

```
.bashrc ef12
build() {
    g++ $@ -o $1.e -DLOC -std=c++11 \
        -Wall -Wextra -Wfatal-errors -Wshadow \
        -Wlogical-op -Wconversion -Wfloat-equal
}

b() { build $@ -O2 }

d() { build $@ -fsanitize=address,undefined \
    -D_GLIBCXX_DEBUG -g }

run() { $1 $2 && echo start >&2 && time ./ $2.e }

loo() {
    set -e; $1 $2; $1 $3
    for ((;;)) {
        ./ $3.e > gen.in
        time ./ $2.e < gen.in > gen.out
    }
}

cmp() {
    set -e; $1 $2; $1 $3; $1 $4
    for ((;;)) {
        ./ $4.e > gen.in;          echo -n 0
        ./ $2.e < gen.in > p1.out;  echo -n 1
        ./ $3.e < gen.in > p2.out;  echo -n 2
        diff p1.out p2.out;        echo -n Y
    }
}

# Other flags:
# -Wformat=2 -Wshift-overflow=2 -Wcast-qual
# -Wcast-align -Wduplicated-cond
# -D_GLIBCXX_DEBUG_PEDANTIC -D_FORTIFY_SOURCE=2
# -fno-sanitize-recover -fstack-protector

# Stacktrace on STL assert:
# g++ -D_GLIBCXX_DEBUG -H test.cpp 2>&1
# >/dev/null | grep "debug/macros.h"
# #ifdef __SANITIZE_ADDRESS__
# extern "C"
# void __sanitizer_print_stack_trace();

.vimrc 8b2d
se ai aw cin cul ic is nocp nohls nu sc scs
se bg=dark sw=4 ts=4 so=7 ttm=9
sy on
vn _ :w !cpp -dD -P -fpreprocessed \ | \
    tr -d '[:space:]' \ | md5sum \ | cut -c-4 <CR>
```

```
template.cpp 8fcc
#include <bits/stdc++.h>
using namespace std;

using ll = long long;
using Vi = vector<int>;
using Pii = pair<int,int>;

#define mp make_pair
#define pb push_back
#define x first
#define y second

#define rep(i,b,e) for(int i=(b); i<(e); i++)
#define each(a,x) for(auto& a : (x))
#define all(x) (x).begin(), (x).end()
#define sz(x) int((x).size())

int main() {
    cin.sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(18);
```

```
// Don't call destructors:
cout << flush; _Exit(0);
} // 4799

// > Debug printer
#define tem template<class t,class u,class...w>
#define pri(x,y,z) tem auto operator<<(t&o,u a) \
    ->decltype(x,o) { o << z; return o << y; }

pri(a.print(), ' '), ' '; a.print()
pri(a.y, ' '), ' (' << a.x << ", " << a.y)

pri(all(a), ' '), '['; auto d="";
    for (auto i : a) (o << d << i, d = ", ")

void DD(...) {}
tem void DD(t s, u a, w... k) {
    for (int b=1; *s && *s - b*44; cerr << *s++)
        b += 2 / (*s*2 - 81);
    cerr << " ": " << a << *s++; DD(s, k...);
} // fd6d

#ifdef LOC
#define deb(...) (DD("[, \b :] "#__VA_ARGS__, \
    __LINE__, __VA_ARGS__), cerr << endl)
#else
#define deb(...)
#endif

#define DBP(...) void print() { \
    DD(__VA_ARGS__, __VA_ARGS__); } // 813f

// > Utils

// Return smallest k such that 2^k > n
// Undefined for n = 0!
int uplg(int n) { return 32-__builtin_clz(n); }
int uplg(ll n) { return 64-__builtin_clzll(n); }

// Compare with certain epsilon (branchless)
// Returns -1 if a < b; 1 if a > b; 0 if equal
// a and b are assumed equal if |a-b| <= eps
int cmp(double a, double b, double eps=1e-9) {
    return (a > b+eps) - (a+eps < b);
} // 81c1
```

```
template.java 4cea
import java.io.OutputStream;
import java.io.IOException;
import java.io.InputStream;
import java.io.PrintWriter;
import java.io.InputStreamReader;
import java.util.StringTokenizer;
import java.io.BufferedReader;
import java.io.InputStream;

public class Main {
    public static void main(String[] args) {
        try (PrintWriter out =
            new PrintWriter(System.out)) {
            new Task().solve(
                new InputReader(System.in), out);
        } // f642
    } // cbc2

    static class InputReader {
        BufferedReader r;
        StringTokenizer t = null;

        public InputReader(InputStream s) {
            r = new BufferedReader(
                new InputStreamReader(s), 32768);
        } // 0a0f
```

```
public String next() {
    while (t == null || !t.hasMoreTokens())
        try {
            t = new StringTokenizer(
                r.readLine());
        } catch (IOException e) {
            throw new RuntimeException(e);
        } // fd4a
        return t.nextToken();
    } // d64b

    public int nextInt() {
        return Integer.parseInt(next());
    } // 7737
    } // 94b9

    static class Task {
        public void solve(InputReader in,
            PrintWriter out) {
            int n = in.nextInt();
            out.printf("hello %d", n);
        } // 477d
    } // 30cf
    } // 75cd
```

```
various.h 51cd
// If math constants like M_PI are not found
// add this at the beginning of file
#define _USE_MATH_DEFINES

// Pragmas
#pragma GCC optimize("Ofast,unroll-loops, \
    no-stack-protector")
#pragma GCC target("popcnt,avx,tune=native")

// Clock
while (clock() < duration*CLOCKS_PER_SEC)

// Automatically implement operators:
// 1. != if == is defined
// 2. >, <= and >= if < is defined
using namespace rel_ops;

// Mersenne twister for randomization.
mt19937_64 rnd(chrono::steady_clock::now()
    .time_since_epoch().count());

// To shuffle randomly use:
shuffle(all(vec), rnd);

// To pick random integer from [A;B] use:
uniform_int_distribution<> dist(A, B);
int value = dist(rnd);

// To pick random real number from [A;B] use:
uniform_real_distribution<> dist(A, B);
double value = dist(rnd);
```

```
various.py a9e9
input().split(' ') # Read and split into words
print('abc!', end='') # Print without newline

>>> from fractions import *
>>> Fraction(16, -10)
Fraction(-8, 5)
>>> Fraction(123)
Fraction(123, 1)
>>> Fraction()
Fraction(0, 1)
>>> Fraction('3/7')
Fraction(3, 7)
>>> Fraction(' -3/7 ')
Fraction(-3, 7)
```

```
>>> Fraction('1.414213 \t\n')
>>> Fraction('7e-6')
Fraction(7, 1000000)
>>> Fraction(2.25)
Fraction(9, 4)
>>> Fraction(1.1)
Fraction(2476979795053773, 2251799813685248)
>>> Fraction('1/2') * Fraction('4/3')
Fraction(2, 3)
>>> Fraction(16, 5).numerator
16
>>> Fraction(16, 5).denominator
5

>>> from decimal import *
>>> getcontext().prec = 28
>>> Decimal(10)
Decimal('10')
>>> Decimal('3.14')
Decimal('3.14')
>>> Decimal(3.14)
Decimal('3.14000000000000001243449787580175...')
>>> Decimal((0, (3, 1, 4), -2))
Decimal('3.14')
>>> Decimal(str(2.0 ** 0.5))
Decimal('1.4142135623730951')
>>> Decimal(2) ** Decimal('0.5')
Decimal('1.414213562373095048801688724')
>>> Decimal('NaN')
Decimal('NaN')
>>> Decimal('-Infinity')
Decimal('-Infinity')
```

```
geometry/convex_hull.h 4a8e
#include "vec2.h"

// Translate points such that lower-left point
// is (0, 0). Returns old point location; O(n)
vec2 normPos(vector<vec2>& points) {
    auto q = points[0].yx();
    each(p, points) q = min(q, p.yx());
    vec2 ret(q.y, q.x);
    each(p, points) p = p-ret;
    return ret;
} // ee96

// Find convex hull of points; time: O(n lg n)
// Points are returned counter-clockwise,
// first point is the lowest-left.
vector<vec2> convexHull(vector<vec2> points) {
    vec2 pivot = normPos(points);
    sort(all(points));
    vector<vec2> hull;

    each(p, points) {
        while (sz(hull) >= 2) {
            vec2 a = hull.back() - hull[sz(hull)-2];
            vec2 b = p - hull.back();
            if (a.cross(b) > 0) break;
            hull.pop_back();
        } // ad91
        hull.pb(p);
    } // 5908

    // Translate back, optional
    each(p, hull) p = p+pivot;
    return hull;
} // 62ed

// Find point p that minimizes dot product p*q.
// Returns point index in hull; time: O(lg n)
```

```
// If multiple points have same dot product
// one with smallest index is returned.
// Points are expected to be in the same order
// as output from convexHull function.
int minDot(const vector<vec2>& hull, vec2 q) {
    auto search = [&](int b, int e, vec2 p) {
        while (b+1 < e) {
            int m = (b+e) / 2;
            (p.dot(hull[m-1]) > p.dot(hull[m])
             ? b : e) = m;
        } // 184c
        return b;
    }; // 62e8

    int m = search(0, sz(hull), {0, -1});
    int i = search(0, m, q);
    int j = search(m, sz(hull), q);
    return q.dot(hull[i]) > q.dot(hull[j])
        ? j : i;
} // 4599
```

## geometry/convex\_hull\_dist.h 2859

```
#include "vec2.h"

// Check if p is inside convex polygon. Hull
// must be given in counter-clockwise order.
// Returns 2 if inside, 1 if on border,
// 0 if outside; time: O(n)
int insideHull(vector<vec2>& hull, vec2 p) {
    int ret = 1;
    rep(i, 0, sz(hull)) {
        auto v = hull[(i+1)%sz(hull)] - hull[i];
        auto t = v.cross(p-hull[i]);
        ret = min(ret, cmp(t, 0)); // For doubles
        //ret = min(ret, (t>0) - (t<0)); // Ints
    } // 0d40
    return int(max(ret+1, 0));
} // 1f39

#include "segment2.h"

// Get distance from point to hull; time: O(n)
double hullDist(vector<vec2>& hull, vec2 p) {
    if (insideHull(hull, p)) return 0;
    double ret = 1e30;
    rep(i, 0, sz(hull)) {
        seg2 seg{hull[(i+1)%sz(hull)], hull[i]};
        ret = min(ret, seg.distTo(p));
    } // f3be
    return ret;
} // a00c

// Compare distance from point to hull
// with sqrt(d2); time: O(n)
// -1 if smaller, 0 if equal, 1 if greater
int cmpHullDist(vector<vec2>& hull,
                 vec2 p, ll d2) {
    if (insideHull(hull, p)) return (d2<0)-(d2>0);
    int ret = 1;
    rep(i, 0, sz(hull)) {
        seg2 seg{hull[(i+1)%sz(hull)], hull[i]};
        ret = min(ret, seg.cmpDistTo(p, d2));
    } // 28cb
    return ret;
} // 30f3
```

## geometry/convex\_hull\_sum.h 7f53

```
#include "vec2.h"

// Get edge sequence for given polygon
// starting from lower-left vertex; time: O(n)
```

```
// Returns start position.
vec2 edgeSeq(vector<vec2> points,
             vector<vec2>& edges) {
    int i = 0, n = sz(points);
    rep(j, 0, n)
        if (points[i].yx() > points[j].yx()) i = j;
    rep(j, 0, n) edges.pb(points[(i+j+1)%n] -
                          points[(i+j)%n]);

    return points[i];
} // 3aa7

// Minkowski sum of given convex polygons.
// Vertices are required to be in
// counter-clockwise order; time: O(n+m)
vector<vec2> hullSum(vector<vec2> A,
                    vector<vec2> B) {
    vector<vec2> sum, e1, e2, es(sz(A) + sz(B));
    vec2 pivot = edgeSeq(A, e1) + edgeSeq(B, e2);
    merge(all(e1), all(e2), es.begin());

    sum.pb(pivot);
    each(e, es) sum.pb(sum.back() + e);
    sum.pop_back();
    return sum;
} // f183
```

## geometry/halfplanes.h 356a

```
#include "vec2.h"
#include "line2.h"

// Intersect halfplanes given by `lines`
// and output hull vertices to `out`
// in counter-clockwise order. Returns true
// if intersection is non-empty and bounded.
// Unbounded cases are not supported,
// add bounding-box if necessary. Works only
// with floating point vec2/line2; O(n lg n)
// PARTIALLY TESTED
bool intersectHalfplanes(vector<line2> in,
                       vector<vec2>& out) {
    sort(all(in), [](line2 a, line2 b) {
        return (a.v.angleCmp(b.v) ? :
                a.c*b.v.len() - b.c*a.v.len()) < 0;
    }); // 82fb

    int a = 0, b = 0, n = sz(in);
    vector<line2> dq(n+5);
    out.resize(n+5);
    dq[0] = in[0];

    rep(i, 1, n+1) {
        if (i == n) in.pb(dq[a]);
        if (!in[i].v.angleCmp(in[i-1].v)) continue;
        while (a < b && in[i].side(out[b-1]) > 0)
            b--;
        while (i!=n && a<b && in[i].side(out[a])>0)
            a++;
        if (in[i].intersect(dq[b], out[b]))
            dq[++b] = in[i];
    } // b9ba

    out.resize(b);
    out.erase(out.begin(), out.begin()+a);
    return b-a > 2;
} // f334
```

## geometry/line2.h 9207

```
#include "vec2.h"

// 2D line/halfplane structure
// PARTIALLY TESTED
```

```
// Base class of versions for ints and doubles
template<class T, class P, class S>
struct bline2 {
    // For lines: v * point == c
    // For halfplanes: v * point <= c
    // (i.e. normal vector points outside)
    P v; // Normal vector [A; B]
    T c; // Offset (C parameter of equation)
    DBP(v, c);

    // Line through 2 points; normal vector
    // points to the right of ab vector
    static S through(P a, P b) {
        return { (a-b).perp(), a.cross(b) };
    } // 9ac7

    // Parallel line through point
    static S parallel(P a, S b) {
        return { b.v, b.v.dot(a) };
    } // 8elc

    // Perpendicular line through point
    static S perp(P a, S b) {
        return { b.v.perp(), b.v.cross(a) };
    } // 7b75

    // Distance from point to line
    double distTo(P a) {
        return fabs(v.dot(a)-c) / v.len();
    } // 79e6
}; // ee4f

// Version for integer coordinates (long long)
struct line2i : bline2<ll, vec2i, line2i> {
    line2i() : bline2{{{}, 0} {}
    line2i(vec2i a, ll b) : bline2{a, b} {}

    // Returns 0 if point a lies on the line,
    // 1 if on side where normal vector points,
    // -1 if on the other side.
    int side(vec2i a) {
        ll d = v.dot(a);
        return (d > c) - (d < c);
    } // 18a7
}; // fc9c
```

```
// Version for double coordinates
// Requires cmp() from template
struct line2d : bline2<double, vec2d, line2d> {
    line2d() : bline2{{{}, 0} {}
    line2d(vec2d a, double b) : bline2{a, b} {}

    // Returns 0 if point a lies on the line,
    // 1 if on side where normal vector points,
    // -1 if on the other side.
    int side(vec2d a) { return cmp(v.dot(a), c); }

    // Intersect this line with line a, returns
    // true on success (false if parallel).
    // Intersection point is saved to `out`.
    bool intersect(line2d a, vec2d& out) {
        double d = v.cross(a.v);
        if (!cmp(d, 0)) return 0;
        out = (v*a.c - a.v*c).perp() / d;
        return 1;
    } // 2e68
}; // ab54
```

using line2 = line2d;

## geometry/rmst.h 476a

```
#include "../structures/find_union.h"

// Rectilinear Minimum Spanning Tree
```

```
// (MST in Manhattan metric); time: O(n lg n)
// Returns MST weight. Outputs spanning tree
// to G, vertex indices match point indices.
// Edge in G is pair (target, weight).
ll rmst(vector<Pii>& points,
        vector<vector<Pii>>& G) {
    int n = sz(points);
    vector<pair<int, Pii>> edges;
    vector<Pii> close;
    Vi ord(n), merged(n);
    iota(all(ord), 0);

    function<void(int,int)> octant =
        [&](int begin, int end) {
            if (begin+1 >= end) return;

            int mid = (begin+end) / 2;
            octant(begin, mid);
            octant(mid, end);

            int j = mid;
            Pii best = {INT_MAX, -1};
            merged.clear();

            rep(i, begin, mid) {
                int v = ord[i];
                Pii p = points[v];

                while (j < end) {
                    int e = ord[j];
                    Pii q = points[e];
                    if (q.x-q.y > p.x-p.y) break;
                    best = min(best, make_pair(q.x+q.y, e));
                    merged.pb(e);
                    j++;
                } // 8576

                if (best.y != -1) {
                    int alt = best.x-p.x-p.y;
                    if (alt < close[v].x)
                        close[v] = {alt, best.y};
                } // 4208
                merged.pb(v);
            } // f3ff

            while (j < end) merged.pb(ord[j++]);
            copy(all(merged), ord.begin()+begin);
        }; // a4e1

    rep(i, 0, 4) {
        rep(j, 0, 2) {
            sort(all(ord), [&](int l, int r) {
                return points[l] < points[r];
            }); // fe33
            close.assign(n, {INT_MAX, -1});
            octant(0, n);
            rep(k, 0, n) {
                Pii p = close[k];
                if (p.y != -1) edges.pb({p.x, {k, p.y}});
                points[k].x *= -1;
            } // 1c1d
        } // 9b38
    } // d06f

    ll sum = 0;
    FAU fau(n);
    sort(all(edges));
    G.assign(n, {});

    each(e, edges) if (fau.join(e.y.x, e.y.y)) {
        sum += e.x;
        G[e.y.x].pb({e.y.y, e.x});
    }
```

```

    G[e.y.y].pb({e.y.x, e.x});
} // b04a
return sum;
} // f586
geometry/segment2.h 6504
#include "vec2.h"

// 2D segment structure; PARTIALLY TESTED

// Base class of versions for ints and doubles
template<class P, class S> struct bseg2 {
    P a, b; // Endpoints

    // Distance from segment to point
    double distTo(P p) const {
        if ((p-a).dot(b-a) < 0) return (p-a).len();
        if ((p-b).dot(a-b) < 0) return (p-b).len();
        return double(abs((p-a).cross(b-a)))
            / (b-a).len();
    } // 62a2
}; // 85bc

// Version for integer coordinates (long long)
struct seg2i : bseg2<vec2i, seg2i> {
    seg2i() {}
    seg2i(vec2i c, vec2i d) : bseg2{c, d} {}

    // Check if segment contains point p
    bool contains(vec2i p) {
        return (a-p).dot(b-p) <= 0 &&
            (a-p).cross(b-p) == 0;
    } // c598

    // Compare distance to p with sqrt(d2)
    // -1 if smaller, 0 if equal, 1 if greater
    int cmpDistTo(vec2i p, ll d2) const {
        if ((p-a).dot(b-a) < 0) {
            ll l = (p-a).len2();
            return (l > d2) - (l < d2);
        } // dla6
        if ((p-b).dot(a-b) < 0) {
            ll l = (p-b).len2();
            return (l > d2) - (l < d2);
        } // 9e65
        ll c = abs((p-a).cross(b-a));
        d2 *= (b-a).len2();
        return (c*c > d2) - (c*c < d2);
    } // 726d
}; // 4df2

// Version for double coordinates
// Requires cmp() from template
struct seg2d : bseg2<vec2d, seg2d> {
    seg2d() {}
    seg2d(vec2d c, vec2d d) : bseg2{c, d} {}

    bool contains(vec2d p) {
        return cmp((a-p).dot(b-p), 0) <= 0 &&
            cmp((a-p).cross(b-p), 0) == 0;
    } // b507
}; // 2036

using seg2 = seg2d;
geometry/vec2.h 6e47
// 2D point/vector structure; PARTIALLY TESTED

// Base class of versions for ints and doubles
template<class T, class S> struct bvec2 {
    T x, y;
    S operator+(S r) const {return{x+r.x,y+r.y};}
    S operator-(S r) const {return{x-r.x,y-r.y};}

```

```

    S operator*(T r) const { return {x*r, y*r}; }
    S operator/(T r) const { return {x/r, y/r}; }

    T dot(S r) const { return x*r.x + y*r.y; }
    T cross(S r) const { return x*r.y - y*r.x; }
    T len2() const { return x*x + y*y; }
    double len() const { return hypot(x, y); }
    S perp() const { return {-y,x}; } // CCW

    pair<T, T> yx() const { return {y, x}; }

    double angle() const { //[0;2*PI] CCW from OX
        double a = atan2(y, x);
        return (a < 0 ? a+2*M_PI : a);
    } // 7095
}; // 17ed

// Version for integer coordinates (long long)
struct vec2i : bvec2<ll, vec2i> {
    vec2i() : bvec2{0, 0} {}
    vec2i(ll a, ll b) : bvec2{a, b} {}

    bool upper() const { return (y ? x) >= 0; }

    int angleCmp(vec2i r) const {
        ll c = cross(r);
        return r.upper()-upper() ? : (c<0) - (c>0);
    } // b35f

    // Compare by angle, length if angles equal
    bool operator<(vec2i r) const {
        return (angleCmp(r) ? :
            len2() - r.len2()) < 0;
    } // 6f78

    bool operator==(vec2i r) const {
        return x == r.x && y == r.y;
    } // 136e
}; // d3f4

// Version for double coordinates
// Requires cmp() from template
struct vec2d : bvec2<double, vec2d> {
    vec2d() : bvec2{0, 0} {}
    vec2d(double a, double b) : bvec2{a, b} {}

    bool upper() const {
        return cmp(y, 0) ? : cmp(x, 0) >= 0;
    } // 086c

    int angleCmp(vec2d r) const {
        return r.upper() - upper() ? :
            cmp(0, cross(r));
    } // 12f3

    // Compare by angle, length if angles equal
    bool operator<(vec2d r) const {
        return (angleCmp(r) ? :
            cmp(len2(), r.len2())) < 0;
    } // f3d7

    bool operator==(vec2d r) const {
        return !cmp(x, r.x) && !cmp(y, r.y);
    } // 81cd

    vec2d unit() const { return *this / len(); }

    vec2d rotate(double a) const { // CCW
        return {x*cos(a) - y*sin(a),
            x*sin(a) + y*cos(a)}; // 1890
    } // 97e3
}; // 08e9

using vec2 = vec2d;

```

```

graphs/2sat.h 2443
// 2-SAT solver; time: O(n+m), space: O(n+m)
// Variables are indexed from 1 and
// negative indices represent negations!
// Usage: SAT2 sat(variable_count);
// (add constraints...)
// bool solution_found = sat.solve();
// sat[i] = value of i-th variable, 0 or 1
// (also indexed from 1!)
// (internally: positive = i*2-1, neg. = i*2-2)
struct SAT2 : Vi {
    vector<Vi> G;
    Vi order, flags;

    // Init n variables, you can add more later
    SAT2(int n = 0) : G(n*2) {}

    // Add new var and return its index
    int addVar() {
        G.resize(sz(G)+2); return sz(G)/2;
    } // 98f3

    // Add (i => j) constraint
    void imply(int i, int j) {
        i = i*2 ^ i >> 31;
        j = j*2 ^ j >> 31;
        G[--i].pb(--j); G[j^1].pb(i^1);
    } // 8e25

    // Add (i v j) constraint
    void either(int i, int j) { imply(-i, j); }

    // Constraint at most one true variable
    void atMostOne(Vi& vars) {
        int y, x = addVar();
        each(i, vars) {
            imply(x, y = addVar());
            imply(i, -x); imply(i, x = y);
        } // 24aa
    } // 3ed7

    // Solve and save assignments in `values`
    bool solve() { // O(n+m), Kosaraju is used
        assign(sz(G)/2+1, -1);
        flags.assign(sz(G), 0);
        rep(i, 0, sz(G)) dfs(i);
        while (!order.empty()) {
            if (!propag(order.back()^1, 1)) return 0;
            order.pop_back();
        } // 5594
        return 1;
    } // 1e58

    void dfs(int i) {
        if (flags[i]) return;
        flags[i] = 1;
        each(e, G[i]) dfs(e);
        order.pb(i);
    } // d076

    bool propag(int i, bool first) {
        if (!flags[i]) return 1;
        flags[i] = 0;
        if (at(i/2+1) >= 0) return first;
        at(i/2+1) = i&1;
        each(e, G[i]) if (!propag(e, 0)) return 0;
        return 1;
    } // 4c1b
}; // d74c

graphs/bellman_ineq.h cd51
struct Ineq {

```

```

    ll a, b, c; // a - b >= c
}; // 663a

// Solve system of inequalities of form a-b>=c
// using Bellman-Ford; time: O(n*m)
bool solveIneq(vector<Ineq>& edges,
    vector<ll>& vars) {
    rep(i, 0, sz(vars)) each(e, edges)
        vars[e.b] = min(vars[e.b], vars[e.a]-e.c);
    each(e, edges)
        if (vars[e.a]-e.c < vars[e.b]) return 0;
    return 1;
} // 241e

graphs/biconnected.h 2b9e
// Biconnected components; time: O(n+m)
// Usage: Biconnected bi(graph);
// bi[v] = indices of components containing v
// bi.verts[i] = vertices of i-th component
// bi.edges[i] = edges of i-th component
// Bridges <=> components with 2 vertices
// Articulation points <=> vertices that belong
// to > 1 component
// Isolated vertex <=> empty component list
struct Biconnected : vector<Vi> {
    vector<Vi> verts;
    vector<vector<Pii>> edges;
    vector<Pii> S;

    Biconnected() {}

    Biconnected(vector<Vi>& G) : S(sz(G)) {
        resize(sz(G));
        rep(i, 0, sz(G)) S[i].x ? : dfs(G, i, -1);
        rep(c, 0, sz(verts)) each(v, verts[c])
            at(v).pb(c);
    } // cfce

    int dfs(vector<Vi>& G, int v, int p) {
        int low = S[v].x = sz(S)-1;
        S.pb({v, -1});

        each(e, G[v]) if (e != p) {
            if (S[e].x < S[v].x) S.pb({v, e});
            low = min(low, S[e].x ? : dfs(G, e, v));
        } // 446d

        if (p+1 && low >= S[p].x) {
            verts.pb({p}); edges.pb({});
            rep(i, S[v].x, sz(S)) {
                if (S[i].y == -1)
                    verts.back().pb(S[i].x);
                else
                    edges.back().pb(S[i]);
            } // 4fab
            S.resize(S[v].x);
        } // 6d66

        return low;
    } // 7fcc
}; // 4fa4

graphs/bip_edge_coloring.h 647c
// Bipartite edge coloring; time: O(nm)
// `edges` is list of (left vert, right vert),
// where vertices on both sides are indexed
// from 0 to n-1. Returns number of used colors
// (which is equal to max degree).
// col[i] = color of i-th edge [0..max_deg-1]
int colorEdges(vector<Pii>& edges,
    int n, Vi& col) {
    int m = sz(edges), c[2] = {}, ans = 0;

```

```

Vi deg[2];
vector<vector<Pii>> has[2];
col.assign(m, 0);
rep(i, 0, 2) {
    deg[i].resize(n+1);
    has[i].resize(n+1, vector<Pii>(n+1));
} // b876

function<void(int,int)> dfs =
[&](int x, int p) {
    Pii i = has[p][x][c[p]];
    if (has[!p][i.x][c[p]].y) dfs(i.x, !p);
    else has[!p][i.x][c[p]] = {};
    has[p][x][c[p]] = i;
    has[!p][i.x][c[p]] = {x, i.y};
    if (i.y) col[i.y-1] = c[p]-1;
}; // 19ad

rep(i, 0, m) {
    int x[2] = {edges[i].x+1, edges[i].y+1};
    rep(d, 0, 2) {
        deg[d][x[d]]++;
        ans = max(ans, deg[d][x[d]]);
        for (c[d] = 1; has[d][x[d]][c[d]].y;)
            c[d]++;
    } // 9454
    if (c[0]-c[1]) dfs(x[1], 1);
    rep(d, 0, 2)
        has[d][x[d]][c[0]] = {x[!d], i+1};
    col[i] = c[0]-1;
} // 5678
return ans;
} // 1bfc

