ECON 0150 | Economic Data Analysis

The economist's data analysis skillset.

Part 3.2 | Sampling and the Central Limit Theorem

A Big Question

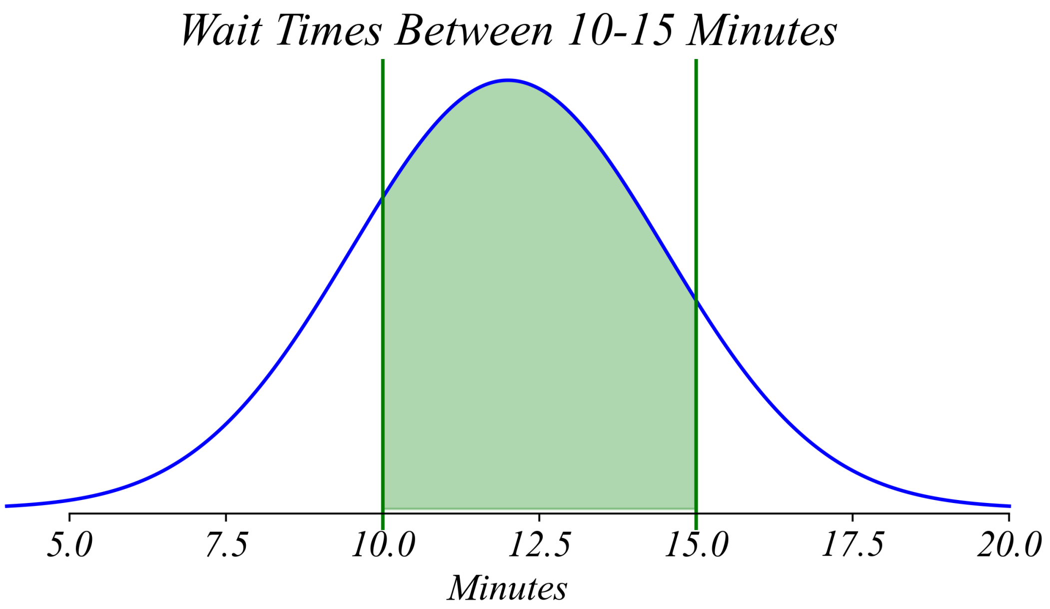
If all we see is the sample, how do we learn about a population?

- *In general, a population's random variables will be unobservable.*
- *If we only see a sample, what can we say about the population?*

Random Variables: Known

If we know the random variable, we can learn many things about the population.

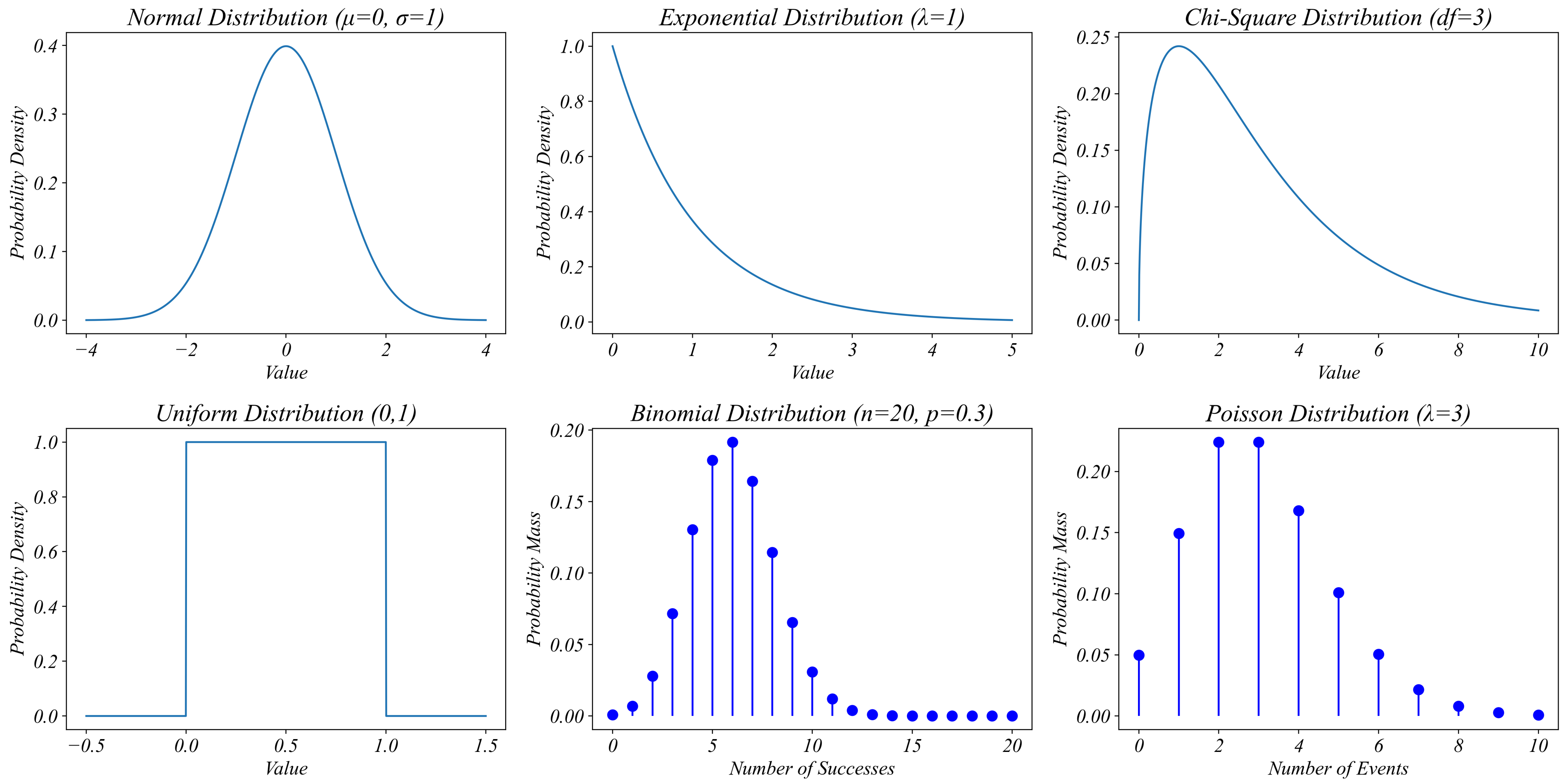
- *Probability wait time < 10:*
  - $P(X < 10) = 0.21$
- *Probability wait time > 15:*
  - $P(X > 15) = 0.11$
- *Probability between 10 - 15:*
  - $P(10 < X < 15) = 0.59$



