrix}$$

$$\begin{pmatrix} \sin t & \cos t \\ \cos t & \sin t \end{pmatrix} \begin{pmatrix} \cos t + \frac{1}{\cos t} \\ -\sin t \end{pmatrix}$$

$$\begin{pmatrix} \sin t\left(\cos t + \frac{1}{\cos t}\right) + -\sin t \cos t \\ \cos t\left(\cos t + \frac{1}{\cos t}\right) + \sin^2 t \end{pmatrix} = \begin{matrix} \cancel{\sin t \cos t} + \frac{\sin t}{\cos t} - \cancel{\sin t \cos t} \\ \cos^2 t + 1 \quad - \sin^2 t \end{matrix}$$

$$\begin{pmatrix} \frac{\sin t}{\cos t} \\ \cos^2 + 1 + \sin^2 t \end{pmatrix} = \begin{pmatrix} \tan x \\ 1 + 1 = 2 \end{pmatrix}$$

answer: $C_1 \begin{pmatrix} \sin t \\ \cos t \end{pmatrix} + C_2 \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} + \begin{pmatrix} \tan t \\ 2 \end{pmatrix}$

ODE hw #4

① ⓐ $A(t) = \begin{pmatrix} \cos 4t \\ 2\sin 4t \end{pmatrix} \Rightarrow \frac{dA}{dt} = \begin{pmatrix} -4\sin 4t \\ 8\cos 4t \end{pmatrix}, \quad \int A(t)\,dt = \begin{pmatrix} \frac{1}{4}\sin 4t \\ -\frac{1}{2}\cos 4t \end{pmatrix}$

ⓓ $A(t) = \begin{pmatrix} e^{2t} & e^{3t} & e^{-t} \\ 2e^{2t} & e^{3t} & e^{-t} \\ 4e^{2t} & -e^{3t} & e^{t} \end{pmatrix} \Rightarrow \frac{dA}{dt} = \begin{pmatrix} 2e^{2t} & 3e^{3t} & -e^{-t} \\ 4e^{2t} & 3e^{3t} & -e^{-t} \\ 8e^{2t} & -3e^{3t} & -e^{-t} \end{pmatrix}, \quad \int A(t)\,dt = \begin{pmatrix} \frac{1}{2}e^{2t} & \frac{1}{3}e^{3t} & -e^{-t} \\ e^{2t} & \frac{1}{3}e^{3t} & -e^{-t} \\ 2e^{2t} & -\frac{1}{3}e^{3t} & -e^{-t} \end{pmatrix}$

② ⓒ $\begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix} = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} \Longrightarrow \vec{x}' = \begin{pmatrix} x_1' \\ x_2' \\ x_3' \end{pmatrix}, \quad A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 2 \end{pmatrix}, \quad \vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$\vec{x}' = A \cdot \vec{x}$

③ ⓓ $A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}, \quad x_1(t) = \begin{pmatrix} e^t \\ 0 \\ 0 \end{pmatrix}, \quad x_2(t) = \begin{pmatrix} te^t \\ e^t \\ 0 \end{pmatrix}, \quad x_3(t) = \begin{pmatrix} (2t + \frac{1}{2}t^2)e^t \\ te^t \\ e^t \end{pmatrix}$

$A \cdot \vec{x}_1 = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} e^t \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} e^t \\ 0 \\ 0 \end{pmatrix} = \vec{x}_1'$

$A \cdot \vec{x}_2 = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} te^t \\ e^t \\ 0 \end{pmatrix} = \begin{pmatrix} te^t + e^t \\ e^t \\ 0 \end{pmatrix} = \vec{x}_2'$

$A \cdot \vec{x}_3 = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} (2t + \frac{1}{2}t^2)e^t \\ te^t \\ e^t \end{pmatrix} = \begin{pmatrix} (2t + \frac{1}{2}t^2 + t + 2)e^t \\ te^t + e^t \\ e^t \end{pmatrix} = \vec{x}_3'$