```

### graphs/boski\_matching.h dbd2

```

// Bosek's algorithm for partially online
// bipartite maximum matching - white vertices
// are fixed, black vertices are added
// one by one; time: O(E*sqrt(V))
// Usage: Matching match(num_white);
// match[v] = index of black vertex matched to
// white vertex v or -1 if unmatched
// match.add(indices_of_white_neighbours);
// Black vertices are indexed in order they
// were added, the first black vertex is 0.
struct Matching : Vi {
    vector<Vi> adj;
    Vi rank, low, pos, vis, seen;
    int k[0];

    // Initialize structure for n white vertices
    Matching(int n = 0) : Vi(n, -1), rank(n) {}

    // Add new black vertex with its neighbours
    // given by `vec`. Returns true if maximum
    // matching is increased by 1.
    bool add(Vi vec) {
        adj.pb(move(vec));
        low.pb(0); pos.pb(0); vis.pb(0);
        if (!adj.back().empty()) {
            int i = k;
nxt:
            seen.clear();
            if (dfs(sz(adj)-1, ++k-i)) return 1;
            each(v, seen) each(e, adj[v])
                if (rank[e] < 1e9 && vis[at(e)] < k)
                    goto nxt;
            each(v, seen) each(w, adj[v])
                rank[w] = low[v] = 1e9;
        } // 6aec

```

```

    return 0;
} // d2a7

bool dfs(int v, int g) {
    if (vis[v] < k) vis[v] = k, seen.pb(v);
    while (low[v] < g) {
        int e = adj[v][pos[v]];
        if (at(e) != v && low[v] == rank[e]) {
            rank[e]++;
            if (at(e) == -1 || dfs(at(e), rank[e]))
                return at(e) = v, 1;
        } else if (++pos[v] == sz(adj[v])) {
            pos[v] = 0; low[v]++;
        } // e532
    } // 3d88
    return 0;
} // 8561
}; // aac1

```

### graphs/bridges\_online.h 4124

```

// Dynamic 2-edge connectivity queries
// Usage: Bridges bridges(vertex_count);
// - bridges.addEdge(u, v); - add edge (u, v)
// - bridges.cc[v] = connected component ID
// - bridges.bi(v) = 2-edge connected comp ID
struct Bridges {
    vector<Vi> G; // Spanning forest
    Vi cc, size, par, bp, seen;
    int cnt[0];

    // Initialize structure for n vertices; O(n)
    Bridges(int n = 0) : G(n), cc(n), size(n, 1),
        par(n, -1), bp(n, -1),
        seen(n) {

        iota(all(cc), 0);
    } // ed70

    // Add edge (u, v); time: amortized O(lg n)
    void addEdge(int u, int v) {
        if (cc[u] == cc[v]) {
            int r = lca(u, v);
            for (int x : {u, v})
                while ((x = root(x)) != r)
                    x = bp[bi(x)] = par[x];
        } else {
            G[u].pb(v); G[v].pb(u);
            if (size[cc[u]] > size[cc[v]]) swap(u, v);
            size[cc[v]] += size[cc[u]];
            dfs(u, v);
        } // abc7
    } // a6fd

    // Get 2-edge connected component ID
    int bi(int v) { // amortized time: < O(lg n)
        return bp[v] + 1 ? bp[v] = bi(bp[v]) : v;
    } // 3206

    int root(int v) {
        return par[v] == -1 || bi(par[v]) != bi(v)
            ? v : par[v] = root(par[v]);
    } // 2d27

    void dfs(int v, int p) {
        cc[v] = cc[par[v] = p];
        each(e, G[v]) if (e != p) dfs(e, v);
    } // 85f5

    int lca(int u, int v) { // Don't use this!
        for (cnt++; swap(u, v) if (u != -1) {
            if (seen[u = root(u)] == cnt) return u;
            seen[u] = cnt; u = par[u];
        } // afed

```

```

    } // 7f56
}; // bd70

```

### graphs/dense\_dfs.h 4fbd

```

#include "../math/bit_matrix.h"

// DFS over bit-packed adjacency matrix
// G = NxN adjacency matrix of graph
// G(i, j) <=> (i, j) is edge
// V = 1xN matrix containing unvisited vertices
// V(0, i) <=> i-th vertex is not visited
// Total DFS time: O(n^2/64)
struct DenseDFS {
    BitMatrix G, V; // space: O(n^2/64)

    // Initialize structure for n vertices
    DenseDFS(int n = 0) : G(n, n), V(1, n) {
        reset();
    } // 79e4

    // Mark all vertices as unvisited
    void reset() { each(x, V.M) x = -1; }

    // Get/set visited flag for i-th vertex
    void setVisited(int i) { V.set(0, i, 0); }
    bool isVisited(int i) { return !V(0, i); }

    // DFS step: func is called on each unvisited
    // neighbour of i. You need to manually call
    // setVisited(child) to mark it visited
    // or this function will call the callback
    // with the same vertex again.
    template<class T>
    void step(int i, T func) {
        ull* E = G.row(i);
        for (int w = 0; w < G.stride;) {
            ull x = E[w] & V.row(0)[w];
            if (x) func((w<<6) | __builtin_ctzll(x));
            else w++;
        } // 4c0a
    } // f045
}; // 8edb

```

### graphs/directed\_mst.h f935

```

#include "../structures/find_union_undo.h"

struct Edge {
    int a, b;
    ll w;
}; // 88dd

struct Node {
    Edge key;
    Node *l, *r;
    ll delta;
    void prop() {
        key.w += delta;
        if (l) l->delta += delta;
        if (r) r->delta += delta;
        delta = 0;
    } // 3f0f
    Edge top() { prop(); return key; }
}; // 019f

Node* merge(Node* a, Node* b) {
    if (!a || !b) return a ? b :
        a->prop(); b->prop();
    if (a->key.w > b->key.w) swap(a, b);
    swap(a->l, (a->r = merge(b, a->r)));
    return a;
} // 34a3

void pop(Node*& a) {

```

```

    a->prop(); a = merge(a->l, a->r);
} // 666b

// Find directed minimum spanning tree
// rooted at vertex `root`; O(m log n)
// Returns weight of found spanning tree.
// out[i] = parent of i-th vertex in the tree,
// out[root] = -1
ll dmst(vector<Edge>& edges,
    int n, int root, Vi& out) {
    RollbackFAU uf(n);
    vector<Node*> heap(n);

    each(e, edges)
        heap[e.b] = merge(heap[e.b],
            new Node{e, 0, 0, 0});

    ll res = 0;
    Vi seen(n, -1), path(n);
    seen[root] = root;

    struct Cycle { int u, t; vector<Edge> e; };
    vector<Edge> Q(n), in(n, {-1, -1, 0}), comp;
    vector<Cycle> cycs;

    rep(s, 0, n) {
        int u = s, qi = 0, w;
        while (seen[u] < 0) {
            if (!heap[u]) return out.clear(), -1;
            Edge e = heap[u]->top();
            heap[u]->delta -= e.w; pop(heap[u]);
            Q[qi] = e; path[qi++] = u; seen[u] = s;
            res += e.w; u = uf.find(e.a);
            if (seen[u] == s) {
                Node* cyc = 0;
                int end = qi, time = uf.time();
                do cyc = merge(cyc, heap[w=path[--qi]]);
                while (uf.join(u, w));
                heap[u = uf.find(u)] = cyc;
                seen[u] = -1;
                cycs.pb({u, time, {&Q[qi], &Q[end]}});
            } // 4ff6
        } // 05d8
        rep(i, 0, qi) in[uf.find(Q[i].b)] = Q[i];
    } // fc95

    reverse(all(cycs));
    each(c, cycs) {
        uf.rollback(c.t);
        Edge tmp = in[c.u];
        each(e, c.e) in[uf.find(e.b)] = e;
        in[uf.find(tmp.b)] = tmp;
    } // 957d
    out.resize(n);
    rep(i, 0, n) out[i] = in[i].a;
    return res;
} // b30d

```

### graphs/dominators.h aec6

```

// Tarjan's algorithm for finding dominators
// in directed graph; time: O(m log n)
// Returns array of immediate dominators idom.
// idom[root] = root
// idom[v] = -1 if v is unreachable from root
Vi dominators(const vector<Vi>& G, int root) {
    int n = sz(G);
    vector<Vi> in(n), bucket(n);
    Vi pre(n, -1), anc(n, -1), par(n), best(n);
    Vi ord, idom(n, -1), sdom(n, n), rdom(n);

    function<void(int,int)> dfs =
[&](int v, int p) {

```



```

    if (pre[v] == -1) {
        par[v] = p;
        pre[v] = sz(ord);
        ord.pb(v);
        each(e, G[v]) in[e].pb(v), dfs(e, v);
    } // 1182
}; // ffd2

function<Pii(int)> find = [&](int v) {
    if (anc[v] == -1) return mp(best[v], v);
    int b; tie(b, anc[v]) = find(anc[v]);
    if (sdom[b] < somd[best[v]]) best[v] = b;
    return mp(best[v], anc[v]);
}; // c07b

rdom[root] = idom[root] = root;
iota(all(best), 0);
dfs(root, -1);

rep(i, 0, sz(ord)) {
    int v = ord[sz(ord)-i-1], b = pre[v];
    each(e, in[v])
        b = min(b, pre[e] < pre[v] ? pre[e] :
                somd[find(e).x]);
    each(u, bucket[v]) rdom[u] = find(u).x;
    somd[v] = b;
    anc[v] = par[v];
    bucket[ord[sdom[v]]].pb(v);
} // 54f4

each(v, ord) idom[v] = (rdom[v] == v ?
    ord[sdom[v]] : idom[rdom[v]]);
return idom;
} // 0656

```

### graphs/edge\_coloring.h 439b

```

// General graph edge coloring; time: O(nm)
// Finds (D+1)-edge-coloring of given graph,
// where D is max vertex degree.
// Returns vector of edge colors `col`.
// col[i] = color of i-th edge [0..D]
Vi vizing(vector<Pii>& edges, int n) {
    Vi cc(n+1), ret(sz(edges)),
        fan(n), fre(n), loc;
    each(e, edges) cc[e.x]++, cc[e.y]++;
    int u, v, cnt = *max_element(all(cc)) + 1;
    vector<Vi> adj(n, Vi(cnt, -1));
    each(e, edges) {
        tie(u, v) = e;
        fan[0] = v;
        loc.assign(cnt, 0);
        int at = u, end = u, d, c = fre[u],
            ind = 0, i = 0;
        while (d = fre[v],
            !loc[d] && (v = adj[u][d]) != -1)
            loc[d] = ++ind, cc[ind] = d, fan[ind] = v;
        cc[loc[d]] = c;
        for (int cd = d; at+1; cd ^= c ^ d,
            at = adj[at][cd])
            swap(adj[at][cd], adj[end=at][cd^c^d]);
        while (adj[fan[i]][d] + 1) {
            int x = fan[i], y = fan[i+1], f = cc[i];
            adj[u][f] = x; adj[x][f] = u;
            adj[y][f] = -1; fre[y] = f;
        } // 0024
        adj[u][d] = fan[i];
        adj[fan[i]][d] = u;
        for (int y : {fan[0], u, end})
            for (int& z = fre[y] = 0; adj[y][z]+1;
                z++);
    }
}

```

```

} // 4240
rep(i, 0, sz(edges))
    for (tie(u, v) = edges[i];
        adj[u][ret[i]] != v; ++ret[i];
    return ret;
} // c259

```

### graphs/edmonds\_karp.h 8326

```

using flow_t = int;
constexpr flow_t INF = 1e9+10;

// Edmonds-Karp algorithm for finding
// maximum flow in graph; time: O(V*E^2)
struct MaxFlow {
    struct Edge {
        int dst, inv;
        flow_t flow, cap;
    }; // a53c

    vector<vector<Edge>> G;
    vector<flow_t> add;
    Vi prev;

    // Initialize for n vertices
    MaxFlow(int n = 0) : G(n) {}

    // Add new vertex
    int addVert() { G.pb({}); return sz(G)-1; }

    // Add edge from u to v with capacity cap
    // and reverse capacity rcap.
    // Returns edge index in adjacency list of u.
    int addEdge(int u, int v,
        flow_t cap, flow_t rcap = 0) {
        G[u].pb({ v, sz(G[v]), 0, cap });
        G[v].pb({ u, sz(G[u])-1, 0, rcap });
        return sz(G[u])-1;
    } // c96a

    // Compute maximum flow from src to dst.
    flow_t maxFlow(int src, int dst) {
        flow_t i, m, f = 0;
        each(v, G) each(e, v) e.flow = 0;

        nxt:
        queue<int> Q;
        Q.push(src);
        prev.assign(sz(G), -1);
        add.assign(sz(G), -1);
        add[src] = INF;

        while (!Q.empty()) {
            m = add[i = Q.front()];
            Q.pop();

            if (i == dst) {
                while (i != src) {
                    auto& e = G[i][prev[i]];
                    e.flow += m;
                    G[i] = e.dst[e.inv].flow += m;
                } // 1f86
                f += m;
                goto nxt;
            } // 43a2

            each(e, G[i])
                if (add[e.dst] < 0 && e.flow < e.cap) {
                    Q.push(e.dst);
                    prev[e.dst] = e.inv;
                    add[e.dst] = min(m, e.cap-e.flow);
                } // 4cdb
        } // 887e

        return f;
    }
}

```

```

} // cec0

// Get flow through e-th edge of vertex v
flow_t getFlow(int v, int e) {
    return G[v][e].flow;
} // 0faf

// Get if v belongs to cut component with src
bool cutSide(int v) { return add[v] >= 0; }
}; // c5ef

```

### graphs/flow\_with\_demands.h 0153

```

#include "edmonds_karp.h"
// #include "push_relabel.h" // if you need

// Flow with demands; time: O(maxflow)
struct FlowDemands {
    MaxFlow net;
    vector<vector<flow_t>> demands;
    flow_t total{0};

    // Initialize for k vertices
    FlowDemands(int k = 0) : net(2) {
        while (k--) addVert();
    } // 7bdf

    // Add new vertex
    int addVert() {
        int v = net.addVert();
        demands.pb({});
        net.addEdge(0, v, 0);
        net.addEdge(v, 1, 0);
        return v-2;
    } // 48b6

    // Add edge from u to v with demand dem
    // and capacity cap (dem <= flow <= cap).
    // Returns edge index in adjacency list of u.
    int addEdge(int u, int v,
        flow_t dem, flow_t cap) {
        demands[u].pb(dem);
        demands[v].pb(0);
        total += dem;
        net.G[0][v].cap += dem;
        net.G[u+2][1].cap += dem;
        return net.addEdge(u+2, v+2, cap-dem) - 2;
    } // a403

    // Check if there exists a flow with value f
    // for source src and destination dst.
    // For circulation, you can set args to 0.
    bool canFlow(int src, int dst, flow_t f) {
        net.addEdge(dst += 2, src += 2, f);
        f = net.maxFlow(0, 1);
        net.G[src].pop_back();
        net.G[dst].pop_back();
        return f == total;
    } // 6285

    // Get flow through e-th edge of vertex v
    flow_t getFlow(int v, int e) {
        return net.getFlow(v+2, e+2)+demands[v][e];
    } // 6cf6
}; // db37

```

### graphs/general\_matching.h f42c

```

// Edmond's Blossom algorithm for maximum
// matching in general graphs; time: O(nm)
// Returns matching size (edge count).
// match[v] = vert matched to v or -1
int blossom(vector<Vi>& G, Vi& match) {
    int n = sz(G), cnt = -1, ans = 0;
}

```

```

match.assign(n, -1);
Vi lab(n), par(n), orig(n), aux(n, -1), q;

auto blos = [&](int v, int w, int a) {
    while (orig[v] != a) {
        par[v] = w; w = match[v];
        if (lab[w] == 1) lab[w] = 0, q.pb(w);
        orig[v] = orig[w] = a; v = par[w];
    } // 319e
}; // ab9e

rep(i, 0, n) if (match[i] == -1)
    each(e, G[i]) if (match[e] == -1) {
        match[match[e] = i] = e; ans++; break;
    } // a22a

rep(root, 0, n) if (match[root] == -1) {
    fill(all(lab), -1);
    iota(all(orig), 0);
    lab[root] = 0;
    q = {root};
    rep(i, 0, sz(q)) {
        int v = q[i];
        each(x, G[v]) if (lab[x] == -1) {
            lab[x] = 1; par[x] = v;
            if (match[x] == -1) {
                for (int y = x; y+1; ) {
                    int p = par[y], w = match[p];
                    match[match[p] = y] = p; y = w;
                } // 30c1
                ans++;
                goto nxt;
            } // 6fd0
            lab[match[x]] = 0; q.pb(match[x]);
        } else if (lab[x] == 0 &&
            orig[v] != orig[x]) {
            int a = orig[v], b = orig[x];
            for (cnt++; swap(a, b); if (a+1) {
                if (aux[a] == cnt) break;
                aux[a] = cnt;
                a = (match[a]+1 ?
                    orig[par[match[a]]] : -1);
            } // 2776
            blos(x, v, a); blos(v, x, a);
        } // 45a1
    } // d488
    nxt++;
} // 8d8a

return ans;
} // d872

```

### graphs/general\_matching\_w.h 8fd8

```

// Edmond's Blossom algorithm for weighted
// maximum matching in general graphs; O(n^3)?
// Weights must be positive (I believe).
struct WeightedBlossom {
    struct edge { int u, v, w; };
    int n, s, nx;
    vector<vector<edge>> g;
    Vi lab, match, slack, st, pa, S, vis;
    vector<Vi> flo, floFrom;
    queue<int> q;

    // Initialize for k vertices
    WeightedBlossom(int k)
        : n(k), s(n*2+1),
          g(s, vector<edge>(s)),
          lab(s), match(s), slack(s), st(s),
          pa(s), S(s), vis(s), flo(s),
          floFrom(s, Vi(n+1)) {}
}

```

```

    rep(u, 1, n+1) rep(v, 1, n+1)
        g[u][v] = {u, v, 0};
} // 5e51

// Add edge between u and v with weight w
void addEdge(int u, int v, int w) {
    u++; v++;
    g[u][v].w = g[v][u].w = max(g[u][v].w, w);
} // d296

// Compute max weight matching.
// `count` is set to matching size,
// `weight` is set to matching weight.
// Returns vector `match` such that:
// match[v] = vert matched to v or -1
Vi solve(int& count, ll& weight) {
    fill(all(match), 0);
    nx = n;
    weight = count = 0;
    rep(u, 0, n+1) flo[st[u] = u].clear();
    int tmp = 0;
    rep(u, 1, n+1) rep(v, 1, n+1) {
        floFrom[u][v] = (u-v ? 0 : v);
        tmp = max(tmp, g[u][v].w);
    } // a881
    rep(u, 1, n+1) lab[u] = tmp;
    while (matching()) count++;
    rep(u, 1, n+1)
        if (match[u] && match[u] < u)
            weight += g[u][match[u]].w;
    Vi ans(n);
    rep(i, 0, n) ans[i] = match[i+1]-1;
    return ans;
} // d611

int delta(edge& e) {
    return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
} // 7b58
void updateSlack(int u, int x) {
    if (!slack[x] || delta(g[u][x]) <
        delta(g[slack[x]][x])) slack[x] = u;
} // 1f7f
void setSlack(int x) {
    slack[x] = 0;
    rep(u, 1, n+1) if (g[u][x].w > 0 &&
        st[u] != x && !S[st[u]])
        updateSlack(u, x);
} // ee9c
void push(int x) {
    if (x <= n) q.push(x);
    else rep(i, 0, sz(flo[x])) push(flo[x][i]);
} // 594d
void setSt(int x, int b) {
    st[x] = b;
    if (x > n) rep(i, 0, sz(flo[x]))
        setSt(flo[x][i], b);
} // c5c8
int getPr(int b, int xr) {
    int pr = int(find(all(flo[b]), xr) -
        flo[b].begin());
    if (pr % 2) {
        reverse(flo[b].begin()+1, flo[b].end());
        return sz(flo[b]) - pr;
    } else return pr;
} // 399f
void setMatch(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    edge e = g[u][v];
    int xr = floFrom[u][e.u], pr = getPr(u, xr);

```

```

    rep(i, 0, pr)
        setMatch(flo[u][i], flo[u][i^1]);
    setMatch(xr, v);
    rotate(flo[u].begin(), flo[u].begin()+pr,
        flo[u].end());
} // f19d
void augment(int u, int v) {
    while (1) {
        int xnv = st[match[u]];
        setMatch(u, v);
        if (!xnv) return;
        setMatch(xnv, st[pa[xnv]]);
        u = st[pa[xnv]], v = xnv;
    } // bd23
} // e61a
int getLca(int u, int v) {
    static int t = 0;
    for (++t; u||v; swap(u, v)) {
        if (!u) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    } // aa78
    return 0;
} // 9f28
void blossom(int u, int lca, int v) {
    int b = n+1;
    while (b <= nx && st[b]) ++b;
    if (b > nx) ++nx;
    lab[b] = S[b] = 0;
    match[b] = match[lca];
    flo[b].clear();
    flo[b].pb(lca);
    for (int x=u, y; x != lca; x = st[pa[y]]) {
        flo[b].pb(x);
        flo[b].pb(y = st[match[x]]);
        push(y);
    } // 63e0
    reverse(flo[b].begin()+1, flo[b].end());
    for (int x=v, y; x != lca; x = st[pa[y]]) {
        flo[b].pb(x);
        flo[b].pb(y = st[match[x]]);
        push(y);
    } // 63e0
    setSt(b, b);
    rep(x, 1, nx+1) g[b][x].w = g[x][b].w = 0;
    rep(x, 1, n+1) floFrom[b][x] = 0;
    rep(i, 0, sz(flo[b])) {
        int xs = flo[b][i];
        rep(x, 1, nx+1) if (!g[b][x].w ||
            delta(g[xs][x]) < delta(g[b][x]))
            g[b][x]=g[xs][x], g[x][b]=g[x][xs];
        rep(x, 1, n+1) if (floFrom[xs][x])
            floFrom[b][x] = xs;
    } // 5833
    setSlack(b);
} // 9000
void blossom(int b) {
    each(e, flo[b]) setSt(e, e);
    int xr = floFrom[b][g[b][pa[b]].u];
    int pr = getPr(b, xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flo[b][i], xns = flo[b][i+1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1; S[xns] = slack[xs] = 0;
        setSlack(xns); push(xns);
    } // f26f

```

```

    S[xr] = 1; pa[xr] = pa[b];
    rep(i, pr+1, sz(flo[b])) {
        int xs = flo[b][i];
        S[xs] = -1; setSlack(xs);
    } // a12a
    st[b] = 0;
} // f750
bool found(const edge& e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u; S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = S[nu] = 0;
        push(nu);
    } else if (!S[v]) {
        int lca = getLca(u, v);
        if (!lca) return augment(u, v),
            augment(v, u), 1;
        else blossom(u, lca, v);
    } // ddbb
    return 0;
} // 1c00
bool matching() {
    fill(S.begin(), S.begin()+nx+1, -1);
    fill(slack.begin(), slack.begin()+nx+1, 0);
    q = {};
    rep(x, 1, nx+1)
        if (st[x] == x && !match[x])
            pa[x] = S[x] = 0, push(x);
    if (q.empty()) return 0;
    while (1) {
        while (q.size()) {
            int u = q.front(); q.pop();
            if (S[st[u]] == 1) continue;
            rep(x, 1, n+1)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (!delta(g[u][v])) {
                        if (found(g[u][v])) return 1;
                        else updateSlack(u, st[v]);
                    } // b782
                } // 4d33
            int d = INT_MAX;
            rep(b, n+1, nx+1)
                if (st[b] == b && S[b] == 1)
                    d = min(d, lab[b]/2);
            rep(x, 1, nx+1)
                if (st[x] == x && slack[x]) {
                    if (S[x] == -1)
                        d = min(d, delta(g[slack[x]][x]));
                    else if (!S[x])
                        d = min(d, delta(g[slack[x]][x])/2);
                } // 2a0e
            rep(u, 1, n+1) {
                if (!S[st[u]]) {
                    if (lab[u] <= d) return 0;
                    lab[u] -= d;
                } else if (S[st[u]] == 1) lab[u] += d;
            } // 4601
            rep(b, n+1, nx+1) if (st[b] == b) {
                if (!S[st[b]]) lab[b] += d*2;
                else if (S[st[b]] == 1) lab[b] -= d*2;
            } // e09b
            q = {};
            rep(x, 1, nx+1)
                if (st[x] == x && slack[x] &&
                    st[slack[x]] != x &&
                    !delta(g[slack[x]][x]) &&
                    found(g[slack[x]][x])) return 1;

```

```

        rep(b, n+1, nx+1)
            if (st[b] == b && S[b] == 1 && !lab[b])
                blossom(b);
    } // a122
    return 0;
} // e966
}; // 3ee2

```

## graphs/global\_min\_cut.h 0eac

```

// Find a minimum cut in an undirected graph
// with non-negative edge weights
// given its adjacency matrix M; time: O(n^3)
// `out` contains vertices on one side.
ll minCut(vector<vector<ll>> M, Vi& out) {
    int n = sz(M);
    ll ans = INT64_MAX;
    vector<Vi> co(n);
    rep(i, 0, n) co[i].pb(i);
    out.clear();
    rep(ph, 1, n) {
        auto w = M[0];
        size_t s = 0, t = 0;
        // O(V^2) -> O(E log V) with priority queue
        rep(it, 0, n-ph) {
            w[t] = INT64_MIN; s = t;
            t = max_element(all(w)) - w.begin();
            rep(i, 0, n) w[i] += M[t][i];
        } // 0831
        ll alt = w[t] - M[t][t];
        if (alt < ans) ans = alt, out = co[t];
        co[s].insert(co[s].end(), all(co[t]));
        rep(i, 0, n) M[s][i] += M[t][i];
        rep(i, 0, n) M[i][s] = M[s][i];
        M[0][t] = INT64_MIN;
    } // df69
    return ans;
} // 2a85

```

## graphs/gomory\_hu.h a520

```

#include "edmonds_karp.h"
// #include "push_relabel.h" // if you need

struct Edge {
    int a, b; // vertices
    flow_t w; // weight
}; // c331

// Build Gomory-Hu tree; time: O(n*maxflow)
// Gomory-Hu tree encodes minimum cuts between
// all pairs of vertices: mincut for u and v
// is equal to minimum on path from u and v
// in Gomory-Hu tree. n is vertex count.
// Returns vector of Gomory-Hu tree edges.
vector<Edge> gomoryHu(vector<Edge>& edges,
    int n) {
    MaxFlow flow(n);
    each(e, edges) flow.addEdge(e.a, e.b, e.w, e.w);

    vector<Edge> ret(n-1);
    rep(i, 1, n) ret[i-1] = {i, 0, 0};

    rep(i, 1, n) {
        ret[i-1].w = flow.maxFlow(i, ret[i-1].b);
        rep(j, i+1, n)
            if (ret[j-1].b == ret[i-1].b &&
                flow.cutSide(j)) ret[j-1].b = i;
    } // 5ae4

    return ret;
} // afd8

```

**graphs/kth\_shortest.h** 1f40

```
constexpr ll INF = 1e18;

// Eppstein's k-th shortest path algorithm;
// time and space: O((m+k) log (m+k))
struct Eppstein {
    using T = ll; // Type for edge weights
    using Edge = pair<int, T>;

    struct Node {
        int E[2] = {}, s{0};
        Edge x;
    }; // 013b

    T shortest; // Shortest path length
    priority_queue<pair<T, int>> Q;
    vector<Node> P{1};
    Vi h;

    // Initialize shortest path structure for
    // weighted graph G, source s and target t;
    // time: O(m log m)
    Eppstein(vector<vector<Edge>>& G,
              int s, int t) {
        int n = sz(G);
        vector<vector<Edge>> H(n);
        rep(i, 0, n) each(e, G[i]) H[e.x].pb({i, e.y});

        Vi ord, par(n, -1);
        vector<T> d(n, -INF);
        Q.push({d[t] = 0, t});

        while (!Q.empty()) {
            auto v = Q.top();
            Q.pop();
            if (d[v.y] == v.x) {
                ord.pb(v.y);
                each(e, H[v.y]) if (v.x - e.y > d[e.x]) {
                    Q.push({d[e.x] = v.x - e.y, e.x});
                    par[e.x] = v.y;
                } // 5895
            } // 1b62
        } // 1a6d

        if ((shortest = -d[s]) >= INF) return;
        h.resize(n);

        each(v, ord) {
            int p = par[v];
            if (p+1) h[v] = h[p];
            each(e, G[v]) if (d[e.x] > -INF) {
                T k = e.y - d[e.x] + d[v];
                if (k || e.x != p)
                    h[v] = push(h[v], {e.x, k});
                else
                    p = -1;
            } // 5e05
        } // 31b9

        P[0].x.x = s;
        Q.push({0, 0});
    } // e00e

    int push(int t, Edge x) {
        P.pb(P[t]);
        if (!P[t] = sz(P)-1).s || P[t].x.y >= x.y)
            swap(x, P[t].x);
        if (P[t].s) {
            int i = P[t].E[0], j = P[t].E[1];
            int d = P[i].s > P[j].s;
            int k = push(d ? j : i, x);
            P[t].E[d] = k; // Don't inline k!
        } // 10e1
```

```
P[t].s++;
return t;
} // a2dc

// Get next shortest path length,
// the first call returns shortest path.
// Returns -1 if there's no more paths;
// time: O(log k), where k is total count
// of nextPath calls.
ll nextPath() {
    if (Q.empty()) return -1;
    auto v = Q.top();
    Q.pop();
    for (int i : P[v.y].E) if (i)
        Q.push({v.x - P[i].x.y + P[v.y].x.y, i});
    int t = h[P[v.y].x.x];
    if (t) Q.push({v.x - P[t].x.y, t});
    return shortest - v.x;
} // 08af
}; // 5326
```