> *when we know the probability function, we can calculate everything exactly*

Random Variables: Known

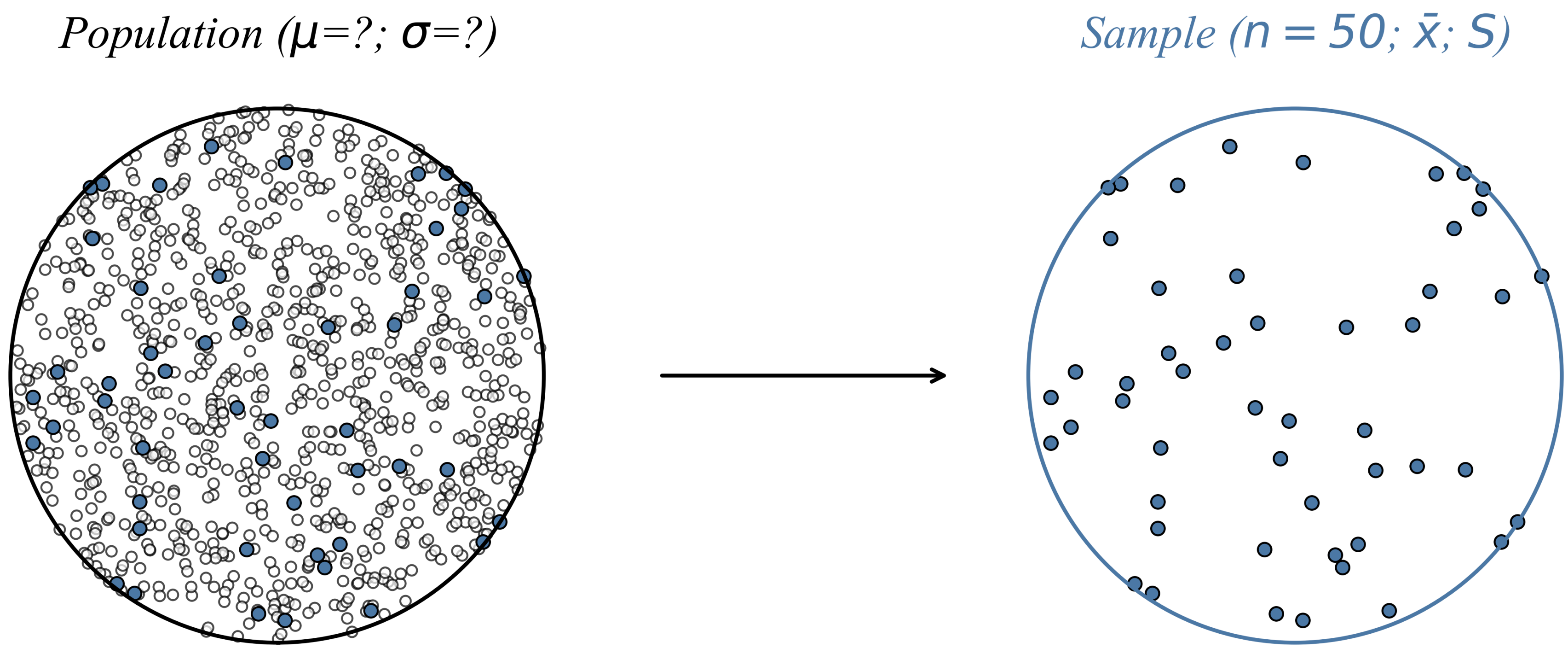
If we know the random variable, we can learn many things about the population.



> but what can we know about the population if we only see the sample?

Random Variables: Unknown

But if all we see is the sample, what can we know about a population?



> *how do we learn about μ if all we have is n, $\bar{x}$, and S?*

Exercise 3.2 | Sampling Dice (sample size: n=1)

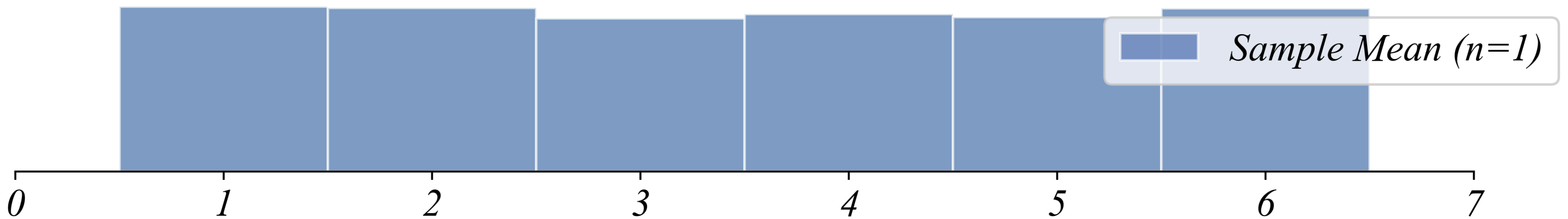
Lets pretend we don't know the probability function for dice.

Lets start with something boring.

1. Roll a dice once (sample size: n=1).
2. We'll plot the distribution of our samples.

Exercise 3.2 | Sampling Variability

Your samples have a lot of variability!



> this variability perfectly matches what we would expect from a fair dice

Exercise 3.2 | Sampling Dice (n=2)

Lets pretend we don't know the probability function for dice.

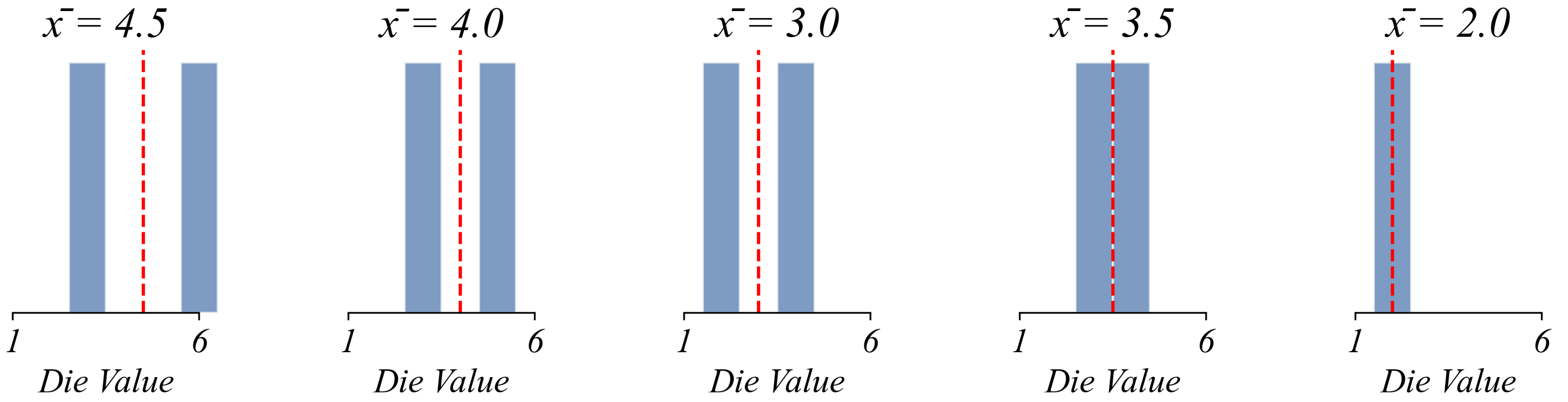
Next is something slighly less boring.

1. Roll a dice once (sample size: n=2).

2. We'll plot the distribution of our samples.

Exercise 3.2 | Sampling Variability

Like before, each sample has a slighly different sample mean.

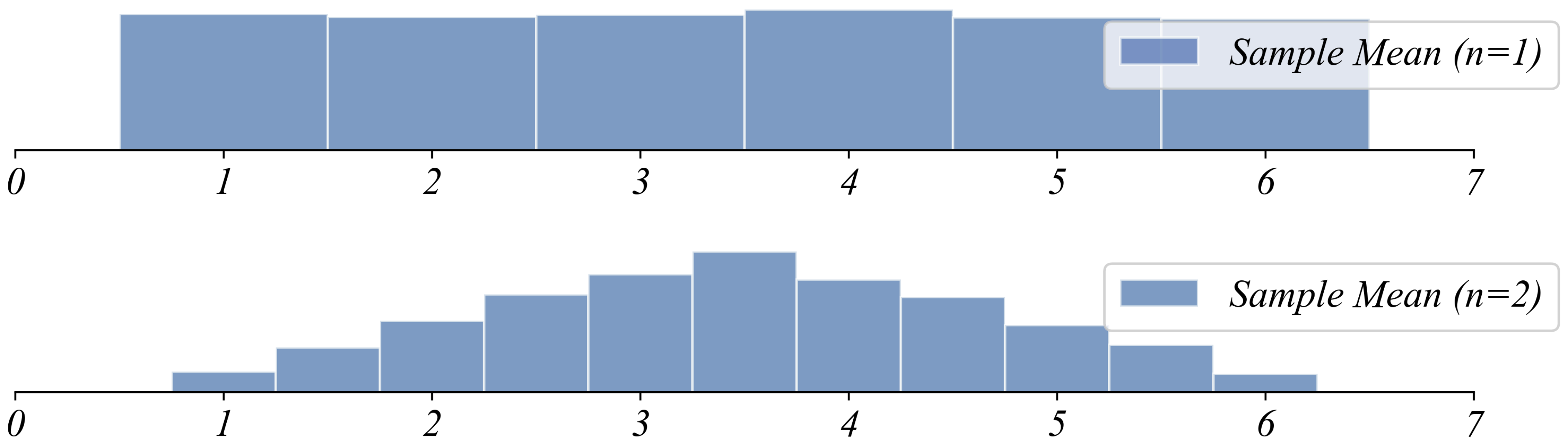


> theres a lot of variability in your sample means!

> what do you expect to see when we plot these sample means ($\bar{x}$)?

Exercise 3.2 | Sampling Variability

The variability in the sample mean with a larger sample size.



