19 $\int \frac{dx}{x+3\sqrt{x}}$ $\left(\begin{aligned} u &= x+3\sqrt{x} \\ du &= 1+\frac{3}{2\sqrt{x}} \end{aligned} \right)$

?

$\downarrow$

$2\int \frac{1 \cdot dx}{2\sqrt{x}(\sqrt{x}+3)}$ $\quad u = \sqrt{x}+3$

$du = \frac{1}{2\sqrt{x}}$

$2\int \frac{du}{u} = 2\ln|u| + c \Rightarrow 2\ln(\sqrt{x}+3)+c$

4 $\int \frac{x^4}{x^2-4}\,dx$ ideas? $\int \frac{x^4-16+16}{x^2+4}\,dx = \int \frac{x^4-16}{x^2+4} + \frac{16}{x^2+4}\,dx$

$\int \frac{(x^2-4)\cancel{(x^2+4)}}{\cancel{x^2+4}}\,dx$

$= \frac{x^3}{3} - 4x + 8\arctan\frac{x}{2} + c$

In notebook

2, 19, 4

calc hw #6

① ③ $\int \frac{(x+3)^2}{x} dx$ $\quad \frac{x^2+6x+9}{x} = \frac{x(x+6+\frac{9}{x})}{x \cdot 1} = \int x+6+\frac{9}{x}\, dx =$

$\frac{x^2}{2} + 6x + \ln|9| + c$

⑥ $\int \sqrt[3]{x}(\sqrt{x}+2)\, dx = \int x^{\frac{1}{3}}(x^{\frac{1}{2}}+2)\, dx = \int x^{\frac{5}{6}} + 2x^{\frac{1}{3}}\, dx$

$\frac{6x^{\frac{11}{6}}}{11} + \frac{6x^{\frac{4}{3}}}{4} + c$

⑧ $\int \cos^2\frac{x}{2}\, dx = \int (\cos\frac{x}{2})(\cos\frac{x}{2})\, dx =$ $\quad$ half angle formula: $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$

$\int \frac{1+\cos x}{2} dx = \int \frac{1}{2} dx + \int \frac{\cos x}{2} dx = \int \frac{1}{2} dx + \frac{1}{2}\int \cos x\, dx =$

$\frac{1}{2}x + \frac{1}{2}\sin x + c$

⑩ $\int \tan^2 x\, dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1-\cos^2 x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} - 1\, dx$

$\int \frac{1}{\cos^2 x} dx - \int 1\, dx = \tan x - x + c$

② ③ $\int \frac{3}{x^2+16} dx = 3\int \frac{1}{x^2+16} dx = 3\int \frac{1}{x^2+4^2} dx = 3 \cdot \frac{1}{4} \arctan \frac{x}{4} + c$

⑧ $\int \sin 2x \cos 3x\, dx = \int \frac{\sin 5x + \sin(-x)}{2} dx$ $\quad$ $\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$

$= \frac{1}{2}\left(\int \sin 5x\, dx + \int \sin(-x)\, dx\right) = \frac{1}{2}\left(-\frac{1}{5}\cos(5x) + \cos x\right) + c$

⑪ $\int \frac{x}{\sqrt[3]{x^2+7}} dx = \int \frac{1}{\sqrt[3]{x^2+7}} \cdot x\, dx = \frac{1}{u^{\frac{1}{3}}} \cdot \frac{du}{2} = \frac{1}{2}\int u^{-\frac{1}{3}}\, du =$

$u = x^2+7$

$du = 2x\, dx$

$\frac{du}{2} = x\, dx$

$\frac{1}{2} \cdot \frac{3u^{\frac{2}{3}}}{2} + C =$

$\frac{3(x^2+7)^{\frac{2}{3}}}{4} + c$

5.2

(13) $\int \frac{e^x dx}{1+e^x}$ $\quad u = 1+e^x$, $du = e^x dx$ $\rightarrow \int \frac{du}{u} = \ln|u| + c = \ln|1+e^x| + c$

(15) $\int \frac{\sin 2x \, dx}{3+\cos 2x}$ $\quad u = 3+\cos 2x$, $du = \sin 2x \cdot -2 \, dx$, $\frac{du}{-2} = \sin 2x \, dx$ $\rightarrow \int \frac{\frac{du}{-2}}{u} = \int \frac{1}{-2u} du = -\frac{1}{2}\int \frac{1}{u} du$

