# MECH 5970/6970 Homework 2

## Due 09/23/2021

1. Pedestrian Navigation System: Use your phone to implement a pedestrian navigation system. Estimate your position by starting at a known location and dead reckoning using an open-source step counter and compass. Make sure that your route is at least 1 km and contains turns in both directions. Use the compass to tracking your orientation. You can record measurements (i.e. step and compass reading) on your phone if you have that functionality. Otherwise use pad and paper to track steps and compass changes.

   You may use a default step length (as discussed in class) or try to estimate your step length (a priori) using a measuring instrument of your choice (e.g. tape measure or GPS). You should update position with each step and orientation at least every time it changes significantly. Plot your route on a map background using GPS visualizer ([https://www.gpsvisualizer.com/](https://www.gpsvisualizer.com/)) or a Matlab toolbox if available. Measure the error in your final position and orientation. Report total error and error relative to distance traveled.

2. Find a scholarly article describing a navigation system for a team of at least two ground vehicles. Describe the sensors used, states estimated, and estimator architecture. Give a brief description of the problem statement/motivation for the work. What results were presented (e.g. plots of positioning accuracy, raw sensor data, etc.)? Did the authors successfully convey the methodology?

3. In class, we developed the basic (elementary) rotation matrix

$$C_{z,\theta} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

that describes the orientation of a coordinate frame rotated away from another coordinate frame by an angle $\theta$ about the $z-$axis.

(a) Derive the basic (elementary) rotation matrix $C_{y,\theta}$ that describes the orientation of a coordinate frame rotated away from another coordinate frame by an angle $\theta$ about the $y-$axis.

4. For each of the matrices below, determine which are valid rotation matrices. Justify your answer based upon expected properties.

(a) $C_b^a = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

(b) $C_c^b = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $C_d^c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(d) $C_e^d = \begin{bmatrix} 0.4330 & -0.7718 & 0.4656 \\ 0.7500 & 0.5950 & 0.2888 \\ -0.5000 & 0.2241 & 0.8365 \end{bmatrix}$

(e) $C_f^e = \begin{bmatrix} 0.5000 & -0.1464 & 0.8536 \\ 0.5000 & -0.8536 & -0.1464 \\ -0.7071 & 0.5000 & 0.5000 \end{bmatrix}$

5. Consider the rotation matrix

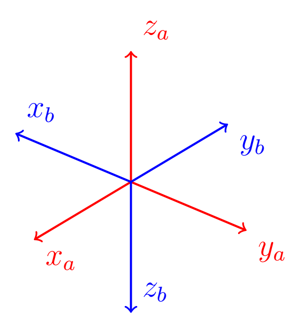
$$C_1^0 = \begin{bmatrix} 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -1 & 0 & 0 \end{bmatrix}.$$

(a) Sketch frames 0 and 1 with their origins co-located.

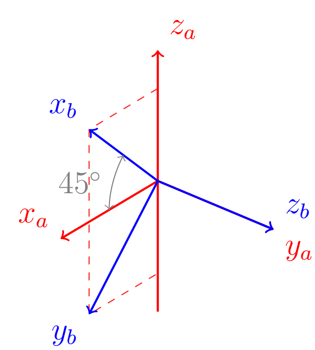
(b) Given a vector $\vec{v}^0 = [1,\ 1,\ 1]^T$ coordinatized in frame 0, re-coordinatize the vector such that it is described in frame 1.

6. For each pair of coordinate frames shown, find the rotation matrix $C_b^a$ that describes their relative orientation.

(a)




(b)



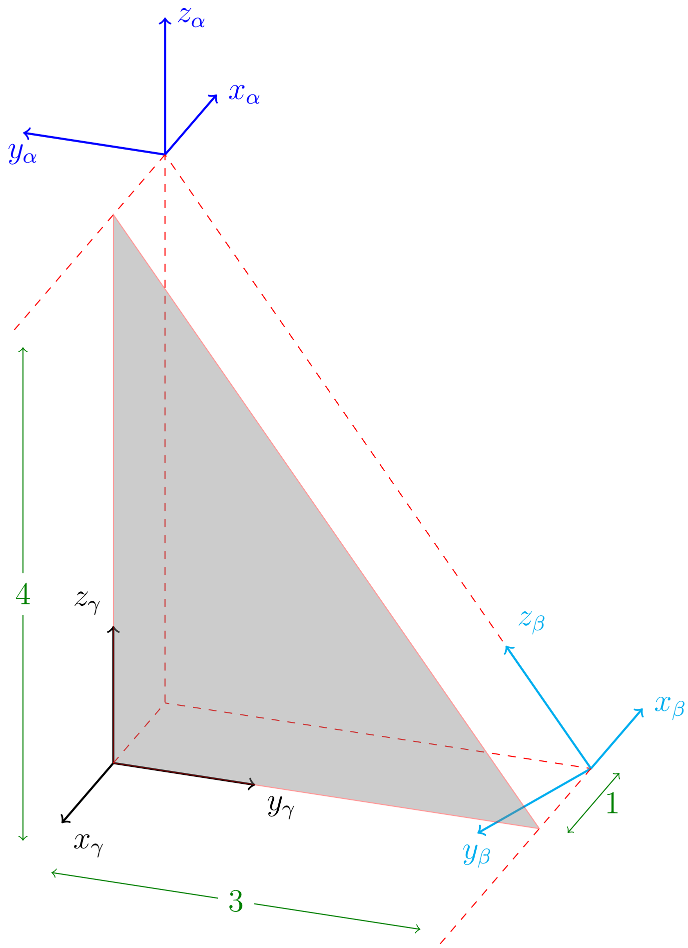
7. Coordinate frame {1} is obtained from frame {0} by the following sequence of rotations:

   (a) $-90°$ about the fixed $z-$axis
   (b) $90°$ about the current $y-$axis
   (c) $-90°$ about the fixed $x-$axis.

   Find the resulting rotation matrix $C_1^0$ and sketch frames {0} and {1} relative to each other.

8. Given the Roll-Pitch-Yaw angles $(\phi, \theta, \psi) = (120°, 45°, -120°)$, find the rotation matrix that describes the same orientation. Assume a ZYX series of rotations. Verify that you have constructed the correct rotation matrix by backing out the angles.

9. (Required for 6790 only) Consider the three coordinate frames $\{\alpha\}$, $\{\beta\}$, and $\{\gamma\}$ shown in the diagram below. Following the notation introduced in the class, find the following Cartesian position vectors (denoted by $\vec{r}$) and rotation matrices (denoted by $C$).




   (a) $\vec{r}^{\,\gamma}_{\gamma\alpha}$
   (b) $\vec{r}^{\,\gamma}_{\gamma\beta}$
   (c) $\vec{r}^{\,\alpha}_{\gamma\alpha}$
   (d) $\vec{r}^{\,\beta}_{\gamma\beta}$
   (e) $C^\gamma_\alpha$
   (f) $C^\gamma_\beta$
   (g) $C^\alpha_\beta$