> our sample means are more bunched (like a pyramid) in the middle! why?

> there are more ways to get 7/2 than 2/2!

Exercise 3.2 | Sampling Dice (n=3)

Lets pretend we don't know the probability function for dice.

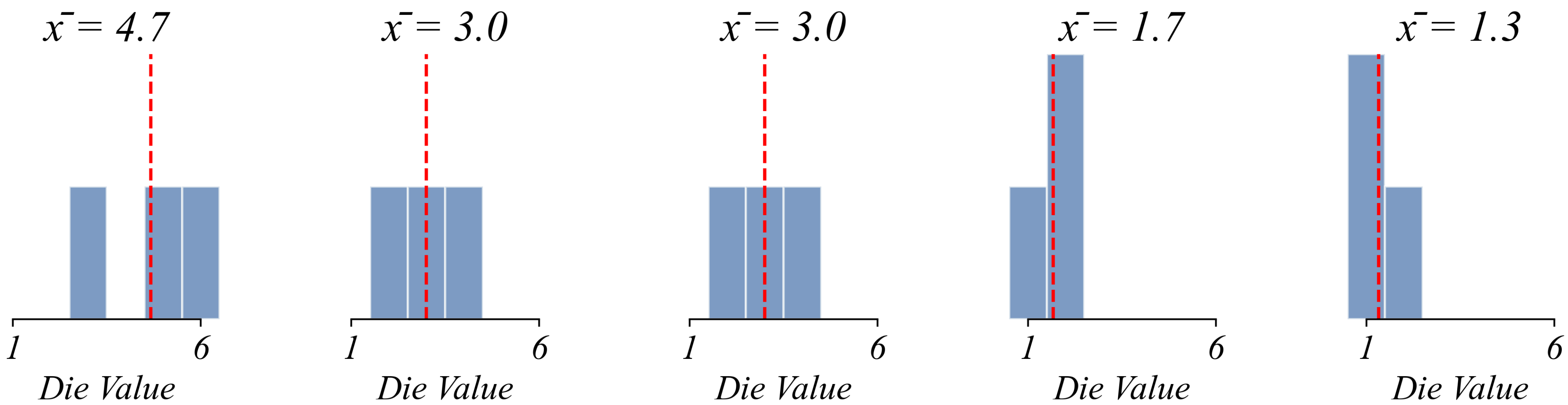
Next is something even less boring.

1. Roll a dice once (sample size: n=3).

2. We'll plot the distribution of our samples.

Exercise 3.2 | Sampling Variability

The variability in the sample mean with a larger sample size.

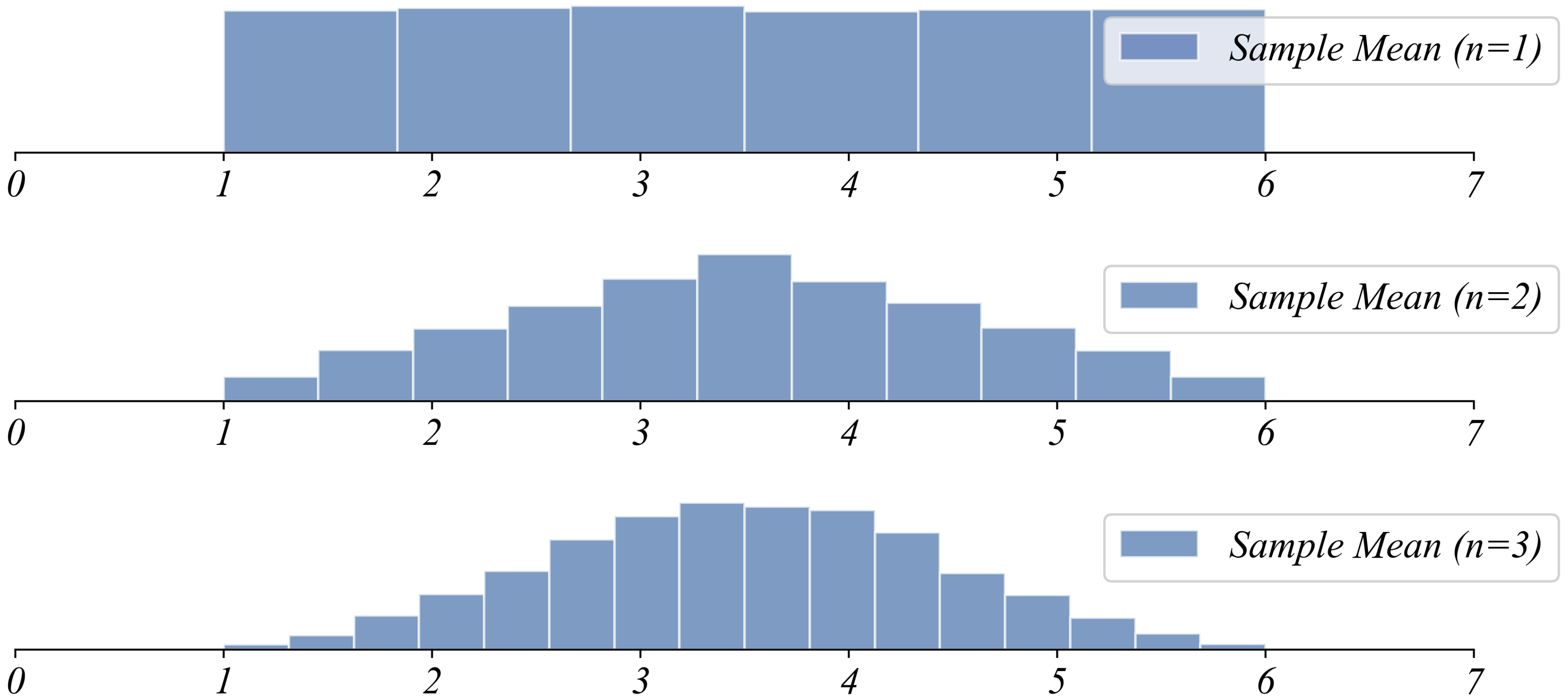


> *theres a even more variability in your sample means!*

> *what do you expect to see when we plot these sample means ($\bar{x}$)?*

Exercise 3.2 | Sampling Variability

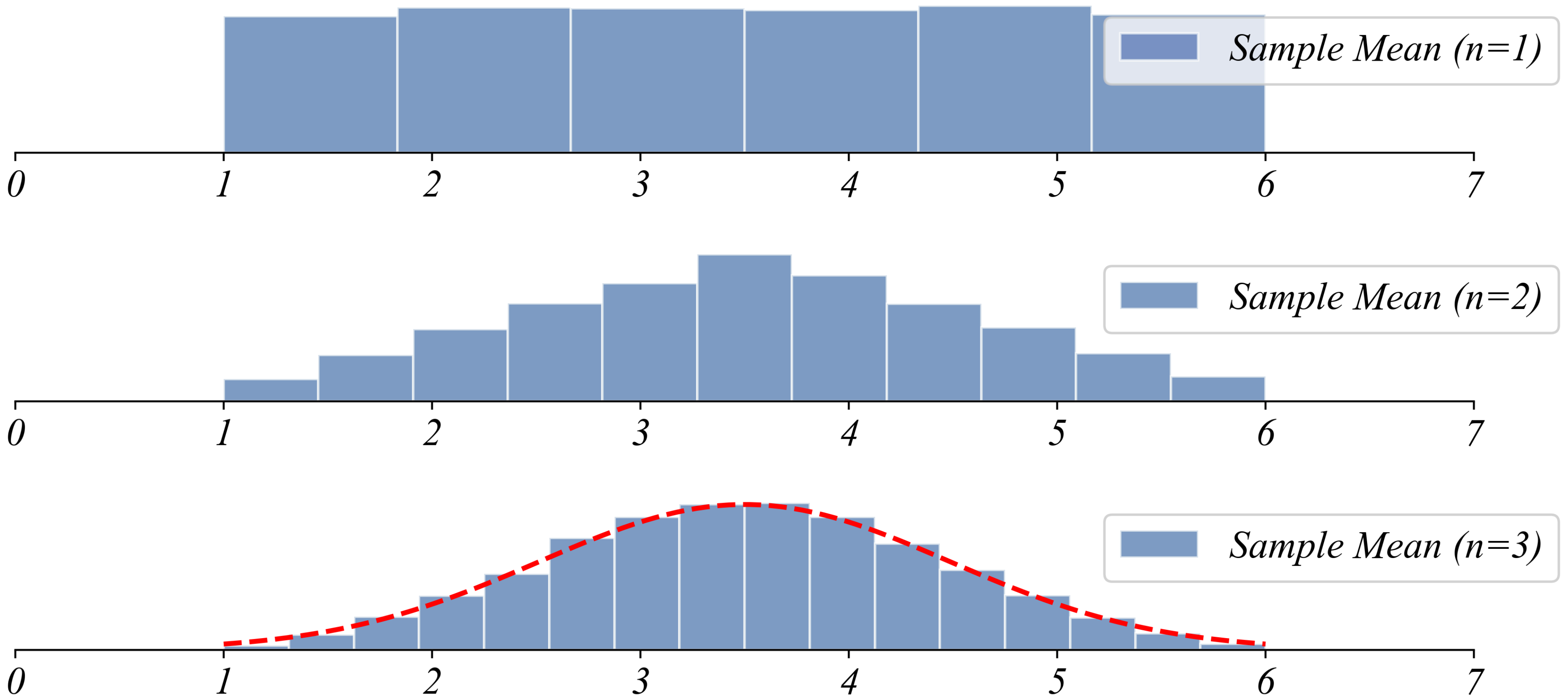
The variability in the sample mean with a larger sample size.



