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Calculus Quiz

① ⓐ $\lim\limits_{x \to -\infty} \dfrac{2-e^x}{2+e^x} = 1$

$$\left| \frac{2-e^x}{2+e^x} - 1 \right| < \varepsilon$$

$$\left| \frac{2-e^x - 2 - e^x}{2+e^x} \right| < \varepsilon$$

$$\left| \frac{-2e^x}{2+e^x} \right| < \varepsilon$$

$$\frac{2e^x}{2+e^x} < \varepsilon$$

$$e^x < \frac{\varepsilon(2+e^x)}{2}$$

$$\boxed{x < \ln \frac{\varepsilon(2+e^x)}{2}}$$

sin

ⓑ $\lim\limits_{x \to 0} \dfrac{\sin 6x - \sin 3x}{\sin 5x} \quad / \cdot \dfrac{5x}{5x}$

$$\frac{\sin 6x - \sin 3x}{\sin 5x} \cdot \frac{5x}{5x} \quad = L = 1 \qquad \frac{\sin 6x - \sin 3x}{5x} = \frac{\sin 6x}{5x} - \frac{\sin 3x}{5x} \quad / \cdot \frac{6x}{6x} \cdot \frac{3x}{3x}$$

$$\frac{\sin 6x}{5x} - \frac{\sin 3x}{5x} \cdot \underset{L=1}{\frac{6x}{6x}} \cdot \underset{L=1}{\frac{3x}{3x}} = \frac{6x}{5x} - \frac{3x}{5x} = \frac{6-3}{5} = \frac{3}{5}$$

② $f(x) = \dfrac{e^x}{e^x-1}$ $x=0$

vertical: $x=0$: $\lim\limits_{x\to\infty} \dfrac{e^x}{e^x-1} = \dfrac{1}{1-1} = \dfrac{1}{0} = \infty$

$x=0$ is a vertical asymptote

horizontal: $\lim\limits_{x\to+\infty} \dfrac{e^x}{e^x-1} = \dfrac{1}{0} = \infty$

$\lim\limits_{x\to-\infty} \dfrac{e^x}{e^x-1} = \dfrac{0}{0-1} = \dfrac{0}{-1} = 0$

not horizontal

slant: $\lim\limits_{x\to\infty} \dfrac{e^x}{\frac{e^x-1}{x}} = \dfrac{e^x}{xe^x-x} = \dfrac{0}{-\infty} = 0$

③ ⓐ true

Rational number is built $\dfrac{n}{m}$ $n\in\mathbb{Z}$ $m\in\mathbb{N}$ $\dfrac{p}{q}$ $p\in\mathbb{Z}$ $q\in\mathbb{N}$

$\dfrac{n}{m}\cdot\dfrac{p}{q} = \dfrac{np}{mq}$ $np\in\mathbb{Z}$ $mq\in\mathbb{N}$

ⓑ False

Irrational number $\mathbb{R}\setminus\mathbb{Q}$ $x\cdot y$

$x = \dfrac{p}{q}\in\mathbb{R}\setminus\mathbb{Q}$ $y = \dfrac{n}{m}\in\mathbb{R}\setminus\mathbb{Q}$

$x\cdot y = \dfrac{p\cdot n}{q\cdot m} = \dfrac{pn}{qm} \in\mathbb{R}$