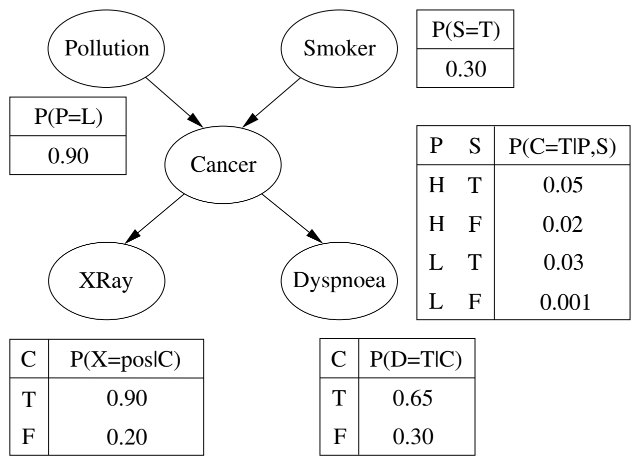
CIS 7414X: Homework Assignment 3



P(S=T)
0.30

P(P=L)
0.90

P	S	P(C=T|P,S)
H	T	0.05
H	F	0.02
L	T	0.03
L	F	0.001

C	P(X=pos|C)
T	0.90
F	0.20

C	P(D=T|C)
T	0.65
F	0.30

Figure 1: A Bayesian Netowrk for the lung cancer problem.

Please write down all intermediate steps in the following computation, and list the grounds of each step: which Kolmogorov's axiom, which definition, which rule and so on.

Question 1 *Compute the following joint probabilities*

$$
\begin{aligned}
P(P = L, S = T, C = T, X = pos, D = T) &= ? \\
P(P = L, S = T, C = T, X = pos, D = F) &= ?
\end{aligned}
$$

using

- *the conditional probabilties listed in Figure 1,*

- *the chain rule*

$$P(x_1, x_2, \ldots, x_n) \;=\; \Pi_{i=1,\ldots,n} P(x_i|x_1, x_2, \ldots, x_{i-1})$$

- *conditional independence represented of the Bayesian network:*

$$P(x_i|x_1, \ldots, x_{i-1}) \;=\; P(x_i|Parents(X_i))$$

Answer 1

$$\begin{aligned}
& P(P = L, S = T, C = T, X = pos, D = T) \\
&= P(P = L) \\
&\quad P(S = T|P = L) \\
&\quad P(C = T|P = L, S = T) \\
&\quad P(X = pos|P = L, S = T, C = T) \\
&\quad P(D = T|P = L, S = T, C = T, X = pos) \\
& \textit{//By the independences in the BN:} \\
&= P(P = L) \\
&\quad P(S = T) \\
&\quad P(C = T|P = L, S = T) \\
&\quad P(X = pos|C = T) \\
&\quad P(D = T|C = T) \\
&= 0.90 \\
&\quad \times 0.30 \\
&\quad \times 0.03 \\
&\quad \times 0.90 \\
&\quad \times 0.65 \\
&= 0.0047385
\end{aligned}$$

$$\begin{aligned}
& P(P = L, S = T, C = T, X = pos, D = F) \\
&= P(P = L) \\
&\quad P(S = T|P = L) \\
&\quad P(C = T|P = L, S = T)
\end{aligned}$$

$$
\begin{aligned}
& P(X = pos|P = L, S = T, C = T) \\
& P(D = F|P = L, S = T, C = T, X = pos)
\end{aligned}
$$

//By the independences in the BN:

$$
\begin{aligned}
= \quad & P(P = L) \\
& P(S = T) \\
& P(C = T|P = L, S = T) \\
& P(X = pos|C = T) \\
& P(D = F|C = T) \\
= \quad & 0.90 \\
& \times 0.30 \\
& \times 0.03 \\
& \times 0.90 \\
& \times 0.30 \\
= \quad & 0.0021870
\end{aligned}
$$

Question 2 *Compute the following conditional probabilities with Bayes' rule*

$$
\begin{aligned}
P(P = L|C = T) &= ? \\
P(P = H|C = F) &= ?
\end{aligned}
$$

Answer 2

$P(P|C)$

//Intruducing hidden variables and then marginalize

//out the introduced variables, so that the

//probabilites can be otained from the CPTs or

//calculation from the CPTs:

$= \quad \Sigma_S P(P, S|C)$

//Bayesian rule on C and $\langle P, S \rangle$:

$= \quad \alpha(C) \cdot \Sigma_S P(C|P, S) \cdot P(P, S)$

//P and S are marginally independence:

$= \quad \alpha(C) \cdot \Sigma_S P(C|P, S) \cdot P(P) \cdot P(S)$

//Move $P(P)$ out of the marginalization as

//$P(P)$ doesn't involve variable S:

$= \quad \alpha(C) \cdot P(P) \cdot \Sigma_S P(C|P, S) \cdot P(S)$

where $\alpha(C) = \frac{1}{P(C)}$

- $P(P=L|C=T)$

$$\begin{aligned}
& P(P=L|C=T) \\
& = \alpha(C=T)\cdot \\
& \quad P(P=L)\cdot \\
& \quad (P(C=T|P=L,S=T)\cdot P(S=T) + P(C=T|P=L,S=F)\cdot P(S=F)) \\
& = \alpha(C=T)\times 0.9 \times (0.03\times 0.30 + 0.001\times 0.70) \\
& = 0.0087300\cdot \alpha(C=T)
\end{aligned}$$

$$\begin{aligned}
& P(P=H|C=T) \\
& = \alpha(C=T)\cdot \\
& \quad P(P=H)\cdot \\
& \quad (P(C=T|P=H,S=T)\cdot P(S=T) + P(C=T|P=H,S=F)\cdot P(S=F)) \\
& = \alpha(C=T)\times 0.10 \times (0.05\times 0.30 + 0.02\times 0.70) \\
& = 0.0029000\cdot \alpha(C=T)
\end{aligned}$$

$$\alpha(C=T) = \frac{1}{0.0087300 + 0.0029000} = 85.985$$

$$P(P=L|C=T) = 0.0087300\times 85.985 = 0.75065$$

- $P(P=H|C=F)$ *can be computed in a similar way.*

Question 3 *Compute the following joint probabilities of*

$$\begin{aligned}
P(P=L,S=T,C=T) &= ? \\
P(P=L,S=T,C=F) &= ? \\
P(P=L,S=F,C=T) &= ? \\
P(P=L,S=F,C=F) &= ?
\end{aligned}$$

$$
\begin{aligned}
P(P=H, S=T, C=T) &= ? \\
P(P=H, S=T, C=F) &= ? \\
P(P=H, S=F, C=T) &= ? \\
P(P=H, S=F, C=F) &= ?
\end{aligned}
$$

Answer 3

$$
\begin{aligned}
&P(P, S, C) \\
&\quad = \quad P(P) \\
&\qquad\quad\; P(S|P) \\
&\qquad\quad\; P(C|P, S) \\
&\textit{//By the independences in the BN:} \\
&\quad = \quad P(P) \\
&\qquad\quad\; P(S) \\
&\qquad\quad\; P(C|P, S)
\end{aligned}
$$

Write down the values of P, S, C in the question and put them into the above equation, look up the CPTs for the corresponding probabilities, and calculate the results.

Question 4 *Compute the following conditional probabilities with the joint probabilties computed in question (3), the definition of conditional probabilities and marginalization*

$$
\begin{aligned}
P(P=L|S=T, C=T) &= ? \\
P(P=L|S=T, C=F) &= ? \\
P(P=L|S=F, C=T) &= ? \\
P(P=L|S=F, C=F) &= ? \\
P(P=H|S=T, C=T) &= ? \\
P(P=H|S=T, C=F) &= ? \\
P(P=H|S=F, C=T) &= ? \\
P(P=H|S=F, C=F) &= ?
\end{aligned}
$$

Hint: Marginalization

$$P(X = a) = \Sigma_{y_i \in Domain(Y)} P(X = a, Y = y_i)$$

Answer 4

- *Solution* 1*:*

$$\begin{aligned} P(P|S, C) &= \frac{P(P, S, C)}{P(S, C)} \\ &= \frac{P(P, S, C)}{\Sigma_P P(P, S, C)} \end{aligned}$$

  Plugin the answers from Question 3 into the above equation.

- *Solution* 2*:*

$$\begin{aligned} & \text{//Take all probabilities being conditional on } S\text{, and} \\ & \text{//use Bayesian rule:} \\ P(P|C, S) &= \frac{P(C|P, S) \cdot P(P|S)}{P(C|S)} \\ & \text{// } P \text{ and } S \text{ are independent:} \\ &= \frac{P(C|P, S) \cdot P(P)}{P(C|S)} \\ & \text{// put } P(C|S) \text{ into a constant facor:} \\ &= \alpha(C|S) \cdot P(C|P, S) \cdot P(P) \end{aligned}$$

  where $\alpha(C|S) = \frac{1}{P(C|S)}$.

Question 5 *Compute the following conditioanl probabilities*

$$\begin{aligned} P(S = T|P = L, C = T) &= ? \\ P(S = T|P = L, C = F) &= ? \\ P(S = F|P = L, C = T) &= ? \\ P(S = F|P = L, C = F) &= ? \\ P(S = T|P = H, C = T) &= ? \\ P(S = T|P = H, C = F) &= ? \\ P(S = F|P = H, C = T) &= ? \\ P(S = F|P = H, C = F) &= ? \end{aligned}$$

Answer 5

- *Solution* 1*:*

$$\begin{aligned} P(S|P,C) &= \frac{P(P,S,C)}{P(P,C)} \\ &= \frac{P(P,S,C)}{\Sigma_S P(P,S,C)} \end{aligned}$$

  Plugin the answers from Question 3 into the above equation.

- *Solution* 2*:*

$$\begin{aligned} & \textit{//Take all probabilities being conditional on } P\textit{, and} \\ & \textit{//use Bayesian rule:} \\ P(S|C,P) &= \frac{P(C|S,P)\cdot P(S|P)}{P(C|P)} \\ & \textit{// } P \textit{ and } S \textit{ are independent:} \\ &= \frac{P(C|S,P)\cdot P(S)}{P(C|P)} \\ & \textit{// put } P(C|P) \textit{ into a constant facor:} \\ &= \alpha(C|P)\cdot P(C|P,S)\cdot P(S) \end{aligned}$$

  where $\alpha(C|P) = \frac{1}{P(C|P)}$.

Question 6 *Use the probabilities computed in question* (4) *and* (5) *to show that* $P \not\perp S|C$.

Answer 6 *Check whether* $P(P|S,C) = P(P|C)$ *for every possible combinations of* $P = \{L,H\}, S = \{T,F\}, C = \{T,F\}$.