> *what do you notice with the shape with n=3?*

Exercise 3.2 | Sampling Variability

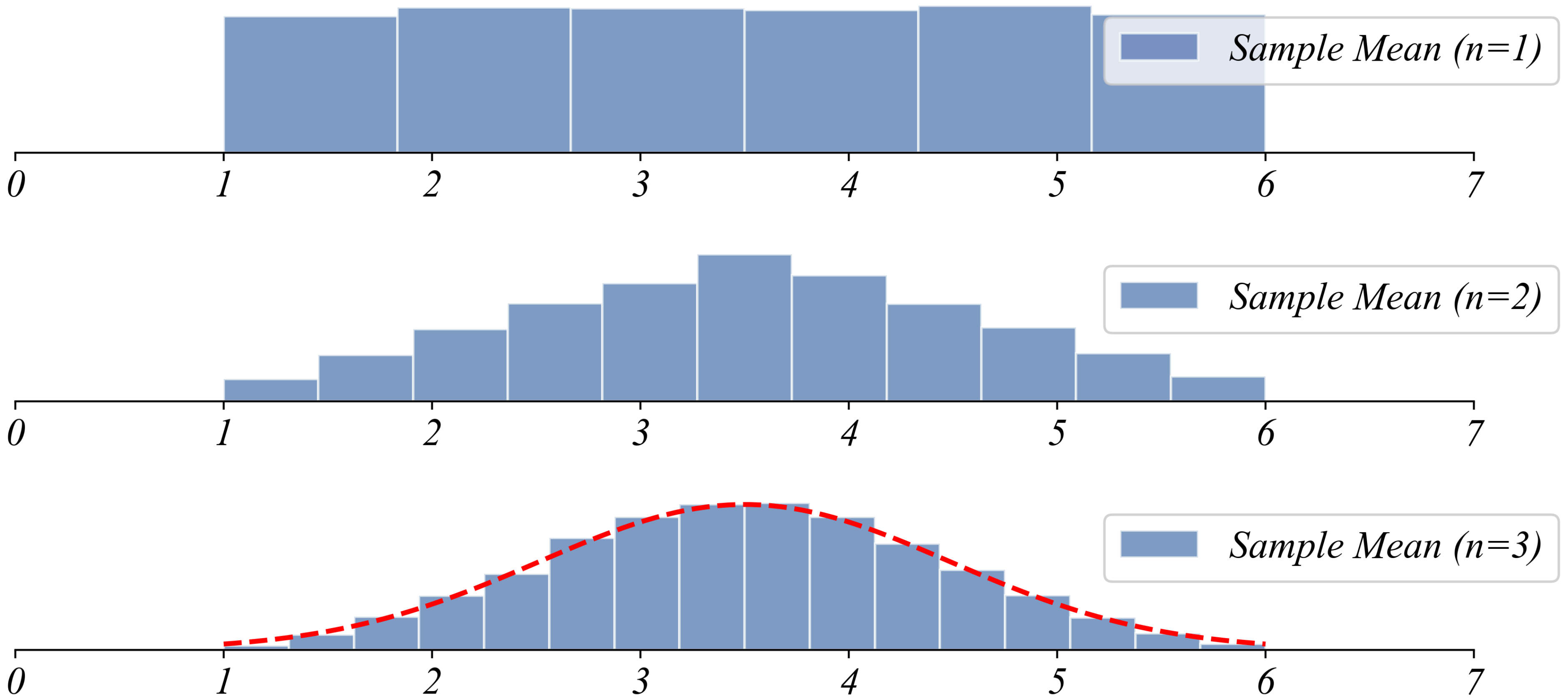
The variability in the sample mean with a larger sample size.



> what do you notice with the shape with n=3?

Exercise 3.2 | Sampling Variability

The variability in the sample mean with a larger sample size.



> there's some curvature to the shape

Exercise 3.2 | Sampling Dice (n=30)

Lets pretend we don't know the probability function for dice.

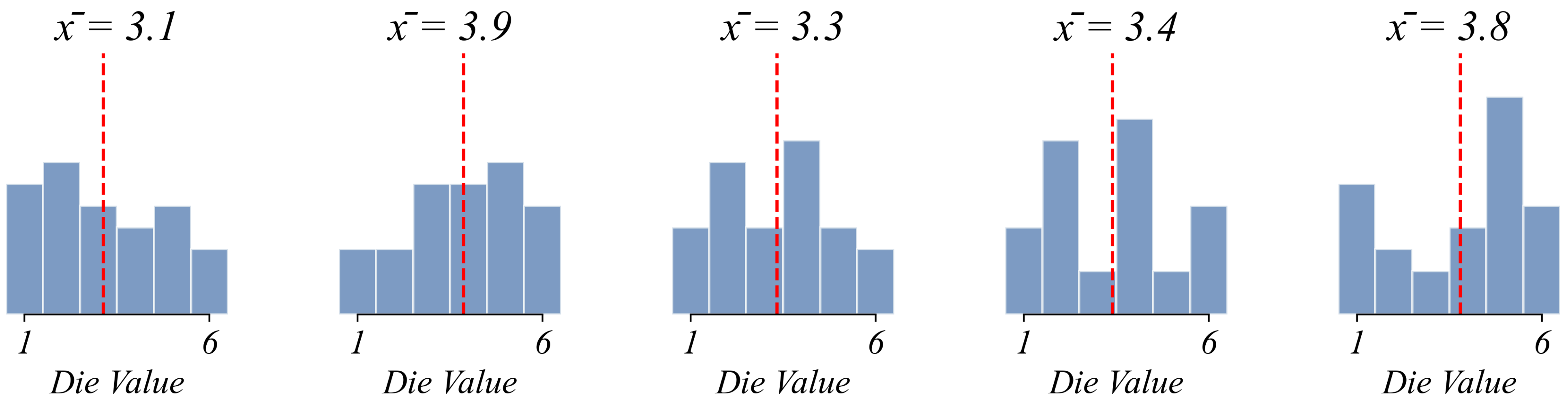
Next is something very un-boring.

1. Roll a dice once (sample size: n=30).

2. We'll plot the distribution of our samples.

Exercise 3.2 | Sampling Variability

The variability in the sample mean with a larger sample size.

