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Preference Incorporation in Multi-objective Evolutionary Algorithms: A Survey

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Abstract— **This paper presents a review of preference incorporation in Multi-Objective Evolutionary Algorithms (MOEA). The incorporation of preference in Evolutionary Multi-objective Optimization (EMO) promotes better decision-making. Introducing preference in MOEAs increases the specificity of selection, leading to solutions which are of higher relevance to the Decision Maker(s). When many objectives are involved, a MOEA based on pure Pareto-optimality criterion may not achieve meaningful search. The incorporation of preference addresses this concern.**

The incorporation of preference is difficult because of uncertainties arising from lack of prior problem knowledge and fuzziness of human preference. Further, decision making is a complex and ill-defined process which at times could not be mathematically characterized. These concerns must be addressed in the incorporation of preference.

I. Introduction

Multi-objective optimization (MOO) is an important class of problem copiously encountered in engineering and industrial context. The conflict of objectives entailed in MOO places the issue of compromise in a central position. Edgeworth and Pareto captured this notion mathematically in the criterion widely known as Pareto-optimality [1]. Solutions belonging to the Pareto-optimal set corresponding to a particular MOO problem perform better in one or more objectives and worse in other objectives. In other words, solutions in a Pareto-optimal set display differing trade-offs, which is readily observable in the image of this set in the objectives space, the Pareto-optimal front.

Since no single solution optimizes all objectives in concert, decision-making based on subjective human preference is an inherent aspect in solving MOO problems. Only a single solution out of the Pareto-optimal set is required. Preference is the basis of tie-breaking between solutions in the Pareto-optimal set.

Human preference entails fuzzy uncertainties. In the areas of Multi-criteria Decision Making (MCDM) and Multi-criteria Decision Aid (MCDA) a variety of frameworks capturing the decision maker(s)' preferences have been proposed. Multi-attribute utility theory (MAUT) [2], Analytic Hierarchy Process [3] and outranking synthesis [4] are some of the most popular preference specification schemes. The multiplicity of preference articulation schemes highlights the complexity of human preference.

Traditionally, multi-objective optimization evolutionary algorithms employ the MAUT-based weighted-sum aggregation of the objective functions. In later developments, especially after the publication of Goldberg's paper [5], other strategies have been proposed to deal with the human preference element in Multi-objective Evolutionary Algorithms. Following Veldhuizen's classification [6], Multi-objective Evolutionary Algorithms (MOEAs) are apriori, interactive, or aposteriori algorithms based on the treatment of preference.

A priori MOEAs involves preference specification prior to the optimization stage, and are traditionally implemented by aggregating objectives [7-10] into a single fitness function with parameters reflecting the preference of the decision maker(s). Interactive MOEA [11-12] allow decision maker(s) to alter parameters during the search, effectively influencing the direction of the search. Aposteriori algorithms [13-15] find the set of Pareto-optimal solutions and relegate decision making based on human preference to the post-optimal stage. The aposteriori approach is to date the most popular. In recent years, researchers have started to look into incorporating preference in the search process of aposteriori MOEA. Development in this area is interesting, potentially the key element to widespread application of MOEA in practical circumstances.

This paper provides a review of preference incorporation in EMO. In section II a survey of representative work in the area is presented. In section III salient research issues are pointed out.

II. Preference In Evolutionary Multi-objective Optimization

The general multi-objective optimization (MOP) problem may be stated as follows [1]:

Definition 1:
Find the vector of decision variables $X^* = \{x_1^*, x_2^*, ...x_n^*\}$ which satisfies the m inequality constraints:

$$g_i(X) \geq 0, i = 1,2,....m$$

p equality constraints:

$$h_i(X) = 0, i = 1,2,....p$$

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and optimizes the k objective functions:

$$\vec{f}(X) = [f_1(X), f_2(X), \ldots f_k(X)]$$

Without loss of generality, the minimization of objectives is considered in the discussion throughout this paper. In general the objectives are non-commensurable and in conflict with each other. The non-commensurability present in the objectives is captured in the Pareto-optimality criterion. Pareto optimality originates from the works of eminent economists Edgeworth and Pareto studying various macro-economic variables in a society's welfare. A vector of decision variables $\boldsymbol{X_j}$ dominates $\boldsymbol{X_i}$ in the Pareto-optimality sense if and only if $\boldsymbol{X_j}$ performs better in at least one objective and at least as good as $\boldsymbol{X_i}$ in the rest.

