

ECON 0150 | Economic Data Analysis

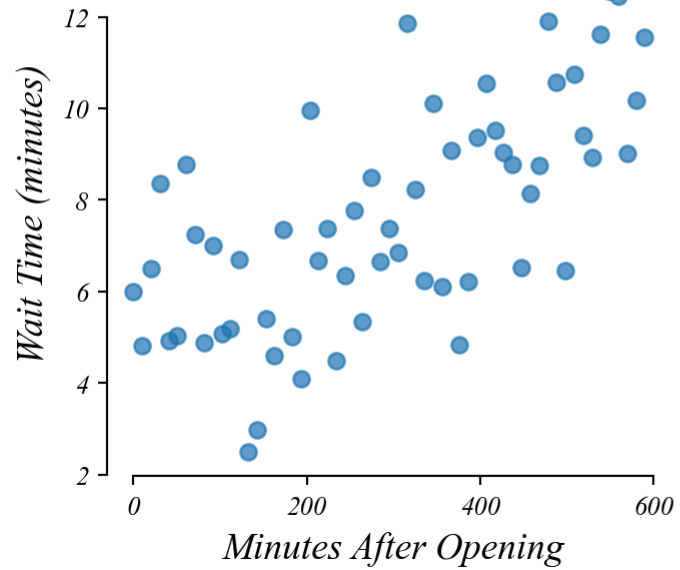
The economist's data analysis stillset.

Part 4.1 | Numerical Predictors

GLM: bivariate data

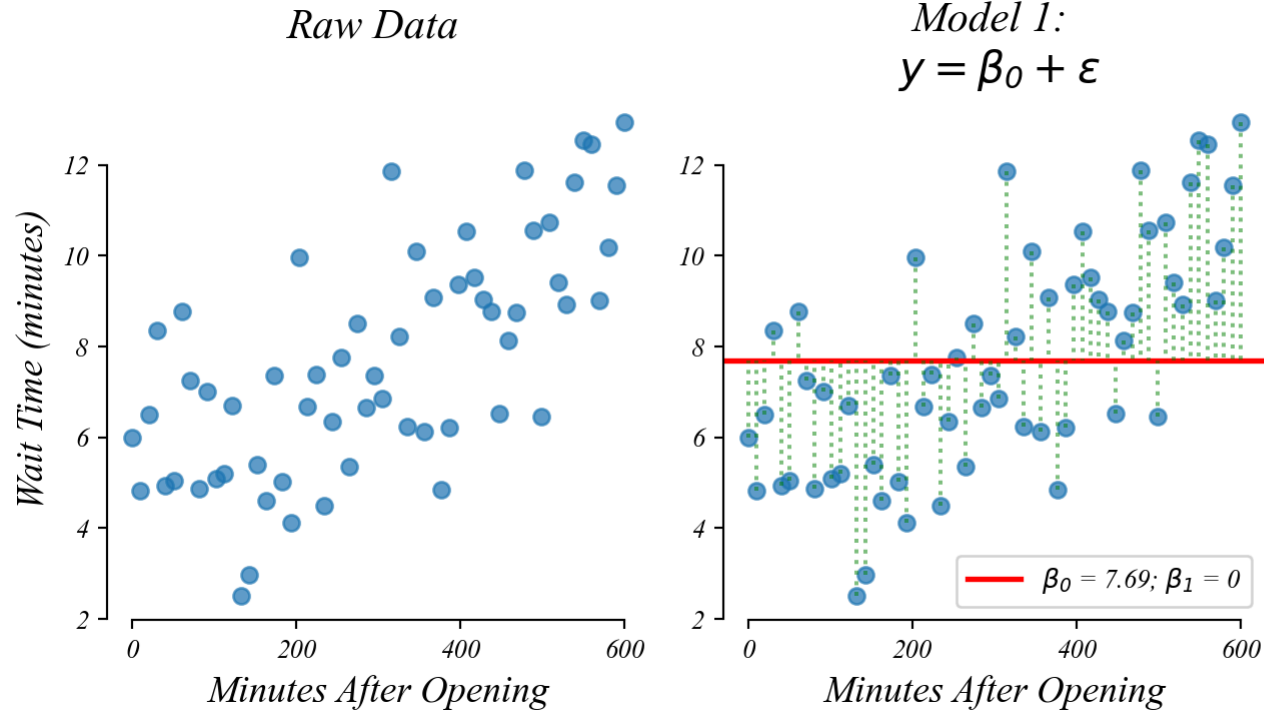
Do people wait longer later in the day?

Raw Data



GLM: bivariate data

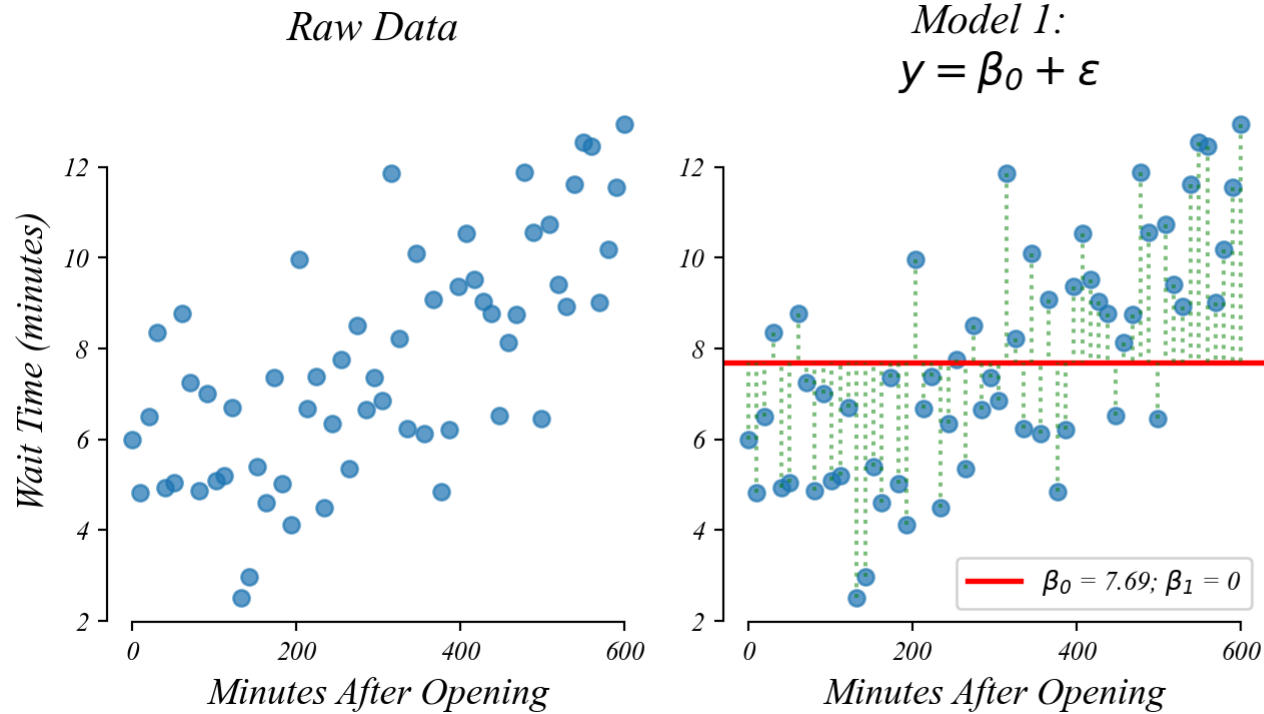
Do people wait longer later in the day?



> *but in general we don't ask many questions about vertical intercepts*

GLM: bivariate data

Do people wait longer later in the day?

