

# ECON 0150 | Economic Data Analysis

*The economist's data analysis skillset.*

## *Part 3.1 | Data vs the Population*

# Inferences From Data

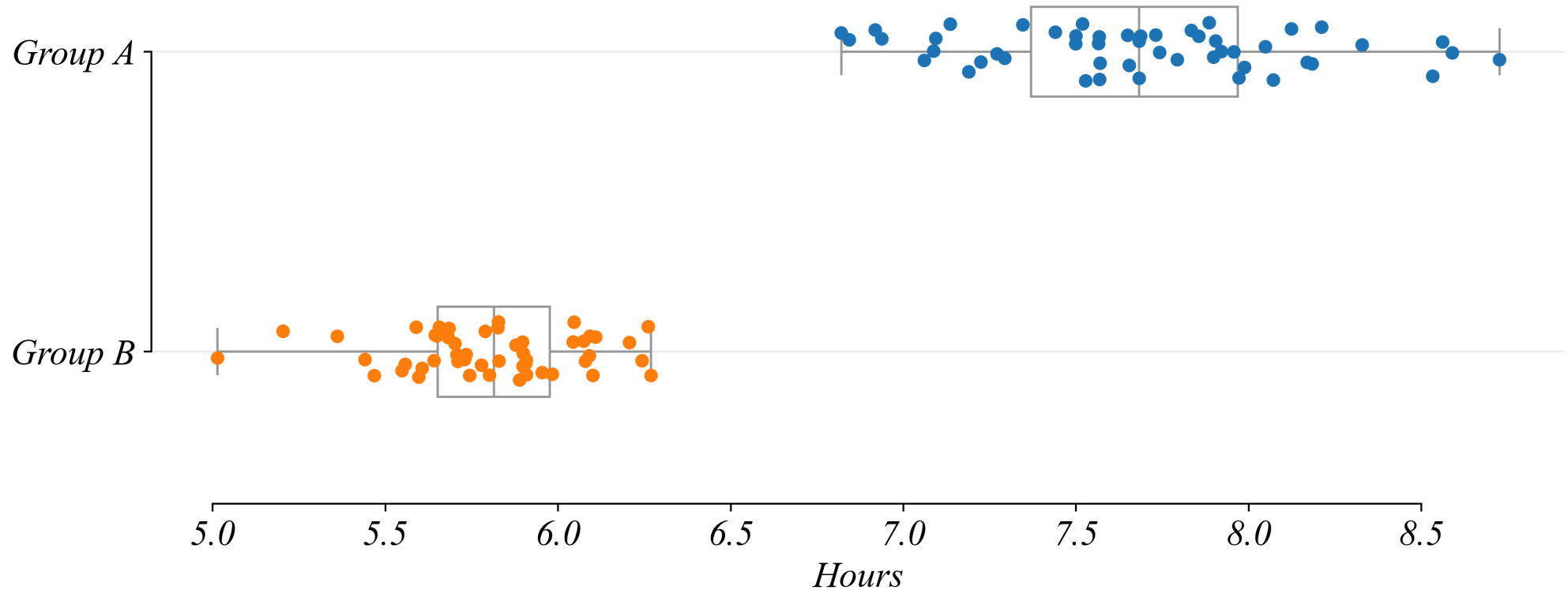
*What can we infer about those not in our data?*

- *We've mastered **summarizing** data*
- *But often we want to say something about the **population**, not just our **data***