Definition 2:

$$X_i \prec X_j \Leftrightarrow$$
$$f_n^j < f_n^i \text{ for } 1 \le n \le k \text{ and } f_m^j \le f_m^i \text{ for all } m \in [1,k], m \ne n$$

If neither of the two candidate solutions dominates the other, they are non-dominated. The Pareto-optimal set of a MOP consists of non-dominated solutions, which dominate other solutions in the feasible region. The image of this set in the objective space is the Pareto-front, which displays the trade-off characteristics between the objectives.

Preference zeroes in on a subset of the Pareto-optimal deemed of high relevance to the decision maker for the particular problem at hand. Preference is vague, perceptive information and is highly dependent on the application context. The vagueness and context-dependence of the DM's preference structure result, unsurprisingly, in various mathematical models as well as techniques proposed for the incorporation of preferences. In section A methods of computationally incorporating preference in MOEA are described. In section B proposed models of preferences are reviewed.

A. Incorporating Preference in MOEA

In *a priori* algorithms, the DM formulates preferences prior to the optimization. The formulation is then incorporated in the fitness function computation and not amenable to change during the optimization. Aggregation of the objective functions into lexicographic order or into a linear/nonlinear combination are the commonly employed a priori methods. In lexicographic aggregation the objectives are optimized in their order of the importance while in fitness combination parameters of the aggregating function reflect the human preferences involved.

When the preference involved could be faithfully captured in the mathematical model employed and no practical computational difficulties arise, a priori approaches are simple and efficient. However, this is rarely the case. The non-commensurability of objectives often cannot be adequately modeled in a priori preference specification. Further, a priori approaches often require sufficient knowledge of the specific problem before associated parameters in the aggregation function could be determined. Practical computational difficulties may also arise. Non-convex Pareto-front induces speciation in MOEA based on the weighted sum approach which leads to the situation that solutions in non-convex regions cannot be found. Scalability problem with respect to increasing number of objective functions are also common to a priori approaches. The difficulty in specifying the right parameters a priori increases considerably for the DM as the number of objectives increase.

Interactive MOEA involve the decision maker(s) in the search process, allowing decision maker(s) to tweak parameters directing the search. An example of an interactive MOEA is I-EMO developed by Deb and Chauduri [11]. Interactive approaches require a priori specification of only a few parameters – the rest may be tailored as the search progresses. As more information of the problem is revealed in the search, decision makers are able to make better judgment to determine the appropriate values of associated parameters. The facility of refining preference articulation as more is revealed on the trade-off characteristic of the particular problem instance is often the most suitable for multi-objective optimization. Interactive approaches are in fact the most favored in MCDM.

The main disadvantage of interactive approaches stem from the intensive effort required from the DM and the challenge faced when more than one decision-maker are involved. Fonseca and Fleming [12] proposed the idea of incorporating an expert system to assume the role of the DM. Fuzzy logic or other established methods in expert systems could be employed to this purpose. However, as is widely recognized, successful expert systems are highly dependent on application context. The construction of a viable expert system is expected to require extensive problem domain knowledge which results from extensive exploration of representative instances of the particular problem at hand.

A posteriori algorithms segregate the optimization and decision-making process. A posteriori MOEAs aim to find a well-distributed approximation which converges to the entire Pareto-front. This is achieved by fitness assignment based on the domination criterion and a niching strategy. Decision-making ensues once all the Pareto-set approximation is obtained. Equipped with the trade-off information the decision maker(s) selects the most preferred solution. The aposteriori approach has gained significantly more popularity than apriori or interactive approach. A number of efficient aposteriori MOEAs [13-15] have been proposed and widely used.

Aposteriori algorithms avoid a number of challenging computational issues by relegating the preference-based decision making to post-optimization stage. However, new problems are introduced. The computational cost incurred in supplying the DM with a set of alternatives exhibiting varying trade-offs is considerable. Population-wide Pareto ranking, archiving strategy, and diversity preservation measures are features commonly found in the MOEAs which are computationally expensive. Streamlining efforts have been introduced (e.g. [16]) to cut down on the computational cost but further research is requisite for significant improvement in this respect. Further, navigating through the Pareto-optimal set could be quite formidable especially where many objectives are concerned such that researchers have developed data mining tools to aid the post-optimal decision making [17-19]. Many-objective optimization with a posteriori MOEA faces yet another more serious challenge. When many objectives are to be optimized or the search space is sufficiently large, algorithms based on the Pareto-optimality criterion could not achieve much progress. So many solutions would be non-dominated with respect to each other that no effective selection pressure is applied in the evolutionary algorithm.

