Calc 2 hw #4

Ⓐ ④ $f(x,y) = x\sin y$

$f'_x : \frac{\partial f}{\partial x} = \lim_{h\to 0} \frac{f(x_0+h, y_0) - f(x_0, y_0)}{h} =$

$\frac{(x_0+h)\cdot\sin(y_0) - x_0\cdot\sin y_0}{h} = \frac{\sin(y_0)\,(x_0+h-x_0)}{h}$

$= \lim_{h\to 0} \frac{h\sin y_0}{h} = \sin(y_0)$

$f_y = \frac{\partial f}{\partial y} = \lim_{h\to 0} \frac{f(x_0, y_0+h) - f(x_0, y_0)}{h}$

$= \frac{x_0\sin(y_0+h) - x_0\cdot\sin(y_0)}{h} = \frac{x_0\cdot(\sin(y_0+h) - \sin(y_0))}{h}$

$-\sin(x) = \sin(-x)$

$\left[\sin(x) - \sin(y) = 2\cdot\sin\left(\frac{x-y}{2}\right)\cdot\cos\left(\frac{x+y}{2}\right)\right]$

$= \lim_{h\to 0} \frac{x_0\cdot 2\cdot\sin\left(\frac{h}{2}\right)\cdot\cos\left(\frac{2y_0+h}{2}\right)\cdot\frac{1}{2}}{h\cdot\frac{1}{2}}$

$= \lim_{h\to 0}\left(\frac{\sin\frac{h}{2}}{\frac{h}{2}}\right)\cdot\lim_{h\to 0}\left(x_0\cdot\cos\left(\frac{2y_0+h}{2}\right)\right) = 1\cdot x_0\cdot\cos(y_0)$

$= x_0\cdot\cos(y_0)$

$f'_x = f(x,y)' = \sin(y) \quad [(ax)' = a]$

$f'_y = f(x,y)' = x\cdot\cos(y) \quad [\sin(y)' = \cos(y)]$

Ⓑ ② $f(x,y) = 2^{xy^2}, \ (1,-1)$

$f'_x = \ln(2)\cdot(xy^2)'\cdot 2^{xy^2} = \ln(2)\cdot y^2\cdot 2^{xy^2}$

$f'_y = \ln(2)\cdot(xy^2)'\cdot 2^{xy^2} = 2xy\cdot\ln(2)\cdot 2^{xy^2}$

$f'_x(1,-1) = \ln(2)\cdot(-1)^2\cdot 2^1 = 2\ln(2) = \ln(4)$

$f'_y(1,-1) = 2\cdot 1\cdot(-1)\cdot\ln(2)\cdot 2^1 = -4\ln(2) = \ln\left(\frac{1}{16}\right)$

Ⓒ ③ $f(x,y) = x\sin(y) + e^y$

$f'_x(x,y) = \sin(y)$

$f''_{xx}(x,y) = 0$

$f''_{xy}(x,y) = \cos(y)$

$f'_y(x,y) = x\cdot\cos(y) + e^y$

$f''_{yy}(x,y) = -x\sin(y) + e^y$

$f''_{yx}(x,y) = \cos(y)$ $\qquad f''_{xy} = f''_{yx}$

$①②$ $f(x,y) = \sqrt{y}$

$f'_x(x,y) = 0$, $f'_x(0,0) = 0$

$f'_y(x,y) = \frac{1}{2\sqrt{x}}$ $f'_y(0,0) =$ değer yok

$$\lim_{\substack{h\to 0\\ y_0 \to 0}} \frac{f(x_0, y_0+h) - f_0(x_0, y_0)}{h} = \frac{(\sqrt{y_0+h} - \sqrt{y_0}) \cdot (\sqrt{y_0+h} + \sqrt{y_0})}{h \cdot (\sqrt{y_0+h} + \sqrt{y_0})}$$

$$= \frac{y_0+h-y_0}{h \cdot (\sqrt{y_0+h} + \sqrt{y_0})} = \frac{1}{\sqrt{y_0+h} + \sqrt{y_0}} = \lim_{h\to 0} \frac{1}{\sqrt{h}} = \infty$$

