Appendix H

Multi-Objective Performance Metrics

In multi-objective optimization processes (MOPs), there are two distinct and orthogonal goals [11] as follows: (1) discover solutions as close to the Pareto-optimal solutions as possible, and (2) find solutions as diverse as possible in the obtained non-dominated front. Therefore, in order to compare two or more MOEAs, at least two performance metrics (one evaluating the progress towards the desired Pareto-optimal front i.e. P^* and the other evaluating the spread of Pareto-front obtained i.e. Q) need to be used and exact definitions of these performance metrics are important.

For this purpose, various performance metrics are reported for MOEAs in the reference [11]. In this work, four different measures such as generational distance (GD), spacing (S), spread (Δ) and hypervolume (HV) are used for numerical comparison of the non-dominated fronts produced by various MOEAs.

The metric GD is used to evaluate the progress towards the Pareto-optimal front and the metrics S and Δ are used to evaluate the spread of the obtained non-dominated solutions. The metric HV provides the combined qualitative information about closeness and diversity in obtained Pareto-fronts.

G.1. Generational distance (GD)

Instead of finding whether a solution of Q belongs to the set of P^* or not, the generational distance metric (GD) recommended by Veldhuizen [170] evaluates an average distance of the solutions of Q from P^*, as follows:

$$GD = \frac{\left(\sum_{i=1}^{|Q|} d_i^p\right)^{1/p}}{|Q|} \tag{H.1}$$

For $p = 2$, the parameter d_i is the Euclidean distance (in the objective space) between the solution $i \in Q$ and the nearest member of P^*:

$$d_i = \min_{k \in |P^*|} \sqrt{\sum_{m=1}^{M} \left(f_m^{(i)} - f_m^{*(i)} \right)^2} \tag{H.2}$$

where $f_m^{*(i)}$ is the m^{th} objective function of the k^{th} member of P^*. *Intuitively, an algorithm having a small value of GD is better.*

G.2. Spacing (S)

The spacing metric (S) suggested by Schott [169] is calculated with a relative distance measure between consecutive solutions in the obtained non-dominated set, as follows:

$$S = \sqrt{\frac{1}{|Q|} \sum_{i=1}^{|Q|} \left(d_i - \bar{d} \right)^2} \tag{H.3}$$

where $d_i = \min_{k \in Q \wedge k \neq i} \left\{ \sum_{m=1}^{M} \left| f_m^i - f_m^k \right| \right\}$ and $\bar{d}$ is the mean value of the above distance measure $\bar{d} = \sum_{i=1}^{|Q|} d_i / |Q|$. The distance measure is the minimum value of the sum of the absolute difference in objective function values between the i^{th} solution and any other solution in the obtained non-dominated set. Notice that this distance measure is different from the minimum Euclidean distance between two solutions.

The above metric measures the standard deviations of different d_i values. When the solutions are nearly spaced, the corresponding distance measure will be small. *Thus, an algorithm finding a set of non-dominated solutions having smaller spacing (S) is better.*

G.3. Spread (Δ)

In MOPs, the main interest is to achieve a set of solutions that spans the entire Pareto-optimal region. The spread metric (Δ) suggested by Deb [165] measures the extent of spread achieved among the obtained solutions. Then, the following metric is to calculate the non-uniformity in the distribution:

$$\Delta = \frac{\sum_{m=1}^{M} d_m^e + \sum_{i=1}^{|Q|} \left| d_i - \bar{d} \right|}{\sum_{m=1}^{M} d_m^e + |Q| \bar{d}} \tag{H.4}$$

where d_i are Euclidean distances between neighbouring solutions having the mean value $\bar{d}$. The parameter d_m^e is the distance between the extreme solutions of P^* and Q corresponding to m^{th} objective function. *An algorithm finding a smaller value of* Δ *is able to find a better diverse set of non-dominated solutions.*

G.4. Hypervolume (HV)

This metric provides a qualitative measure of convergence as well as diversity in a combined sense. Nevertheless, it can be used along with one of the above two metrics to get a better overall picture of algorithm performance.

It calculates the volume (in the objective space) covered by members of Q for problems where all objectives are to be minimized [170]-[171]. Mathematically, for each solution $i \in Q$, a hypercube v_i is constructed with reference point W and the solution i as the diagonal corners of the hypercube. The reference point can simply be the found by constructing a vector of worst objective function values. Thereafter, a union of all hypervolume (HV) is calculated as follows:

$$HV = volume\left(\bigcup_{i=1}^{|Q|} v_i\right) \tag{H.5}$$

Obviously, *an algorithm with large value of HV metric is desirable.* This metric is free from arbitrary scaling of objectives. For example, if the first objective function takes values an order of magnitude more than of the second objective, a unit improvement in f_1 would reduce HV much more than that a unit improvement in f_2. Thus, this metric will favour a set Q which has better converged solution set for the least-scaled objective function. To eliminate this difficulty, the above metric can be evaluated by using normalized objective function values.