Algorithms amalgamating the principle of apriori and aposteriori approaches have also been proposed in recent years [20-21]. These algorithms characterize decision maker(s) preference and concentrate the search effort to solutions on the Pareto-optimal front with the desired feature. At the end of the optimization stage, the decision maker(s) are provided with a set of solutions to select from. By introducing a bias to specified regions of the Pareto-front the algorithms avail decision maker(s) with higher number of solutions likely to be relevant to the decision maker(s). The preference-biased search is often accomplished by modifying the Pareto-domination criterion, either by adding a secondary selection criterion based on the preference model or direct reformulation of domination criterion.

B. Modeling Decision Maker(s) Preference

The mathematical model captures the characteristic of the subset of Pareto-optimal candidate solutions on which the search is to be concentrated. The model must be able to represent faithfully the intuitive preference of the Decision Maker(s). Different model for preferences in EMO have been developed, including specification of the priority of objectives, acceptable trade-off between objectives, optimization goal, and specific properties of the solution.

Once the preference model is specified, the designer selects a way of incorporating this model into the search process. Below are some representative research works in this area:

1. Trade-off between objectives.

Trade-off specifies the amount of improvement in one or more objective(s) which the DM is ready to relinquish to attain a unit improvement in other objective. Trade-off is an intuitive formulation of preference.

Specifying preferred trade-off between objectives is the basis of Branke's [20] guided evolutionary algorithm. Two sets of weights corresponding to the minimum and maximum trade-offs are pre-specified and the linear summation of weighted objective functions - $w_{\max}(f_1(X), f_2(X))$ and $w_{\min}(f_1(X), f_2(X))$ corresponding to the maximum and minimum trade-offs respectively, are employed in a modified domination criteria as follows:

$$
\begin{aligned}
& X_j \succ X_i \Leftrightarrow \\
& w_{\max}(f_1(X_j), f_2(X_j)) \le w_{\max}(f_1(X_i), f_2(X_i)) \\
& \wedge w_{\min}(f_1(X_j), f_2(X_j)) \le w_{\min}(f_1(X_i), f_2(X_i))
\end{aligned}
$$

with strict inequality in at least one of the two cases.

The modification resulted, for any point in the objective space, the narrowing of the dominating region. With fitness sharing, the method displayed convergence in the desired segment of the convex Pareto front. One of the weaknesses of the approach is that the linear utility function arithmetic leads to speciation in cases of a concave front. Further, dependence between objectives cannot be modeled in this approach. Like the standard MAUT-based weighted-sum approach, an extension to higher numbers of objective functions is encumbered by the difficulty of specifying minimum and maximum tradeoffs. An interactive implementation of this approach would address some of the problems.

2. Goal Specification

A priori goal specification, when possible, could contribute to the efficiency of the MOEA. In 1993 Fonseca and Fleming proposed MOGA with interactive goal attainment which allows the Decision Maker(s) to modify the goals as more problem knowledge becomes available to them [12]. In 1998 Fonseca and Fleming [21] proposed added a facility for specifying priorities of the objectives. Multiple objectives may be assigned the same priority. Solutions are compared pair-wise in terms of the objectives in order of priority. Objectives in the highest priority are considered first and if the two solutions being compared violate the goals in the same way, objectives in the next priority level are considered. The approach is also applicable where no specific goals are available, or where all objectives are of the same priority.

The specification of a viable goal in the above approaches requires involved interaction with the DM during the search progression. The concern is addressed in the approached proposed by Tan et al. [31]. By allowing multiple goals specified by conjunctions "AND" and "OR" the most appropriate from the set of goals defined a priori would effectively guide the search. The proposed approach also incorporates goal and priority-of-objectives information which are combined to form a goal-sequence matrix.

The implementation in the above algorithms directs the solutions already satisfying pre-specified goals to the utopian infinum by employing the standard Pareto-domination criterion. The primary disadvantage of this approach is the added computational complexity. Further, the model, unlike models in (1) and (3) does not allow the notion of trade-off between the objectives or any other relation between the objectives.

3. Ranking of objectives

Relative importance between objectives has historically been a popular model of preference in multi-objective optimization. Lexicographic optimization of objectives does not allow trade-off information to be captured. With the aid of fuzzy logic, this difficulty could be surmounted.