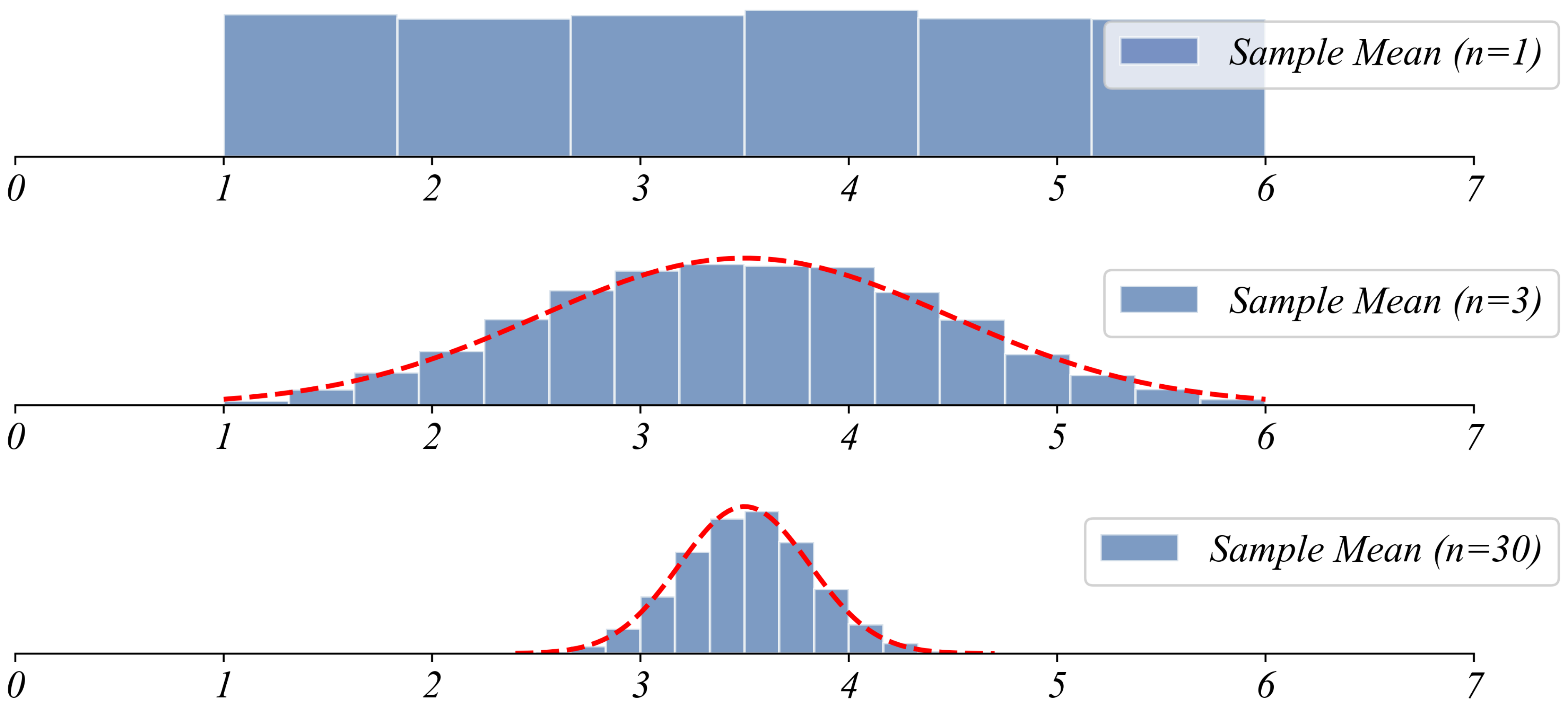


> *theres a even more ways your sample could look!*

> *what do you expect to see when we plot these sample means ($\bar{x}$)?*

Exercise 3.2 | Sampling Variability

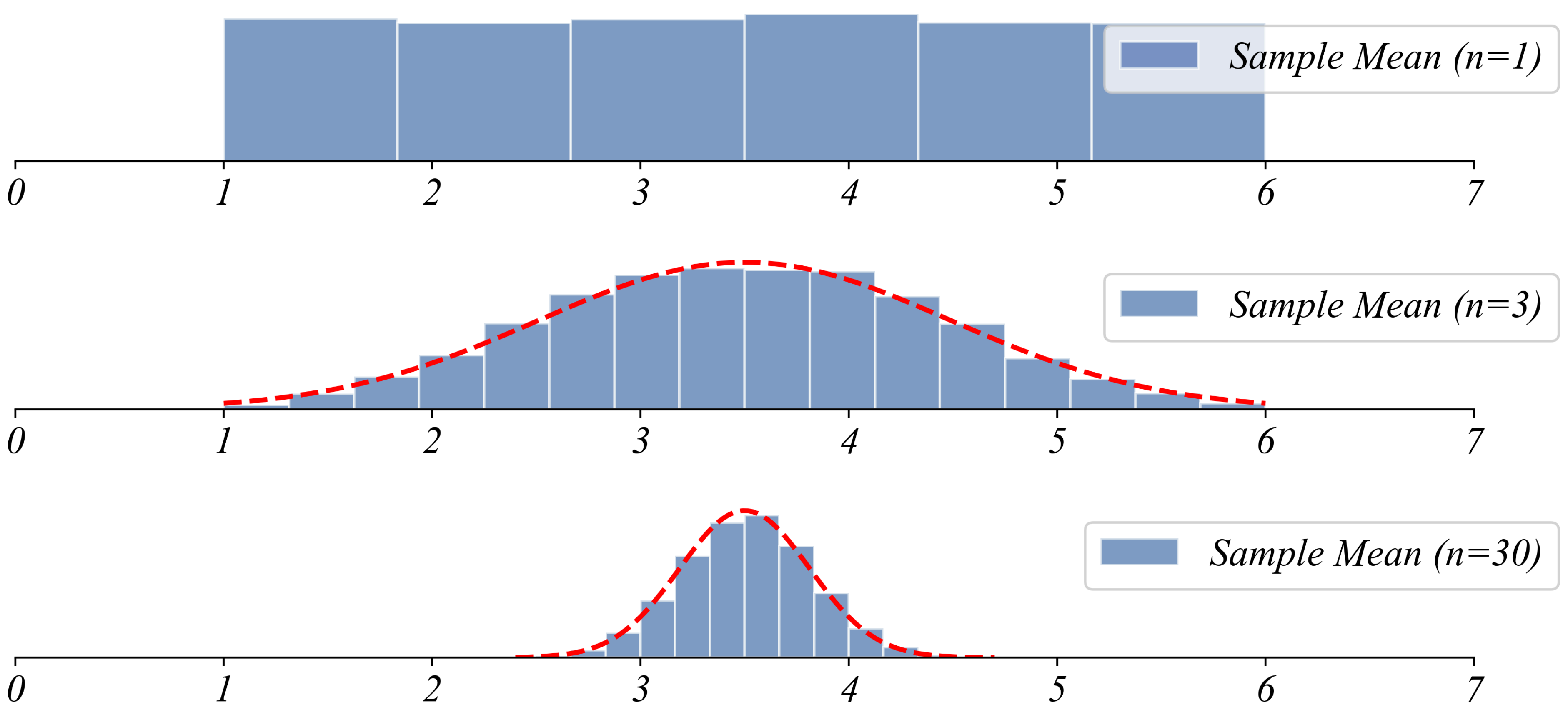
What happens when we really increase the sample size?



> what do you notice with the shape with n=30?

