

.bashrc	1	math/phi_precompute.h	9	trees/lca_linear.h	18
.vimrc	1	math/phi_prefix_sum.h	9	trees/link_cut_tree.h	18
template.cpp	1	math/pi_large.h	9	util/arc_interval_cover.h	18
various.h	1	math/pi_large_precomp.h	10	util/bit_hacks.h	18
geometry/convex_hull.h	1	math/pollard_rho.h	10	util/bump_alloc.h	19
geometry/convex_hull_dist.h	1	math/polynomial.h	10	util/compress_vec.h	19
geometry/convex_hull_sum.h	1	math/polynomial_interp.h	10	util/inversion_vector.h	19
geometry/halfplanes.h	1	math/sieve.h	11	util/longest_inc_subseq.h	19
geometry/line2.h	2	math/sieve_factors.h	11	util/max_rects.h	19
geometry/rmst.h	2	math/sieve_segmented.h	11	util/mo.h	19
geometry/segment2.h	2	structures/bitset_plus.h	11	util/parallel_binsearch.h	19
geometry/vec2.h	2	structures/fenwick_tree.h	11	util/radix_sort.h	19
graphs/2sat.h	3	structures/fenwick_tree_2d.h	11		
graphs/bellman_ineq.h	3	structures/find_union.h	11		
graphs/biconnected.h	3	structures/hull_offline.h	11		
graphs/boski_matching.h	3	structures/hull_online.h	11		
graphs/bridges_online.h	3	structures/max_queue.h	12		
graphs/dense_dfs.h	3	structures/pairing_heap.h	12		
graphs/dominators.h	4	structures/rmq.h	12		
graphs/edmonds_karp.h	4	structures/segtree_config.h	12		
graphs/gomory_hu.h	4	structures/segtree_general.h	13		
graphs/kth_shortest.h	4	structures/segtree_persist.h	13		
graphs/matroids.h	4	structures/segtree_point.h	13		
graphs/push_relabel.h	5	structures/treap.h	13		
graphs/scc.h	6	structures/wavelet_tree.h	14		
graphs/turbo_matching.h	6	structures/ext/hash_table.h	14		
graphs/weighted_matching.h	6	structures/ext/rope.h	14		
math/berlekamp_massey.h	6	structures/ext/tree.h	14		
math/bit_gauss.h	6	structures/ext/trie.h	14		
math/bit_matrix.h	6	text/aho_corasick.h	14		
math/crt.h	6	text/kmp.h	14		
math/fft_complex.h	7	text/kmr.h	14		
math/fft_mod.h	7	text/lcp.h	15		
math/fwht.h	7	text/lyndon_factorization.h	15		
math/gauss.h	7	text/main_lorentz.h	15		
math/matrix.h	8	text/manacher.h	15		
math/miller_rabin.h	8	text/min_rotation.h	15		
math/modinv_precompute.h	8	text/palindromic_tree.h	15		
math/modular.h	8	text/suffix_array_linear.h	15		
math/modular64.h	8	text/suffix_automaton.h	16		
math/modular_generator.h	8	text/suffix_tree.h	16		
math/modular_log.h	9	text/z_function.h	16		
math/modular_sqrt.h	9	trees/centroid_decomp.h	16		
math/montgomery.h	9	trees/centroid_offline.h	17		
math/nimber.h	9	trees/heavylight_decomp.h	17		
math/phi_large.h	9	trees/lca.h	17		

```

.bashrc ef12

build() {
    g++ $@ -o $1.e -DLOC -std=c++11 \
        -Wall -Wextra -Wfatal-errors -Wshadow \
        -Wlogical-op -Wconversion -Wfloat-equal
}

b() { build $@ -O2 }

d() { build $@ -fsanitize=address,undefined \
    -D_GLIBCXX_DEBUG -g }

run() { $1 $2 && echo start >&2 && time ./ $2.e }

loo() {
    set -e; $1 $2; $1 $3
    for (((i));) {
        ./ $3.e > gen.in
        time ./ $2.e < gen.in > gen.out
    }
}

cmp() {
    set -e; $1 $2; $1 $3; $1 $4
    for (((i));) {
        ./ $4.e > gen.in;          echo -n 0
        ./ $2.e < gen.in > p1.out; echo -n 1
        ./ $3.e < gen.in > p2.out; echo -n 2
        diff p1.out p2.out;       echo -n Y
    }
}

# Other flags:
# -Wformat=2 -Wshift-overflow=2 -Wcast-qual
# -Wcast-align -Wduplicated-cond
# -D_GLIBCXX_DEBUG_PEDANTIC -D_FORTIFY_SOURCE=2
# -fno-sanitize-recover -fstack-protector

.vimrc 8b2d

se ai aw cin cul ic is nocp nohls nu sc scs
se bg=dark sw=4 ts=4 so=7 ttm=9
sy on
vn _ :w !cpp -dD -P -fpreprocessed \ | \
    tr -d '[:space:]' \ | md5sum \ | cut -c-4 <CR>

template.cpp 8fcc

#include <bits/stdc++.h>
using namespace std;

using ll = long long;
using Vi = vector<int>;
using Pii = pair<int,int>;

#define mp make_pair
#define pb push_back
#define x first
#define y second

#define rep(i,b,e) for(int i=(b); i<(e); i++)
#define each(a,x) for(auto& a : (x))
#define all(x) (x).begin(), (x).end()
#define sz(x) int((x).size())

int main() {
    cin.sync_with_stdio(0); cin.tie(0);
    cout << fixed << setprecision(18);

    // Don't call destructors:
    cout << flush; _Exit(0);
} // 4799

// > Debug printer

#define tem template<class t,class u,class...w>

