ECON 0150 | Economic Data Analysis

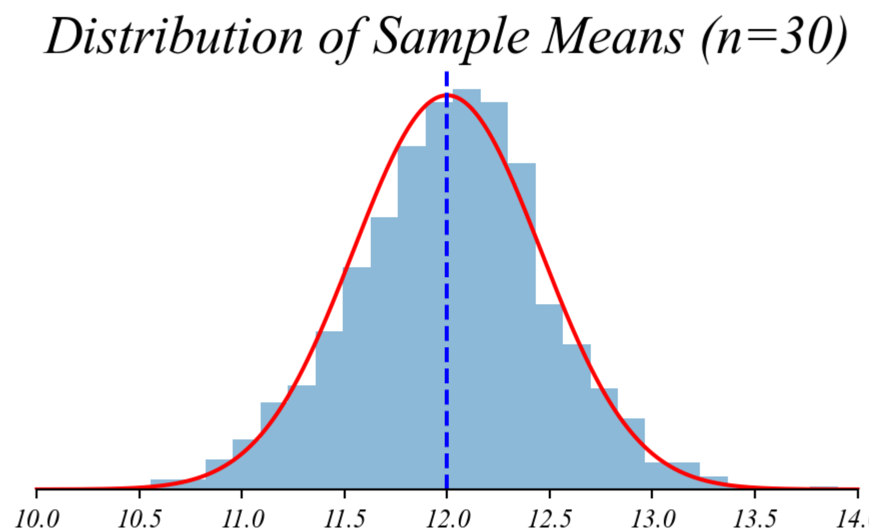
The economist's data analysis pipeline.

Part 3.4 | Testing Hypotheses

Confidence Intervals Recap

We used the distribution of sample means to create confidence intervals around $\bar{x}$.

- *Sample mean $\bar{x}$ follows a normal distribution*
- *Centered at population mean μ*
- *Standard error = $\frac{\sigma}{\sqrt{n}}$*
- *95% of samples will have $\bar{x}$ and μ roughly 1.96 standard errors apart*



> in the wait time example, we asked "where is the true mean wait time?"

> but what if we want to test a specific claim about the mean?

Flipping The Question

What if we want to test a specific claim about the mean?

