ECON 0150 | Spring 2026 | Homework 3.1

Due: Friday, February 27 at 5 PM

Homework is designed to both test your knowledge and challenge you to apply familiar concepts in new applications. Answer clearly and completely. You are welcomed and encouraged to work in groups so long as your work is your own. Submit your figures and answers to Gradescope.

Q1. Working with a Normal Distribution

The wait times (in minutes) at a restaurant follow a normal distribution with mean $\mu = 12$ minutes and standard deviation $\sigma = 2.5$ minutes. Start each question with a rough sketch of your answer, then find the number using code. Two useful functions from Exercise 3.1:

```
from scipy import stats
# stats.norm.cdf()
# stats.norm.ppf()
```

a) What is the theoretical mean (μ) wait time *(warm up question)*?

b) What is the theoretical variance for the wait time *(warm up question)*?

c) What is the wait time which is longer than exactly 77 percent of wait times *(use python)*?

d) What is the probability that a wait time will be greater than 10 minutes *(use python)*?

e) What is the probability the wait time will be between 10 and 14 minutes *(use python)*?

f) What is the probability the wait time will be less than 7 or greater than 17 *(use python)*?

Q2. Sample vs. Population

We know the restaurant's population parameters: $\mu = 12$ and $\sigma = 2.5$. But in the real world, we don't know the population. We only have a sample. In this question, you'll draw samples from the known population and see how well sample statistics approximate population parameters.

a) Use `np.random.normal(12, 2.5, 10)` to draw a sample of $n = 10$ wait times. Compute the sample mean and sample standard deviation (`np.std(sample, ddof=1)`). How do they compare to μ and σ?

b) Repeat part (a) for $n = 100$ and $n = 1{,}000$. How do the sample statistics change as n increases?

c) Plot a histogram of your $n = 1{,}000$ sample. On the same figure, overlay the population PDF using:

```
x = np.linspace(4, 20, 200)
plt.plot(x, stats.norm.pdf(x, 12, 2.5))
```

How well does the sample histogram approximate the population?

d) In your own words, what is the relationship between a sample and a population? Why do larger samples give us more information about the population?

Q3. A Different Distribution

Not all random variables are normal. The time between buses at a city bus stop follows a **uniform distribution** between 0 and 20 minutes. Any wait time in that range is equally likely. Use `stats.uniform(loc=0, scale=20)` for CDF/PPF calculations and `np.random.uniform(0, 20, n)` to draw samples.

a) The theoretical mean of a uniform distribution on $[a, b]$ is $\mu = (a + b)/2$ and the variance is $\sigma^2 = (b - a)^2/12$. Compute μ and σ for this bus stop.

b) What is the probability of waiting more than 15 minutes? You can reason this out from the shape of the distribution, then verify with `stats.uniform.cdf()`.

c) What is the probability of waiting between 5 and 12 minutes?

d) Draw a sample of $n = 1{,}000$ from this distribution and plot the histogram. On the same figure, plot the population PDF. How does the shape of this distribution compare to the normal distribution in Q1?

e) In your own words, what does it mean to say "random variables come in many shapes"? How do the normal and uniform distributions differ in what they tell us about the data-generating process?