ECON 0150 | Economic Data Analysis

The economist's data analysis skillset.

Part 4.3 | Model Residuals and Diagnostics

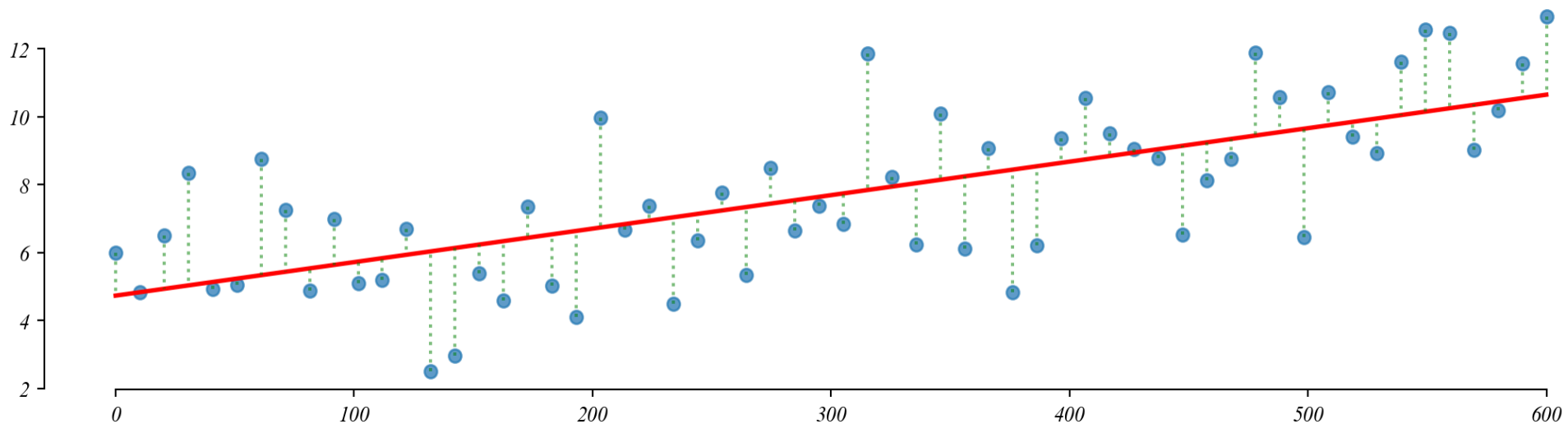
General Linear Model

... a flexible approach to run many statistical tests.

The Linear Model: $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

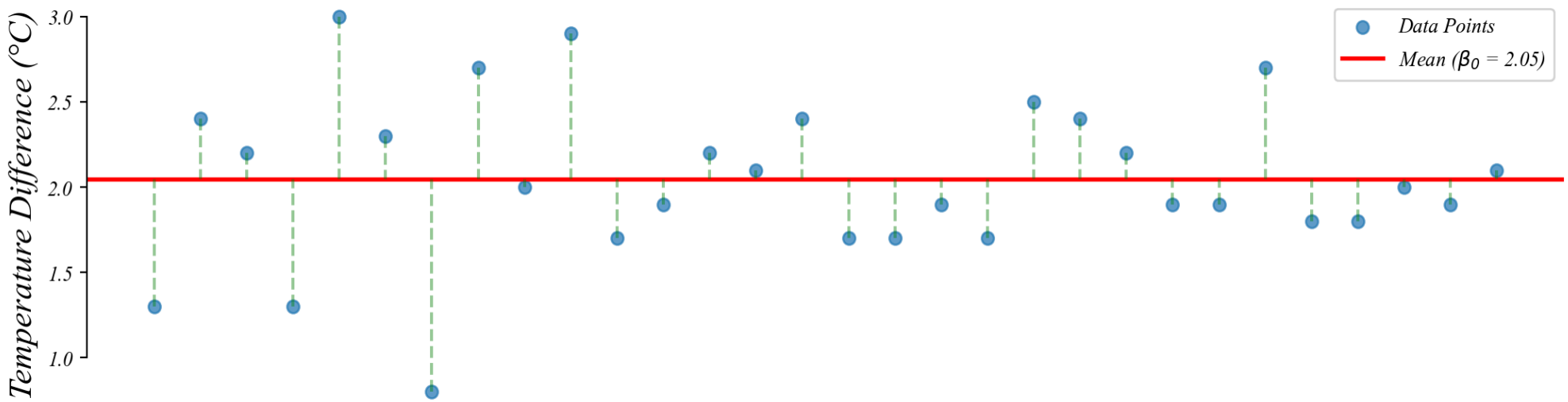
- β_0 *is the intercept (value of* $\bar{y}$ *when* $x = 0$*)*
- β_1 *is the slope (change in y per unit change in x)*
- ε_i *is the error term (random noise around the model)*

OLS Estimation: Minimizes $\sum_{i=1}^{n} \varepsilon_i^2$



GLM: Intercept Model

A one-sample t-test is a horizontal line model.