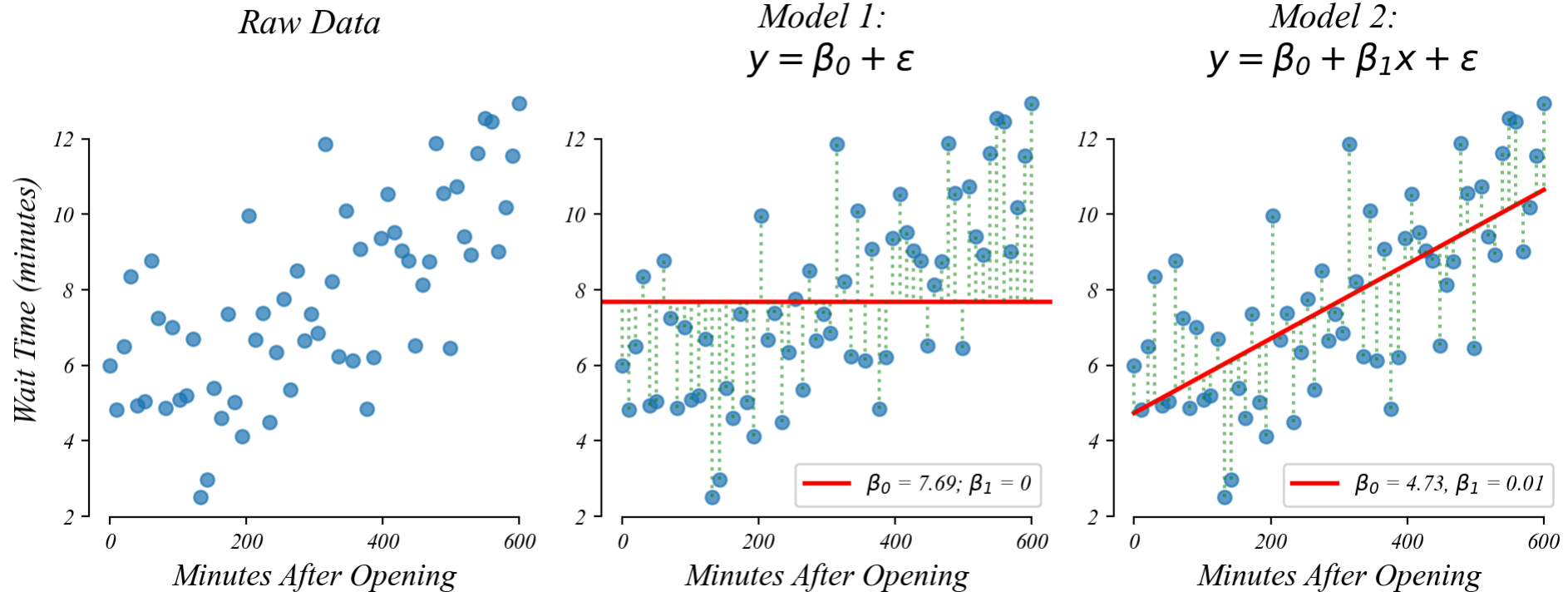


Lets compare two models.

- *Model 1 (Intercept Only):* $y = b$
- *Model 2 (Intercept+Slope):* $y = mx + b$

GLM: bivariate data

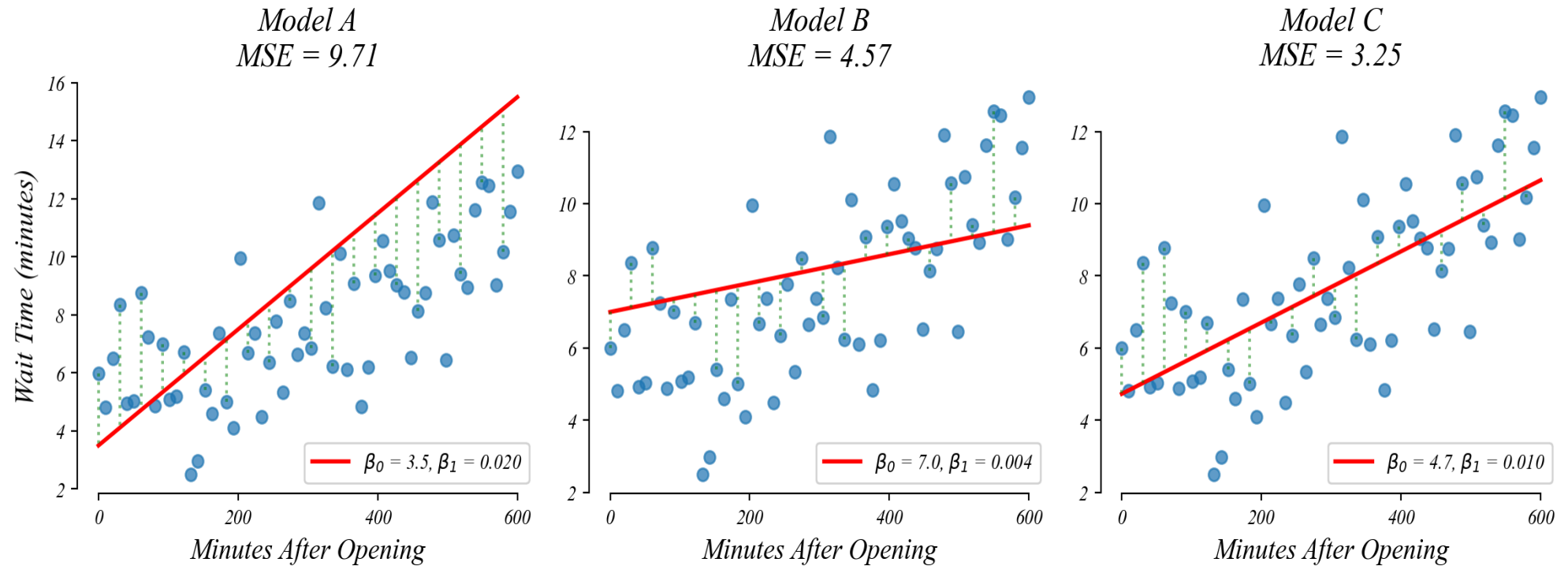
Do people wait longer later in the day?



- > a slope (β_1) improves model fit (MSE; 'wrongness') when there's a relationship
- > the intercept is no longer the mean

Bivariate GLM: minimizing MSE

Which model minimizes the models' 'wrongness' (Mean Squared Error)?

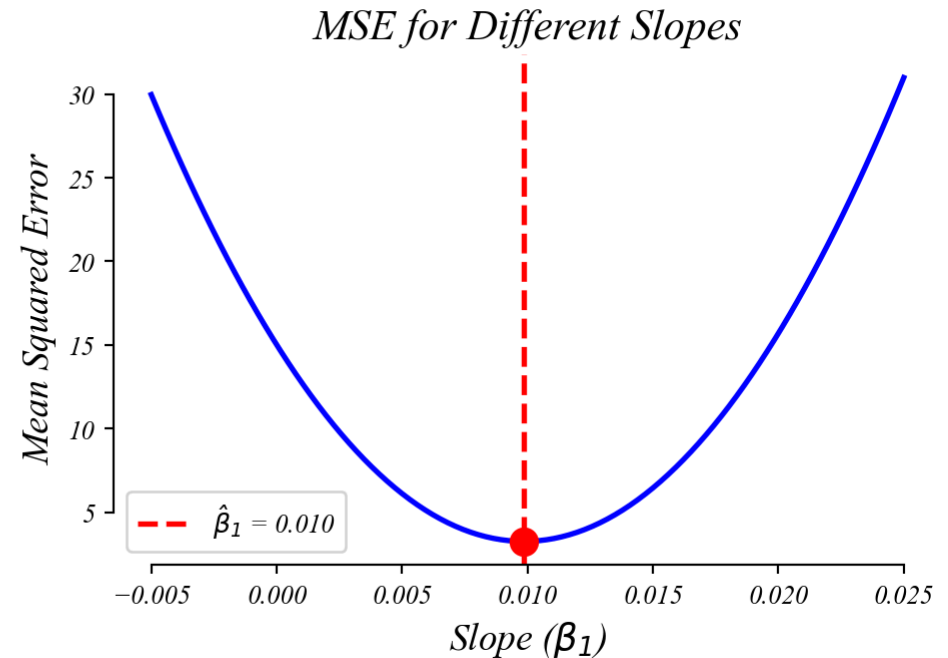
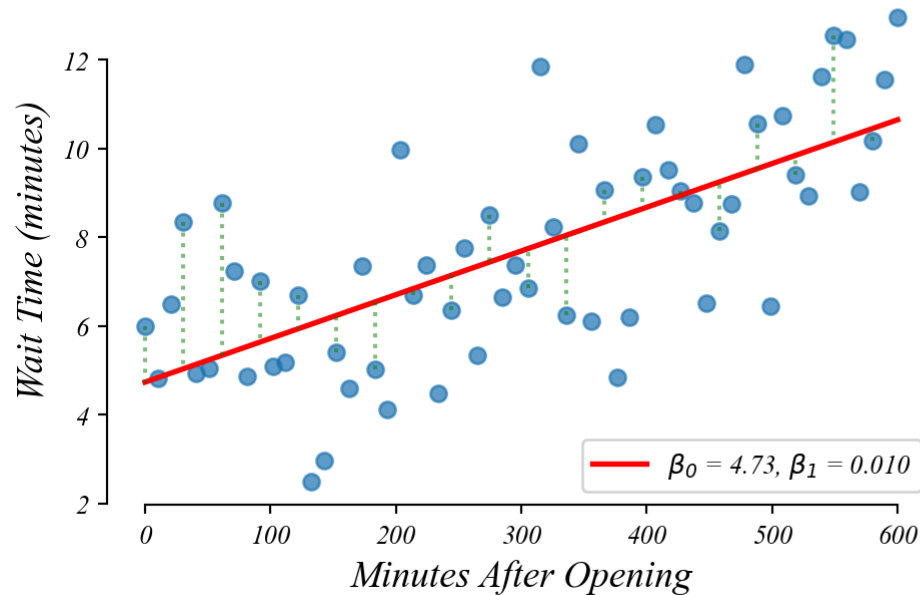


> *Model C minimizes MSE!*

Bivariate GLM: minimizing MSE

GLM selects the β_1 with the smallest MSE.