# Data Question 1: Sleep Time in Two Samples

*Which group sleeps longer?*

*Sleep Patterns*

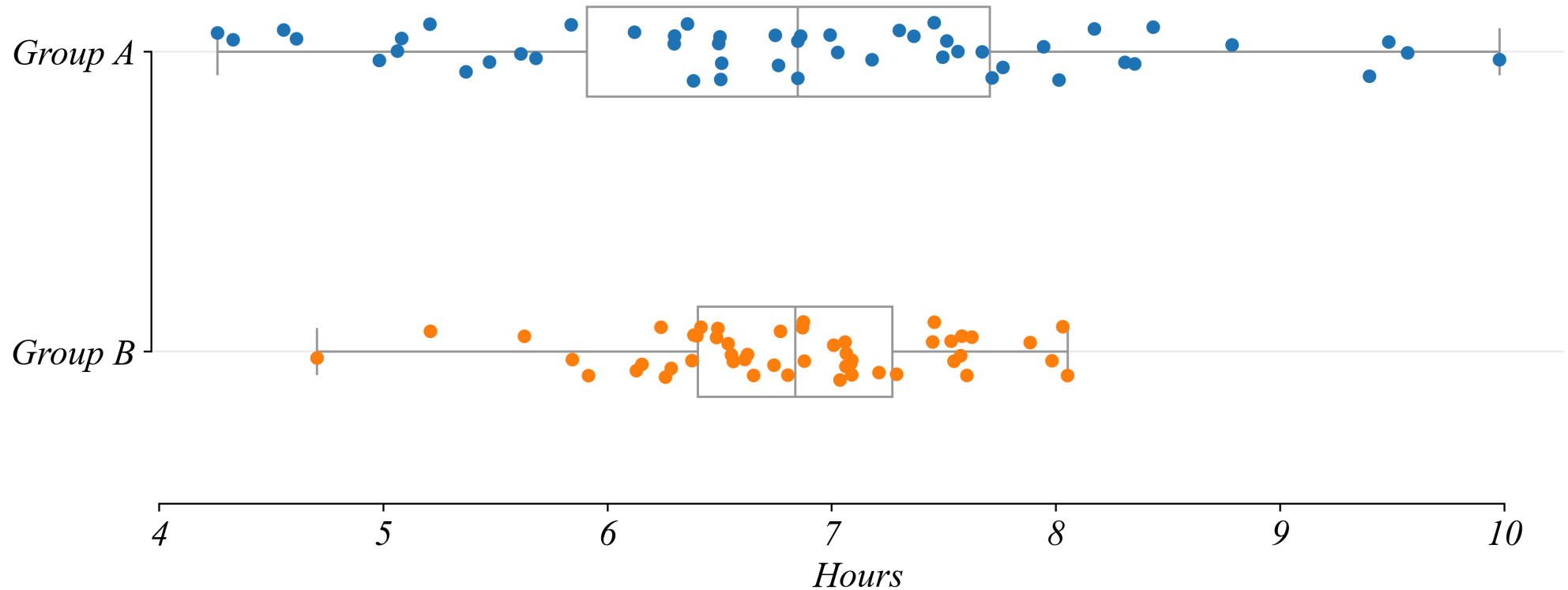


*> everyone in Group A sleeps longer than anyone in Group B*

# Data Question 2: Sleep Time in Two Samples

*Which group sleeps longer?*

*Sleep Patterns*



> *these distributions overlap... lets compare them more precisely*

# Measures of Location

*Where is the “center” of each group?*

**Mean:** The average value

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_N}{N}$$

# Measures of Location

*Where is the “center” of each group?*

**Mean:** The average value

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$$

```
1 # Calculate means
2 mean_A = group_A.mean()
3 mean_B = group_B.mean()
```