**graphs/matroids.h** 55ef

```
// Find largest subset S of [n] such that
// S is independent in both matroid A and B.
// A and B are given by their oracles,
// see example implementations below.
// Returns vector V such that V[i] = 1 iff
// i-th element is included in found set;
// time: O(r^2*init + r^2*n*add),
// where r is max independent set,
// `init` is max time of oracles init
// and `add` is max time of oracles canAdd.
template<class T, class U>
vector<bool> intersectMatroids(T& A, U& B,
                              int n) {
    vector<bool> ans(n);
    bool ok = 1;

    // NOTE: for weighted matroid intersection
    // find shortest augmenting paths
    // first by weight change, then by length
    // using Bellman-Ford, and skip this speedup:
    A.init(ans);
    B.init(ans);
    rep(i, 0, n) if (A.canAdd(i) && B.canAdd(i))
        ans[i] = 1, A.init(ans), B.init(ans);

    while (ok) {
        vector<Vi> G(n);
        vector<bool> good(n);
        queue<int> que;
        Vi prev(n, -1);

        A.init(ans);
        B.init(ans);
        ok = 0;

        rep(i, 0, n) if (!ans[i]) {
            if (A.canAdd(i)) que.push(i), prev[i] = -2;
            good[i] = B.canAdd(i);
        } // 9581

        rep(i, 0, n) if (ans[i]) {
            ans[i] = 0;
            A.init(ans);
            B.init(ans);

            rep(j, 0, n) if (i != j && !ans[j]) {
                if (A.canAdd(j)) G[i].pb(j);
                if (B.canAdd(j)) G[j].pb(i);
            } // bd2a
            ans[i] = 1;
```

```
} // bf3e

while (!que.empty()) {
    int i = que.front();
    que.pop();

    if (good[i]) {
        ans[i] = 1;
        while (prev[i] >= 0) {
            ans[i] = prev[i] = 0;
            ans[i] = prev[i] = 1;
        } // 51c8
        ok = 1;
        break;
    } // 384b

    each(j, G[i]) if (prev[j] == -1)
        que.push(j), prev[j] = i;
} // 6eb6
} // e092

return ans;
} // ae8e

// Matroid where each element has color
// and set is independent iff for each color c
// #elements of color c) <= maxAllowed[c].
struct LimOracle {
    Vi color; // color[i] = color of i-th element
    Vi maxAllowed; // Limits for colors
    Vi tmp;

    // Init oracle for independent set S; O(n)
    void init(vector<bool>& S) {
        tmp = maxAllowed;
        rep(i, 0, sz(S)) tmp[color[i]] -= S[i];
    } // 4dfb

    // Check if S+{k} is independent; time: O(1)
    bool canAdd(int k) {
        return tmp[color[k]] > 0;
    } // e312
}; // 7b5d

// Graphic matroid - each element is edge,
// set is independent iff subgraph is acyclic.
struct GraphOracle {
    vector<Pii> elems; // Ground set: graph edges
    int n; // Number of vertices, indexed [0;n-1]
    Vi par;

    int find(int i) {
        return par[i] == -1 ? i
            : par[i] = find(par[i]);
    } // b8b7

    // Init oracle for independent set S; ~O(n)
    void init(vector<bool>& S) {
        par.assign(n, -1);
        rep(i, 0, sz(S)) if (S[i])
            par[find(elems[i].x)] = find(elems[i].y);
    } // 1827

    // Check if S+{k} is independent; time: ~O(1)
    bool canAdd(int k) {
        return
            find(elems[k].x) != find(elems[k].y);
    } // 8ca4
}; // c506

// Co-graphic matroid - each element is edge,
// set is independent iff after removing edges
// from graph number of connected components
// doesn't change.
```

```
struct CographOracle {
    vector<Pii> elems; // Ground set: graph edges
    int n; // Number of vertices, indexed [0;n-1]
    vector<Vi> G;
    Vi pre, low;
    int cnt;

    int dfs(int v, int p) {
        pre[v] = low[v] = ++cnt;
        each(e, G[v]) if (e != p)
            low[v] = min(low[v], pre[e] ? : dfs(e, v));
        return low[v];
    } // 9d30

    // Init oracle for independent set S; O(n)
    void init(vector<bool>& S) {
        G.assign(n, {});
        pre.assign(n, 0);
        low.resize(n);
        cnt = 0;
        rep(i, 0, sz(S)) if (!S[i]) {
            Pii e = elems[i];
            G[e.x].pb(e.y);
            G[e.y].pb(e.x);
        } // 79a1
        rep(v, 0, n) if (!pre[v]) dfs(v, -1);
    } // 1200

    // Check if S+{k} is independent; time: O(1)
    bool canAdd(int k) {
        Pii e = elems[k];
        return max(pre[e.x], pre[e.y])
            != max(low[e.x], low[e.y]);
    } // 2550
}; // a5cc

// Matroid equivalent to linear space with XOR
struct XorOracle {
    vector<ll> elems; // Ground set: numbers
    vector<ll> base;

    // Init for independent set S; O(n+r^2)
    void init(vector<bool>& S) {
        base.assign(63, 0);
        rep(i, 0, sz(S)) if (S[i]) {
            ll e = elems[i];
            rep(j, 0, sz(base)) if ((e >> j) & 1) {
                if (!base[j]) {
                    base[j] = e;
                    break;
                } // 1df5
                e ^= base[j];
            } // 8495
        } // 655e
    } // b68c

    // Check if S+{k} is independent; time: O(r)
    bool canAdd(int k) {
        ll e = elems[k];
        rep(i, 0, sz(base)) if ((e >> i) & 1) {
            if (!base[i]) return 1;
            e ^= base[i];
        } // 49d1
        return 0;
    } // 66ff
}; // 4af3

graphs/min_cost_max_flow.h 1fa6

using flow_t = ll;
constexpr flow_t INF = 1e18;

// Min cost max flow using cheapest paths;
```

```
// time: O(nm + |f|*(m log n))
// or O(|f|*(m log n)) if costs are nonnegative
struct MCMF {
    struct Edge {
        int dst, inv;
        flow_t flow, cap, cost;
    }; // 20f7

    vector<vector<Edge>> G;
    vector<flow_t> add;

    // Initialize for n vertices
    MCMF(int n = 0) : G(n) {}

    // Add new vertex
    int addVert() { G.pb({}); return sz(G)-1; }

    // Add edge from u to v.
    // Returns edge index in adjacency list of u.
    int addEdge(int u, int v,
        flow_t cap, flow_t cost) {
        G[u].pb({ v, sz(G[v]), 0, cap, cost });
        G[v].pb({ u, sz(G[u])-1, 0, 0, -cost });
        return sz(G[u])-1;
    } // 1095

    // Compute minimum cost maximum flow
    // from src to dst. `f` is set to flow value,
    // `c` is set to total cost value.
    // Returns false iff negative cycle
    // is reachable from from source.
    bool maxFlow(int src, int dst,
        flow_t& f, flow_t& c) {
        flow_t i, m, d;
        f = c = 0;
        each(v, G) each(e, v) e.flow = 0;

        // [If costs are nonnegative]
        // vector<flow_t> pot(sz(G));
        // [/end]

        // [If costs can be negative] O(n*m)
        vector<flow_t> pot(sz(G), INF);
        pot[src] = 0;
        int it = sz(G), ch = 1;
        while (ch-- && it--)
            rep(s, 0, sz(G)) if (pot[s] != INF)
                each(e, G[s]) if (e.cap)
                    if ((d = pot[s]+e.cost) < pot[e.dst])
                        pot[e.dst] = d, ch = 1;
        if (it < 0) return 0;
        // [/end]

nxt:
        Vi prev(sz(G), -1);
        vector<flow_t> dist(sz(G), INF);
        priority_queue<pair<flow_t, int>> Q;
        add.assign(sz(G), -1);
        Q.push({0, src});
        add[src] = INF;
        dist[src] = 0;

        while (!Q.empty()) {
            tie(d, i) = Q.top();
            Q.pop();
            if (d != -dist[i]) continue;
            m = add[i];

            if (i == dst) {
                f += m;
                c += m * (dist[i]-pot[src]+pot[i]);
                while (i != src) {
                    auto& e = G[i][prev[i]];

```

```
                e.flow -= m;
                G[i] = e.dst[e.inv].flow += m;
            } // 1f86
            rep(j, 0, sz(G))
                pot[j] = min(pot[j]+dist[j], INF);
            goto nxt;
        } // 36d4

        each(e, G[i]) if (e.flow < e.cap) {
            d = dist[i]+e.cost+pot[i]-pot[e.dst];
            if (d < dist[e.dst]) {
                Q.push({-d, e.dst});
                prev[e.dst] = e.inv;
                add[e.dst] = min(m, e.cap-e.flow);
                dist[e.dst] = d;
            } // 5ee6
        } // b6b2
    } // d47c
    return 1;
} // 0bc3

// Get flow through e-th edge of vertex v
flow_t getFlow(int v, int e) {
    return G[v][e].flow;
} // 0faf

// Get if v belongs to cut component with src
bool cutSide(int v) { return add[v] >= 0; }
}; // 1867
```

### graphs/push\_relabel.h 07d1

```
using flow_t = int;

// Push-relabel algorithm for maximum flow;
// O(V^2*sqrt(E)), but very fast in practice.
struct MaxFlow {
    struct Edge {
        int to, inv;
        flow_t rem, cap;
    }; // bc77

    vector<basic_string<Edge>> G;
    vector<flow_t> extra;
    Vi hei, arc, prv, nxt, act, bot;
    queue<int> Q;
    int n, high, cut, work;

    // Initialize for k vertices
    MaxFlow(int k = 0) : G(k) {}

    // Add new vertex
    int addVert() { G.pb({}); return sz(G)-1; }

    // Add edge from u to v with capacity cap
    // and reverse capacity rcap.
    // Returns edge index in adjacency list of u.
    int addEdge(int u, int v,
        flow_t cap, flow_t rcap = 0) {
        G[u].pb({ v, sz(G[v]), 0, cap });
        G[v].pb({ u, sz(G[u])-1, 0, rcap });
        return sz(G[u])-1;
    } // c96a

    void raise(int v, int h) {
        prv[nxt[prv[v]] = nxt[v]] = prv[v];
        hei[v] = h;
        if (extra[v] > 0) {
            bot[v] = act[h]; act[h] = v;
            high = max(high, h);
        } // d7ee
        if (h < n) cut = max(cut, h+1);
        nxt[v] = nxt[prv[v]] = h += n;
        prv[nxt[nxt[h] = v]] = v;

```

```
    } // 5274

    void global(int s, int t) {
        hei.assign(n, n*2);
        act.assign(n*2, -1);
        iota(all(prv), 0);
        iota(all(nxt), 0);
        hei[t] = high = cut = work = 0;
        hei[s] = n;
        for (int x : {t, s})
            for (Q.push(x); !Q.empty(); Q.pop()) {
                int v = Q.front();
                each(e, G[v])
                    if (hei[e.to] == n*2 &&
                        G[e.to][e.inv].rem)
                        Q.push(e.to), raise(e.to, hei[v]+1);
            } // 1901
    } // 3181

    void push(int v, Edge& e, bool z) {
        auto f = min(extra[v], e.rem);
        if (f > 0) {
            if (z && !extra[e.to]) {
                bot[e.to] = act[hei[e.to]];
                act[hei[e.to]] = e.to;
            } // 9d90
            e.rem -= f; G[e.to][e.inv].rem += f;
            extra[v] -= f; extra[e.to] += f;
        } // 0ffb
    } // da44

    void discharge(int v) {
        int h = n*2, k = hei[v];

        rep(j, 0, sz(G[v])) {
            auto& e = G[v][arc[v]];
            if (e.rem)
                if (k == hei[e.to]+1) {
                    push(v, e, 1);
                    if (extra[v] <= 0) return;
                } else h = min(h, hei[e.to]+1);
            } // 87c1
            if (++arc[v] >= sz(G[v])) arc[v] = 0;
        } // 9741

        if (k < n && nxt[k+n] == prv[k+n]) {
            rep(j, k, cut) while (nxt[j+n] < n)
                raise(nxt[j+n], n);
            cut = k;
        } else raise(v, h), work++;
    } // b64f
};
```

```
// Compute maximum flow from src to dst
flow_t maxFlow(int src, int dst) {
    extra.assign(n = sz(G), 0);
    arc.assign(n, 0);
    prv.resize(n*3);
    nxt.resize(n*3);
    bot.resize(n);
    each(v, G) each(e, v) e.rem = e.cap;

    each(e, G[src])
        extra[src] = e.cap, push(src, e, 0);
    global(src, dst);

    for (; high; high--)
        while (act[high] != -1) {
            int v = act[high];
            act[high] = bot[v];
            if (v != src && hei[v] == high) {
                discharge(v);
                if (work > 4*n) global(src, dst);
            }
        }
    return cut;
}
```

```
    } // 7dcc
    } // 26d4

    return extra[dst];
} // aa5e

// Get flow through e-th edge of vertex v
flow_t getFlow(int v, int e) {
    return G[v][e].cap - G[v][e].rem;
} // 812c

// Get if v belongs to cut component with src
bool cutSide(int v) { return hei[v] >= n; }
}; // 2d6b
```

### graphs/scc.h 1c43

```
// Tarjan's SCC algorithm; time: O(n+m)
// Usage: SCC scc(graph);
// scc[v] = index of SCC for vertex v
// scc.comps[i] = vertices of i-th SCC
// Components are in reversed topological order
struct SCC : Vi {
    vector<Vi> comps;
    Vi S;

    SCC() {}

    SCC(vector<Vi>& G) : Vi(sz(G), -1), S(sz(G)) {
        rep(i, 0, sz(G)) if (!S[i]) dfs(G, i);
    } // f0fa

    int dfs(vector<Vi>& G, int v) {
        int low = S[v] = sz(S);
        S.pb(v);

        each(e, G[v]) if (at(e) < 0)
            low = min(low, S[e] ?: dfs(G, e));

        if (low == S[v]) {
            comps.pb({});
            rep(i, S[v], sz(S)) {
                at[S[i]] = sz(comps)-1;
                comps.back().pb(S[i]);
            } // 8ed0
            S.resize(S[v]);
        } // ecc7

        return low;
    } // f3c6
}; // 215e
```

### graphs/turbo\_matching.h d400

```
// Find maximum bipartite matching; time: ?
// G must be bipartite graph!
// Returns matching size (edge count).
// match[v] = vert matched to v or -1
int matching(vector<Vi>& G, Vi& match) {
    vector<bool> seen;
    int n = 0, k = 1;
    match.assign(sz(G), -1);

    function<int(int)> dfs = [&](int i) {
        if (seen[i]) return 0;
        seen[i] = 1;
        each(e, G[i]) {
            if (match[e] < 0 || dfs(match[e])) {
                match[i] = e; match[e] = i;
                return 1;
            } // 893d
        } // 9532
        return 0;
    }; // d332

    while (k) {

```



```

seen.assign(sz(G), 0);
k = 0;
rep(i, 0, sz(G)) if (match[i] < 0)
    k += dfs(i);
    n += k;
} // 1128
return n;
} // 0d38

// Convert maximum matching to vertex cover
// time: O(n+m)
Vi vertexCover(vector<Vi>& G, Vi& match) {
    Vi ret, col(sz(G)), seen(sz(G));

    function<void(int, int)> dfs =
        [&](int i, int c) {
            if (col[i]) return;
            col[i] = c+1;
            each(e, G[i]) dfs(e, !c);
        }; // 1f1b

    function<void(int)> aug = [&](int i) {
        if (seen[i] || col[i] != 1) return;
        seen[i] = 1;
        each(e, G[i]) seen[e] = 1, aug(match[e]);
    }; // 2465

    rep(i, 0, sz(G)) dfs(i, 0);
    rep(i, 0, sz(G)) if (match[i] < 0) aug(i);
    rep(i, 0, sz(G))
        if (seen[i] == col[i]-1) ret.pb(i);
    return ret;
} // 6f72

```

## graphs/weighted\_matching.h 8264

```

// Minimum cost bipartite matching; O(n^2*m)
// Input is n x m cost matrix, where n <= m.
// Returns matching weight.
// L[i] = right vertex matched to i-th left
// R[i] = left vertex matched to i-th right
ll hungarian(const vector<vector<ll>>& cost,
              Vi& L, Vi& R) {
    if (cost.empty())
        return L.clear(), R.clear(), 0;
    int b, c = 0, n = sz(cost), m = sz(cost[0]);
    assert(n <= m);

    vector<ll> x(n), y(m+1);
    L.assign(n, -1);
    R.assign(m+1, -1);

    rep(i, 0, n) {
        vector<ll> sla(m, INT64_MAX);
        Vi vis(m+1), prv(m, -1);
        for (R[b = m] = i; R[b]+1; b = c) {
            int a = R[b];
            ll d = INT64_MAX;
            vis[b] = 1;
            rep(j, 0, m) if (!vis[j]) {
                ll cur = cost[a][j] - x[a] - y[j];
                if (cur < sla[j])
                    sla[j] = cur, prv[j] = b;
                if (sla[j] < d) d = sla[j], c = j;
            } // 6717
            rep(j, 0, m+1) {
                if (vis[j]) x[R[j]] += d, y[j] -= d;
                else sla[j] -= d;
            } // 8bb3
        } // 01c6
        while (b-m) c = b, R[c] = R[b = prv[b]];
    } // 71f5
}

```

```

rep(j, 0, m) if (R[j]+1) L[R[j]] = j;
R.resize(m);
return -y[m];
} // 349d

```

## math/berlekamp\_massey.h 7d12

```

constexpr int MOD = 998244353;

ll modInv(ll a, ll m) { // a^(-1) mod m
    if (a == 1) return 1;
    return ((a - modInv(m%a, a))*m + 1) / a;
} // c437

// Find shortest linear recurrence that matches
// given starting terms of recurrence; O(n^2)
// Returns vector C such that for each i >= |C|
// A[i] = sum A[i-j-1]*C[j] for j = 0..|C|-1
vector<ll> massey(vector<ll>& A) {
    if (A.empty()) return {};
    int n = sz(A), len = 0, k = 0;
    ll s = 1;
    vector<ll> B(n), C(n), tmp;
    B[0] = C[0] = 1;

    rep(i, 0, n) {
        ll d = 0;
        k++;
        rep(j, 0, len+1)
            d = (d + C[j] * A[i-j]) % MOD;

        if (d) {
            ll q = d * modInv(s, MOD) % MOD;
            tmp = C;

            rep(j, k, n)
                C[j] = (C[j] - q * B[j-k]) % MOD;

            if (len*2 <= i) {
                B.swap(tmp);
                len = i-len+1;
                s = d + (d < 0) * MOD;
                k = 0;
            } // c350
        } // 79c7
    } // f70c

    C.resize(len+1);
    C.erase(C.begin());
    each(x, C) x = (MOD - x) % MOD;
    return C;
} // 20ce

```

## math/bit\_gauss.h 13eb

```

constexpr int MAX_COLS = 2048;

// Solve system of linear equations over Z_2
// time: O(n^2*m/W), where W is word size
// - A - extended matrix, rows are equations,
//       columns are variables,
//       m-th column is equation result
//       (A[i][j] - i-th row and j-th column)
// - ans - output for variables values
// - m - variable count
// Returns 0 if no solutions found, 1 if one,
// 2 if more than 1 solution exist.
int bitGauss(vector<bitset<MAX_COLS>>& A,
              vector<bool>& ans, int m) {

    Vi col;
    ans.assign(m, 0);

    rep(i, 0, sz(A)) {
        int c = int(A[i]._Find_first());
        if (c >= m) {

```

```

            if (c == m) return 0;
            continue;
        } // a6bb

        rep(k, i+1, sz(A)) if (A[k][c]) A[k]^=A[i];
        swap(A[i], A[sz(col)]);
        col.pb(c);
    } // a953

    for (int i = sz(col); i--;) if (A[i][m]) {
        ans[col[i]] = 1;
        rep(k, 0, i) if (A[k][col[i]]) A[k][m].flip();
    } // 4ca1

    return sz(col) < m ? 2 : 1;
} // 996e

```

## math/bit\_matrix.h 2e3f

```

using ull = uint64_t;

// Matrix over Z_2 (bits and xor)
// TODO: arithmetic operations
struct BitMatrix {
    vector<ull> M;
    int rows, cols, stride;

    // Create matrix with n rows and m columns
    BitMatrix(int n = 0, int m = 0) {
        rows = n; cols = m;
        stride = (m+63)/64;
        M.resize(n*stride);
    } // 7ef0

    // Get pointer to bit-packed data of i-th row
    ull* row(int i) { return &M[i*stride]; }

    // Get value in i-th row and j-th column
    bool operator()(int i, int j) {
        return (row(i)[j/64] >> (j%64)) & 1;
    } // 28bd

    // Set value in i-th row and j-th column
    void set(int i, int j, bool val) {
        ull &w = row(i)[j/64], m = 1ull << (j%64);
        if (val) w |= m;
        else w &= ~m;
    } // 98a8
}; // 4df7

```

## math/crt.h 8a85

```

using Pll = pair<ll, ll>;

ll egcd(ll a, ll b, ll& x, ll& y) {
    if (!a) return x=0, y=1, b;
    ll d = egcd(b%a, a, y, x);
    x -= b/a*y;
    return d;
} // 23c8

// Chinese Remainder Theorem; time: O(lg lcm)
// Solves x = a.x (mod a.y), x = b.x (mod b.y)
// Returns pair (x mod lcm, lcm(a.y, b.y))
// or (-1, -1) if there's no solution.
// WARNING: a.x and b.x are assumed to be
// in [0;a.y) and [0;b.y) respectively.
// Works properly if lcm(a.y, b.y) < 2^63.
Pll crt(Pll a, Pll b) {
    if (a.y < b.y) swap(a, b);
    ll x, y, g = egcd(a.y, b.y, x, y);
    ll c = b.x-a.x, d = b.y/g, p = a.y*d;
    if (c % g) return {-1, -1};
    ll s = (a.x + c/g*x % d * a.y) % p;
    return {s < 0 ? s+p : s, p};
}

```

```

} // 35a8
math/fft_complex.h 31ee

using dbl = double;
using cmpl = complex<dbl>;

// Default std::complex multiplication is slow.
// You can use this to achieve small speedup.
cmpl operator*(cmpl a, cmpl b) {
    dbl ax = real(a), ay = imag(a);
    dbl bx = real(b), by = imag(b);
    return {ax*bx-ay*by, ax*by+ay*bx};
} // 3b78

cmpl operator+=(cmpl& a, cmpl b) {return a=a+b;}

// Compute DFT over complex numbers; O(n lg n)
// Input size must be power of 2!
void fft(vector<cmpl>& a) {
    static vector<cmpl> w(2, 1);
    int n = sz(a);

    for (int k = sz(w); k < n; k *= 2) {
        w.resize(n);
        rep(i, 0, k) w[k+i] = exp(cmpl(0, M_PI*i/k));
    } // 92a9

    Vi rev(n);
    rep(i, 0, n) rev[i] = (rev[i/2] | i%2*n) / 2;
    rep(i, 0, n) if (i < rev[i]) swap(a[i], a[rev[i]]);

    for (int k = 1; k < n; k *= 2) {
        for (int i=0; i < n; i += k*2) rep(j, 0, k) {
            auto d = a[i+j+k] * w[j+k];
            a[i+j+k] = a[i+j] - d;
            a[i+j] += d;
        } // b389
    } // 84bf
} // adf8

// Convolve complex-valued a and b,
// store result in a; time: O(n lg n), 3x FFT
void convolve(vector<cmpl>& a, vector<cmpl> b) {
    int len = max(sz(a) + sz(b) - 1, 0);
    int n = 1 << (32 - __builtin_clz(len));
    a.resize(n); b.resize(n);
    fft(a); fft(b);
    rep(i, 0, n) a[i] *= b[i] / dbl(n);
    reverse(a.begin()+1, a.end());
    fft(a);
    a.resize(len);
} // 7987

// Convolve real-valued a and b, returns result
// time: O(n lg n), 2x FFT
// Rounding to integers is safe as long as
// (max_coeff^2)*n*log_2(n) < 9*10^14
// (in practice 10^16 or higher).
vector<dbl> convolve(vector<dbl>& a,
                     vector<dbl>& b) {
    int len = max(sz(a) + sz(b) - 1, 0);
    int n = 1 << (32 - __builtin_clz(len));

    vector<cmpl> in(n), out(n);
    rep(i, 0, sz(a)) in[i].real(a[i]);
    rep(i, 0, sz(b)) in[i].imag(b[i]);

    fft(in);
    each(x, in) x *= x;
    rep(i, 0, n) out[i] = in[-i+(n-1)]-conj(in[i]);
    fft(out);

    vector<dbl> ret(len);
    rep(i, 0, len) ret[i] = imag(out[i]) / (n*4);
}

```

```

    return ret;
} // 19ed

constexpr ll MOD = 1e9+7;

// High precision convolution of integer-valued
// a and b mod MOD; time: O(n lg n), 4x FFT
// Input is expected to be in range [0;MOD)!
// Rounding is safe if MOD*n*log_2(n) < 9*10^14
// (in practice 10^16 or higher).
vector<ll> convMod(vector<ll>& a,
                  vector<ll>& b) {
    vector<ll> ret(sz(a) + sz(b) - 1);
    int n = 1 << (32 - __builtin_clz(sz(ret)));
    ll cut = ll(sqrt(MOD))+1;

    vector<cmpl> c(n), d(n), g(n), f(n);

    rep(i, 0, sz(a))
        c[i] = {dbl(a[i]/cut), dbl(a[i]%cut)};
    rep(i, 0, sz(b))
        d[i] = {dbl(b[i]/cut), dbl(b[i]%cut)};

    fft(c); fft(d);

    rep(i, 0, n) {
        int j = -i & (n-1);
        f[j] = (c[i]+conj(c[j])) * d[i] / (n*2.0);
        g[j] =
            (c[i]-conj(c[j])) * d[i] / cmpl(0, n*2);
    } // e877

    fft(f); fft(g);

    rep(i, 0, sz(ret)) {
        ll t = llround(real(f[i])) % MOD * cut;
        t += llround(imag(f[i]));
        t = (t + llround(real(g[i]))) % MOD * cut;
        t = (t + llround(imag(g[i]))) % MOD;
        ret[i] = (t < 0 ? t+MOD : t);
    } // e75d

    return ret;
} // 513f

```

**math/fft\_mod.h**

17aa

```

// Number Theoretic Transform (NTT)
// For functions below you can choose 2 params:
// 1. M - prime modulus that MUST BE of form
//      a*2^k+1, computation is done in Z_M
// 2. R - generator of Z_M

```

```

// Modulus often seen on Codeforces:
// M = (119<<23)+1, R = 62; M is 998244353

// Parameters for ll computation with CRT:
// M = (479<<21)+1, R = 62; M is > 10^9
// M = (483<<21)+1, R = 62; M is > 10^9

```

```

ll modPow(ll a, ll e, ll m) {
    ll t = 1 % m;
    while (e) {
        if (e % 2) t = t*a % m;
        e /= 2; a = a*a % m;
    } // 66ca
    return t;
} // 1973

```

```

// Compute DFT over Z_M with generator R.
// Input size must be power of 2; O(n lg n)
// Input is expected to be in range [0;MOD)!
// dit == true <=> inverse transform * 2^n
// (without normalization)
template<ll M, ll R, bool dit>
void ntt(vector<ll>& a) {

```

```

static vector<ll> w(2, 1);
int n = sz(a);

for (int k = sz(w); k < n; k *= 2) {
    w.resize(n, 1);
    ll c = modPow(R, M/2/k, M);
    if (dit) c = modPow(c, M-2, M);
    rep(i, k+1, k*2) w[i] = w[i-1]*c % M;
} // 0d98

for (int t = 1; t < n; t *= 2) {
    int k = (dit ? t : n/t/2);
    for (int i=0; i < n; i += k*2) rep(j, 0, k) {
        ll &c = a[i+j], &d = a[i+j+k];
        ll e = w[j+k], f = d;
        d = (dit ? c - (f*f*e%M) : (c-f)*e % M);
        if (d < 0) d += M;
        if ((c += f) >= M) c -= M;
    } // e4a6
} // 8d38
} // 01f5

```

```

// Convolve a and b mod M (R is generator),
// store result in a; time: O(n lg n), 3x NTT
// Input is expected to be in range [0;MOD)!
template<ll M = (119<<23)+1, ll R = 62>
void convolve(vector<ll>& a, vector<ll> b) {

```

```

    int len = max(sz(a) + sz(b) - 1, 0);
    int n = 1 << (32 - __builtin_clz(len));
    void convolve(vector<ll>& a, vector<ll> b) {
        ll t = modPow(n, M-2, M);
        a.resize(n); b.resize(n);
        ntt<M,R,0>(a); ntt<M,R,0>(b);
        rep(i, 0, n) a[i] = a[i]*b[i] % M * t % M;
        ntt<M,R,1>(a);
        a.resize(len);
    } // 4b4d

```

```

ll egcd(ll a, ll b, ll& x, ll& y) {
    if (!a) return x=0, y=1, b;
    ll d = egcd(b%a, a, y, x);
    x -= b/a*y;
    return d;
} // 23c8

```

```

// Convolve a and b with 64-bit output,
// store result in a; time: O(n lg n), 6x NTT
// Input is expected to be non-negative!
void convLong(vector<ll>& a, vector<ll> b) {
    const ll M1 = (479<<21)+1, M2 = (483<<21)+1;
    const ll MX = M1*M2, R = 62;

```

```

    vector<ll> c = a, d = b;
    each(k, a) k %= M1; each(k, b) k %= M1;
    each(k, c) k %= M2; each(k, d) k %= M2;

```

```

    convolve<M1, R>(a, b);
    convolve<M2, R>(c, d);

```

```

    ll x, y; egcd(M1, M2, x, y);

```

```

    rep(i, 0, sz(a)) {
        a[i] += (c[i]-a[i])*x % M2 * M1;
        if ((a[i] %= MX) < 0) a[i] += MX;
    } // 2279
} // ef93

```

**math/fwht.h**

3e6f

```

// Fast Walsh-Hadamard Transform; O(n lg n)
// Input must be power of 2!
// Uncommented version is for XOR.
// OR version is equivalent to sum-over-subsets
// (Zeta transform, inverse is Moebius).
// AND version is same as sum-over-supersets.