Exercise 3.2 | Sampling Variability

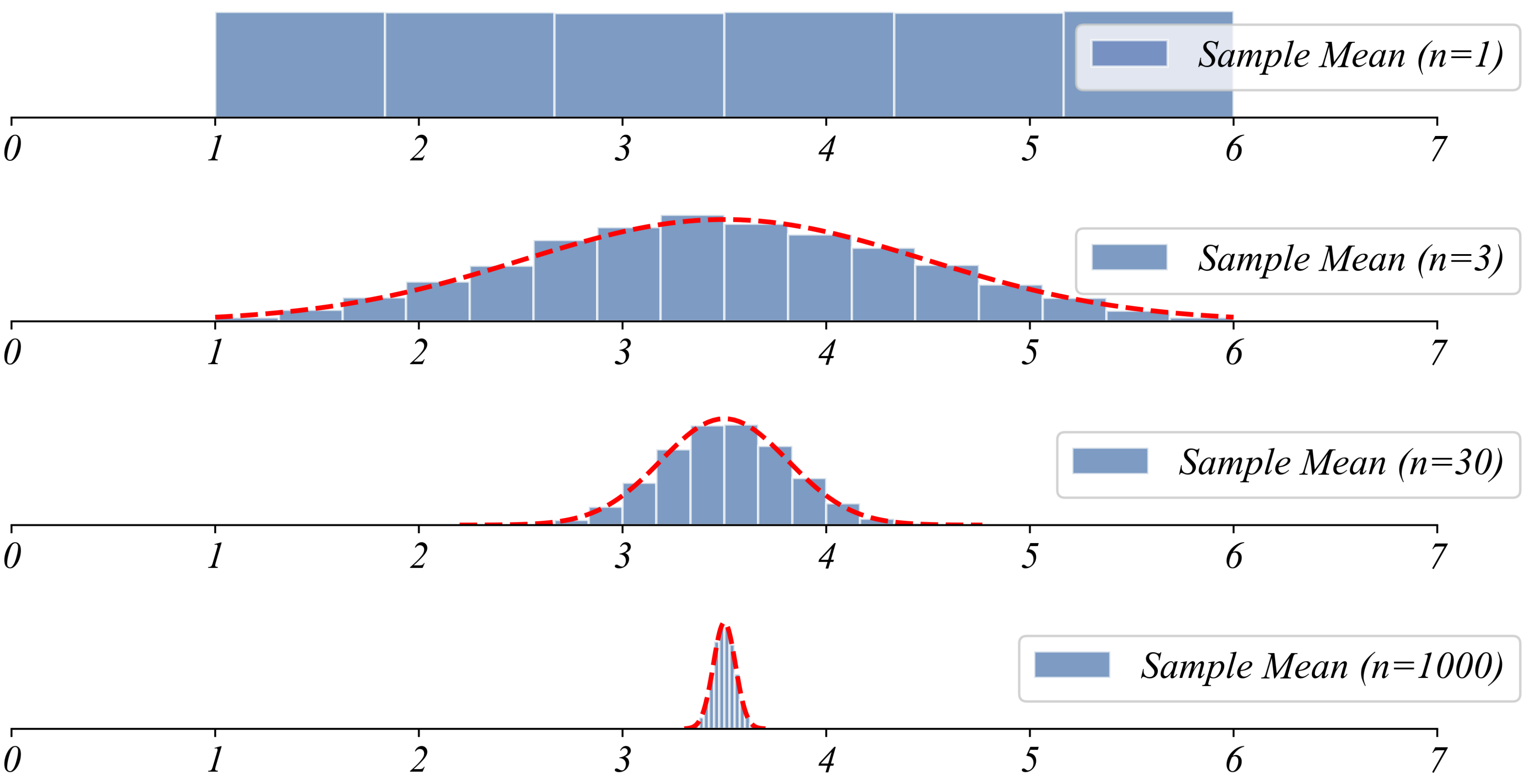
What happens when we really increase the sample size?



> the distribution of sample means gets tighter and more bell-shaped

Exercise 3.2 | Sampling Variability

What happens when we really increase the sample size?

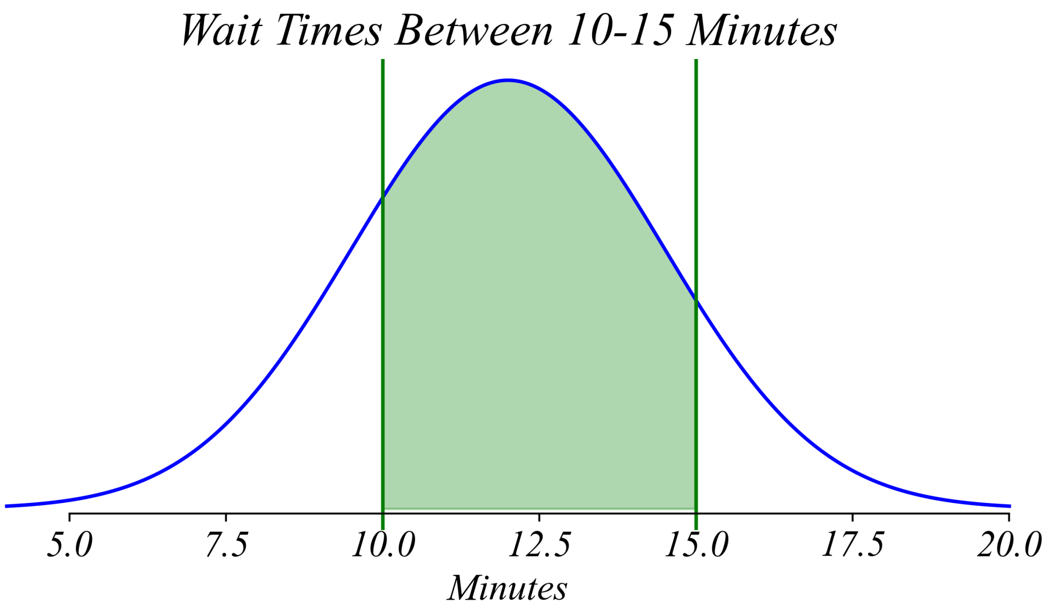


> what is this probability function in red?

Random Variables: Known

If we know the random variable, we can learn many things about the population.

- *Probability wait time < 10:*
  - $P(X < 10) = 0.21$
- *Probability wait time > 15:*
  - $P(X > 15) = 0.11$
- *Probability between 10 - 15:*
  - $P(10 < X < 15) = 0.59$