$-\frac{1}{2} \cdot \ln|u| + c = -\frac{\ln|3+\cos 2x|}{2} + c$

(17) $\int \frac{x+e^{2x}}{x^2+e^{2x}} dx$ $\quad u = x^2+e^{2x}$, $du = 2x+2e^{2x} dx$, $\frac{du}{2} = x+e^{2x} dx$ $\Rightarrow \int \frac{\frac{du}{2}}{u} = \int \frac{du}{2u}$

$\frac{1}{2}\int \frac{1}{u} du = \frac{1}{2} \cdot \ln|u| + c = \frac{\ln|x^2+e^{2x}|}{2} + c$

(21) $\int \frac{(2x+3)}{(x^2+3x+1)^{10}} dx$ $\quad u = x^2+3x+1$, $du = 2x+3 \, dx$ $\Rightarrow \int \frac{du}{u^{10}} = \int u^{-10} du =$

$\frac{u^{-9}}{-9} + c = \frac{1}{-9(x^2+3x+1)^9} + c$

(25) $\int \sin x \cdot \cos^5 x \, dx$ $\quad u = \cos x$, $du = -\sin x \, dx$, $-du = \sin x \, dx$ $= \int u^5 \, -du = -1\int u^5 du =$

$-\frac{u^6}{6} + c = - \; -\frac{\cos^6 x}{6} + c$

(26) $\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx \Rightarrow$ $\quad u = \cos x$, $du = -\sin x \, dx$, $-du = \sin x \, dx$ $\Rightarrow \int \frac{-du}{u} = -1\int \frac{1}{u} du$

$-\ln|u| + c = -\ln|\cos x| + c$

$$\text{(29)} \int \cos^3 x\, dx = \int \cos^2 x \cos x\, dx = \quad \cos^2 x = 1-\sin^2 x$$

$$\int (1-\sin^2 x) \cos x\, dx \qquad u = \sin x \qquad du = \cos x\, dx$$

$$\int 1-u^2\, du = \int 1\, du - \int u^2\, du = u - \frac{u^3}{3} + C = \sin x - \frac{\sin^3(x)}{3} + C$$

$$\text{(30)} \int x^2 \sqrt{x+3}\, dx = \int x^2 (x+3)^{\frac{1}{2}}\, dx \qquad u = x+3 \rightarrow u-3 = x \qquad du = 1\,dx$$

$$\int (u-3)^2 (u)^{\frac{1}{2}}\, du = (u^2-6u+9)(u)^{\frac{1}{2}} = \int u^{2.5} - 6u^{1.5} + 9u^{\frac{1}{2}}\, du =$$

$$\frac{u^{\frac{7}{2}}}{\frac{7}{2}} - \frac{12}{5}u^{\frac{5}{2}} + 6u^{\frac{3}{2}} + c = \frac{2}{7}u^{\frac{7}{2}} - \frac{12}{5}u^{\frac{5}{2}} + 6u^{\frac{3}{2}} + c =$$

$$\frac{2}{7}(x+3)^{\frac{7}{2}} - \frac{12}{5}(x+3)^{\frac{5}{2}} - 6(x+3)^{\frac{3}{2}} + c$$

$$\text{(3) (3)} \int \underset{u}{x} \underset{dv}{e^{-x}}\, dx \qquad u = x \quad v = e^{-x} \qquad du = dx \quad dv = e^{-x}\, dx \qquad \int u\,dv = uv - \int v\,du$$

$$= xe^{-x} - \int e^{-x}\, dx$$

$$= xe^{-x} - e^{-x}\, dx$$

$$\text{(6)} \int \frac{\ln x}{x^2}\, dx \qquad u = \ln x \quad v = \frac{x^{-1}}{-1} = -\frac{1}{x} \qquad du = \frac{1}{x}dx \quad dv = x^{-2}\, dx$$

$$= \frac{-\ln x}{x} - \int -\frac{1}{x} \cdot \frac{1}{x}\, dx$$

$$-\frac{1}{x^2} \qquad = -\frac{\ln x}{x} + \int \frac{1}{x^2}\, dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

$$\text{(8)} \int \frac{x}{\cos^2 x}\, dx \qquad u = x \quad v = \tan x \qquad du = dx \quad dv = \cos^{-2} x\, dx$$

$$x\tan x - \int \tan x\, dx \qquad \frac{\sin x}{\cos x}dx \qquad u = \cos x \quad du = -\sin x\, dx \quad -du = \sin x\, dx \qquad \int \frac{-du}{u} = -1\int \frac{1}{u}\, du$$