```

```

#define pri(x,y,z)tem auto operator<<(t&o,u a) \
    ->decltype(x,o) { o << z; return o << y; }

pri(a.print(), '|', '|'; a.print())
pri(a.y, '|', '(' << a.x << ", " << a.y)
pri(all(a), '|', '|'; auto d="";
    for (auto i : a) (o << d << i, d = ", "))

void DD(...) {}
tem void DD(t s, u a, w... k) {
    for (int b=1; *s && *s - b*44; cerr << *s++)
        b += 2 / (*s*2 - 81);
    cerr << ": " << a << *s++; DD(s, k...);
} // fd6d

#ifdef LOC
#define deb(...) (DD("[, \b :]" #__VA_ARGS__, \
    __LINE__, __VA_ARGS__), cerr << endl)
#else
#define deb(...)
#endif

#define DBP(...) void print() { \
    DD(__VA_ARGS__, __VA_ARGS__); } // 813f

// > Utils

// Return smallest k such that 2^k > n
// Undefined for n = 0!
int uplg(int n) { return 32 - __builtin_clz(n); }
int uplg(ll n) { return 64 - __builtin_clzll(n); }

// Compare with certain epsilon (branchless)
// Returns -1 if a < b; 1 if a > b; 0 if equal
// a and b are assumed equal if |a-b| <= eps
int cmp(double a, double b, double eps=1e-9) {
    return (a > b+eps) - (a+eps < b);
} // 81cl

various.h cbc4

// If math constants like M_PI are not found
// add this at the beginning of file
#define _USE_MATH_DEFINES

// Pragmas
#pragma GCC optimize("Ofast,unroll-loops, \
    no-stack-protector")
#pragma GCC target("popcnt,avx,tune=native")

// Clock
while (clock() < duration*CLOCKS_PER_SEC)

// Automatically implement operators:
// 1. != if == is defined
// 2. >, <= and >= if < is defined
using namespace rel_ops;

// Mersenne twister for randomization.
mt19937_64 rnd(chrono::steady_clock::now()
    .time_since_epoch().count());

// To shuffle randomly use:
shuffle(all(vec), rnd)

// To pick random integer from [A;B] use:
uniform_int_distribution<> dist(A, B);
int value = dist(rnd);

// To pick random real number from [A;B] use:
uniform_real_distribution<> dist(A, B);
double value = dist(rnd);

geometry/convex_hull.h 4a8e

#include "vec2.h"

