Probability HW #5

② ⓐ $X \sim B(6, 0.6)$. $P(X=K) = \binom{6}{K}(0.6)^K(0.4)^{6-K}$, $E[X] = 3.6$, $V(X) = 3.6(0.4) = 1.44$

ⓑ $P(X=K) = \binom{6}{3}(0.6)^3(0.4)^3 = 0.27648$

ⓒ $P(X \leq 4) = 1 - P(X=5) - P(X=6) = 1 - \binom{6}{5}(0.6)^5(0.4) - \binom{6}{6}(0.6)^6(0.4)^0$

$= 1 - 0.18624 - 0.0466 = 0.76672$

ⓓ $P(X \leq 4 \mid X \geq 2) = \dfrac{P(X \geq 2 \cap X \leq 4)}{P(X \geq 2)} = \dfrac{P(X=2) + P(X=3) + P(X=4)}{1 - P(X=0) - P(X=1)}$

$$\frac{\binom{6}{2}(0.6)^2(0.4)^4 + \binom{6}{3}(0.6)^3(0.4)^3 + \binom{6}{4}(0.6)^4(0.4)^2}{1 - \binom{6}{0}(0.6)^0(0.4)^6 - \binom{6}{1}(0.6)^1(0.4)^5} = \frac{\frac{432+864+972}{3125}}{\frac{15625-64-576}{15625}}$$

$$= \frac{2268}{3125} \cdot \frac{15625}{14985} = \frac{28}{37} = 0.\overline{756}$$

ⓔ $X \sim B(4, 0.6)$. $P(X=k) = \binom{4}{k}(0.6)^k(0.4)^{4-k}$

$P(X \leq 2) = P(X=0) + P(X=1) + P(X=2)$

$\binom{4}{0}(0.6)^0(0.4)^4 + \binom{4}{1}(0.6)(0.4)^3 + \binom{4}{2}(0.6)^2(0.4)^2 = \frac{16+96+216}{625} = \frac{328}{625} = 0.524$

③ ⓐ $P(A \cup B) = 0.2 + 0.04 - 0.01 = 0.23$. $X \sim B(8, 0.77)$. $P(X=K) = \binom{8}{K}(0.77)^K(0.23)^{8-K}$

$\binom{8}{8}(0.77)^8(0.23)^0 = (0.77)^8 = 0.12357$

ⓑ given $P(A \cap B) = 0.01 \neq P(A) \cdot P(B) = 0.2 \cdot 0.04 = 0.008 \rightarrow$ dependent.

This shows that location errors and depth errors depend on each other.

ⓒ let X = # of points where some errors occur $X \sim B(8, 0.23)$

let Y = # of points where location errors occurs $Y \sim B(8, 0.2)$

$E[X] = 8 \cdot 0.23 = 1.84$ $E[Y] = 8 \cdot 0.2 = 1.6$

ⓓ define X, Y as in part (c). let Z = # of points where depth error occurs

$Z \sim B(8, 0.04)$

$P(Y=3 \mid X \geq 2) = \dfrac{P(Y=3) \cdot P(0 \leq Z \leq 5)}{P(X \geq 2)} = \dfrac{P(Y=3) \cdot (1 - P(Z=8) - P(Z=7) - P(Z=6))}{1 - P(X=0) - P(X=1)}$

$$\frac{\binom{8}{3}(0.2)^3(0.8)^5 \cdot \left(1 - \binom{8}{8}(0.04)^8 - \binom{8}{7}(0.04)^7(0.96) - \binom{8}{6}(0.04)^6(0.96)^2\right)}{1 - \binom{8}{0}(0.23)^0(0.77)^8 - \binom{8}{1}(0.23)(0.77)^7}$$

$$= \frac{0.1468 \cdot 0.999999893}{0.5811335427} = 0.2526$$

⑦ $X \sim G(0.8)$

ⓐ $P(X=3) = 0.8(0.2)^2 = 0.032$

ⓑ $P(X=1 \mid X \leq 4) = \dfrac{P(X=1 \cap X \leq 4)}{P(X \leq 4)} = \dfrac{P(X=1)}{P(X=1) + P(X=2) + P(X=3) + P(X=4)}$

$$= \frac{(0.8)}{0.8 + (0.8)(0.2) + (0.8)(0.2)^2 + (0.8)(0.2)^3} = \frac{(0.8)}{(0.8) + (0.16) + (0.032) + (0.0064)}$$

$$= \frac{0.8}{0.9984} = 0.80128$$

9 a 1 $X \sim H(20, 6, \frac{5}{20})$ $\quad P(X=k) = \frac{\binom{5}{k}\binom{15}{6-k}}{\binom{20}{6}}$ $\quad E[X] = \frac{5}{20} \cdot 6 = \frac{3}{2}$

$$V(X) = \frac{14}{19} \cdot 6 \cdot \frac{5}{20} \cdot \frac{15}{20} = \frac{63}{76}$$