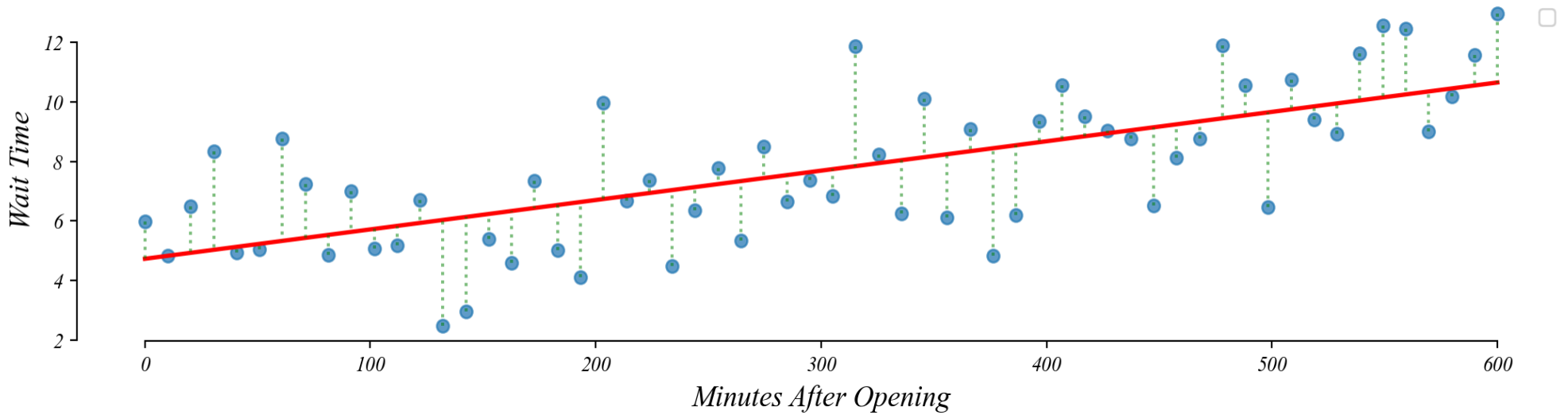
$$Temperature = \beta_0 + \varepsilon$$

> *the intercept β_0 is the estimated mean temperature*

> *the p-value is the probability of seeing β_0 if the null is true*

GLM: Intercept + Slope

A regression is a test of relationships.



$$\text{WaitTime} = \beta_0 + \beta_1 \text{MinutesAfterOpening} + \epsilon$$

> *the intercept parameter β_0 is the estimated temperature at 0 on the horizontal*

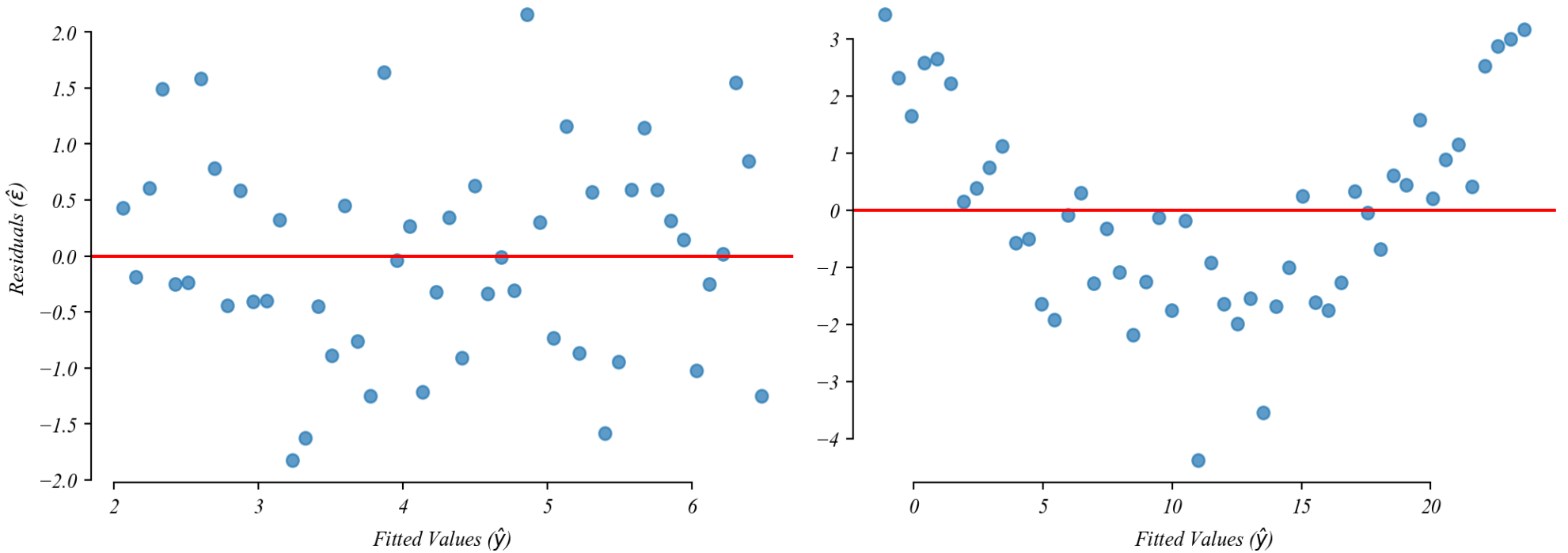
> *the slope parameter β_1 is the estimated change in y for a 1 unit change in x*

