ECON 0150 | Economic Data Analysis

The economist's data analysis pipeline.

Part 4.2 | A t-test is a simple linear model

Lets Draw Some Lines: Key Concepts

The general linear model is just drawing lines through data points.

Linear Model Equation: $y = mx + b$

- *Basically just a line*
- *If you want to be fancy, write it like:* $y_i = mx_i + b + \epsilon_i$

Mean Sqaured Error: $MSE = \frac{1}{n} \sum_i \epsilon_i^2$

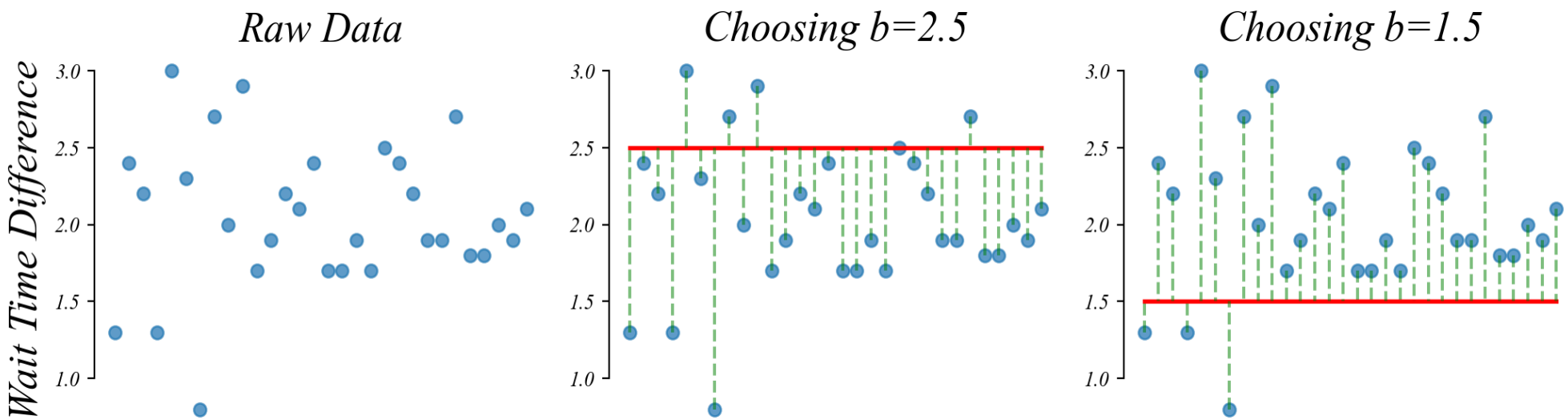
- *Basically just the average distance between the line and a data point*

The Intercept-Only Model

Lets start with a model with no x (basically: x=0).

This simple model look like: $y = b$.

