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classmate
Date
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Assignment - 2

1. Given $H(z) = \dfrac{3\sqrt{2} - 6z^{-1} + 3\sqrt{2}z^{-2}}{4\sqrt{2} - 6z^{-1} + 2\sqrt{2}z^{-2}}$

$$\Rightarrow H(z) = \frac{3\sqrt{2}\left(1 - \sqrt{2}z^{-1} + 1z^{-2}\right)}{4\sqrt{2}\left(1 - \frac{3}{2\sqrt{2}}z^{-1} + \frac{1}{2}z^{-2}\right)}$$

$$\Rightarrow H(z) = \frac{\left(1+\frac{1}{2}\right)}{2} \quad \frac{1 - 2\cdot\frac{1}{\sqrt{2}}z^{-1} + z^{-2}}{1 - \frac{1}{\sqrt{2}}\left(1+\frac{1}{2}\right)z^{-1} + \frac{1}{2}z^{-2}}$$

b) $\therefore$ it is a <u>NOTCH FILTER</u> with $\alpha = \frac{1}{2}$, $\beta = \frac{1}{\sqrt{2}}$

c) 3 dB bandwidth of a notch filter $= \cos^{-1}\left(\dfrac{2\alpha}{1+\alpha^2}\right)$

$$\Rightarrow B_{3dB} = \cos^{-1}\left(\frac{4}{5}\right) = 0.6435 \text{ or } 36.8^\circ$$

Centre frequency $= \cos^{-1}(\beta) = \dfrac{\pi}{4}$ or 45°

2. Given $y[n] = 3y[n-1] + x[n]$

a) Taking Z-transform, $Y(z) = 3z^{-1}Y(z) + X(z)$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1}{1 - 3z^{-1}}$$

Pole of $H(z)$: $D^r = 0 \Rightarrow 1 - 3z^{-1} = 0 \Rightarrow z = 3$

Now, $\because$ $z = 3$ lies outside unit circle ($|z| = 1$),
the system is <u>unstable</u>.

2b) $y[n] = 3y[n-1] + x[n]$

Let $x[n] = u[n] \Rightarrow y[n] = 3y[n-1] + u[n]$

Assuming $y[n]$ to be causal,

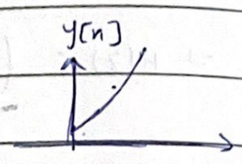
$$y[0] = 3\cancel{y[-1]}^{\to 0} + u[0] = 1$$

$$y[1] = 3y[1] + u[1] = 4$$

$$y[2] = 3 \times 4 + 1 = 13$$

$$y[3] = 3 \times 13 + 1 = 40$$

$$\vdots$$



It could be observed that as $n \to \infty$, $y[n] \to \infty$.

$\therefore$ $y[n]$ is unbounded for $x[n] = u[n]$ (bounded i/p)

3a) Poles : $0.5, -0.75$

$$\therefore D^r = (z-0.5)(z+0.75) = z^2(1 + 0.25z^{-1} - 0.375z^{-2})$$

$\therefore$ the N^r : $z^2(-0.375 + 0.25z + z^{-2})$

Transfer func^n of all-pass filter, $H(z) = \dfrac{-0.375 + 0.25z^{-1} + z^{-2}}{1 + 0.25z^{-1} - 0.375z^{-2}}$

4. $h[n] = \left(\frac{1}{4}\right)^n \cos\left(\frac{\pi}{4}n\right) u[n]$

a) Taking z-transform,

$$H(z) = \frac{1 - (1/4)z^{-1}\cos(\pi/4)}{1 - (2/4)z^{-1}\cos(\pi/4) + (1/4)^2 z^{-2}}$$