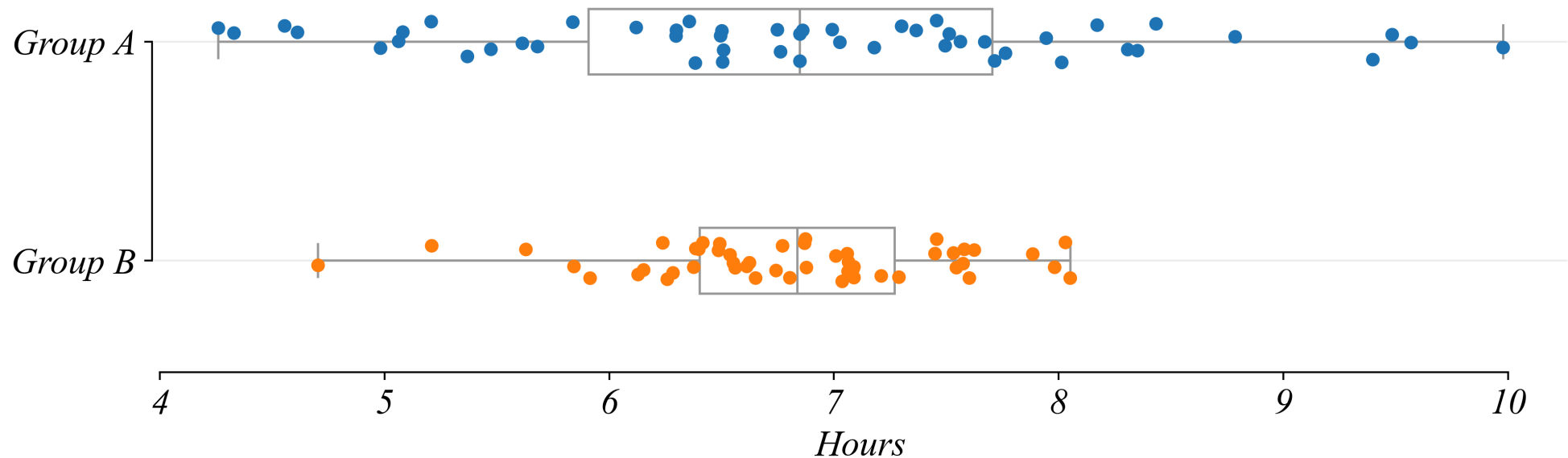
Group A mean: 7.14 hours

Group B mean: 6.98 hours

# Data Question 2: Sleep Time in Two Samples

*Which group sleeps longer?*

*Sleep Patterns*



Group A mean: 7.14 hours

Group B mean: 6.98 hours

> *group A sleeps longer on average*

> *but some in Group B sleep longer than most in Group A!*

# Measures of Dispersion

*How spread out is the data?*

**Range:** difference between the largest and smallest value in the data

- *Simple but doesn't respond to changes near the middle of the distribution*



# Measures of Dispersion

*How spread out is the data?*

**Mean Deviation:** difference between each value and the average

$$\sum \frac{x_i - \bar{x}}{n}$$

- *Simple but the average of the difference is zero...*

# Measures of Dispersion

*How spread out is the data?*

**Mean Absolute Deviation:** absolute value of the difference from the average

$$\sum \frac{|x_i - \bar{x}|}{n}$$

- *The mean isn't zero*
- *A little more complex and isn't so nice mathematically*

# Measures of Dispersion

*How spread out is the data?*

**Variance:** average squared difference from the mean

$$Var_X = \sum \frac{(x_i - \bar{x})^2}{n}$$

- *Treats negatives appropriately*
- *The mean isn't zero*
- *Mathematically nice*
- *Units are uninformative*

# Measures of Dispersion

*How spread out is the data?*

**Standard Deviation:** A measure of spread

$$S_X = \sqrt{\sum \frac{(x_i - \bar{x})^2}{n}}$$

- *Treats negatives appropriately*
- *The mean isn't zero*
- *Mathematically nice*
- *Units are roughly average deviation from the mean*

# Measures of Dispersion

*How spread out is the data?*

**Standard Deviation:** A measure of spread

$$S_X = \sqrt{\sum \frac{(x_i - \bar{x})^2}{n}}$$

```
1 # Calculate standard deviations
2 std_A = group_A.std()
3 std_B = group_B.std()
```