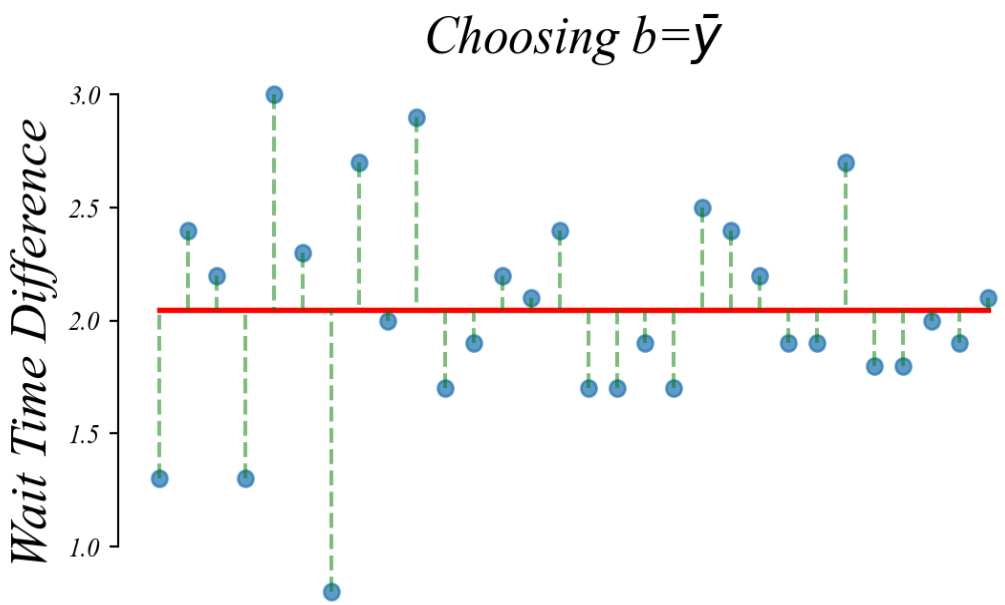
> *there's only an incercept term!*

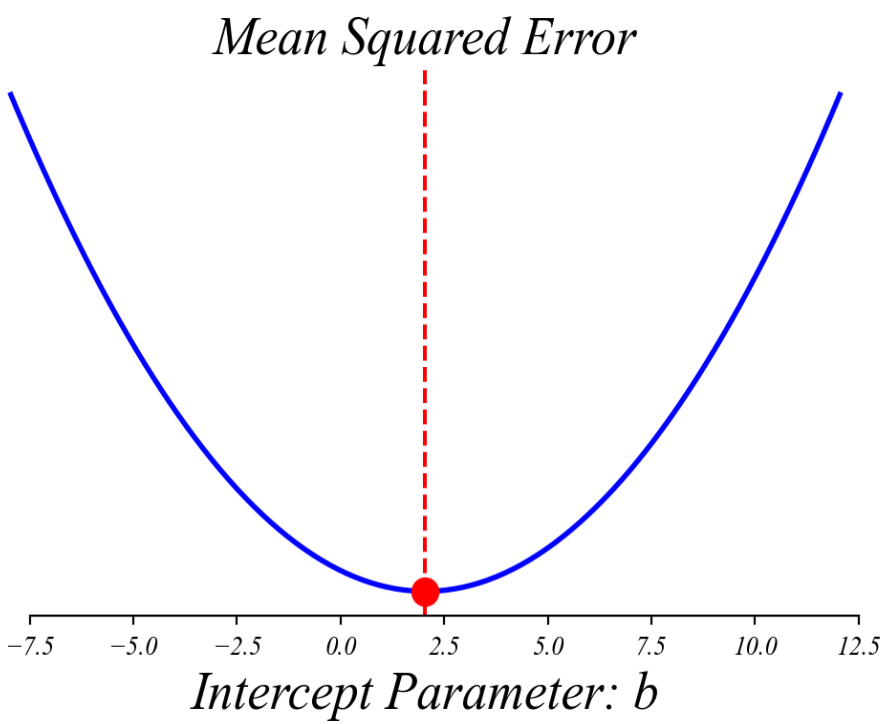


> *what should we choose for b to minimize the model's error?*

Line Fitting and the Sample Mean

The sample mean minimizes the MSE.

