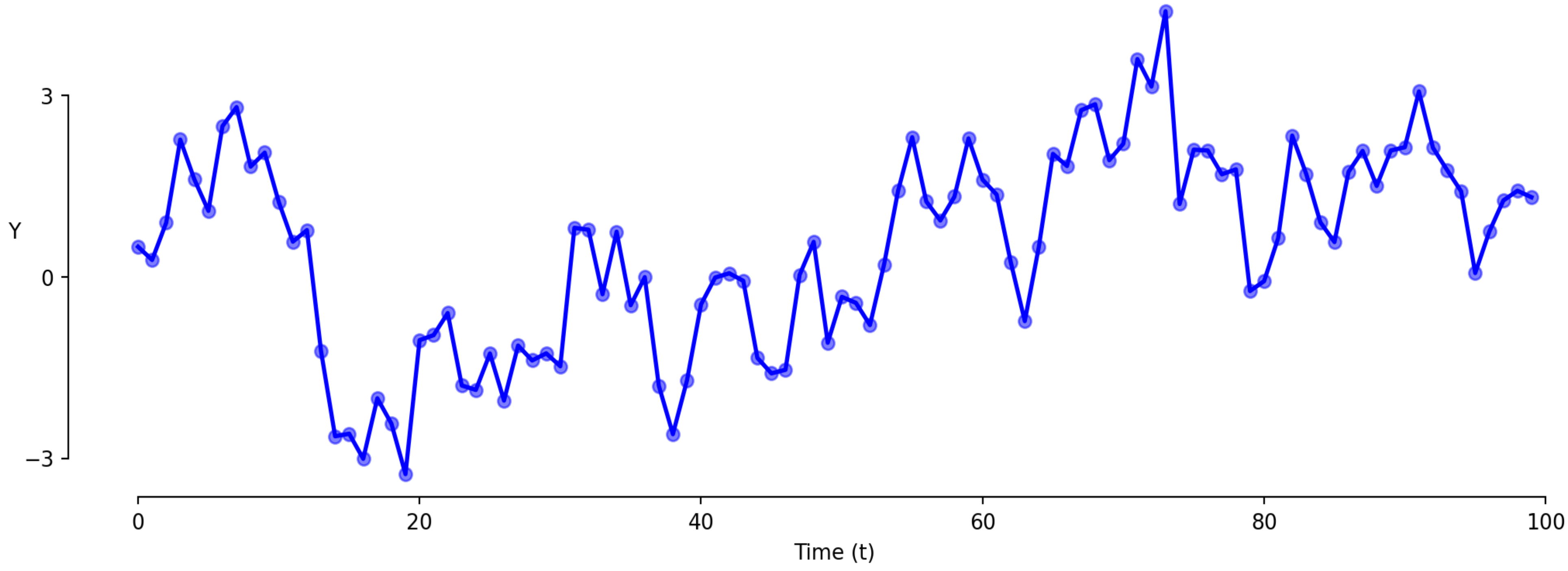
ECON 0150 | Economic Data Analysis

The economist's data analysis workflow.

Part 4.4 | The Problem of Timeseries

Timeseries Analysis

We often want to model relationships through time.



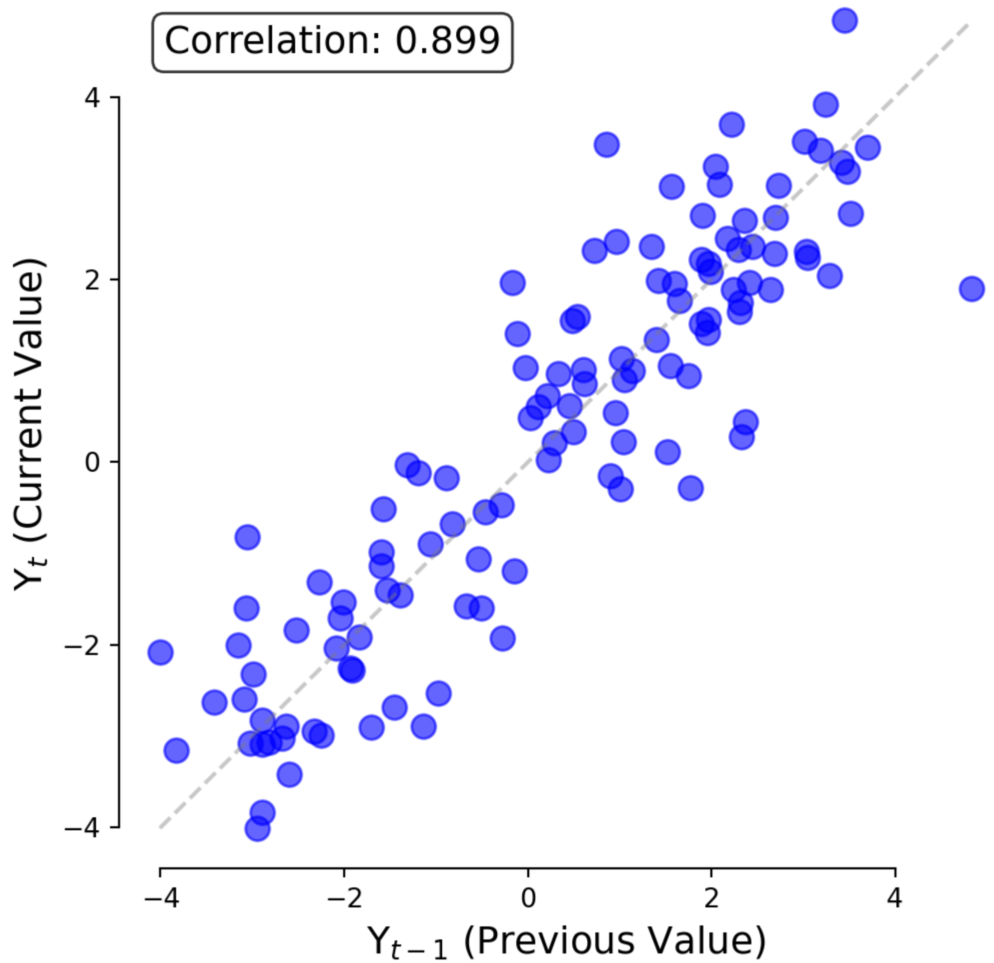
Data related in time has a special problem:

- *Observations are related to their past values (autocorrelation).*
- *This violates **Assumption 4: Independence**.*

Timeseries Analysis: Autocorrelation

We can check whether values in timeseries are related to their own past values.

