Comprehensive and Automatic Fitness Landscape Analysis using HeuristicLab

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Abstract. Many different techniques have been developed for fitness landscape analysis. We present a consolidated and uniform implementation inside the heuristic optimization platform HeuristicLab. On top of these analysis methods a new approach to empirical measurement of isotropy is presented that can be used to extend existing methods. Results are shown using the existing implementation within HeuristicLab 3.3.

Keywords: Fitness Landscape Analysis, Isotropy, HeuristicLab

1 Introduction

The original idea of a fitness landscape stems from the descriptions of adaptive populations in [16]. Since then, this metaphor has not only been used in biology but in genetic algorithms and other heuristic methods. It provides a vivid metaphor of the search space as perceived by an optimization process. Hence, it is the basis for understanding typical problems that are subject to heuristic problems solving.

This paper is structured as follows: First the purpose of fitness landscape analysis is reiterated followed by a formal definition. Next, fitness landscape analysis methods are introduced, followed by the presentation of a new method for isotropy measurement as an extension to existing analysis methods. Finally several fitness landscapes are analyzed using standard methods followed by a measurement of isotropy using the newly introduced technique.

1.1 Purpose

In the course of fitness landscape *analysis* many different methods have been proposed. These methods aim to provide complementary views of the fitness landscape and enable the researcher to gain a better understanding of the problem at hand.

At a first glance, fitness landscape analysis might seem like the holy grail of heuristic problem solving. Understanding the fitness landscape of a given problems should yield immediate insight that enables us to easily find the global optimum. In reality, it is bound by the same limits as any other heuristic optimization method even though it has a different focus. While typical metaheuristics provide an effort versus quality trade-off giving you a choice between a more sophisticated analysis and, in return, a solution of higher quality, fitness landscape analysis provides you with an effort versus insight trade-off. Here, the aim is not to solve a given problem as efficiently as possible but to gain maximum insight with feasible effort.

1.2 Definition

Informally, a fitness landscape is often assumed to be given merely by the fitness function $f : \mathcal{S} \rightarrow \mathbb{R}$ alone that assigns a real value to every solution candidate in the solution space $\mathcal{S}$. However, this does not yet give rise to a cohesive structure. To fully capture the landscape, a notion of connectivity is needed [7, 3]. This can range from a direct connectivity through neighbors or mutation operators to distances measures or hypergraphs for modeling crossover [11] . In summary, a fitness landscape is, therefore, defined by the triple

$$\mathcal{F} := \{\mathcal{S}, f, \mathcal{X}\} \tag{1}$$

given by the solution space $\mathcal{S}$, a fitness function f and a notion of connectivity $\mathcal{X}$. A more detailed explanation and comprehensive survey of existing methods can be found in [6].

2 Existing Fitness Landscape Analysis Methods

2.1 Local Methods

The first form of fitness landscape analysis is based on move or manipulation operators and closely examines the immediate structure that is encountered during an optimization process. These local measures examine the immediate fitness changes that occur by applying a certain operator on a fitness landscape and are very often based on trajectories of the fitness landscape created by a random walk.

Ruggedness One of the first measures for fitness landscape analysis was a measure of ruggedness. Several more or less equivalent measures are derived from the autocorrelation function shown in Eq. (2) where $\{f\}_{i=0}^{n}$ is a series of neighboring fitness values that is typically derived through a *random walk* in the landscape, $\overline{f}$ and σ_f are the mean and variance of the fitness values in the random walk and $\mathrm{E}(x)$ is the expected value of x.

$$R(\tau) := \frac{\mathrm{E}[(f_i - \overline{f})(f_{i+\tau} - \overline{f})]}{{\sigma_f}^2} \tag{2}$$

Typically $R(1)$, the autocorrelation after one step is used as an indicator of the ruggedness of a fitness landscape [15]. Additionally, the average maximum distance to a statistically significantly correlated point, called the *autocorrelation length* can be used instead.

Information Analysis Similar to the notion of ruggedness, information analysis as described in [12] can be used to explore local properties of fitness landscapes. First, the series of fitness values obtained from e.g. a random walk is changed into a series of slope directions $d_i = \mathrm{sign}(f_i - f_{i-1})$. Instead of using a strict sign function, a certain interval $[-\varepsilon, \varepsilon]$ around zero can also be regarded as being equal to zero, allowing to "zoom" in and out and vary the amount of detail being analyzed.