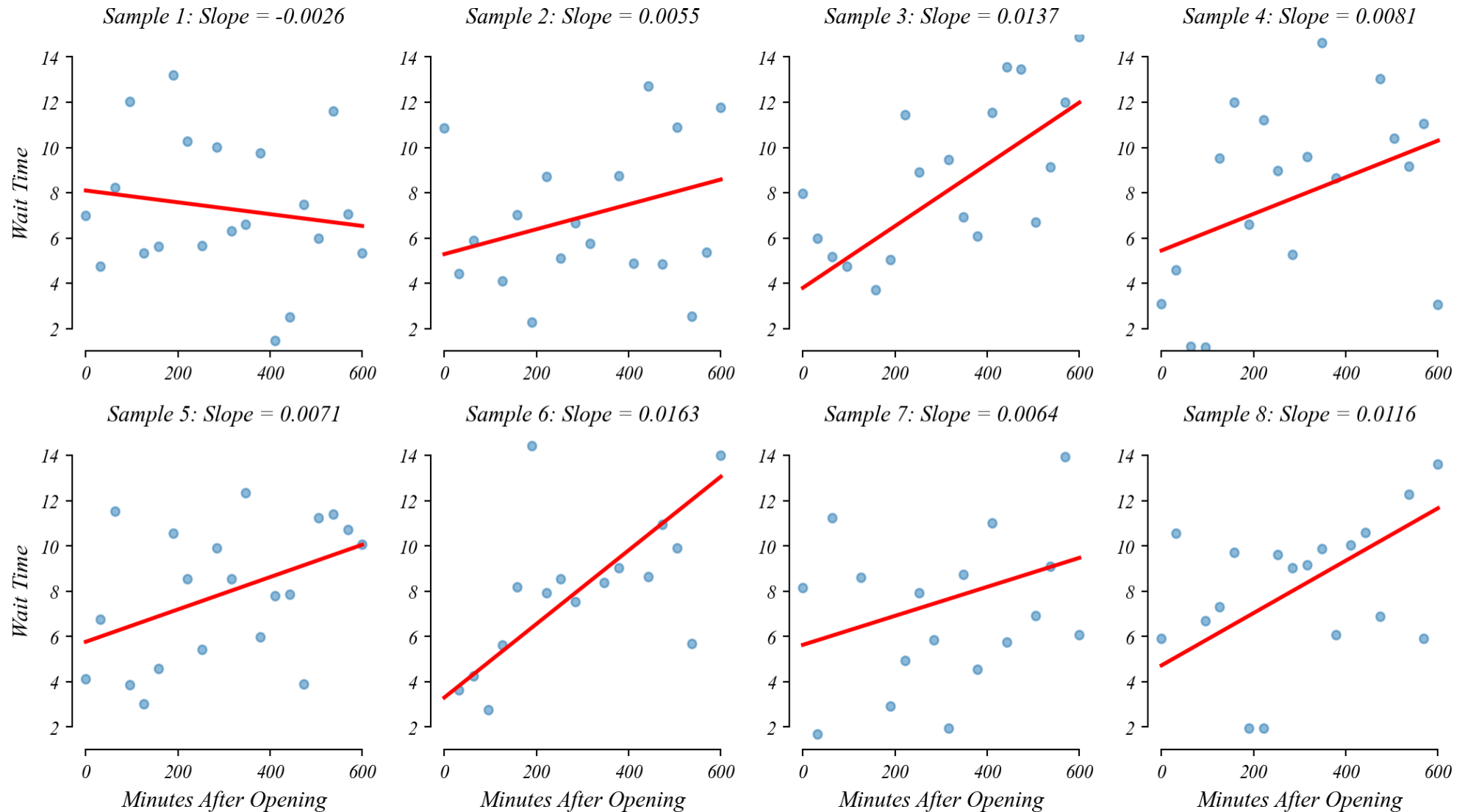
Model C:
 $y = \beta_0 + \beta_1 x + \varepsilon$



- > *this slope (β_1) gives the best guess of the relationship between x and y*
- > *but what if the true slope is zero ... could this slope be just sampling error?*

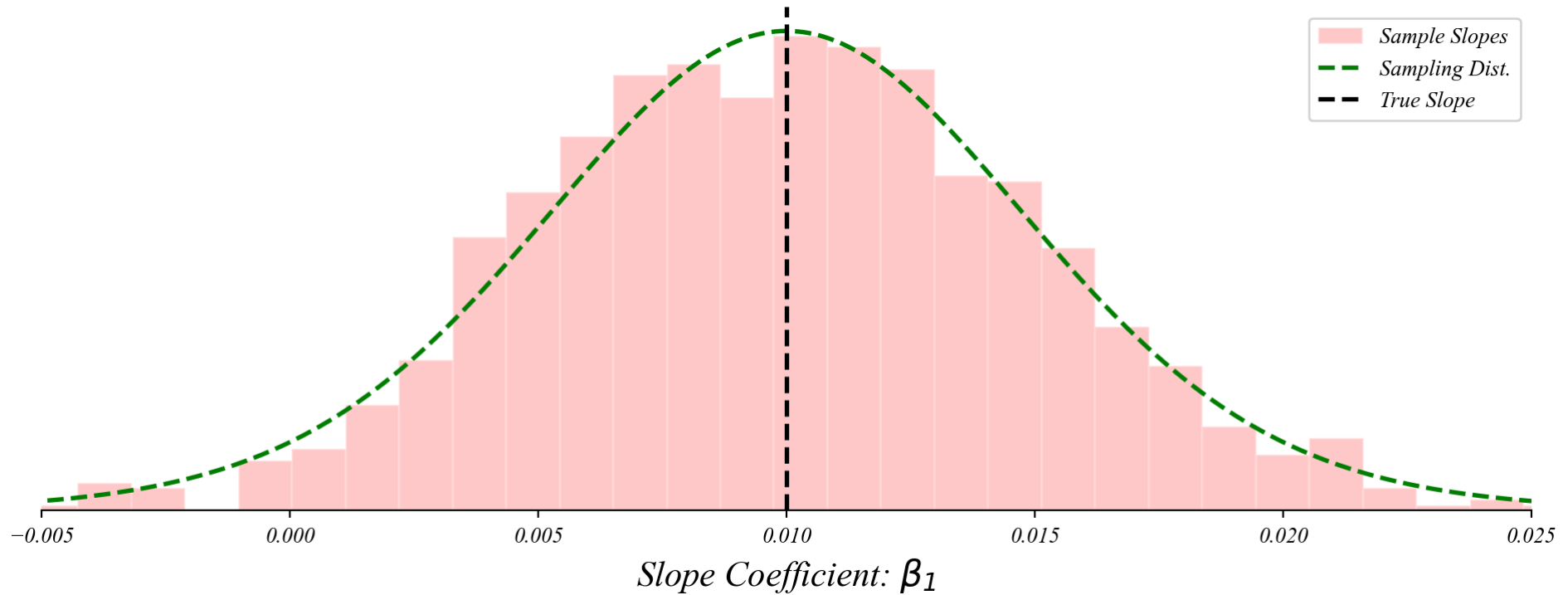
Bivariate GLM: sampling error

Like before, if we take many samples, we get slightly different slopes and slightly different fits.