> *the p-value is the probability of seeing parameter (β_0 or β_1) if the null is true*

GLM: Intercept + Slope

A regression is a test of relationships.

Do you think the model on the right offers good predictions?



> no… our model isn't set up correctly to handle the data!

GLM Assumptions

Our test results are only valid when the model assumptions are valid.

1. ***Linearity***: *The relationship between X and Y is linear*

2. ***Homoskedasticity***: *Equal error variance across all values of X*

3. ***Normality***: *Errors are normally distributed*

4. ***Independence***: *Observations are independent from each other*

GLM Assumptions: why check?

Assumption violations affect our inferences

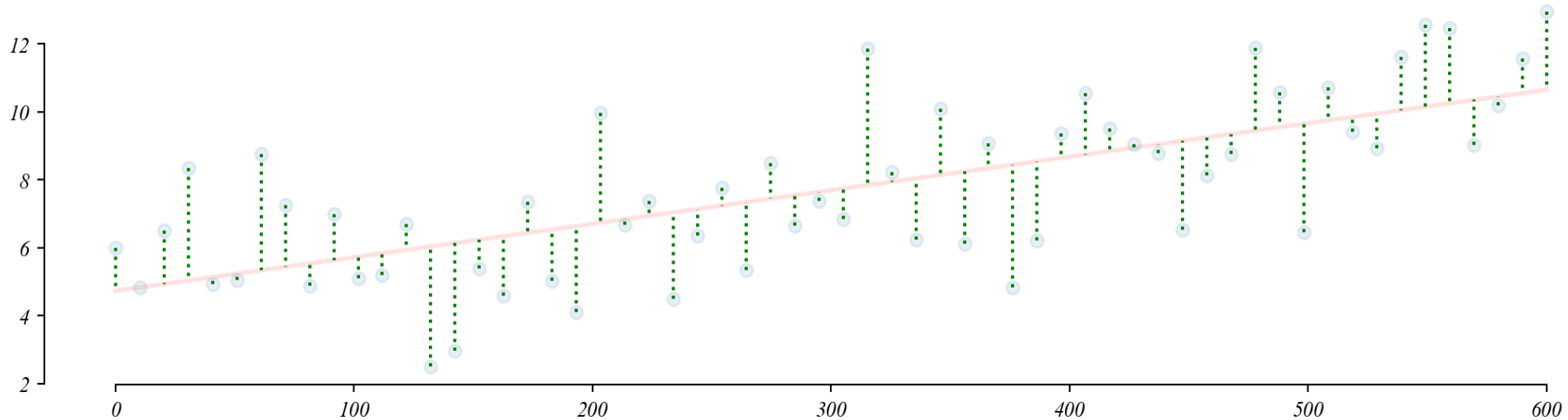
If assumptions are violated:

- *Coefficient estimates may be biased*
- *Standard errors may be wrong*
- *p-values may be misleading*
- *Predictions may be unreliable*

> to test whether the model is 'specified', we can calculate the residuals and the model predictions

Model Residuals

... we can directly examine the error of the model.

