The Polycal Package

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Abstract

The polycal package adds calculus macros to polynom package. It can differentiate or integrate polynomials.

1 Introduction

The polynom package manipulates polynomials. It supports multiplication, division, getting GCD of polynomials with Euclidean algorithm, factorization with coefficients in rational numbers. But it lacks in calculus operation, and substitution commands. Polycal package adds these commands to polynom package.

For one variable polynomial P(x), substitution P(a) can be obtained as a remainder of division with x - a. Also dviding with $(x - a)^2$, one can obtain P'(a)(x-a)+P(a) as a remainder, and can get P'(a). But polynom package can't deal with multivariate polynomial enough, so sometimes division fails. One can't specify main variable. So I implemented direct substitution commands.

2 Acknowledgments

I wish to thank Carsten Heinz, and Hendri Adriaens for creation and maintainance of polynom package.

3 Usage

See the manual of polynom package for a basic usage of original commands. This manual explains newly added commands.

3.1 Definition of polynomials

First, specify variables you'll use.

\polyset{vars=xab}

Define some polynomials using \polydefine.

 $\verb|\polydefine| & \langle macro \rangle \{ \langle polynomial \rangle \} \\$

 $\displaystyle \frac{x^2}{polydefine} x^2 \ polydefine \ (a+b)x-ab}$

Polynomials are set to macros \fx and \gx. You can print them using \polyprint:

 $[f(x)=\sqrt{x},\quad g(x)=\sqrt{x}.]$

$$f(x) = x^2, \qquad g(x) = bx + ax - ab.$$

Never print \fx or \gx directly.

3.2 Differentiation

You can differentiate polynomials using \polydiff.

 $\verb|\polydiff| & \langle macro\rangle \{ \langle variable \rangle \} \{ \langle polynomial \rangle \} \\$

 $\displaystyle \begin{array}{l} \begin{array}{l} \\ \\ \end{array} \end{array}$

 $\[\frac{df(x)}{dx}=\sqrt{Dfx. } \]$

$$\frac{df(x)}{dx} = 2x.$$

If you define \x as $\polydefine \x{x}$, you can write this operation as follows.

\[\polyprint\aa, \polyprint\bb, \polyprint\cc, \polyprint\dd.\]

$$2x, 2x, 2x, 2x$$
.

You can differentiate with another variable:

\polydiff\Dag{a}\gx \polydiff\Dbg{b}\gx
\[\frac{\partial g(x,a,b)}{\partial a} =\polyprint\Dag,\quad
\frac{\partial g(x,a,b)}{\partial b} =\polyprint\Dbg. \]

$$\frac{\partial g(x,a,b)}{\partial a} = x - b, \quad \frac{\partial g(x,a,b)}{\partial b} = x - a.$$

But in this manual, g(x) will be treated as one variable function.

 $\displaystyle \int polydiff \gx{x} \gx \[\frac{dg(x)}{dx}= polyprint \gx. \]$

$$\frac{dg(x)}{dx} = b + a.$$

3.3 Integration

Now we will calculate area between y = f(x) and y = g(x).

$$h(x) = q(x) - f(x) = -x^2 + bx + ax - ab.$$

We will integrate this function.

 $\verb|\polyint| \langle macro \rangle \{ \langle variable \rangle \} \{ \langle polynomial \rangle \}$

This macro calculates indefinite integral of $\langle polynomial \rangle$ from $\langle variable \rangle = 0$.

$$H(x) = \int_0^x h(t)dt = -\frac{1}{3}x^3 + \frac{1}{2}bx^2 + \frac{1}{2}ax^2 - abx.$$

3.4 Substitution

And definite integral can be obtained by substitution.

 $\verb|\polysubst| \langle macro\rangle \{\langle variable\rangle\} \{\langle polynomial-1\rangle\} \{\langle polynomial-2\rangle\}|$

This macro substitute $\langle polynomial-1 \rangle$ for $\langle variable \rangle$ in $\langle polynomial-2 \rangle$.

$$S = \int_{a}^{b} h(x)dx = H(b) - H(a) = \frac{1}{6}b^{3} - \frac{1}{6}a^{3} + \frac{1}{2}a^{2}b - \frac{1}{2}ab^{2}.$$

Unfortunately, factorization of multivariate polynomial is not currently supported, so this can not be arranged to $S=\frac{(b-a)^3}{6}$ automatically.