Group A std dev: 1.50 hours

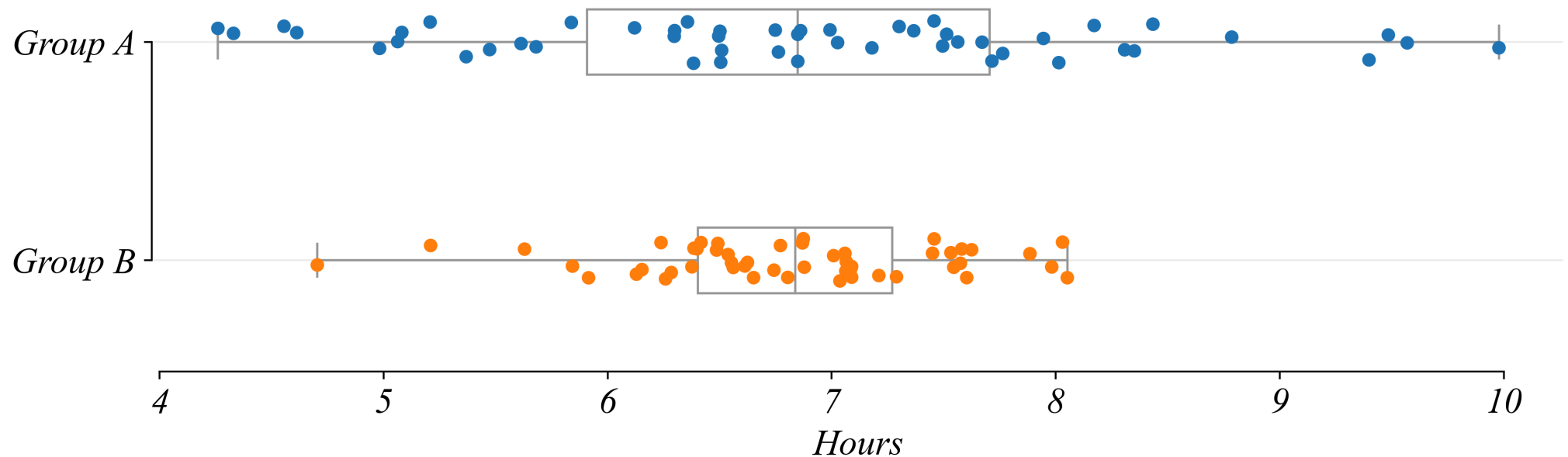
Group B std dev: 0.78 hours

> *Group A has **more variability** - some sleep much less, some much more*

# Sample vs Population

*Both groups are 50 people selected from two different counties.*

## *Sleep Patterns*



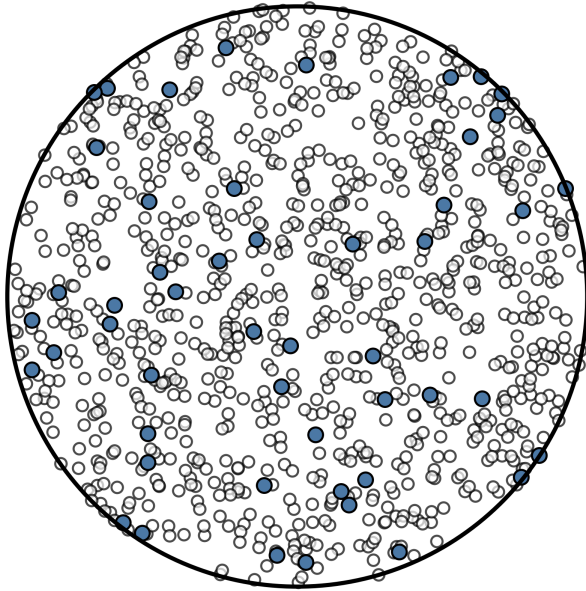
Old question: “Which **group** sleeps longer?” (*about the **data***)

New question: “Which **county** sleeps longer?” (*about the **population***)

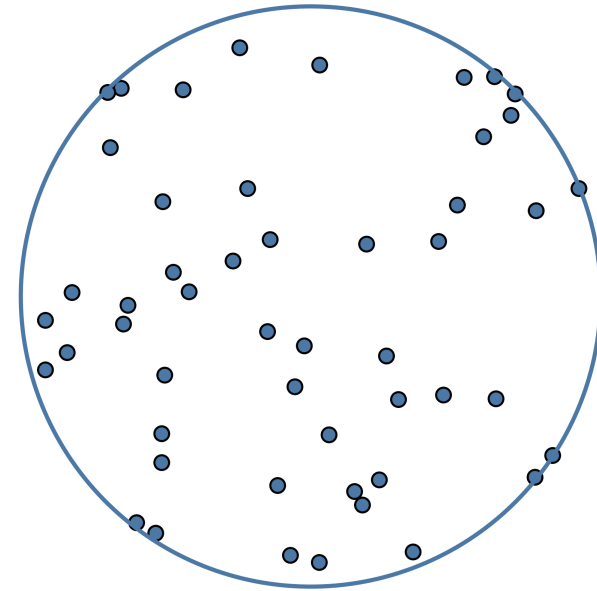
# Sample vs Population

*The data is a sample drawn from a population.*

*Population ( $\mu=?; \sigma=?$ )*



*Sample ( $n = 50; \bar{x}; S$ )*



# Sample vs Population

*We observe **samples** but want to understand **populations**.*

- **Data:** *50 individuals we happened to sample from both counties*
- **Population:** *All people who could live in these counties*
  - *Even if we surveyed everyone today, tomorrow would bring new residents*
  - *The population is a theoretical concept - an infinite pool of possibilities*



