# Probability HW #1

1 b $P(2 \text{ in } 1 \text{ box}, 1 \text{ in } 1 \text{ box}, 2 \text{ empty boxes})$

$= P(\text{same box as first, then an empty box}) + P(\text{diff box, then 1 of 1st two})$

$= P(2^{nd} \text{ in same a 1st}) * P(3^{rd} \text{ in empty}) + P(\text{place 2nd in empty}) * P(3^{rd} \text{ in box with 1})$

$= \frac{1}{4} \cdot \frac{3}{4} + \frac{3}{4} \cdot \frac{1}{2} = \frac{3}{16} + \frac{3}{8} = \frac{9}{16}$

3 a $\{A,B,C,D,1,2,3\}$ in 7 places, 1 each

all digits precede letters

$\{1,2,3\}, \{A,B,C,D\}$

$|E| = 3! \cdot 4! = 144$

$|\Omega| = 7! = 5040$

$P(E) = \frac{144}{5040} = \frac{1}{35}$

b digits adjacent, letters adjacent

$|E| = 2! \cdot 3! \cdot 4! = 288$

$P(E) = \frac{288}{5040} = \frac{2}{35}$

c all digits adjacent

L $\{1,2,3\}$ L L L

$|E| = 5! \cdot 3! = 720$

$P(E) = \frac{720}{5040} = \frac{1}{7}$

d No 2 letters adjacent

L N L N L N L

$4! \cdot 3! = 144$

$P(E) = \frac{144}{5040} = \frac{1}{35}$

5 15 workers including 3 staff 5 chosen

$|\Omega| = \binom{15}{5} = 3003 = \frac{15!}{5!\,10!}$

b $|E| = \binom{12}{4} \cdot \binom{3}{1} = \frac{12! \cdot 3!}{4! \cdot 8! \cdot 1!\, 2!} = 1485$

$P(E) = \frac{1485}{3003} = \frac{45}{91}$

d $|E| = \binom{12}{4}$ $\quad P(E) = \frac{495}{3003} = \frac{15}{91}$

f $|E| = \binom{12}{3} \cdot \binom{3}{2} = 220 \cdot 3 = 660$

$P(E) = \frac{660}{3003} = \frac{20}{91}$

h $|\bar{E}| = P(\text{no staff}) + P(\text{one staff}) = \binom{12}{5} + \binom{12}{4}\binom{3}{1} = 792 + 495 \cdot 3 = 2277$

$|E| = 3003 - 2277 = 726$

$P(E) = \frac{726}{3003} = \frac{22}{91}$

6 $|\Omega| = 7! = 5040$

b Dad right of mom

$|E| = 21 \cdot 5! = 2520$

$6+5+4+3+2+1$

$P(E) = \frac{2520}{5040} = \frac{1}{2}$

207

⑥ ⓓ Mom and Dad on ends, Arie Benny adjacent

$|E| = 2 \cdot 4! \cdot 2 = 96$

mom/dad permutations; kids counting AB as 1; A, B permutations

$P(E) = \frac{96}{5040} = \frac{2}{105}$

ⓕ Mom, A adj, A, B, C, D adj

$\{Mom, A\}$ $\{A, B, C, D\}$ $\{Dad\}$

$|E| = 3! \cdot 2! \cdot 4! = 288$

$P(E) = \frac{288}{5040} = \frac{2}{35}$

⑦ ⓒ $|\Omega| = 6!$

$\{Mom, Dad\}$ A, B, C, D, E

$|E| = 5! \cdot 2! = 240$

$P(E) = \frac{240}{720} = \frac{1}{3}$

⑨ $\{5,5,5,2,2, [346789], [346789]\}$

$\text{combinations} = \binom{7}{3}\binom{4}{2} \cdot 6 \cdot 6 = 7560$

choose 3 spots for 5's; choose 2 of remaining for 2's; 36 ways to choose 2 numbers from $[3,4,6,7,8,9]$

$P(E) = \frac{1}{7560}$

⑪ ⓐ $\times \{A, E\}, B, C, D$ $|\Omega| = 5!$

$|\bar{E}| = 4! \cdot 2! = 48$

$|E| = 5! - 48 = 72$

$P(E) = \frac{72}{120} = \frac{3}{5}$

ⓒ $|E| = 6 \cdot 3! = 36$

$P(E) = \frac{36}{120} = \frac{3}{10}$

$\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$

ⓑ AE... ·AE·- --AE· ---A·E ---AE

A·E-- ·A·E·

A--E· ·A--E

A···E

$|E| = 10 \cdot 3! = 60$

$P(E) = \frac{60}{120} = \frac{1}{2}$

13 (b) $5 \times$ (R) $6 \times$ (W) $|\Omega| = 11 \cdot 10 = 110$

$|E| = 4 \cdot 5 = 20$

$P(E) = \frac{20}{110} = \frac{2}{11}$

(d) $P(1 \text{ or both red}) = 1 - P(\text{both white})$

$|\bar{E}| = 6 \cdot 5 = 30$ (ways to match 2 white balls)

