LA2 HW #5

① ③ (1) $A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \Rightarrow |A - \lambda I| = \left|\begin{pmatrix} 1-\lambda & -1 & 4 \\ 3 & 2-\lambda & -1 \\ 2 & 1 & -1-\lambda \end{pmatrix}\right| = (1-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ 1 & -1-\lambda \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ 2 & -1-\lambda \end{vmatrix}$

$+4\begin{vmatrix} 3 & 2-\lambda \\ 2 & 1 \end{vmatrix} = (1-\lambda)((2-\lambda)(-1-\lambda)+1) + 3(-1-\lambda)+2+4(3-2(2-\lambda))$

$= (1-\lambda)(\lambda^2-\lambda-1) - 3-3\lambda+2+12-16+8\lambda = -\lambda^3+2\lambda^2+5\lambda-6 \Rightarrow \boxed{|A-\lambda I| = -\lambda^3+2\lambda^2+5\lambda-6}$

(2) check when its ~~equal~~ to 0

$-\lambda^3+2\lambda^2+5\lambda-6=0 \Rightarrow (\lambda-1)(-\lambda^2+\lambda+6)=0 \Rightarrow (\lambda-1)(\lambda+2)(3-\lambda)=0$

$\Rightarrow \boxed{\lambda_1=1,\ \lambda_2=-2,\ \lambda_3=3}$

(3) $(A-I)V=0 \Rightarrow \begin{pmatrix} 0 & -1 & 4 \\ 3 & 1 & -1 \\ 2 & 1 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix} \Rightarrow B = \left\{\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}\right\} : \lambda=1$

$(A+2I)V=0 \Rightarrow \begin{pmatrix} 3 & -1 & 4 \\ 3 & 4 & -1 \\ 2 & 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix} \Rightarrow B = \left\{\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right\} : \lambda=2$

$(A+3I)V=0 \Rightarrow \begin{pmatrix} -2 & -1 & 4 \\ 3 & -1 & -1 \\ 2 & 1 & -4 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \Rightarrow B = \left\{\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}\right\} : \lambda=3$

(4) $P = (\bar{f_1}, \bar{f_2}, \bar{f_3}) = \begin{pmatrix} -1 & -1 & 1 \\ 4 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow D = P^{-1}AP = \begin{pmatrix} -\frac{1}{6} & \frac{1}{3} & -\frac{1}{2} \\ -\frac{1}{3} & -\frac{1}{3} & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} -1 & -1 & 1 \\ 4 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}$ yes it is

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

(5) $\Rightarrow A^{2019+3} = A^{2022} = (PDP^{-1})^{2022} = P \cdot D^{2022} \cdot P^{-1}$

$= \begin{pmatrix} -1 & -1 & 1 \\ 4 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1^{2022} & 0 & 0 \\ 0 & (-2)^{2022} & 0 \\ 0 & 0 & 3^{2022} \end{pmatrix}\begin{pmatrix} -\frac{1}{6} & \frac{1}{3} & -\frac{1}{2} \\ -\frac{1}{3} & -\frac{1}{3} & 1 \\ \frac{1}{2} & 0 & \frac{1}{2} \end{pmatrix} = \frac{1}{6}\begin{pmatrix} 1+2^{2023}+3^{2023} & 2^{2023}-2 & 3+3^{2023}-3\cdot 2^{2023} \\ -4-2^{2023}+2\cdot 3^{2023} & 8-2^{2023} & -12+3^{2023}+2\cdot 3^{2023} \\ -1-2^{2023}+3^{2023} & 2-2^{2023} & -3+3^{2023}+3\cdot 2^{2023} \end{pmatrix}$

⑥ (1) $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow |A-\lambda I| = \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2-1$

(2) $\lambda^2-1=0 \Rightarrow (\lambda+1)(\lambda-1)=0 \Rightarrow \boxed{\lambda_1=1,\ \lambda_2=-1}$

(3) $\lambda=1$: $(A-I)V=0 \Rightarrow \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = t\begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow B = \left\{\begin{pmatrix} 1 \\ 1 \end{pmatrix}\right\}$

$\lambda=-1$: $(A+I)V=0 \Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = t\begin{pmatrix} -1 \\ 1 \end{pmatrix} \Rightarrow B = \left\{\begin{pmatrix} -1 \\ 1 \end{pmatrix}\right\}$

(4) $P = (\bar{f_1}, \bar{f_2}) = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \Rightarrow D = P^{-1}AP = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ yes it is