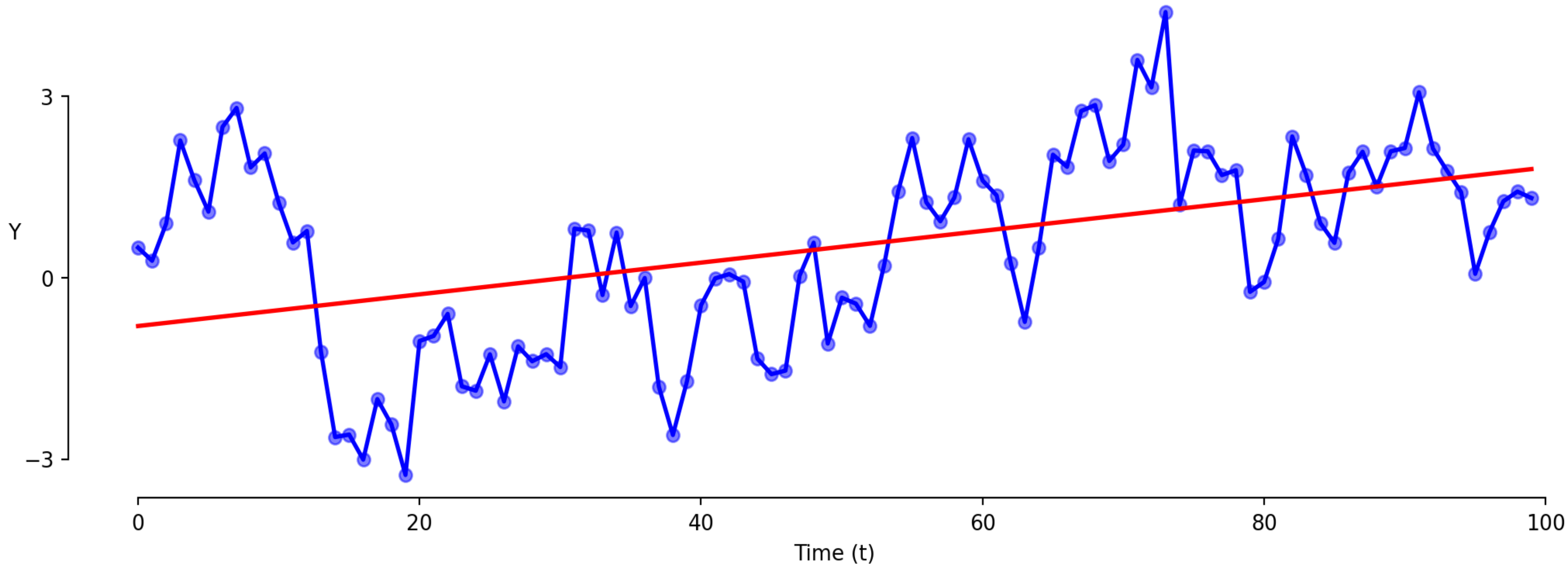
A **Lag Plot** shows the value today (t) against the value yesterday ($t-1$).



Timeseries: Model 1 (Levels)

The standard approach has problems with time series.

$$Y = \beta_0 + \beta_1 \cdot t + \varepsilon$$



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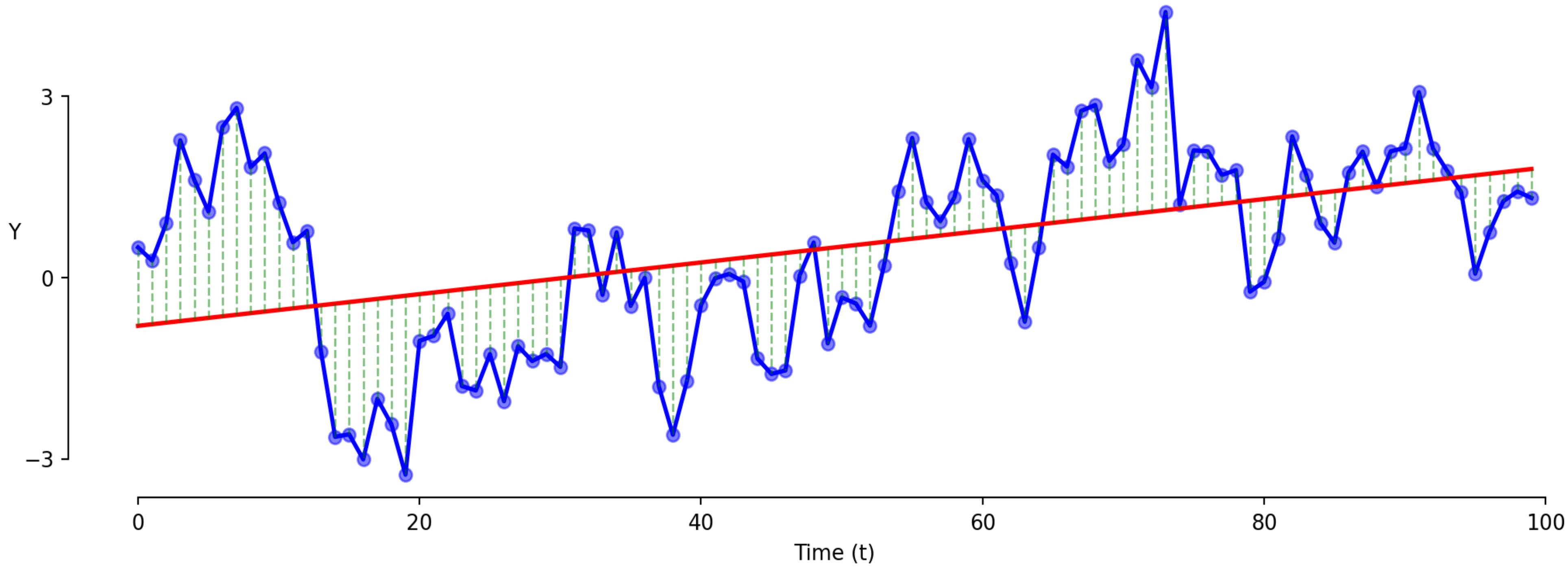
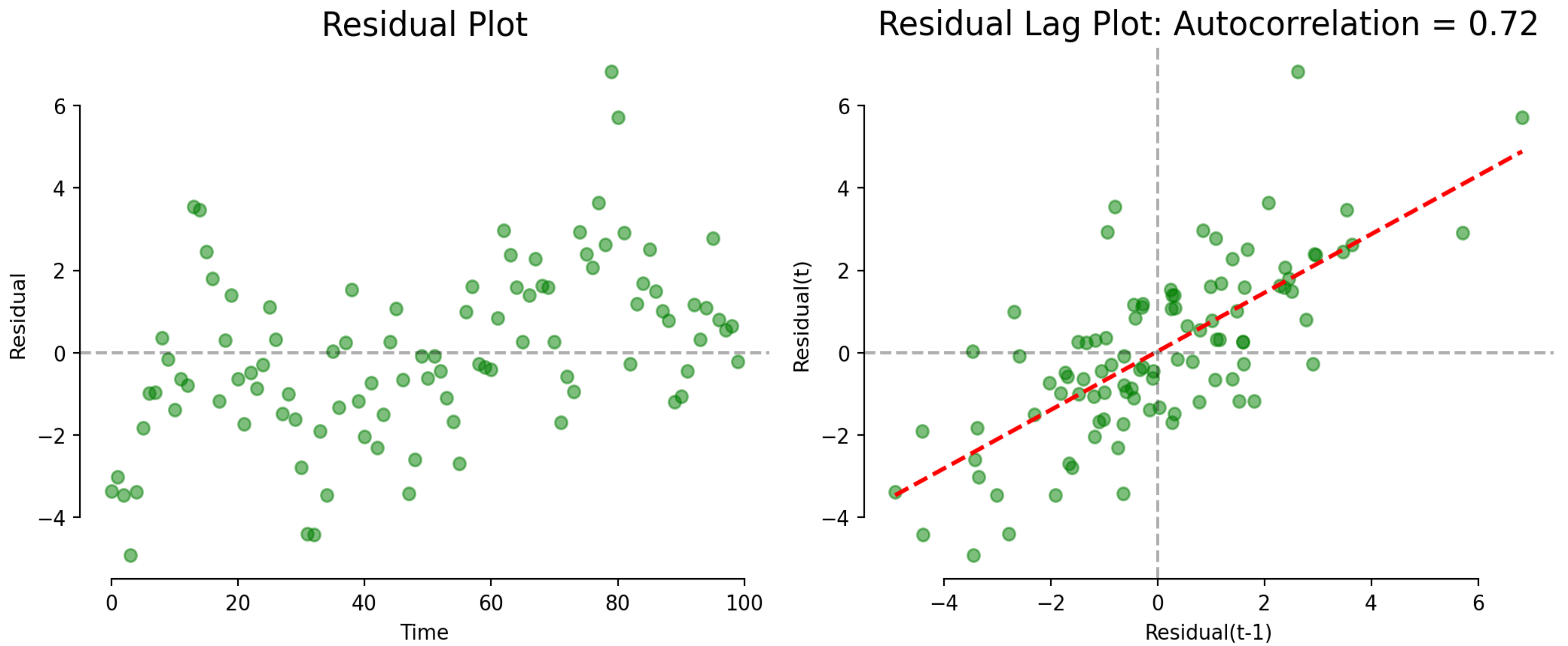