```

```

template<bool inv, class T>
void fwht(vector<T>& b) {
    for (int s = 1; s < sz(b); s *= 2) {
        for (int i = 0; i < sz(b); i += s*2) {
            rep(j, i, i+s) {
                auto &x = b[j], &y = b[j+s];
                tie(x, y) =
                    mp(x+y, x-y); //XOR
                // inv ? mp(x-y, y) : mp(x+y, y); //AND
                // inv ? mp(x, y-x) : mp(x, x+y); //OR
            } // eea9
        } // a3d5
    } // 95ed

    // ONLY FOR XOR:
    if (inv) each(e, b) e /= sz(b);
} // 0779

```

```

// Compute convolution of a and b such that
// ans[i#j] += a[i]*b[j], where # is OR, AND
// or XOR, depending on FWHT version.
// Stores result in a; time: O(n lg n)
// Both arrays must be of same size = 2^n!

```

```

template<class T>
void bitConv(vector<T>& a, vector<T> b) {
    fwht<0>(a);
    fwht<0>(b);
    rep(i, 0, sz(a)) a[i] *= b[i];
    fwht<1>(a);
} // 7b82

```

**math/gauss.h**

7f0b

```

// Solve system of linear equations; O(n^2*m)
// - A - extended matrix, rows are equations,
//   columns are variables,
//   m-th column is equation result
// (A[i][j] - i-th row and j-th column)
// - ans - output for variables values
// - m - variable count
// Returns 0 if no solutions found, 1 if one,
// 2 if more than 1 solution exist.

```

```

int gauss(vector<vector<double>>& A,
          vector<double>& ans, int m) {

```

```

    Vi col;
    ans.assign(m, 0);

    rep(i, 0, sz(A)) {
        int c = 0;
        while (c <= m && !cmp(A[i][c], 0)) c++;
        // For Zp:
        //while (c <= m && !A[i][c].x) c++;

        if (c >= m) {
            if (c == m) return 0;
            continue;
        } // a6bb
    }

    rep(k, i+1, sz(A)) {
        auto mult = A[k][c] / A[i][c];
        rep(j, 0, m+1) A[k][j] -= A[i][j]*mult;
    } // 8dd5

    swap(A[i], A[sz(col)]);
    col.pb(c);
} // ea2c

```

```

    for (int i = sz(col); i--;) {
        ans[col[i]] = A[i][m] / A[i][col[i]];
        rep(k, 0, i)
            A[k][m] -= ans[col[i]] * A[k][col[i]];
    } // 31b9

```

```

    return sz(col) < m ? 2 : 1;
} // 0b76

```

**math/linear\_rec.h**

60be

```

constexpr ll MOD = 998244353;

using Poly = vector<ll>;

// Compute k-th term of an n-order linear
// recurrence C[i] = sum C[i-j-1]*D[j],
// given C[0..n-1] and D[0..n-1]; O(n^2 log k)
ll linearRec(const Poly& C,
             const Poly& D, ll k) {
    int n = sz(D);

    auto mul = [&](Poly a, Poly b) {
        Poly ret(n*2+1);
        rep(i, 0, n+1) rep(j, 0, n+1)
            ret[i+j] = (ret[i+j] + a[i]*b[j]) % MOD;
        for (int i = n*2; i > n; i--) rep(j, 0, n)
            ret[i-j-1] =
                (ret[i-j-1] + ret[i]*D[j]) % MOD;
        ret.resize(n+1);
        return ret;
    }; // e722

    Poly pol(n+1), e(n+1);
    pol[0] = e[1] = 1;

    for (k++; k; k /= 2) {
        if (k % 2) pol = mul(pol, e);
        e = mul(e, e);
    } // 13af

    ll ret = 0;
    rep(i, 0, n) ret = (ret + pol[i+1]*C[i]) % MOD;
    return ret;
} // 3fd1

```

**math/linear\_rec\_fast.h**

58a8

```

#include "polynomial.h"

// Compute k-th term of an n-order linear
// recurrence C[i] = sum C[i-j-1]*D[j],
// given C[0..n-1] and D[0..n-1];
// time: O(n log n log k)
Zp linearRec(const Poly& C,
             const Poly& D, ll k) {
    Poly f(sz(D)+1, 1);
    rep(i, 0, sz(D)) f[i] = -D[sz(D)-i-1];
    f = pow({0, 1}, k, f);
    Zp ret = 0;
    rep(i, 0, sz(f)) ret += f[i]*C[i];
    return ret;
} // 5b8d

```

**math/matrix.h**

9bf7

```

#include "modular.h"

using Row = vector<Zp>;
using Matrix = vector<Row>;

// Create n x n identity matrix
Matrix ident(int n) {
    Matrix ret(n, Row(n));
    rep(i, 0, n) ret[i][i] = 1;
    return ret;
} // addd

// Add matrices
Matrix& operator+=(Matrix& l, const Matrix& r){
    rep(i, 0, sz(l)) rep(k, 0, sz(l[0]))
        l[i][k] += r[i][k];

```

```
    return l;
} // b6bf
Matrix operator+(Matrix l, const Matrix& r) {
    return l += r;
} // d9b3

// Subtract matrices
Matrix& operator-=(Matrix& l, const Matrix& r){
    rep(i, 0, sz(l)) rep(k, 0, sz(l[0]))
        l[i][k] -= r[i][k];
    return l;
} // 90a1
Matrix operator-(Matrix l, const Matrix& r) {
    return l -= r;
} // dc4f

// Multiply matrices
Matrix operator*(const Matrix& l,
                  const Matrix& r) {
    Matrix ret(sz(l), Row(sz(r[0])));
    rep(i, 0, sz(l)) rep(j, 0, sz(r[0]))
        rep(k, 0, sz(r))
            ret[i][j] += l[i][k] * r[k][j];
    return ret;
} // 52ca
Matrix& operator*=(Matrix& l, const Matrix& r){
    return l = l*r;
} // da8a

// Square matrix power; time: O(n^3 * lg e)
Matrix pow(Matrix a, ll e) {
    Matrix t = ident(sz(a));
    while (e) {
        if (e % 2) t *= a;
        e /= 2; a *= a;
    } // 4400
    return t;
} // 65ea

// Transpose matrix
Matrix transpose(const Matrix& m) {
    Matrix ret(sz(m[0]), Row(sz(m)));
    rep(i, 0, sz(m)) rep(j, 0, sz(m[0]))
        ret[j][i] = m[i][j];
    return ret;
} // 5650

// Transform matrix to echelon form
// and compute its determinant sign and rank.
int echelon(Matrix& A, int& sign) { // O(n^3)
    int rank = 0;
    sign = 1;
    rep(c, 0, sz(A[0])) {
        if (rank >= sz(A)) break;
        rep(i, rank+1, sz(A)) if (A[i][c].x) {
            swap(A[i], A[rank]);
            sign *= -1;
            break;
        } // f98a
        if (A[rank][c].x) {
            rep(i, rank+1, sz(A)) {
                auto mult = A[i][c] / A[rank][c];
                rep(j, 0, sz(A[0]))
                    A[i][j] -= A[rank][j]*mult;
            } // f519
            rank++;
        } // 4cd8
    } // 36e9
    return rank;
} // 6882
```

```
// Compute matrix rank; time: O(n^3)
#define rank_rank_
int rank(Matrix A) {
    int s; return echelon(A, s);
} // c599

// Compute square matrix determinant; O(n^3)
Zp det(Matrix A) {
    int s; echelon(A, s);
    Zp ret = s;
    rep(i, 0, sz(A)) ret *= A[i][i];
    return ret;
} // b252

// Invert square matrix if possible; O(n^3)
// Returns true if matrix is invertible.
bool invert(Matrix& A) {
    int s, n = sz(A);
    rep(i, 0, n) A[i].resize(n*2), A[i][n+i] = 1;
    echelon(A, s);
    for (int i = n; i--;) {
        if (!A[i][i].x) return 0;
        auto mult = A[i][i].inv();
        each(k, A[i]) k *= mult;
        rep(k, 0, i) rep(j, 0, n)
            A[k][n+j] -= A[i][n+j]*A[k][i];
    } // 1e97
    each(r, A) r.erase(r.begin(), r.begin()+n);
    return 1;
} // 65b9
```

math/miller\_rabin.h 7005

```
#include "modular64.h"

// Miller-Rabin primality test
// time O(k*lg^2 n), where k = number of bases

// Deterministic for p <= 10^9
// constexpr ll BASES[] = {
//     336781006125, 9639812373923155
// }; // d41d

// Deterministic for p <= 2^64
constexpr ll BASES[] = {
    2,325,9375,28178,450775,9780504,1795265022
}; // b8e0

bool isPrime(ll p) {
    if (p <= 2) return p == 2;
    if (p%2 == 0) return 0;

    ll d = p-1, t = 0;
    while (d%2 == 0) d /= 2, t++;

    each(a, BASES) if (a%p) {
        // ll a = rand() % (p-1) + 1;
        ll b = modPow(a%p, d, p);
        if (b == 1 || b == p-1) continue;
        rep(i, 1, t)
            if ((b = modMul(b, b, p)) == p-1) break;
        if (b != p-1) return 0;
    } // 9342
    return 1;
} // bec2

math/modinv_precompute.h 2882

constexpr ll MOD = 234567899;
vector<ll> modInv(MOD); // You can lower size

// Precompute modular inverses; time: O(MOD)
void initModInv() {
    modInv[1] = 1;
```

```
    rep(i, 2, sz(modInv)) modInv[i] =
        (MOD - (MOD/i) * modInv[MOD%i]) % MOD;
} // 22c1

math/modular.h 77ce

// Modulus often seen on Codeforces:
constexpr int MOD = 998244353;
// Some big prime: 15*(1<27)+1 ~ 2*10^9

ll modInv(ll a, ll m) { // a^(-1) mod m
    if (a == 1) return 1;
    return ((a - modInv(m%a, a))*m + 1) / a;
} // c437

ll modPow(ll a, ll e, ll m) { // a^e mod m
    ll t = 1 % m;
    while (e) {
        if (e % 2) t = t*a % m;
        e /= 2; a = a*a % m;
    } // 66ca
    return t;
} // 1973

// Wrapper for modular arithmetic
struct Zp {
    ll x; // Contained value, in range [0;MOD-1]
    Zp() : x(0) {}
    Zp(ll a) : x(a%MOD) { if (x < 0) x += MOD; }

    #define OP(c,d) Zp& operator c##=(Zp r) { \
        x = x d; return *this; } \
        Zp operator c(Zp r) const { \
            Zp t = *this; return t c##= r; } // e986

    OP(+, +r.x - MOD*(x+r.x >= MOD));
    OP(-, -r.x + MOD*(0 > x-r.x));
    OP(*, *r.x % MOD);
    OP(/, *r.inv().x % MOD);
    Zp operator-()
        const { Zp t; t.x = MOD-x; return t; }

    // For composite modulus use modInv, not pow
    Zp inv() const { return pow(MOD-2); }
    Zp pow(ll e) const{ return modPow(x,e,MOD); }
    void print() { cerr << x; } // For deb()
}; // ee2c

// Extended Euclidean Algorithm
ll egcd(ll a, ll b, ll& x, ll& y) {
    if (!a) return x=0, y=1, b;
    ll d = egcd(b%a, a, y, x);
    x -= b/a*y;
    return d;
} // 23c8

math/modular64.h 4b73

// Modular arithmetic for modulus < 2^62

ll modAdd(ll x, ll y, ll m) {
    x += y;
    return x < m ? x : x-m;
} // b653

ll modSub(ll x, ll y, ll m) {
    x -= y;
    return x >= 0 ? x : x+m;
} // b073

// About 4x slower than normal modulo
ll modMul(ll a, ll b, ll m) {
    ll c = ll((long double)a * b / m);
    ll r = (a*b - c*m) % m;
    return r < 0 ? r+m : r;
```

```
} // 1815

ll modPow(ll x, ll e, ll m) {
    ll t = 1;
    while (e) {
        if (e & 1) t = modMul(t, x, m);
        e >>= 1;
        x = modMul(x, x, m);
    } // bd61
    return t;
} // c8ba

math/modular_generator.h 845b

#include "modular.h" // modPow

// Get unique prime factors of n; O(sqrt n)
vector<ll> factorize(ll n) {
    vector<ll> fac;
    for (ll i = 2; i*i <= n; i++) {
        if (n%i == 0) {
            while (n%i == 0) n /= i;
            fac.pb(i);
        } // 6069
    } // a0cc
    if (n > 1) fac.pb(n);
    return fac;
} // 4a2a

// Find smallest primitive root mod n;
// time: O(sqrt(n) + g*log^2 n)
// Returns -1 if generator doesn't exist.
// For n <= 10^7 smallest generator is <= 115.
// You can use faster factorization algorithm
// to get rid of sqrt(n).
ll generator(ll n) {
    if (n <= 1 || (n > 4 && n%4 == 0)) return -1;

    vector<ll> fac = factorize(n);
    if (sz(fac) > (fac[0] == 2)+1) return -1;

    ll phi = n;
    each(p, fac) phi = phi / p * (p-1);
    fac = factorize(phi);

    for (ll g = 1; g++;) if (__gcd(g, n) == 1) {
        each(f, fac) if (modPow(g, phi/f, n) == 1)
            goto nxt;
        return g;
    } // db24
} // 7641

math/modular_log.h ac62

#include "modular.h" // modInv

// Baby-step giant-step algorithm; O(sqrt(p))
// Finds smallest x such that a^x = b (mod p)
// or returns -1 if there's no solution.
ll dlog(ll a, ll b, ll p) {
    int m = int(min(ll(sqrt(p))+2, p-1));
    unordered_map<ll, int> small;
    ll t = 1;

    rep(i, 0, m) {
        int& k = small[t];
        if (!k) k = i+1;
        t = t*a % p;
    } // fld0

    t = modInv(t, p);

    rep(i, 0, m) {
        int j = small[b];
```

```

    if (j) return i*ll(m) + j - 1;
    b = b*t % p;
} // c7ed

return -1;
} // 5c26

```

## math/modular\_sqrt.h db16

```

#include "modular.h" // modPow

// Tonelli-Shanks algorithm for modular sqrt
// modulo prime; O(lg^2 p), O(lg p) for most p
// Returns -1 if root doesn't exists or else
// returns square root x (the other one is -x).
ll modSqrt(ll a, ll p) {
    a %= p;
    if (a < 0) a += p;
    if (a <= 1) return a;
    if (modPow(a, p/2, p) != 1) return -1;
    if (p%4 == 3) return modPow(a, p/4+1, p);

    ll s = p-1, n = 2;
    int r = 0, j;
    while (s%2 == 0) s /= 2, r++;
    while (modPow(n, p/2, p) != p-1) n++;

    ll x = modPow(a, (s+1)/2, p);
    ll b = modPow(a, s, p), g = modPow(n, s, p);

    for (; r = j) {
        ll t = b;
        for (j = 0; j < r && t != 1; j++)
            t = t*t % p;
        if (!j) return x;
        ll gs = modPow(g, 1LL << (r-j-1), p);
        g = gs*gs % p;
        x = x*gs % p;
        b = b*g % p;
    } // f83f
} // 7a97

```

## math/montgomery.h a4ba

```

#include "modular.h" // modInv

// Montgomery modular multiplication
// MOD < MG_MULT, gcd(MG_MULT, MOD) must be 1
// Don't use if modulo is constexpr; UNTESTED
constexpr ll MG_SHIFT = 32;
constexpr ll MG_MULT = 1LL << MG_SHIFT;
constexpr ll MG_MASK = MG_MULT - 1;
const ll MG_INV = MG_MULT-modInv(MOD, MG_MULT);

// Convert to Montgomery form
ll MG(ll x) { return (x*MG_MULT) % MOD; }

// Montgomery reduction
// redc(mg * mg) = Montgomery-form product
ll redc(ll x) {
    ll q = (x * MG_INV) & MG_MASK;
    x = (x + q*MOD) >> MG_SHIFT;
    return (x >= MOD ? x-MOD : x);
} // d0f5

```

## math/nimber.h 474f

```

// Nimbers are defined as sizes of Nim heaps.
// Operations on nimbers are defined as:
// a+b = mex({a'+b : a' < a} u {a+b' : b' < b})
// ab = mex({a'b+ab'+a'b' : a' < a, b' < b})
// Nimbers smaller than M = 2^2^k form a field.
// Addition is equivalent to xor, meanwhile
// multiplication can be evaluated
// in O(lg^2 M) after precomputing.

```

```

using ull = uint64_t;
ull nbuf[64][64]; // Nim-products for 2^i * 2^j

// Multiply nimbers; time: O(lg^2 M)
// WARNING: Call initNimMul() before using.
ull nimMul(ull a, ull b) {
    ull ret = 0;
    for (ull s = a; s; s &= (s-1))
        for (ull t = b; t; t &= (t-1))
            ret ^= nbuf[__builtin_ctzll(s)]
                [__builtin_ctzll(t)];

    return ret;
} // 25be

// Initialize nim-products lookup table
void initNimMul() {
    rep(i, 0, 64)
        nbuf[i][0] = nbuf[0][i] = 1ull << i;
    rep(b, 1, 64) rep(a, 1, b+1) {
        int i = 1 << (63 - __builtin_clzll(a));
        int j = 1 << (63 - __builtin_clzll(b));
        ull t = nbuf[a-i][b-j];
        if (i < j)
            t = nimMul(t, 1ull << i) << j;
        else
            t = nimMul(t, 1ull << (i-1)) ^ (t << i);
        nbuf[a][b] = nbuf[b][a] = t;
    } // ca24
} // 1811

// Compute a^e under nim arithmetic; O(lg^3 M)
// WARNING: Call initNimMul() before using.
ull nimPow(ull a, ull e) {
    ull t = 1;
    while (e) {
        if (e & 2) t = nimMul(t, a);
        e /= 2; a = nimMul(a, a);
    } // da53
    return t;
} // c06c

// Compute inverse of a in 2^64 nim-field;
// time: O(lg^3 M)
// WARNING: Call initNimMul() before using.
ull nimInv(ull a) {
    return nimPow(a, ull(-2));
} // c6d9

// If you need to multiply many nimbers by
// the same value you can use this to speedup.
// WARNING: Call initNimMul() before using.
struct NimMult {
    ull M[64] = {0};

    // Initialize lookup; time: O(lg^2 M)
    NimMult(ull a) {
        for (ull t=a; t; t &= (t-1)) rep(i, 0, 64)
            M[i] ^= nbuf[__builtin_ctzll(t)][i];
    } // ea88

    // Multiply by b; time: O(lg M)
    ull operator()(ull b) {
        ull ret = 0;
        for (ull t = b; t; t &= (t-1))
            ret ^= M[__builtin_ctzll(t)];
        return ret;
    } // e480
}; // 1b80

```

## math/phi\_large.h 8703

```
#include "pollard_rho.h"
```

```

// Compute Euler's totient of large numbers
// time: O(n^(1/4)) <- factorization
ll phi(ll n) {
    each(p, factorize(n)) n = n / p.x * (p.x-1);
    return n;
} // 798e

math/phi_precompute.h 728b

Vi phi(1e7+1);

// Precompute Euler's totients; time: O(n lg n)
void calcPhi() {
    iota(all(phi), 0);
    rep(i, 2, sz(phi)) if (phi[i] == i)
        for (int j = i; j < sz(phi); j += i)
            phi[j] = phi[j] / i * (i-1);
} // 3c65

math/phi_prefix_sum.h a9e0

#include "phi_precompute.h"

vector<ll> phiSum; // [k] = sum from 0 to k-1

// Precompute Euler's totient prefix sums
// for small values; time: O(n lg n)
void calcPhiSum() {
    calcPhi();
    phiSum.resize(sz(phi)+1);
    rep(i, 0, sz(phi))
        phiSum[i+1] = phiSum[i] + phi[i];
} // 3855

// Get prefix sum of phi(0) + ... + phi(n-1).
// WARNING: Call calcPhiSum first!
// For n > 4*10^9, answer will overflow.
// If you wish to get answer mod M use
// commented lines.
ll getPhiSum(ll n) { // time: O(n^(2/3))
    static unordered_map<ll, ll> big;
    if (n < sz(phiSum)) return phiSum[n];
    if (big.count(--n)) return big[n];

    ll ret = n*(n+1)/2;
    // ll ret = (n%2 ? n%M * ((n+1)/2 % M)
    //           : n/2%M * (n%M+1) % M;

    for (ll s, i = 2; i <= n; i = s+1) {
        s = n / (n/i);
        ret -= (s-i+1) * getPhiSum(n/i+1);
        // ret -= (s-i+1)%M * getPhiSum(n/i+1) % M;
    } // e792

    // ret = (ret%M + M) % M;
    return big[n] = ret;
} // 820b

math/pi_large.h fcdb

constexpr int MAX_P = 1e7;
vector<ll> pis, prl;

// Precompute prime counting function
// for small values; time: O(n lg lg n)
void initPi() {
    pis.assign(MAX_P+1, 1);
    pis[0] = pis[1] = 0;

    for (int i = 2; i*i <= MAX_P; i++)
        if (pis[i])
            for (int j = i*i; j <= MAX_P; j += i)
                pis[j] = 0;

    rep(i, 1, sz(pis)) {

```

```

        if (pis[i]) prl.pb(i);
        pis[i] += pis[i-1];
    } // 0672
} // 6d92

ll partial(ll x, ll a) {
    static vector<unordered_map<ll, ll>> big;
    big.resize(sz(prl));
    if (!a) return (x+1) / 2;
    if (big[a].count(x)) return big[a][x];
    ll ret = partial(x, a-1)
        - partial(x / prl[a], a-1);
    return big[a][x] = ret;
} // 774f

// Count number of primes <= x;
// time: O(n^(2/3) * log(n)^(1/3))
// Set MAX_P to be > sqrt(x) and call initPi
// before using!
ll pi(ll x) {
    static unordered_map<ll, ll> big;
    if (x < sz(pis)) return pis[x];
    if (big.count(x)) return big[x];

    ll a = 0;
    while (prl[a]*prl[a]*prl[a]*prl[a] < x) a++;

    ll ret = 0, b = --a;

    while (++b < sz(prl) && prl[b]*prl[b] < x) {
        ll w = x / prl[b];
        ret -= pi(w);
        for (ll j = b; prl[j]*prl[j] <= w; j++)
            ret -= pi(w / prl[j]) - j;
    } // a584

    ret += partial(x, a) + (b+a-1)*(b-a)/2;
    return big[x] = ret;
} // eald

```

## math/pi\_large\_precomp.h 7fc0

```

#include "sieve.h"

// Count primes in given interval
// using precomputed table.
// Set MAX_P to sqrt(MAX_N) and run sieve()!
// Precomputed table will contain N_BUCKETS
// elements - check source size limit.

constexpr ll MAX_N = 1e11+1;
constexpr ll N_BUCKETS = 10000;
constexpr ll BUCKET_SIZE = (MAX_N/N_BUCKETS)+1;
constexpr ll precomputed[] = { /* ... */ };

ll sieveRange(ll from, ll to) {
    bitset<BUCKET_SIZE> elems;
    from = max(from, 2LL);
    to = max(from, to);
    each(p, primesList) {
        ll c = max((from+p-1) / p, 2LL);
        for (ll i = c*p; i < to; i += p)
            elems.set(i-from);
    } // a29f
    return to-from-elems.count();
} // c646

// Run once on local computer to precompute
// table. Takes about 10 minutes for n = 1e11.
// Sanity check (for default params):
// 664579, 606028, 587253, 575795, ...
void localPrecompute() {
    for (ll i = 0; i < MAX_N; i += BUCKET_SIZE) {
        ll to = min(i+BUCKET_SIZE, MAX_N);

```



```

    cout << sieveRange(i, to) << ',' << flush;
} // f6a7
cout << endl;
} // 2b1e

// Count primes in [from;to) using table.
// O(N_BUCKETS + BUCKET_SIZE*lg lg n + sqrt(n))
ll countPrimes(ll from, ll to) {
    ll bFrom = from/BUCKET_SIZE+1,
    bTo = to/BUCKET_SIZE;
    if (bFrom > bTo) return sieveRange(from, to);
    ll ret = accumulate(precomputed+bFrom,
        precomputed+bTo, 0);
    ret += sieveRange(from, bFrom*BUCKET_SIZE);
    ret += sieveRange(bTo*BUCKET_SIZE, to);
    return ret;
} // cced

```

**math/pollard\_rho.h**

ef01

```

#include "modular64.h"
#include "miller_rabin.h"

using Factor = pair<ll, int>;

void rho(vector<ll>& out, ll n) {
    if (n <= 1) return;
    if (isPrime(n)) out.pb(n);
    else if (n%2 == 0) rho(out, 2), rho(out, n/2);
    else for (ll a = 2; a++) {
        ll x = 2, y = 2, d = 1;
        while (d == 1) {
            x = modAdd(modMul(x, x, n), a, n);
            y = modAdd(modMul(y, y, n), a, n);
            y = modAdd(modMul(y, y, n), a, n);
            d = __gcd(abs(x-y), n);
        } // 3378
        if (d != n) return rho(out, d), rho(out, n/d);
    } // 047e
} // ba89

// Pollard's rho factorization algorithm
// Las Vegas version; time: n^(1/4)
// Returns pairs (prime, power), sorted
vector<Factor> factorize(ll n) {
    vector<Factor> ret;
    vector<ll> raw;
    rho(raw, n);
    sort(all(raw));
    each(f, raw) {
        if (ret.empty() || ret.back().x != f)
            ret.pb({f, 1});
        else
            ret.back().y++;
    } // 2ab1
    return ret;
} // 471c

```

**math/polynomial.h**

d8c4

```

#include "modular.h"
#include "fft_mod.h"

using Poly = vector<Zp>;

// Cut off trailing zeroes; time: O(n)
void norm(Poly& P) {
    while (!P.empty() && !P.back().x)
        P.pop_back();
} // 8a8a

// Evaluate polynomial at x; time: O(n)
Zp eval(const Poly& P, Zp x) {
    Zp n = 0, y = 1;