Let's try another substitution. If you substitute $b = a^2 - 2$ in S, then

$$b = a^2 - 2 \implies S = \frac{1}{6}a^6 - \frac{1}{2}a^5 - \frac{1}{2}a^4 + \frac{11}{6}a^3 + a^2 - 2a - \frac{4}{3} = \frac{1}{6}(a+1)^3(a-2)^3$$

 $\label{eq:condition} $$\operatorname{polysubstnum} \ \langle macro \rangle \{\langle variable \rangle\} \{\langle rational\ number \rangle\} \{\langle polynomial \rangle\} $$$

To substitute a number, there's another command. Compared with \polysubst, it is based on different algorithm, but not so fast. Experimental.

 $\verb|\polysubstsqrt| \langle macro\rangle \{\langle variable\rangle\} \{\langle rational\ number\rangle\} \{\langle 1-variable\ polynomial\rangle\} \}|$

This macro substitute $\sqrt{\langle rational\ number\rangle}$ for $\langle variable\rangle$. Note that polynomial should be one variable. Let substitute $a=\sqrt{5}$ in $S=\frac{1}{6}\left(a+1\right)^3\left(a-2\right)^3$.

\polysubstsqrt\SSS{a}{5}\SS \[S=\polyprint\SSS. \]

$$S = \left(12 + \frac{-16}{3}\sqrt{5}\right).$$

To eliminate parentheses, set \polyset{delims={}}}.

\[\polyset{delims={}}}S=\polyprint\SSS. \]

$$S = 12 + \frac{-16}{3}\sqrt{5}.$$

4 Implementation

Let's start with identification.

- $1 \langle *package \rangle$
- 2 \NeedsTeXFormat{LaTeX2e}
- 3 \ProvidesPackage{polycal}[2014/01/11 v0.01b (TS)]
- 4 \RequirePackage{polynom}

4.1 The user commands

\polydefine $\langle macro \rangle \{\langle polynomial \rangle \}$

Transform $\langle polynomial \rangle$ to internal representation, and set it to $\langle macro \rangle$.

 $\label{lem:command*polydefine} \begin{tabular}{l} $$ \end{tabular} $$ \end{tabular} $$ \end{tabular} $$ in excommand* \polydefine [2] {\pld@GetPoly#1{}{#2}} $$$

 $\polydiff \ \langle macro \rangle \{\langle variable \rangle\} \{\langle polynomial \rangle\}$

Differentiate $\langle polynomial \rangle$ with respect to $\langle variable \rangle$, and set it to $\langle macro \rangle$.

- $\label{eq:command*polydiff[1]} 6 \texttt{\newcommand*\polydiff[1]} \$
- 7 \pld@GetPoly{\pld@polya\pld@polyb}%
- 8 {\pld@DiffPoly#1\pld@polya\pld@polyb
- 9 \ignorespaces}}

 $\verb|\polyint| \langle macro\rangle \{\langle variable\rangle\} \{\langle polynomial\rangle\}|$

Integrate $\langle polynomial \rangle$ from 0, with respect to $\langle variable \rangle$.