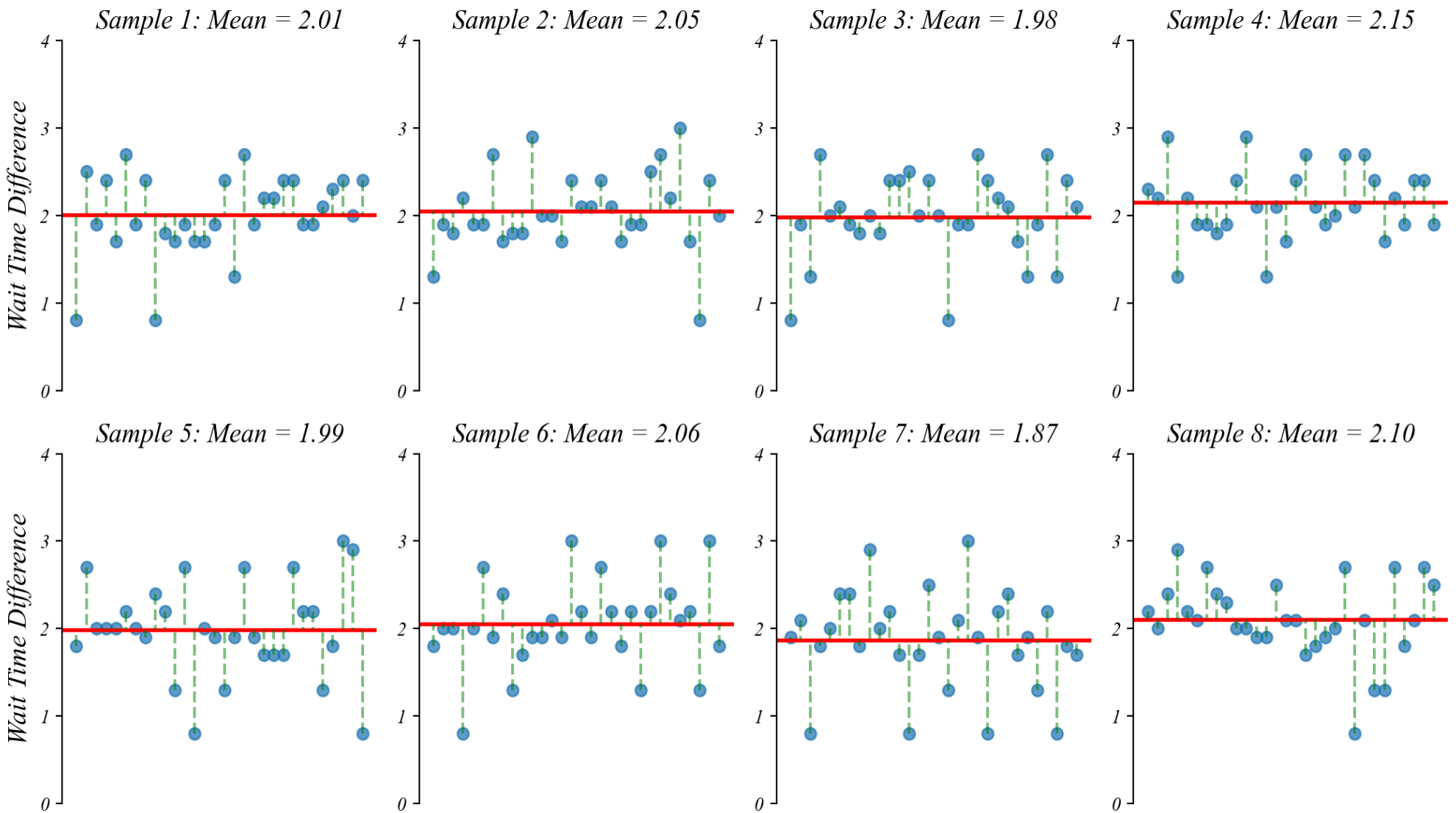




> *this also means that the MSE equals the Variance!*

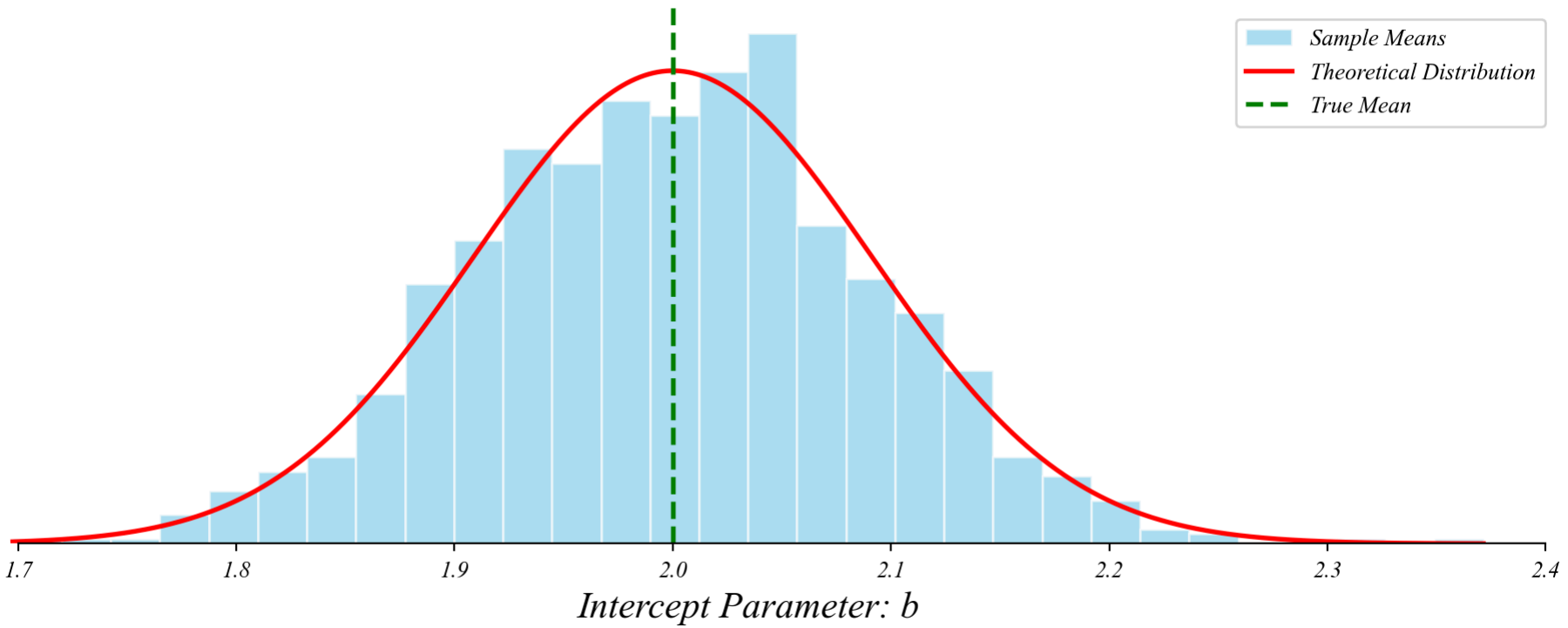
Sampling Error and Line Fitting

Like before, if we take many samples, we get slighly different means and slighly different fits.



Simulating the Sampling Distribution

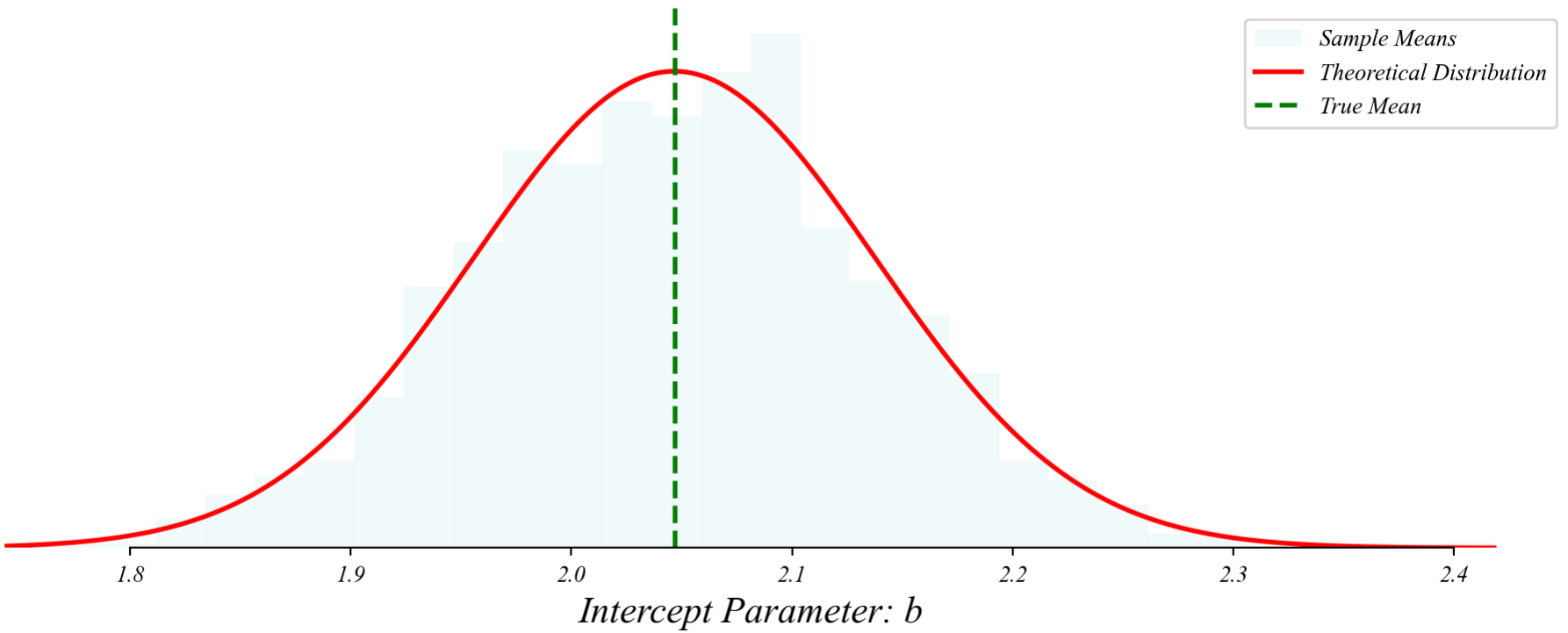
The mean follows a normal distribution centered on the true mean



> so if i were to ask you the probability of getting this sample mean under the standard null of b=0, what would you say?