Next, several entropic measures are derived. While the *information content* $H(\varepsilon)$ measures the number of slope shapes in the random walk, the *density basin information* $h(\varepsilon)$ analyzes the smooth regions as shown in Eq. (3), where $P_{[pq]}$ is the frequency of the consecutive occurrence the two slopes p and q.

$$H(\varepsilon) := -\sum_{p \neq q} P_{[pq]} \log_6 P_{[pq]} \qquad h(\varepsilon) := -\sum_{p = q} P_{[pq]} \log_3 P_{[pq]} \tag{3}$$

In addition, the *partial information content* $M(\varepsilon)$ measures the number of slope direction changes, hence, also helps in determining the ruggedness. Finally, the smallest ε for which the landscape becomes completely flat, i.e. the maximum fitness difference of consecutive steps is called the *information stability*.

2.2 Global Methods

As a complement to local methods that primarily analyze the immediate effect on optimization algorithms as they proceed step by step, the following *global* methods can help to obtain an overview of the fitness landscape. It has to be emphasized, however, that neither view is superior to the other in general.

Fitness Distance Correlation A very insightful global analysis method is the fitness distance correlation. It visualizes the relation between fitness of a solution candidate and its distance to the global optimum. While this can give very good insight into whether the fitness function together with the selected stepping algorithm yields a meaningful succession towards the global optimum it has one severe drawback: One needs to know the global optimum in advance.

The fitness distance correlation coefficient is defined in Eq. (4). This single number can provide a quick look at whether increased fitness actually leads the search process closer to the global optimum. However, as any correlation coefficient this does not work properly for non-linear correlations.

$$\mathrm{FDC} := \frac{\mathrm{E}[(f_i - \overline{f})(d_i - \overline{d})]}{\sigma_f \, \sigma_d} \tag{4}$$

On the other hand, looking at a fitness versus distance plot can provide some more insight into the global structure of the fitness landscape and help in determining certain trends.

Evolvability Another approach for a more global analysis is to take the neighboring fitness values but regroup them according to their base fitness. It is like looking at the individual contour lines in a contour plot of the landscape. In [9] a selection of different measures of evolvability is presented called the *Evolvability Portraits.*

We start with a basic definition of evolvability itself in Eq. (5), where $N(x)$ is the set of all neighbors of x and N^* is the set of all better neighbors of x or, in the continuous definition, the integral over all neighbors with higher fitness $(f(n) \geq f(x))$ over the probability of selecting a certain neighbor $p(n|x)$.

$$\mathcal{E}(x) := \frac{|N^*(x)|}{|N(x)|} \qquad \text{or} \qquad \mathcal{E}(x) := \int_{f(n)\geq f(x)} p(n|x)\mathrm{d}n \tag{5}$$

Based on this definition Smith and coworkers define four measures for every solution candidate x that together form the so-called *evolvability portraits* [9]: First, E_a is simply the probability of a non-deleterious mutation. Second, E_b is the average expected offspring fitness $\int p(n|x)f(x)\mathrm{d}n$. Finally, the two measures E_c and E_d are a certain top and bottom percentile of the expected offspring fitness or in our implementation simply the minimum and maximum expected offspring fitness.

3 Isotropy

It is very convenient to assume that the fitness landscape looks exactly the same everywhere. This assumption is called *isotropy.* Many local analysis methods implicitly or explicitly assume isotropy [12, 2]. While many real world fitness landscapes seem to be almost isotropic this assumption has to be verified. One obvious formulation for a measure of isotropy has been described in [10] as *empiric isotropy* as shown in Eq. (6), where $\langle\cdot\rangle_d$ is the average over all values with $|x-y| = d$ and $\langle\cdot\rangle_d^A$ is a restriction of the former to an arbitrary subset A.

$$\langle (f(x) - f(y))^2 \rangle)_d^A \cong \langle (f(x) - f(y))^2 \rangle_d \tag{6}$$

