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# Probability HW#4

3 a

| K | $P(X=K)$ |
|---|---|
| -3 (0 heads) | $\frac{1}{8}$ |
| -1 (1 head) | $\frac{3}{8}$ |
| 1 (2 heads) | $\frac{3}{8}$ |
| 3 (3 heads) | $\frac{1}{8}$ |

$E[X] = -3\cdot\frac{1}{8} + -1\cdot\frac{3}{8} + 1\cdot\frac{3}{8} + 3\cdot\frac{1}{8}$
$= 0$

b $P(H)=\frac{1}{3}$ $P(T)=\frac{2}{3}$

| K | $P(X=K)$ |
|---|---|
| -3 | $(\frac{2}{3})^3 = \frac{8}{27}$ |
| -1 | $\frac{1}{3}(\frac{2}{3})^2\cdot\binom{3}{1} = \frac{12}{27}$ |
| 1 | $(\frac{1}{3})^2(\frac{2}{3})\binom{3}{2} = \frac{6}{27}$ |
| 3 | $(\frac{1}{3})^3 = \frac{1}{27}$ |

$E[X] = -3\cdot(\frac{8}{27}) + -1\cdot(\frac{12}{27}) + 1\cdot(\frac{6}{27}) + 3\cdot(\frac{1}{27})$
$= -\frac{24}{27} - \frac{12}{27} + \frac{6}{27} + \frac{3}{27}$

4 $X$ = profit from a single product

| K | $P(X=K)$ |
|---|---|
| 50 | $(0.98)\cdot(0.95)\cdot(0.99) = 0.92169$ |
| 20 | $(0.02)(0.95)(0.99) + (0.98)(0.05)(0.99) + (0.98)(0.95)(0.01) = 0.07663$ |
| 10 | $(0.02)(0.05)(0.99) + (0.02)(0.95)(0.01) + (0.98)(0.05)(0.01) = 0.00167$ |
| 0 | $(0.02)(0.05)(0.01) = 0.0001$ |

$E[X] = 50(0.92169) + 20(0.07663) + 10(0.00167) = 47.6338$

$E[X^2] = 2500(0.92169) + 400(0.07663) + 100(0.00167) = 2335.044$

$V(X) = E[X^2] - (E[X])^2 = 2335.04 - (47.6338)^2 = 66.26109$

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| K | $P(X=K)$ |
|---|---|
| 0 | $(\frac{1}{2})(\frac{2}{3})(\frac{3}{4}) = \frac{6}{24} = \frac{1}{4}$ |
| 1 | $(\frac{1}{2})(\frac{2}{3})(\frac{3}{4}) + (\frac{1}{2})(\frac{1}{3})(\frac{3}{4}) + (\frac{1}{2})(\frac{2}{3})(\frac{1}{4}) = \frac{6}{24} + \frac{3}{24} + \frac{2}{24} = \frac{11}{24}$ |
| 2 | $(\frac{1}{2})(\frac{1}{3})(\frac{3}{4}) + (\frac{1}{2})(\frac{2}{3})(\frac{1}{4}) + (\frac{1}{2})(\frac{1}{3})(\frac{1}{4}) = \frac{6}{24} = \frac{1}{4}$ |
| 3 | $(\frac{1}{2})(\frac{1}{3})(\frac{1}{4}) = \frac{1}{24}$ |

a $Y = 3 - X$, $X = 3 - Y$

b $E[X] = 1\cdot\frac{11}{24} + 2\cdot\frac{6}{24} + 3\cdot\frac{1}{24} = \frac{26}{24} = \frac{13}{12}$