Distribution Around the Sample Mean

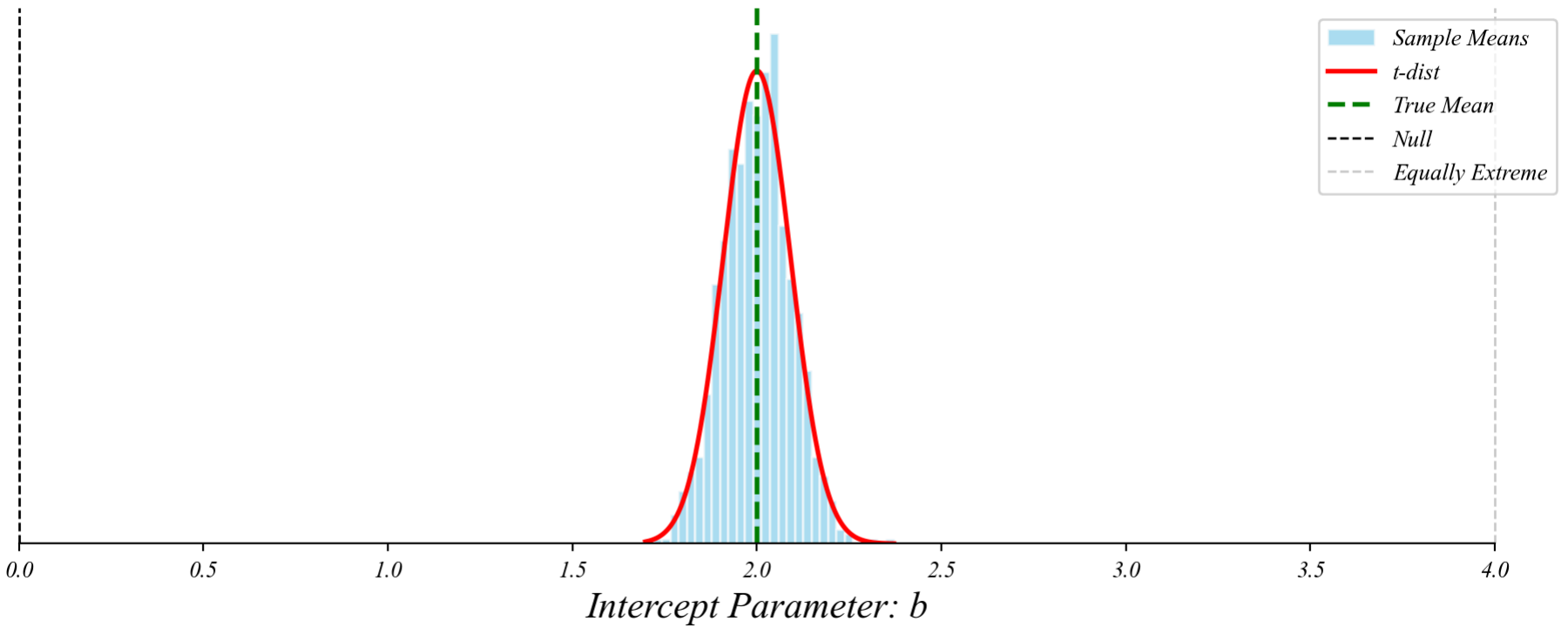
We don't know the true mean, just our sample mean. Center the distribution there.



> so if i were to ask you the probability of getting this sample mean under the standard null of b=0, what would you say?

Finding p-values in the t-distribution

Testing whether the mean is significantly different from zero

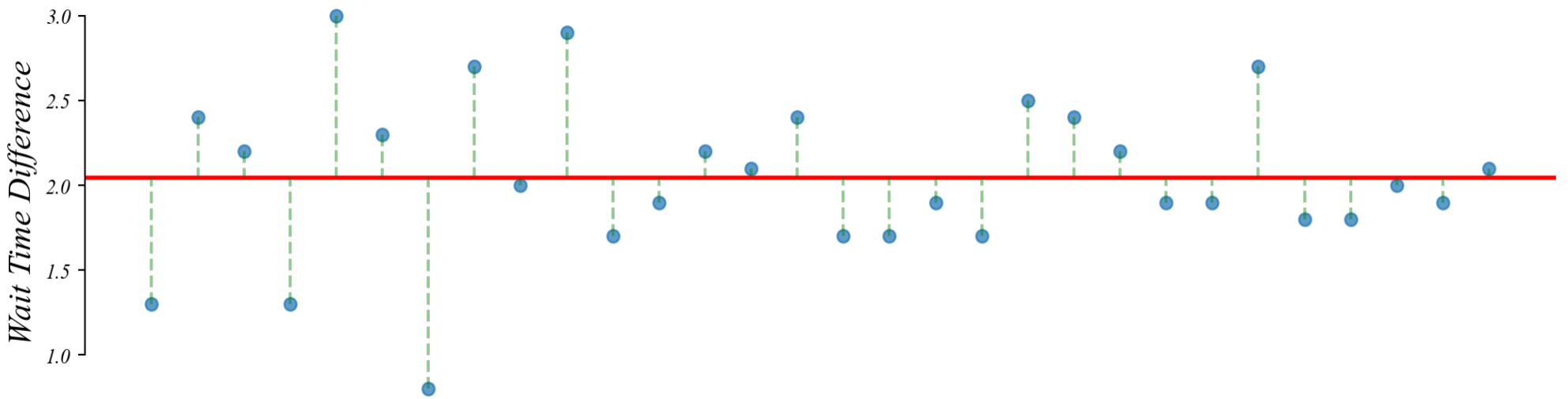


> *here we're centering the t-distribution on the observed sample mean*

> *as before, this is mathematically equivalent to centering it on the null*

Regression: Horizontal Line Model

A t-test a linear model with only an intercept: $y = \beta_0 + \epsilon$



> *the sample mean* β_0 *minimizes the sum of squared errors*

> *the p-value tells us the probability of the data given the default null*

> *the best guess of the true mean is* β_0

> *this is the simplest version of an OLS regression model*

Example: Difference in Wait Times

Are wait times different in the morning and afternoon?