- 10 \newcommand*\polyint[1]{%
- 11 \pld@GetPoly{\pld@polya\pld@polyb}%
- 12 {\pld@IntPoly#1\pld@polya\pld@polyb
- 13 \ignorespaces}}

```
\label{eq:condition} $$\operatorname{polysubstnum} \ \langle macro \rangle \{\langle variable \rangle\} \{\langle rational\ number \rangle\} \{\langle polynomial \rangle\}$$
                       Substitute \langle rational\ number \rangle for \langle variable \rangle in \langle polynomial \rangle.
                       14 \newcommand*\polysubstnum[1]{%
                             \pld@GetPoly{\pld@polya\pld@polyb\pld@polyc}%
                             {\pld@SubstNumPoly#1\pld@polya\pld@polyb\pld@polyc
                                \ignorespaces}}
                       \langle macro \rangle \{\langle variable \rangle\} \{\langle polynomial \rangle\} \{\langle non\ negative\ integer \rangle\}
      \polypower
                       Set \langle non \ negative \ integer \rangleth power of \langle polynomial \rangle to \langle macro \rangle.
                       18 \newcommand*\polypower[1]{%
                             \pld@GetPoly{\pld@polya}%
                             {\pld@PowerPoly#1\pld@polya}}
                       \langle macro \rangle \{\langle variable \rangle\} \{\langle polynomial-1 \rangle\} \{\langle polynomial-2 \rangle\}
      \polysubst
                       Substitute \langle polynomial-1 \rangle for \langle variable \rangle in \langle polynomial-2 \rangle.
                       21 \newcommand*\polysubst[1]{%
                             \pld@GetPoly{\pld@polya\pld@polyb\pld@polyc}%
                             {\pld@SubstPolyPoly#1\pld@polya\pld@polyb\pld@polyc
                                 \ignorespaces}}
                       \langle macro \rangle \{ \langle variable \rangle \} \{ \langle rational\ number \rangle \} \{ \langle 1-variable\ polynomial \rangle \}
 \polysubstsqrt
                       Substitute non negative square root of \langle rational \ number \rangle for \langle variable \rangle in
                       \langle 1-variable polynomial\rangle.
                       25 \newcommand*\polysubstsqrt[1]{%
                             \pld@GetPoly{\pld@polya\pld@polyb\pld@polyc}%
                       27
                             {\pld@SubstSqrtPoly#1\pld@polya\pld@polyb\pld@polyc
                                 \ignorespaces}}
                       4.2
                                 Internal routines
                       The following three macros gets parameter in internal representation of factors.
    \@fifthofsix For example, \polydefine\X{X} is equivalent to \def\X{\pld@R{1}{1}\pld@V{X}{1}},
                       and \Ofifthofsix\X gets variable letter X.
\@secondofthree
 \@thirdofthree
                       29 \providecommand\@fifthofsix[6]{#5}
                       30 \providecommand\@secondofthree[3]{#2}
                       31 \providecommand\@thirdofthree[3]{#3}
                       4.2.1 Differentiation
                       \langle macro\ a \rangle \langle macro\ b \rangle \langle macro\ c \rangle
  \pld@DiffPoly
                       \langle macro\ a \rangle gets the derivative \frac{d\langle macro\ c \rangle}{d\langle macro\ b \rangle}.
```

```
32 \def\pld@DiffPoly#1#2#3{%
                                                            \let\polycalc@tempb\@empty%
                                        33
                                                            \ifx\@empty#3\relax\else
                                        34
                                                                      \expandafter\pld@DiffPoly@\expandafter#2#3+\relax+%
                                        35
                                        36
                                                            \fi
                                        37
                                                            \let#1\polycalc@tempb \pld@Simplify{#1}%
                                        38
 \pld@DiffPolv@
                                        This macro cuts away a monomial in the third argument \langle macro \ c \rangle of \pld@DiffPoly,
                                        send it to \pld@DiffMonom.
                                        39 \def\pld@DiffPoly@#1#2+{%
                                                  \let\polycalc@tempa\@empty
                                        40
                                                        \ifx\relax#2 \relax
                                        41
                                        42
                                                                 \pld@DiffMonom\polycalc@tempa#1{#2}%
                                        43
                                                                 \pld@ExtendPoly\polycalc@tempb{\polycalc@tempa}%
                                         44
                                                                 \expandafter\pld@DiffPoly@\expandafter#1%
                                        45
                                        46 \fi}
                                        \langle macro \ a \rangle \langle macro \ b \rangle \langle macro \ c \rangle
 \pld@DiffMonom
                                        This macro decrease the degree of the monomial \langle macro \ c \rangle by 1 with respect to
                                        variable \langle macro b \rangle and send it to \pld@DiffMonom@.
                                        47 \def\pld@DiffMonom#1#2#3{%
                                                        \let#1\@empty
                                        48
                                                        \edef\pldc@var@letter{\expandafter\@fifthofsix#2}%
                                        49
                                        50
                                                        \def\@tempb{#3}\edef\@tempc{\noexpand\pld@V{\pldc@var@letter}{-1}}%
                                                        \pld@MultiplyPoly\@tempa\@tempb\@tempc%
                                        51
                                                        \def\pldc@CallMono@##1{%
                                        52
                                                        \expandafter\pld@DiffMonom@\expandafter #1\expandafter##1%
                                        53
                                        54
                                                        \@tempa\relax\@empty\@empty\relax\relax}%
                                                        \expandafter\pldc@CallMono@\pldc@var@letter
                                        56 }
                                        \langle macro\ a \rangle \langle variable\ letter \rangle \langle macro\ b \rangle \{\langle content\ c \rangle\} \{\langle content\ d \rangle\}
\pld@DiffMonom@
                                        This macro cuts away a factor \langle macro b \rangle \{\langle content c \rangle\} \{\langle content d \rangle\}, of the mono-
                                        mial argument \langle macro \ c \rangle of \pld@DiffMonom. If the factor is a power of \langle variable
                                        letter\rangle, do x^{n-1} \rightarrow nx^{n-1}.
                                        57 \def\pld@DiffMonom@#1#2#3#4#5{%
                                                        \ifx\relax#3
                                        58
                                                            \relax \expandafter\@gobbletwo
                                        59
                                        60
                                                        \else
                                                            ifx #3\pld@V if#4#2
                                        61
                                                                           \@tempcnta #5 \advance\@tempcnta 1
                                        62
                                        63
                                                                           \expandafter\pld@Extend\expandafter#1{\expandafter\pld@R\expandafter{\the\@tempcnta
                                        64
                                                            \fi\fi
                                                            \label{local_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_extend_pld_ext
                                        65
                                                        \expandafter\pld@DiffMonom@\expandafter#1\expandafter#2%
                                        66
                                                        \fi}
                                        67
```