Cvetkovic [22] and Jin and Sendhoff [23] model the preference of the decision maker as the relative importance of objectives. In both work, the decision maker exploits linguistic variables and modifiers in fuzzy logic to characterize the relative importance of two objectives at a time. From the characterization, a fuzzy preference matrix is constructed and used to compute weight parameters in an aggregative fitness function. Cvetkovic applied the method to the weighted sum method, weighted Pareto optimization, weighted co-evolutionary optimization and weighted scenario and constraint handling. The method performed well for a bi-objective maximization problem with convex Pareto front. The authors demonstrated that solutions converge to the part of the Pareto front favoring the more important objective. In cases of a non-convex Pareto front, however, speciation will occur. Sensitivity to parameters requiring an involved process of tuning is also a problem.

To address the problem of parameter tuning, Jin and Sendhoff [23] used weight intervals, which are incorporated in the MOEA using Random Weighted Aggregation and Dynamic Weighted Aggregation [24] techniques, designed on the basis of linear weighted sum to cover the whole Pareto front. The methods are simple to implement and the lack of diversity preservation measure saves computation time. The performance of the algorithm for convex as well as concave Pareto front corresponding to a bi-objective minimization was presented. In the former case, a concentration on the desired segment of the Pareto front, but in the latter case, the convergence was not as good. Another difficulty with the preference model is that all objectives are assumed to be independent of each other.

Greenwood et al. proposed a slightly different approach [25] in which decision-maker(s) are required to rank a small number of candidate solutions instead of the objective functions. From the ranking of candidate solutions, the relative importance of objectives are inferred and employed in an imprecisely specified multi-attribute value theory (ISMAUT) formulation. A linear programming problem is solved to infer the ranking between two solutions. The mutual independence of objectives is again a basic assumption in the preference model.

A specification of the relative importance of objectives with the aid of fuzzy logic allows the quantification of relative importance. However, the process of extracting the preferences of the Decision Maker(s) could be quite daunting when a large number of objectives are involved.

4. Outranking

The outranking method [4] is a special scheme of ranking objectives which allow non-transitivity to occur. Outranking takes into account the non-commensurability of objectives and therefore could be implemented directly without Pareto-domination criterion in a MOEA. The method caters to DM preferences in terms of weights specifying the importance of objectives employed in a weighted sum determining the preference index of one candidate solution over another. The preference index is computed for every pair of candidate solutions and determines a preference flow figure for every candidate solution. PROMETHEE II [32-34], an implementation of outranking method where complete preorder is constructed from the preference flow, is the version employed in the preference-based MOEA by Rekiek et al. [35] and Filomeno et al. [36]. Besides the weights, preference and indifference indexes for each objective involved are also parameters to be pre-specified by the DM in PROMETHEE II.

The complete preorder achieved means a single best solution is discovered by the algorithm at the end of the run and could be directly used by the DM. Rekiek et al. implemented PROMETHEE II at every iteration of a MOEA solving hybrid assembly line problems. The outranking parameters in the approach are determined a priori. If the solution obtained at the end of the run is unsatisfactory, a different set of weights could be used to re-run the algorithm.

Filomeno et al. [36] introduced an adaptive weighting scheme based on constraint satisfaction for solving the problem of mechanical structures design. As constraints figure significantly in such problems, an additional objective function is synthesized which represents violated constraints. The predefined set of weight values for each true objective function is scaled by the proportion of the population satisfying the constraint. Filomeno et al. compared their algorithm to the weighted-sum approach and obtained consistently better results for several test problems. However, in the case of an interesting simulation result where the proposed method failed to converge to the true Pareto-front the author suggested that Pareto-based approaches may produce the desired convergence. In that light, a preference-based MOEA which combines the outranking-synthesized fitness figure with a Pareto-domination based criterion may introduce the desired convergence and focus on preferred solutions. Alternatively, an a posteriori approach where the set of Pareto-optimal solutions are evolved and later ranked according to the outranking criterion as proposed by Massebeuf et. al. [37] could also be employed to ensure better convergence properties.

The main disadvantage of the outranking method remains the high number of parameters to be specified. Whether defined a priori or in an interactive manner, the effort required of the DM in setting the parameters will be overwhelming as the number of objectives increases.