```

```

    each(a, P) n += a*y, y *= x;
    return n;
} // d865

// Add polynomial; time: O(n)
Poly& operator+=(Poly& l, const Poly& r) {
    l.resize(max(sz(l), sz(r)));
    rep(i, 0, sz(r)) l[i] += r[i];
    norm(l);
    return l;
} // 656e

Poly operator+(Poly l, const Poly& r) {
    return l += r;
} // d9b3

// Subtract polynomial; time: O(n)
Poly& operator-=(Poly& l, const Poly& r) {
    l.resize(max(sz(l), sz(r)));
    rep(i, 0, sz(r)) l[i] -= r[i];
    norm(l);
    return l;
} // c68b

Poly operator-(Poly l, const Poly& r) {
    return l -= r;
} // dc4f

// Multiply by polynomial; time: O(n lg n)
Poly& operator*=(Poly& l, const Poly& r) {
    if (min(sz(l), sz(r)) < 50) {
        // Naive multiplication
        Poly P(sz(l)+sz(r));
        rep(i, 0, sz(l)) rep(j, 0, sz(r))
            P[i+j] += l[i]*r[j];
        l.swap(P);
    } else {
        // FFT multiplication
        vector<ll> a, b;
        each(k, l) a.pb(k.x);
        each(k, r) b.pb(k.x);
        // Choose appropriate convolution method,
        // see fft_mod.h and fft_complex.h
        convolve<MOD, 62>(a, b);
        l.assign(all(a));
    } // f730
    norm(l);
    return l;
} // 28de

Poly operator*(Poly l, const Poly& r) {
    return l *= r;
} // 2de3

// Derivate polynomial; time: O(n)
Poly derivate(Poly P) {
    if (!P.empty()) {
        rep(i, 1, sz(P)) P[i-1] = P[i]*i;
        P.pop_back();
    } // bd78
    return P;
} // c6c5

// Integrate polynomial; time: O(n)
Poly integrate(Poly P) {
    if (!P.empty()) {
        P.pb(0);
        for (int i = sz(P); --i;) P[i] = P[i-1]/i;
        P[0] = 0;
    } // eec1
    return P;
} // e2f3

// Compute inverse series mod x^n; O(n lg n)

```

```

Poly invert(const Poly& P, int n) {
    assert(!P.empty() && P[0].x != 0);
    Poly tmp, ret = {P[0].inv()};

    for (int i = 1; i < n; i *= 2) {
        tmp.clear();
        rep(j, 0, min(i*2, sz(P))) tmp.pb(-P[j]);
        tmp *= ret;
        tmp[0] += 2;
        ret *= tmp;
        ret.resize(i*2);
    } // 139b

    ret.resize(n);
    return ret;
} // 4d3e

// Floor division by polynomial; O(n lg n)
Poly operator/(Poly l, Poly r) {
    norm(l); norm(r);
    int d = sz(l)-sz(r)+1;
    if (d <= 0) return {};
    reverse(all(l));
    reverse(all(r));
    l.resize(d);
    l *= invert(r, d);
    l.resize(d);
    reverse(all(l));
    return l;
} // d4a4

Poly& operator/=(Poly& l, const Poly& r) {
    return l = l/r;
} // e12e

// Compute modulo by polynomial; O(n lg n)
Poly operator%(const Poly& l, const Poly& r) {
    return l - r*(l/r);
} // 4fc8

Poly& operator%=(Poly& l, const Poly& r) {
    return l -= r*(l/r);
} // 80bb

// Compute a^e mod m, where a and m are
// polynomials; time: O(m log m log e)
Poly pow(Poly a, ll e, Poly m) {
    Poly t = {1};
    while (e) {
        if (e % 2) t = t*a % m;
        e /= 2; a = a*a % m;
    } // 66ca
    return t;
} // 6f9c

// Evaluate polynomial P in given points;
// time: O(n lg^2 n)
Poly eval(const Poly& P, Poly points) {
    int len = 1;
    while (len < sz(points)) len *= 2;

    vector<Poly> tree(len*2, {1});
    rep(i, 0, sz(points))
        tree[len+i] = {points[i], 1};

    for (int i = len; --i;)
        tree[i] = tree[i*2] * tree[i*2+1];

    tree[0] = P;
    rep(i, 1, len*2)
        tree[i] = tree[i/2] % tree[i];

    rep(i, 0, sz(points)) {
        auto& vec = tree[len+i];
        points[i] = vec.empty() ? 0 : vec[0];
    }

```

```

    } // clc2
    return points;
} // 69b0

// Given n points (x, f(x)) compute n-1-degree
// polynomial f that passes through them;
// time: O(n lg^2 n)
// For O(n^2) version see polynomial_interp.h.
Poly interpolate(const vector<pair<Zp, Zp>>& P) {
    int len = 1;
    while (len < sz(P)) len *= 2;

    vector<Poly> mult(len*2, {1}), tree(len*2);
    rep(i, 0, sz(P))
        mult[len+i] = {-P[i].x, 1};

    for (int i = len; --i;)
        mult[i] = mult[i*2] * mult[i*2+1];

    tree[0] = derivate(mult[1]);
    rep(i, 1, len*2)
        tree[i] = tree[i/2] % mult[i];

    rep(i, 0, sz(P))
        tree[len+i][0] = P[i].y / tree[len+i][0];

    for (int i = len; --i;)
        tree[i] = tree[i*2]*mult[i*2+1]
            + mult[i*2]*tree[i*2+1];
    return tree[1];
} // b706

```

**math/polynomial\_interp.h**

a4cc

```

// Interpolate set of points (i, vec[i])
// and return it evaluated at x; time: O(n)
template<class T>
T polyExtend(vector<T>& vec, T x) {
    int n = sz(vec);
    vector<T> fac(n, 1), suf(n, 1);

    rep(i, 1, n) fac[i] = fac[i-1] * i;
    for (int i=n; --i;) suf[i-1] = suf[i]*(x-i);

    T pref = 1, ret = 0;
    rep(i, 0, n) {
        T d = fac[i] * fac[n-i-1] * ((n-i)%2*2-1);
        ret += vec[i] * suf[i] * pref / d;
        pref *= x-i;
    } // 681d
    return ret;
} // dd92

// Given n points (x, f(x)) compute n-1-degree
// polynomial f that passes through them;
// time: O(n^2)
// For O(n lg^2 n) version see polynomial.h
template<class T>
vector<T> polyInterp(vector<pair<T, T>> P) {
    int n = sz(P);
    vector<T> ret(n), tmp(n);
    T last = 0;
    tmp[0] = 1;

    rep(k, 0, n-1) rep(i, k+1, n)
        P[i].y = (P[i].y-P[k].y) / (P[i].x-P[k].x);

    rep(k, 0, n) rep(i, 0, n) {
        ret[i] += P[k].y * tmp[i];
        swap(last, tmp[i]);
        tmp[i] -= last * P[k].x;
    } // af1c
    return ret;
} // 7c2c

```

**math/sieve.h** 3f3d

```
constexpr int MAX_P = 1e6;
bitset<MAX_P+1> primes;
Vi primesList;

// Erathostenes sieve; time: O(n lg lg n)
void sieve() {
    primes.set();
    primes.reset(0);
    primes.reset(1);

    for (int i = 2; i*i <= MAX_P; i++)
        if (primes[i])
            for (int j = i*i; j <= MAX_P; j += i)
                primes.reset(j);

    rep(i, 0, MAX_P+1) if (primes[i])
        primesList.pb(i);
} // d5ca
```

**math/sieve\_factors.h** 312d

```
constexpr int MAX_P = 1e6;
Vi factor(MAX_P+1);

// Erathostenes sieve with saving smallest
// factor for each number; time: O(n lg lg n)
void sieve() {
    for (int i = 2; i*i <= MAX_P; i++)
        if (!factor[i])
            for (int j = i*i; j <= MAX_P; j += i)
                if (!factor[j])
                    factor[j] = i;

    rep(i, 0, MAX_P+1) if (!factor[i]) factor[i] = i;
} // 82b6

// Factorize n <= MAX_P; time: O(lg n)
// Returns pairs (prime, power), sorted
vector<Pii> factorize(ll n) {
    vector<Pii> ret;
    while (n > 1) {
        int f = factor[n];
        if (ret.empty() || ret.back().x != f)
            ret.pb({f, 1});
        else
            ret.back().y++;
        n /= f;
    } // 664c
    return ret;
} // bb65
```

**math/sieve\_segmented.h** 849b

```
constexpr int MAX_P = 1e9;
bitset<MAX_P/2+1> primes; // Only odd numbers

// Cache-friendly Erathostenes sieve
// ~1.5s on Intel Core i5 for MAX_P = 10^9
// Memory usage: MAX_P/16 bytes
void sieve() {
    constexpr int SEG_SIZE = 1<<18;
    int pSqrt = int(sqrt(MAX_P)+0.5);
    vector<Pii> dels;
    primes.set();
    primes.reset(0);

    for (int i = 3; i <= pSqrt; i += 2) {
        if (primes[i/2]) {
            int j;
            for (j = i*i; j <= pSqrt; j += i*2)
                primes.reset(j/2);
            dels.pb({i, j/2});
        } // 9e62
```

```
} // ff49

for (int seg = pSqrt/2;
     seg <= sz(primes); seg += SEG_SIZE) {
    int lim = min(seg+SEG_SIZE, sz(primes));
    each(d, dels) for (; d.y < lim; d.y += d.x)
        primes.reset(d.y);
} // 97ae
} // 6456

bool isPrime(int x) {
    return x == 2 || (x%2 && primes[x/2]);
} // 422c
```

**math/simplex.h** c4cf

```
using dbl = double;
using Row = vector<dbl>;
using Matrix = vector<Row>;

#define ltj(X) if (s == -1 || \
    mp(X[j], N[j]) < mp(X[s], N[s])) s = j

// Simplex algorithm; time: O(nm * pivots)
// Given m x n matrix A, vector b of length m,
// vector c of length n solves the following:
// maximize c^T x, Ax <= b, x >= 0
// Output vector `x` contains optimal solution
// or some feasible solution in unbounded case.
// Returns objective value if bounded,
// +inf if unbounded, and -inf if no solution.
// You can test if double is inf using `isinf`.
// PARTIALLY TESTED
dbl simplex(const Matrix& A,
            const Row& b, const Row& c,
            Row& x, dbl eps = 1e-8) {
    int m = sz(b), n = sz(c);
    x.assign(n, 0);
    if (!n) return
        *min_element(all(b)) < -eps ? -1/.0 : 0;

    Vi N(n+1), B(m);
    Matrix D(m+2, Row(n+2));

    auto pivot = [&](int r, int s) {
        dbl inv = 1 / D[r][s];
        rep(i, 0, m+2)
            if (i != r && abs(D[i][s]) > eps) {
                dbl tmp = D[i][s] * inv;
                rep(j, 0, n+2) D[i][j] -= D[r][j] * tmp;
                D[i][s] = D[r][s] * tmp;
            } // 5281
        each(k, D[r]) k *= inv;
        each(k, D) k[s] *= -inv;
        D[r][s] = inv;
        swap(B[r], N[s]);
    }; // f56b

    auto solve = [&](int phase) {
        for (int y = m+phase-1; ) {
            int s = -1, r = -1;
            rep(j, 0, n+1)
                if (N[j] != -phase) ltj(D[y]);
            if (D[y][s] >= -eps) return 1;
            rep(i, 0, m)
                if (D[i][s] > eps && (r == -1 ||
                    mp(D[i][n+1] / D[i][s], B[i]) <
                    mp(D[r][n+1] / D[r][s], B[r]))) r=i;
            if (r == -1) return 0;
            pivot(r, s);
        } // 3bef
    }; // 614a
```

```
rep(i, 0, m) {
    copy(all(A[i]), D[i].begin());
    B[i] = n+i; D[i][n] = -1; D[i][n+1] = b[i];
} // b705

rep(j, 0, n) D[m][N[j]] = j;
N[n] = -1; D[m+1][n] = 1;

int r = 0;
rep(i, 1, m) if (D[i][n+1] < D[r][n+1]) r = i;
if (D[r][n+1] < -eps) {
    pivot(r, n);
    if (!solve(2) || D[m+1][n+1] < -eps)
        return -1/.0;
    rep(i, 0, m) if (B[i] == -1) {
        int s = 0;
        rep(j, 1, n+1) ltj(D[i]);
        pivot(i, s);
    } // 78fd
} // b52b
bool ok = solve(1);
rep(i, 0, m) if (B[i] < n) x[B[i]] = D[i][n+1];
return ok ? D[m][n+1] : 1/.0;
} // fe6f
```

**structures/bitset\_plus.h** 6737

```
// Undocumented std::bitset features:
// - _Find_first() - returns first bit = 1 or N
// - _Find_next(i) - returns first bit = 1
// after i-th bit
// or N if not found

// Bitwise operations for vector<bool>
// UNTESTED

#define OP(x) vector<bool>& operator x##=( \
    vector<bool>& l, const vector<bool>& r) { \
    assert(sz(l) == sz(r)); \
    auto a = l.begin(); auto b = r.begin(); \
    while (a<l_end()) *a._M_p++ x##= *b._M_p++; \
    return l; } // fl64

OP(&)OP(!)OP(^)
```

**structures/fenwick\_tree.h** b467

```
// Fenwick tree (BIT tree); space: O(n)
// Default version: prefix sums
struct Fenwick {
    using T = int;
    T ID = 0;
    T f(T a, T b) { return a+b; }

    vector<T> s;
    Fenwick(int n = 0) : s(n, ID) {}

    // A[i] = f(A[i], v); time: O(lg n)
    void modify(int i, T v) {
        for (; i < sz(s); i |= i+1) s[i] = f(s[i], v);
    } // a047

    // Get f(A[0], ..., A[i-1]); time: O(lg n)
    T query(int i) {
        T v = ID;
        for (; i > 0; i &= i-1) v = f(v, s[i-1]);
        return v;
    } // 9810

    // Find smallest i such that
    // f(A[0], ..., A[i-1]) >= val; time: O(lg n)
    // Prefixes must have non-decreasing values.
    int lowerBound(T val) {
        if (val <= ID) return 0;
        int i = -1, mask = 1;
        while (mask <= sz(s)) mask *= 2;
```

```
T off = ID;

while (mask /= 2) {
    int k = mask+i;
    if (k < sz(s)) {
        T x = f(off, s[k]);
        if (val > x) i=k, off=x;
    } // de7f
} // 929c
return i+2;
} // 4be9
}; // 1470
```

**structures/fenwick\_tree\_2d.h** 4945

```
// Fenwick tree 2D (BIT tree 2D); space: O(n*m)
// Default version: prefix sums 2D
// Change s to hashmap for O(q lg^2 n) memory
struct Fenwick2D {
    using T = int;
    T ID = 0;
    T f(T a, T b) { return a+b; }

    vector<T> s;
    int w, h;

    Fenwick2D(int n = 0, int m = 0)
        : s(n*m, ID), w(n), h(m) {}

    // A[i, j] = f(A[i, j], v); time: O(lg^2 n)
    void modify(int i, int j, T v) {
        for (; i < w; i |= i+1)
            for (int k = j; k < h; k |= k+1)
                s[i*h+k] = f(s[i*h+k], v);
    } // d46b

    // Query prefix; time: O(lg^2 n)
    T query(int i, int j) {
        T v = ID;
        for (; i>0; i&=i-1)
            for (int k = j; k > 0; k &= k-1)
                v = f(v, s[i*h+k-h-1]);
        return v;
    } // 08cf
}; // 36b4
```

**structures/find\_union.h** b4de

```
// Disjoint set data structure; space: O(n)
// Operations work in amortized O(alpha(n))
struct FAU {
    Vi G;
    FAU(int n = 0) : G(n, -1) {}

    // Get size of set containing i
    int size(int i) { return -G[find(i)]; }

    // Find representative of set containing i
    int find(int i) {
        return G[i] < 0 ? i : G[i] = find(G[i]);
    } // 5bcl

    // Union sets containing i and j
    bool join(int i, int j) {
        i = find(i); j = find(j);
        if (i == j) return 0;
        if (G[i] > G[j]) swap(i, j);
        G[i] += G[j]; G[j] = i;
        return 1;
    } // c721
}; // 62a4
```

**structures/find\_union\_undo.h** ce37

```
// Disjoint set data structure
```

```
// with rollback; space: O(n)
// Operations work in O(log(n)) time.
struct RollbackFAU {
    Vi G;
    vector<Pii> his;

    RollbackFAU(int n = 0) : G(n, -1) {}

    // Get size of set containing i
    int size(int i) { return -G[find(i)]; }

    // Find representative of set containing i
    int find(int i) {
        return G[i] < 0 ? i : find(G[i]);
    } // e478

    // Current time (for rollbacks)
    int time() { return sz(his); }

    // Rollback all operations after time `t`
    void rollback(int t) {
        for (int i = time(); t < i--;)
            G[his[i].x] = his[i].y;
        his.resize(t);
    } // 3ef3

    // Union sets containing i and j
    bool join(int i, int j) {
        i = find(i); j = find(j);
        if (i == j) return 0;
        if (G[i] > G[j]) swap(i, j);
        his.pb({i, G[i]});
        his.pb({j, G[j]});
        G[i] += G[j]; G[j] = i;
        return 1;
    } // 1491
}; // da8c
```

## structures/hull\_offline.h 3030

```
constexpr ll INF = 2e18;
// constexpr double INF = 1e30;
// constexpr double EPS = 1e-9;

// MAX of linear functions; space: O(n)
// Use if you add lines in increasing `a` order
// Default uncommented version is for int64
struct Hull {
    using T = ll; // Or change to double

    struct Line {
        T a, b, end;
        T intersect(const Line& r) const {
            // Version for double:
            //if (r.a-a < EPS) return b>r.b?INF:-INF;
            //return (b-r.b) / (r.a-a);
            if (a==r.a) return b > r.b ? INF : -INF;
            ll u = b-r.b, d = r.a-a;
            return u/d + ((u^d) >= 0 || !(u^d));
        } // f27f
    }; // 10dc

    vector<Line> S;
    Hull() { S.pb({ 0, -INF, INF }); }

    // Insert f(x) = ax+b; time: amortized O(1)
    void push(T a, T b) {
        Line l{a, b, INF};
        while (true) {
            T e = S.back().end=S.back().intersect(l);
            if (sz(S) < 2 || S[sz(S)-2].end < e)
                break;
            S.pop_back();
        } // 044f
```

```
S.pb(l);
    } // 978e

    // Query max(f(x) for each f): time: O(lg n)
    T query(T x) {
        auto t = *upper_bound(all(S), x,
            [](int l, const Line& r) {
                return l < r.end;
            }); // de77
        return t.a*x + t.b;
    } // b8de
}; // ld64
```

## structures/hull\_online.h 2a7b

```
constexpr ll INF = 2e18;
// MAX of linear functions online; space: O(n)
struct Hull {
    static bool modeQ; // Toggles operator< mode

    struct Line {
        mutable ll a, b, end;

        ll intersect(const Line& r) const {
            if (a==r.a) return b > r.b ? INF : -INF;
            ll u = b-r.b, d = r.a-a;
            return u/d + ((u^d) >= 0 || !(u^d));
        } // f27f

        bool operator<(const Line& r) const {
            return modeQ ? end < r.end : a < r.a;
        } // cfab
    }; // 6046

    multiset<Line> S;
    Hull() { S.insert({ 0, -INF, INF }); }

    // Updates segment end
    bool update(multiset<Line>::iterator it) {
        auto cur = it++; cur->end = INF;
        if (it == S.end()) return false;
        cur->end = cur->intersect(*it);
        return cur->end >= it->end;
    } // 63b8

    // Insert f(x) = ax+b; time: O(lg n)
    void insert(ll a, ll b) {
        auto it = S.insert({ a, b, INF });
        while (update(it)) it = --S.erase(++it);
        rep(i, 0, 2)
            while (it != S.begin() && update(--it))
                update(it = --S.erase(++it));
    } // 4f69

    // Query max(f(x) for each f): time: O(lg n)
    ll query(ll x) {
        modeQ = 1;
        auto l = *S.upper_bound({ 0, 0, x });
        modeQ = 0;
        return l.a*x + l.b;
    } // 7533
}; // 037e
```

bool Hull::modeQ = false;

## structures/max\_queue.h 3e9e

```
// Queue with max query on contained elements
struct MaxQueue {
    using T = int;
    deque<T> Q, M;

    // Add v to the back; time: amortized O(1)
    void push(T v) {
```

```
    while (!M.empty() && M.back() < v)
        M.pop_back();
    M.pb(v); Q.pb(v);
} // 57a2

// Pop from the front; time: O(1)
void pop() {
    if (M.front() == Q.front()) M.pop_front();
    Q.pop_front();
} // 101c

// Get max element value; time: O(1)
T max() const { return M.front(); }
}; // b6c4
```

## structures/pairing\_heap.h b2a7

```
// Pairing heap implementation; space O(n)
// Elements are stored in vector for faster
// allocation. It's MINIMUM queue.
// Allows to merge heaps in O(1)
template<class T, class Cmp = less<T>>
struct PHeap {
    struct Node {
        T val;
        int child{-1}, next{-1}, prev{-1};

        Node(T x = T()) : val(x) {}
    }; // 1lee

    using Vnode = vector<Node>;
    Vnode& M;
    int root{-1};

    int unlink(int& i) {
        if (i >= 0) M[i].prev = -1;
        int x = i; i = -1;
        return x;
    } // d9f6

    void link(int host, int& i, int val) {
        if (i >= 0) M[i].prev = -1;
        i = val;
        if (i >= 0) M[i].prev = host;
    } // 47d5

    int merge(int l, int r) {
        if (l < 0) return r;
        if (r < 0) return l;
        if (Cmp()(M[l].val, M[r].val)) swap(l, r);

        link(l, M[l].next, unlink(M[r].child));
        link(r, M[r].child, l);
        return r;
    } // fc42

    int mergePairs(int v) {
        if (v < 0 || M[v].next < 0) return v;
        int v2 = unlink(M[v].next);
        int v3 = unlink(M[v2].next);
        return merge(merge(v, v2), mergePairs(v3));
    } // 2eea

    // ---

    // Initialize heap with given node storage
    // Just declare 1 Vnode and pass it to heaps
    PHeap(Vnode& mem) : M(mem) {}

    // Add given key to heap, returns index; O(1)
    int push(const T& x) {
        int index = sz(M);
        M.emplace_back(x);
        root = merge(root, index);
        return index;
    }
```

```
} // e744

// Change key of i to smaller value; O(1)
void decrease(int i, T val) {
    assert(!Cmp()(M[i].val, val));
    M[i].val = val;

    int prev = M[i].prev;
    if (prev < 0) return;

    auto& p = M[prev];
    link(prev, (p.child == i ? p.child
        : p.next), unlink(M[i].next));

    root = merge(root, i);
} // 1a67

bool empty() { return root < 0; }
const T& top() { return M[root].val; }

// Merge with other heap. Must use same vec.
void merge(PHeap& r) { // time: O(1)
    assert(&M == &r.M);
    root = merge(root, r.root); r.root = -1;
} // 9623

// Remove min element; time: O(lg n)
void pop() {
    root = mergePairs(unlink(M[root].child));
} // 5b13
}; // 09f3
```

## structures/rmq.h f749

```
// Range Minimum Query; space: O(n lg n)
struct RMQ {
    using T = int;
    T ID = INT_MAX;
    T f(T a, T b) { return min(a, b); }

    vector<vector<T>> s;

    // Initialize RMQ structure; time: O(n lg n)
    RMQ(const vector<T>& vec = {}) {
        s = {vec};
        for (int h = 1; h <= sz(vec); h *= 2) {
            s.pb({});
            auto& prev = s[sz(s)-2];
            rep(i, 0, sz(vec)-h*2+1)
                s.back().pb(f(prev[i], prev[i+h]));
        } // 7c37
    } // 14ed

    // Query f(s[b], ... ,s[e-1]); time: O(1)
    T query(int b, int e) {
        if (b >= e) return ID;
        int k = 31 - __builtin_clz(e-b);
        return f(s[k][b], s[k][e - (1<<k)]);
    } // caaa
}; // elce
```

## structures/segtree\_config.h 7ef8

```
// Segment tree configurations to be used
// in segtree_general and segtree_persistent.
// See comments in TREE_PLUS version
// to understand how to create custom ones.
// Capabilities notation: (update; query)

#if TREE_PLUS // (+; sum, max, max count)
// time: O(lg n)
using T = int; // Data type for update
// operations (lazy tag)

T ID = 0; // Neutral value for
// updates and lazy tags
```

```
// This structure keeps aggregated data
struct Agg {
    // Aggregated data: sum, max, max count
    // Default values should be neutral
    // values, i.e. "aggregate over empty set"
    T sum{0}, vMax{INT_MIN}, nMax{0};

    // Initialize as leaf (single value)
    void leaf() { sum = vMax = 0; nMax = 1; }

    // Combine data with aggregated data
    // from node to the right
    void merge(const Agg& r) {
        if (vMax < r.vMax) nMax = r.nMax;
        else if (vMax == r.vMax) nMax += r.nMax;
        vMax = max(vMax, r.vMax);
        sum += r.sum;
    } // 8850

    // Apply update provided in `x`:
    // - update aggregated data
    // - update lazy tag `lazy`
    // - `size` is amount of elements
    // - return 0 if update should branch
    // (to be used in "segment tree beats")
    // - if you change value of `x` changed
    // value will be passed to next node
    // to the right during updates
    bool apply(T& lazy, T& x, int size) {
        lazy += x;
        sum += x*size;
        vMax += x;
        return 1;
    } // 4858
}; // 9bf5

#ifdef TREE_MAX // (max; max, max count)
// time: O(lg n)
using T = int;
T ID = INT_MIN;

struct Agg {
    // Aggregated data: max value, max count
    T vMax{INT_MIN}, nMax{0};
    void leaf() { vMax = 0; nMax = 1; }

    void merge(const Agg& r) {
        if (vMax < r.vMax) nMax = r.nMax;
        else if (vMax == r.vMax) nMax += r.nMax;
        vMax = max(vMax, r.vMax);
    } // f56b

    bool apply(T& lazy, T& x, int size) {
        if (vMax <= x) nMax = size;
        lazy = max(lazy, x);
        vMax = max(vMax, x);
        return 1;
    } // 8bd5
}; // 15b6

#ifdef TREE_SET // (=; sum, max, max count)
// time: O(lg n)
// Set ID to some unused value.
using T = int;
T ID = INT_MIN;

struct Agg {
    // Aggregated data: sum, max, max count
    T sum{0}, vMax{INT_MIN}, nMax{0};
    void leaf() { sum = vMax = 0; nMax = 1; }

    void merge(const Agg& r) {
        if (vMax < r.vMax) nMax = r.nMax;
        else if (vMax == r.vMax) nMax += r.nMax;
```

```
        vMax = max(vMax, r.vMax);
        sum += r.sum;
    } // 8850

    bool apply(T& lazy, T& x, int size) {
        lazy = x;
        sum = x*size;
        vMax = x;
        nMax = size;
        return 1;
    } // 845b
}; // 7488

#ifdef TREE_BEATS // (+, min; sum, max)
// time: amortized O(lg n) if not using +
// amortized O(lg^2 n) if using +
// Lazy tag is pair (add, min).
// To add x: run update with {x, INT_MAX},
// to min x: run update with {0, x}.
// When both parts are provided addition
// is applied first, then minimum.
using T = Pii;
T ID = {0, INT_MAX};

struct Agg {
    // Aggregated data: max value, max count,
    // second max value, sum
    int vMax{INT_MIN}, nMax{0}, max2{INT_MIN};
    int sum{0};
    void leaf() { sum = vMax = 0; nMax = 1; }

    void merge(const Agg& r) {
        if (r.vMax > vMax) {
            max2 = vMax;
            vMax = r.vMax;
            nMax = r.nMax;
        } else if (r.vMax == vMax) {
            nMax += r.nMax;
        } else if (r.vMax > max2) {
            max2 = r.vMax;
        } // b074
        max2 = max(max2, r.max2);
        sum += r.sum;
    } // 3124

    bool apply(T& lazy, T& x, int size) {
        if (max2 != INT_MIN && max2+x.x >= x.y)
            return 0;

        lazy.x += x.x;
        sum += x.x*size;
        vMax += x.x;
        if (max2 != INT_MIN) max2 += x.x;

        if (x.y < vMax) {
            sum -= (vMax-x.y) * nMax;
            vMax = x.y;
        } // 7025
        lazy.y = vMax;
        return 1;
    } // fe0c
}; // 2924
#endif

structures/segtree_general.h 725a

// Highly configurable statically allocated
// (interval; interval) segment tree;
// space: O(n)
struct SegTree {
    // Choose/write configuration
    #include "segtree_config.h"

    // Root node is 1, left is i*2, right i*2+1
```

```
vector<Agg> agg; // Aggregated data for nodes
vector<T> lazy; // Lazy tags for nodes
int len{1}; // Number of leaves

// Initialize tree for n elements; time: O(n)
SegTree(int n = 0) {
    while (len < n) len *= 2;
    agg.resize(len*2);
    lazy.resize(len*2, ID);
    rep(i, 0, n) agg[len+i].leaf();
    for (int i = len; --i;)
        (agg[i] = agg[i*2]).merge(agg[i*2+1]);
} // 4417

void push(int i, int s) {
    if (lazy[i] != ID) {
        agg[i*2].apply(lazy[i*2], lazy[i], s/2);
        agg[i*2+1].apply(lazy[i*2+1],
                        lazy[i], s/2);

        lazy[i] = ID;
    } // 3ba9
} // 5d19

// Modify interval [vb;ve) with val; O(lg n)
T update(int vb, int ve, T val, int i = 1,
        int b = 0, int e = -1) {
    if (e < 0) e = len;
    if (vb >= e || b >= ve) return val;

    if (b >= vb && e <= ve &&
        agg[i].apply(lazy[i], val, e-b))
        return val;

    int m = (b+e) / 2;
    push(i, e-b);
    val = update(vb, ve, val, i*2, b, m);
    val = update(vb, ve, val, i*2+1, m, e);
    (agg[i] = agg[i*2]).merge(agg[i*2+1]);
    return val;
} // aa8e

// Query interval [vb;ve); time: O(lg n)
Agg query(int vb, int ve, int i = 1,
        int b = 0, int e = -1) {
    if (e < 0) e = len;
    if (vb >= e || b >= ve) return {};
    if (b >= vb && e <= ve) return agg[i];

    int m = (b+e) / 2;
    push(i, e-b);
    Agg t = query(vb, ve, i*2, b, m);
    t.merge(query(vb, ve, i*2+1, m, e));
    return t;
} // 1a1e
}; // db5c

structures/segtree_persist.h dcfc

// Highly configurable (interval; interval)
// persistent segment tree;
// space: O(queries lg n)
// First tree version number is 0.
struct SegTree {
    // Choose/write configuration
    #include "segtree_config.h"

    vector<Agg> agg; // Aggregated data for nodes
    vector<T> lazy; // Lazy tags for nodes
    vector<bool> cow; // Copy children on push?
    Vi L, R; // Children links
    int len{1}; // Number of leaves

    // Initialize tree for n elements; O(lg n)
    SegTree(int n = 0) {
```

```
    int k = 1;
    while (len < n) len *= 2, k++;

    agg.resize(k);
    lazy.resize(k, ID);
    cow.resize(k, 1);
    L.resize(k);
    R.resize(k);
    agg[1].leaf();

    while (k-->1) {
        (agg[k] = agg[k+1]).merge(agg[k+1]);
        L[k] = R[k] = k+1;
    } // 211f
} // 83cf

// New version from version `i`; time: O(1)
// First version number is 0.
int fork(int i) {
    L.pb(L[i]); R.pb(R[i]); cow.pb(cow[i] = 1);
    agg.pb(agg[i]); lazy.pb(lazy[i]);
    return sz(L)-1;
} // a21b

void push(int i, int s, bool w) {
    bool has = (lazy[i] != ID);
    if ((has || w) && cow[i]) {
        int a = fork(L[i]), b = fork(R[i]);
        L[i] = a; R[i] = b; cow[i] = 0;
    } // 1a3e
    if (has) {
        agg[L[i]].apply(lazy[L[i]], lazy[i], s/2);
        agg[R[i]].apply(lazy[R[i]], lazy[i], s/2);
        lazy[i] = ID;
    } // eca6
} // 9f84

// Modify interval [vb;ve) with val
// in tree version `i`; time: O(lg n)
T update(int i, int vb, int ve, T val,
        int b = 0, int e = -1) {
    if (e < 0) e = len;
    if (vb >= e || b >= ve) return val;

    if (b >= vb && e <= ve &&
        agg[i].apply(lazy[i], val, e-b))
        return val;

    int m = (b+e) / 2;
    push(i, e-b, 1);
    val = update(L[i], vb, ve, val, b, m);
    val = update(R[i], vb, ve, val, m, e);
    (agg[i] = agg[L[i]]).merge(agg[R[i]]);
    return val;
} // 776e

// Query interval [vb;ve)
// in tree version `i`; time: O(lg n)
Agg query(int i, int vb, int ve,
        int b = 0, int e = -1) {
    if (e < 0) e = len;
    if (vb >= e || b >= ve) return {};
    if (b >= vb && e <= ve) return agg[i];

    int m = (b+e) / 2;
    push(i, e-b, 0);
    Agg t = query(L[i], vb, ve, b, m);
    t.merge(query(R[i], vb, ve, m, e));
    return t;
} // abf4
}; // 8a44
```