4.2.2 Integration

```
\pld@IntPoly \langle macro\ a \rangle \langle macro\ b \rangle \langle macro\ c \rangle
                     \langle macro~a\rangle~{\rm gets~the~integration}~\int_0^{\langle macro~b\rangle} \langle macro~c\rangle d\langle macro~b\rangle.
                     68 \def\pld@IntPoly#1#2#3{%
                                \let\polycalc@tempb\@empty
                     69
                                \ifx\@empty#3\relax\else
                     70
                                     \expandafter\pld@IntPoly@\expandafter#2#3+\relax+%
                     71
                     72
                                \fi
                     73
                             \let#1\polycalc@tempb \pld@Simplify{#1}%
 \pld@IntPoly@
                    This macro cuts away a monomial in the third argument \langle macro c \rangle of \pld@IntPoly,
                     send it to \pld@IntMonom.
                     75 \def\pld@IntPoly@#1#2+{%
                          \let\polycalc@tempa\@empty
                             \ifx\relax#2 \relax
                     77
                     78
                                   \pld@IntMonom\polycalc@tempa#1{#2}%
                     79
                                   \pld@ExtendPoly\polycalc@tempb{\polycalc@tempa}%
                     80
                                   \expandafter\pld@IntPoly@\expandafter#1%
                     81
                     82 \fi}
                     \langle macro \ a \rangle \langle macro \ b \rangle \langle macro \ c \rangle
 \pld@IntMonom
                     This macro increase the degree of the monomial \langle macro \ c \rangle by 1 with respect to
                     variable \langle macro b \rangle and send it to \pld@IntMonom@.
                     83 \def\pld@IntMonom#1#2#3{%
                     84
                             \let#1\@empty
                     85
                             \def\@tempb{#3}\def\@tempc{#2}%
                             \pld@MultiplyPoly\@tempa\@tempb\@tempc
                     86
                             \edef\pldc@var@letter{\expandafter\@fifthofsix#2}%
                     87
                             \def\pldc@CallMono@##1{%
                     88
                             \expandafter\pld@IntMonom@\expandafter #1\expandafter##1%
                     89
                             \@tempa\relax\@empty\@empty\relax\relax}%
                     90
                             \expandafter\pldc@CallMono@\pldc@var@letter
                     91
                     92 }
                     \langle macro\ a \rangle \langle variable\ letter \rangle \langle macro\ b \rangle \{\langle content\ c \rangle\} \{\langle content\ d \rangle\}
\pld@IntMonom@
                     This macro cuts away a factor \langle macro\ b \rangle \{\langle content\ c \rangle\} \{\langle content\ d \rangle\}, of the mono-
                     mial argument \langle macro\ c \rangle of \pld@IntMonom. If the factor is a power of \langle variable
                     |letter\rangle, do x^{n+1} \rightarrow \frac{x^{n+1}}{n+1}.
                     93 \def\pld@IntMonom@#1#2#3#4#5{%
                             \ifx\relax#3 \relax
```

```
\relax \expandafter\@gobbletwo
                                                 95
                                                 96
                                                                  \else
                                                                      \ifx #3\pld@V \if#4#2
                                                 97
                                                                           98
                                                                      \fi\fi
                                                 99
                                                100
                                                                      \plue{20} \plu
                                                101
                                                                  \expandafter\pld@IntMonom@\expandafter#1\expandafter#2%
                                               102
                                                                  Substitution of number
                                                 4.2.3
                                                 \langle macro\ a \rangle \langle macro\ b \rangle \langle macro\ c \rangle \langle macro\ d \rangle
  \pld@SubstNumPoly
                                                 Substitute rational number \langle macro\ c \rangle for variable \langle macro\ b \rangle in the polynomial
                                                 \langle macro \ d \rangle, and stores it to \langle macro \ a \rangle.
                                               103 \def\pld@SubstNumPoly#1#2#3#4{%
                                                                      \let\polycalc@tempb\@empty
                                               104
                                               105
                                                                      \ifx\@empty#4\relax\else
                                                                                \expandafter\pld@SubstNumPoly@\expandafter#2\expandafter#3#4+\relax+%
                                                106
                                                                      \fi
                                               107
                                                                  \let#1\polycalc@tempb \pld@Simplify{#1}%
                                               108
                                               109
                                                 This macro cuts away a monomial in the fourth argument \langle macro \ d \rangle of
\pld@SubstNumPoly@
                                                  \pld@SubstNumPoly, send it to \pld@SubstNumMonom.
                                               110 \def\pld@SubstNumPoly@#1#2#3+{%
                                                            \let\polycalc@tempa\@empty
                                               111
                                               112
                                                                 \ifx\relax#3 \relax
                                               113
                                                                  \else
                                                                           \pld@SubstNumMonom\polycalc@tempa#1#2{#3}%
                                               114
                                               115
                                                                           \pld@ExtendPoly\polycalc@tempb{\polycalc@tempa}%
                                               116
                                                                           \expandafter\pld@SubstNumPoly@\expandafter#1\expandafter#2%
                                               117 \fi}
                                                 \langle macro \ a \rangle \langle macro \ b \rangle \langle macro \ c \rangle \langle macro \ d \rangle
\pld@SubstNumMonom
                                                 This macro only replace \langle macro \ b \rangle with its variable letter, and sends four argu-
                                                 ments to \pld@SubstNumMonom@.
                                               118 \def\pld@SubstNumMonom#1#2#3#4{%
                                                                  \let#1\@empty
                                               119
                                                                  \edef\pldc@var@letter{\expandafter\@fifthofsix#2}%
                                               120
                                               121
                                                                  \def\pldc@CallMono@##1{%
                                               122
                                                                  \expandafter\pld@SubstNumMonom@\expandafter#1\expandafter##1\expandafter#3\expandafter%
                                               123
                                                                 #4\relax\@empty\@empty\relax\relax}%
                                               124
                                                                  \expandafter\pldc@CallMono@\pldc@var@letter
                                               125 }
```