This notion of empirical isotropy is defined over fitness differences at certain distances in the solution space. We propose to extend this idea to any measure by requiring that the result of a measurement on a subspace has to be approximately the result of the measurement on the whole solution space within reasonable statistical bounds as shown in Eq. (7) where $\mathcal{M}$ is any fitness landscape measure and A is an arbitrary subset of the solution space $\mathcal{S}$.

$$\mathcal{M}(A) \cong \mathcal{M}(\mathcal{S}) \tag{7}$$

As an extension, Eq. (8) also has to hold in an isotropic landscape where A_i and A_j are suitable subsets of the solution space $\mathcal{S}$.

$$\mathcal{M}(A_i) \cong \mathcal{M}(A_j) \tag{8}$$

By repeatedly comparing different subsets A_i and A_j the amount of isotropy can be estimated. An important trade-off to consider in this scenario is the choice of number and sizes of the subsets A_i. While a larger subset gives a statistically more robust estimate of the local measure a smaller subset will elucidate a much more local view of the landscape.

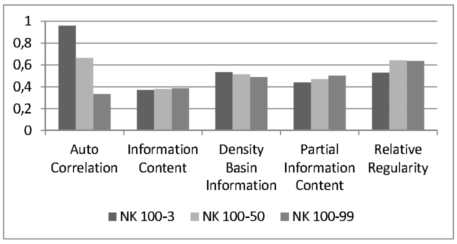
Usually these subsets will be selected as the trajectory of a random walk at a random starting point. As a random walk introduces a large amount of variability especially if it is short, the random walks are concentrated around the sample points by repeatedly restarting from the same point several times and averaging over these repeats. This gives a much more robust analysis while still enabling a local view of the fitness landscape.

4 Implementation

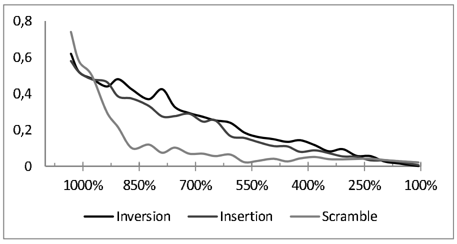
The paradigm-invariant open-source optimization framework HeuristicLab [13, 14] provides an optimal starting point for the extension with a fitness landscape analysis toolkit. An additional plugin has been implemented containing the following features. Most analysis methods have been implemented as *analyzers* that can be attached to existing algorithms and provide information about fitness landscape characteristics during execution. In addition new *algorithm-like drivers* have been implemented that enable the use of custom trajectories for analysis, such as random walks or adaptive up-down walks which are frequently used for analysis. Moreover, the functionality of the analyzers is available as operators which can be reused in user-defined algorithms together with additional operators aimed at fitness landscape analysis. Additionally, an implementation of the NK fitness landscapes [4], which are the defacto playground problem for fitness landscape analysis methods, using a more general block model as described in [1] is also included. A HeuristicLab version with the new plugin together with the demonstration runs is available at `http://dev.heursiticlab.com/AdditionalMaterial`.

5 Results

Figure 1 shows a selection of scalar fitness landscape analysis results as obtained by HeuristicLab. It was derived using the local analysis algorithm configured to perform a random walk on different NK fitness landscapes [4] using a bit flip operator (Figure 1(a)) and a traveling salesman problem [5] instance (ch130 from the TSPlib [8]) using different permutation mutation operators (Figure 1(b)). It is easy to see how increasing complexity of the NK landscapes also increases measures for ruggedness, (partial) information content and regularity, while at