**structures/segtree\_point.h** 2b0f

```
// Segment tree (point, interval)
// Configure by modifying:
// - T - stored data type
// - ID - neutral element for query operation
// - f(a, b) - combine results
struct SegTree {
    using T = int;
    T ID = INT_MIN;
    T f(T a, T b) { return max(a,b); }

    vector<T> V;
    int len;

    // Initialize tree for n elements; time: O(n)
    SegTree(int n = 0, T def = 0) {
        for (len = 1; len < n; len *= 2);
        V.resize(len*2, ID);
        rep(i, 0, n) V[len+i] = def;
        for (int i = len; --i;)
            V[i] = f(V[i*2], V[i*2+1]);
    } // 459e

    // Set element 'i' to 'val'; time: O(lg n)
    void set(int i, T val) {
        V[i += len] = val;
        while (i /= 2)
            V[i] = f(V[i*2], V[i*2+1]);
    } // 4bcd

    // Query interval [b;e]; time: O(lg n)
    T query(int b, int e) {
        b += len; e += len-1;
        if (b > e) return ID;
        if (b == e) return V[b];
        T x = f(V[b], V[e]);

        while (b/2 < e/2) {
            if (~b&1) x = f(x, V[b^1]);
            if (e&1) x = f(x, V[e^1]);
            b /= 2; e /= 2;
        } // 444a
        return x;
    } // de36
}; // d571
```

**structures/treap.h** 0da3

```
// "Set" of implicit keyed treaps; space: O(n)
// Nodes are keyed by their indices in array
// of all nodes. Treap key is key of its root.
// "Node x" means "node with key x".
// "Treap x" means "treap with key x".
// Key -1 is "null".
// Put any additional data in Node struct.
struct Treap {
    struct Node {
        // E[0] = left child, E[1] = right child
        // weight = node random weight (for treap)
        // size = subtree size, par = parent node
        int E[2] = {-1, -1}, weight{rand()};
        int size[1], par{-1};
        bool flip{0}; // Is interval reversed?
    }; // c082

    vector<Node> G; // Array of all nodes

    // Initialize structure for n nodes
    // with keys 0, ..., n-1; time: O(n)
    // Each node is separate treap,
    // use join() to make sequence.
    Treap(int n = 0) : G(n) {}
```

```
// Create new treap (a single node),
// returns its key; time: O(1)
int make() { G.pb({}); return sz(G)-1; }

// Get size of node x subtree. x can be -1.
int size(int x) { // time: O(1)
    return (x >= 0 ? G[x].size : 0);
} // 81cf

// Propagate down data (flip flag etc).
// x can be -1; time: O(1)
void push(int x) {
    if (x >= 0 && G[x].flip) {
        G[x].flip = 0;
        swap(G[x].E[0], G[x].E[1]);
        each(e, G[x].E) if (e>=0) G[e].flip ^= 1;
    } // + any other lazy operations
} // ed19

// Update aggregates of node x.
// x can be -1; time: O(1)
void update(int x) {
    if (x >= 0) {
        int& s = G[x].size = 1;
        G[x].par = -1;
        each(e, G[x].E) if (e >= 0) {
            s += G[e].size;
            G[e].par = x;
        } // f7a7
    } // + any other aggregates
} // 46a3

// Split treap x into treaps l and r
// such that l contains first i elements
// and r the remaining ones.
// x, l, r can be -1; time: ~O(lg n)
void split(int x, int& l, int& r, int i) {
    push(x); l = r = -1;
    if (x < 0) return;
    int key = size(G[x].E[0]);
    if (i <= key) {
        split(G[x].E[0], l, G[x].E[0], i);
        r = x;
    } else {
        split(G[x].E[1], G[x].E[1], r, i-key-1);
        l = x;
    } // fe19
    update(x);
} // 8211

// Join treaps l and r into one treap
// such that elements of l are before
// elements of r. Returns new treap.
// l, r and returned value can be -1.
int join(int l, int r) { // time: ~O(lg n)
    push(l); push(r);
    if (l < 0 || r < 0) return max(l, r);

    if (G[l].weight < G[r].weight) {
        G[l].E[1] = join(G[l].E[1], r);
        update(l);
        return l;
    } // 18c7

    G[r].E[0] = join(l, G[r].E[0]);
    update(r);
    return r;
} // b559

// Find i-th node in treap x.
// Returns its key or -1 if not found.
// x can be -1; time: ~O(lg n)
```

```
int find(int x, int i) {
    while (x >= 0) {
        push(x);
        int key = size(G[x].E[0]);
        if (key == i) return x;
        x = G[x].E[key < i];
        if (key < i) i -= key+1;
    } // 054c
    return -1;
} // 0b9b

// Get key of treap containing node x
// (key of treap root). x can be -1.
int root(int x) { // time: ~O(lg n)
    while (G[x].par >= 0) x = G[x].par;
    return x;
} // be8b

// Get position of node x in its treap.
// x is assumed to NOT be -1; time: ~O(lg n)
int index(int x) {
    int p, i = size(G[x].E[G[x].flip]);
    while ((p = G[x].par) >= 0) {
        if (G[p].E[1] == x) i += size(G[p].E[0])+1;
        if (G[p].flip) i = G[p].size-i-1;
        x = p;
    } // 3f81
    return i;
} // ddad

// Reverse interval [l;r] in treap x.
// Returns new key of treap; time: ~O(lg n)
int reverse(int x, int l, int r) {
    int a, b, c;
    split(x, b, c, r);
    split(b, a, b, l);
    if (b >= 0) G[b].flip ^= 1;
    return join(join(a, b), c);
} // e418
}; // 73f2
```

**structures/wavelet\_tree.h** 69f4

```
// Wavelet tree ("merge-sort tree over values")
// Each node represent interval of values.
// seq[l] = original sequence
// seq[i] = subsequence with values
// represented by i-th node
// left[i][j] = how many values in seq[0:j]
// go to left subtree
struct WaveletTree {
    vector<Vi> seq, left;
    int len;

    // Build wavelet tree for sequence `elems`;
    // time and space: O((n+maxVal) log maxVal)
    // Values are expected to be in [0;maxVal].
    WaveletTree(const Vi& elems, int maxVal) {
        for (len = 1; len < maxVal; len *= 2);
        seq.resize(len*2);
        left.resize(len*2);
        seq[1] = elems;
        build(1, 0, len);
    } // a5e9

    void build(int i, int b, int e) {
        if (i >= len) return;
        int m = (b+e) / 2;
        left[i].pb(0);
        each(x, seq[i]) {
            left[i].pb(left[i].back() + (x < m));
            seq[i*2 + (x >= m)].pb(x);
        }
```

```
} // ac25
    build(i*2, b, m);
    build(i*2+1, m, e);
} // 8153

// Find k-th smallest element in [begin;end)
// [begin;end); time: O(log maxVal)
int kth(int begin, int end, int k, int i=1) {
    if (i >= len) return seq[i][0];
    int x = left[i][begin], y = left[i][end];
    if (k < y-x) return kth(x, y, k, i*2);
    return kth(begin-x, end-y, k-y+x, i*2+1);
} // 7861

// Count number of elements >= vb and < ve
// in [begin;end); time: O(log maxVal)
int count(int begin, int end, int vb, int ve,
          int i = 1, int b = 0, int e = -1) {
    if (e < 0) e = len;
    if (b >= ve || vb >= e) return 0;
    if (b >= vb && e <= ve) return end-begin;
    int m = (b+e) / 2;
    int x = left[i][begin], y = left[i][end];
    return count(x, y, vb, ve, i*2, b, m) +
        count(begin-x, end-y, vb, ve, i*2+1, m, e);
} // 71cf
}; // dd48
```

**structures/ext/hash\_table.h** 2d30

```
#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
// gp_hash_table<K, V> = faster unordered_set

// Anti-anti-hash
const size_t HXOR = mt19937_64(time(0))();
template<class T> struct SafeHash {
    size_t operator()(const T& x) const {
        return hash<T>()(x ^ T(HXOR));
    } // 3a78
}; // 7d0e
```

**structures/ext/rope.h** 051f

```
#include <ext/rope>
using namespace __gnu_cxx;
// rope<T> = implicit cartesian tree
```

**structures/ext/tree.h** a3bc

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;
```

```
template<class T, class Cmp = less<T>>
using ordered_set = tree<
    T, null_type, Cmp, rb_tree_tag,
    tree_order_statistics_node_update
>;
```

```
// Standard set functions and:
// t.order_of_key(key) - index of first >= key
// t.find_by_order(i) - find i-th element
// t1.join(t2) - assuming t1<t2 merge t2 to t1
```

**structures/ext/trie.h** 5cc2

```
#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/trie_policy.hpp>
using namespace __gnu_pbds;

using pref_trie = trie<
    string, null_type,
    trie_string_access_traits<>, pat_trie_tag,
    trie_prefix_search_node_update
```

```
>;
text/aho_corasick.h 697d
constexpr char AMIN = 'a'; // Smallest letter
constexpr int ALPHA = 26; // Alphabet size

// Aho-Corasick algorithm for linear-time
// multiple pattern matching.
// Add patterns using add(), then call build().
struct Aho {
    vector<array<int, ALPHA>> nxt{1};
    Vi suf = {-1}, accLink = {-1};
    vector<Vi> accept{1};

    // Add string with given ID to structure
    // Returns index of accepting node
    int add(const string& str, int id) {
        int i = 0;
        each(c, str) {
            if (!nxt[i][c-AMIN]) {
                nxt[i][c-AMIN] = sz(nxt);
                nxt.pb({}); suf.pb(-1);
                accLink.pb(1); accept.pb({});
            } // 5ead
            i = nxt[i][c-AMIN];
        } // ace9
        accept[i].pb(id);
        return i;
    } // 27c8

    // Build automata; time: O(V*ALPHA)
    void build() {
        queue<int> que;
        each(e, nxt[0]) if (e) {
            suf[e] = 0; que.push(e);
        } // c34d
        while (!que.empty()) {
            int i = que.front(), s = suf[i], j = 0;
            que.pop();
            each(e, nxt[i]) {
                if (e) que.push(e);
                (e ? suf[e] : e) = nxt[s][j++];
            } // 8521
            accLink[i] = (accept[s].empty() ?
                accLink[s] : s);
        } // 1e8a
    } // 2561

    // Append `c` to state `i`
    int next(int i, char c) {
        return nxt[i][c-AMIN];
    } // 6bb7

    // Call `f` for each pattern accepted
    // when in state `i` with its ID as argument.
    // Return true from `f` to terminate early.
    // Calls are in decreasing length order.
    template<class F> void accepted(int i, F f) {
        while (i != -1) {
            each(a, accept[i]) if (f(a)) return;
            i = accLink[i];
        } // c175
    } // 1f0d
}; // 2768

text/kmp.h 5729

// Computes prefsuf array; time: O(n)
// ps[i] = max prefsuf of [0;i); ps[0] := -1
template<class T> Vi kmp(const T& str) {
    Vi ps; ps.pb(-1);
    each(x, str) {
```

```
        int k = ps.back();
        while (k >= 0 && str[k] != x) k = ps[k];
        ps.pb(k+1);
    } // 05aa
    return ps;
} // 8a6c

// Finds occurrences of pat in vec; time: O(n)
// Returns starting indices of matches.
template<class T>
Vi match(const T& str, T pat) {
    int n = sz(pat);
    pat.pb(-1); // SET TO SOME UNUSED CHARACTER
    pat.insert(pat.end(), all(str));
    Vi ret, ps = kmp(pat);
    rep(i, 0, sz(ps)) {
        if (ps[i] == n) ret.pb(i-2*n-1);
    } // ale9
    return ret;
} // f986

text/kmr.h ee8c

// KMR algorithm for O(1) lexicographical
// comparison of substrings.
struct KMR {
    vector<Vi> ids;

    KMR() {}

    // Initialize structure; time: O(n lg^2 n)
    // You can change str type to Vi freely.
    KMR(const string& str) {
        ids.clear();
        ids.pb(Vi(all(str)));

        for (int h = 1; h <= sz(str); h *= 2) {
            vector<pair<Pii, int>> tmp;

            rep(j, 0, sz(str)) {
                int a = ids.back()[j], b = -1;
                if (j+h < sz(str)) b = ids.back()[j+h];
                tmp.pb({a, b}, j);
            } // a210

            sort(all(tmp));
            ids.emplace_back(sz(tmp));

            int n = 0;
            rep(j, 0, sz(tmp)) {
                if (j > 0 && tmp[j-1].x != tmp[j].x)
                    n++;
                ids.back()[tmp[j].y] = n;
            } // bd2e
        } // 969a
    } // a40e

    // Get representative of [begin;end); O(1)
    Pii get(int begin, int end) {
        if (begin >= end) return {0, 0};
        int k = 31 - __builtin_clz(end-begin);
        return {ids[k][begin], ids[k][end-(1<<k)]};
    } // 85f3

    // Compare [b1;e1) with [b2;e2); O(1)
    // Returns -1 if <, 0 if ==, 1 if >
    int cmp(int b1, int e1, int b2, int e2) {
        int l1 = e1-b1, l2 = e2-b2;
        int l = min(l1, l2);
        Pii x = get(b1, b1+l), y = get(b2, b2+l);

        if (x == y) return (l1 > l2) - (l1 < l2);
        return (x > y) - (x < y);
    } // bf42
```

```
// Compute suffix array of string; O(n)
Vi sufArray() {
    Vi sufs(sz(ids.back()));
    rep(i, 0, sz(ids.back()))
        sufs[ids.back()[i]] = i;
    return sufs;
} // d98d
}; // 457e

text/lcp.h 0c65

// Compute Longest Common Prefix array for
// given string and it's suffix array; O(n)
// In order to compute suffix array use kmr.h
// or suffix_array_linear.h
template<class T>
Vi lcpArray(const T& str, const Vi& sufs) {
    int n = sz(str), k = 0;
    Vi pos(n), lcp(n-1);
    rep(i, 0, n) pos[sufs[i]] = i;
    rep(i, 0, n) {
        if (pos[i] < n-1) {
            int j = sufs[pos[i]+1];
            while (i+k < n && j+k < n &&
                str[i+k] == str[j+k]) k++;
            lcp[pos[i]] = k;
        } // 2cba
        if (k > 0) k--;
    } // 8b22
    return lcp;
} // d438

text/lyndon_factorization.h 688c

// Compute Lyndon factorization for s; O(n)
// Word is simple iff it's stricly smaller
// than any of it's nontrivial suffixes.
// Lyndon factorization is division of string
// into non-increasing simple words.
// It is unique.
vector<string> duval(const string& s) {
    int n = sz(s), i = 0;
    vector<string> ret;
    while (i < n) {
        int j = i+1, k = i;
        while (j < n && s[k] <= s[j])
            k = {s[k] < s[j] ? i : k+1}, j++;
        while (i <= k)
            ret.pb(s.substr(i, j-k)), i += j-k;
    } // 3f17
    return ret;
} // 0e48

text/main_lorentz.h 9afb

#include "z_function.h"

struct Sqr {
    int begin, end, len;
}; // f012

// Main-Lorentz algorithm for finding
// all squares in given word; time: O(n lg n)
// Results are in compressed form:
// (b, e, l) means that for each b <= i < e
// there is square at position i of size 2l.
// Each square is present in only one interval.
vector<Sqr> lorentz(const string& s) {
    int n = sz(s);
    if (n <= 1) return {};
    auto a = s.substr(0, n/2), b = s.substr(n/2);
    auto ans = lorentz(a);
```

```
each(p, lorentz(b))
    ans.pb({p.begin+n/2, p.end+n/2, p.len});

string ra(a.rbegin(), a.rend());
string rb(b.rbegin(), b.rend());

rep(j, 0, 2) {
    Vi z1 = prefPref(ra), z2 = prefPref(b+a);
    z1.pb(0); z2.pb(0);

    rep(c, 0, sz(a)) {
        int l = sz(a)-c;
        int x = c - min(l-1, z1[l]);
        int y = c - max(l-z2[sz(b)+c], j);
        if (x > y) continue;

        if (j)
            ans.pb({n-y-l*2, n-x-l*2+1, l});
        else
            ans.pb({x, y+1, l});
    } // 6c51

    a.swap(rb);
    b.swap(ra);
} // b6b2

return ans;
} // 5567

text/manacher.h 8680

// Manacher algorithm; time: O(n)
// Finds largest radiuses for palindromes:
// r[2*i] = for center at i (single letter = 1)
// r[2*i+1] = for center between i and i+1
template<class T> Vi manacher(const T& str) {
    int n = sz(str)*2, c = 0, e = 1;
    Vi r(n, 1);
    auto get = [&](int i) { return i%2 ? 0 :
        (i >= 0 && i < n ? str[i/2] : i); }; // 3d98

    rep(i, 0, n) {
        if (i < e) r[i] = min(r[c*2-i], e-i);
        while (get(i-r[i]) == get(i+r[i])) r[i]++;
        if (i+r[i] > e) c = i, e = i+r[i]-1;
    } // 0f87

    rep(i, 0, n) r[i] /= 2;
    return r;
} // a300

text/min_rotation.h e4d6

// Find lexicographically smallest
// rotation of s; time: O(n)
// Returns index where shifted word starts.
// You can use std::rotate to get the word:
// rotate(s.begin(), s.begin()+minRotation(s),
// s.end());
int minRotation(string s) {
    int a = 0, n = sz(s); s += s;
    rep(b, 0, n) rep(i, 0, n) {
        if (a+i == b || s[a+i] < s[b+i]) {
            b += max(0, i-1); break;
        } // 865b
        if (s[a+i] > s[b+i]) {
            a = b; break;
        } // 7628
    } // 40be
    return a;
} // 9ed8

text/palindromic_tree.h 1207

constexpr int ALPHA = 26; // Set alphabet size
```

```
// Tree of all palindromes in string,
// constructed online by appending letters.
// space: O(n*ALPHA); time: O(n)
// Code marked with [EXT] is extension for
// calculating minimal palindrome partition
// in O(n lg n). Can also be modified for
// similar dynamic programmings.
struct PalTree {
    Vi txt; // Text for which tree is built

    // Node 0 = empty palindrome (root of even)
    // Node 1 = "-1" palindrome (root of odd)
    Vi len{0, -1}; // Lengths of palindromes
    Vi link{1, 0}; // Suffix palindrome links
    // Edges to next palindromes
    vector<array<int, ALPHA>> to{ {}, {} };
    int last{0}; // Current node (max suffix pal)

    Vi diff{0, 0}; // len[i]-len[link[i]] [EXT]
    Vi slink{0, 0}; // Serial links [EXT]
    Vi series{0, 0}; // Series DP answer [EXT]
    Vi ans{0}; // DP answer for prefix[EXT]

    int ext(int i) {
        while (len[i]+2 > sz(txt) ||
            txt[sz(txt)-len[i]-2] != txt.back())
            i = link[i];
        return i;
    } // d442

    // Append letter from [0;ALPHA); time: O(1)
    // (or O(lg n) if [EXT] is enabled)
    void add(int x) {
        txt.pb(x);
        last = ext(last);

        if (!to[last][x]) {
            len.pb(len[last]+2);
            link.pb(to[ext(link[last])][x]);
            to[last][x] = sz(to);
            to.pb({});

            // [EXT]
            diff.pb(len.back() - len[link.back()]);
            slink.pb(diff.back() == diff[link.back()]
                ? slink[link.back()] : link.back());
            series.pb(0);
            // [/EXT]
        } // 8c1b
        last = to[last][x];

        // [EXT]
        ans.pb(INT_MAX);
        for (int i=last; len[i] > 0; i=slink[i]) {
            series[i] = ans[sz(ans) - len[slink[i]]
                - diff[i] - 1];

            if (diff[i] == diff[link[i]])
                series[i] = min(series[i],
                    series[link[i]]);

            // If you want only even palindromes
            // set ans only for sz(txt)%2 == 0
            ans.back() = min(ans.back(), series[i]+1);
        } // ab3b
        // [/EXT]
    } // 66d3
}; // 595d
```

**text/suffix\_array\_linear.h** 1341

```
#include "../util/radix_sort.h"
```

```
// KS algorithm for suffix array; time: O(n)
// Input values are assumed to be in [1;k]
```

```
Vi sufArray(Vi str, int k) {
    int n = sz(str);
    Vi suf(n);
    str.resize(n+15);

    if (n < 15) {
        iota(all(suf), 0);
        rep(j, 0, n) countSort(suf,
            [&](int i) { return str[i+n-j-1]; }, k);
        return suf;
    } // 5fcf

    // Compute triples codes
    Vi tmp, code(n+2);
    rep(i, 0, n) if (i % 3) tmp.pb(i);

    rep(j, 0, 3) countSort(tmp,
        [&](int i) { return str[i-j+2]; }, k);

    int mc = 0, j = -1;

    each(i, tmp) {
        code[i] = mc += (j == -1 ||
            str[i] != str[j] ||
            str[i+1] != str[j+1] ||
            str[i+2] != str[j+2]);
        j = i;
    } // bfdc

    // Compute suffix array of 2/3
    tmp.clear();
    for (int i=1; i < n; i += 3) tmp.pb(code[i]);
    tmp.pb(0);
    for (int i=2; i < n; i += 3) tmp.pb(code[i]);
    tmp = sufArray(move(tmp), mc);

    // Compute partial suffix arrays
    Vi third;
    int th = (n+4) / 3;
    if (n%3 == 1) third.pb(n-1);

    rep(i, 1, sz(tmp)) {
        int e = tmp[i];
        tmp[i-1] = (e < th ? e*3+1 : (e-th)*3+2);
        code[tmp[i-1]] = i;
        if (e < th) third.pb(e*3);
    } // f9f1

    tmp.pop_back();
    countSort(third,
        [&](int i) { return str[i]; }, k);

    // Merge suffix arrays
    merge(all(third), all(tmp), suf.begin(),
        [&](int l, int r) {
            while (l%3 == 0 || r%3 == 0) {
                if (str[l] != str[r])
                    return str[l] < str[r];
                l++; r++;
            } // 2f8a
            return code[l] < code[r];
        }); // 4cb3

    return suf;
} // 9165

// KS algorithm for suffix array; time: O(n)
Vi sufArray(const string& str) {
    return sufArray(Vi(all(str)), 255);
} // 593f
```

**text/suffix\_automaton.h** d00d

```
constexpr char AMIN = 'a'; // Smallest letter
constexpr int ALPHA = 26; // Set alphabet size
```

```
// Suffix automaton - minimal DFA that
// recognizes all suffixes of given string
// (and encodes all substrings);
// space: O(n*ALPHA); time: O(n)
// Paths from root are equivalent to substrings
// Extensions:
// - [OCC] - count occurrences of substrings
// - [PATHS] - count paths from node
struct SufDFA {
    // State v represents endpos-equivalence
    // class that contains words of all lengths
    // between link[len[v]]+1 and len[v].
    // len[v] = longest word of equivalence class
    // link[v] = link to state of longest suffix
    // in other equivalence class
    // to[v][c] = automaton edge c from v
    Vi len{0}, link{-1};
    vector<array<int, ALPHA>> to{ {}, {} };
    int last{0}; // Current node (whole word)

    vector<Vi> inSufs; // [OCC] Suffix-link tree
    Vi cnt{0}; // [OCC] Occurrence count
    vector<ll> paths; // [PATHS] Out-path count

    SufDFA() {}

    // Build suffix automaton for given string
    // and compute extended stuff; time: O(n)
    SufDFA(const string& s) {
        each(c, s) add(c);
        finish();
    } // ec2e

    // Append letter to the back
    void add(char c) {
        int v = last, x = c-AMIN;
        last = sz(len);
        len.pb(len[v]+1);
        link.pb(0);
        to.pb({});
        cnt.pb(1); // [OCC]

        while (v != -1 && !to[v][x]) {
            to[v][x] = last;
            v = link[v];
        } // 4cfc

        if (v != -1) {
            int q = to[v][x];
            if (len[v]+1 == len[q]) {
                link[last] = q;
            } else {
                len.pb(len[v]+1);
                link.pb(link[q]);
                to.pb(to[q]);
                cnt.pb(0); // [OCC]
                link[last] = link[q] = sz(len)-1;
                while (v != -1 && to[v][x] == q) {
                    to[v][x] = link[q];
                    v = link[v];
                } // 784f
            } // 90aa
        } // af69
    } // 345a

    // Compute some additional stuff (offline)
    void finish() {
        // [OCC]
        inSufs.resize(sz(len));
        rep(i, 1, sz(link)) inSufs[link[i]].pb(i);
        dfsSufs(0);
        // [/OCC]
```

```
        // [PATHS]
        paths.assign(sz(len), 0);
        dfs(0);
        // [/PATHS]
    } // 3f75

    // Only for [OCC]
    void dfsSufs(int v) {
        each(e, inSufs[v]) {
            dfsSufs(e);
            cnt[v] += cnt[e];
        } // 2469
    } // 0c60