> *you can see it will often be wrong in the same direction repeatedly*

Timeseries: Model 1 (Levels)

GLM's confidence levels requres that the error terms are independent.

$$Y = \beta_0 + \beta_1 \cdot t + \varepsilon$$



> this 'levels' model shows strong patterns in residuals (autocorrelation)

Exercise 4.4 | Model 1 (Levels)

Examine a levels model of the relationship between GDP and unemployment.

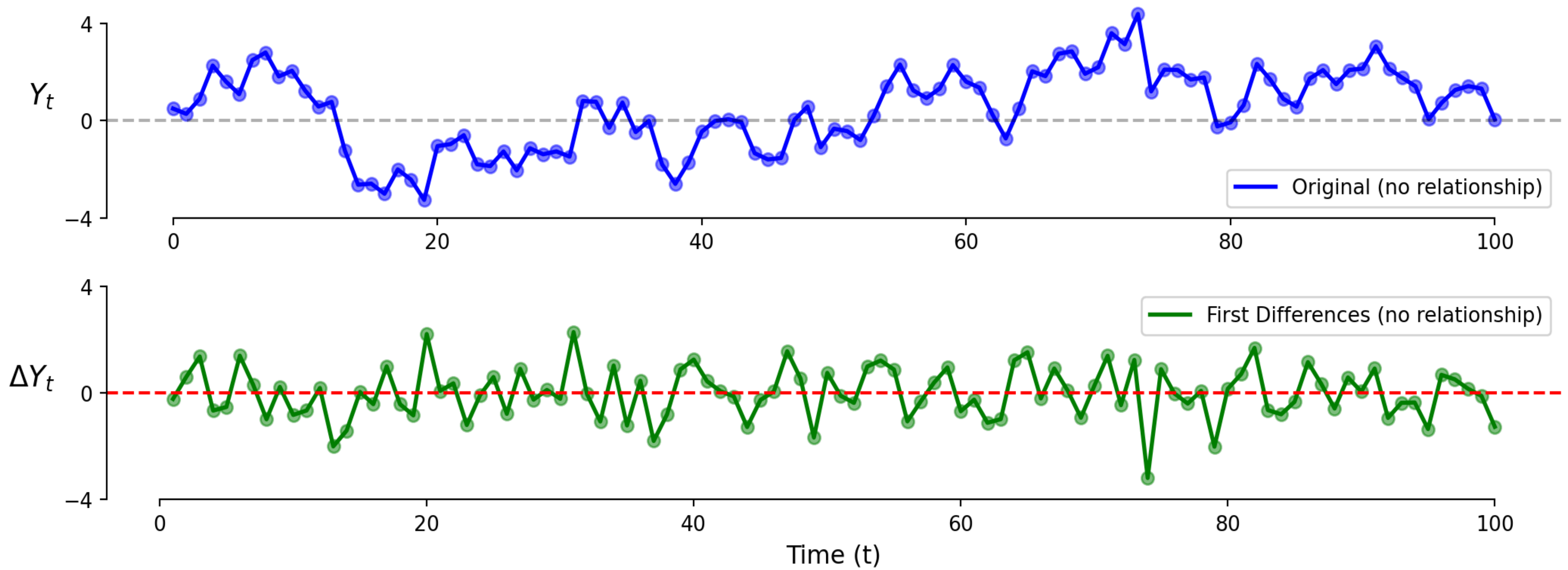
$$Y = \beta_0 + \beta_1 \times t + \varepsilon$$

```
# 1. Fit the levels model
model1 = smf.ols('gdp ~ unemployment', data=data).fit()
print(model1.summary().tables[1])
```

Timeseries: Model 2 (First Differences)

We can fix some issues of autocorrelation by looking at changes instead of levels.