> *imports*

```
# Imports
import numpy as np
import statsmodels.api as sm
import scipy.stats as stats
```

> *the dataset*

```
# Paired Differences: Afternoon Wait Minus Morning Wait
data = [1.3, 2.4, 2.2, 1.3, 3. , 2.3, 0.8, 2.7, 2. , 2.9, 1.7, 1.9, 2.2,
        2.1, 2.4, 1.7, 1.7, 1.9, 1.7, 2.5, 2.4, 2.2, 1.9, 1.9, 2.7, 1.8,
        1.8, 2. , 1.9, 2.1]
```

> *regression model*

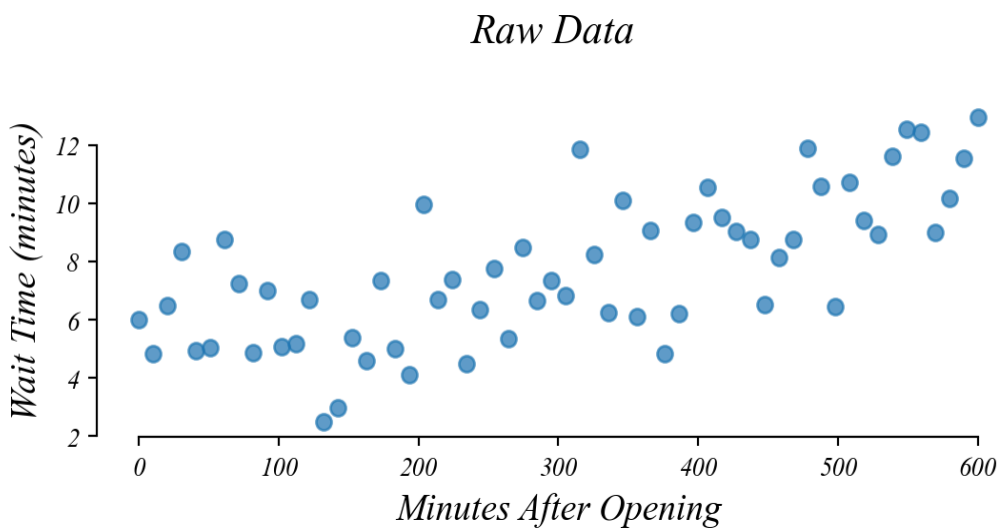
```
# Run the model
model = sm.OLS(data, np.ones(len(diff))).fit()
print(model.summary().tables[1])
```

> *one sample t-test*

```
t_stat, p_value = stats.ttest_1samp(data, 0)
print(t_stat, p_value)
```

Modeling Relationships Between Variables

Let's introduce a (potential) relationship: $y = \beta_0 + \beta_1 x + \epsilon$



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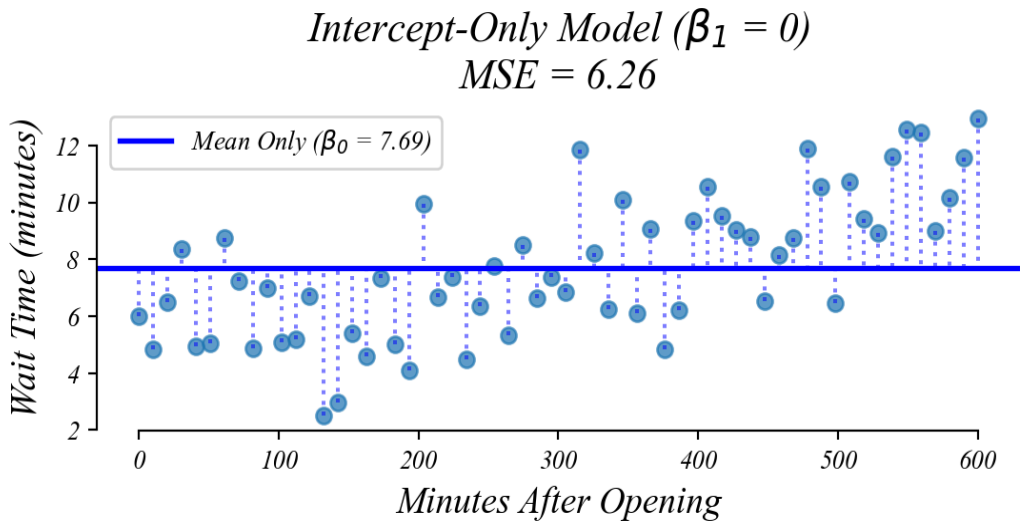