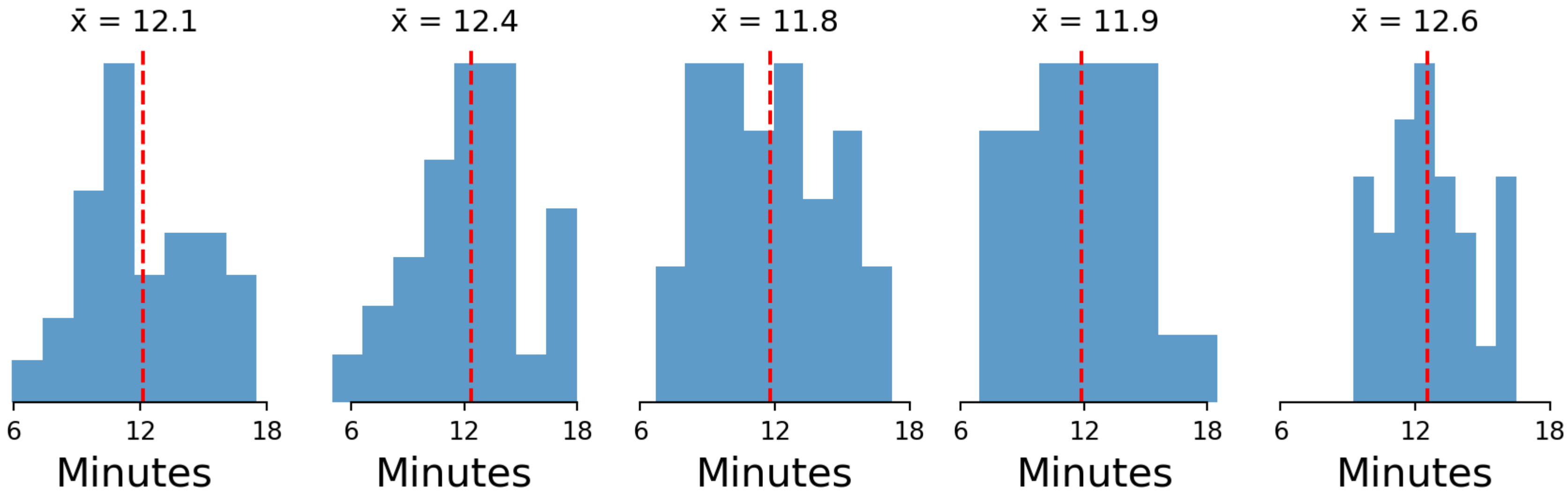


> when we know the probability function, we can calculate everything exactly

Random Variables: Unknown

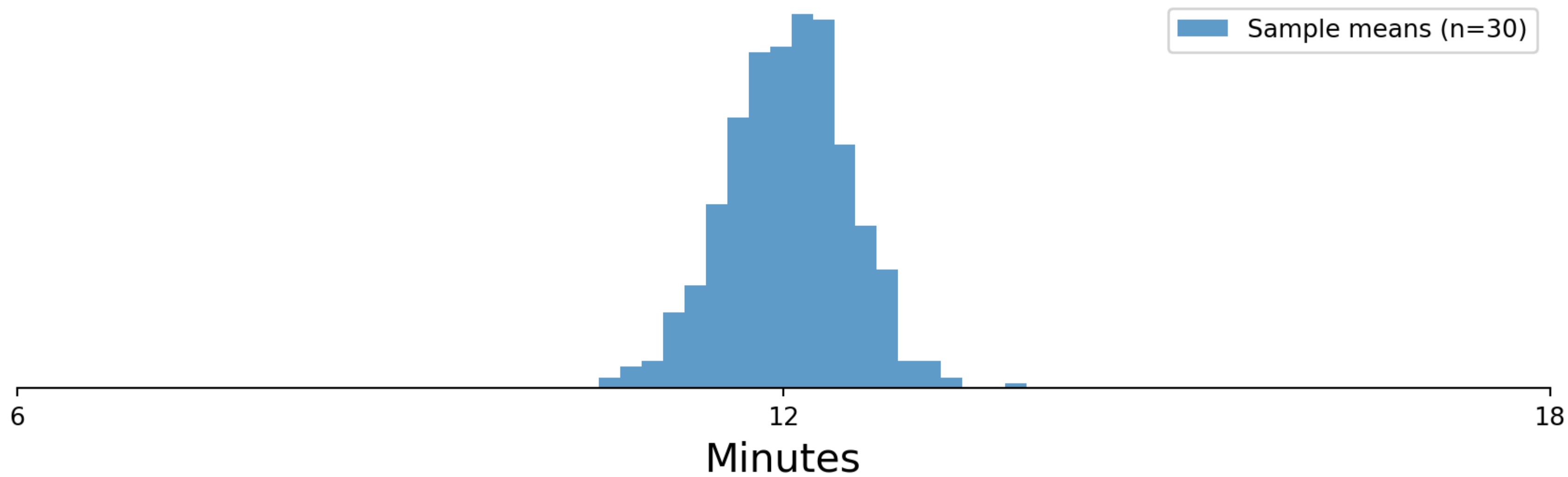
If we take multiple samples, we get different sample means.

Each sample gives us a different estimate of the population mean.



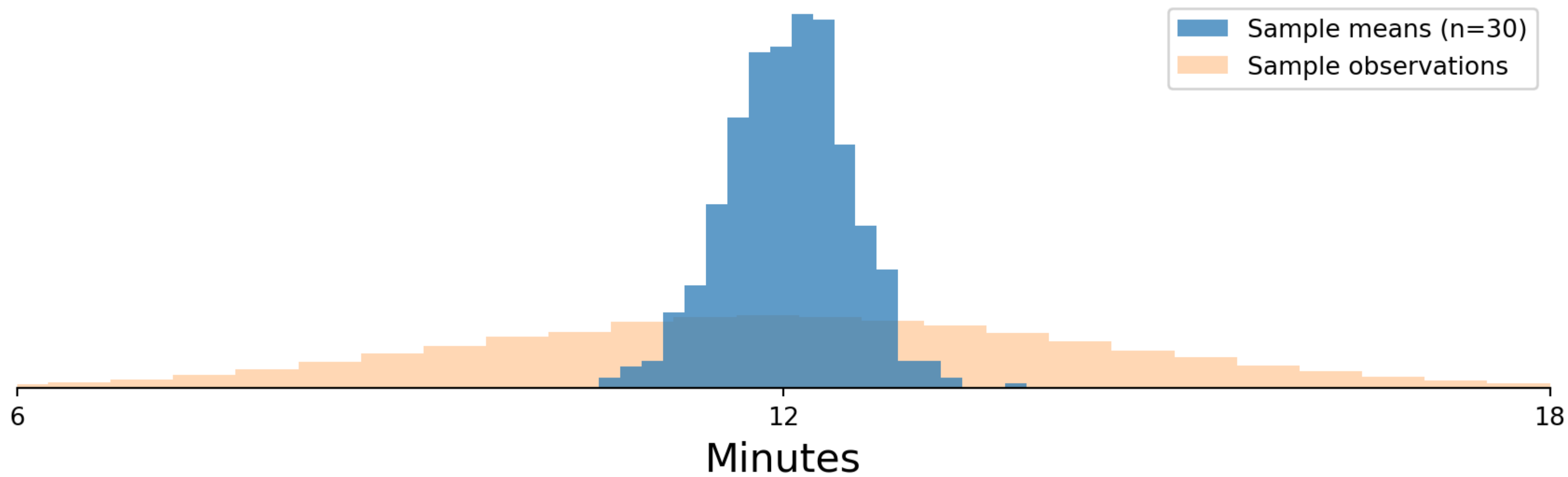
Random Variables: Unknown

If we take multiple samples, their means will vary.



Random Variables: Unknown

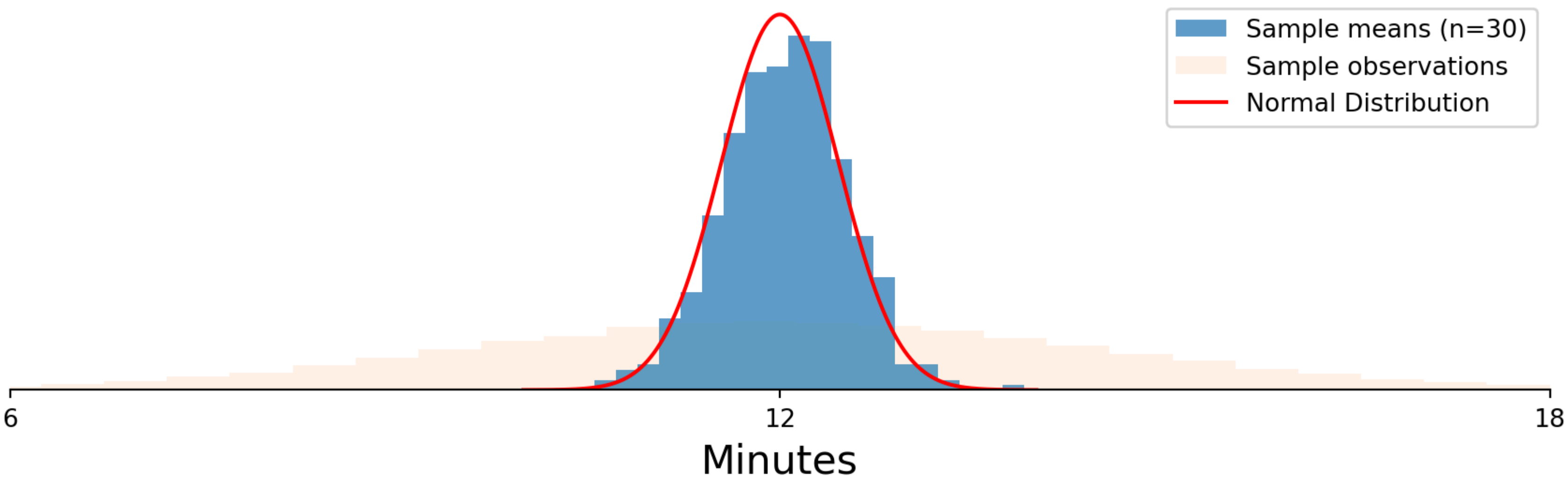
If we take multiple samples, their means will vary, and by much less than the original distribution.