$|E| = 110 - 30 = 80$

$P(E) = \frac{80}{110} = \frac{8}{11}$

(f) $|\Omega| = 11 \cdot 11 = 121$

$|E| = 5 \cdot 6 + 6 \cdot 5 = 60$

$P(E) = \frac{60}{121}$

15 (b) 5 math, 4 physics, 6 computers - 4 chosen at random

2 comp, 2 physics

$|\Omega| = \binom{15}{4} = 1365$ $|E| = \binom{6}{2}\binom{4}{2} = 15 \cdot 6$ $P(E) = \frac{90}{1365} = \frac{6}{91}$

(d) at least 1 of each

$P(2,1,1) + P(1,2,1) + P(1,1,2)$

$|E| = \binom{5}{2}\binom{4}{1}\binom{6}{1} + \binom{5}{1}\binom{4}{2}\binom{6}{1} + \binom{5}{1}\binom{4}{1}\binom{6}{2} =$

$10 \cdot 4 \cdot 6 + 5 \cdot 6 \cdot 6 + 5 \cdot 4 \cdot 15 = 720$

$P(E) = \frac{720}{1365} = \frac{48}{91}$

17 (b) 2 grades 1-7, 4 grades 1-5, find $P(\text{all} \geq 4, \text{ at least } 1 > 4)$

$|\Omega| = 7 \cdot 7 \cdot 5 \cdot 5 \cdot 5 \cdot 5 = 30{,}625$

$|E| = |\text{all} \geq 4| - |\text{all} = 4|$

$= 4 \cdot 4 \cdot 2 \cdot 2 \cdot 2 \cdot 2 - 1 = 255$

$P(E) = \frac{255}{30{,}625} = \frac{51}{6125}$

(d) $P(\text{all \#'s different}) = P(7 \text{ and } 6, \text{ then 4 of } 1\text{-}5) + P(\text{one of 1st two is 6 or 7, then 4 of 1-5})$

$$= \Big(\underbrace{\tfrac{1}{7} \cdot \tfrac{1}{7}}_{\text{getting 6 \& 7}} \cdot \underbrace{\tfrac{5}{5} \cdot \tfrac{4}{5} \cdot \tfrac{3}{5} \cdot \tfrac{2}{5}}_{\text{getting 4 diff numbers 1-5}}\Big) \cdot \underbrace{2}_{\text{permutations of 6,7}} + \underbrace{\Big(\underbrace{\tfrac{2}{7} \cdot \tfrac{5}{7}}_{\text{getting a 6 or 7 and a 1-5}} \cdot \underbrace{\tfrac{4}{5} \cdot \tfrac{3}{5} \cdot \tfrac{2}{5} \cdot \tfrac{1}{5}}_{\text{getting the other 4 unique \#'s 1-5}}\Big) \cdot \underbrace{2}_{\text{permutations of 6,7 before or after 1-5}}} = \frac{48}{6125} + \frac{96}{6125} = \frac{144}{6125}$$

(18) #1-600 selected

(a) not divisable by 2 or 3

$|\Omega| = 600$

$|E| = 600 - (|\#\text{'s div by } 2| + |\#\text{'s div by } 3| - |\#\text{'s div by } 2 \text{ and } 3|)$

$= 600 - (300 + 200 - 100)$

$= 200$

$\frac{1}{2} 200$

$P(E) = \frac{200}{600} = \frac{1}{3}$

(b) not divisable by 2, 3, or 5

$|E| = 600 - (|\#\text{'s div by } 2| + |\text{div by } 3| + |\text{div by } 5| - |\text{div by } 2,3|$
$- |\text{div by } 3,5| - |\text{div by } 2,5| + |\text{div by } 2,3,5|)$

$= 600 - (300 + 200 + 120 - \frac{200}{2} - \frac{200}{5} - \frac{120}{2} + \frac{120}{2 \cdot 3})$

$= 600 - (300 + 200 + 120 - 100 - 40 - 60 + 20)$

$= 600 - 160$

$= 440$

$P(E) = \frac{440}{600} = \frac{11}{15}$