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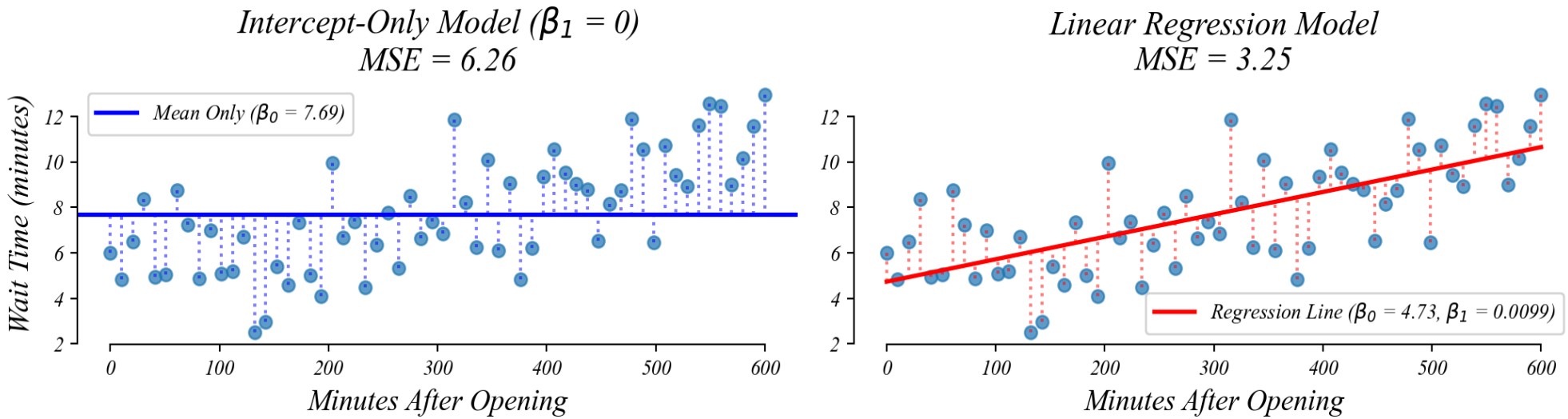