Bivariate GLM: sampling distribution of slopes

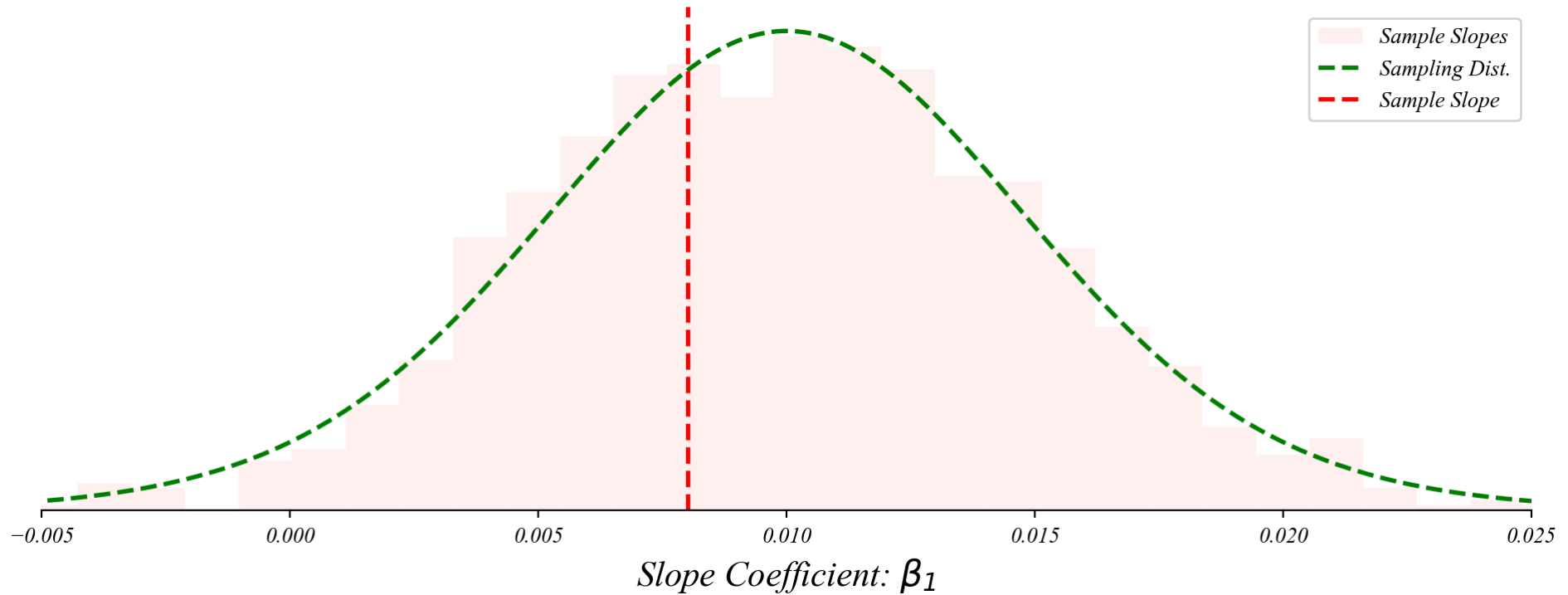
The slope coefficient follows a normal distribution centered on the population slope.



- > *the slopes follow a normal distribution around the population relationship!*
- > *this lets us perform a t-test on the slope!*

Bivariate GLM: sampling distribution of slopes

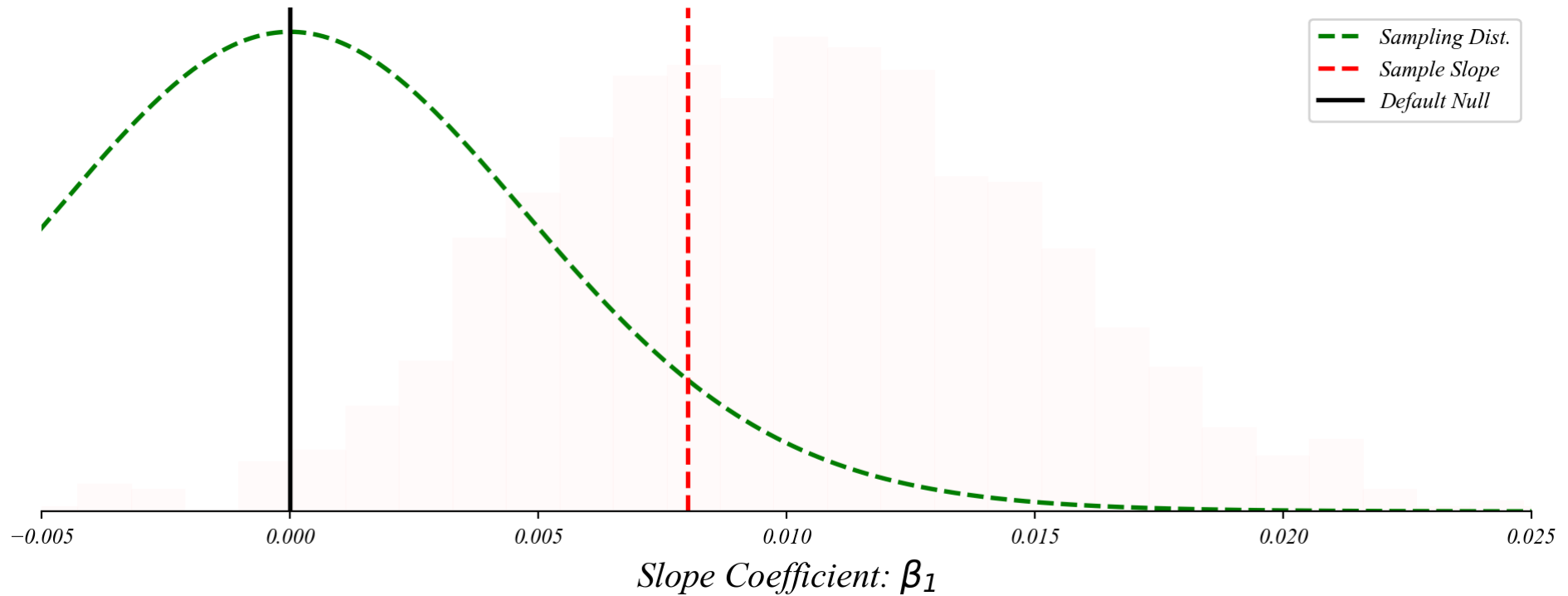
The slope coefficient follows a normal distribution centered on the population slope.



> *we don't know the entire distribution, just our sample slope*

Bivariate GLM: sampling distribution of slopes

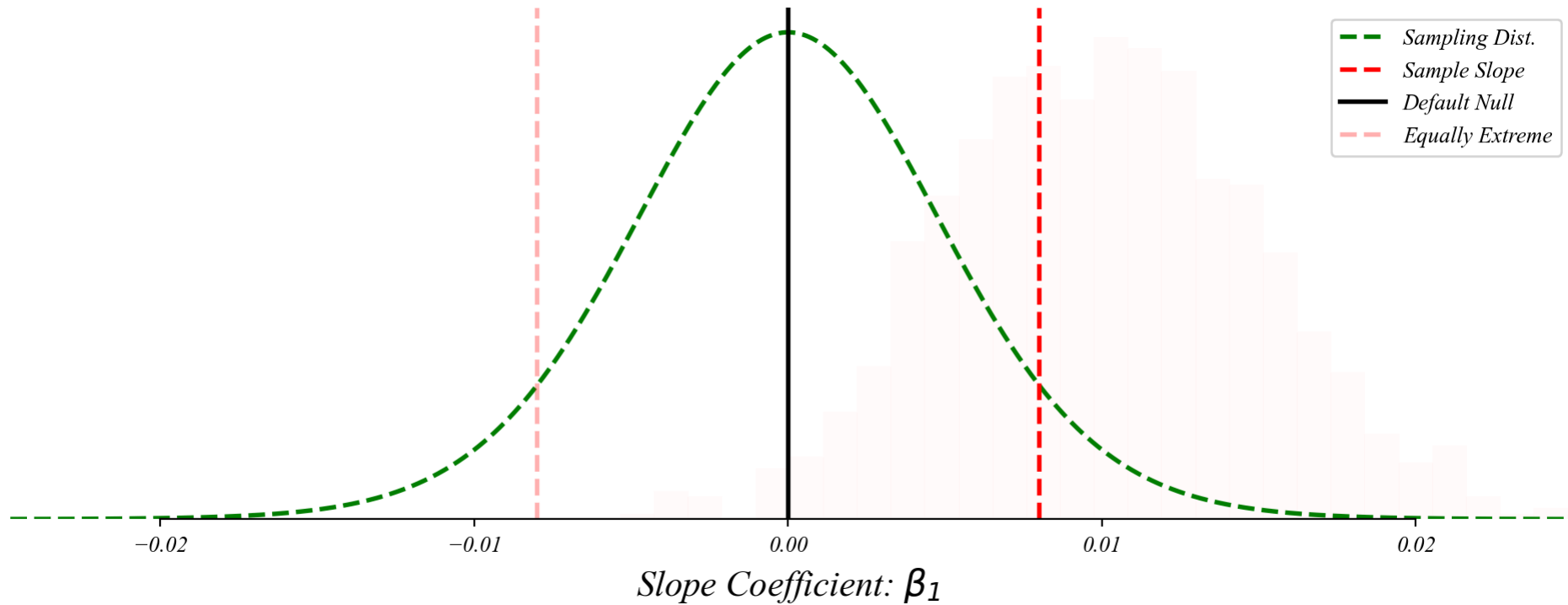
The slope coefficient follows a normal distribution centered on the population slope.