2. $P(X<5) = 1 - P(X=5) = 1 - \frac{\binom{5}{5}\binom{15}{1}}{\binom{20}{6}} = 1 - \frac{15}{38760} = 0.9996$

b 1 $Y \sim H(20, 6, \frac{8}{20})$ $\quad P(Y=k) = \frac{\binom{8}{k}\binom{12}{6-k}}{\binom{20}{6}}$ $\quad E[Y] = \frac{8}{20} \cdot 6 = \frac{48}{20} = \frac{12}{5}$

$$V(Y) = \frac{14}{19} \cdot 6 \cdot \frac{8}{20} \cdot \frac{12}{20} = 1.06$$

2 $P(Y=6) = \frac{\binom{8}{6}\binom{12}{0}}{\binom{20}{6}} = \frac{7}{9690}$

3 $P(Y \geq 3 \cap Y < 6) = P(Y=3) + P(Y=4) + P(Y=5) = \frac{\binom{8}{3}\binom{12}{3}}{\binom{20}{6}} + \frac{\binom{8}{4}\binom{12}{2}}{\binom{20}{6}} + \frac{\binom{8}{5}\binom{12}{1}}{\binom{20}{6}}$

$$= \frac{12320 + 4620 + 672}{38760} = \frac{259}{570} = 0.454$$

c $X \sim H(12, 6, \frac{7}{12})$ $\quad P(X=6) = \frac{\binom{7}{6}\binom{5}{0}}{\binom{12}{6}} = \frac{7}{924} = \frac{1}{132}$

12 a $0.80 \cdot 0.86 + 0.30 \cdot 0.14 = 0.73$

b let A_1 = tourist staying < month $\quad A_2$ = tourist staying ≥ month

B = hotel - staying tourist $\quad P(B) = 0.73$ $\quad P(A_1 \mid B) = \frac{P(A_1) \cdot P(B \mid A_1)}{P(B)}$

$$= \frac{0.86 \cdot 0.8}{0.73} = \frac{344}{365} = 0.942$$

c $X \sim B(5, 0.942)$ $\quad P(X \geq 3) = 1 - P(X=0) - P(X=1) - P(X=2)$

$$= 1 - \binom{5}{0}(0.058)^5 - \binom{5}{1}(0.942)(0.058)^4 - \binom{5}{2}(0.942)^2(0.058)^3 = 0.998$$

d $P(B \mid A_2) = 0.3$ let X = # of tourists staying ≥ month who are hotel-staying $\quad X \sim B(10, 0.3)$ $\quad E[X] = 10 \cdot 0.3 = 3$

e 1 $P(A_2 \mid B) = \frac{P(A_2) \cdot P(B \mid A_2)}{P(B)} = \frac{0.14 \cdot 0.3}{0.73} = \frac{21}{365} = 0.0575$, $X \sim G(0.0575)$

$$E[X] = \frac{365}{21}$$

2 $P(X \geq 7) = 1 - P(X=1) - P(X=2) - P(X=3) - P(X=4) - P(X=5) - P(X=6)$

$1-(0.0575)-(0.0575)(0.9425)-(0.0575)(0.9425)^2-(0.0575)(0.9425)^3$
$-(0.0575)(0.9425)^4-(0.0575)(0.9425)^5=0.7=(0.9425)^6$ (6 failures)

(18) $X \sim B(300, \frac{1}{12})$ $E[X]=25$ $V(X)=25\cdot\frac{11}{12}=\frac{275}{12}=22.91\overline{6}$

(20) (a) $X \sim G(p)$. $P(X=k)=(1-p)^{k-1}\cdot p$ $(X=1,2,3,\ldots)$

$P(X \text{ is even})=P(X=2)+P(X=4)+\ldots=(1-p)p+(1-p)^3p+(1-p)^5p+\ldots$
$=p(1-p)\sum_{k=0}^{\infty}(1-p)^{2k}$! $\frac{a}{1-r}=\frac{1}{1-(1-p)^2}=\frac{1}{p^2-2p}=\frac{1}{p(p-2)}$

(b) $X \sim B(n,p)$ $P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}$ $(X=0,1,2,3,\ldots)$
for 4 even: $\frac{1}{2}(1+(2p-1)^n)$ for n odd: $1-(\frac{1}{2}(1+(2p-1)^n))$

(22) $X \sim NB(0.3,5)$ $P(X=12)=\binom{11}{4}(0.3)^5(0.7)^7=0.066$
$E[X]=\frac{5}{0.3}=\frac{50}{3}=16.\overline{6}$ $V(X)=\frac{5(0.7)}{(0.3)^2}=38.\overline{8}$

(25) (a) $X \sim H(10,5,\frac{2}{10})$ $E[X]=5\cdot\frac{2}{10}=1$ $V(X)=\frac{5}{9}\cdot 5\cdot\frac{2}{10}\cdot\frac{8}{10}=\frac{4}{9}$
(b) $X \sim B(5,0.2)$ $E[X]=1$ $V(X)=1\cdot(0.8)=0.8$
(c) $X \sim G(0,2)$ $E[X]=5$ $V(X)=\frac{0.8}{(0.2)^2}=20$
(d) $X \sim NB(0.2,4)$ $E[X]=\frac{4}{0.2}=20$ $V(X)=\frac{4(0.8)}{(0.2)^2}=80$

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