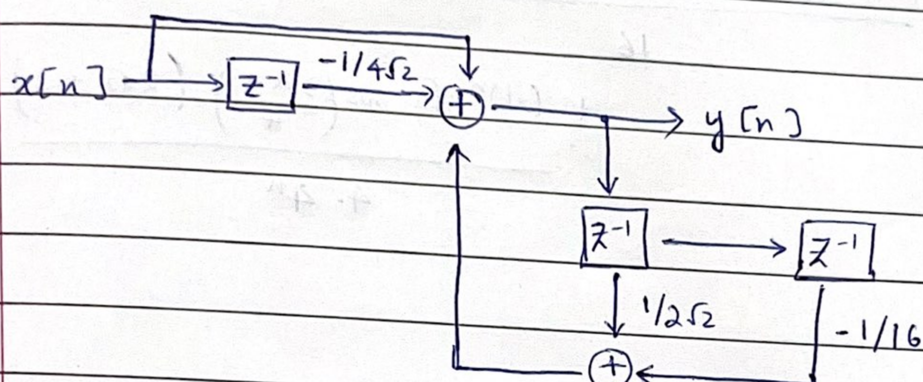
$$\Rightarrow H(z) = \frac{1 - \frac{1}{4\sqrt{2}}z^{-1}}{1 - \frac{1}{2\sqrt{2}}z^{-1} + \frac{1}{16}z^{-2}} = \frac{16(4\sqrt{2} - z^{-1})}{4\sqrt{2}(16 - 4\sqrt{2}z^{-1} + z^{-2})}$$

$$\Rightarrow H(z) = \frac{2\sqrt{2}\,(4\sqrt{2} - z^{-1})}{(16 - 4\sqrt{2}z^{-1} + z^{-2})} = \frac{16 - 2\sqrt{2}\,z^{-1}}{16 - 4\sqrt{2}\,z^{-1} + z^{-2}} = \frac{Y(z)}{X(z)}$$

$$\Rightarrow 16Y(z) - 4\sqrt{2}\,z^{-1}Y(z) + z^{-2}Y(z) = 16X(z) - 2\sqrt{2}\,z^{-1}X(z)$$

$$\Rightarrow 16y[n] - 4\sqrt{2}\,y[n-1] + y[n-2] = 16x[n] - 2\sqrt{2}\,x[n-1].$$

$$\Rightarrow y[n] = x[n] - \frac{1}{4\sqrt{2}}x[n-1] + \frac{1}{2\sqrt{2}}y[n-1] - \frac{1}{16}y[n-2]$$



4c) Given input $x[n] = \left(\frac{1}{4}\right)^n u[n]$.

$$\therefore X(z) = \frac{1}{1 - \frac{1}{4}z^{-1}} = \frac{4}{4 - z^{-1}}$$

Now,

$$Y(z) = X(z).H(z) = \left(\frac{4}{4 - z^{-1}}\right)\cdot\left(\frac{16 - 2\sqrt{2}\,z^{-1}}{16 - 4\sqrt{2}z^{-1} + z^{-2}}\right)$$

$$\Rightarrow Y(z) = \frac{4(2\sqrt{2}\,z^{-1} - 16)}{(-4 + z^{-1})\left(\frac{2}{2}z^{-2} - 4\sqrt{2}\,z^{-1} + 16\right)}$$

$$\Rightarrow Y(z) = \frac{4(2\sqrt{2}z^{-1} - 16)}{(z^{-1} - 4)(z^{-2} - 4\sqrt{2}z^{-1} + 16)}$$

Using MATLAB, inverse z-transform:

$$y[n] = \frac{\left(\frac{1}{4}\right)^n}{2} + \frac{(-1)^n\, 16^{(1-n)} \left(\sqrt{2}(-2-2i)\right)^{n-1} (\sqrt{2}+2)\, i}{16}$$

$$- \frac{(-1)^n\, 16^{(1-n)} \left(\sqrt{2}(-2+2i)\right)^{n-1} (\sqrt{2}+2)\, i}{16}$$

$$+ \frac{(-1)^n \sqrt{2} \cos\left(\frac{3\pi n}{4}\right) \left(2\sqrt{2}+2\right)}{4 \cdot 4^n}$$