- > *center the distribution on our null*
- > *check the distance from the sample*

Bivariate GLM: sampling distribution of slopes

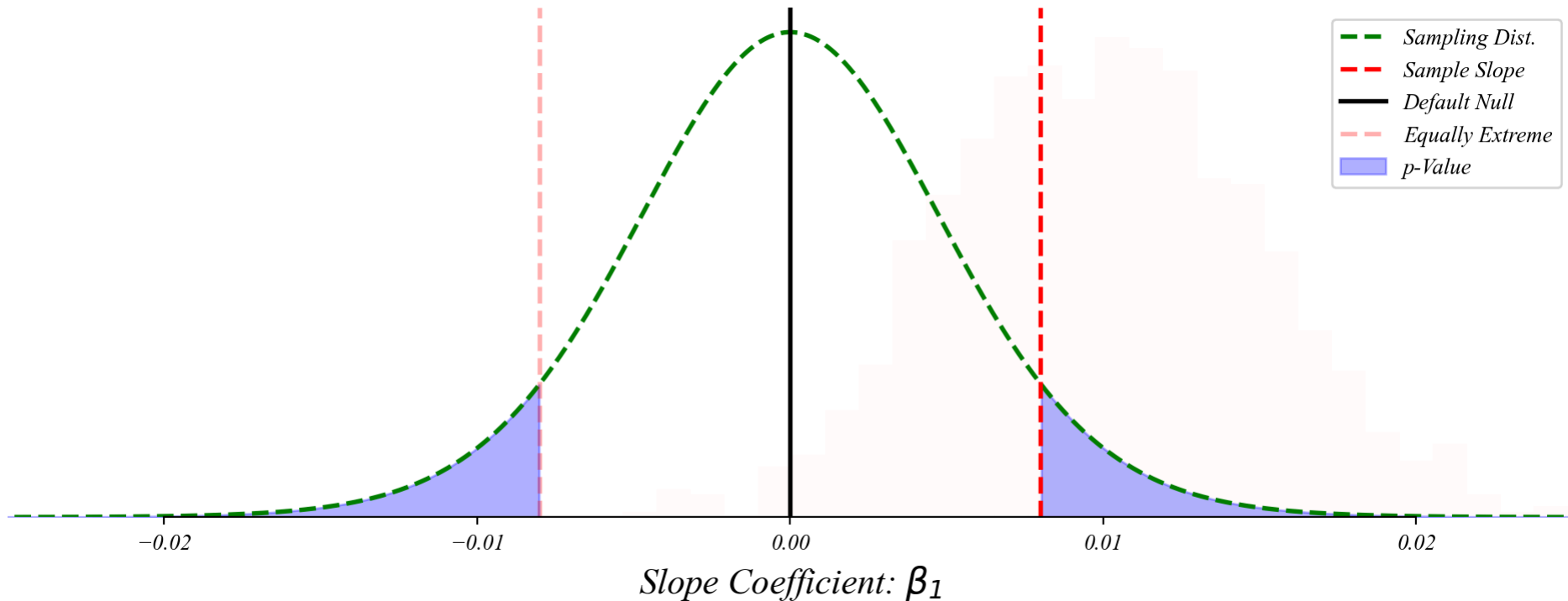
The slope coefficient follows a normal distribution centered on the population slope.



> *the p-value is the probability of something as far from the null as our sample*

Bivariate GLM: sampling distribution of slopes

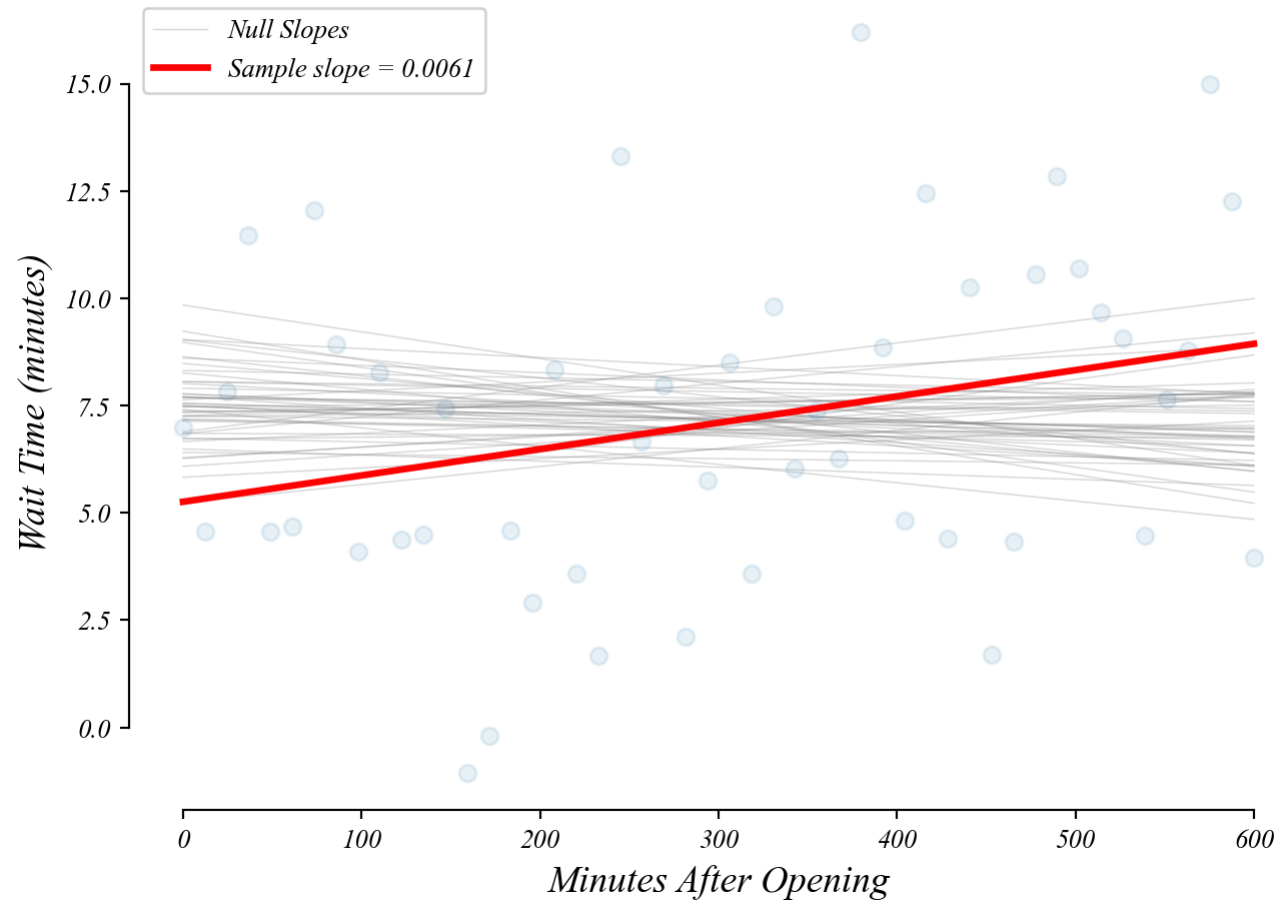
The slope coefficient follows a normal distribution centered on the population slope.



- > *p-value: the ‘surprisingness’ of our sample if $\beta_1 = 0$*
- > *the probability of seeing our sample by chance if there is no relationship*
- > *a small p-value is evidence against the null hypothesis ($\beta_1 = 0$)*

Bivariate GLM: sampling distribution of slopes

Many possible models we might observe by chance if the null ($\beta_1 = 0$) were true.