$$\Delta Y_t = Y_t - Y_{t-1}$$

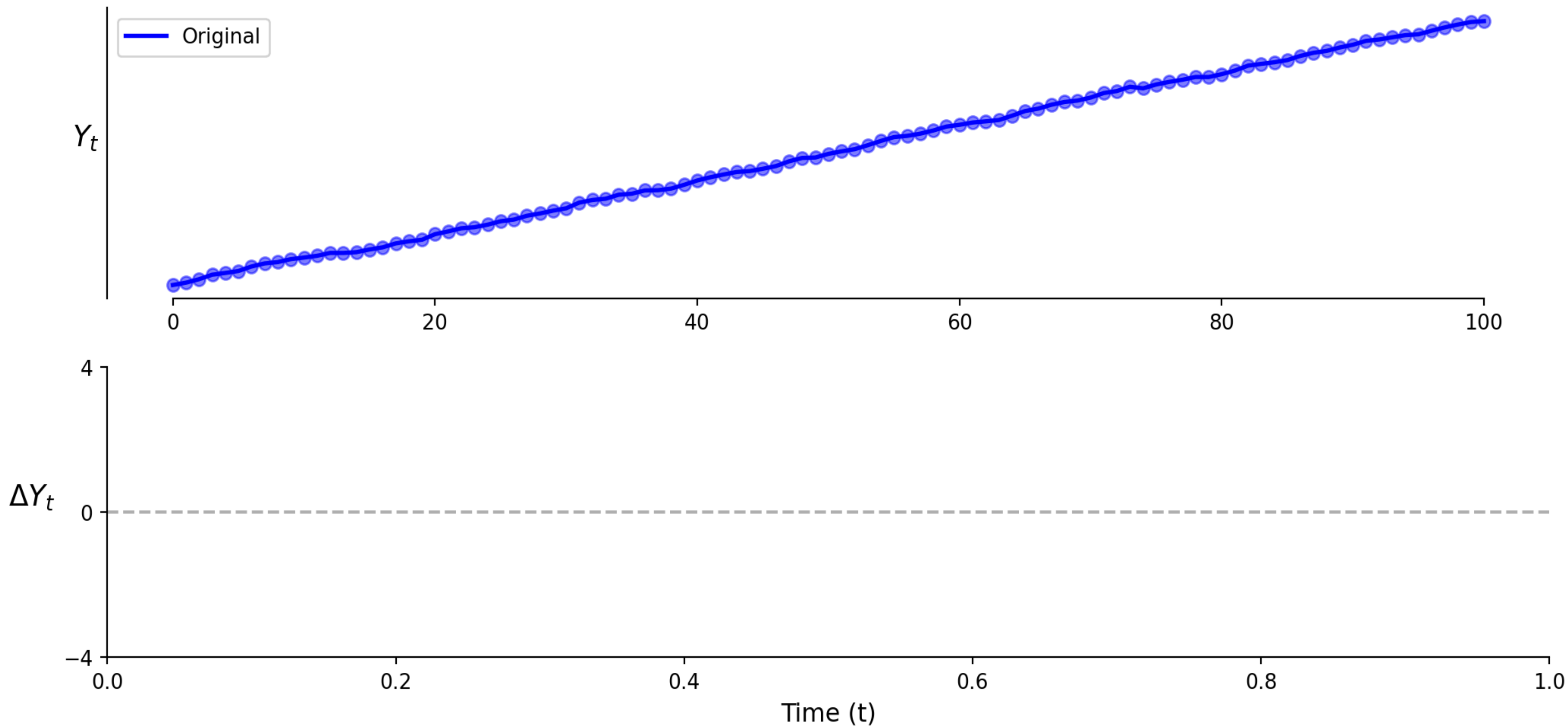


> *differences (correctly in this case) shows no relationship*

> *what would first differences look like if there WAS a positive trend?*

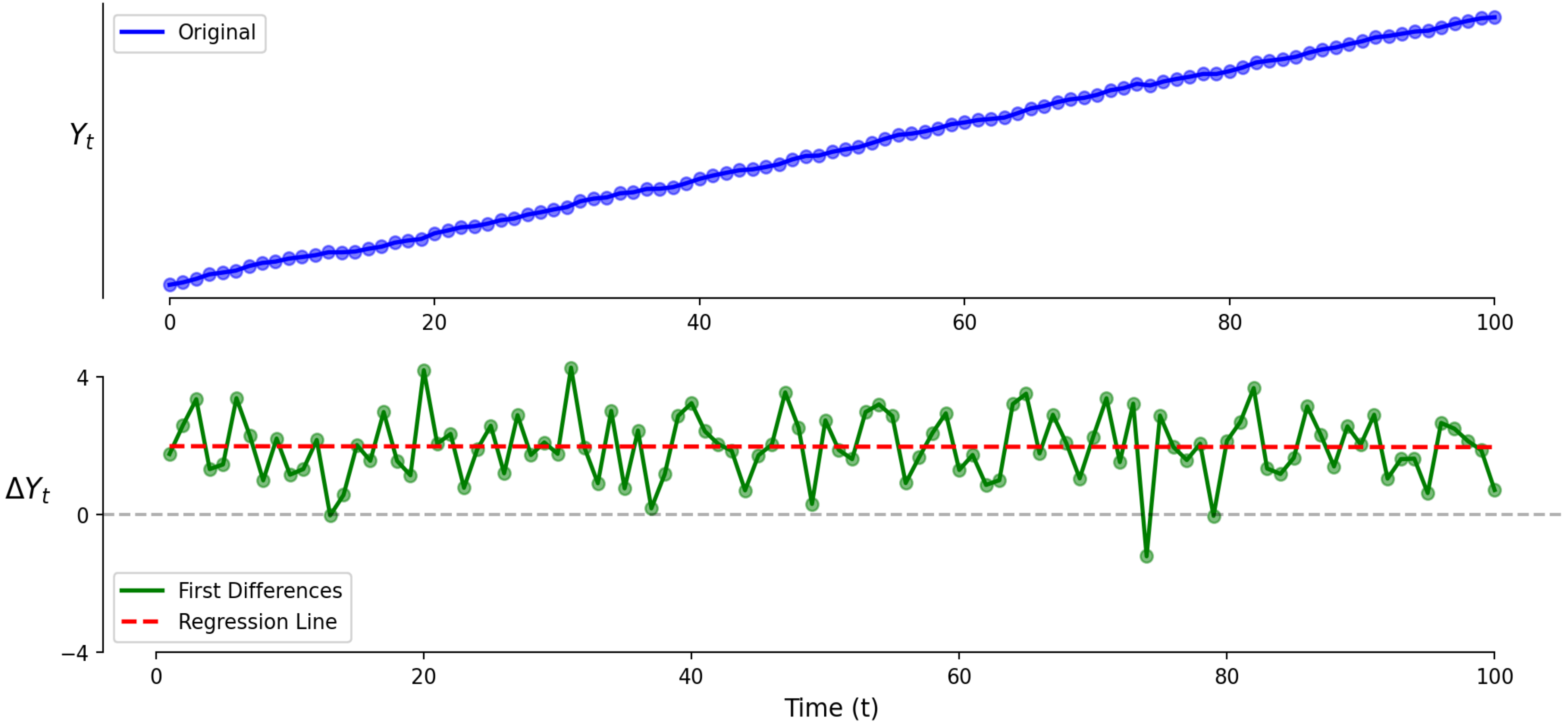
Timeseries: Model 2 (First Differences)

