ECON 0150 | MiniExam 05 | Spring 2025

Solutions Guide

Q1. A restaurant's customer wait times follow a normal distribution with mean (μ) = 12 minutes and standard deviation (σ) = 2 minutes.

a) What is the standard error of the sampling distribution if you take a sample of 64 customers?

✓ □ 0.25 minutes

Solution: The standard error of the sampling distribution is calculated as $\sigma/\sqrt{n} = 2/\sqrt{64} = 2/8 = 0.25$ minutes.

b) If you increase your sample size, what happens to the standard error?

✓ □ The standard error decreases

Solution: As sample size (n) increases, the standard error ($\sigma/\sqrt{n}$) decreases because we're dividing by a larger number. Intuitively, larger samples give more precise estimates of the population mean.

Q2. For a sample of 30 customers at a restaurant:

a) The sample mean ($\bar{x}$) wait time is 14 minutes and the sample standard deviation (S) is 3 minutes. Which of these is a statistic?

✓ □ The sample mean ($\bar{x}$)

Partial credit for:

□ The standard error of the mean

Solution: A statistic is a quantity calculated from sample data. The sample mean ($\bar{x}$) is calculated from the sample and is therefore a statistic. Population parameters (μ and σ) are fixed values of the population, not statistics. The standard error is derived from both a statistic (n) and a parameter (σ).

b) According to the Central Limit Theorem, which statement is correct?

✓ □ The distribution of sample means will approach a normal distribution

Solution: The Central Limit Theorem states that the sampling distribution of means approaches a normal distribution as sample size increases, regardless of the shape of the original population distribution. The theorem does not imply that individual sample values become normal, that the sample mean equals the population mean, or that the population must be normal.

Q3. You roll a fair six-sided die 50 times and calculate the average. You repeat this process many times and create a histogram of all these sample averages.

Which of these statements is true about the histogram?

✓ □ It will be centered at 3.5

Solution: The expected value (mean) of a single fair die roll is $(1+2+3+4+5+6)/6 = 3.5$. According to the Central Limit Theorem, the sampling distribution of means will be centered at the population mean, which is 3.5 in this case.

The histogram will not look like a uniform distribution (it will be approximately normal). It will not have the same spread as the original distribution (it will have less spread by a factor of $\sqrt{50}$).

Full explanation: When we roll a die many times and take the average, we're creating a sampling distribution of means. The Central Limit Theorem tells us that this distribution will:

1. Be centered at the population mean (3.5 for a fair die)
2. Approach a normal distribution regardless of the original distribution (which is uniform for a die)
3. Have a standard error of $\sigma/\sqrt{n} = (\sqrt{35}/12)/\sqrt{50} \approx 0.24$, which is much smaller than the standard deviation of the original distribution ($\sqrt{35}/12 \approx 1.71$)

The key insight is that averaging across many rolls reduces variability and produces a bell-shaped distribution centered at the true population mean.