$x\tan x + \ln|u| + c = x\tan x + \ln|\cos x| + c$

(10) $\int x \sinh 2x \, dx$ $\quad u = x \quad v = \frac{\cosh 2x}{2}$

$du = 1\,dx \quad dv = \sinh 2x \, dx$

$x\frac{\cosh 2x}{2} - \int \frac{\cosh 2x \, dx}{2} \Rightarrow -\frac{1}{2}\int \cosh 2x \, dx$

$\frac{x\cosh 2x}{2} - \frac{\sinh 2x}{4} + c$

(14) $\int t^2 \sin 0.5t \, dt$ $\quad u = t^2 \quad v = -2\cos\frac{t}{2}$

$du = 2t\,dt \quad dv = \sin\frac{t}{2}\,dt$

$t^2 - 2\cos\frac{t}{2} - \int -2\cos\frac{t}{2}\, 2t \, dt = -2t^2\cos\frac{t}{2} + \int 4t\cos\frac{t}{2}\,dt$

$u = 4t \quad v = 2\sin\frac{t}{2}$

$du = 4\,dt \quad dv = \cos\frac{t}{2}\,dt$

$-2t^2\cos\frac{t}{2} + 4t\,2\sin\frac{t}{2} - \int 2\sin\frac{t}{2}\,4\,dt =$

$-2t^2\cos\frac{t}{2} + 8t\sin\frac{t}{2} - 8\int \sin\frac{t}{2}\,dt =$

$-2t^2\cos\frac{t}{2} + 8t\sin\frac{t}{2} + 16\cos\frac{t}{2} + c$

(17) $\int e^{\sqrt{x}} \, dx$ $\quad t = \sqrt{x}$

$dt = \frac{1}{2\sqrt{x}}\,dx \Rightarrow 2\sqrt{x}\,dt = dx \rightarrow 2t\,dt = dx$

$\int e^t\, 2t\,dt = 2\int te^t\,dt$ $\quad u = t \quad v = e^t$

$du = dt \quad dv = e^t\,dt$

$2\left(te^t - \int e^t\,dt\right) = 2te^t - 2\int e^t\,dt = 2te^t - 2e^t + c = 2e^t(t-1) + c =$

$2e^{\sqrt{x}}(\sqrt{x}-1) + c$

(20) $\int e^x \sin 2x \, dx$ $\quad u = e^x \quad v = -\frac{1}{2}\cos 2x$

$du = e^x\,dx \quad dv = \sin 2x\,dx$

$e^x - \frac{1}{2}\cos 2x - \int -\frac{1}{2}\cos 2x\, e^x\,dx = -\frac{1}{2}e^x\cos 2x + \frac{1}{2}\int \cos 2x\, e^x\,dx$

$u = e^x \qquad v = \frac{1}{2}\sin 2x$

$du = e^x dx \qquad dv = \cos 2x \, dx$

$-\frac{1}{2}e^x \cos 2x + \frac{1}{2}\left(e^x \frac{1}{2}\sin 2x - \int \frac{1}{2}\sin 2x \, e^x dx\right) =$

$-\frac{1}{2}e^x \cos 2x + e^x \frac{1}{4}\sin 2x - \frac{1}{4}\int \sin 2x \, e^x dx = \int e^x \sin 2x \, dx$

$-\frac{1}{2}e^x \cos 2x + e^x \frac{1}{4}\sin 2x = \frac{5}{4}\int e^x \sin 2x \, dx$

$\Downarrow \qquad /:\frac{5}{4}$

$-\frac{2}{5}e^x \cos 2x + \frac{1}{5}e^x \sin 2x + c$

4 2 $\int \frac{4x^2-3x-4}{x(x-1)(x+2)} dx$

$\frac{4x^2-3x-4}{x(x-1)(x+2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2} \qquad A(x-1)(x+2) + Bx(x+2) + Cx(x-1) = 4x^2-3x-4$