# Sample vs Population

*What is data? A sample.*

**Random Variable:** a random process about a population

- *the random variable is like a deck of cards*

**Probability (Mass/Density) Function:** a function that assigns probabilities to each possible outcome

- *the probability function is like which cards are in the deck*

**Observation:** a realization of a random variable . . .

- *the observation is the card you drew*

**Sample:** a collection of observations

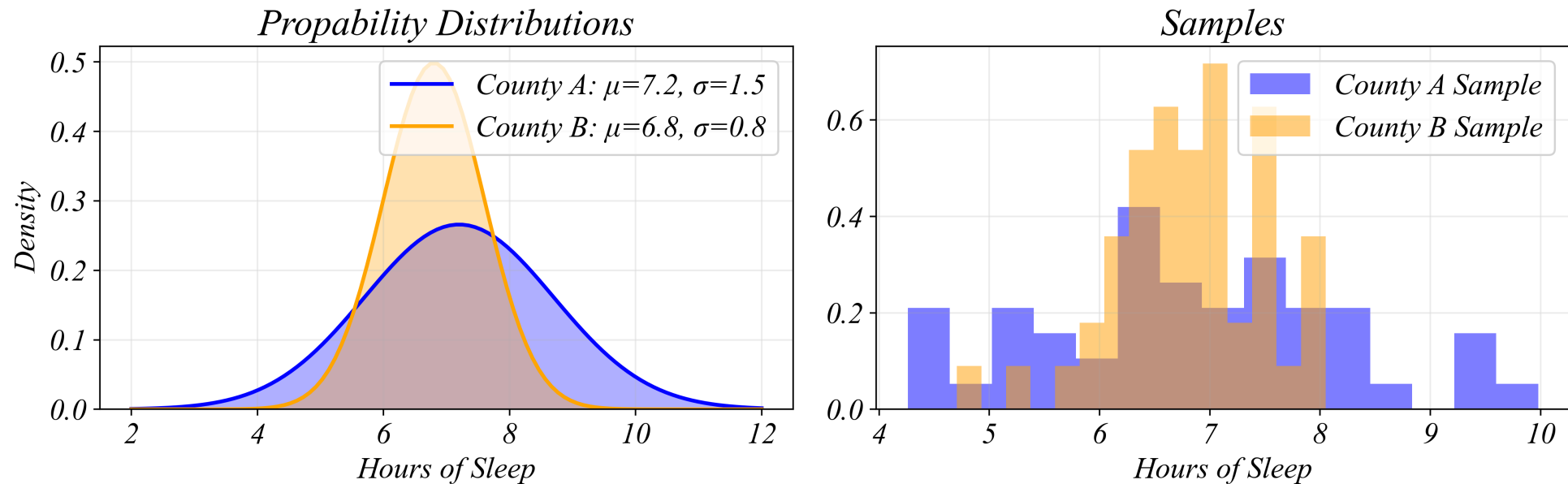
- *the sample is the record of cards you've drawn*