③ $f(x,y) = \ln(e^x + e^y)$

$$f_x = \frac{e^x}{e^x+e^y} \cdot f_y \frac{e^y}{e^x+e^y} \quad \frac{e^x}{e^x+e^y} + \frac{e^y}{e^x+e^y} = \frac{e^x+e^y}{e^x+e^y} = 1$$

$$f_{xx} \frac{e^x}{e^x+e^y} = \frac{(e^x+e^y)(e^x) - (e^x e^x)}{(e^x+e^y)^2} \Rightarrow \frac{e^{x+y}}{(e^x+e^y)^2}$$

$$f_{yy} \frac{e^y}{e^x+e^y} : \frac{(e^x+e^y)(e^y) - (e^y)(e^y)}{(e^x+e^y)^2} \Rightarrow \frac{e^{x+y}}{(e^x+e^y)^2}$$

$$\left(\frac{e^{x+y}}{(e^x+e^y)^2}\right) \cdot \left(\frac{e^{x+y}}{(e^x+e^y)^2}\right) - \left(\frac{e^{x+y}}{(e^x+e^y)^2}\right)^2 = 0$$

$$f_{xy} \ e^x (e^x + e^y)^{-1} = \frac{e^x \cdot e^y}{(e^x+e^y)^2}$$

$⑤①$ $z = x^2y + y^2x$, $x = t^3+1$, $y = 1-t^4$

$f'(t) = f(x) \cdot x'(t) + f_y \cdot y'(t)$

$x'(t) = 3t^2 \quad y'(t) = -4t^3$

$f_x = 2xy + y^2 = 2(t^3+1)(1-t^4) + (1-t^4)^2$

$f_y = x^2 + 2xy = (t^3+1)^2 + 2(t^3+1)(1-t^4)$

$6t^2 (t^3+1)(1-t^4) + 3 + 2(1-t^4) - 4 + 3(t^3+1)^2)$

$+ -8 + 3(t^3+1)(1-t^4)$

$⑤②$ $F(x,y)$ $x(t) = u^3+2v$ $y(t) = u$

$f_x = ye^{xy}$ $f_y = xe^{xy}$ $x'(t) = 3x^2$ $y'(t) = 1$

$x'(v) = 2$ $y'(v) = 0$

$\Downarrow$ $\Downarrow$

$ue^{(u^3+2v)u}$ $u^3+2v\, e^{(u^3+2v)u}$

$$\frac{\partial z}{\partial u} \ 3u^3 e^{(u^3+2v)u} + u^3+2v\, e^{(u^3+2v)u}$$

⑨ $x = y = t$ $g(t) = f(x(t), y(t))$

$$\frac{dg}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} \qquad \frac{dg}{dt} = \frac{dg}{dx}$$

$x = y = t$

$$g(x)' = \frac{\partial f}{\partial x} \cdot 1 + \frac{\partial f}{\partial y} \cdot 1$$

$$g(x)' = f_x'(x,y) + f_y'(x,y)$$

$$g'(x) = f_x'(x,y) + f_y'(x,x)$$

$$g'(x) = f_x'(x,x)$$

4) 1) $2\ln\sqrt{x^2+y^2} + x = 1$

$$f(x,y) = \ln(x^2+y^2) + x - 1 = 0$$

$x = t$

$$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t}$$

$$0 = f_x(x,y) \cdot 1 + f_y(x,y) \cdot y'$$

$$-y' \cdot f_y(x,y) = f_x(x,y)$$

$$y' = -\frac{f_x(x,y)}{f_y(x,y)}$$

$$y' = -\frac{\frac{2x}{x^2+y^2} + 1}{\frac{2y}{x^2+y^2}} = -\frac{\frac{2x + x^2+y^2}{x^2+y^2}}{\frac{2y}{x^2+y^2}} = -\frac{2x + x^2+y^2}{2y}$$

$$\frac{dy}{dx} = -\frac{2x + x^2+y^2}{2y}$$

$$\left[2\ln\left(\sqrt{x^2+y^2} + x\right]' = [1]'$$

$$[\ln(x^2+y^2) + x]' = 0$$

$$\frac{1}{x^2+y^2} \cdot (2x + 2yy') + 1 = 0$$

$$(2x + 2yy') = -(x^2+y^2)$$

$$2yy' = -(x^2+y^2+2x)$$

$$y' = -\frac{x^2+y^2+2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x^2+y^2+2x}{2y}$$

5) 1) $$\vec{\nabla} f(x,y) = \begin{pmatrix} f_x(x,y) \\ f_y(x,y) \end{pmatrix} = \begin{pmatrix} \frac{2x}{x^2+y^2} \\ \frac{2y}{x^2+y^2} \end{pmatrix}$$

$$\vec{\nabla} f(1,0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix} \qquad \vec{\nabla} f(2,-1) = \begin{pmatrix} \frac{4}{5} \\ -\frac{2}{5} \end{pmatrix}$$

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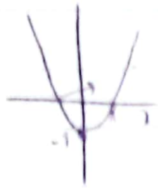
(J) (2) $f_x = -2x \quad f_y = 1$

$f(1,1)$



(K) (3) $f(x,y) = x^2 - y^2$

$\nabla f(x,y) = \begin{pmatrix} f_x(x,y) \\ f_y(x,y) \end{pmatrix} = \begin{pmatrix} 2x \\ -2y \end{pmatrix}$ graph (b)

