Offline Reinforcement Learning with Implicit Q-Learning

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What is Offline RL?

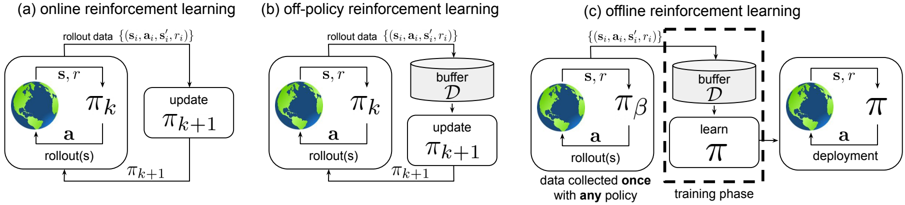
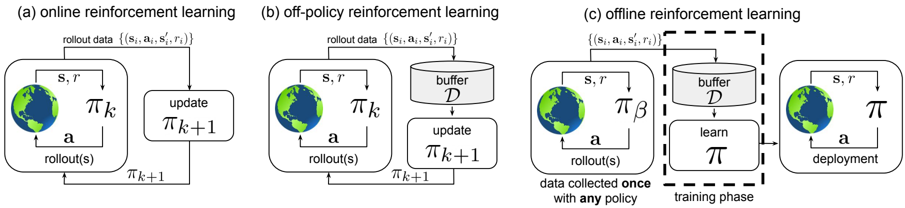


Figure: Different paradigms of Reinforcement Learning [Levine et al., 2020]



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  1. There is no feedback from the environment.
  2. The distribution on which we minimize our loss comes from the behaviour policy (π_β) however, the distribution over which we will run the policy will be a new policy (π_θ)

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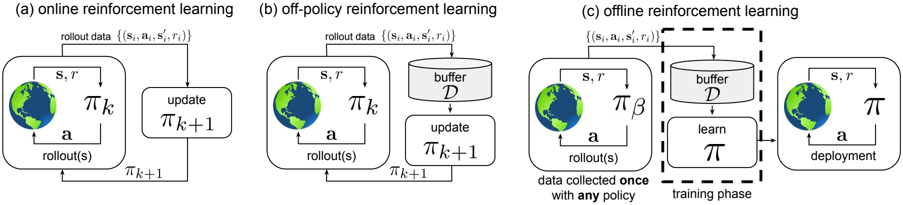

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  1. Distribution Constraint based
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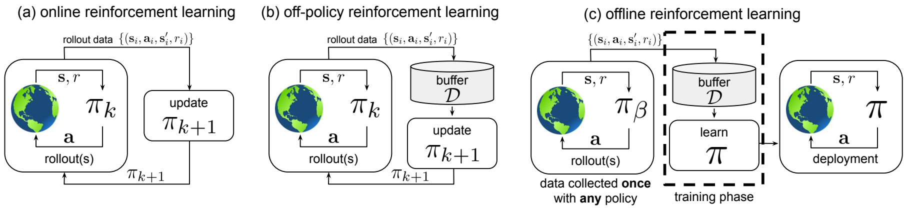

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Proposed Methodology

Offline reinforcement learning objective

$$L_{TD}(\theta) = \mathbb{E}_{(s,a,s')\sim\mathcal{D}}\left[\left(r(s,a) + \gamma \max_{a'} Q_{\hat{\theta}}\left(s', a'\right) - Q_{\theta}(s,a)\right)^2\right]$$
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  Why?: $\max_{a'} Q_{\hat{\theta}}(s',a')$ is not constrained to in-dataset actions, hence maximization over unseen actions would give arbitrary values.
- In other words, this maximization may lead to cases where for a given state, unseen actions have the maximum Q value.

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- SARSA-like objective function:

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- We approximate "$\max_{\substack{a'\in\mathcal{A} \\ \text{s.t. } \pi_\beta(a'|s')>0}} Q_{\hat{\theta}}(s',a')$" using Expectile Regression.