 $\langle macro\ a \rangle \langle variable\ letter \rangle \langle macro\ b \rangle \langle macro\ c \rangle \{ \langle contents\ d \rangle \} \{ \langle contents\ e \rangle \}$

\pld@SubstNumMonom@

This macro cuts away a factor $\langle macro\ c \rangle \{\langle content\ d \rangle\} \{\langle content\ e \rangle\}$, of the monomial argument $\langle macro\ d \rangle$ of \pld@SubstNumMonom. If the factor contains $\langle variable\ letter \rangle$ as variable, this macro substitute the rational number $\langle macro\ b \rangle$ for that variable.

```
126 \def\pld@SubstNumMonom@#1#2#3#4#5#6{%
                   \ifx\relax#4 \relax \expandafter\@gobbletwo
127
128
                    \else
                        \ifx #4\pld@V
129
                                   \if#5#2
130
                                           \label{lem:lempb} $$ \end{condense} $$ \end{co
131
                                           \pld@MultiplyPoly\@tempa\@tempb\@tempc
132
                                           133
                                           \pld@ExtendPoly\@tempb\@tempa \pld@Simplify\@tempb
134
                                           \def\@tempc{\pld@V{#5}{#6}}%
135
136
                                           \let\pld@stage\maxdimen
137
                                           \pld@DividePoly\@tempc\@tempb \pld@Simplify\pld@remainder
138
                                           \pld@Extend#1{\pld@remainder}%
                                   \else
139
                                           140
                                   \fi
141
142
                        \else
143
                                    144
145
                    \expandafter\pld@SubstNumMonom@\expandafter#1\expandafter#2\expandafter#3%
146
                    \fi}
                     Substitution of polynomial
  \langle macro\ a \rangle \langle macro\ b \rangle \langle macro\ c \rangle \langle macro\ d \rangle
  Substitute polynomial \langle macro\ c \rangle for variable \langle macro\ b \rangle in the polynomial \langle macro\ c \rangle
  d\rangle, and stores it to \langle macro\ a\rangle.
147 \def\pld@SubstPolyPoly#1#2#3#4{%
148
                        \let\polycalc@tempb\@empty
149
                        \ifx\@empty#4\relax\else
                                    \expandafter\pld@SubstPolyPoly@\expandafter#2\expandafter#3#4+\relax+%
150
151
                   \let#1\polycalc@tempb \pld@Simplify{#1}%
152
                   }
153
  This macro cuts away a monomial in the fourth argument \langle macro \ d \rangle of
  \pld@SubstPolyPoly, send it to \pld@SubstPolyMonom.
154 \def\pld@SubstPolyPoly@#1#2#3+{%
             \let\polycalc@tempa\@empty
155
                   \ifx\relax#3 \relax
156
157
```