$4x^2-3x-4 = A(x^2+x-2) + Bx^2 + 2Bx + Cx^2 - Cx$

$4x^2-3x-4 = Ax^2 + Ax - 2A + Bx^2 + 2Bx + Cx^2 - Cx$

$4x^2-3x-4 = (A+B+C)x^2 + (A+2B-C)x + -2A$

$4 = A+B+C$

$-3 = A+2B-C$

$-4 = -2A$

$\boxed{A=2}$

$4-2-C = B$

$2-C = B \Rightarrow -3 = A+4-2C-C$

$-3 = 2+4-2C-C$

$-9 = -3C \qquad \boxed{C=3}$

$4 = 2+3+B \qquad \boxed{B=-1}$

$\int \frac{4x^2-3x-4}{x(x-1)(x+2)} dx = \int \frac{2}{x} + \frac{3}{x-1} - \frac{1}{x-2} dx = 2\ln|x| + 3\ln|x-1| - \ln|x-2| + c$

④ ③ $\int \frac{x^2+5x+5}{x(x^2-25)}\,dx$

$\frac{x^2+5x+5}{x(x-5)(x+5)} \quad \frac{A}{(x)} + \frac{B}{(x-5)} + \frac{C}{(x+5)}$

$x^2+5x+5 = A(x-5)(x+5) + Bx(x+5) + Cx(x-5)$

$x^2+5x+5 = Ax^2 - 25A + Bx^2 + B5x + Cx^2 - C5x$

$x^2+5x+5 = x^2(A+B+C) + x(5B-C5) - 25A$

$1 = A+B+C$

$5 = 5B - C5$

$5 = -25A \Rightarrow A = -0.2$

$1.2 = B+C \Rightarrow -C = B-1.2 \Rightarrow C = 1.2 - B$

$5 = 5B - 5(1.2-B)$

$5 = 5B - 6 + 5B \Rightarrow 11 = 10B \Rightarrow \frac{11}{10} = B$

$1 = -0.2 + \frac{11}{10} + C$

$C = \frac{1}{10}$

$\int \frac{x^2+5x+5}{x(x^2-25)}\,dx = \int -\frac{1}{5x} + \frac{11}{10(x-5)} + \frac{1}{10(x+5)} = -\frac{1}{5}\ln|x| + \frac{11}{10}\ln|x-5| + \frac{1}{10}\ln|x+5| + C$

⑤ $\int \frac{2x^3-4x+3}{x^2+1}\,dx$ numerator has higher degree; divide

$$\begin{array}{r} 2x \\ x^2+1 \overline{)\,2x^3-4x+3} \\ 2x^3+2x \\ \hline -6x+3 \end{array}$$

$= \int 2x + \frac{-6x+3}{x^2+1}\,dx$

$= \int 2x - \frac{6x}{x^2+1} + \frac{3}{x^2+1}\,dx$

x^2

Substitute

$u = x^2+1$

$du = 2x\,dx$

$-3\int \frac{2x}{x^2+1}\,dx$

$-3\int \frac{du}{u}$

$-3\ln|u|$

$+3\arctan x$

$= x^2 - 3\ln|u| + 3\arctan x + C$

$= x^2 - 3\ln|x^2+1| + 3\arctan x + C$

5) 2) $\int \frac{\sin 2x - \cos x}{\sin^2 x - 2\sin x}\,dx = \int \frac{2\sin x\cos x - \cos x}{\sin^2 x - 2\sin x}\,dx$

$= \int \frac{\cos x(\sin x - 1)}{\sin x(\sin x - 2)}\,dx$ $\quad u = \sin x$, $du = \cos x\,dx$ $\quad \int \frac{2u-1}{u(u-2)}\,du$

$\frac{2u-1}{u(u-2)} = \frac{A}{u} + \frac{B}{u-2} \Rightarrow 2u - 1 = A(u-2) + Bu$

$u=2$: $3 = 2B \Rightarrow B = \frac{3}{2}$

$u=0$: $-1 = -2A = A = \frac{1}{2}$

$\frac{1}{2}\int \frac{1}{u}\,du + \frac{3}{2}\int \frac{1}{u-2}\,du$

$= \frac{1}{2}\ln|u| + \frac{3}{2}\ln|u-2| + c$

$= \frac{1}{2}\ln|\sin x| + \frac{3}{2}\ln|\sin x - 2| + c$

4) 7) $\int \frac{x+3}{x^2+6x+13}\,dx$

$$\begin{array}{r} x+3 \\ x+3 \,\overline{)\, x^2+6x+13} \\ x^2+3x \\ \hline 3x+13 \\ 3x+9 \\ \hline 4 \end{array}$$