What would first differences look like if there WAS a positive trend?



Timeseries: Model 2 (First Differences)

What would first differences look like if there WAS a positive trend?

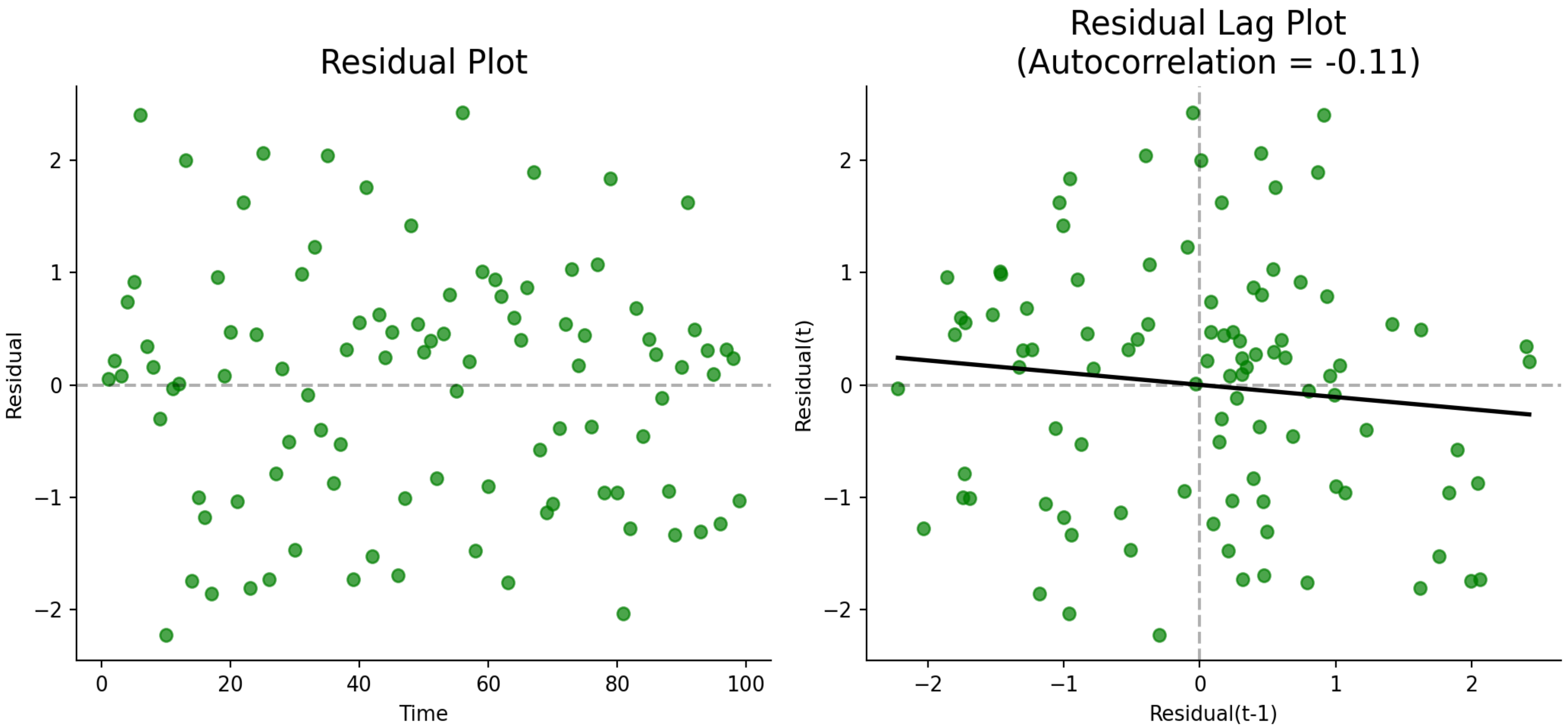


> *the vertical intercept* β_0 *is positive!*

> *the slope coefficient* β_1 *is zero!*

Timeseries: Model 2 (First Differences)

First differences reduces but does not eliminate the problem of autocorrelation.



Exercise 4.4 | Model 2

Examine a first difference model of the relationship between GDP and unemployment.

```
# Step 1. Create first differences variables
data['gdp_diff'] = data['gdp'].diff()
data['unemployment_diff'] = data['unemployment'].diff()
```