(5) $A^{2019+6} = A^{2025} = (PDP^{-1})^{2025} = PD^{2025}P^{-1} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

⑨ (1) $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \Rightarrow |A-\lambda I| = \begin{vmatrix} 1-\lambda & 0 & 1 \\ 0 & 1-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^3 = -\lambda^3+3\lambda^2-3\lambda+1$ 3.7

(2) $(1-\lambda)^3 = 0 \Rightarrow \boxed{\lambda = 1}$

(3) $(A-I)v = 0 \Rightarrow \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + s\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \Rightarrow B = \{e_1, e_2\}$

(4) the algebraic plurality is 3 and the gometric is 2, therefore its not

(5) not relevent

③ ⓑ ② $\lambda x = Ax \Rightarrow x\lambda A^{-1} = x \Rightarrow x\frac{1}{\lambda} = xA^{-1} \Rightarrow \frac{1}{\lambda}$

here you can see $\frac{1}{\lambda}$ is an eigenvector of A^{-1}

④ $Ax = \lambda x \Rightarrow A^2x = A\lambda x \Rightarrow x = \lambda \cdot Ax = \lambda \cdot \lambda x \Rightarrow \lambda^2 = 1 \Rightarrow \boxed{\lambda = \pm 1}$

③ ⓒ not true

ⓓ if A is [illegible] then $A = P^{-1}BP$

$AV = \lambda V \Rightarrow P^{-1}BPV = \lambda V \Rightarrow BPV = P\lambda V = \lambda PV \Rightarrow$ therefore its ~~false~~ true

⑥ ⓐ $T\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 + 4x_3 \\ 3x_1 + 2x_2 - x_3 \\ 2x_1 + x_2 - x_3 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$|A-\lambda I| = \begin{vmatrix} 1-\lambda & 1 & 4 \\ 3 & 2-\lambda & -1 \\ 2 & 1 & -1-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ 1 & -1-\lambda \end{vmatrix} + \begin{vmatrix} 3 & -1 \\ 2 & -1-\lambda \end{vmatrix} + 4\begin{vmatrix} 3 & 2-\lambda \\ 2 & 1 \end{vmatrix}$

$= (1-\lambda)((2-\lambda)(-1-\lambda)+1) + 2-3-3\lambda+4(3-4+2\lambda) = (1-\lambda)(\lambda^2-\lambda-2)-5+5\lambda$

$= (1-\lambda)(\lambda^2-\lambda-1)-5(1-\lambda) = (1-\lambda)(\lambda^2-\lambda-6) = (1-\lambda)(\lambda-3)(\lambda+2) \Rightarrow \lambda_1 = 1, \lambda_2 = 3, \lambda_3 = -2$

ⓒ $\lambda = 1$: $(A-I)v = 0 \Rightarrow \left(\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 3 & 1 & -1 & 0 \\ 2 & 1 & -2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 3 & 0 & 3 & 0 \\ 2 & 1 & -2 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & -1 & 4 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 0 \end{array}\right) \Rightarrow \bar{f}_1 = \begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}$

$\lambda = 3$: $(A-3I)V = 0 \Rightarrow \left(\begin{array}{ccc|c} -2 & -1 & 4 & 0 \\ 3 & -1 & -1 & 0 \\ 2 & 1 & -4 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 3 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & 5 & -10 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \bar{f}_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\lambda = -2$: $(A+2I)V = 0 \Rightarrow \left(\begin{array}{ccc|c} 3 & -1 & 4 & 0 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & -5 & 5 & 0 \\ 1 & 3 & -2 & 0 \\ 2 & 1 & 1 & 0 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 0 & 1 & -1 & 0 \\ 1 & 3 & -2 & 0 \\ 0 & -5 & 5 & 0 \end{array}\right) \Rightarrow \bar{f}_3 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$B = \left\{\begin{pmatrix} -1 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}\right\}$

ⓓ $M^E_F = (M^F_E)^{-1} = \begin{pmatrix} -1 & 1 & -1 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -\frac{1}{6} & \frac{1}{3} & -\frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{3} & -\frac{1}{3} & 1 \end{pmatrix}$

$[T]^F_F = [I]^E_F [T]^E_E [I]^F_E = \begin{pmatrix} -\frac{1}{6} & \frac{1}{3} & -\frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{3} & -\frac{1}{3} & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 1 & -1 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