> *how likely does it look like this slope was drawn from the null slopes?*

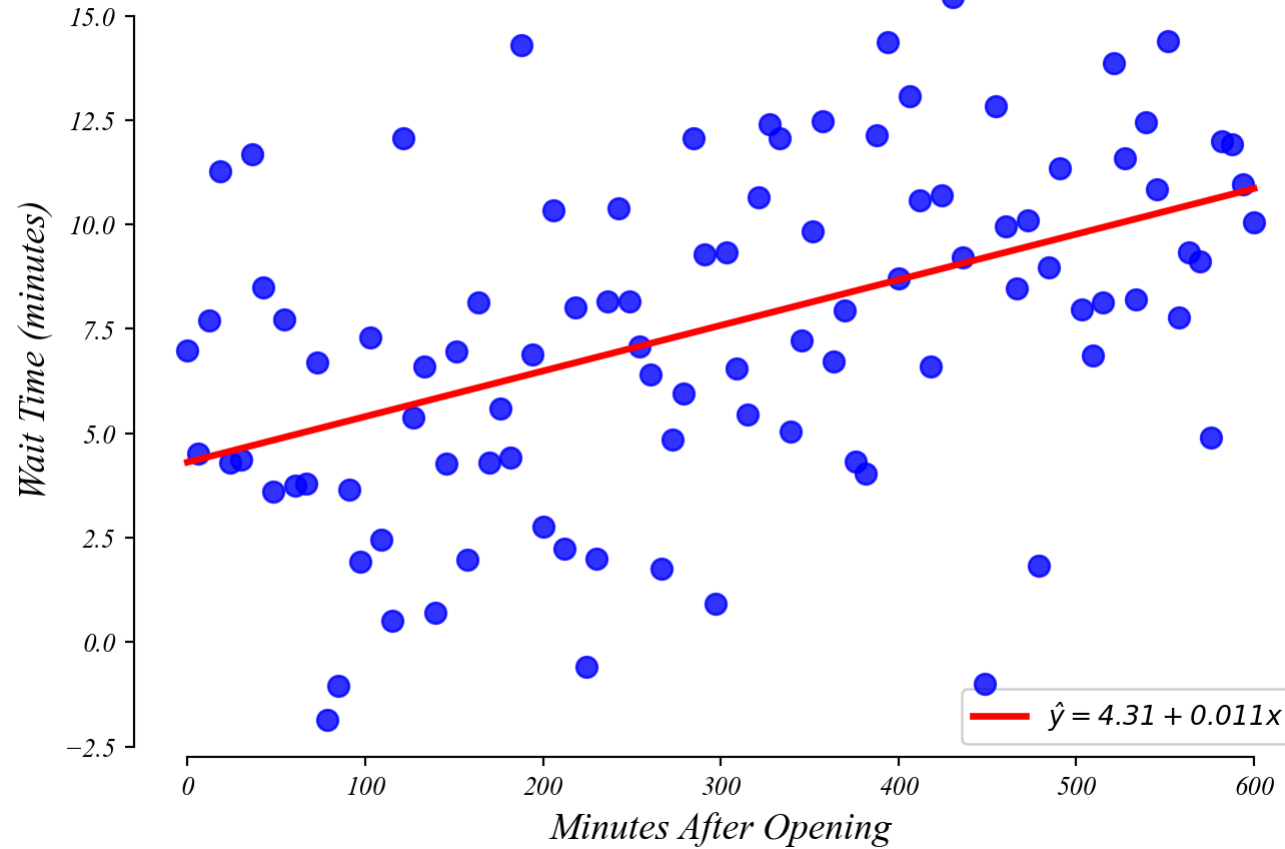
> *p-value: the probability a slope as extreme as ours under the null ($\beta_1 = 0$)*

Exercise 4.1 | Happiness and Per Capita GDP

Are wealthier countries happier?

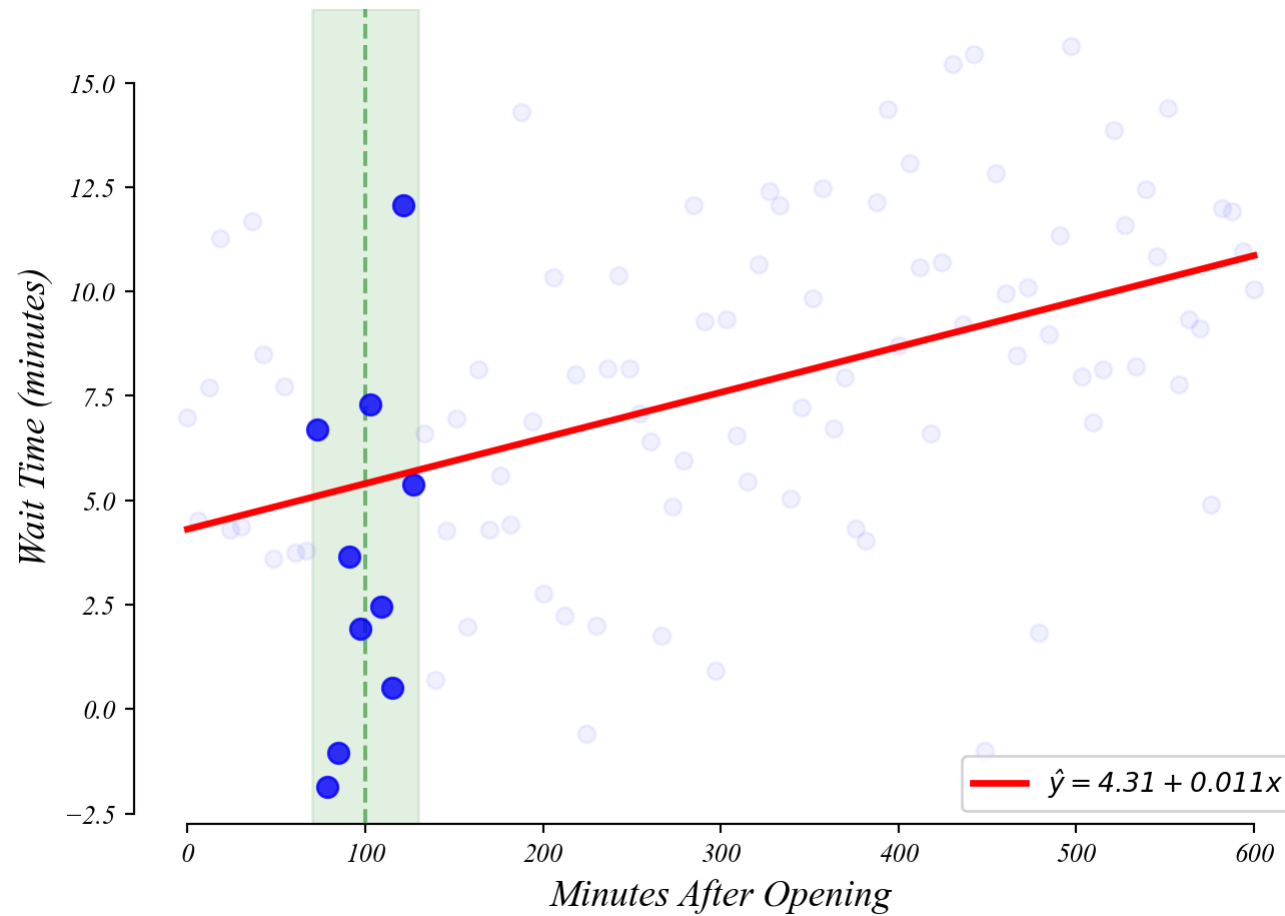
GLM: predictions

What wait time should we expect at 100 minutes after open?