# Data is a Sample

*A random variable generates our data.*

**Random Variable:** a random process about a population

**Probability Function:** a function that assigns probabilities to each possibility

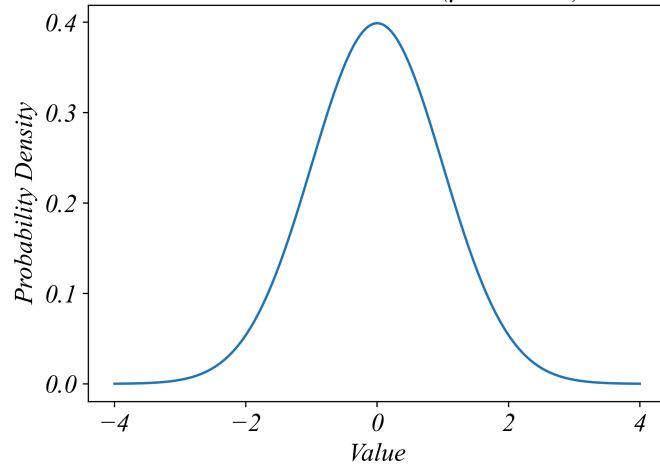


> *data is a sample drawn from a random variable*

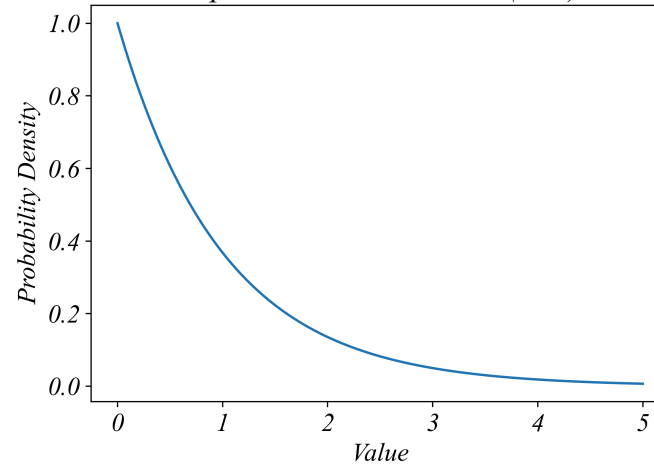
# Probability Functions

*Random variables can have many kinds of probability functions.*

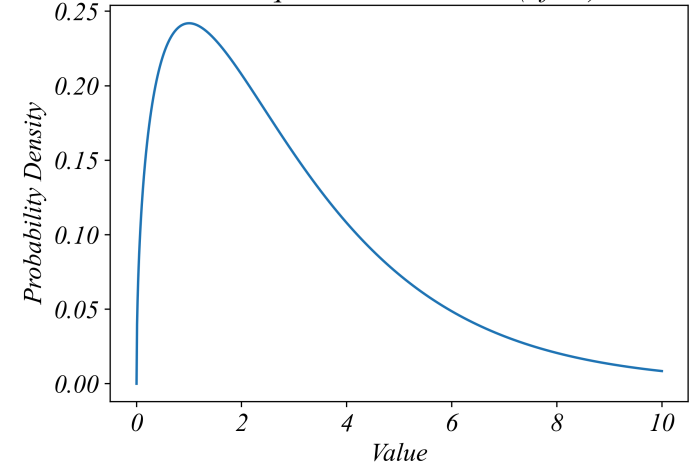
*Normal Distribution ( $\mu=0, \sigma=1$ )*



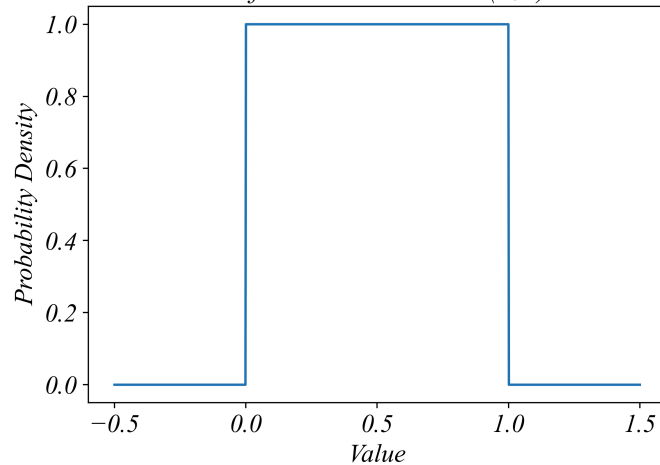
*Exponential Distribution ( $\lambda=1$ )*