> *"my boss claims the mean wait time is 10 minutes"*

> *is our data consistent with that specific claim?*

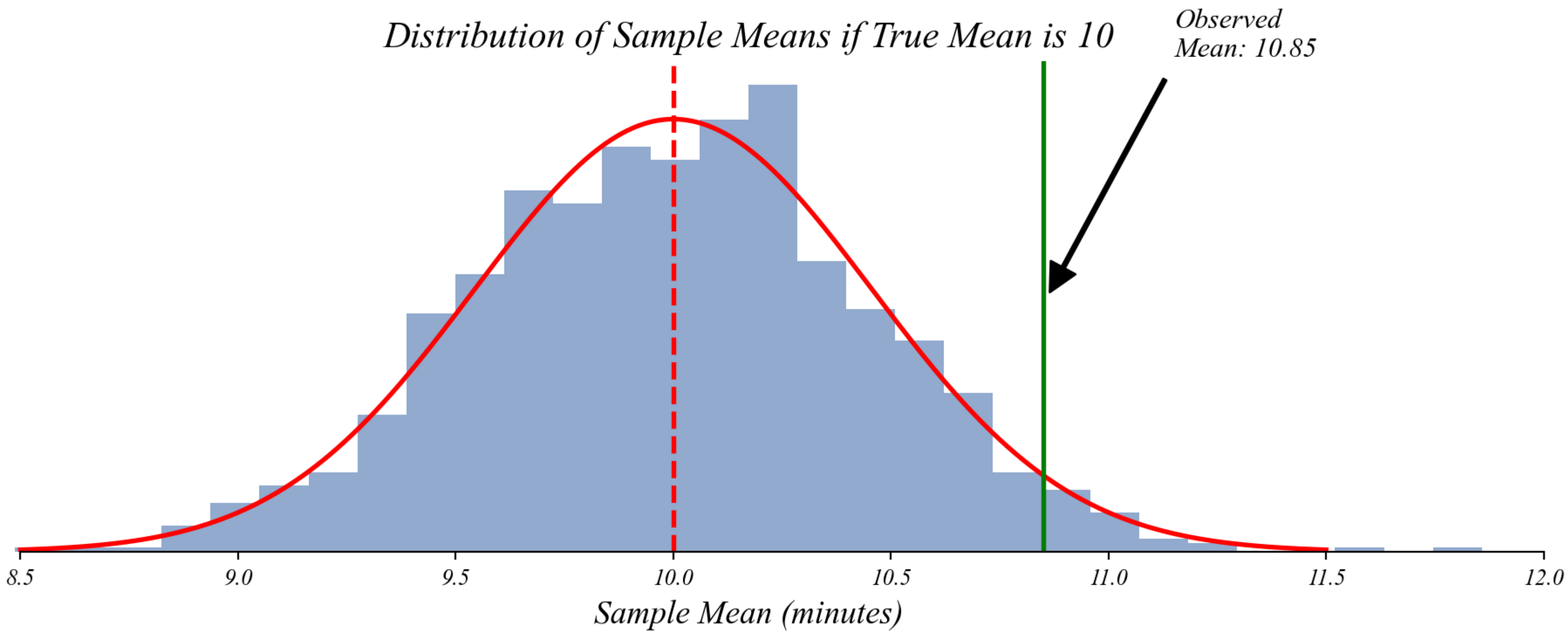
> *same math as last time, but a different question…*

> *instead of finding where some μ might be, we're testing a specific value of μ*

Testing a Specific Value

If $\bar{x}$ = 10.85, is that consistent with μ_0 = 10?

> *let's simulate data where $\mu = 10$ and see what sample means we'd get*



> *how "surprising" would our observed $\bar{x}$ be if μ actually was 10?*

Hypotheses and Confidence Intervals

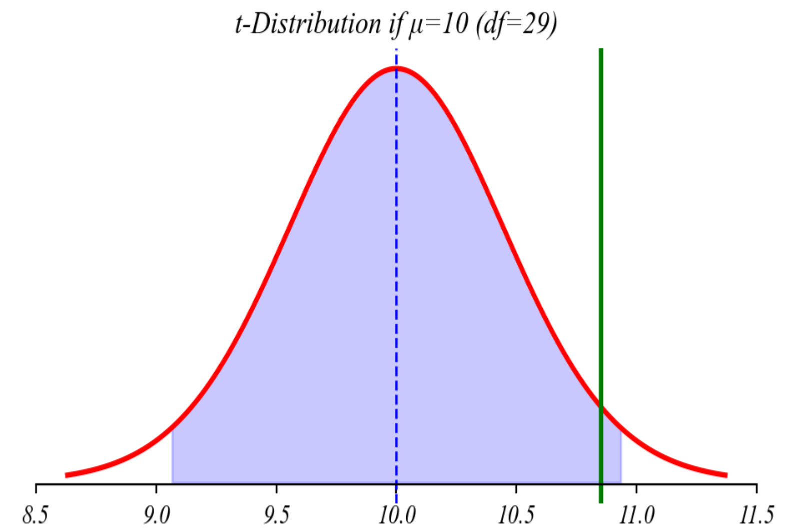
The math to answer this question is identical to confidence intervals.