$\int \frac{x+3}{(x+3)^2+4}\,dx$ $\quad t = x+3$, $dt = dx$ $\quad u = t^2+4$, $du = 2t\,dt$

$\int \frac{t}{t^2+4}\,dt = \frac{1}{2}\int \frac{2t}{t^2+4}\,dt = \frac{1}{2}\int \frac{1}{t^2+4}\,du = \frac{1}{2}\int \frac{1}{u}\,du =$

$\frac{1}{2}\ln|u| + c = \frac{1}{2}\ln|t^2+4| + c = \frac{1}{2}\ln|(x+3)^2+4| + c$

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(13) $$\int \frac{(x^2-3x-8)}{(x^2+4x+5)(x+1)}\,dx$$

$$\frac{x^2-3x-8}{(x^2+4x+5)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4x+5} \quad \Big/ \cdot (x+1)(x^2+4x+5)$$

$$x^2-3x-8 = A(x^2+4x+5) + (Bx+C)(x+1)$$
$$= A(x^2+4x+5) + Bx(x+1) + C(x+1)$$

$\downarrow x=-1$ $\qquad\qquad\qquad$ $\downarrow x=0$

$-4 = 2A$ $\qquad\qquad\qquad$ $-8 = 5A + C$

$\downarrow$ $\qquad\qquad\qquad\qquad$ $\Downarrow$

$\boxed{A=-2}$ $\qquad\qquad\qquad$ $-8 = -10 + C$

$\qquad\qquad\qquad\qquad\qquad$ $\Downarrow$

$\qquad\qquad\qquad\qquad\qquad$ $\boxed{C=2}$

$$x^2-3x-8 = -2(x^2+4x+5) + (Bx+2)(x+1)$$

$\downarrow x=1$

$-10 = -20 + (B+2)2$

$10 = 2B+4$

$6 = 2B$

$\boxed{B=3}$

$$\int \frac{(x^2-3x-8)}{(x^2+4x+5)(x+1)}\,dx = -2\int \frac{1}{x+1} + \int \frac{3x+2}{x^2+4x+5} = -2\int \frac{1}{x+1}\,dx + \int \frac{3x+2}{(x+2)^2+1}\,dx$$

$$\begin{cases} t = x+2 \Rightarrow x = t-2 \\ dt = dx \end{cases}$$

$\Downarrow$

$$* \quad \frac{3x+2}{(x+2)^2+1}\,dx = \int \frac{3(t-2)+2}{t^2+1}\,dt = \int \frac{3t-4}{t^2+1}\,dt = 3\int \frac{t}{t^2+1}\,dt - 4\int \frac{1}{t^2+1}\,dt$$

$$\begin{cases} u = t^2 \\ du = 2t\,dt \end{cases}$$

$$** \quad \int \frac{t}{t^2+1}\,dt = \frac{1}{2}\cdot\int \frac{2t}{t^2+1}\,dt = \frac{1}{2}\int \frac{1}{u+1}\,du = \frac{1}{2}\ln|u+1| = \frac{1}{2}\ln|t^2+1|$$

$$* \quad \int \frac{3x+2}{(x+2)^2+1}\,dx = 3\cdot\frac{1}{2}\ln|t^2+1| - \arctan(t) = \frac{3}{2}\ln|(x+2)^2| - 4\arctan(x+2)$$

$$\int \frac{(x^2-3x-5)}{(x^2+4x+5)(x+1)}\,dx = -2\ln|x+1| + \frac{3}{2}\ln|(x+2)^2+1| - 4\arctan(x+2) + C$$

$$= -2\ln|x+1| + \frac{3}{2}\ln|x^2+4x+5| - 4\arctan(x+2) + C$$

(14) $\int \frac{x^2-4}{(x-1)(x+5)^2}dx$

$\frac{x^2-4}{(x-1)(x+5)^2} = \frac{A}{x-1} + \frac{B}{x+5} + \frac{C}{(x+5)^2} \Big/ \cdot (x-1)(x+5)^2$