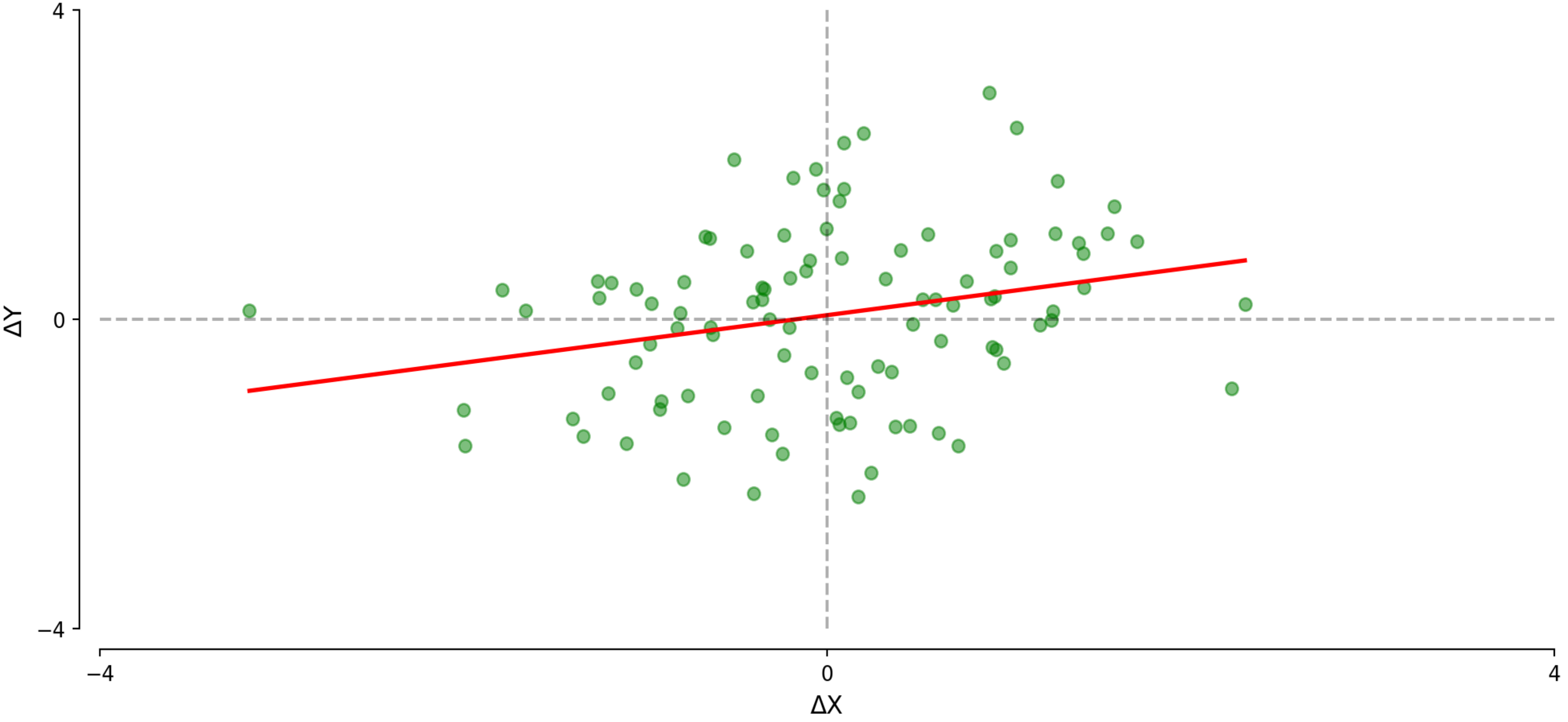
```
# Step 2. Drop the first row which has NaN due to differencing
data = data.dropna()
```

```
# Step 3. Fit the differences model
model2 = smf.ols('gdp_diff ~ unemployment_diff', data=data).fit()
print(model2.summary().tables[1])
```

Timeseries: Model 3 (Double First Differences)

Sometimes we want to measure how two variables move together.

$$\Delta Y_t = \beta_0 + \beta_1 \times \Delta X_t + \varepsilon_t$$



Timeseries: Model 3 (Double First Differences)

Relating changes in X to changes in Y.

$$\Delta Y_t = \beta_0 + \beta_1 \times \Delta X_t + \varepsilon_t$$

- *Further reduces serial correlation in the error terms.*
- β_0 *captures time trend in Y*
- β_1 *captures the short-term relationship between variables.*
- *Clear interpretation: how do changes in X relate to changes in Y?*

Exercise 4.4 | Model 3

Examine a double first difference model of the relationship between GDP and unemployment.

```python
# Step 1. Create first differences variables
data['gdp_diff'] = df['gdp'].diff()
df['unemployment_diff'] = df['unemployment'].diff()
```

```python
# Drop the first row which has NaN due to differencing
data = data.dropna()
```

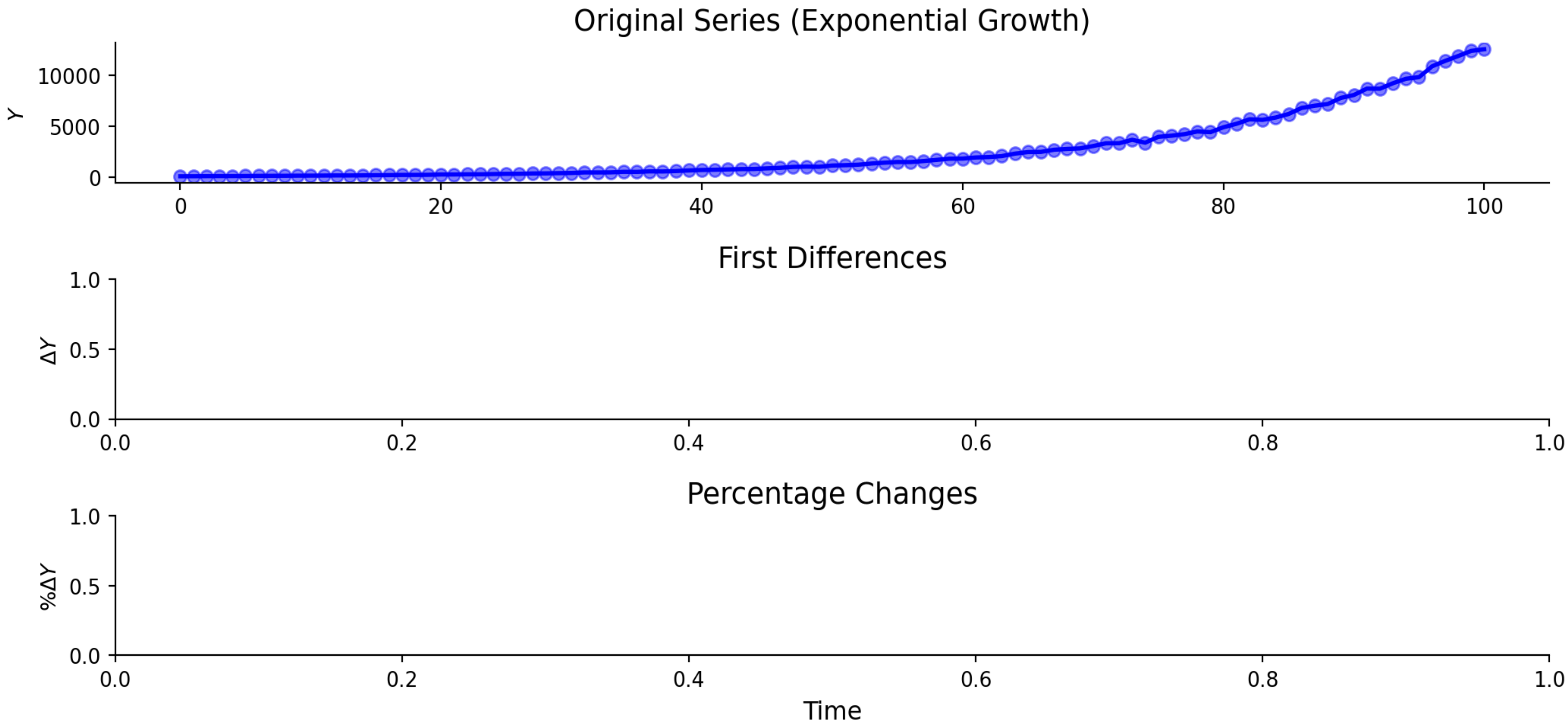
```python
# Fit the differences model
model3 = smf.ols('gdp_diff ~ unemployment_diff', data=data).fit()
print(model3.summary())
```