If true mean is $\mu_0 = 10$:

$$\bar{x} \sim t_{n-1}\left(\mu_0, \frac{s}{\sqrt{n}}\right)$$

$$\bar{x} \sim t_{29}\left(10, \frac{2.5}{\sqrt{30}}\right)$$

$$\bar{x} \sim t_{29}(10, 0.456)$$



A 95% confidence interval around μ_0 would be:

$$[10 - 2.045 \times 0.456, 10 + 2.045 \times 0.456]$$

$$[9.07, 10.93]$$

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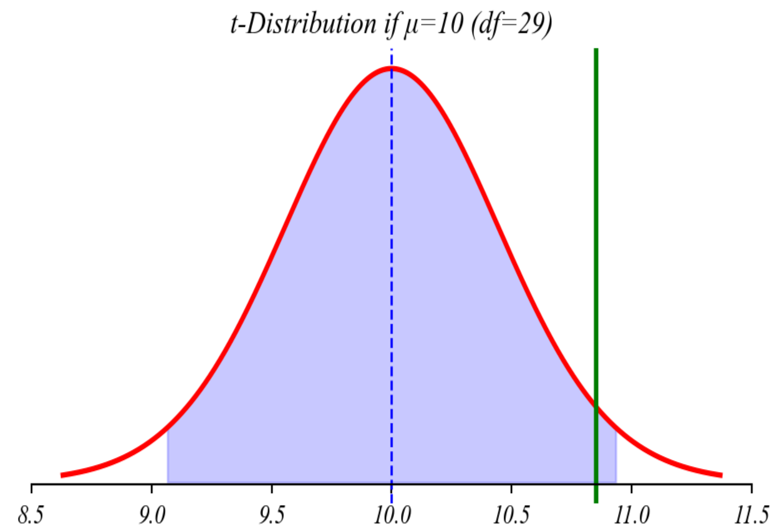


> *our observed mean ($\bar{x} = 10.85$) is within this interval — not surprising if μ=10*

> *but if we observed $\bar{x} = 11.5$, that would be outside the interval — surprising!*

The Null Hypothesis

We formalize this approach by setting up a "null hypothesis"

Null Hypothesis (H_0): *The specific value or claim we're testing*

- $H_0 : \mu = 10$ *(wait time is 10 minutes)*

Alternative Hypothesis (H_1 or H_a): *What we accept if we reject the null*

- $H_1 : \mu \neq 10$ *(wait time is not 10 minutes)*

Testing Approach:

- *Calculate how "surprising" our data would be if* H_0 *were true*
- *If sufficiently surprising, we reject* H_0

Quantifying “Surprise”: p-values

The p-value measures how compatible our data is with the null hypothesis

p-value: *The probability of observing a test statistic at least as extreme as ours, if the null hypothesis were true*

For our example:

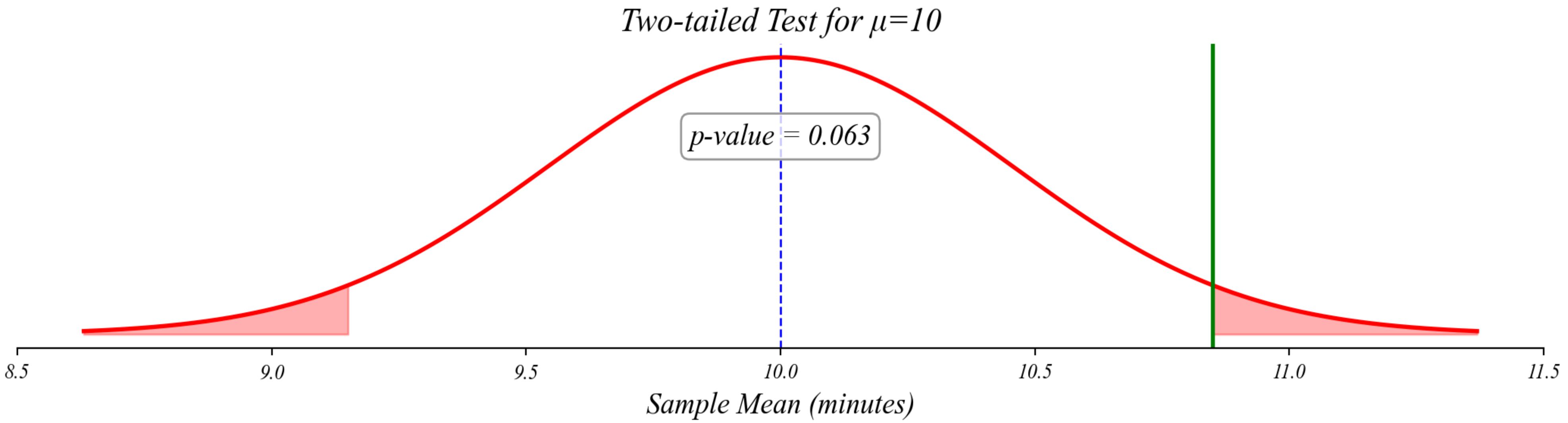
- *Null:* $\mu = 10$
- *Observed:* $\bar{x} = 10.85$