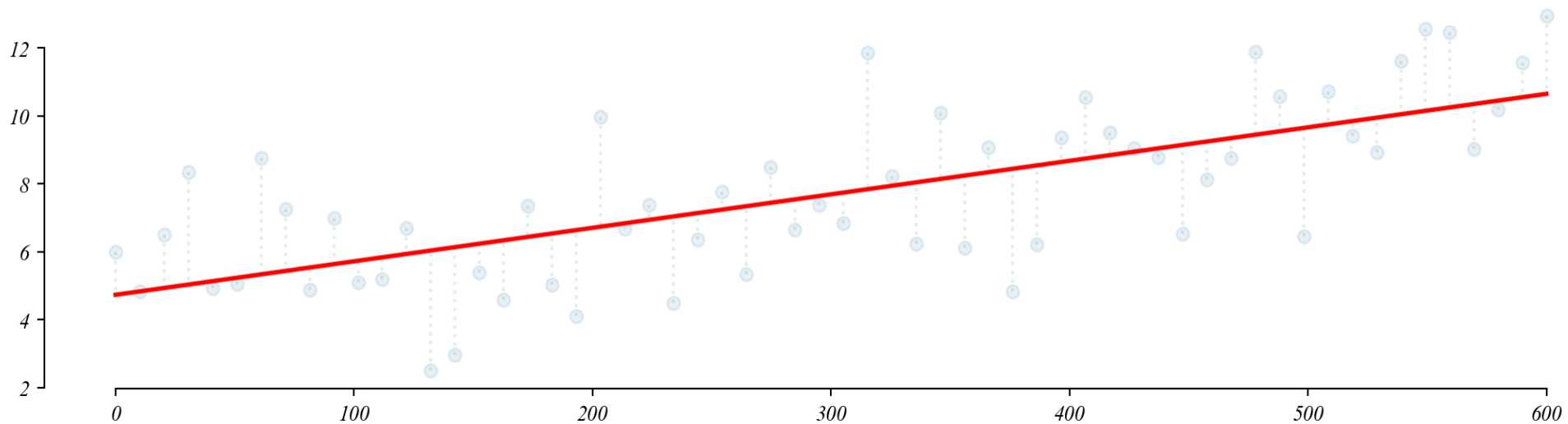



```
# Calculate residuals
residuals = model.resid
sns.histplot(residuals)
```

> *this is ε*

Model Predictions

... we can directly examine the predictions of the model.




```
# Calculate predictions
predictions = model.predict()
sns.histplot(predictions)
```

> *this is $\hat{y}$, the model prediction*

Exercise 4.2 | Residual Plot of Happiness and GDP

Is income higher for those more highly educated?

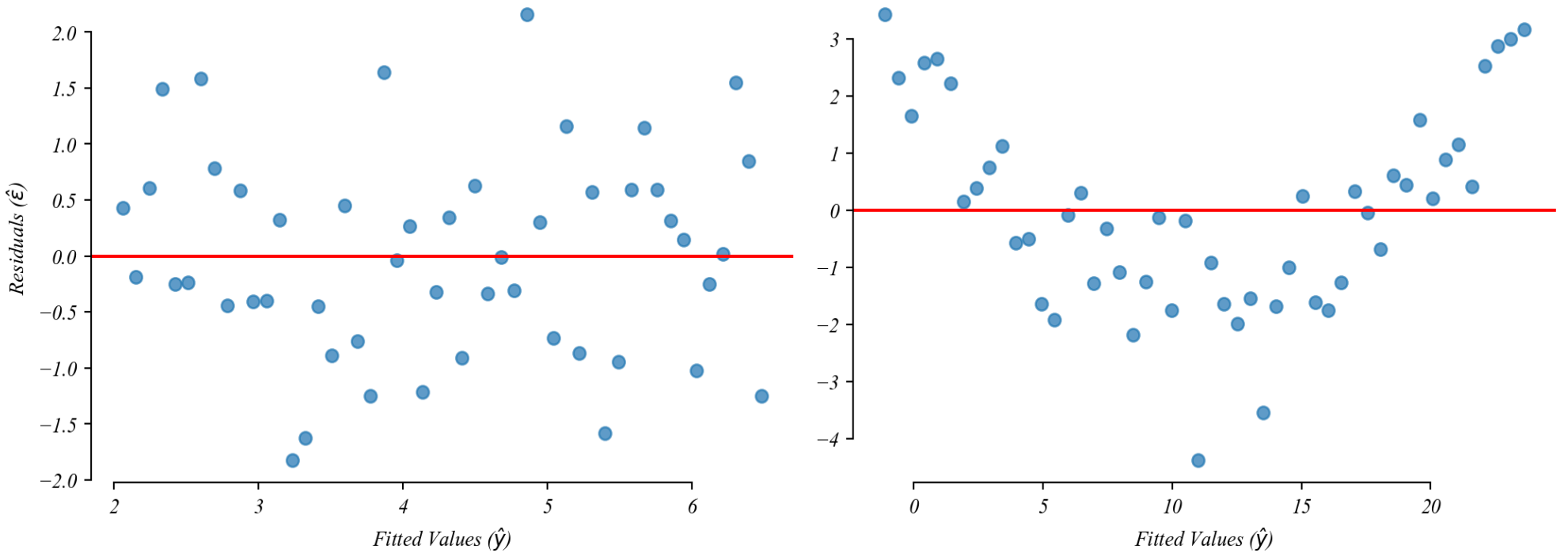
A **Residual Plot** directly visualizes the error for each model estimate.

```
1 plt.scatter(predictions, residuals)
```

