CIS 7414X: Homework Assignment 3

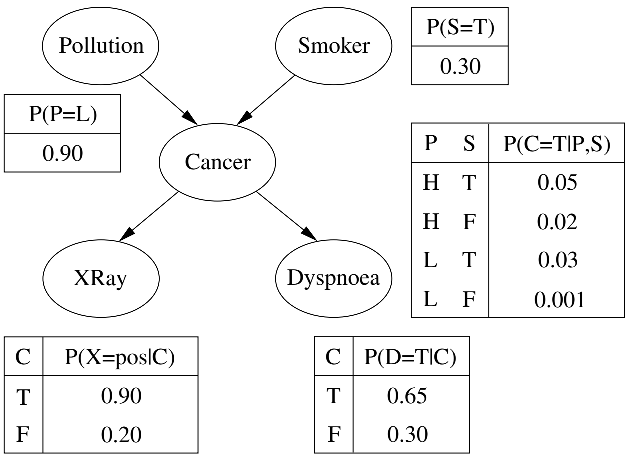

Figure 1: A Bayesian Netowrk for the lung cancer problem.

Please write down all intermediate steps in the following computation, and list the grounds of each step: which Kolmogorov's axiom, which definition, which rule and so on.

1. Compute the following joint probabilities

$$
\begin{aligned}
P(P = L, S = T, C = T, X = pos, D = T) &= ? \\
P(P = L, S = T, C = T, X = pos, D = F) &= ?
\end{aligned}
$$

using

- the conditional probabilties listed in Figure 1,

- the chain rule

$$P(x_1, x_2, \ldots, x_n) \quad = \quad \Pi_{i=1,\ldots,n} P(x_i | x_1, x_2, \ldots, x_{i-1})$$

- conditional independence represented of the Bayesian network:

$$P(x_i | x_1, \ldots, x_{i-1}) \quad = \quad P(x_i | Parents(X_i))$$

2. Compute the following conditional probabilities with Bayes' rule

$$\begin{aligned} P(P = L | C = T) &= ? \\ P(P = H | C = F) &= ? \end{aligned}$$

3. Compute the following joint probabilities of

$$\begin{aligned} P(P = L, S = T, C = T) &= ? \\ P(P = L, S = T, C = F) &= ? \\ P(P = L, S = F, C = T) &= ? \\ P(P = L, S = F, C = F) &= ? \\ P(P = H, S = T, C = T) &= ? \\ P(P = H, S = T, C = F) &= ? \\ P(P = H, S = F, C = T) &= ? \\ P(P = H, S = F, C = F) &= ? \end{aligned}$$

4. Compute the following conditional probabilities with the joint probabilities computed in question (3), the definition of conditional probabilities and marginalization

$$\begin{aligned} P(P = L | S = T, C = T) &= ? \\ P(P = L | S = T, C = F) &= ? \\ P(P = L | S = F, C = T) &= ? \\ P(P = L | S = F, C = F) &= ? \\ P(P = H | S = T, C = T) &= ? \\ P(P = H | S = T, C = F) &= ? \\ P(P = H | S = F, C = T) &= ? \\ P(P = H | S = F, C = F) &= ? \end{aligned}$$

Hint: Marginalization

$$P(X = a) = \Sigma_{y_i \in Domain(Y)} P(X = a, Y = y_i)$$

5. Compute the following conditioanl probabilities

$$\begin{aligned}
P(S = T|P = L, C = T) &= ? \\
P(S = T|P = L, C = F) &= ? \\
P(S = F|P = L, C = T) &= ? \\
P(S = F|P = L, C = F) &= ? \\
P(S = T|P = H, C = T) &= ? \\
P(S = T|P = H, C = F) &= ? \\
P(S = F|P = H, C = T) &= ? \\
P(S = F|P = H, C = F) &= ?
\end{aligned}$$

6. Use the probabilities computed in question (4) and (5) to show that $P \not\perp S|C$.