Probability hw #9

④ $x \sim U(7, 10)$

ⓐ $P(9.5 < x < 7.4) = \dfrac{9.5 - 7.4}{10 - 7} = \dfrac{2.1}{3} = 0.7$

ⓑ $P(x \geq 9.4) = \dfrac{10 - 9.4}{10 - 7} = \dfrac{0.6}{3} = 0.2$

let $Y = \#$ of workdays with $x \geq 9.4$ $Y \sim B(6, 0.2)$

$P(Y \leq 4) = 1 - P(Y = 5) - P(Y = 6) = 1 - \binom{6}{5}(0.2)^5(0.8) - (0.2)^6 = \dfrac{624}{625} = 0.9984$

ⓒ $P(x > 9) = \dfrac{10 - 9}{10 - 7} = \dfrac{1}{3}$ let $Y = \#$ of days until $x > 9$ $Y \sim G(\tfrac{1}{3})$

$P(Y = 5) = \left(\dfrac{2}{3}\right)^4\left(\dfrac{1}{3}\right) = \dfrac{16}{243} = 0.0658$

⑤ $x \sim Exp(2)$ $F(x) = 1 - e^{-2x}$ $f(x) = 2e^{-2x}$

$P(x > 1) = 1 - F(1) = 1 - (1 - e^{-2}) = e^{-2}$

$P(x > 4 \mid x > 3) = P(x > 4 - 3) = P(x > 1) = e^{-2}$

$P(x > 1 \mid 0.5 < x < 1.5) = \dfrac{P(1 < x < 1.5)}{P(0.5 < x < 1.5)} = \dfrac{F(1.5) - F(1)}{F(1.5) - F(0.5)} = \dfrac{(1 - e^{-2(1.5)}) - (1 - e^{-2})}{(1 - e^{-2(1.5)}) - (1 - e^{-2(0.5)})}$

$= \dfrac{e^{-2} - e^{-3}}{e^{-1} - e^{-3}} = 0.2689$

$P(0.5 < x < 1.5 \mid x > 1) = \dfrac{P(1 < x < 1.5)}{P(x > 1)} = \dfrac{e^{-2} - e^{-3}}{e^{-2}} = 0.6321$

⑦ $x \sim Exp\left(\dfrac{1}{1000}\right)$ $F(x) = 1 - e^{\frac{-x}{1000}}$

ⓐ $P(x > 1000) = 1 - F(1000) = 1 - (1 - e^{-1}) = \dfrac{1}{e}$

ⓑ $P(x < 250) = F(250) = 1 - e^{\frac{-250}{1000}} = 0.2211$

ⓒ let $Y = \#$ of party sold until $x < 250$ $Y \sim G(0.2211)$

$E[Y] = \dfrac{1}{p} = \dfrac{1}{0.2211} = 4.5228$

⑫ $P(x > 100) = 0.5 \rightarrow 1 - (1 - e^{-100\lambda}) = 0.5 \rightarrow -100\lambda = \ln(0.5) \rightarrow \lambda = \dfrac{\ln 2}{100}$

$P(x > 300) = 1 - \left(1 - e^{-300\left(\frac{\ln 0.5}{-100}\right)}\right) = e^{3\ln 0.5} = \dfrac{1}{8}$