(a) Key figures of different NK landscapes



(b) Evolvability analysis of different mutation operators on a TSP landscapes

Fig. 1. Fitness landscape analysis results using HeuristicLab

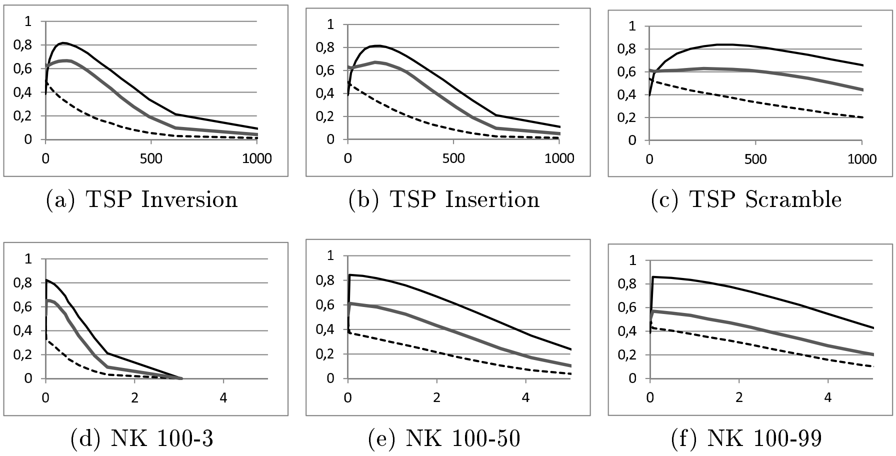
the same time decreases measures for smoothness such as auto correlation and density basin information. In the TSP examples (Figure 1(b)) the evolvability average for different parent quality levels is plotted. The probability of producing fitter offspring starts at around 80% while still 1000% away from the best known solution and reduces to almost zero when approaching the best known solution. The initially superior scramble operator quickly becomes worse as higher quality base fitness values are approached. However, closer to the optimum it performs better than the others because (2% vs. 0,5% and 0,03% for insertion and inversion respectively).

While the previous analysis gives a quick insight into which mutation operators might be beneficial or analyze which landscape variant is more difficult, the information analysis in Figure 2 can provide more insight into the structure of the landscape. These plots show several information analysis values for the previously analyzed NK and TSP landscapes. We can observe how higher quality jumps occur for the more rugged landscapes as described previously. Moreover, this time we can also see the fundamental difference in more global structures of their fitness landscapes by looking at the landscape from a more coarse grained perspective. This shows, that the TSP landscape is much more structured as it shows a smooth transition between different zoom levels while the NK landscape immediately jumps to very information rich structures. Moreover, the NK landscape reaches higher levels of information content and lower levels for density basin information, hinting at a richer formation of rugged and less prevalent smooth areas in the landscape.

Figure 3 shows an example of isotropy estimation. The pronounced difference of the autocorrelation curves throughout the landscape for the sine of the exponential function demonstrate the much higher anisotropy in this case.

6 Conclusions

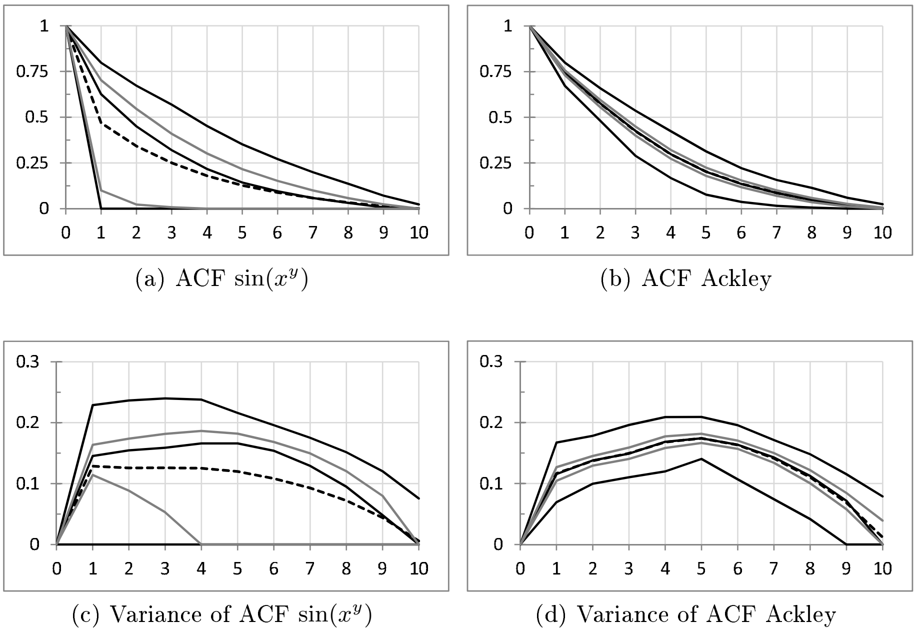
We have created a simple toolkit that allows plugging typical fitness landscape analysis methods into standard algorithms and provides new algorithm-like tra-



(a) TSP Inversion (b) TSP Insertion (c) TSP Scramble

(d) NK 100-3 (e) NK 100-50 (f) NK 100-99

Fig. 2. Information analysis: The X-axis contains the ε-threshold used to smooth the quality jumps in the random walk, while the Y-axis contains the entropy of information content (solid black), partial information content (dashed) and density basin information (solid gray).