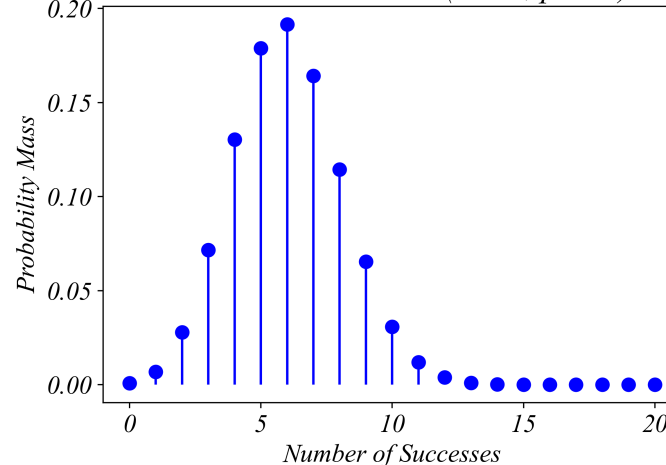
*Chi-Square Distribution ( $df=3$ )*



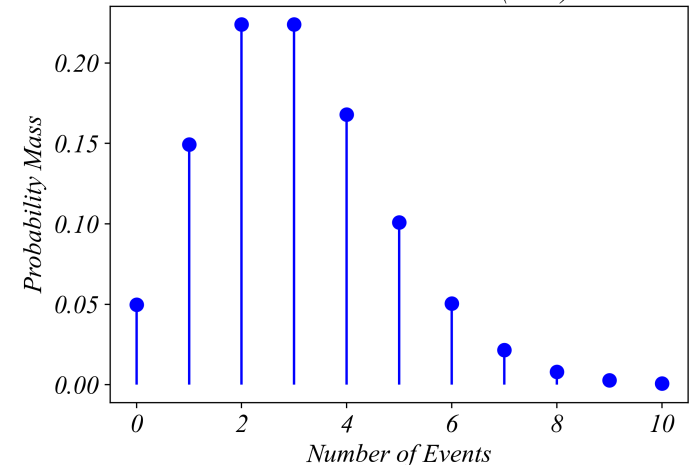
*Uniform Distribution (0,1)*



*Binomial Distribution ( $n=20, p=0.3$ )*



*Poisson Distribution ( $\lambda=3$ )*



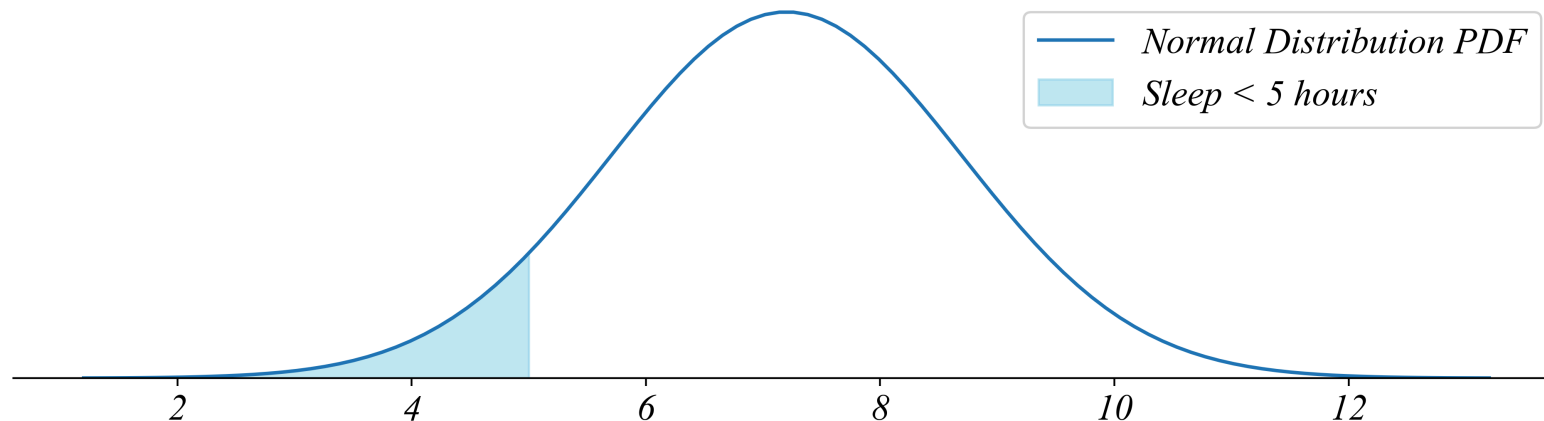
# Exercise 3.1 | Known Distribution

*We can answer **many** kinds of probability questions when we know the distribution.*

County A's probability function:

$$x_i \sim N(\mu = 7.2, \sigma = 1.5)$$

*1. What proportion of the population sleeps less than 5 hours?*



```
1 stats.norm.cdf(5, loc=mu, scale=sigma).item()
```

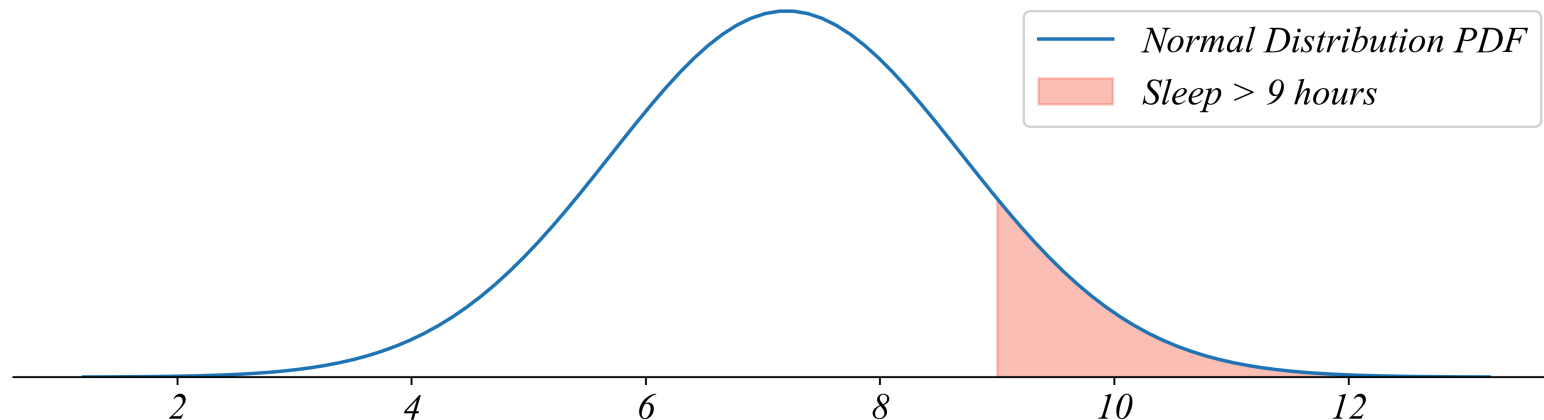
# Exercise 3.1 | Known Distribution

*We can answer **many** kinds of probability questions when we know the distribution.*