\pld@SubstPolyPoly

\pld@SubstPolyPoly@

158

159

\pld@SubstPolyMonom\polycalc@tempa#1#2{#3}%

\pld@ExtendPoly\polycalc@tempb{\polycalc@tempa}%

```
160 \expandafter\pld@SubstPolyPoly@\expandafter#1\expandafter#2% 161 \fi}
```

\pld@SubstPolyMonom

```
\langle macro \ a \rangle \langle macro \ b \rangle \langle macro \ c \rangle \langle macro \ d \rangle
```

This macro only replace $\langle macro\ b \rangle$ with its variable letter, and sends four arguments to $<page-header> \rho d \ subst PolyMonom @$.

```
162 \def\pld@SubstPolyMonom#1#2#3#4{%
163
       \def#1{\left( Pld@R{1}{1}}\right) 
        \edef\@tempb{\expandafter\@fifthofsix#2}%
164
165
        \edef\pldc@var@letter{\expandafter\@fifthofsix#2}%
166
        \def\pldc@CallMono@##1{%
        \expandafter\pld@SubstPolyMonom@\expandafter#1\expandafter##1\expandafter#3\expandafter%
167
       #4\relax\@empty\@empty\relax\relax}%
168
169
      \expandafter\pldc@CallMono@\pldc@var@letter
170 }
```

\pld@SubstPolyMonom@

```
\langle macro\ a \rangle \langle variable\ letter \rangle \langle macro\ b \rangle \langle macro\ c \rangle \{\langle contents\ d \rangle\} \{\langle contents\ e \rangle\}
```

This macro cuts away a factor $\langle macro\ c \rangle \{\langle content\ d \rangle\} \{\langle content\ e \rangle\}$, of the monomial argument $\langle macro\ d \rangle$ of $\pld@SubstPolyMonom$. If the factor contains $\langle variable\ letter \rangle$ as variable, this macro substitute the polynomial $\langle macro\ b \rangle$ for that variable.

```
171 \def\pld@SubstPolyMonom@#1#2#3#4#5#6{%
```

```
172
       \ifx\relax#4 \relax \expandafter\@gobbletwo
173
       \else
174
         \ifx #4\pld@V
175
            \if#5#2
176
                 \edef\@tempa{\number#6}%
177
                 \pld@PowerPoly\@tempb#3\@tempa
                 \expandafter\def\expandafter\0tempa\expandafter{#1}%
178
179
                 \pld@MultiplyPoly#1\@tempa\@tempb
180
             \else
181
                \expandafter\def\expandafter\@tempa\expandafter{#1}%
                \edgn(0) = \frac{4{\#5}{\#6}}%
182
183
                \pld@MultiplyPoly#1\@tempa\@tempb
             \fi
184
         \else
185
186
              \expandafter\def\expandafter\@tempa\expandafter{#1}%
187
              \edf\edge {\noexpand#4{#5}{#6}}
              \pld@MultiplyPoly#1\@tempa\@tempb
188
189
         \fi
       \expandafter\pld@SubstPolyMonom@\expandafter#1\expandafter#2\expandafter#3%
190
191
       \fi}
```

\pld@PowerPoly

```
\langle macro\ a \rangle \langle macro\ b \rangle \{\langle number\ a \rangle \}
```