(a) ACF $\sin(x^y)$ (b) ACF Ackley

(c) Variance of ACF $\sin(x^y)$ (d) Variance of ACF Ackley

Fig. 3. Autocorrelation analysis of 50 random walks of length 50 at 10,000 points. The solid lines are minimum, median and maximum values, while the gray lines are the quartiles and the dashed line is the average. The wider spread of the autocorrelation families shows the anisotropy of the sine exp function in Figure 3(a) and 3(c).

jectory generators for more traditional fitness landscape analysis using random or partly adaptive walks. Isotropy is a convenient assumption in most cases. Its extent has only sparingly been tested in the past. We have demonstrated a simple method to empirically measure isotropy extending the well established autocorrelation analysis. Based on this method it is possible to extend other analysis methods and obtain other perspectives of isotropy. We argue that isotropy estimation cannot be determined universally but has to be linked to a particular underlying measure.

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References

1. Altenberg, L.: The Handbook of Evoluationary Computation, chap. NK Fitness Landscapes, pp. B2.7.2:1–11. Oxford University Press (1997)
2. Czech, Z.: Statistical measures of a fitness landscape for the vehicle routing problem. In: IEEE International Symposium on Parallel and Distributed Processing, 2008. pp. 1–8 (2008)
3. Jones, T.: Evolutionary Algorithms, Fitness Landscapes and Search. Ph.D. thesis, University of New Mexico, Albuquerque, New Mexico (1995)
4. Kauffman, S.A.: The Origins of Order. Oxford University Press (1993)
5. Lawler, E.L., Lenstra, J.K., Kan, A.H.G.R., Shmoys, D.B. (eds.): The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization. Wiley (1985)
6. Pitzer, E., Affenzeller, M.: Recent Advances in Intelligent Engineering Systems, chap. A Comprehensive Survey on Fitness Landscape Analysis, pp. 167–196. Springer (2011)
7. Reeves, C.R., Rowe, J.E.: Genetic Algorithms. Kluwer (2003)
8. Reinelt, G.: Tsplib–a traveling salesman problem library. ORSA Journal on Computing 3(4), 376–384 (1991)
9. Smith, T., Husbands, P., Layzell, P., O'Shea, M.: Fitness landscapes and evolvability. Evol. Comput. 10(1), 1–34 (2002)
10. Stadler, P.F., Grüner, W.: Anisotropy in fitness landscapes. J. Theor. Biol. 165(3), 373–388 (1993)
11. Stadler, P., Wagner, G.: The algebraic theory of recombination spaces. Evol.Comp. 5, 241–275 (1998)
12. Vassilev, V.K., Fogarty, T.C., Miller, J.F.: Information characteristics and the structure of landscapes. Evol. Comput. 8(1), 31–60 (2000)
13. Wagner, S.: Heuristic Optimization Software Systems - Modeling of Heuristic Optimization Algorithms in the HeuristicLab Software Environment. Ph.D. thesis, Johannes Kepler University, Linz, Austria (2009)
14. Wagner, S., Kronberger, G., Beham, A., Winkler, S., Affenzeller, M.: Modeling of heuristic optimization algorithms. In: Proceedings of the 20th European Modeling and Simulation Symposium. pp. 106–111. DIPTEM University of Genova (2008)
15. Weinberger, E.: Correlated and uncorrelated fitness landscapes and how to tell the difference. Biological Cybernetics 63(5), 325–336 (1990)
16. Wright, S.: The roles of mutation, inbreeding, crossbreeding and selection in evolution. In: Proceedings of the Sixth International Congress of genetics. vol. 1, pp. 356–366 (1932)