    // Only for [PATHS]
    void dfs(int v) {
        if (paths[v] != 0) return;
        paths[v] = 1;
        each(e, to[v]) if (e) {
            dfs(e);
            paths[v] += paths[e];
        } // 22b3
    } // d004

    // Go using edge `c` from state `i`.
    // Returns 0 if edge doesn't exist.
    int next(int i, char c) {
        return to[i][c-AMIN];
    } // c363

    // Get lexicographically k-th substring
    // of represented string; time: O(|substr|)
    // Empty string has index 0.
    // Requires [PATHS] extension.
    string lex(ll k) {
        string s;
        int v = 0;
        while (k-->0) rep(i, 0, ALPHA) {
            int e = to[v][i];
            if (e) {
                if (k < paths[e]) {
                    s.pb(char(AMIN+i));
                    v = e;
                    break;
                } // f307
                k -= paths[e];
            } // 29be
        } // 4600
        return s;
    } // e4af
}; // 7135
```

**text/suffix\_tree.h** 40a4

```
constexpr int ALPHA = 26;
```

```
// Ukkonen's algorithm for online suffix tree
// construction; space: O(n*ALPHA); time: O(n)
// Real tree nodes are called dedicated nodes.
// "Nodes" lying on compressed edges are called
// implicit nodes and are represented
// as pairs (lower node, label index).
// Labels are represented as intervals [L;R)
// which refer to substrings [L;R) of txt.
// Leaves have labels of form [L;infinity),
// use getR to get current right endpoint.
// Suffix links are valid only for internal
// nodes (non-leaves).
struct SufTree {
    Vi txt; // Text for which tree is built
    // to[v][c] = edge with label starting with c
```

```

// from node v
vector<array<int, ALPHA>> to{ {} };
Vi L[0], R[0]; // Parent edge label endpoints
Vi par[0]; // Parent link
Vi link[0]; // Suffix link
Pii cur[0, 0]; // Current state

// Get current right end of node label
int getR(int i) { return min(R[i], sz(txt)); }

// Follow edge `e` of implicit node `s`.
// Returns (-1, -1) if there is no edge.
Pii next(Pii s, int e) {
    if (s.y < getR(s.x))
        return txt[s.y] == e ? mp(s.x, s.y+1) : mp(-1, -1);
    e = to[s.x][e];
    return e ? mp(e, L[e]+1) : mp(-1, -1);
} // f430

// Create dedicated node for implicit node
// and all its suffixes
int split(Pii s) {
    if (s.y == R[s.x]) return s.x;
    int t = sz(to); to.pb({});
    to[t][txt[s.y]] = s.x;
    L.pb(L[s.x]);
    R.pb(L[s.x] = s.y);
    par.pb(par[s.x]);
    par[s.x] = to[par[t]][txt[L[t]]] = t;
    link.pb(-1);
    int v = link[par[t]], l = L[t] + !par[t];
    while (l < R[t]) {
        v = to[v][txt[l]];
        l += getR(v) - L[v];
    } // 0393
    v = split({v, getR(v)-l+R[t]});
    link[t] = v;
    return t;
} // 10bb

// Append letter from [0;ALPHA) to the back
void add(int x) { // amortized time: O(1)
    Pii t; txt.pb(x);
    while ((t = next(cur, x)).x == -1) {
        int m = split(cur);
        to[m][x] = sz(to);
        to.pb({});
        par.pb(m);
        L.pb(sz(txt)-1);
        R.pb(INT_MAX);
        link.pb(-1);
        cur = {link[m], getR(link[m])};
        if (!m) return;
    } // 60c2
    cur = t;
} // 6f4e
}; // dbfb

```

**text/z\_function.h** 0466

```

// Computes Z function array; time: O(n)
// zf[i] = max common prefix of str and str[i:]
template<class T> Vi prefPref(const T& str) {
    int n = sz(str), b = 0, e = 1;
    Vi zf(n);
    rep(i, 1, n) {
        if (i < e) zf[i] = min(zf[i-b], e-i);
        while (i+zf[i] < n && str[zi[i]] == str[i+zf[i]]) zf[i]++;
    }
}

```

```

    if (i+zf[i] > e) b = i, e = i+zf[i];
} // e906
zf[0] = n;
return zf;
} // b88d

```

**trees/centroid\_decomp.h** 5247

```

// Centroid decomposition; space: O(n lg n)
struct CentroidTree {
    // child[v] = children of v in centroid tree
    // par[v] = parent of v in centroid tree
    // (-1 for root)
    // depth[v] = depth of v in centroid tree
    // (0 for root)
    // ind[v][i] = index of vertex v in i-th
    // centroid subtree from root
    // size[v] = size of centroid subtree of v
    // subtree[v] = list of vertices
    // in centroid subtree of v
    // dists[v] = distances from v to vertices
    // in its centroid subtree
    // (in the order of subtree[v])
    // neigh[v] = neighbours of v
    // in its centroid subtree
    // dir[v][i] = index of centroid neighbour
    // that is first vertex on path
    // from centroid v to i-th vertex
    // of centroid subtree
    // (-1 for centroid)
    vector<Vi> child, ind, dists, subtree,
    neigh, dir;
    Vi par, depth, size;
    int root; // Root centroid
    CentroidTree() {}
    CentroidTree(vector<Vi>& G)
        : child(sz(G)), ind(sz(G)), dists(sz(G)),
        subtree(sz(G)), neigh(sz(G)),
        dir(sz(G)), par(sz(G), -2),
        depth(sz(G)), size(sz(G)) {
        root = decomp(G, 0, 0);
    } // 026c

    void dfs(vector<Vi>& G, int v, int p) {
        size[v] = 1;
        each(e, G[v]) if (e != p && par[e] == -2)
            dfs(G, e, v), size[v] += size[e];
    } // bbed

    void layer(vector<Vi>& G, int v,
        int p, int c, int d) {
        ind[v].pb(sz(subtree[c]));
        subtree[c].pb(v);
        dists[c].pb(d);
        dir[c].pb(sz(neigh[c])-1);
        each(e, G[v]) if (e != p && par[e] == -2) {
            if (v == c) neigh[c].pb(e);
            layer(G, e, v, c, d+1);
        } // dc82
    } // 37ee

    int decomp(vector<Vi>& G, int v, int d) {
        dfs(G, v, -1);
        int p = -1, s = size[v];
        loop:
        each(e, G[v]) {
            if (e != p && par[e] == -2 &&
                size[e] > s/2) {
                p = v; v = e; goto loop;
            } // e0a5
        }
    }
}

```

```

    } // 3533
    par[v] = -1;
    size[v] = s;
    depth[v] = d;
    layer(G, v, -1, v, 0);

    each(e, G[v]) if (par[e] == -2) {
        int j = decomp(G, e, d+1);
        child[v].pb(j);
        par[j] = v;
    } // 70b5
    return v;
} // 217c
}; // 71d6

```

**trees/centroid\_offline.h** ac92

```

// Helper for offline centroid decomposition
// Usage: CentroidDecomp(G);
// Constructor calls method `process`
// for each centroid subtree.
struct CentroidDecomp {
    vector<Vi>& G; // Reference to target graph
    vector<bool> on; // Is vertex enabled?
    Vi size; // Used internally

    // Run centroid decomposition for graph g
    CentroidDecomp(vector<Vi>& g)
        : G(g), on(sz(g), 1), size(sz(g)) {
        decomp(0);
    } // 8677

    // Compute subtree sizes for subtree rooted
    // at v, ignoring p and disabled vertices
    void computeSize(int v, int p) {
        size[v] = 1;
        each(e, G[v]) if (e != p && on[e])
            computeSize(e, v), size[v] += size[e];
    } // 1c0d

    void decomp(int v) {
        computeSize(v, -1);
        int p = -1, s = size[v];
        loop:
        each(e, G[v]) {
            if (e != p && on[e] && size[e] > s/2) {
                p = v; v = e; goto loop;
            } // e0a5
        } // f31d
        process(v);
        on[v] = 0;
        each(e, G[v]) if (on[e]) decomp(e);
    } // f170

    // Process current centroid subtree:
    // - v is centroid
    // - boundary vertices have on[x] = 0
    // Formally: Let H be subgraph induced
    // on vertices such that on[v] = 1.
    // Then current centroid subtree is
    // connected component of H that contains v
    // and v is its centroid.
    void process(int v) {
        // Do your stuff here...
    } // d41d
}; // a923

```

**trees/heavylight\_decomp.h** 9739

```

#include "../structures/segtree_point.h"
// Heavy-Light Decomposition of tree
// with subtree query support; space: O(n)

```

```

struct HLD {
    // Subtree of v = [pos[v]; pos[v]+size[v])
    // Chain with v = [chBegin[v]; chEnd[v])
    Vi par; // Vertex parent
    Vi size; // Vertex subtree size
    Vi depth; // Vertex distance to root
    Vi pos; // Vertex position in "HLD" order
    Vi chBegin; // Begin of chain with vertex
    Vi chEnd; // End of chain with vertex
    Vi order; // "HLD" preorder of vertices
    SegTree tree; // Verts are in HLD order

    HLD() {}

    // Initialize structure for tree G
    // and given root; time: O(n lg n)
    // MODIFIES ORDER OF EDGES IN G!
    HLD(vector<Vi>& G, int root)
        : par(sz(G)), size(sz(G)),
        depth(sz(G)), pos(sz(G)),
        chBegin(sz(G)), chEnd(sz(G)) {
        dfs(G, root, -1);
        decomp(G, root, -1, 0);
        tree = {sz(order)};
    } // 8263

    void dfs(vector<Vi>& G, int v, int p) {
        par[v] = p;
        size[v] = 1;
        depth[v] = p < 0 ? 0 : depth[p]+1;
        int& fs = G[v][0];
        if (fs == p) swap(fs, G[v].back());
        each(e, G[v]) if (e != p) {
            dfs(G, e, v);
            size[v] += size[e];
            if (size[e] > size[fs]) swap(e, fs);
        } // 9872
    } // 12fe

    void decomp(vector<Vi>& G,
        int v, int p, int chb) {
        pos[v] = sz(order);
        chBegin[v] = chb;
        chEnd[v] = pos[v]+1;
        order.pb(v);
        each(e, G[v]) if (e != p) {
            if (e == G[v][0]) {
                decomp(G, e, v, chb);
                chEnd[v] = chEnd[e];
            } else {
                decomp(G, e, v, sz(order));
            } // c84a
        } // f707
    } // eb89

    // Get root of chain containing v
    int chRoot(int v) {return order[chBegin[v]];}

    // Level Ancestor Query; time: O(lg n)
    int laq(int v, int level) {
        while (true) {
            int k = pos[v] - depth[v] + level;
            if (k >= chBegin[v]) return order[k];
            v = par[chRoot(v)];
        } // 8c18
    } // 675e

    // Lowest Common Ancestor; time: O(lg n)
    int lca(int a, int b) {
        while (chBegin[a] != chBegin[b]) {

```



```

    int ha = chRoot(a), hb = chRoot(b);
    if (depth[ha] > depth[hb]) a = par[ha];
    else b = par[hb];
} // 5620
return depth[a] < depth[b] ? a : b;
} // c168

// Call func(chBegin, chEnd) on each path
// segment; time: O(lg n * time of func)
template<class T>
void iterPath(int a, int b, T func) {
    while (chBegin[a] != chBegin[b]) {
        int ha = chRoot(a), hb = chRoot(b);
        if (depth[ha] > depth[hb]) {
            func(chBegin[a], pos[a]+1);
            a = par[ha];
        } else {
            func(chBegin[b], pos[b]+1);
            b = par[hb];
        } // f9a5
    } // 563c

    if (pos[a] > pos[b]) swap(a, b);
    // Remove +1 from pos[a]+1 for vertices
    // queries (with +1 -> edges).
    func(pos[a]+1, pos[b]+1);
} // 17e5

// Query path between a and b; O(lg^2 n)
SegTree::T queryPath(int a, int b) {
    auto ret = tree.ID;
    iterPath(a, b, [&](int i, int j) {
        ret = tree.f(ret, tree.query(i, j));
    }); // 1113
    return ret;
} // 1bc9

// Query subtree of v; time: O(lg n)
SegTree::T querySubtree(int v) {
    return tree.query(pos[v], pos[v]+size[v]);
} // 23db
}; // a062

```

## trees/lca.h

294f

```

// LAQ and LCA using jump pointers
// space: O(n lg n)

struct LCA {
    vector<Vi> jumps;
    Vi level, pre, post;
    int cnt{0}, depth;

    LCA() {}

    // Initialize structure for tree G
    // and root r; time: O(n lg n)
    LCA(vector<Vi>& G, int root)
        : jumps(sz(G)), level(sz(G)),
          pre(sz(G)), post(sz(G)) {
        dfs(G, root, root);
        depth = int(log2(sz(G))) + 2;
        rep(j, 0, depth) each(v, jumps)
            v.pb(jumps[v[j]][j]);
    } // d6ce

    void dfs(vector<Vi>& G, int v, int p) {
        level[v] = p == v ? 0 : level[p]+1;
        jumps[v].pb(p);
        pre[v] = ++cnt;
        each(e, G[v]) if (e != p) dfs(G, e, v);
        post[v] = ++cnt;
    } // e286

```

```

// Check if a is ancestor of b; time: O(1)
bool isAncestor(int a, int b) {
    return pre[a] <= pre[b] &&
           post[b] <= post[a];
} // 5514

// Lowest Common Ancestor; time: O(lg n)
int operator()(int a, int b) {
    for (int j = depth; j--;)
        if (!isAncestor(jumps[a][j], b))
            a = jumps[a][j];
    return isAncestor(a, b) ? a : jumps[a][0];
} // 27d8

// Level Ancestor Query; time: O(lg n)
int laq(int a, int lvl) {
    for (int j = depth; j--;)
        if (lvl <= level[jumps[a][j]])
            a = jumps[a][j];
    return a;
} // 75b3

// Get distance from a to b; time: O(lg n)
int distance(int a, int b) {
    return level[a] + level[b] -
           level[operator()(a, b)]*2;
} // 07e0

// Get k-th vertex on path from a to b,
// a is 0, b is last; time: O(lg n)
// Returns -1 if k > distance(a, b)
int kthVertex(int a, int b, int k) {
    int c = operator()(a, b);
    if (level[a]-k >= level[c])
        return laq(a, level[a]-k);
    k += level[c]*2 - level[a];
    return (k > level[b] ? -1 : laq(b, k));
} // 46c9
}; // 2254

```

## trees/lca\_linear.h

7aa5

```

// LAQ and LCA using jump pointers
// with linear memory; space: O(n)
struct LCA {
    Vi par, jmp, depth, pre, post;
    int cnt{0};

    LCA() {}

    // Initialize structure for tree G
    // and root v; time: O(n lg n)
    LCA(vector<Vi>& G, int v)
        : par(sz(G), -1), jmp(sz(G), v),
          depth(sz(G)), pre(sz(G)), post(sz(G)) {
        dfs(G, v);
    } // 94cf

    void dfs(vector<Vi>& G, int v) {
        int j = jmp[v], k = jmp[j], x =
            depth[v]+depth[k] == depth[j]*2 ? k : v;
        pre[v] = ++cnt;
        each(e, G[v]) if (!pre[e]) {
            par[e] = v; jmp[e] = x;
            depth[e] = depth[v]+1;
            dfs(G, e);
        } // b123
        post[v] = ++cnt;
    } // 3280

    // Level Ancestor Query; time: O(lg n)
    int laq(int v, int d) {
        while (depth[v] > d)

```

```

        v = depth[jmp[v]] < d ? par[v] : jmp[v];
        return v;
    } // f509

    // Lowest Common Ancestor; time: O(lg n)
    int operator()(int a, int b) {
        if (depth[a] > depth[b]) swap(a, b);
        b = laq(b, depth[a]);
        while (a != b) {
            if (jmp[a] == jmp[b])
                a = par[a], b = par[b];
            else
                a = jmp[a], b = jmp[b];
        } // fe08
        return a;
    } // 25ff

    // Check if a is ancestor of b; time: O(1)
    bool isAncestor(int a, int b) {
        return pre[a] <= pre[b] &&
               post[b] <= post[a];
    } // 5514

    // Get distance from a to b; time: O(lg n)
    int distance(int a, int b) {
        return depth[a] + depth[b] -
               depth[operator()(a, b)]*2;
    } // a340

    // Get k-th vertex on path from a to b,
    // a is 0, b is last; time: O(lg n)
    // Returns -1 if k > distance(a, b)
    int kthVertex(int a, int b, int k) {
        int c = operator()(a, b);
        if (depth[a]-k >= depth[c])
            return laq(a, depth[a]-k);
        k += depth[c]*2 - depth[a];
        return (k > depth[b] ? -1 : laq(b, k));
    } // 34ed
}; // a221

```

## trees/link\_cut\_tree.h

6bd6

```

constexpr int INF = 1e9;

// Link/cut tree; space: O(n)
// Represents forest of (un)rooted trees.
struct LinkCutTree {
    vector<array<int, 2>> child;
    Vi par, prev, flip, size;

    // Initialize structure for n vertices; O(n)
    // At first there's no edges.
    LinkCutTree(int n = 0)
        : child(n, {-1, -1}), par(n, -1),
          prev(n, -1), flip(n, -1), size(n, 1) {}

    void push(int x) {
        if (x >= 0 && flip[x]) {
            flip[x] = 0;
            swap(child[x][0], child[x][1]);
            each(e, child[x]) if (e >= 0) flip[e] ^= 1;
        } // + any other lazy path operations
    } // bae2

    void update(int x) {
        if (x >= 0) {
            size[x] = 1;
            each(e, child[x]) if (e >= 0)
                size[x] += size[e];
        } // + any other path aggregates
    } // 8ec0

    void auxLink(int p, int i, int ch) {

```

```

        child[p][i] = ch;
        if (ch >= 0) par[ch] = p;
        update(p);
    } // 0a9a

    void rot(int p, int i) {
        int x = child[p][i], g = par[x] = par[p];
        if (g >= 0) child[g][child[g][1] == p] = x;
        auxLink(p, i, child[x][!i]);
        auxLink(x, !i, p);
        swap(prev[x], prev[p]);
        update(g);
    } // 4c76

    void splay(int x) {
        while (par[x] >= 0) {
            int p = par[x], g = par[p];
            push(g); push(p); push(x);
            bool f = (child[p][1] == x);
            if (g >= 0) {
                if (child[g][f] == p) { // zig-zig
                    rot(g, f); rot(p, f);
                } else { // zig-zag
                    rot(p, f); rot(g, !f);
                } // 2ebb
            } else { // zig
                rot(p, f);
            } // f8a2
        } // 446b
        push(x);
    } // 55a7

    // After this operation x becomes the end
    // of preferred path starting in root;
    void access(int x) { // amortized O(lg n)
        while (true) {
            splay(x);
            int p = prev[x];
            if (p < 0) break;

            prev[x] = -1;
            splay(p);

            int r = child[p][1];
            if (r >= 0) swap(par[r], prev[r]);
            auxLink(p, 1, x);
        } // 2b87
    } // 30be

    // Make x root of its tree; amortized O(lg n)
    void makeRoot(int x) {
        access(x);
        int& l = child[x][0];
        if (l >= 0) {
            swap(par[l], prev[l]);
            flip[l] ^= 1;
            update(l);
            update(x);
        } // 0064
    } // b246

    // Find root of tree containing x
    int find(int x) { // time: amortized O(lg n)
        access(x);
        while (child[x][0] >= 0)
            push(x = child[x][0]);
        splay(x);
        return x;
    } // d78d

    // Add edge x-y; time: amortized O(lg n)

```

```
// Root of tree containing y becomes
// root of new tree.
void link(int x, int y) {
    makeRoot(x); prev[x] = y;
} // fb4f

// Remove edge x-y; time: amortized O(lg n)
// x and y become roots of new trees!
void cut(int x, int y) {
    makeRoot(x); access(y);
    par[x] = child[y][0] = -1;
    update(y);
} // 1908

// Get distance between x and y,
// returns INF if x and y there's no path.
// This operation makes x root of the tree!
int dist(int x, int y) { // amortized O(lg n)
    makeRoot(x);
    if (find(y) != x) return INF;
    access(y);
    int t = child[y][0];
    return t >= 0 ? size[t] : 0;
} // ae69
}; // 4480
```

## util/arc\_interval\_cover.h 7507

```
using dbl = double;

// Find size of smallest set of points
// such that each arc contains at least one
// of them; time: O(n lg n)
int arcCover(vector<pair<dbl, dbl>>& inters,
              dbl wrap) {
    int n = sz(inters);

    rep(i, 0, n) {
        auto& e = inters[i];
        e.x = fmod(e.x, wrap);
        e.y = fmod(e.y, wrap);
        if (e.x < 0) e.x += wrap, e.y += wrap;
        if (e.x > e.y) e.x += wrap;
        inters.pb({e.x+wrap, e.y+wrap});
    } // b87d

    Vi nxt(n);
    deque<dbl> que;
    dbl r = wrap*4;
    sort(all(inters));

    for (int i = n*2-1; i--;) {
        r = min(r, inters[i].y);
        que.push_front(inters[i].x);
        while (!que.empty() && que.back() > r)
            que.pop_back();
        if (i < n) nxt[i] = i+sz(que);
    } // 5e6c

    int a = 0, b = 0;
    do {
        a = nxt[a] % n;
        b = nxt[nxt[b]%n] % n;
    } while (a != b);

    int ans = 0;
    while (b < a+n) {
        b += nxt[b%n] - b%n;
        ans++;
    } // 7350
    return ans;
} // 7871
```

## util/bit\_hacks.h 599a

```
// __builtin_popcount - count number of 1 bits
// __builtin_clz - count most significant 0s
// __builtin_ctz - count least significant 0s
// __builtin_ffs - like ctz, but indexed from 1
// returns 0 for 0
// For ll version add ll to name

using ull = uint64_t;

#define T64(s,up) \
    for (ull i=0; i<64; i+=s*2) \
        for (ull j = i; j < i+s; j++) { \
            ull a = (M[j] >> s) & up; \
            ull b = (M[j+s] & up) << s; \
            M[j] = (M[j] & up) | b; \
            M[j+s] = (M[j+s] & (up<<s)) | a; \
        } // a290

// Transpose 64x64 bit matrix
void transpose64(array<ull, 64>& M) {
    T64(1, 0x5555555555555555);
    T64(2, 0x3333333333333333);
    T64(4, 0xF0F0F0F0F0F0F0F);
    T64(8, 0xFF00FF00FF00FF);
    T64(16, 0xFFFF0000FFFF);
    T64(32, 0xFFFFFFFFLL);
} // 6889

// Lexicographically next mask with same
// amount of ones.
int nextSubset(int v) {
    int t = v | (v - 1);
    return (t + 1) | (((~t & -~t) - 1) >>
        (__builtin_ctz(v) + 1));
} // 4c0c
```

## util/bump\_alloc.h 09f9

```
// Allocator, which doesn't free memory.

char mem[400<<20]; // Set memory limit
size_t nMem;

void* operator new(size_t n) {
    nMem += n; return &mem[nMem-n];
} // fba6

void operator delete(void*) {}
```

## util/compress\_vec.h bc5d

```
// Compress integers to range [0;n) while
// preserving their order; time: O(n lg n)
// Returns mapping: compressed -> original
Vi compressVec(vector<int*>& vec) {
    sort(all(vec),
        [](int* l, int* r) { return *l < *r; });
    Vi old;
    each(e, vec) {
        if (old.empty() || old.back() != *e)
            old.pb(*e);
        *e = sz(old)-1;
    } // 7eb0
    return old;
} // 2b60
```

## util/inversion\_vector.h 01f9

```
// Get inversion vector for sequence of
// numbers in [0;n); ret[i] = count of numbers
// greater than perm[i] to the left; O(n lg n)
Vi encodeInversions(Vi perm) {
    Vi odd, ret(sz(perm));
    int cont = 1;
```

```
while (cont) {
    odd.assign(sz(perm)+1, 0);
    cont = 0;

    rep(i, 0, sz(perm)) {
        if (perm[i] % 2) odd[perm[i]]++;
        else ret[i] += odd[perm[i]+1];
        cont += perm[i] /= 2;
    } // 4ed0
    } // a4f0
    return ret;
} // c2e1

// Count inversions in sequence of numbers
// in [0;n); time: O(n lg n)
ll countInversions(Vi perm) {
    ll ret = 0, cont = 1;
    Vi odd;

    while (cont) {
        odd.assign(sz(perm)+1, 0);
        cont = 0;

        rep(i, 0, sz(perm)) {
            if (perm[i] % 2) odd[perm[i]]++;
            else ret += odd[perm[i]+1];
            cont += perm[i] /= 2;
        } // 916f
    } // c9b5
    return ret;
} // laaf
```

## util/longest\_inc\_subseq.h 0bd1

```
// Longest Increasing Subsequence; O(n lg n)
int lis(const Vi& seq) {
    Vi dp(sz(seq), INT_MAX);
    each(c, seq) *lower_bound(all(dp), c) = c;
    return int(lower_bound(all(dp), INT_MAX)
        - dp.begin());
} // d0e9
```

## util/max\_rects.h 2a16

```
struct MaxRect {
    // begin = first column of rectangle
    // end = first column after rectangle
    // hei = height of rectangle
    // touch = columns of height hei inside
    int begin, end, hei;
    Vi touch; // sorted increasing
}; // 41fe

// Given consecutive column heights find
// all inclusion-wise maximal rectangles
// contained in "drawing" of columns; time O(n)
vector<MaxRect> getMaxRects(Vi hei) {
    hei.insert(hei.begin(), -1);
    hei.pb(-1);
    Vi reach(sz(hei), sz(hei)-1);
    vector<MaxRect> ans;

    for (int i = sz(hei)-1; --i;) {
        int j = i+1, k = i;
        while (hei[j] > hei[i]) j = reach[j];
        reach[i] = j;

        while (hei[k] > hei[i-1]) {
            ans.pb({i-1, 0, hei[k], {}});
            auto& rect = ans.back();

            while (hei[k] == rect.hei) {
                rect.touch.pb(k-1);
```

```
            k = reach[k];
        } // 6e7e
        rect.end = k-1;
    } // e03f
} // 2796
return ans;
} // 0e49
```

## util/mo.h caeb

```
// Modified MO's queries sorting algorithm,
// slightly better results than standard.
// Allows to process q queries in O(n*sqrt(q))

struct Query {
    int begin, end;
}; // b76d

// Get point index on Hilbert curve
ll hilbert(int x, int y, int s, ll c = 0) {
    if (s <= 1) return c;
    s /= 2; c *= 4;
    if (y < s)
        return hilbert(x&(s-1), y, s, c+(x>=s)+1);
    if (x < s)
        return hilbert(2*s-y-1, s-x-1, s, c);
    return hilbert(y-s, x-s, s, c+3);
} // 0fb9

// Get good order of queries; time: O(n lg n)
Vi moOrder(vector<Query>& queries, int maxN) {
    int s = 1;
    while (s < maxN) s *= 2;

    vector<ll> ord;
    each(q, queries)
        ord.pb(hilbert(q.begin, q.end, s));

    Vi ret(sz(ord));
    iota(all(ret), 0);
    sort(all(ret), [&](int l, int r) {
        return ord[l] < ord[r];
    }); // 9aea
    return ret;
} // ecec
```

## util/parallel\_binsearch.h bc5d

```
// Run `n` binary searches on [b;e) parallelly.
// `cmp` should be lambda with arguments:
// 1) vector<Pii>& - pairs (v, i)
// which are queries if value for index i
// is greater or equal to v;
// pairs are sorted by v
// 2) vector<bool>& - output vector,
// set true at index i if value
// for i-th query is >= queried value
// Returns vector of found values;
// time: O((n+c) lg range), where c is cmp time
template<class T>
Vi multiBS(int b, int e, int n, T cmp) {
    vector<Pii> que(n), rng(n, {b, e});
    vector<bool> ans(n);

    rep(i, 0, n) que[i] = {(b+e)/2, i};

    for (int k = 32-__builtin_clz(e-b); k--;) {
        int last = 0, j = 0;
        cmp(que, ans);
        rep(i, 0, sz(que)) {
            Pii &q = que[i], &r = rng[q.y];
            if (q.x != last) last = q.x, j = i;
            (ans[i] ? r.x : r.y) = q.x;
            q.x = (r.x+r.y) / 2;
```

```

    if (!ans[i]) swap(que[i], que[j++]);
} // 6c4e
} // 622c

Vi ret;
each(p, rng) ret.pb(p.x);
return ret;
} // 1ad7

```

util/radix\_sort.h 6fb4