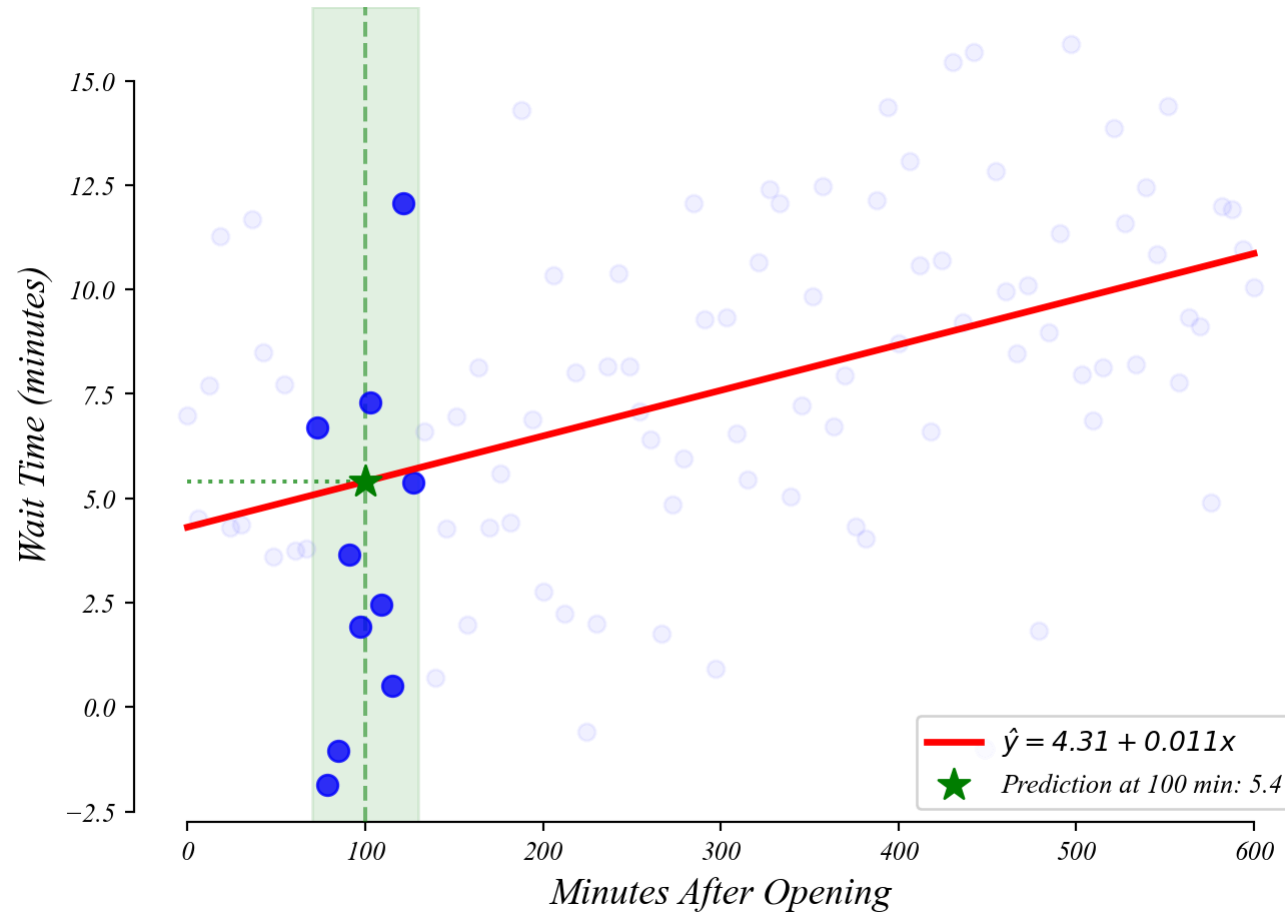
GLM: predictions

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GLM: predictions

What wait time should we expect at 100 minutes after open?