Assumption 1: Checking for Linearity

The error term should be unrelated to the fitted value.

Which figure shows linearity?

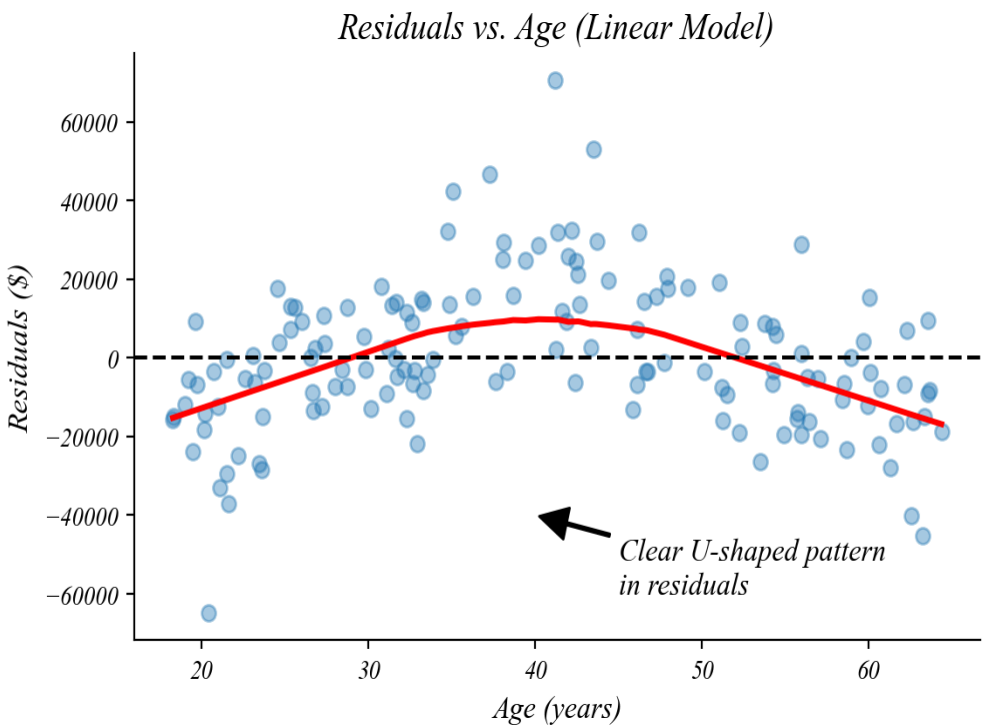


> *the left figure shows that the model is equally wrong everywhere*

> *the right figure shows that the model is a good fit at only some values*

Assumption 1: Checking for Linearity

A non-linear relationship will produce non-linear residuals.



> *linear model misses curvature, leading to systematic errors*

> *check your residuals*

Handling Non-Linear Relationships

Transform variables to become linear

Adding a square term or performing a log transformation can fix the problem.