 $\langle macro\ a \rangle$ gets a $\langle number\ a \rangle$ th power of polynomial $\langle macro\ b \rangle$. $\langle number\ a \rangle$ should be a non negative integer.

```
192 \def\pld@PowerPoly#1#2#3{%
                                                    \label{local_condition} $$ \int_{\mathbb{R}^2} \left( \frac{1}{1} \right)^{2} d^{2} d^{2} \right) d^{2} d^{2}
193
                                                    \else
194
                                                                          \@tempcnta#3\advance\@tempcnta-1
195
196
                                                                          \edef\@tempa{\the\@tempcnta}%
 197
                                                                          \pld@PowerPoly#1#2\@tempa
 198
                                                                          \let\@tempb#1
199
                                                                          \pld@MultiplyPoly#1\@tempb#2%
200
                                                  \fi
201 }
```

4.2.5 Substitution of square root

\pld@SubstSqrtPoly

```
\langle macro \ a \rangle \langle macro \ b \rangle \langle macro \ c \rangle \langle macro \ d \rangle
```

Substitute non negative square root of rational number $\langle macro\ c \rangle$ for variable $\langle macro\ b \rangle$ in the **one variable** polynomial $\langle macro\ d \rangle$, and stores it to $\langle macro\ a \rangle$.

```
202 \def\pld@SubstSqrtPoly#1#2#3#4{%
     \edef\pldc@var@letter{\expandafter\@fifthofsix#2}%
203
     \edef\pldc@numer@sqrt{\expandafter\@secondofthree#3}%
204
     \edef\pldc@denom@sqrt{\expandafter\@thirdofthree#3}%
205
206
     \edef\pld@polyd{%
207
       \noexpand\pld@R{1}{1}\noexpand\pld@V{\pldc@var@letter }{2}%
208
       +\noexpand\pld@R{-\pldc@numer@sqrt }{\pldc@denom@sqrt }}%
209
     \let\pld@stage\maxdimen
210
     \pld@DividePoly#4\pld@polyd
     \let\pldc@main@formula\pld@remainder
211
     \pld@FDefSqrt{\pldc@numer@sqrt}{\pldc@denom@sqrt}\let\pldc@sqrt@inner\pld@temp
212
     \def\pldc@sqrt@outer{\pld@R {1}{1}\pld@S {\pldc@sqrt@inner }{1}}%
213
214
     \pld@SubstNumPoly#1#2\pldc@sqrt@outer\pldc@main@formula%
215 }
```

4.3 Miscellany

\pld@IfMonomL@ A bug fix of \pld@IfMonomL@ in polynom.sty.

```
216 %%%%% The following is a bug fix of \pld@IfMonomL@ in polynom.sty.
217 \end{figure} $$217 \end{fi
                                     \let\pld@next\@empty
218
                                                                   \ifx #6\relax \aftergroup\@secondoftwo
219
                                     \else \ifx #2\relax \aftergroup\@firstoftwo
220
221
                                                          222
                                                          \ifx\pld@va\pld@vb
223
                                                                              224
                                                                                                  \def\pld@next{\pld@IfMonomL@#4\@empty#8\@empty}%
225
                                                                              \else
226
                                                                                                  \ifnum#3<#7\relax \aftergroup\@firstoftwo
227
228
                                                                                                                                                                \else \aftergroup\@secondoftwo \fi
229
                                                                              \fi
```

```
230
           \else
               % \pld@IfVarL#2\relax\@empty#6\relax\@empty
231
               \pld@IfVarL#2#6%
232
                   233
                   {\aftergroup\@secondoftwo}%
234
           \fi
235
       \fi \fi
236
237
       \pld@next}
238 \langle /package \rangle
```

5 History

0.01 2014/01/09 (private test version)

 $- \ Implemented \verb|\pld@DiffPoly|, \verb|\pld@SubstNumPoly|, \verb|\pld@SubstPlyPoly|, \verb|\pld@SubstSqrtPoly|.$

 $0.01a \ 2014/01/11$

- Fixed a bug in \pld@IfMonomL@ of original polynom package.

0.01b 2014/01/12

- Wrote .dtx.
- Minor fix in \pld@SubstSqrtPoly.

TODO:

- User definable order of variables.
- Rational functions and its derivatives.
- Vector calculus.
- Differential geometric operations.