Expectile Regression

Definition

The $\tau \in (0,1)$ expectile of some random variable X is defined as a solution to the asymmetric least squares problem:

$$\arg\min_{m_\tau} \quad \mathbb{E}_{x \sim X}\left[L_2^\tau\left(x - m_\tau\right)\right],$$

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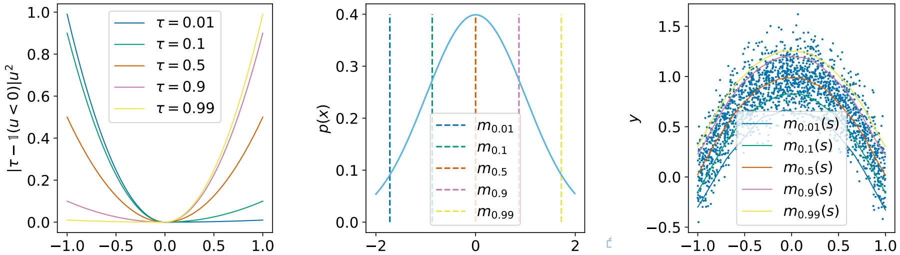

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- IDEA: Approximate $\max\limits_{\substack{a' \in \mathcal{A} \\ \text{s.t. } \pi_\beta\left(a'|s'\right)>0}} Q_{\dot{\theta}}\left(s', a'\right)$ by using Expectile Regression.

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Value function loss

$$L_V(\psi) = \mathbb{E}_{(s,a)\sim\mathcal{D}}\left[L_2^\tau(Q_\theta(s,a) - V_\psi(s))\right] \tag{3}$$

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- Hence, if the value of τ is kept high (≈ 1),

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Q-Value function loss

$$L(\theta) = \mathbb{E}_{(s,a,s')\sim\mathcal{D}}\left[\left(r(s,a) + \gamma V_\psi(s') - Q_\theta(s,a)\right)^2\right] \tag{4}$$

Policy Extraction & Algorithm

Algorithm

Algorithm Implicit Q-learning

```
Initialize parameters ψ, θ, θ̂, φ.
TD learning (IQL):
for each gradient step do
    ψ ← ψ − λ_V ∇_ψ L_V(ψ)
    θ ← θ − λ_Q ∇_θ L_Q(θ)
    θ̂ ← (1 − α)θ̂ + αθ
end for
Policy extraction (AWR):
for each gradient step do
    φ ← φ + λ_π ∇_φ L_π(φ)
end for
```

Policy Extraction & Algorithm

- By design, IQL does not give us an optimal policy. A policy extraction step is required to learn an optimal policy from the learnt Q function.

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2: TD learning (IQL):
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- Authors have used Advantage Weighted Regression which finds an optimal policy by maximizing the following objective.

$$L_\pi(\phi) = \mathbb{E}_{(s,a)\sim\mathcal{D}}\Big[\exp\left(\beta\left(Q_{\hat{\theta}}(s,a) - V_\psi(s)\right)\right) \times \log \pi_\phi(a \mid s)\Big]$$

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- This is similar to Cross-Entropy loss where $\beta \in [0, \infty)$ is a hyper-parameter.

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Theoretical Analysis

Notations

- $V_\tau(s) = \mathbb{E}^{\tau}_{a\sim\mu(\cdot|a)}\left[Q_\tau(s,a)\right]$
- $Q_\tau(s,a) = r(s,a) + \gamma\mathbb{E}_{s'\sim p(\cdot|s,a)}\left[V_\tau(s')\right]$
- m_τ : τ^{th} expectile
- $\mathbb{E}^{\tau}_{a\sim\mu(\cdot|a)}(\cdot)$: Expectile operator

Theoretical Analysis

Lemma 1

Let X be a real-valued random variable with a bounded support and supremum of the support is x^*. Then,

$$\lim_{\tau \to 1} m_\tau = x^* \quad (1)$$

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Proof

By the definition of m_τ, we have the following property:

$$m_\tau - \mu = \frac{(2\tau - 1)}{1 - \tau} \int_{m_\tau}^{+\infty} (x - m_\tau) dF(x)$$

where $F(x)$ is distribution function of X and $\mu = \mathbb{E}[X]$.

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Let $T(m) = \int_m^{+\infty} (x - m) dF(x)$ and $\alpha(\tau) = \frac{(2\tau - 1)}{(1 - \tau)}$. It can be shown that $T(m)$ is a convex function in m.

Proof (Contd.)

Now, $T'(m) = \frac{dT(m)}{dm} = \int_m^\infty \frac{d}{dm}(x-m)dF(x) = \int_m^\infty (-1)dF(x) = F(m) - 1$

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Now, $\alpha(\tau) > -1$ and $\frac{d\alpha(\tau)}{d\tau} = \frac{1}{(1-\tau)^2} > 0 \implies \alpha(\tau)$ is monotonically increasing in τ.

Now, using the fact that $\alpha(\tau)$ is monotonically increasing in τ and $T(m)$ is monotonically decreasing in m, one can show that $\alpha(\tau)T(m_\tau)$ is monotonically increasing in τ.