(4) $f(x,y) = xy$

$\nabla f(x,y) \begin{pmatrix} f_x(x,y) \\ f_y(x,y) \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$ graph (c)

(L) (3) $\nabla f(x,y) = \begin{pmatrix} f_x(x,y) \\ f_y(x,y) \end{pmatrix} = \begin{pmatrix} 2x(y-1) \\ x^2 \end{pmatrix}$

$\vec{u} = 1\cdot\left(\cos\left(\frac{\pi}{6}\right)\vec{i} + \sin\left(\frac{\pi}{6}\right)\vec{j}\right) = \left(\frac{\sqrt{3}}{2}\vec{i} + \frac{1}{2}\vec{j}\right)$

$(D_{\vec{u}} f)(x_0,y_0) = \lim_{h\to 0} \frac{f(x_0 + hu_1, y_0 + hu_2) - f(x_0,y_0)}{h}$

$(x_0,y_0) = (1,-1)$

$= \frac{f(1+\frac{\sqrt{3}}{2}h, -1+\frac{1}{2}h) - f(1,-1)}{h}$

$= \frac{(1+\frac{\sqrt{3}}{2}h)^2\cdot((-1+\frac{1}{2}h)-1)+2}{h}$

$= \frac{(\frac{3}{4}h^2+\sqrt{3}h+1)\cdot(\frac{1}{2}h-2)+2}{h}$

$= \frac{\frac{3}{8}h^3 + \frac{\sqrt{3}}{2}h^2 + \frac{1}{2}h - \frac{3}{2}h^2 - 2\sqrt{3}h - 2 + 2}{h}$

$= \frac{h\left(\frac{3}{8}h^2 + \frac{\sqrt{3}}{2}h + \frac{1}{2} - \frac{3}{2}h - 2\sqrt{3}\right)}{h}$

$= \lim_{h\to 0} \frac{3}{8}h^2 + \frac{\sqrt{3}}{2}h - \frac{3}{2}h + \frac{1}{2} - 2\sqrt{3} = \frac{1}{2} - 2\sqrt{3}$

$(D_{\vec{u}} f)_{(1,-1)} = \vec{u}\cdot\nabla f(1,-1) = \begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{2} \end{pmatrix}\cdot\begin{pmatrix} -4 \\ 1 \end{pmatrix} = \frac{\sqrt{3}}{2}\cdot(-4) + \frac{1}{2}\cdot 1 = \frac{1}{2} - 2\sqrt{3}$

(M) (3)
B high to low $D_u f < 0$
A no change $D_u f = 0$
C middle climb
D steep climb

(N) (3)
$f_x = 2x - 3y$ $\quad a = (-4, 13)$
$f_y = -3x + 8y$ $\quad b = (4, -13)$
$\nabla f\ (0,0) = (-4, 13)$ $\quad C = (-13, -4)$

1) no since $D_u(f) = \|\nabla f\|$

$|\nabla f(1,1)| = \|(-4,13)\|$ $\sqrt{4^2+13^2}$

$\sqrt{185} \approx 13.6 \neq 14$

2)

1) no

2) yes

3) no

4) yes

5) no

5) 2) $xy = -4$; $(2,-2)$

$f(x,y) = xy$ $P = (2,-2)$

$$\nabla f(x,y) = \begin{pmatrix} f_x'(x,y) \\ f_y'(x,y) \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

$$\nabla f(2,-2) = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

$ax + by + c = 0$

$-2x + 2y - (-2 \cdot 2 + 2 \cdot -2) = 0$

$-2x + 2y + 8 = 0$

$y = x - 4$

7) 1) $f(x,y) = x^2 - xy + y^2$

$$\nabla f(x,y) = \begin{pmatrix} f_x'(x,y) \\ f_y'(x,y) \end{pmatrix} = \begin{pmatrix} 2x - y \\ 2y - x \end{pmatrix}$$

$$\nabla f(-1,2) = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$$

$a(x - x_0) + b(y - y_0) = 0$

$-4(x+1) + 5(y-2) = 0$

$-4x - 4 + 5y - 10 = 0$

$5y = 4x + 14$

$y = \frac{4}{5}x + \frac{14}{5}$

4) 2) $f_x(1,1,3) \cdot (x-1) + f_y(1,1,3) \cdot (y-1) + f_z(1,1,3)(z-3) = 0$

$(3x + 6xy^2 + 4y)$ $16yx^2 + 3y^2 + 4x$ $(-2z)$

$\nabla f(1,1,3) = (13, 13, -6)$

$13(x-1) + 13(y-1) - 6(z-3) = 0 \Rightarrow 13x + 13y - 6z - 8 = 0$

$(13, 13, -6) = \nabla f(1,1,3) \Rightarrow (1,1,3) + t(13, 13, -6)$

$(1 + 13t, 1 + 13t, 3 - 6t)$