$E[X^2] = 1\cdot\frac{11}{24} + 4\cdot\frac{6}{24} + 9\cdot\frac{1}{24} = \frac{44}{24} = \frac{22}{12}$ $V(X) = \frac{22}{12} - (\frac{13}{12})^2 = \frac{95}{144}$

c $E[3-X] = -E[X] + 3 = -\frac{13}{12} + \frac{36}{12} = \frac{23}{12}$

$V(3-X) = V(-X+3) = V(-X) = (-1)^2 V(X) = V(X) = \frac{95}{144}$

8 a

| K | $P(X=K)$ |
|---|---|
| 0 | $(\frac{5}{7})^2 = \frac{25}{49}$ |
| 1 | $(\frac{5}{7})(\frac{2}{7})\binom{2}{1} = \frac{20}{49}$ |
| 2 | $(\frac{2}{7})^2 = \frac{4}{49}$ |

$E[X] = 1\cdot\frac{20}{49} + 2\cdot\frac{4}{49}$
$= \frac{28}{49} = \frac{4}{7}$

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ⓑ

| $k$ | $P(X=k)$ |
|---|---|
| 0 | $(\frac{5}{7})(\frac{4}{6}) = \frac{20}{42} = \frac{10}{21}$ |
| 1 | $(\frac{5}{7})(\frac{2}{6}) + (\frac{2}{7})(\frac{5}{6}) = \frac{20}{42} = \frac{10}{21}$ |
| 2 | $(\frac{2}{7})(\frac{1}{6}) = \frac{2}{42} = \frac{1}{21}$ |

$E[X] = 1 \cdot \frac{10}{21} + 2 \cdot \frac{1}{21}$
$= \frac{12}{21} = \frac{4}{7}$

ⓒ i. both

| $k$ | $P(X_1, X_2 = k)$ |
|---|---|
| 0 | $\frac{5}{7}$ |
| 1 | $\frac{2}{7}$ |

ii. $E[X_1] = \frac{2}{7}$ $\quad Y = \frac{2}{7} + \frac{2}{7} = \frac{4}{7}$

iii. $Y$ is the # of chosen white balls, and the probability is unchanged regardless of parts a,b with replacement or not. we look at the whole.

⑩

| $k$ | $P(X=k)$ |
|---|---|
| 0 | $(\frac{3}{4})^4 = \frac{81}{256}$ |
| 1 | $(\frac{3}{4})^3(\frac{1}{4})\binom{4}{1} = \frac{27}{64} = \frac{108}{256}$ |
| 2 | $(\frac{3}{4})^2(\frac{1}{4})^2\binom{4}{2} = \frac{9}{16} \cdot \frac{1}{16} \cdot 6 = \frac{54}{256}$ |
| 3 | $(\frac{3}{4})(\frac{1}{4})^3\binom{4}{3} = \frac{3}{64} = \frac{12}{256}$ |
| 4 | $(\frac{1}{4})^4 = \frac{1}{256}$ |

$E[X] = 1 \cdot \frac{108}{256} + 2 \cdot \frac{54}{256} + 3 \cdot \frac{12}{256} + 4 \cdot \frac{1}{256}$
$= \frac{108}{256} + \frac{108}{256} + \frac{36}{256} + \frac{4}{256} = 1$

⑫

| $k$ | $P(X=k)$ (sum) |
|---|---|
| 2 | $(\frac{1}{6})^2 = \frac{1}{36}$ |
| 3 | $\frac{2}{36}$ |
| 4 | $\frac{3}{36}$ |
| 5 | $\frac{4}{36}$ |
| 6 | $\frac{5}{36}$ |
| 7 | $\frac{6}{36}$ |
| 8 | $\frac{5}{36}$ |
| 9 | $\frac{4}{36}$ |
| 10 | $\frac{3}{36}$ |
| 11 | $\frac{2}{36}$ |
| 12 | $\frac{1}{36}$ |

| $k$ | $P(Y=k)$ (max) |
|---|---|
| 1 | $\frac{1}{36}$ |
| 2 | $\frac{3}{36}$ |
| 3 | $\frac{5}{36}$ |
| 4 | $\frac{7}{36}$ |
| 5 | $\frac{9}{36}$ |
| 6 | $\frac{11}{36}$ |

| $k$ | $P(Z=k)$ (min) |
|---|---|
| 1 | $\frac{11}{36}$ |
| 2 | $\frac{9}{36}$ |
| 3 | $\frac{7}{36}$ |
| 4 | $\frac{5}{36}$ |
| 5 | $\frac{3}{36}$ |
| 6 | $\frac{1}{36}$ |

