ECON 0150 | Fall 2025 | Homework 3.2

Due: Friday, March 6, 5PM

Homework is designed to both test your knowledge and challenge you to apply familiar concepts in new applications. Answer clearly and completely. You are welcomed and encouraged to work in groups so long as your work is your own. Submit your figures and answers to Gradescope.

Q1. The Central Limit Theorem with a Normal Population

The wait times (in minutes) at a restaurant follow a normal distribution with mean (μ) = 12 minutes and standard deviation (σ) = 2.5 minutes. In this question you will explore what happens to the distribution of **sample means** as the sample size increases.

a) Take 1,000 samples of size $n = 5$ from this distribution. Compute the mean of each sample and plot a histogram of the 1,000 sample means.

```
sample_means_5 = [np.mean(np.random.normal(loc=12, scale=2.5, size=5)) for _ in
range(1000)]
```

b) Repeat part (a) for sample sizes $n = 30$ and $n = 100$. Plot all three histograms. Describe how the distribution of sample means changes as n increases.

c) Compute the standard deviation of the sample means for each sample size ($n = 5$, $n = 30$, $n = 100$).

d) Compute the theoretical standard error $\sigma/\sqrt{n}$ for each sample size. Compare these values to the standard deviations you computed in part (c).

e) In your own words, why does the spread of the sampling distribution decrease as n increases?

Q2. The Central Limit Theorem with a Skewed Population

Now consider a different scenario. Wait times at a busy food truck follow an **exponential distribution** with a mean of 5 minutes. This distribution is skewed right — most waits are short, but some are very long. Use `np.random.exponential(scale=5, size=n)` to draw from this population.

a) Draw 10,000 observations from this population and plot the histogram. Describe the shape — is it normal?

b) Take 1,000 samples of size $n = 5$ from this population. Compute the mean of each sample and plot a histogram of the sample means.

c) Repeat part (b) for $n = 30$. Plot the histogram and describe how the shape changed compared to $n = 5$.

d) For both $n = 5$ and $n = 30$, compare the standard deviation of the sample means to the theoretical standard error $\sigma/\sqrt{n}$. For the exponential distribution, the population standard deviation equals the mean, so $\sigma = 5$.

e) In your own words, what does the Central Limit Theorem guarantee about the shape of the sampling distribution, regardless of the population shape?

Q3. The Central Limit Theorem with a Bimodal Population

A coffee shop has two types of customers: those who order just a drink (mean spending $\mu_1 = 4$ dollars, $\sigma_1 = 1$) and those who order a drink plus food (mean spending $\mu_2 = 12$ dollars, $\sigma_2 = 1.5$). About half of customers fall into each group. The function below draws `n` observations from this bimodal population.

```
def bimodal_sample(n):
    group = np.random.choice([0, 1], size=n)
    return np.where(group == 0, np.random.normal(4, 1, n), np.random.normal(12, 1.5, n))
```

a) Draw 10,000 observations from this population and plot the histogram. Describe the shape — how many peaks does it have?

b) Take 1,000 samples of size $n = 5$ from this population. Compute the mean of each sample and plot a histogram of the sample means.

c) Repeat part (b) for $n = 30$. Plot the histogram and describe how the shape changed compared to the population and to $n = 5$.

d) Compute the standard deviation of your 10,000 population observations from part (a). Use this as σ. Then compare the standard deviation of your sample means (for $n = 5$ and $n = 30$) to the theoretical standard error $\sigma/\sqrt{n}$
.

e) Across all three questions, you started with a normal population, a skewed population, and a bimodal population. What happened to the sampling distribution of the mean in each case as n increased? What does this tell you about the generality of the Central Limit Theorem?