> *allowing a slope (β_1) improves model fit (MSE) when there's a relationship*

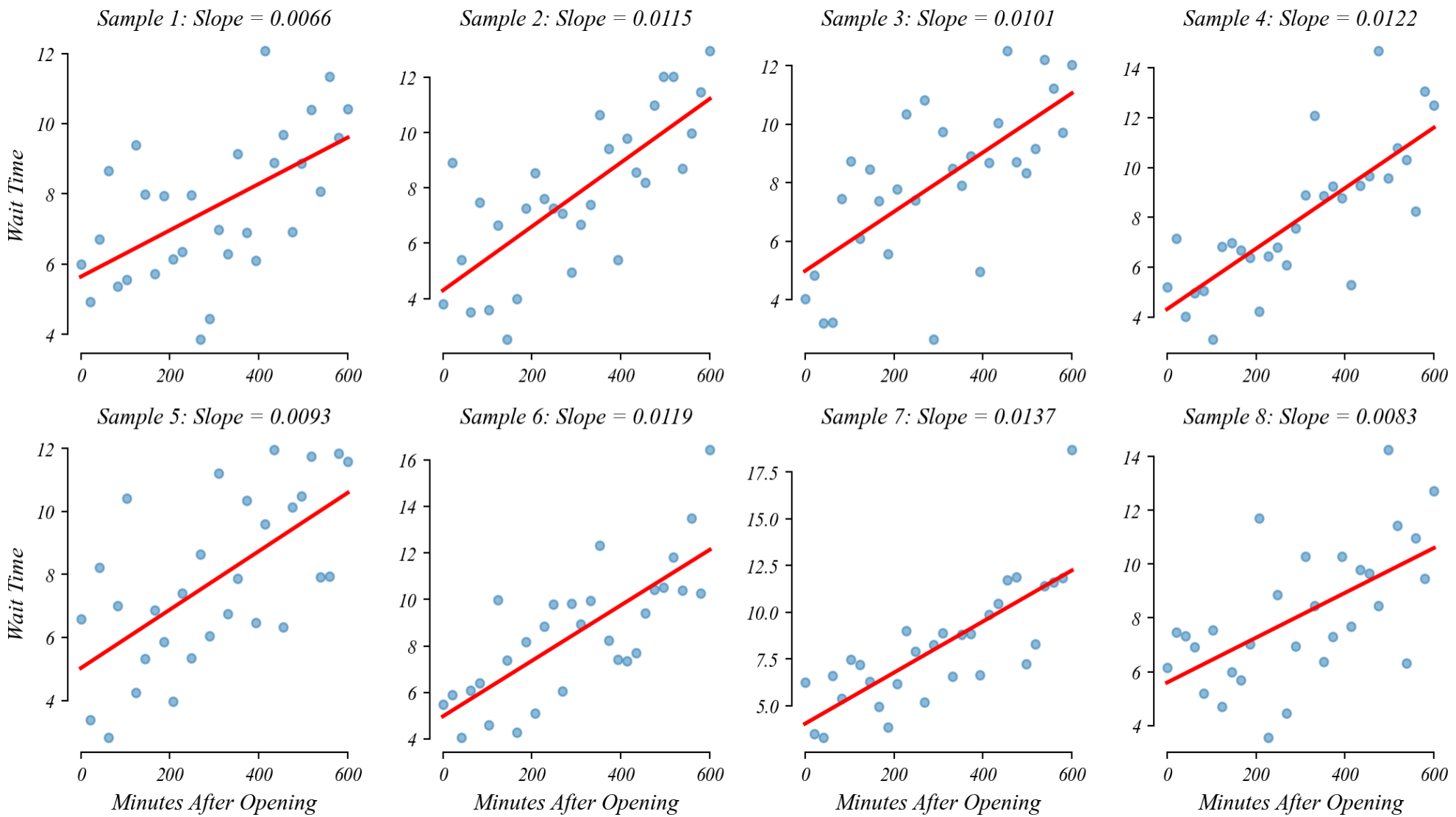
> *the intercept is no longer the mean*

> *the slope (β_1) gives the best guess of the relationship between x and y*

> *but could this slope be just sampling error?*

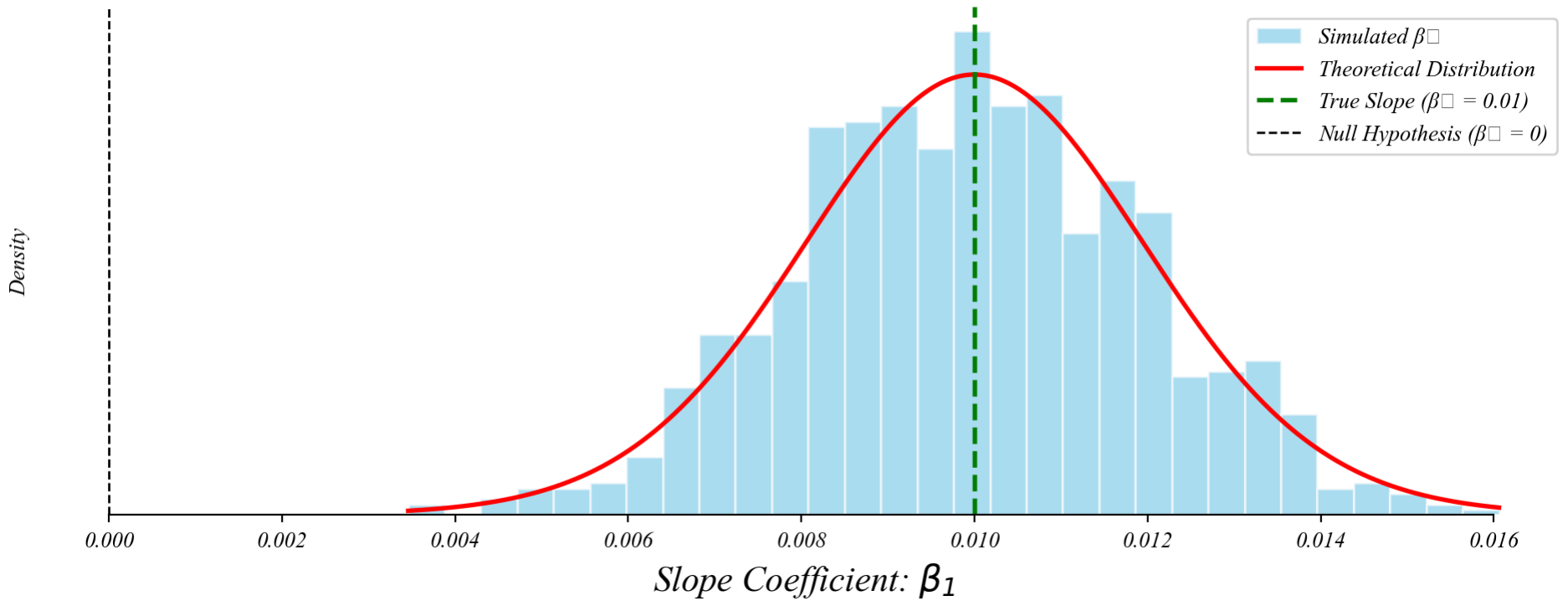
Sampling Error and Line Fitting

Like before, if we take many samples, we get slighly different slopes and slighly different fits.



Sampling Distribution of the Slope Coefficient

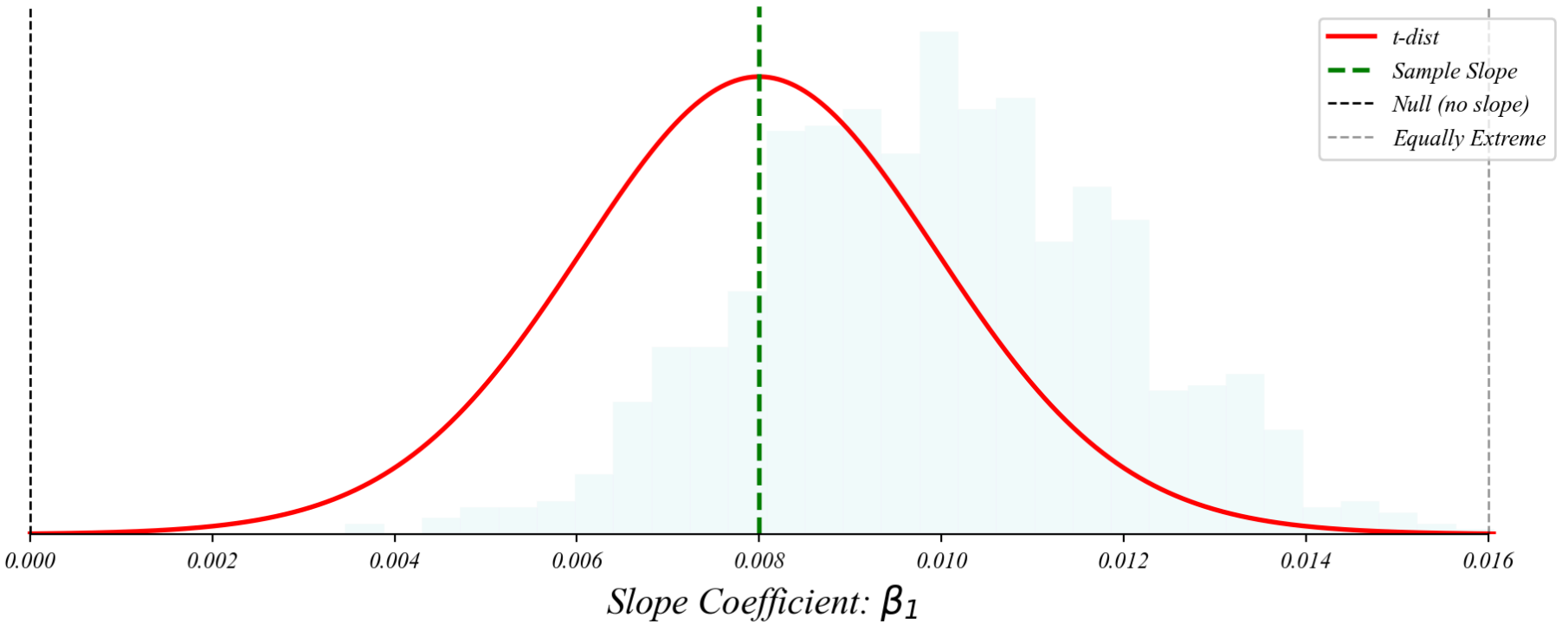
The slope coefficient follows a t-distribution centered on the true slope