```

// Stable countingsort; time: O(k+sz(vec))
// See example usage in radixSort for pairs.
template<class F>
void countSort(Vi& vec, F key, int k) {
    static Vi buf, cnt;
    vec.swap(buf);
    vec.resize(sz(buf));
    cnt.assign(k+1, 0);
    each(e, buf) cnt[key(e)]++;
    rep(i, 1, k+1) cnt[i] += cnt[i-1];
    for (int i = sz(vec)-1; i >= 0; i--)
        vec[--cnt[key(buf[i])]] = buf[i];
} // cc6f

// Compute order of elems, k is max key; O(n)
Vi radixSort(const vector<Pii>& elems, int k) {
    Vi order(sz(elems));
    iota(all(order), 0);
    countSort(order,
        [&](int i) { return elems[i].y; }, k);
    countSort(order,
        [&](int i) { return elems[i].x; }, k);
    return order;
} // e8f6

```

xyz/kactl.h b5ec

```

// --- POINT 3D
template<class T> struct Point3D {
    typedef Point3D P;
    typedef const P& R;
    T x, y, z;
    explicit Point3D(T a=0, T b=0, T c=0)
        : x(a), y(b), z(c) {}
    bool operator<(R p) const {
        return tie(x, y, z) < tie(p.x, p.y, p.z);
    }
    bool operator==(R p) const {
        return tie(x, y, z) == tie(p.x, p.y, p.z);
    }
    P operator+(R p) const {
        return P(x+p.x, y+p.y, z+p.z);
    } // 6cad
    P operator-(R p) const {
        return P(x-p.x, y-p.y, z-p.z);
    } // 01e2
    P operator*(T d) const {
        return P(x*d, y*d, z*d);
    } // 071d
    P operator/(T d) const {
        return P(x/d, y/d, z/d);
    } // 40df
    T dot(R p) const {
        return x*p.x + y*p.y + z*p.z;
    } // 466c
    P cross(R p) const {
        return P(y*p.z - z*p.y, z*p.x - x*p.z,
            x*p.y - y*p.x);
    } // 923b
    T dist2() const { return x*x + y*y + z*z; }
    double dist() const {
        return sqrt((double)dist2());
    } // de26
    //Azimuthal angle (longitude) to x-axis
    double phi() const { // in interval [-pi, pi]
        return atan2(y, x);
    }
    //Zenith angle (latitude) to the z-axis
    double theta() const { // in interval [0, pi]

```

```

    return atan2(sqrt(x*x+y*y), z);
} // ed29
P unit() const {
    return *this/(T)dist();
} //makes dist()=1
//returns unit vector normal to *this and p
P normal(P p) const {return cross(p).unit();}
//returns point rotated 'angle' radians
// ccw around axis
P rotate(double angle, P axis) const {
    double s = sin(angle), c = cos(angle);
    P u = axis.unit();
    return u*dot(u)*(1-c) +
        (*this)*c - cross(u)*s;
} // bc6c
} // ce68

// --- HULL 3D (requires POINT 3D) O(n^2)
typedef Point3D<double> P3;
struct PR {
    void ins(int x) { (a == -1 ? a : b) = x; }
    void rem(int x) { (a == x ? a : b) = -1; }
    int cnt() { return (a != -1) + (b != -1); }
    int a, b;
} // 8ad1

struct F { P3 q; int a, b, c; };
vector<F> hull3d(const vector<P3>& A) {
    assert(sz(A) >= 4);
    vector<vector<PR>> E(sz(A), vector<PR>(sz(A),
        {-1, -1}));
#define E(x,y) E[f.x][f.y]
    vector<F> FS;
    auto mf = [&](int i, int j, int k, int l) {
        P3 q = (A[j] - A[i]).cross((A[k] - A[i]));
        if (q.dot(A[l]) > q.dot(A[i]))
            q = q * -1;
        F f{q, i, j, k};
        E(a,b).ins(k); E(a,c).ins(j);
        E(b,c).ins(i);
        FS.push_back(f);
    } // 51be
    rep(i,0,4) rep(j,i+1,4) rep(k,j+1,4)
        mf(i, j, k, 6 - i - j - k);

    rep(i,4,sz(A)) {
        rep(j,0,sz(FS)) {
            F f = FS[j];
            if (f.q.dot(A[i]) > f.q.dot(A[f.a])) {
                E(a,b).rem(f.c);
                E(a,c).rem(f.b);
                E(b,c).rem(f.a);
                swap(FS[j-1], FS.back());
                FS.pop_back();
            } // 5d69
        } // eeb6
        int nw = sz(FS);
        rep(j,0,nw) {
            F f = FS[j];
#define C(a,b,c) if (E(a,b).cnt() != 2) \
                mf(f.a, f.b, i, f.c);
            C(a, b, c); C(a, c, b); C(b, c, a);
        } // 9578
    } // c66b
    for (F& it : FS) if ((A[it.b]-A[it.a]).cross(
        A[it.c] - A[it.a]).dot(it.q) <= 0)
        swap(it.c, it.b);
    return FS;
} // 2116

// --- DELAUNAY (requires POINT 3D and HULL 3D)

```

```

// Calls trifun for every triangle
template<class P, class F> // O(n^2)
void delaunay(vector<P>& ps, F trifun) {
    if (sz(ps) == 3) { int d =
        ((ps[1]-ps[0]).cross(ps[2]-ps[0]) < 0);
        trifun(0,1,d,2-d);
    } // 1266
    vector<P3> p3;
    for (P p : ps)
        p3.emplace_back(p.x, p.y, p.len2());
    if (sz(p3) > 3) for(auto t:hull3d(p3))
        if ((p3[t.b]-p3[t.a]).
            cross(p3[t.c]-p3[t.a]).dot(P3(0,0,1)) < 0)
            trifun(t.a, t.c, t.b);
} // 95dc

// --- FAST DELAUNAY (requires our vec2)
// O(n log n)
#include ".../geometry/vec2.h"
typedef vec2i P;
typedef struct Quad* Q;
// (can be 11 if coords are < 2e4)
typedef __int128_t l1l;
// not equal to any other point
P arb(LLONG_MAX, LLONG_MAX);

struct Quad {
    Q rot, o; P p = arb; bool mark;
    Quad(Q q) {rot=q;}
    P& F() { return r()->p; }
    Q& r() { return rot->rot; }
    Q prev() { return rot->o->rot; }
    Q next() { return r()->prev(); }
} *H;

l1l cross(P x, P a, P b) {
    return (a-x).cross(b-x);
} // 0647

bool circ(P p, P a, P b, P c) {
    l1l p2 = p.len2(), A = a.len2()-p2,
        B = b.len2()-p2, C = c.len2()-p2;
    return cross(p,a,b)*C + cross(p,b,c)*A +
        cross(p,c,a)*B > 0;
} // fe20

Q makeEdge(P orig, P dest) {
    Q r = H ? H : new Quad{new Quad{new Quad{
        new Quad{0}}}}; // 08c9
    H = r->o; r->r()->r() = r;
    rep(i,0,4) r = r->rot, r->p = arb,
        r->o = i & 1 ? r : r->r();
    r->p = orig; r->F() = dest;
    return r;
} // 8da8

void splice(Q a, Q b) {
    swap(a->o->rot->o, b->o->rot->o);
    swap(a->o, b->o);
} // 6867

Q connect(Q a, Q b) {
    Q q = makeEdge(a->F(), b->p);
    splice(q, a->next());
    splice(q->r(), b);
    return q;
} // af0b

pair<Q,Q> rec(const vector<P>& s) {
    if (sz(s) <= 3) {
        Q a = makeEdge(s[0], s[1]), b =
            makeEdge(s[1], s.back());
        if (sz(s) == 2) return { a, a->r() };
        splice(a->r(), b);
        auto side = cross(s[0], s[1], s[2]);

```

```

        Q c = side ? connect(b, a) : 0;
        return {side < 0 ? c->r() : a, side < 0 ?
            c : b->r() }; // d148
    } // d2fb

#define H(e) e->F(), e->p
#define valid(e) (cross(e->F(), H(base)) > 0)
Q A, B, ra, rb;
int half = sz(s) / 2;
tie(ra, A) = rec({all(s) - half});
tie(B, rb) = rec({sz(s) - half + all(s)});
while ((cross(B->p, H(A)) < 0 &&
    (A = A->next()) ||
    (cross(A->p, H(B)) > 0 &&
    (B = B->r()->o))))
    Q base = connect(B->r(), A);
    if (A->p == ra->p) ra = base->r();
    if (B->p == rb->p) rb = base;

#define DEL(e, init, dir) Q e = init->dir; \
    if (valid(e)) \
        while (circ(e->dir->F(), H(base), e->F())) { \
            Q t = e->dir; \
            splice(e, e->prev()); \
            splice(e->r(), e->r()->prev()); \
            e->o = H; H = e; e = t; \
        } // a3f9
    for (;;) {
        DEL(LC, base->r(), o); DEL(RC, base, prev());
        if (!valid(LC) && !valid(RC)) break;
        if (!valid(LC) || (valid(RC) &&
            circ(H(RC), H(LC))))
            base = connect(RC, base->r());
        else
            base = connect(base->r(), LC->r());
    } // 53db
    return { ra, rb };
} // 4baa

vector<P> triangulate(vector<P> pts) {
    sort(all(pts)); assert(unique(all(pts))
        == pts.end());
    if (sz(pts) < 2) return {};
    Q e = rec(pts).first;
    vector<Q> q = {e};
    int qi = 0;
    while (cross(e->o->F(), e->F(), e->p)
        < 0) e = e->o;
#define ADD { Q c = e; do { c->mark = 1; \
        pts.push_back(c->p); \
        q.push_back(c->r()); c = c->next(); } \
        while (c != e); } // 889e
    ADD; pts.clear();
    while (qi < sz(q))
        if (!(e = q[qi++])->mark) ADD;
    return pts;
} // ef04

// --- CIRCUMCIRCLE (requires our vec2)
#include ".../geometry/vec2.h"
typedef vec2d P;
double ccRadius(const P& A, const P& B,
    const P& C) {
    return (B-A).len()*(C-B).len()*(
        (A-C).len()/abs((B-A).cross(C-A)))/2;
} // 95ee

P ccCenter(const P& A, const P& B, const P& C) {
    P b = C-A, c = B-A;
    return A + (b*c.len2()-c*b.len2()).perp()
        / b.cross(c)/2;

```

```
// 2880
// --- MINIMUM ENCLOSING CIRCLE
// requires CIRCUMCIRCLE; expected O(n)
pair<P, double> mec(vector<P> ps) {
    shuffle(all(ps), mt19937(uint32_t(time(0))));
    P o = ps[0];
    double r = 0, EPS = 1 + 1e-8;
    rep(i, 0, sz(ps)) if ((o - ps[i]).len() >
        r * EPS) {
        o = ps[i], r = 0;
        rep(j, 0, i) if ((o - ps[j]).len() >
            r * EPS) {
            o = (ps[i] + ps[j]) / 2;
            r = (o - ps[i]).len();
            rep(k, 0, j) if ((o - ps[k]).len() >
                r * EPS) {
                o = ccCenter(ps[i], ps[j], ps[k]);
                r = (o - ps[i]).len();
            } // b992
        } // 942d
    } // 0a5c
    return {o, r};
} // faa0

// --- POLYGON CENTER OF MASS
typedef vec2d P; // (requires our vec2d)
P polygonCenter(const vector<P>& v) {
    P res(0, 0); double A = 0;
    for (int i = 0, j = sz(v) - 1;
        i < sz(v); j = i++) {
        res = res + (v[i]+v[j]) * v[j].cross(v[i]);
        A += v[j].cross(v[i]);
    } // eaf6
    return res / A / 3;
} // 405a
```

## xyz/uj.h

33d1

```
// P = punkt 2D lub 3D, K = double
// --- GEOMETRIA OKREGOW W 2D ~Bartosz Walczak
struct circle { // okrag w 2D
    P c; K r; // srodek, promien
    circle(const P &ci, K ri=0) : c(ci), r(ri) {}
    circle() {}
    K length() const { return 2*M_PI*r; }
    K area() const { return M_PI*r*r; }
}; // 2a8d
// Czy punkt lezy na okregu?
bool on_circle(const P &a, const circle &c)
{ return fabs((a-c.c).norm()-c.r*c.r) < EPS; }
// Czy kolo/punkt lezy wewnatrz lub na brzegu?
bool operator<(const circle &a, const circle &b)
{ return b.r+EPS > a.r && (a.c-b.c).norm() <
    (b.r-a.r)*(b.r-a.r)+EPS; } // 1f58
// Srodek okragu opisanego na trojkacie
circle circumcircle(P a, P b, P c) {
    if ((a-b).norm() > (c-b).norm()) swap(a, c);
    if ((b-c).norm() > (a-c).norm()) swap(a, b);
    if (fabs(det(b-a, c-b)) < EPS)
        throw "zdegenerowany";
    P v=intersection(median(a, b), median(b, c));
    return circle(v, sqrt((a-v).norm()));
} // 1327
// Przeciecie okregu i prostej.
// Zwraca liczbe punktow
int intersection(const circle &c,
    const line &p, P I[]/*OUT*/) {
    K d = p.n.norm(), a = (p.n*c-p.c)/d;
    P u = c.c-p.n*a; a *= a; K r = c.r*c.r/d;
```

```
if (a >= r+EPS) return 0;
if (a > r-EPS) { I[0]=u; return 1; }
K h = sqrt(r-a);
I[0] = u+cross(p.n)*h; I[1] = u-cross(p.n)*h;
return 2;
} // 51f4
// Przeciecie dwuch okregow.
// Zwraca liczbe punktow. Zalozenie: c1.c!=c2.c
int intersection(const circle &c1,
    const circle &c2, P I[]/*OUT*/) {
    K d = (c2.c-c1.c).norm(), r1 =
        c1.r*c1.r/d, r2 = c2.r*c2.r/d;
    P u = c1.c*((r2-r1+1)*0.5) +
        c2.c*((r1-r2+1)*0.5);
    if (r1 > r2) swap(r1, r2);
    K a = (r1-r2+1)*0.5; a *= a;
    if (a >= r1+EPS) return 0;
    if (a > r1-EPS) { I[0] = u; return 1; }
    P v = cross(c2.c - c1.c); K h = sqrt(r1-a);
    I[0] = u+v*h; I[1] = u-v*h; return 2;
} // 45ea

// --- GEOMETRIA 3D ~Bartosz Walczak
// Kat pomiedzy dwoma wektorami. Zawsze >=0.
// Zalozenie: a,b!=0
K angle3(const xyz &a, const xyz &b)
{ return atan2(sqrt(cross(b,a).norm()), a*b); }
struct plane { // plaszczyzna {v: n*v=c}
    xyz n; K c; // (n - wektor normalny)
    plane(const xyz &ni, K ci) : n(ni), c(ci) {}
    plane() {}
}; // 93a3
// Czy punkt lezy na plaszczyzynie?
bool on_plane(const xyz &a, const plane &p)
{ return fabs(p.n*a - p.c) < EPS; }
// Plaszczyzna rozpieta przez 3 punkty ccw.
// Zalozenie: a,b,c niezależne
plane span3(const xyz &a, const xyz &b,
    const xyz &c)
{ xyz n = cross(c-a, b-a);
    return plane(n, n*a); } // 2d8e
// Plaszczyzna symetryczna odcinka.
// Zalozenie: a!=b
plane median3(const xyz &a, const xyz &b)
{ return plane(b-a, (b-a)*(b+a)*0.5); }
// Plaszczyzna rownolegla przez punkt
plane parallel3(const xyz &a, const plane &p)
{ return plane(p.n, p.n*a); }
// Odleglosc punktu od plaszczyzny
K dist3(const xyz &a, const plane &p)
{ return fabs(p.n*a-p.c)/sqrt(p.n.norm()); }
struct line3 { // prosta {v: cross(v,u)=w}
    xyz u, w; // (u - wektor kierunku)
    // UWAGA! konstruktor dwuargumentowy
    // nie tworzy prostej przechodzacej
    // przez 2 punkty,
    // w tym celu nalezyc uzyc span3!
    line3(const xyz &ui, const xyz &wi)
        : u(ui), w(wi) {}
    line3() {}
}; // 8ba4
// Czy punkt lezy na prostej?
bool on_line3(const xyz &a, const line3 &p)
{ return (cross(a,p.u)-p.w).norm() < EPS; }
// Prosta rozpieta przez 2 punkty. Za_u.: a!=b
line3 span3(const xyz &a, const xyz &b)
{ return line3(b-a, cross(a, b-a)); }
// Plaszczyzna rozpieta przez prosta i
// punkt ccw. Zalozenie: cross(a,p.u)!=p.w
```

```
plane span3(const line3 &p, const xyz &a)
{ return plane(cross(a,p.u)-p.w, p.w*a); }
// Prosta przeciecia dwuch plaszczyzn
line3 intersection3(const plane &p,
    const plane &q) {
    xyz u=cross(q.n, p.n);
    if (u.norm() < EPS) throw "rownolegle";
    return line3(u, q.n*p.c-p.n*q.c);
} // 08c7
// Punkt przeciecia plaszczyzny i prostej
xyz intersection3(const plane &p,
    const line3 &q) {
    K d = q.u*p.n;
    if (fabs(d) < EPS) throw "rownolegle";
    return (q.u*p.c + cross(p.n, q.w))/d;
} // 7f03
// Prosta prostopadla do plaszczyzny
// przechodzaca przez punkt
line3 perp3(const xyz &a, const plane &p)
{ return line3(p.n, cross(a, p.n)); }
// Plaszczyzna prostopadla do prostej
// przechodzaca przez punkt
plane perp3(const xyz &a, const line3 &p)
{ return plane(p.u, p.u*a); }
// Odleglosc punktu od prostej
K dist3(const xyz &a, const line3 &p)
{ return sqrt((cross(a,p.u)-p.w).norm()) /
    sqrt(p.u.norm())); } // a713
// Odleglosc 2 prostych od siebie.
// Zalozenie: cross(q,u,p.u)!=0
// (niestabilne przy bliskim 0)
K dist3(const line3 &p, const line3 &q)
{ return fabs(p.u*q.w + q.u*p.w) /
    sqrt(cross(q,u, p.u).norm()); } // 88d8

// --- GEOMETRIA SFER W 3D ~Bartosz Walczak
struct sphere {
    xyz c; K r; // srodek, promien
    sphere(const xyz &ci, K ri=0)
        : c(ci), r(ri) {}
    sphere() {}
    // pole powierzchni
    K area() const { return 4*M_PI*r*r; }
    // objetosc kuli
    K volume() const { return 4*M_PI*r*r*r/3; }
}; // 029e
// Czy punkt lezy na sferze?
bool on_sphere(const xyz &a, const sphere &s)
{ return fabs((a-s.c).norm()-s.r*s.r) < EPS; }
// Czy sfera/punkt lezy wewnatrz lub na brzegu?
bool in_sphere(const sphere &a, const sphere &b)
{ return b.r+EPS > a.r && (a.c-b.c).norm() <
    (b.r-a.r)*(b.r-a.r)+EPS; } // 1f58
// Przeciecie sfery i prostej.
// Zwraca liczbe punktow przeciecia
int intersection3(const sphere &s,
    const line3 &p, xyz I[]/*OUT*/) {
    K d = p.u.norm(), a = (cross(s.c,p.u)-p.w).
        norm()/(d*d), r = s.r*s.r/d;
    if (a >= r+EPS) return 0;
    xyz u = (p.u*(p.u*s.c)+cross(p.u,p.w))/d;
    if (a > r-EPS) { I[0] = u; return 1; }
    K h = sqrt(r-a);
    I[0] = u+p.u*h; I[1] = u-p.u*h; return 2;
} // 14d7
// Przeciecie sfery i plaszczyzny.
// Zwraca true, jesli sie przecinaja. Wtedy u, r
// sa odp. srodkiem i promieniem okregu
// przeciecia. Zalozenie: s1.c!=s2.c
```

```
bool intersection3(const sphere &s,
    const plane &p, xyz &u, K &r) {
    K d = p.n.norm(), a = (p.n*s.c-p.c)/d;
    u = s.c-p.n*a; a *= a; K r1 = s.r*s.r/d;
    if (a >= r1+EPS) return false;
    r = a > r1-EPS ? 0 : sqrt(r1-a)*sqrt(d);
    return true;
} // d90d
// Przeciecie dwuch sfer.
// Zwraca true, jesli sie przecinaja.
// Wtedy u, r sa odp. srodkiem i promieniem
// okregu przeciecia. Zalozenie: s1.c!=s2.c
bool intersection3(const sphere &s1,
    const sphere &s2, xyz &u, K &r) {
    K d = (s2.c-s1.c).norm(), r1 = s1.r*s1.r/d,
        r2 = s2.r*s2.r/d;
    u = s1.c*((r2-r1+1)*0.5) +
        s2.c*((r1-r2+1)*0.5);
    if (r1 > r2) swap(r1, r2);
    K a = (r1-r2+1)*0.5; a *= a;
    if (a >= r1+EPS) return false;
    r = a > r1-EPS ? 0 : sqrt(r1-a)*sqrt(d);
    return true;
} // db4b

// --- GEOMETRIA NA SFERZE ~Bartosz Walczak
// Odleglosc dwuch punktow na sferze
K distS(const xyz &a, const xyz &b)
{ return atan2(sqrt(cross(b,a).norm()), a*b); }
struct circleS { // okrag na sferze
    xyz c; K r; // srodek, promien katowy
    circleS(const xyz &ci, K ri) : c(ci), r(ri) {}
    circleS() {}
    K area() const { return 2*M_PI*(1-cos(r)); }
}; // faf2
// Okrag rozpiety przez 3 punkty.
// Zalozenie: punkty sa parami rozne
circleS spanS(xyz a, xyz b, xyz c) {
    int tmp = 1;
    if ((a-b).norm() > (c-b).norm())
        { swap(a, c); tmp = -tmp; }
    if ((b-c).norm() > (a-c).norm())
        { swap(a, b); tmp = -tmp; }
    xyz v = cross(c-b, b-a);
    v = v*(tmp/sqrt(v.norm()));
    return circleS(v, distS(a,v));
} // 7374
// Przeciecie 2 okregow na sferze.
// Zalozenie: cross(c2.c,c1.c)!=0
int intersection3(const circleS &c1,
    const circleS &c2, xyz I[]/*OUT*/) {
    xyz n = cross(c2.c, c1.c),
        w = c2.c*cos(c1.r)-c1.c*cos(c2.r);
    K d = n.norm(), a = w.norm()/d;
    if (a >= 1+EPS) return 0;
    xyz u = cross(n,w)/d;
    if (a > 1-EPS) { I[0] = u; return 1; }
    K h = sqrt(1-a)/sqrt(d);
    I[0] = u+n*h; I[1] = u-n*h; return 2;
} // cbcb
```



**Tutte Matrix.** For a simple undirected graph  $G$ , Let  $M$  be a matrix with entries  $A_{i,j} = 0$  if  $(i,j) \notin E$  and  $A_{i,j} = -A_{j,i} = X$  if  $(i,j) \in E$ .  $X$  could be any random value. If the determinants are non-zero, then a perfect matching exists, while other direction might not hold for very small probability.

**Kirchhoff’s Theorem.** For a multigraph  $G$  with no loops, define Laplacian matrix as  $L = D - A$ .  $D$  is a diagonal matrix with  $D_{i,i} = deg(i)$ , and  $A$  is an adjacency matrix. If you remove any row and column of  $L$ , the determinant gives a number of spanning trees.

**Burnside’s lemma / Pólya enumeration theorem.** let  $G$  and  $H$  be groups of permutations of finite sets  $X$  and  $Y$ . Let  $c_m(g)$  denote the number of cycles of length  $m$  in  $g \in G$  when permuting  $X$ . The number of colorings of  $X$  into  $|Y| = n$  colors with exactly  $r_i$  occurrences of the  $i$ -th color is the coefficient of  $w_1^{r_1} \dots w_n^{r_n}$  in the following polynomial:

$$P(w_1, \dots, w_n) = \frac{1}{|H|} \sum_{h \in H} \frac{1}{|G|} \sum_{g \in G} \prod_{m \geq 1} (\sum_{h^m(b)=b} (w_b^m))^{c_m(g)}$$

When  $H = \{I\}$  (No color permutation):  
 $P(w_1, \dots, w_n) = \frac{1}{|G|} \sum_{g \in G} \prod_{m \geq 1} (w_1^m + \dots + w_n^m)^{c_m(g)}$

Total number of partitions of  $n$  distinct elements.  $B(n) = 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, \dots$  For  $p$  prime,

Without the occurrence restriction:  
 $P(1, \dots, 1) = \frac{1}{|G|} \sum_{g \in G} n^{c(g)}$

$B(p^m + n) \equiv mB(n) + B(n + 1) \pmod{p}$

where  $c(g)$  could also be interpreted as the number of elements in  $X$  that are fixed up to  $g$ .

**Pick’s Theorem.**  $A = i + \frac{b}{2} - 1$ , where:  $P$  is a simple polygon whose vertices are grid points,  $A$  is area of  $P$ ,  $i$  is # of grid points in the interior of  $P$ , and  $b$  is # of grid points on the boundary of  $P$ . If  $h$  is # of holes of  $P$  ( $h + 1$  simple closed curves in total),  $A = i + \frac{b}{2} + h - 1$ .

**Xudyh Sieve.**  $F(n) = \sum_{d|n} f(d)$   
 $S(n) = \sum_{i \leq n} f(i) = \sum_{i \leq n} F(i) - \sum_{d=2}^n S(\lfloor \frac{n}{d} \rfloor)$   
Preprocess  $S(1)$  to  $S(M)$  (Set  $M = n^{\frac{2}{3}}$  for complexity)  
 $S(n) = \sum f(i) = \sum_{i \leq n} [F(i) - \sum_{j|i, j \neq i} f(j)] = \sum F(i) - \sum_{i/j=d=2}^n \sum_{dj \leq n} f(j)$   
 $S(n) = \sum i f(i) = \sum_{i \leq n} i [F(i) - \sum_{j|i, j \neq i} f(j)] = \sum i F(i) - \sum_{i/j=d=2}^n \sum_{dj \leq n} d j f(j)$   
 $\sum_{d|n} \varphi(d) = n$   $\sum_{d|n} \mu(d) = \text{if } (n > 1) \text{ then } 0 \text{ else } 1$   $\sum_{d|n} (\mu(\frac{n}{d}) \sum_{e|d} f(e)) = f(n)$

**Labeled unrooted trees**  
# on  $n$  vertices:  $n^{n-2}$   
# on  $k$  existing trees of size  $n_i$ :  $n_1 n_2 \dots n_k n^{k-2}$   
# with degrees  $d_i$ :  $(n-2)! / ((d_1-1)! \dots (d_n-1)!)$   
 $x_1 x_2 x_3 \dots x_n (x_1 + x_2 + \dots x_n)^{n-2} = \sum_T x_1^{d_{T(1)}} x_2^{d_{T(2)}} \dots x_n^{d_{T(n)}}$

**Partition function**  
Number of ways of writing  $n$  as a sum of positive integers, disregarding the order of the summands.  
 $p(0) = 1, p(n) = \sum_{k \in \mathbb{Z} \setminus \{0\}} (-1)^{k+1} p(n - k(3k-1)/2)$

where the sum is over all spanning trees  $T$  in  $K_n$  and  $d_{T(i)}$  is the degree of  $i$  in  $T$

**Taylor’s theorem**<sup>[4][5][6]</sup> — Let  $k \geq 1$  be an [integer](#) and let the [function](#)  $f: \mathbf{R} \rightarrow \mathbf{R}$  be  $k$  times [differentiable](#) at the point  $a \in \mathbf{R}$ . Then there exists a function  $h_k: \mathbf{R} \rightarrow \mathbf{R}$  such that

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \dots + \frac{f^{(k)}(a)}{k!}(x-a)^k + h_k(x)(x-a)^k,$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots, (-\infty < x < \infty)$$
$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots, (-1 < x \leq 1)$$
$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{2x^3}{32} - \frac{5x^4}{128} + \dots, (-1 \leq x \leq 1)$$

$$\frac{1}{1-x} = \sum_{n \geq 0} x^n$$
$$\log \frac{1}{1-x} = \sum_{n \geq 1} \frac{x^n}{n}$$

$$\sin x = \sum_{n \geq 0} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$
$$\cos x = \sum_{n \geq 0} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\tan^{-1} x = \sum_{n \geq 0} (-1)^n \frac{x^{2n+1}}{2n+1}$$
$$\frac{1}{(1-x)^{k+1}} = \sum_n \binom{n+k}{n} x^n$$

$$\frac{1}{\sqrt{1-4x}} \left( \frac{1-\sqrt{1-4x}}{2x} \right)^k = \sum_n \binom{2n+k}{n} x^n$$
$$\frac{1}{2x} (1-\sqrt{1-4x}) = \sum_n \frac{1}{n+1} \binom{2n}{n} x^n$$
$$\frac{1}{\sqrt{1-4x}} = \sum_k \binom{2k}{k} x^k$$