5. Fuzzy Logic

This section presents methods which employ fuzzy logic in an implementation of preference in the MOEA. Fuzzy logic was proposed as a framework of human reasoning and information representation. Rachmawati and Srinivasan [39] proposed a Fuzzy Inference aggregation of multiple objectives to solve a resource allocation problem. The method in effect models fitness as various local functions, each of which aggregates the objectives with parameters reflecting the desirability of that particular locality. The advantage of the method is in the simplicity of the fitness function once the fuzzy inference system is constructed. The main weakness of the method is in the formulation of the fuzzy inference system which requires intensive tuning of membership functions and inference operators. Variations of the underlying aggregation function could approximate complex preference structures, but again, translating the human preference into the appropriate function and parameters would prove to be difficult.

6. Attributes in the solution.

A number of researchers recognized that in some engineering applications specific features of the solutions are desirable. Robustness and marginal rate of return are some of the characteristics which have been incorporated into the selection criterion of MOEAs.

Robustness is a measure of the sensitivity of a solution's performance in objective functions to perturbations in the decision variables. In some real life applications, it is impossible to precisely implement the obtained solution without any perturbation. Deb and Gupta [26] proposed a MOEA focusing on the robust Pareto-optimal front. Two measures of robustness involving the averaged objective functions in a neighborhood of a solution replace the usual objective functions in the dominance criteria. A straightforward extension of the domination criteria, it is more computationally expensive than non-robust algorithms due to the neighborhood averaging.

Branke et al. [27] proposed a MOEA which emphasizes convergence to knee regions in the Pareto-front. Solutions characterized by steep trade-off (small improvement in one objective is accompanied by considerable deterioration in another) constitute regions of interest to the decision maker because of the high marginal rates of return. The angle a solution makes in the objective space with adjacent solutions on the Pareto-front or the expected marginal utility given by the solution are incorporated as a secondary criteria after the domination criteria. As the author acknowledged, complexity is a deterrent to application of the former method to optimizations of more objectives while non-convex Pareto front renders the latter method invalid. The focus on knee regions tries to incorporate the amount of improvement and degradation (extent of trade-off) in the optimality criteria with assumption of equal importance among objectives. Farina and Amato [28] presented a fuzzy domination criteria considering the extent of trade-off which includes also the number of objectives degraded and improved while assuming equal importance among objectives.

Zitzler and Kunzli [29] proposed the use of a quality indicator as a general facility for expressing the preference of the decision maker with respect to the property of the Pareto-optimal set obtained. The quality indicators could be defined according to the characteristic of the solutions which are of importance to the decision maker(s). Two binary quality indicators comparing two Pareto set approximations were presented, with reference to other possible quality indicators in [29]. Simulations on benchmark problems demonstrated the superior performance of the Indicator-Based Evolutionary Algorithm (IBEA) with respect to the desired property compared to two Pareto-ranking based MOEA. The indicator function is an aggregation scheme which must be formulated by the decision maker(s) according to his/her preference, making the scheme not widely accessible for non-expert users. It is also too general for widespread use without further study.

The algorithms above were implemented as a combination of apriori and aposteriori approaches. Deb and Chaudhuri proposed a multi-stage interactive algorithm encapsulating preference based on robustness, marginal rate of return and limiting trade-off (the preference model described in point 1) in [12].

III. The Incorporation of Preference in MOEA: Salient Research Issues

The optimality criterion in an optimization algorithm imposes a single-dimensional order to the feasible region in the objective space. Pareto-optimality criterion captures the non-commensurability of objectives in the partial order induced by the domination relation. The incorporation of preference modifies the order defined by Pareto-optimality and the associated Pareto-domination relation. Let the new preference-based relation be denoted by the symbol $\succ_{pref}$.

In his review paper, Coello Coello identified four issues to consider in the incorporation of preference [30]:

1. Preservation of Pareto-domination
2. Transitivity of preference
3. Scalability with respect to the number of objectives.
4. Multiple decision makers

To the four the authors would like to add the issue of performance measures. Various models have been proposed but none was noted to employ an explicit measure of how well the obtained solutions match the preference of the decision maker. The difficulty in formulating such a performance measure stems from the vagueness and imprecision of human preference. However, for various models of the same preference (ranking of objectives, etc) to be compared, a performance measure is necessary.

The preservation of Pareto domination relation implies that if solution $\boldsymbol{X_j}$ dominates $\boldsymbol{X_i}$, solution $\boldsymbol{X_j}$ is preferred over solution $\boldsymbol{X_i}$.

$$X_j \succ X_i \Leftrightarrow X_j \succ_{pref} X_i$$

The inclusion of preference only modifies relation between non-dominated solutions, thereby producing more specific selection.