> β_1 *now represents the short-term relationship between changes in* X *and* Y

Timeseries: Model 4 (Growth Rates)

Proportional changes provide interpretable coefficients:

$$g_Y = \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \frac{\Delta Y_t}{Y_{t-1}}$$



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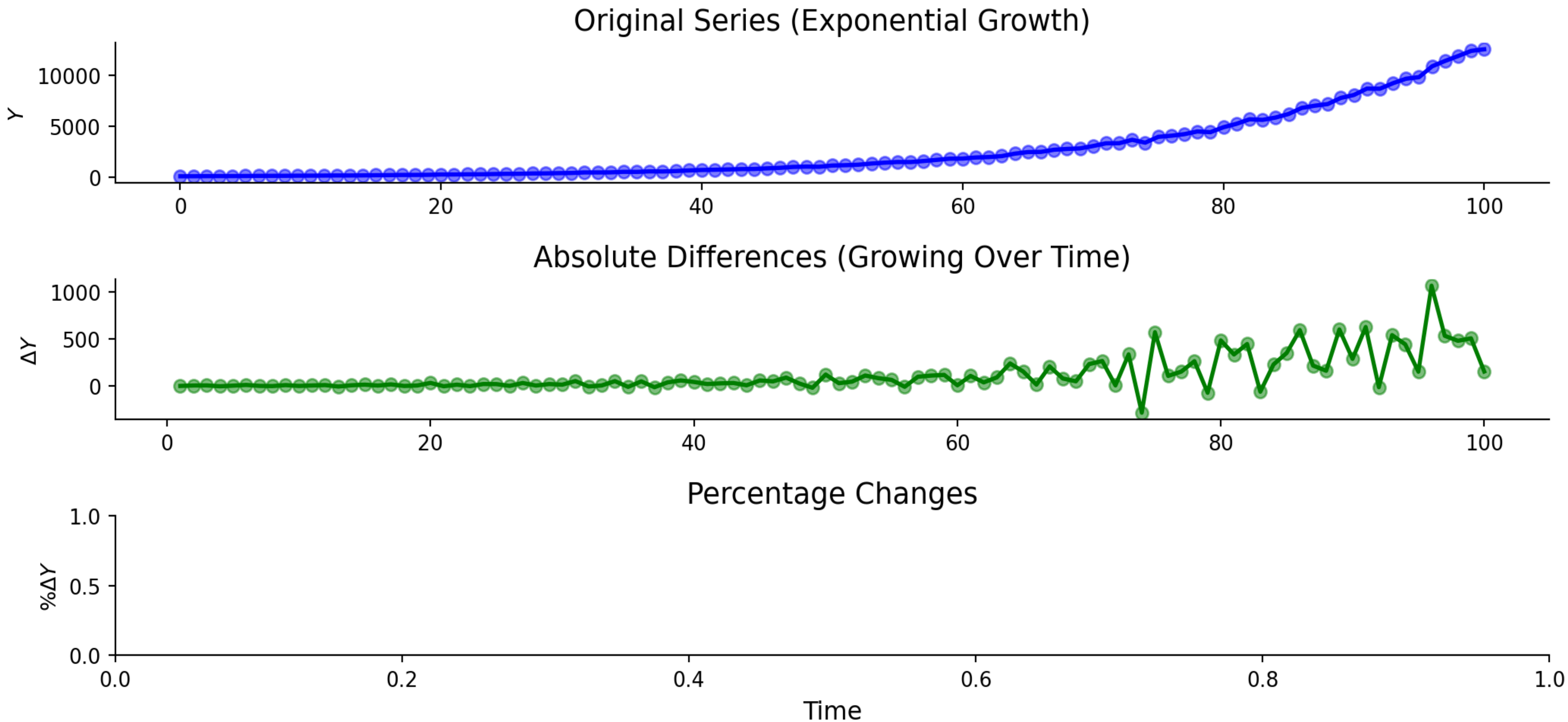