instead of

$$\text{income} = \beta_0 + \beta_1 \text{age} + \varepsilon$$

we could use

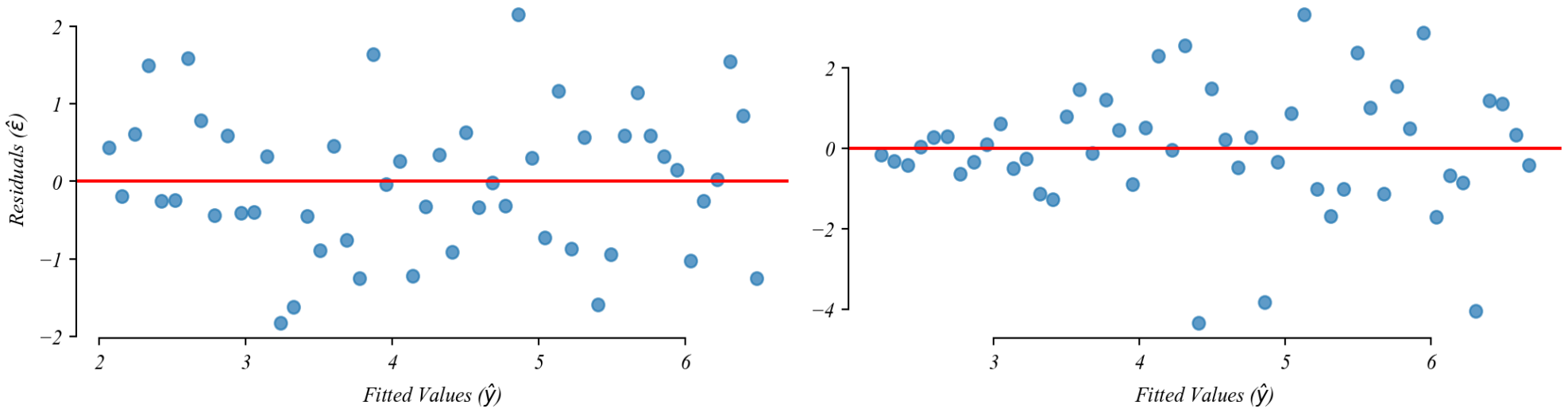
$$\text{income} = \beta_0 + \beta_1 \text{age} + \beta_2 \text{age}^2 + \varepsilon$$

It's also common to log transform either the x or y variable.

Assumption 2: Homoskedasticity

Residuals should be spread out the same everywhere.

Which one of these figures shows homoskedasticity?



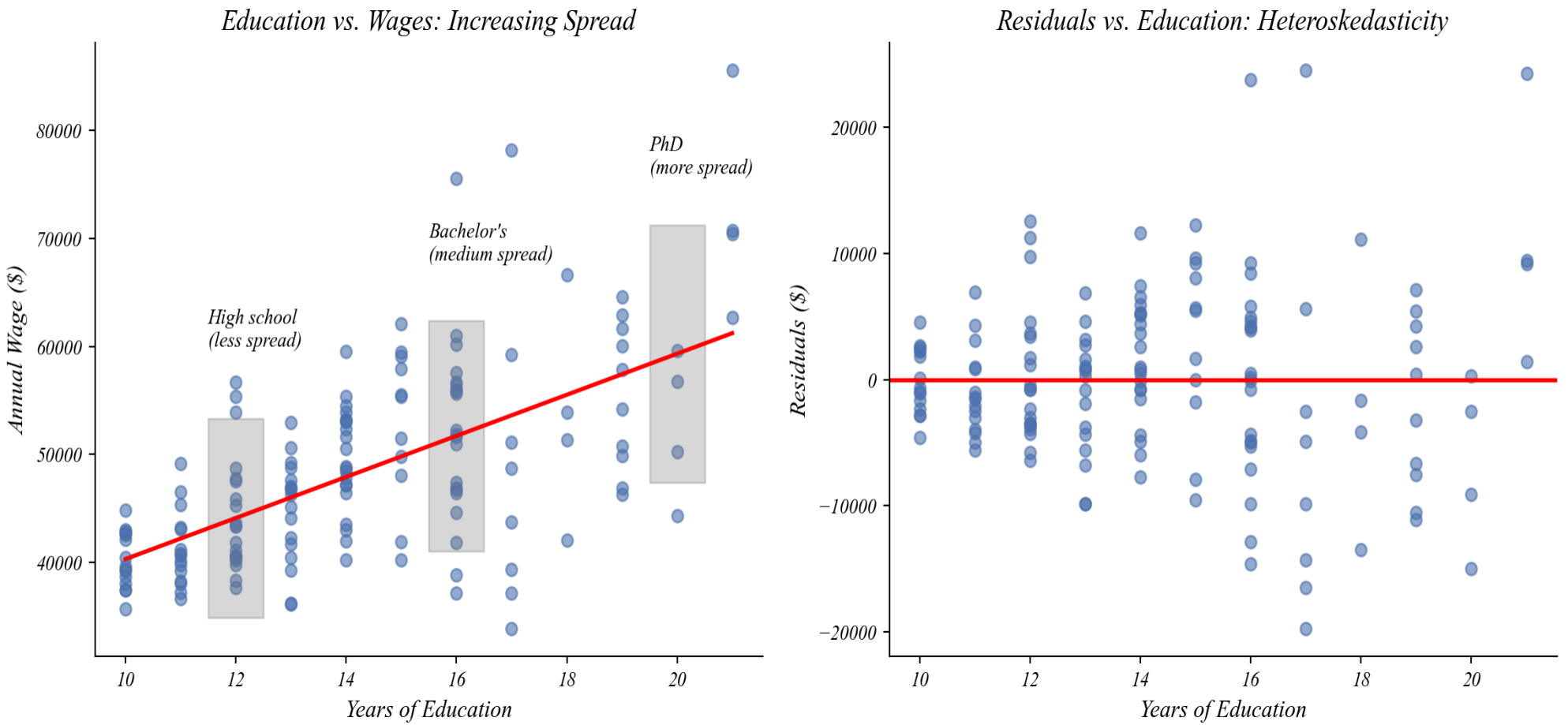
> *the left figure shows constant variability (homoskedasticity)*