④ ⓑ $A = \begin{pmatrix} 0 & 1 & 2 \\ 4 & 0 & 1 \\ 3 & -1 & 1 \end{pmatrix} \Rightarrow |A - r \cdot I| = 0 \Rightarrow \begin{vmatrix} -r & 1 & 2 \\ 4 & -r & 1 \\ 3 & -1 & 1-r \end{vmatrix} = 0$

$= -r\begin{vmatrix} -r & 1 \\ -1 & 1-r \end{vmatrix} - \begin{vmatrix} 4 & 1 \\ 3 & 1-r \end{vmatrix} + 2\begin{vmatrix} 4 & -r \\ 3 & -1 \end{vmatrix} = -r(r^2 - r + 1) - (4 - 4r - 3) + 2(-4 + 3r) = -r^3 + r^2 + 9r - 9$

$= (r-1)(-r^2 + 9) = (r-1)(3-r)(3+r) \Rightarrow r_1 = 1, \ r_2 = 3, \ r_3 = -3$

$(A - I)\vec{u}_1 = \vec{0} = \begin{pmatrix} -1 & 1 & 2 \\ 4 & -1 & 1 \\ 3 & -1 & 0 \end{pmatrix}\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{ll} u_{12} = 3u_{11} & = 3t \\ u_{13} = -u_{11} & = -t \\ u_{11} = t & \end{array} \Rightarrow \vec{u}_1 = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$

$(A - 3I)\vec{u}_2 = \vec{0} \Rightarrow \begin{pmatrix} -3 & 1 & 2 \\ 4 & -3 & 1 \\ 3 & -1 & -2 \end{pmatrix}\begin{pmatrix} u_{21} \\ u_{22} \\ u_{23} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{array}{l} u_{21} = 3u_{21} - 2u_{23} \\ u_{23} = 3u_{22} - 4u_{21} = 5u_{21} - 6u_{22} \\ u_{22} = \frac{5}{7}u_{21} \Rightarrow u_{22} = \frac{11}{7}u_{21} \end{array} \Rightarrow \vec{u}_2 = \begin{pmatrix} 1 \\ \frac{11}{7} \\ \frac{5}{7} \end{pmatrix}$

$n{:}0$ $\quad n{:}2$

$$(A+3I)\vec{u}_3=\vec{0} \Rightarrow \begin{pmatrix} 3 & 1 & 2 \\ 4 & 3 & 1 \\ 3 & -1 & 4 \end{pmatrix}\begin{pmatrix} u_{31} \\ u_{32} \\ u_{33} \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{matrix} u_{32}=u_{33} \\ u_{31}=-u_{33} \\ u_{33}=t \end{matrix} \Rightarrow \vec{u_3}=\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

$$\vec{x}(t)=\sum_{k=1}^{3} c_i \cdot \vec{u}_i \cdot e^{n \cdot t}=c_1\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}e^{t}+c_2\begin{pmatrix} 1 \\ \frac{11}{7} \\ \frac{5}{7} \end{pmatrix}e^{3t}+c_3\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}e^{-3t}$$

8 a) $y''-9xy=3$, $y=\sum_{n=0}^{\infty} a_n x^n \Rightarrow y''=\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}=\sum_{n=0}^{\infty}(n+2)(n+1)a_{n+2}x^n$

$$\Rightarrow \sum_{n=0}^{\infty}\left((n+2)(n+1)a_{n+2}x^n\right)-9x\sum_{n=0}^{\infty}a_n x^n=\sum_{n=0}^{\infty}\left((n+2)(n+1)a_{n+2}x^n\right)-\sum_{n=1}^{\infty}9a_{n-1}x^n$$

$$=2a_2+\sum_{n=1}^{\infty}\left((n+2)(n+1)a_{n+2}-9a_{n-1}\right)x^n=3 \Rightarrow a_2=\frac{3}{2}$$