> *this lets us perform a t-test on the slope!*

The p-value for the Slope Coefficient

Testing whether the slope is significantly different from zero



> *very small p-value = strong evidence against the null hypothesis ($\beta_1 = 0$)*

Exmaple: Wait Times Throughout The Day

Does wait time change throughout the day?

```
import pandas as pd
import statsmodels.api as sm
```

> *load minutes data*

```
minutes_after_open = [  0.        ,  10.16949153,  20.33898305,  30.50847458,
        40.6779661 ,  50.84745763,  61.01694915,  71.18644068,
        81.3559322 ,  91.52542373, 101.69491525, 111.86440678,
       122.03389831, 132.20338983, 142.37288136, 152.54237288,
       162.71186441, 172.88135593, 183.05084746, 193.22033898,
       203.38983051, 213.55932203, 223.72881356, 233.89830508,
       244.06779661, 254.23728814, 264.40677966, 274.57627119,
       284.74576271, 294.91525424, 305.08474576, 315.25423729,
       325.42372881, 335.59322034, 345.76271186, 355.93220339,
       366.10169492, 376.27118644, 386.44067797, 396.61016949,
       406.77966102, 416.94915254, 427.11864407, 437.28813559,
       447.45762712, 457.62711864, 467.79661017, 477.96610169,
       488.13559322, 498.30508475, 508.47457627, 518.6440678 ,
       528.81355932, 538.98305085, 549.15254237, 559.3220339 ,
       569.49152542, 579.66101695, 589.83050847, 600.        ]
```

Exmaple: Wait Times Throughout The Day

Does wait time change throughout the day?

> *load wait time data*

```
wait_times = [ 5.99342831,  4.82516631,  6.49876691,  8.35114446,  4.93847291,
        5.04020066,  8.76859512,  7.24673387,  4.87461055,  7.00037432,
        5.09011377,  5.18718456,  6.70426353,  2.49547341,  2.97389315,
        5.40084867,  4.6014564 ,  7.35730822,  5.01446032,  4.10759599,
        9.96519584,  6.68404062,  7.37234454,  4.48948668,  6.35191252,
        7.76421806,  5.34208064,  8.49715875,  6.64618025,  7.36576504,
        6.84743423, 11.85709874,  8.22724284,  6.24051035, 10.10271694,
        6.11763473,  9.07874414,  4.84337162,  6.20803468,  9.35982417,
       10.54472977,  9.51222809,  9.03988988,  8.77067396,  6.51753229,
        8.13658277,  8.75668856, 11.89390547, 10.56859251,  6.45697054,
       10.7329137 ,  9.41627612,  8.93429159, 11.61318309, 12.55352447,
       12.45578058,  9.01648021, 10.17818542, 11.56083195, 12.95109025]
```

> *merge into a dataframe*

```
data = pd.DataFrame({'minutes_after_open': minutes_after_open, 'wait_times': wait_times})
```

Exmaple: Wait Times Throughout The Day

Does wait time change throughout the day?

> *run the model: predict wait time using time of day*

```
# Add a constant for the intercept
X = sm.add_constant(data['minutes_after_open'])

# Fit the regression model
model = sm.OLS(data['wait_times'], X).fit()

# Use the built-in summary table method for a simple display
print(model.summary().tables[1])
```

```
==============================================================================
                  coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          4.7328      0.467     10.128      0.000       3.797       5.668
minutes_after_open     0.0099      0.001      7.340      0.000       0.007       0.013
==============================================================================
```

> *every minute later in the day sees 0.01 minutes more of wait time*

> *this is very unlikely to be due to chance*

The General Linear Model

Regression is a flexible t-test

One-sample t-test:

- *Regression with only an intercept:* $y = \beta_0 + \varepsilon$
- *Tests whether* $\beta_0 = \mu_0$ *(null value)*

Continuous Predictor:

- *Regression with an intercept and continuous predictor:* $y = \beta_0 + \beta_1 \cdot x + \epsilon$
- x *is a continuous predictor (like age, income, temperature, etc.)*
- *Tests whether* $\beta_1 = 0$ *(no relationship between x and y)*

Multiple regression:

- *Adds more predictor variables:* $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots + \varepsilon$
- *Each coefficient has its own t-test against the null that it equals zero*

Economic Applications

Regression is the workhorse of empirical economics

Labor Economics:

- *Effect of education on wages:* $wage = \beta_0 + \beta_1 \cdot education + \varepsilon$
- *Tests whether education significantly affects wages (*$\beta_1 \neq 0$*)*

Policy Analysis:

- *Impact of minimum wage on employment:*
  $employment = \beta_0 + \beta_1 \cdot min\_wage + \varepsilon$
- *Tests whether minimum wage affects employment (*$\beta_1 \neq 0$*)*

Finance:

- *Asset pricing model:* $return = \beta_0 + \beta_1 \cdot market\_return + \varepsilon$
- *Tests whether asset moves with the market (*$\beta_1 \neq 0$*)*

Key Takeaways

Connecting t-tests to regression

1. ***Unified Framework:*** *T-tests and regression are part of the same general linear model framework.*

2. ***Continuous Predictors:*** *Regression extends t-tests by allowing continuous predictors.*

3. ***Multiple Variables:*** *Regression lets us include multiple predictors and control variables.*

4. ***Same Interpretation:*** *The p-values have the same interpretation: probability of seeing results this extreme if the null is true.*

5. ***Same Distribution:*** *Coefficient estimates follow t-distributions centered on true values.*

Looking Forward

Extending our regression framework

We will explore:

- *Two-Sample t-Test*
- *ANOVA: checking for differences between many groups*
- *Multiple regression with several predictors*
- *Controlling for confounding variables*
- *Categorical variables and dummy coding*
- *Interaction effects*

> all built on the same statistical foundation we explored today