ⓐ $X = Y + Z$

ⓑ $E[X] = \frac{2}{36} + \frac{6}{36} + \frac{11}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{36}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36} = \frac{252}{36} = 7$

Scanned with CamScanner

$$E[x^2] = \frac{4}{36} + \frac{18}{36} + \frac{48}{36} + \frac{100}{36} + \frac{180}{36} + \frac{294}{36} + \frac{320}{36} + \frac{324}{36} + \frac{300}{36} + \frac{242}{36} + \frac{144}{36} = \frac{1974}{36}$$

$$v(x) = \frac{329}{6} - 49 = \frac{35}{6} = 5.8\overline{3}$$

$$E[Y] = \frac{1}{36} + \frac{6}{36} + \frac{15}{36} + \frac{28}{36} + \frac{45}{36} + \frac{66}{36} = \frac{161}{36}$$

$$E[Y^2] = \frac{1}{36} + \frac{12}{36} + \frac{45}{36} + \frac{112}{36} + \frac{225}{36} + \frac{396}{36} = \frac{791}{36}$$

$$V(Y) = \frac{791}{36} - \left(\frac{161}{36}\right)^2 = \frac{791}{36} - \frac{25921}{1296} = 1.971$$

$$E[Z] = \frac{11}{36} + \frac{18}{36} + \frac{21}{36} + \frac{20}{36} + \frac{15}{36} + \frac{6}{36} = \frac{91}{36}$$

$$E[Z^2] = \frac{11}{36} + \frac{36}{36} + \frac{63}{36} + \frac{80}{36} + \frac{75}{36} + \frac{36}{36} = \frac{301}{36}$$

$$V(Z) = \frac{301}{36} - \left(\frac{91}{36}\right)^2 = 1.971$$

(c) $E[x] = E[Y] + E[Z] \rightarrow 7 = \frac{161}{36} + \frac{91}{36} = \frac{252}{36}$ ✓

variance is not satisfied because Y, Z are dependent

(16)

| k | $P(x=k)$ |
|---|---|
| 1 | $\frac{a}{3}$ |
| 2 | $\frac{a}{9}$ |
| 3 | $\frac{a}{27}$ |
| 4 | $\frac{a}{81}$ |
| 5 | $\frac{a}{243}$ |

$$\sum_k P(x=k) = 1$$

$$a\left(\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243}\right) = 1$$

$$a\left(\frac{121}{243}\right) = 1$$

$$a = \frac{243}{121}$$

(20) (a)

| k | $P(x=k)$ |
|---|---|
| 2 | $\left(\frac{2}{6}\right)\left(\frac{1}{5}\right) = \frac{2}{30}$ |
| 3 | $\left(\frac{4}{6}\right)\left(\frac{2}{5}\right)\left(\frac{1}{4}\right) + \left(\frac{2}{6}\right)\left(\frac{4}{5}\right)\left(\frac{1}{4}\right) = \frac{8}{120} + \frac{8}{120} = \frac{16}{120} = \frac{4}{30}$ |
| 4 | $\left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{4}{6}\right)\left(\frac{2}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) + \left(\frac{2}{6}\right)\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{1}{3}\right) = \frac{96}{360} = \frac{8}{30}$ |
| 5 | $\left[\left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\right]2 + \left[\left(\frac{4}{6}\right)\left(\frac{3}{5}\right)\left(\frac{2}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\right]2 + \left[\left(\frac{4}{6}\right)\left(\frac{2}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\right]2 + \left[\left(\frac{2}{6}\right)\left(\frac{4}{5}\right)\left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right)\right]2 = \frac{192}{360} = \frac{16}{30}$ |