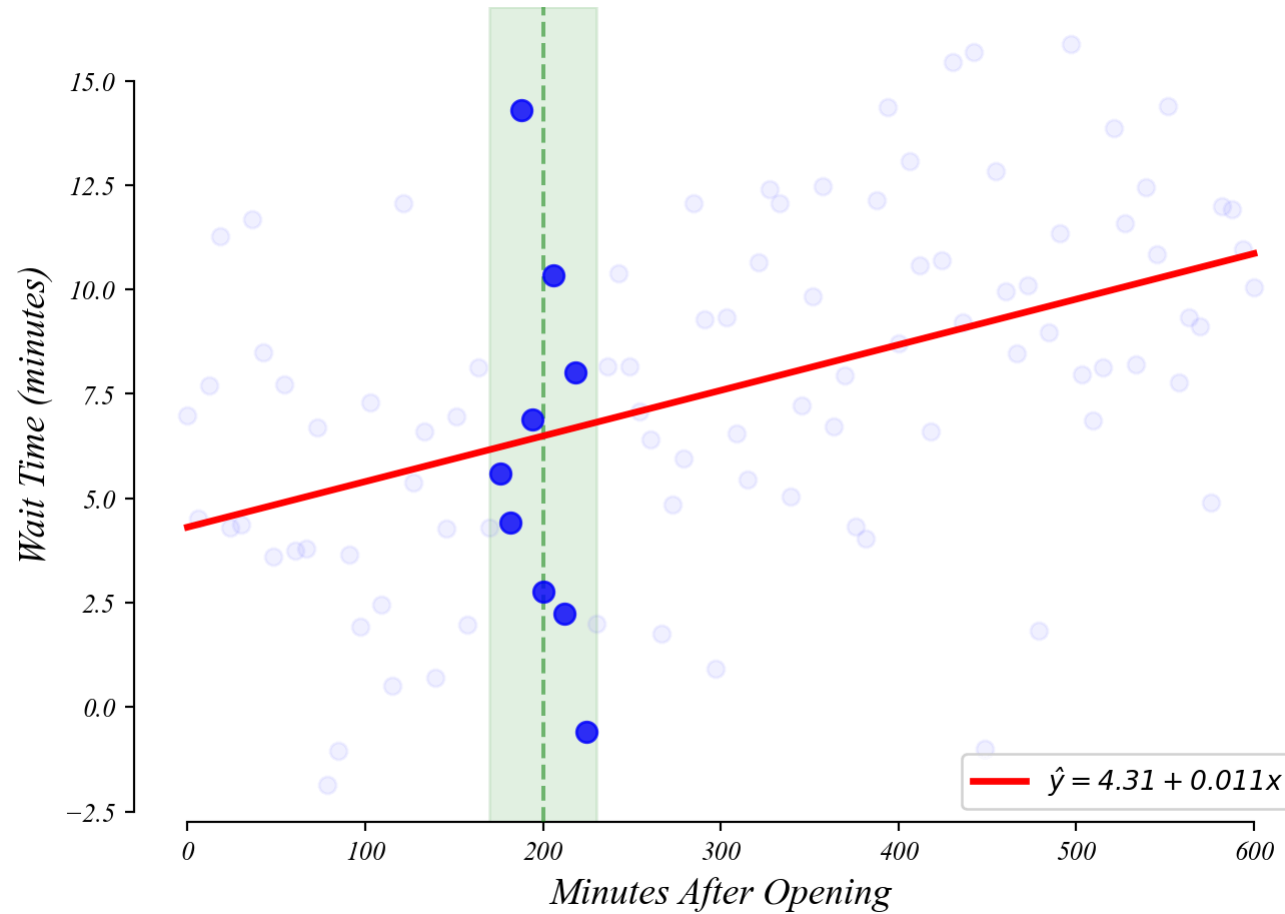


> *you can find this with a calculator!*

> *plug $x = 100$ into the equation $y = 4.31 + 0.011x$*

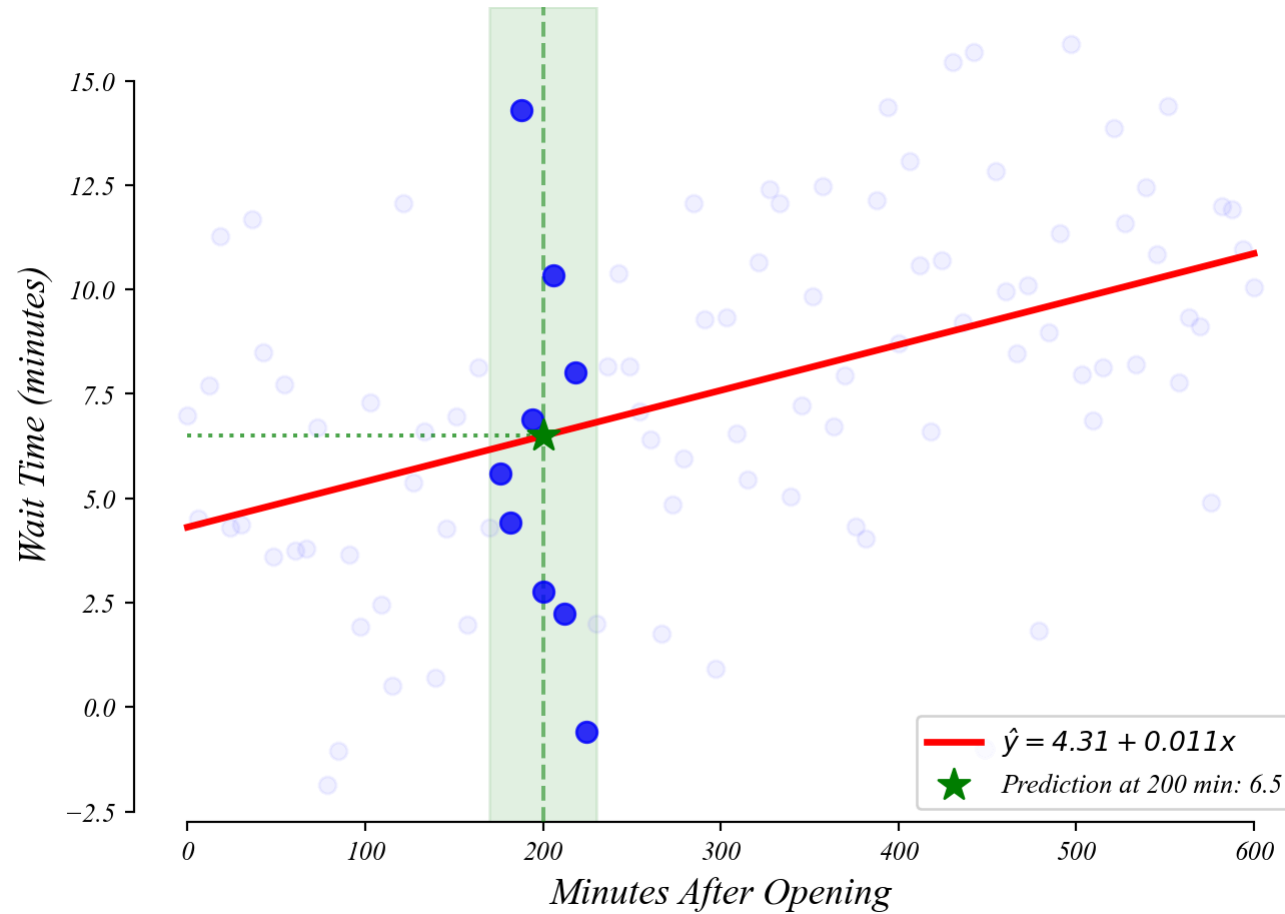
GLM: predictions

What wait time should we expect at 200 minutes after open?



GLM: predictions

What wait time should we expect at 200 minutes after open?

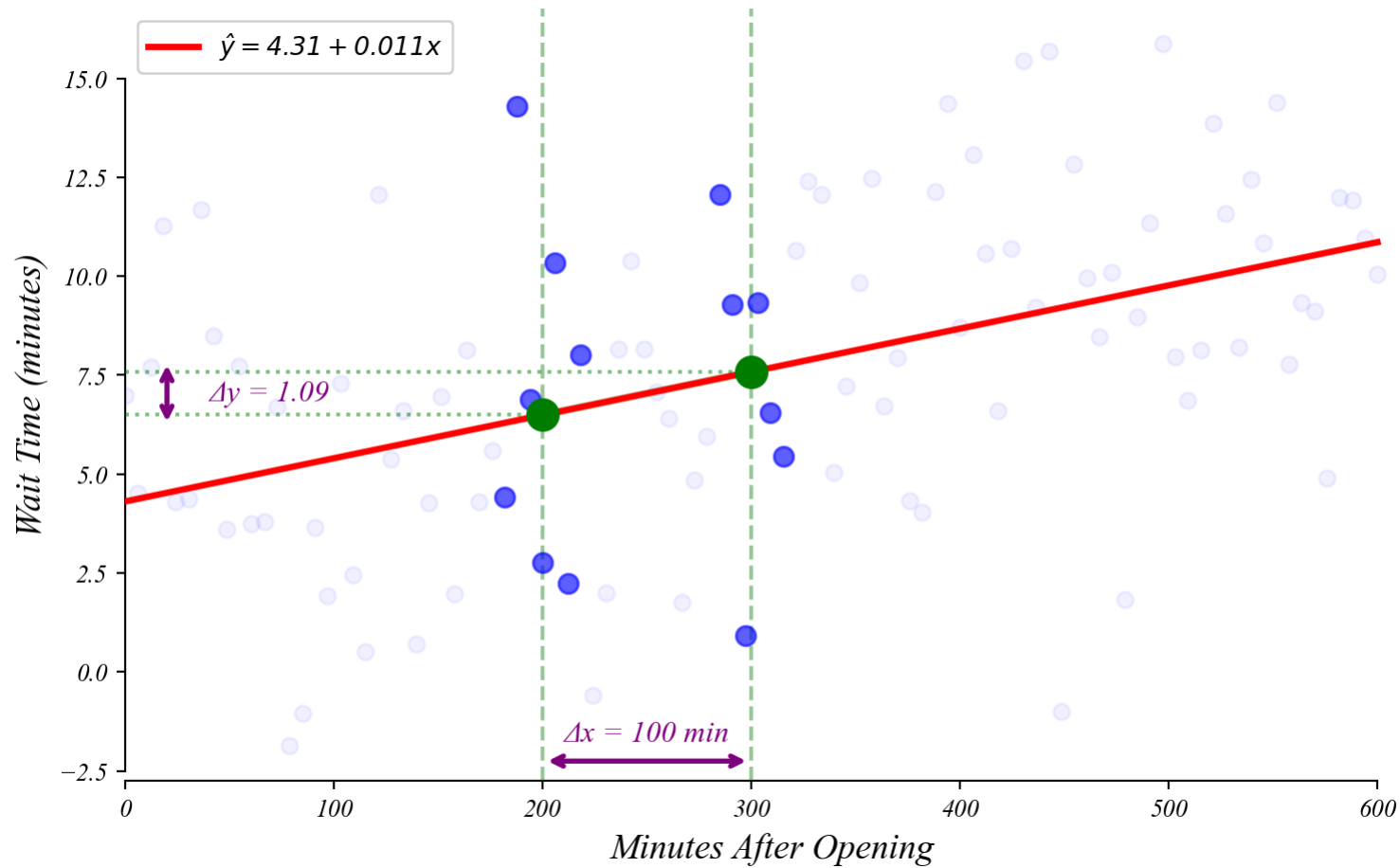


Exercise 4.1 | Happiness and Per Capita GDP

Are wealthier countries happier?

GLM: interpretation

How much does wait time increase every minute after open?



> β_1 tells us how much y increases with every 1 unit increase in x

Exercise 4.1 | Happiness and Per Capita GDP

How much does happiness increase for each additional \$1,000 of per capita GDP?

The General Linear Model

GLM performs a t-test on all model coefficients.

Univariate (Part 3): $y = \beta_0 + \epsilon$

- *Equivalent to a one-sample t-test*
- *Tests whether $\beta_0 = \mu_0$ (default null)*

Numerical Predictor: $y = \beta_0 + \beta_1 x + \epsilon$

- *x is a numerical variable (like age, income, temperature, etc.)*
- *Tests both intercept ($\beta_0 = 0$) and slope ($\beta_1 = 0$)*
- *Null hypothesis on slope: no relationship between x and y ($\beta_1 = 0$)*

The General Linear Model

GLM uses the idea of a t-test with any coefficient.

Categorical Predictor (*next time*): $y = \beta_0 + \beta_1 x + \epsilon$

- *x is a categorical variable (like age, income, temperature, etc.)*
- *Equivalent to a two-sample t-test (when x is binary)*

Multivariate GLM (*Part 5*):

- *Adds more predictor variables: $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \epsilon$*
- *Each coefficient has its own t-test against the null that it equals zero*

Economic Applications

GLM is the workhorse statistical tool in empirical economics.

Labor Economics: *relationship between education and wages.*

$$\text{wage} = \beta_0 + \beta_1 \text{education} + \varepsilon$$

Policy Analysis: *relationship between minimum wages and employment.*

$$\text{employment} = \beta_0 + \beta_1 \text{minimum_wage} + \varepsilon$$

Political Economy: *relationship between neighbor's party and voter turnout*

$$\text{voted} = \beta_0 + \beta_1 \text{neighborhood_politics} + \varepsilon$$

Bivariate GLM: Numerical Predictors

Summary

GLM Framework:

- *T-tests and regression are part of the same very flexible framework.*

Numerical Predictors:

- *Bivariate GLM extends the t-test by allowing continuous predictors.*

Same Distribution:

- *Coefficient estimates follow t-distributions centered on the true population values.*

Same Interpretation:

- *The p-values have the same interpretation: probability of seeing results this extreme if the null is true.*

Looking Forward

Extending the GLM framework

Next Up:

- *Part 4.2 | Bad Models*
- *Part 4.3 | Categorical Predictors*
- *Part 4.4 | Timeseries*
- *Part 4.5 | Causality*

Later:

- *Part 5.1 | Numerical Controls*
- *Part 5.2 | Categorical Controls*
- *Part 5.3 | Interactions*
- *Part 5.4 | Model Selection*

> all built on the same statistical foundation