> why? think about rolling two dice… it's much less likely to get a 2 than a 7

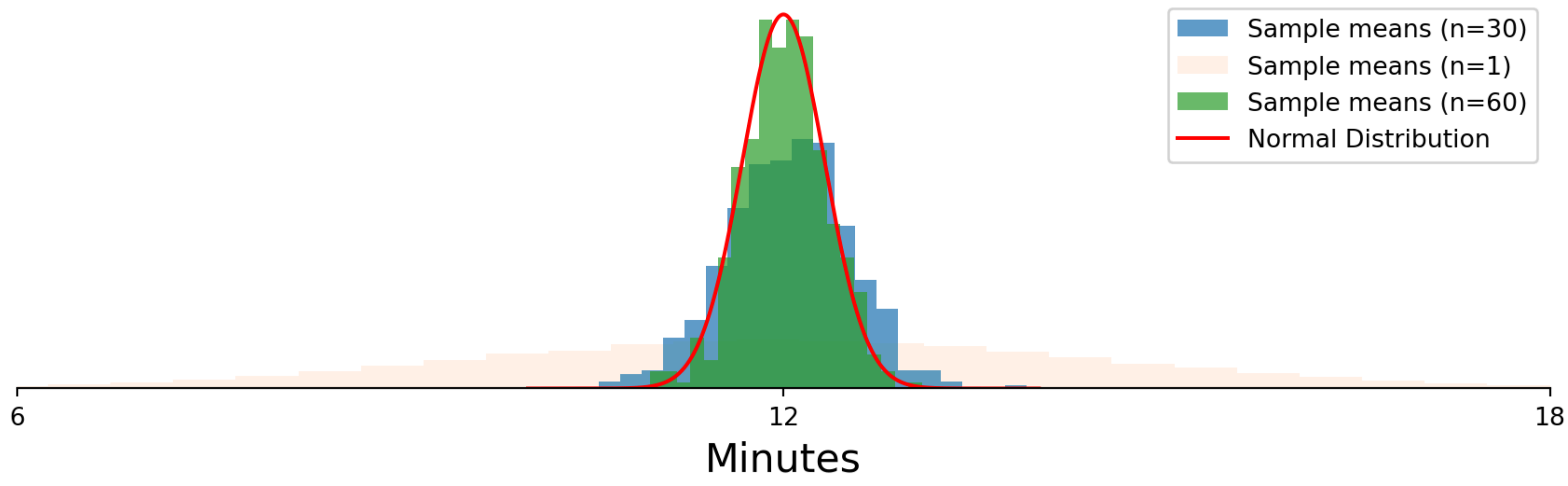
Random Variables: Unknown

As sample size grows, the distribution of the sample means approaches a normal distribution.



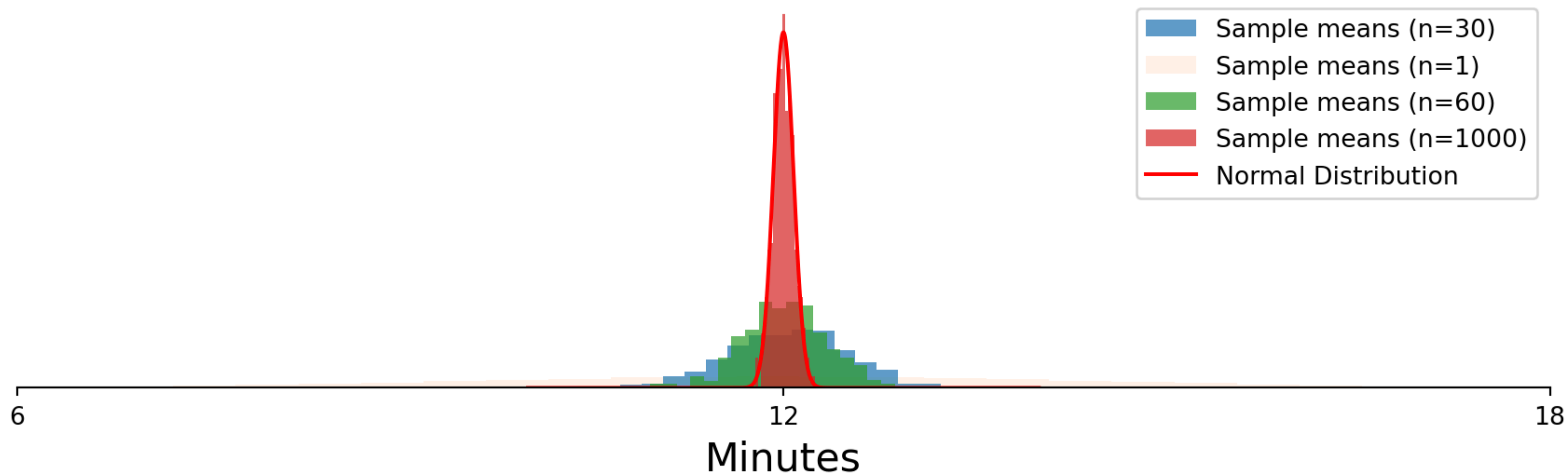
Random Variables: Unknown

As sample size grows, the normal distribution the sample means approach gets narrower.



Random Variables: Unknown

The normal distribution the sample means approach is centered on the population mean!

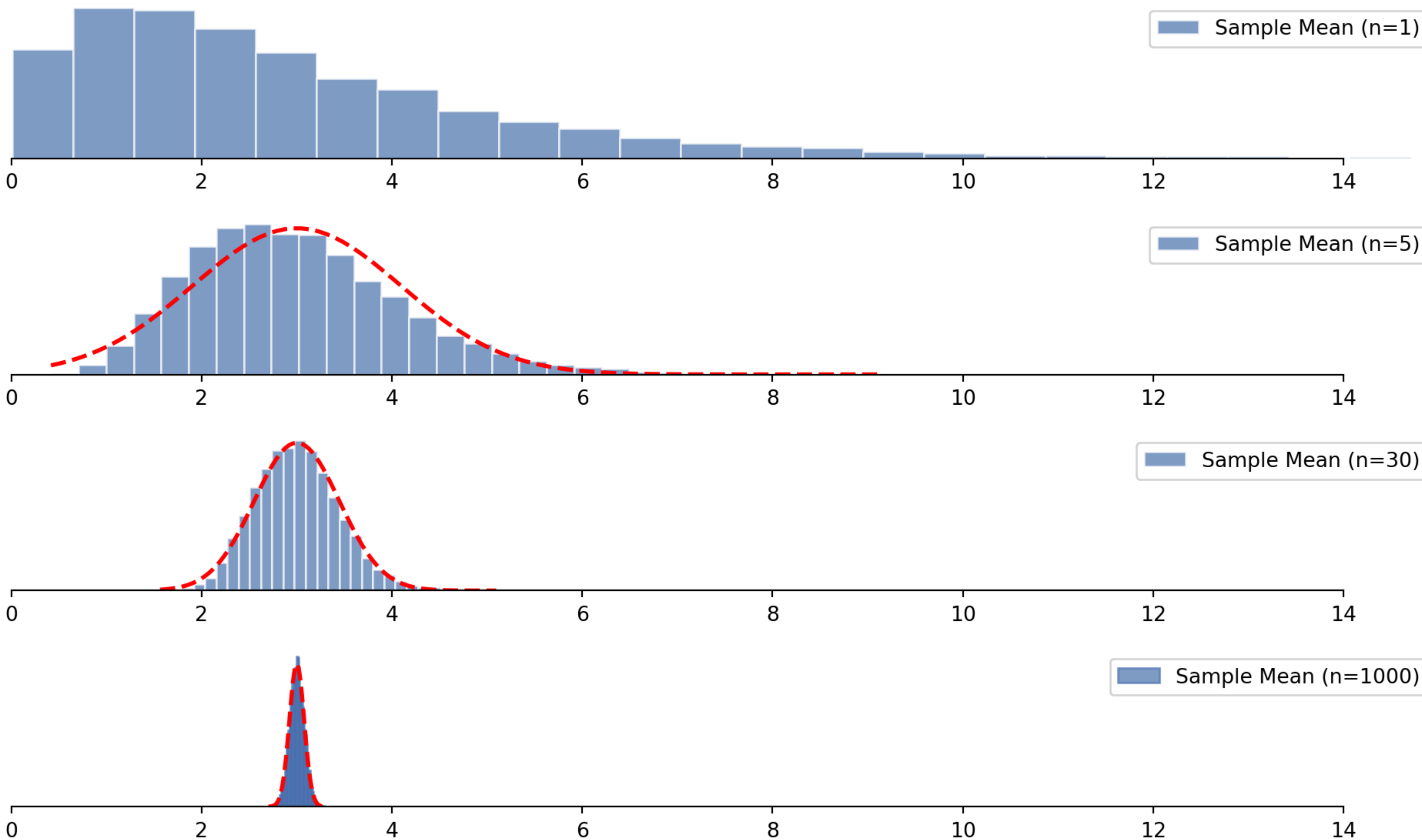


> *the sample mean $\bar{x}$ follows a normal distribution around the truth* 😱

$$\bar{x} \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

Random Variables: Unknown

This works for (nearly) any distribution shape as sample size increases.



The Central Limit Theorem

The distribution of sample means approximates a normal distribution as sample size increases, regardless of the population's distribution.

Key insights:

- *Sample means cluster around μ*
- *Standard error = $\sigma/\sqrt{n}$*
- *Normal shape emerges*

Implications:

- *We can predict the behavior of $\bar{x}$*
- *This works for (nearly) ANY distribution*