$x^2-4 = A(x+5)^2 + B(x-1)(x+5) + C(x-1)$

$\downarrow x=1 \qquad \downarrow x=-5$

$-3 = 36A \qquad 21 = -6C$

$\boxed{A = -\frac{1}{12}} \qquad \boxed{C = -\frac{7}{2}}$

$x^2-4 = -\frac{1}{12}(x+5)^2 + B(x-1)(x+5) - \frac{7}{2}(x-1)$

$\downarrow x=0$

$-4 = -\frac{25}{12} - 5B + \frac{7}{2} \qquad 5B = \frac{65}{12} \qquad \boxed{B = \frac{13}{12}}$

$\int \frac{x^2-4}{(x-1)(x+5)^2}dx = -\frac{1}{12}\int \frac{1}{x-1}dx + \frac{13}{12}\int \frac{1}{x+5}dx - \frac{7}{2}\int \frac{1}{(x+5)^2}dx$

$= \frac{-\ln|x-1|}{12} + \frac{13}{12}\ln|x+5| + \frac{7}{2}\int \frac{1}{(x+5)^2}dx$

$\int \frac{1}{(x+5)^2}dx = \int \frac{1}{t^2}dt = \int t^{-2}dt = -t^{-1} = -(x+5)^{-1} = -\frac{1}{x+5}$

$t = x+5 \quad dt = dx$

$\int \frac{x^2-4}{(x-1)(x+5)^2}dx = \frac{-\ln|x-1|}{12} + \frac{13}{12}\ln|x+5| + \frac{7}{2(x+5)} + C$

(18) $\int \frac{x^2-4x+3}{x^2+4x+5}dx = \int 1 + \frac{-8x-2}{x^2+4x+5}dx = x - \int \frac{8x+2}{(x+2)^2+1}$

$\begin{array}{l} \quad 1 \\ x^2-4x+3 \,|\, x^2+4x+5 \\ x^2+4x+5 \\ \hline -8x-2 \end{array}$

$t = x+2 \qquad x = t-2$

$dt = dx$

$= x - \int \frac{8(t-2)+2}{t^2+1}dt = x - \int \frac{8t-16+2}{t^2+1}dt = x - \int \frac{8t-14}{t^2+1}dt$

$= x - 8\int \frac{t}{t^2+1} - 14\int \frac{1}{t^2+1}$

$u = t^2$

$du = 2t\,dt \qquad = x - 8\cdot\frac{1}{2}\int \frac{2t}{t^2+1}dt - 14\arctan(t)$

$= x - 4\int \frac{1}{u+1}du - 14\arctan(t)$

$= x - 4\ln|u+1| - 14\arctan(t) + c$

$= x - 4\ln|t^2+1| - 14\arctan(t) + c$

$= x - 4\ln|(x+2)^2+1| - 14\arctan(x+2) + c$

$= x - 4\ln|x^2+4x+5| - 14\arctan(x+2) + c$

④ $\int \frac{3e^x}{\sqrt{1-e^{2x}}}dx = \int \frac{3}{\sqrt{1-t^2}}dt = 3\int \frac{1}{\sqrt{1-t^2}}dt = 3\cdot \arcsin t + c$

$t = e^x$

$dt = e^x\,dx$

$= 3\arcsin(e^x) + c$

⑤ $\int \frac{3+2\ln x}{x\ln^2 x - 9x}dx = \int \frac{1}{x}\cdot\frac{3+2\ln x}{\ln^2 x - 9}dx = \int \frac{3+2t}{t^2-9}dt = \int \frac{2t+3}{(t+3)(t-3)}dt$

$t = \ln x$

$dt = \frac{1}{x}dx$

$\frac{2t+3}{(t+3)(t-3)} = \frac{A}{t+3} + \frac{B}{t-3} \quad /\cdot (t-3)(t+3)$

$2t+3 = A(t-3) + B(t+3)$

$t=-3 \downarrow \qquad \downarrow t=3$

$-3 = -6A \qquad 9 = 6B$

$\Downarrow \qquad \Downarrow$

$A = \frac{1}{2} \qquad B = \frac{3}{2}$

$\int \frac{3+2\ln x}{x\ln^2 x - 9x}dx = \frac{1}{2}\int\frac{1}{t+3}dt + \frac{3}{2}\int\frac{1}{t-3}dt = \frac{1}{2}\ln|t+3| + \frac{3}{2}\ln|t-3| + c$

$= \frac{1}{2}\ln|\ln x+3| + \frac{3}{2}\ln|\ln x-3| + c$