> *the right figure shows increasing variability (heteroskedasticity)*

> *residual plots should show that the model is equally wrong everywhere*

Assumption 2: Homoskedasticity

When the spread of residuals changes across values of X

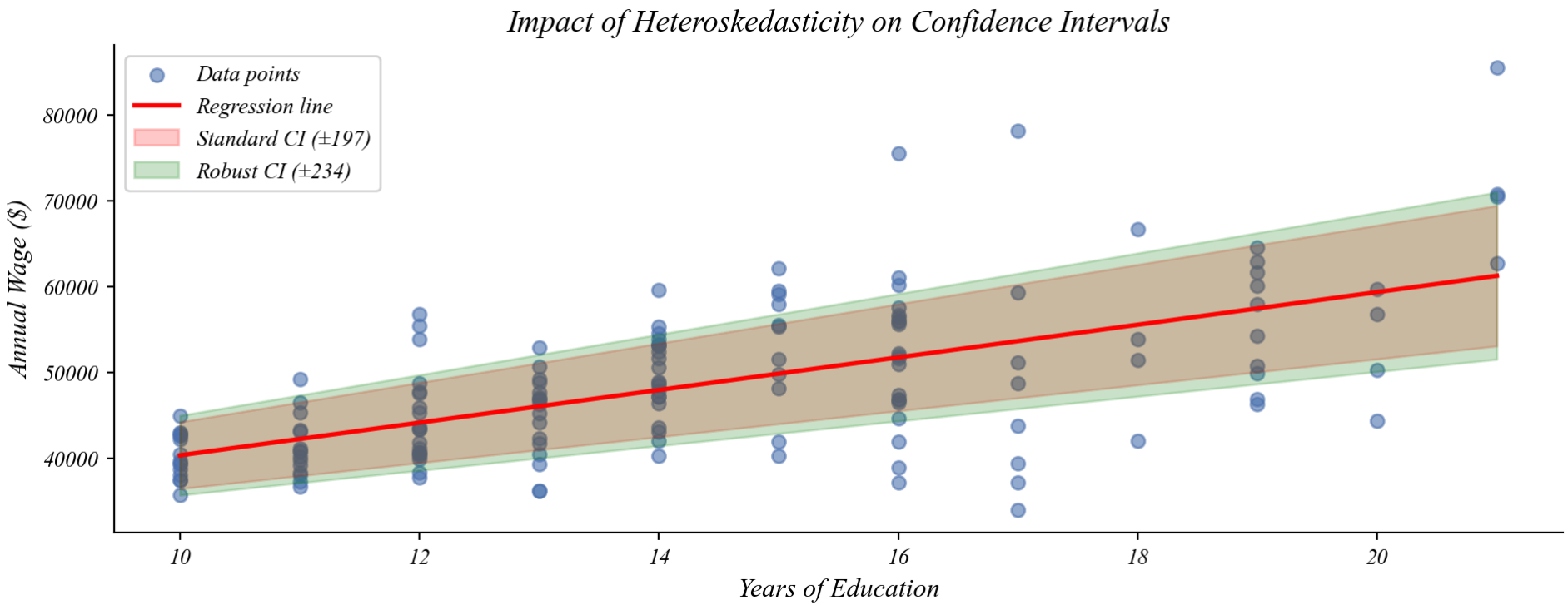


> *notice how the spread of points increases with more education*

> *PhD wages vary more than high school wages*

Assumption 2: Homoskedasticity

Heteroskedasticity affects how we measure uncertainty in our estimates



> *standard methods assume constant spread (homoskedasticity)*

> *like using the wrong ruler to measure uncertainty*

> *with heteroskedasticity, we need robust standard errors*

> *these adjust for the changing spread in our data*

Handling Heteroskedasticity

Robust standard errors give more accurate measures of uncertainty

```
# Fit the model with robust standard errors (HC3: heteroskedastic-constant)
robust_model = smf.ols('wages ~ education', data=df).fit(cov_type='HC3')
```