$n=1$ $\quad 3\cdot2\cdot a_3-9a_0=0 \Rightarrow a_3=\frac{9a_0}{2\cdot3}$

$n=2$ $\quad 4\cdot3\,a_4-9a_1=0 \Rightarrow a_4=\frac{9a_1}{3\cdot4}$

$n=3$ $\quad 5\cdot4\,a_5-9a_2=0 \Rightarrow a_5=\frac{9a_2}{4\cdot5}$

$(n+2)(n+1)a_{n+2}-9a_{n-1}=0 \Rightarrow a_n=\frac{9a_{n-3}}{n(n-1)}$, $a_2=\frac{3}{2}$ $\quad 3\leq n$

b) $y''-9xy'+y=x^5$, $y(0)=-1$, $y'(0)=3$

$$y=\sum_{n=0}^{\infty}a_n x^n \Rightarrow y'=\sum_{n=1}^{\infty}na_n x^{n-1} \Rightarrow y''=\sum_{n=2}^{\infty}n(n-1)a_n x^{n-2}$$

$$\sum_{n=2}^{\infty}n(n-1)a_n x^{n-2}-\sum_{n=1}^{\infty}9na_n x^n+\sum_{n=0}^{\infty}a_n x^n=x^5$$

$$\Rightarrow \sum_{n=0}^{\infty}\left((n+1)(n+2)a_{n+2}x^n\right)-\sum_{n=1}^{\infty}\left(9na_n x^n\right)+\sum_{n=0}^{\infty}a_n x^n=2a_2+a_0+\sum_{n=1}^{\infty}\left((n+1)(n+2)a_{n+2}-9na_n+a_n\right)x^n=x^5$$

$y(0)=-1 \Rightarrow a_0=-1$, $y'(0)=3 \Rightarrow a_1=3$, $2a_2+a_0=0 \Rightarrow a_2=\frac{1}{2}$

$2\cdot3a_3-9a_1+a_1=0 \Rightarrow 6a_3=8a_1=3\cdot8 \Rightarrow a_3=4$ $\quad \underline{n=1}$

$3\cdot4a_4-18a_2+a_2=0 \Rightarrow 12a_4-17\cdot\frac{1}{2}=0 \Rightarrow a_4=\frac{17}{24}$ $\quad \underline{n=2}$

$\Rightarrow a_0=-1$, $a_1=3$, $a_2=\frac{1}{2}$, $a_3=4$, $a_4=\frac{17}{24}$

c) $y=\sum_{n=0}^{\infty}a_n x^n \Rightarrow y'=\sum_{n=1}^{\infty}na_n x^{n-1} \Rightarrow y'+x^2y=\sum_{n=1}^{\infty}a_n n x^{n-1}+\sum_{n=0}^{\infty}a_n x^{n+2}$

$$=\sum_{n=0}^{\infty}a_{n+1}(n+1)x^n+\sum_{n=2}^{\infty}a_{n-2}x^n=a_1+2a_2x+\sum_{n=2}^{\infty}\left(a_{n+1}(n+1)+a_{n-2}\right)x^n$$

$$= xe^{3x} = x \cdot \sum_{n=0}^{\infty} \frac{(3x)^n}{n!} = \sum_{n=0}^{\infty} \frac{3^n \cdot x^{n+1}}{n!} = x + 3x^2 + \sum_{n=2}^{\infty} \frac{3^n \cdot x^{n+1}}{n!}$$

$$= x + 3x^2 + \sum_{n=3}^{\infty} \frac{3^{n-1}}{(n-1)!} \cdot x^n \Rightarrow a_1 = 0 \;,\; a_2 = \frac{1}{2} \;,\; 3a_3 + a_0 = 3 \Rightarrow a_3 = \frac{3 - a_0}{3}$$

$$\left(a_{n+1}(n+1) + a_{n-2} \right) = \frac{3^{n-1}}{(n-1)!} \Rightarrow a_{n+1} = \frac{\frac{3^{n-1}}{(n-1)!} - a_{n-2}}{n+1} \quad ; 3 \leq n$$

$$= \frac{n \cdot 3^{n-1}}{(n+1)!} - \frac{a_{n-2}}{n+1}$$

$$\Rightarrow a_n = \frac{(n-1)\,3^{n-2}}{n!} - \frac{a_{n-3}}{n} \qquad \begin{array}{l} a_1 = 0 \\ a_2 = \frac{1}{2} \end{array} \quad n \geq 3$$