County A's probability function:

$$x_i \sim N(\mu = 7.2, \sigma = 1.5)$$

2. *What proportion of the population sleeps more than 9 hours?*



```
1 1 - stats.norm.cdf(9, loc=mu, scale=sigma).item()
```

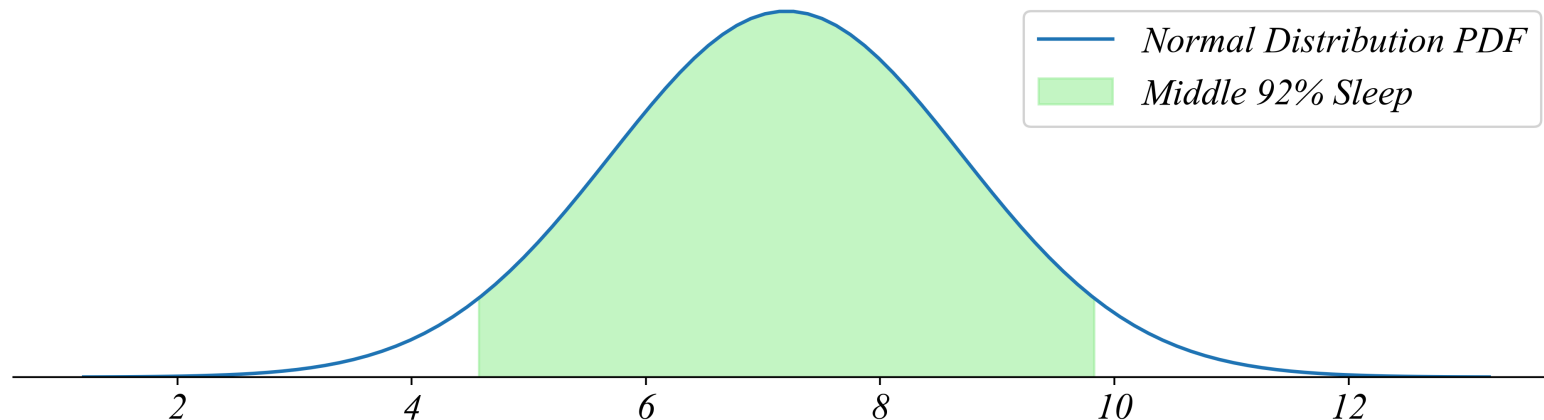
# Exercise 3.1 | Known Distribution

*We can answer **many** kinds of probability questions when we know the distribution.*

County A's probability function:

$$x_i \sim N(\mu = 7.2, \sigma = 1.5)$$

*3. How much sleep does the middle 92% of the population get?*



```
1 lower_bound = stats.norm.ppf(0.04, loc=mu, scale=sigma)
2 upper_bound = stats.norm.ppf(0.96, loc=mu, scale=sigma)
```

# Unknown Distributions

*What can we say about an unknown population if all we see is the sample?*

## What we observe:

- *Sample size:  $n = 50$*
- *Sample mean:  $\bar{x} = 7.24$  hours*
- *Sample standard deviation:  $s = 1.48$  hours*

## What we want to know:

- *Population mean:  $\mu = ?$*
- *Population standard deviation:  $\sigma = ?$*
- *Population distribution:  $f(x) = ?$*

# Unknown Distributions

*What can we say about an unknown population if all we see is the sample?*

The sample statistics  $(\bar{x}, S)$  are **not** the population parameters  $(\mu, \sigma)$ .

$$\bar{x} \neq \mu$$

$$S \neq \sigma$$



# The Central Question

*What can we say about an unknown population if all we see is the sample?*

- *Part 3.2 | **Central Limit Theorem** - the distribution of the sample mean*
- *Part 3.3 | **Confidence Intervals** - the closeness of the sample mean to the truth*
- *Part 3.4 | **Hypothesis Testing** - the probability we are wrong*

*> we can answer questions about an unknown population using just a sample*