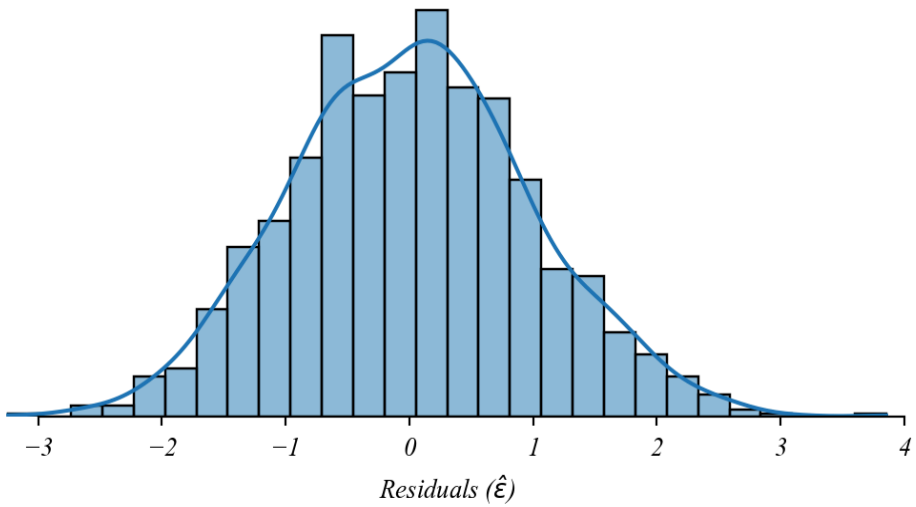
> *robust standard errors give more accurate confidence intervals*

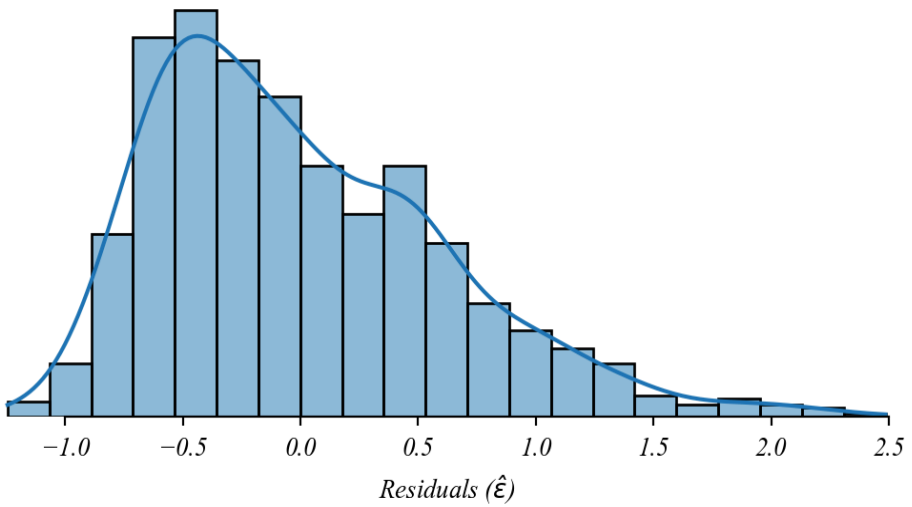
> *and more reliable hypothesis tests*

> *especially important when heteroskedasticity is pronounced*

Assumption 3: Normality

Residuals should be normally distributed





> *left shows a nice bell shape (roughly normally distributed)*

> *right shows a skewed distribution (not normally distributed)*

> *by the CLT we can still use regression without this if the sample is large*

Assumption 4: Indepedence

Observations are independent from each other

We'll return to this assumption in **Part 4.4 | Timeseries**.

Looking Forward

Extending the GLM framework

Next Up:

- *Part 4.3 | Categorical Predictors*
- *Part 4.4 | Timeseries*
- *Part 4.5 | Causality*

Later:

- *Part 5.1 | Numerical Controls*
- *Part 5.2 | Categorical Controls*
- *Part 5.3 | Interactions*
- *Part 5.4 | Model Selection*

> all built on the same statistical foundation