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$$\lim_{\tau \to 1} m_\tau = x^*$$

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Lemma 2

For all s, τ_1 and τ_2 such that $\tau_1 < \tau_2$ we get

$$V_{\tau_1}(s) \leq V_{\tau_2}(s) \tag{2}$$

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$$V_{\tau_1}(s) = \mathbb{E}^{\tau_1}_{a \sim \mu(\cdot|s)} \left[r(s, a) + \gamma \mathbb{E}_{s' \sim p(\cdot|s,a)} \left[V_{\tau_1}\left(s'\right) \right] \right]$$

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For all s, τ_1 and τ_2 such that $\tau_1 < \tau_2$ we get

$$V_{\tau_1}(s) \leq V_{\tau_2}(s) \tag{2}$$

Proof

$$\begin{aligned} V_{\tau_1}(s) &= \mathbb{E}^{\tau_1}_{a\sim\mu(\cdot|s)}\left[r(s,a) + \gamma\mathbb{E}_{s'\sim p(\cdot|s,a)}\left[V_{\tau_1}\left(s'\right)\right]\right] \\ &\leq \mathbb{E}^{\tau_2}_{a\sim\mu(\cdot|s)}\left[r(s,a) + \gamma\mathbb{E}_{s'\sim p(\cdot|s,a)}\left[V_{\tau_1}\left(s'\right)\right]\right] \quad \text{[Using Lemma 1]} \end{aligned}$$

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Theoretical Analysis

Corollary 2.1

For any τ and s we have

$$V_\tau(s) \leq \max_{\substack{a \in \mathcal{A} \\ s.t\ \pi_\beta(a|s)>0}} Q^*(s,a) \tag{3}$$

where $V_\tau(s)$ is as defined earlier and $Q^*(s,a)$ is an optimal state-action value function constrained to the dataset and defined as:

$$Q^*(s,a) = r(s,a) + \gamma \mathbb{E}_{s' \sim p(\cdot|s,a)} \left[\max_{\substack{a' \in \mathcal{A} \\ s.t.\ \pi_\beta(a'|s')>0}} Q^*(s',a') \right]$$

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Proof

From the definition of $V_\tau(s)$:

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$$\lim_{\tau \to 1} V_\tau(s) = \max_{\substack{a \in \mathcal{A} \\ s.t\ \pi_\beta(a|s) > 0}} Q^*(s, a) \tag{4}$$

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Consider a set $\mathcal{T} = \{\tau_i \mid \tau_i \in (0, 1) \forall i\}$. Then for a sequence $\tau_1 < \tau_2 < \tau_3 < \ldots < \tau_n$, from Lemma 2, we have:

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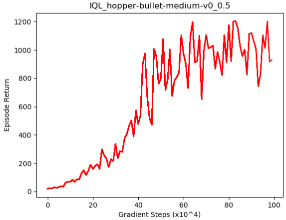
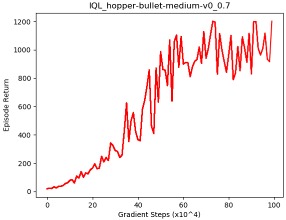
$$\lim_{\tau \to 1} V_\tau(s) = \max_{\substack{a \in \mathcal{A} \\ s.t.\ \pi_\beta(a|s) > 0}} Q^*(s,a)$$

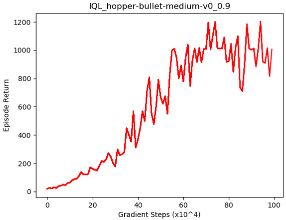
Experiments



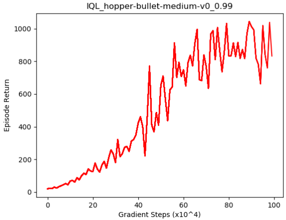
(a) $\tau = 0.5$



(b) $\tau = 0.7$



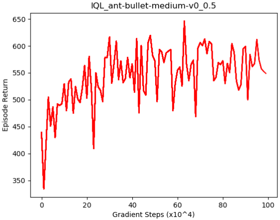
(c) $\tau = 0.9$



(d) $\tau = 0.99$

Figure: hopper-bullet-medium-v0

Experiments



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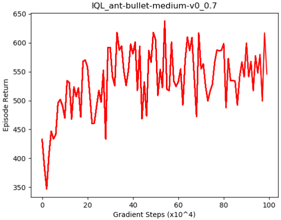

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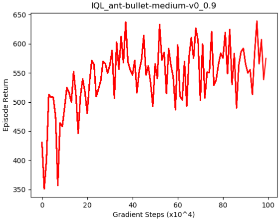