(e) $T^8(v) = [T]^8 \cdot v = [I]^f_e \cdot (T)^{f8}_f \cdot [I]^e_f \cdot v = \begin{pmatrix} -1 & 1 & -1 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1^8 & 0 & 0 \\ 0 & 3^8 & 0 \\ 0 & 0 & (-2)^8 \end{pmatrix} \begin{pmatrix} -\frac{1}{6} & \frac{1}{3} & -\frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ -\frac{1}{3} & -\frac{1}{3} & 1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 15722 \\ 33479 \\ 17083 \end{pmatrix}$

(8) (a) $T(p(x)) = (2x-1)p'(x) + 3p''(x)$, $B = (1, x, x^2)$

$[T]^B_B = (T(1), T(x), T(x^2)) = (0, 2x-1, 4x^2-2x+6) = \begin{pmatrix} 0 & -1 & 6 \\ 0 & 2 & -2 \\ 0 & 0 & 4 \end{pmatrix}$

(b) $|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} -\lambda & -1 & 6 \\ 0 & 2-\lambda & -2 \\ 0 & 0 & 4-\lambda \end{vmatrix} = 0 \Rightarrow \lambda(\lambda-2)(4-\lambda) = 0 \Rightarrow \lambda_1 = 0 \quad \lambda_2 = 2 \quad \lambda_3 = 4$

$Av = 0 \Rightarrow \left(\begin{array}{ccc|c} 0 & -1 & 6 & 0 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right) \Rightarrow v = t\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \Rightarrow B_1 = \{1\} \quad : \lambda_1 = 0$

$(A - 2I)v = 0 \Rightarrow \left(\begin{array}{ccc|c} -2 & 1 & 6 & 0 \\ 0 & 0 & -2 & 0 \\ 0 & 0 & -2 & 0 \end{array}\right) \Rightarrow v = t\begin{pmatrix} -\frac{1}{2} \\ 1 \\ 0 \end{pmatrix} \Rightarrow B_2 = \{-\frac{1}{2} + x\} : \lambda_2 = 2$

$(A - 4I)v = 0 \Rightarrow \left(\begin{array}{ccc|c} -4 & -1 & 6 & 0 \\ 0 & -2 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow v = t\begin{pmatrix} \frac{7}{4} \\ -1 \\ 1 \end{pmatrix} \Rightarrow B_3 = \{\frac{7}{4} - x + x^2\} : \lambda_3 = 4$

(c)

$[T]^F_F = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

(10) $T\begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ -2 & -2 \end{pmatrix}$, $T\begin{pmatrix} 3 & 0 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 9 & 0 \\ 9 & 9 \end{pmatrix}$, $T\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 4 & 4 \end{pmatrix}$, $T\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$

$B = \left\{ \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$, $G = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right\}$

(a) $[T]^B_B = ([T(b_1)]_B, [T(b_2)]_B, [T(b_3)]_B, [T(b_4)]_B) = \left(\begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}_B, \begin{bmatrix} 3 & 0 \\ 3 & 3 \end{bmatrix}_B, \begin{bmatrix} 0 & 4 \\ 4 & 4 \end{bmatrix}_B, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}_B \right)$

$= ((2,0,0,0), (0,3,0,0), (0,0,4,0), (0,0,0,-1)) = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

(b)

$\lambda_1 = 2 \quad Av = 2v \Rightarrow T(v) = 2v \Rightarrow v_1 = \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$

$\lambda_2 = 3 \quad Av = 3v \Rightarrow T(v) = 3v \Rightarrow v_2 = \begin{pmatrix} 3 & 0 \\ 3 & 3 \end{pmatrix}$

$\lambda_3 = 4 \quad Av = 4v \Rightarrow T(v) = 4v \Rightarrow v_3 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$

$\lambda_4 = -1 \quad Av = -v \Rightarrow T(v) = -v \Rightarrow v_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$[T]^G_G = [I]^B_G [T]^B_B [I]^G_B = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 2 \end{pmatrix}$

(12) $T(v) = \begin{pmatrix} 0 & 1 \\ 1 & a \end{pmatrix} v$

(a) False $T(1,0) = (0,1)$ λ doesn't exist s.t $(0,1) = \lambda(1,0)$

(b) False $T(0,1) = (1,a)$ λ doesn't exist s.t $(1,a) = \lambda(0,1)$

(c) False The determinent is -1 for every a therefore $|A - 0 \cdot I| = 0$ doesn't exist and $0 = J$

(d) True

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