Pareto domination relation imposes a partial order on the objective space [9]. Domination is a transitive, irreflexive and anti-symmetric binary relation. The equivalence relation induced, non-domination, is however non-transitive. Fig. 1 shows three points, A, B and C in the objective space of a bi-objective minimization problem to illustrate this. Points A and C correspond to non-dominated solutions, and likewise B and C. Points A however correspond to a solution dominating that corresponding to point B.

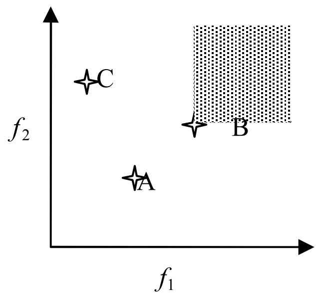

Fig. 1 Non-transitivity in Pareto non-domination relation

The non-transitivity of non-domination relation is the reason behind population-wide comparisons in Pareto-ranking procedures used in pure aposteriori MOEAs. Intransitivity prohibits the formulation of a concise mathematical model and efficient algorithms. Preservation of Pareto-domination relation could be achieved while ignoring the non-transitivity. Preserving Pareto-domination may be easily accomplished by adapting the Pareto-domination criteria in the fitness assignment scheme or employing variants of the MAUT-based weighted-sum aggregation. The main concern rather should be placed on constructing a preference model which differentiates non-dominated solutions according to the true preference of the DM. Variants of the weighted-sum methods, for instance, places a severe limit on the expression of equivalence between solutions. Non-dominated solutions are equivalent if and only if they lie on the same linear hyper-plane defined by the weighted sum.

To obtain higher discriminatory power the non-transitivity must be propagated to preference-based algorithms, regardless of the transitivity or non-transitivity of the preference model itself. This is also important in preference models where objectives are considered to be of the same importance. The propagation of the non-transitivity of non-domination relation has prompted reliance on Pareto-domination based selection criterion. Most of the preference models described in the previous section employ modified Pareto-domination criterion. Only the outranking-based methods ([35-36]) and those employing weighted-sum variants ([22-23]) do not.

The scalability issue is still the formidable challenge common across most preference models surveyed. Increase in computational complexity and difficulties in setting required parameters as the number of objectives increases are issues in a priori as well as interactive preference formulation. The use of an expert system to substitute the DM would be highly beneficial but constructing the appropriate expert system for such application would be an issue worth investigating on its own.

While imprecision is an acknowledged artifact, few models employ fuzzy logic directly in the fitness function. The presence of multiple decision maker(s) is also a significant issue in real-life applications. Coello summarized common features of group decision making in the irrationality of choice which may occur [30]. A consistent mathematical model of preference cannot be constructed where irrationality of choice needs to be taken into account. While irrationality is hard to deal with, in practice it may be localized in the following sense: Suppose particular attributes have been agreed upon by the group of the decision makers as preferred attributes and have been captured in an appropriate mathematical model *M*. In the resulting solutions, a solution *A* may score higher compared to *B* according to *M*. In the group decision making process *B* may be selected instead of *A* if the discrepancy in the score is tolerably small. Under that circumstance, fuzzy logic tools may be used to model the possible region of irrationality as a region of imprecision. A partial specification of preference assuming rationality is accomplished which allows solutions *A* and *B* to be equally preferred. In the optimization stage, the strategy allows for a region of uncertainty to cover the possible final solutions. Within this region of uncertainty, irrationality of choice is catered to.

IV. Conclusion

In the paper a review of preference incorporation in Multi-objective Evolutionary Algorithms was presented in terms of the preference model and implementation strategy. Proposed mathematical models of preference are highly application dependent. The main concern in the approaches reviewed remains the scalability with respect to the number of objectives.

The implementation of preference is to a certain degree independent of the preference model. In setting the various parameters in the preference model two main strategies which could be identified include educated a priori setting by the decision maker and extraction of simple preferences using fuzzy logic method [22, 23]. In the former, prior problem knowledge or extensive exploration by the DM is required. Interactive approaches could alternatively be employed although it has the drawback of taxing computational time requirement. In the latter, special care must be taken in constructing the method of extracting preferences so that the numerical levels truly correspond to the DM preferences.

The use of fuzzy logic in particular requires futher investigation. Fuzzy logic could be employed in an expert system assuming the role of the DM in an interactive approach. As a medium of representing information granulations recognized as the means of human cognition it may be used to extract preference information as in [22,23] or as a computational tool involved directly in the fitness function formulation The main challenge lies probably in the definition of membership function and operators employed in rules and inference such that the resulting fuzzy system represents the DM's reasoning.

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