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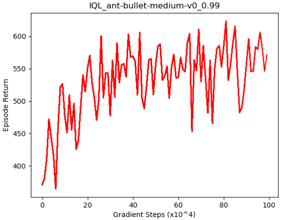

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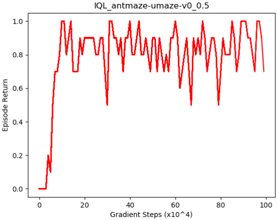
Figure: ant-bullet-medium-v0



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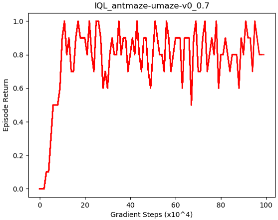

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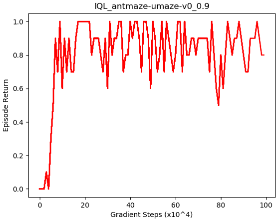

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Figure: antmaze-umaze-v0

Experiments (Online Fine-Tuning)

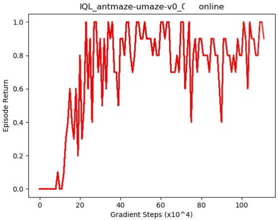

Figure: Online Tuning with $\tau = 0.7$

For fine-tuning experiments, we first run offline RL for 1M gradient steps. Then we continue training while collecting data actively in the environment and adding that data to the replay buffer, running 1 gradient update / environment step. All other training details are kept the same between the offline RL phase and the online RL phase

Drawback

- The theoretical analysis claims that $\tau \to 1$ closely approximates Q^*. However, during the experiments the authors use $\tau = 0.9$ or 0.7.

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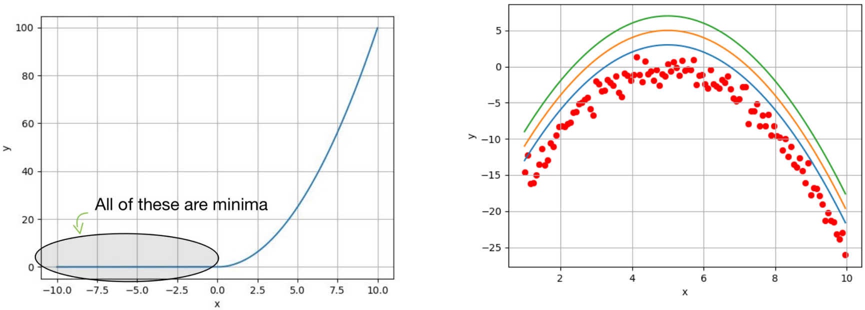

Figure: Drawbacks of Proposed Method

Potential Improvements

Idea 1

Fit $V_{\hat{\psi}}(s)$ using expectile regression with $\tau \approx 0$, so that it is an under-estimate of Q-values. Now, use $V_{\hat{\psi}}(s)$ as initialization for expectile regression with $\tau \approx 1$.

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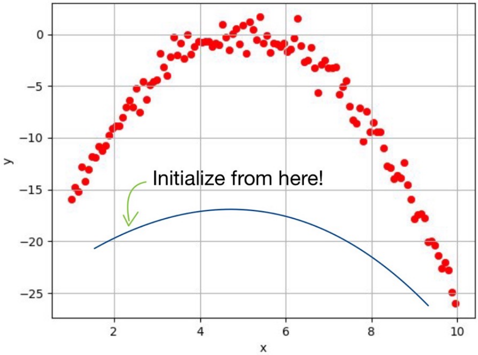

Potential Improvements

Idea 2

Penalize/Regularize $V_\psi(s)$ based on how far they are from $\max_a Q(s,a)$. To get this, choose some exemplar/candidate points for $\max_a Q(s,a)$ i.e, minimize

$$L'_V(\psi) = \mathbb{E}_{(s,a)\sim\mathcal{D}}\left[L_2^\tau(Q_{\hat{\theta}}(s,a) - V_\psi(s)) + \lambda_R(V_\psi(s) - \hat{Q}(s,a))\right]$$

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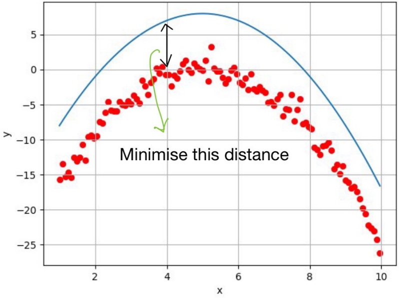

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Offline reinforcement learning: Tutorial, review, and perspectives on open problems.
arXiv preprint arXiv:2005.01643.

Link to our experiments: https://github.com/tayalmanan28/Offline˙learning˙IQL