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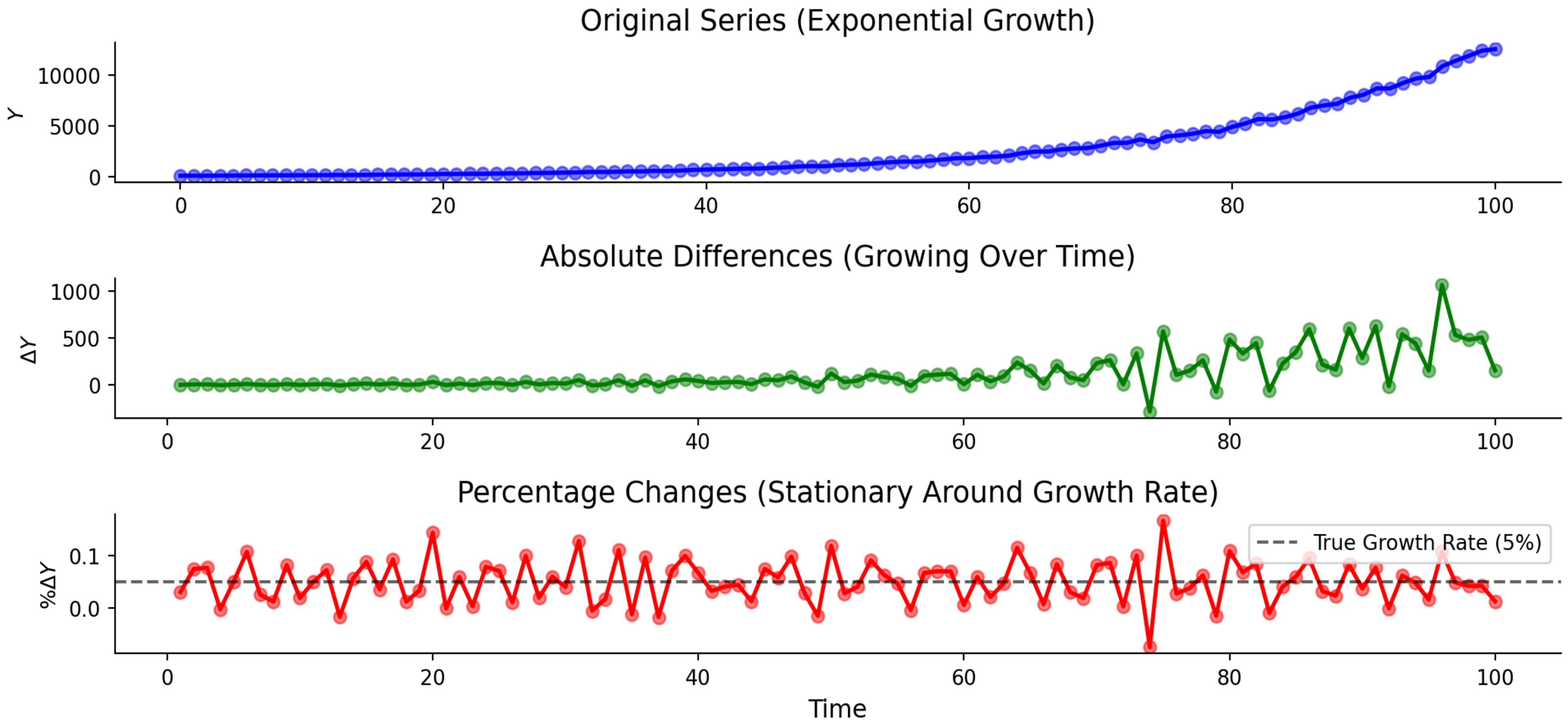
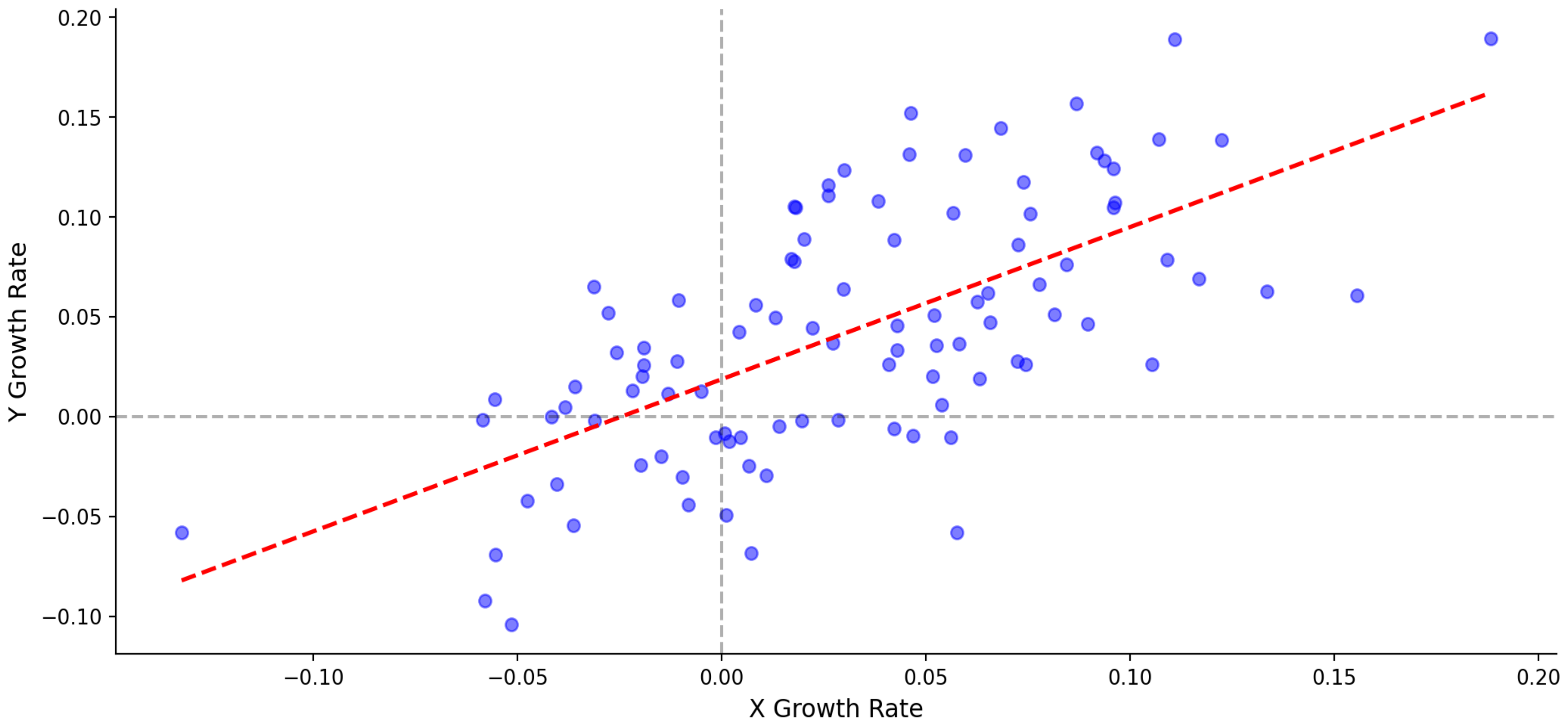


Timeseries: Model 4 (Growth Rates)

Is the growth in Y related to the growth in X?

$$g_Y = \beta_0 + \beta_1 \times g_X + \varepsilon_t$$



Timeseries: Model 4 (Growth Rates)

Is the growth in Y related to the growth in X?

$$g_Y = \beta_0 + \beta_1 \times g_X + \varepsilon_t$$

Growth rate models have the advantages of first differences and can scale better.

- *This is natural for variables with exponential growth.*
- β_0 *is Y's baseline growth rate.*
- β_1 *is how Y's growth responds to a 1 percentage point increase in X's growth.*

Exercise 4.4 | Model 4

Examine a growth rates model of the relationship between GDP and unemployment.

```
# Step 1. Calculate growth rates (percentage changes)
data['gdp_growth'] = data['gdp'].pct_change() # in percentage points
data['unemployment_growth'] = data['unemployment'].pct_change()
```

```
# Step 2. Drop rows with NaN values
data = data.dropna()
```

```
# Step 3. Fit the growth rate model
model4 = smf.ols('gdp_growth ~ unemployment_growth', data=data).fit()
print(model4.summary())
```

> β_1 *is now expressed in percentage point terms*