(b) $E[x] = \frac{4}{30} + \frac{12}{30} + \frac{32}{30} + \frac{80}{30} = \frac{128}{30} = \frac{64}{15}$

$$E[x^2] = \frac{8}{30} + \frac{36}{30} + \frac{128}{30} + \frac{400}{30} = \frac{572}{30} = \frac{286}{15}$$

$$V(x) = \frac{286}{15} - \left(\frac{64}{15}\right)^2 = 0.86\overline{2} \qquad \sigma(x) = \sqrt{0.86\overline{2}} = 0.9286$$

(c) i. $\frac{3}{4}$ ii. $\frac{1}{4}$

(d) define events:

let B = all remaining balls are blue

let $A_2, \ldots, A_5 = x = 2, \ldots, 5$

sol

$$P(B) = P(A_2)\cdot P(B/A_2) + \dots + P(A_5)\cdot P(B/A_5)$$
$$= \frac{2}{30}\cdot 1 + \frac{4}{30}\cdot 1 + \frac{8}{30}\cdot\frac{3}{4} + \frac{16}{30}\cdot\frac{1}{2} = \frac{2}{30} + \frac{4}{30} + \frac{24}{30} + \frac{8}{30} = \frac{2}{3}$$

i. $P(A_2/B) = \dfrac{P(A_2)\cdot P(B/A_2)}{P(B)} = \dfrac{\frac{2}{30}\cdot 1}{\frac{2}{3}} = \dfrac{1}{10}$

ii. $P(A_4/B) = \dfrac{P(A_4)\cdot P(B/A_4)}{P(B)} = \dfrac{\frac{8}{30}\cdot\frac{3}{4}}{\frac{2}{3}} = \dfrac{3}{10}$

iii. 0

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| K | P(X=K) |
|---|---|
| 0 | $\left(\frac{5}{8}\right)^5 = \frac{3125}{32768}$ |
| 1 | $\left(\frac{5}{8}\right)^4\left(\frac{3}{8}\right)\binom{5}{1} = \frac{9375}{32768}$ |
| 2 | $\left(\frac{5}{8}\right)^3\left(\frac{3}{8}\right)^2\binom{5}{2} = \frac{11250}{32768}$ |
| 3 | $\left(\frac{5}{8}\right)^2\left(\frac{3}{8}\right)^3\binom{5}{3} = \frac{6750}{32768}$ |
| 4 | $\left(\frac{5}{8}\right)\left(\frac{3}{8}\right)^4\binom{5}{4} = \frac{2025}{32768}$ |
| 5 | $\left(\frac{3}{8}\right)^5 = \frac{243}{32768}$ |

| K | P(Y=K) |
|---|---|
| 0 | $\left(\frac{5}{8}\right)\left(\frac{4}{7}\right)\left(\frac{3}{6}\right)\left(\frac{2}{5}\right)\left(\frac{1}{4}\right) = \frac{\binom{5}{5}}{\binom{8}{5}} = \frac{1}{56}$ |
| 1 | $\frac{\binom{5}{4}\binom{3}{1}}{\binom{8}{5}} = \frac{15}{56}$ |
| 2 | $\frac{\binom{5}{3}\binom{3}{2}}{\binom{8}{5}} = \frac{30}{56}$ |
| 3 | $\frac{\binom{5}{2}\binom{3}{3}}{\binom{8}{5}} = \frac{10}{56}$ |

$$E[X] = \frac{9375 + 2\cdot(11250) + 3\cdot(6750) + 4\cdot(2025) + 5\cdot(243)}{32768} = \frac{61440}{32768} = \frac{15}{8}$$

$$E[Y] = \frac{15}{56} + \frac{60}{56} + \frac{30}{56} = \frac{105}{56} = \frac{15}{8}$$