§1 SAT-COLOR-EXCLUSION INTRO 1

May 19, 2018 at 02:30

1. Intro. This little program outputs clauses that are satisfiable if and only if the graph g can be c-colored, given g and c.

```
(It generalizes SAT-PIGEONS, which is the case where g = K_m and c = n.)
```

Suppose the graph has m edges and n vertices. Then there are nc variables v.k, meaning that vertex v gets color k. And there are n clauses of size c (to ensure that each vertex gets at least one color), plus mc clauses of size 2 (to ensure that adjacent vertices don't share a color). Plus $n\binom{c}{2}$ "exclusion clauses," to ensure that no vertex gets more than one color.

```
#include <stdio.h>
#include <stdlib.h>
#include "gb_graph.h"
#include "gb_save.h"
  int c:
  main(\mathbf{int} \ argc, \mathbf{char} * argv[])
     register int i, j, k;
     register Arc *a;
     register Graph *g;
     register Vertex *v;
      \langle \text{Process the command line 2} \rangle;
      (Generate the positive clauses 3);
      (Generate the negative clauses 4);
      (Generate the exclusion clauses 5);
  }
2. \langle \text{Process the command line } 2 \rangle \equiv
  if (argc \neq 3 \lor sscanf(argv[2], "%d", \&c) \neq 1) {
     fprintf(stderr, "Usage: \_\%s\_foo.gb\_c\n", argv[0]);
     exit(-1);
  }
  g = restore\_graph(argv[1]);
  if (\neg g) {
     fprintf(stderr, "I_{\square}couldn't_{\square}reconstruct_{\square}graph_{\square}%s!\n", argv[1]);
     exit(-2);
  if (c \le 0) {
     fprintf(stderr, "c_{\perp}must_{\perp}be_{\perp}positive!\n");
     exit(-3);
  printf(\verb""" usat-color-exclusion u%s u%d n", argv[1], c);
This code is used in section 1.
3. \langle Generate the positive clauses 3\rangle \equiv
  for (v = g \neg vertices; v < g \neg vertices + g \neg n; v \leftrightarrow) {
     for (k = 1; k \le c; k++) printf("\_%s.%d", v \rightarrow name, k);
     printf("\n");
This code is used in section 1.
```

```
4. ⟨Generate the negative clauses 4⟩ ≡
for (k = 1; k ≤ c; k++)
for (v = g¬vertices; v < g¬vertices + g¬n; v++)
for (a = v¬arcs; a; a = a¬next)
if (a¬tip > v) printf("¬%s.%d¬¬%s.%d¬", v¬name, k, a¬tip¬name, k);
This code is used in section 1.
5. ⟨Generate the exclusion clauses 5⟩ ≡
for (j = 1; j ≤ c; j++)
for (k = j + 1; k ≤ c; k++)
for (v = g¬vertices; v < g¬vertices + g¬n; v++) printf("¬%s.%d¬¬%s.%d¬¬, v¬name, j, v¬name, k);</li>
This code is used in section 1.
```

6. Index.

 $\begin{array}{lll} a: & \underline{1}. \\ \textbf{Arc:} & 1. \\ arcs: & 4. \\ argc: & \underline{1}, \ 2. \\ argv: & \underline{1}, \ 2. \\ c: & \underline{1}. \\ exit: & 2. \\ fprintf: & 2. \\ g: & \underline{1}. \end{array}$

Graph: 1.

i: <u>1</u>.

j: <u>1</u>.

k: <u>1</u>.

 $\begin{array}{ll} \textit{main:} & \underline{1}. \\ \textit{name:} & 3, \ 4, \ 5. \\ \textit{next:} & 4. \end{array}$

 $\begin{array}{lll} \textit{printf}\colon & 2,\ 3,\ 4,\ 5.\\ \textit{restore_graph}\colon & 2. \end{array}$

sscanf: 2. stderr: 2. tip: 4. v: <u>1</u>.

Vertex: 1. *vertices*: 3, 4, 5.

4 NAMES OF THE SECTIONS

SAT-COLOR-EXCLUSION

 $\left\langle \begin{array}{ll} \text{Generate the exclusion clauses 5} \right\rangle & \text{Used in section 1.} \\ \left\langle \begin{array}{ll} \text{Generate the negative clauses 4} \right\rangle & \text{Used in section 1.} \\ \left\langle \begin{array}{ll} \text{Generate the positive clauses 3} \right\rangle & \text{Used in section 1.} \\ \left\langle \begin{array}{ll} \text{Process the command line 2} \right\rangle & \text{Used in section 1.} \\ \end{array} \right.$

SAT-COLOR-EXCLUSION

	Section	Page
Intro	 1	1
Index	6	3