> How likely is it to get $\bar{x}$ this far or farther from 10, if the true mean is 10?

Calculating a p-value

Use the sampling distribution to find the probability.

> *How likely is it to get $\bar{x}$ this far or farther from 10, if the true mean is 10?*



> *interpretation: if $\mu=10$, we'd see $\bar{x}$ this far from 10 about 6.4% of the time*

> *often, we reject H_0 if p-value < 0.05 (5%)*

> *here, p-value > 0.05, so we don't reject the claim that $\mu=10$*

Test Statistic: The t-statistic

We can standardize our result for easier interpretation

The **t-statistic** measures how many standard errors our sample mean is from the null value:

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Where:

- $\bar{x}$ *is our sample mean (10.85)*
- μ_0 *is our null value (10)*
- s *is our sample standard deviation (2.5)*
- n *is our sample size (30)*

$$t = \frac{10.85 - 10}{2.5/\sqrt{30}} = \frac{0.85}{0.456} = 1.86$$

The t-test

This example has become a formal hypothesis test.

One-sample t-test:

- $H_0 : \mu = 10$
- $H_1 : \mu \neq 10$
- *Test statistic:* $t = 1.86$
- *Degrees of freedom: 29*
- *p-value: 0.064*

Decision rule:

- *If p-value* < 0.05*, reject* H_0
- *Otherwise, fail to reject* H_0

```python
import numpy as np
from scipy import stats

# Sample data
sample_mean = 10.85
pop_mean = 10    # null hypothesis
std_dev = 2.5
sample_size = 30

# Calculate t-statistic
t_stat = (sample_mean - pop_mean) / (std_dev / np.sqrt

# Calculate p-value
p_value = 2 * (1 - stats.t.cdf(abs(t_stat), df=sample_

print(f"t-statistic: {t_stat:.3f}")
print(f"p-value: {p_value:.3f}")
```

```
t-statistic: 1.862
p-value: 0.073
```

> not all test statistics use a t-distribution — depends on the test and sample size

> *but t-tests are extremely common, especially in regression (coming soon!)*

Statistical vs. Practical Significance

A caution about hypothesis testing

Statistical significance:

- *Formal rejection of the null hypothesis ($p < 0.05$)*
- *Only tells us if the effect is unlikely due to chance*

Practical significance:

- *Whether the effect size matters in the real world*
- *A statistically significant result can still be tiny*

> *with large samples, even tiny differences can be statistically significant*

> *always consider the magnitude of the effect, not just the p-value*

Common Misinterpretations

What a p-value is NOT

❌ The probability that H_0 is true

❌ The probability that the results occurred by chance

❌ The probability that H_1 is true

✅ **Correct**: The probability of observing a test statistic at least as extreme as ours, if H_0 were true

Looking Forward

The t-test framework extends to many scenarios

Next time:

- *Comparing means between two groups*
- *Two-sample t-tests*
- *Paired t-tests*

Coming soon:

- *This same framework underlies regression analysis*
- *Regression coefficients are tested using t-tests*
- *ANOVA uses the same fundamental approach*

> *the hypothesis testing framework is foundational for modern science*