```

```

// Translate points such that lower-left point
// is (0, 0). Returns old point location; O(n)
vec2 normPos(vector<vec2>& points) {
    auto q = points[0].yx();
    each(p, points) q = min(q, p.yx());
    vec2 ret{q.y, q.x};
    each(p, points) p = p-ret;
    return ret;
} // ee96

// Find convex hull of points; time: O(n lg n)
// Points are returned counter-clockwise,
// first point is the lowest-left.
vector<vec2> convexHull(vector<vec2> points) {
    vec2 pivot = normPos(points);
    sort(all(points));
    vector<vec2> hull;

    each(p, points) {
        while (sz(hull) >= 2) {
            vec2 a = hull.back() - hull[sz(hull)-2];
            vec2 b = p - hull.back();
            if (a.cross(b) > 0) break;
            hull.pop_back();
        } // ad91
        hull.pb(p);
    } // 5908

    // Translate back, optional
    each(p, hull) p = p+pivot;
    return hull;
} // 62ed

// Find point p that minimizes dot product p*q.
// Returns point index in hull; time: O(lg n)
// If multiple points have same dot product
// one with smallest index is returned.
// Points are expected to be in the same order
// as output from convexHull function.
int minDot(const vector<vec2>& hull, vec2 q) {
    auto search = [&](int b, int e, vec2 p) {
        while (b+1 < e) {
            int m = (b+e) / 2;
            (p.dot(hull[m-1]) > p.dot(hull[m])
                ? b : e) = m;
        } // 184c
        return b;
    }; // 62e8

    int m = search(0, sz(hull), {0, -1});
    int i = search(0, m, q);
    int j = search(m, sz(hull), q);
    return q.dot(hull[i]) > q.dot(hull[j])
        ? j : i;
} // 4599

geometry/convex_hull_dist.h 2859

#include "vec2.h"

// Check if p is inside convex polygon. Hull
// must be given in counter-clockwise order.
// Returns 2 if inside, 1 if on border,
// 0 if outside; time: O(n)
int insideHull(vector<vec2>& hull, vec2 p) {
    int ret = 1;
    rep(i, 0, sz(hull)) {
        auto v = hull[(i+1)%sz(hull)] - hull[i];
        auto t = v.cross(p-hull[i]);
        ret = min(ret, cmp(t, 0)); // For doubles
        // ret = min(ret, (t>0) - (t<0)); // Ints
    } // 0d40

```

```

    return int(max(ret+1, 0));
} // 1f39

#include "segment2.h"

// Get distance from point to hull; time: O(n)
double hullDist(vector<vec2>& hull, vec2 p) {
    if (insideHull(hull, p)) return 0;
    double ret = 1e30;
    rep(i, 0, sz(hull)) {
        seg2 seg[hull[(i+1)%sz(hull)], hull[i]];
        ret = min(ret, seg.distTo(p));
    } // f3be
    return ret;
} // a00c

// Compare distance from point to hull
// with sqrt(d2); time: O(n)
// -1 if smaller, 0 if equal, 1 if greater
int cmpHullDist(vector<vec2>& hull,
    vec2 p, ll d2) {
    if (insideHull(hull,p)) return (d2<0)-(d2>0);
    int ret = 1;
    rep(i, 0, sz(hull)) {
        seg2 seg[hull[(i+1)%sz(hull)], hull[i]];
        ret = min(ret, seg.cmpDistTo(p, d2));
    } // 28cb
    return ret;
} // 30f3

geometry/convex_hull_sum.h 7f53

#include "vec2.h"

// Get edge sequence for given polygon
// starting from lower-left vertex; time: O(n)
// Returns start position.
vec2 edgeSeq(vector<vec2> points,
    vector<vec2>& edges) {
    int i = 0, n = sz(points);
    rep(j, 0, n)
        if (points[i].yx() > points[j].yx()) i = j;
    rep(j, 0, n) edges.pb(points[(i+j+1)%n] -
        points[(i+j)%n]);
    return points[i];
} // 3aa7

// Minkowski sum of given convex polygons.
// Vertices are required to be in
// counter-clockwise order; time: O(n+m)
vector<vec2> hullSum(vector<vec2> A,
    vector<vec2> B) {
    vector<vec2> sum, e1, e2, es(sz(A) + sz(B));
    vec2 pivot = edgeSeq(A, e1) + edgeSeq(B, e2);
    merge(all(e1), all(e2), es.begin());

    sum.pb(pivot);
    each(e, es) sum.pb(sum.back() + e);
    sum.pop_back();
    return sum;
} // f183

geometry/halfplanes.h 356a

#include "vec2.h"
#include "line2.h"

// Intersect halfplanes given by `lines`
// and output hull vertices to `out`
// in counter-clockwise order. Returns true
// if intersection is non-empty and bounded.
// Unbounded cases are not supported,
// add bounding-box if necessary. Works only
// with floating point vec2/line2; O(n lg n)

```

```
// PARTIALLY TESTED
bool intersectHalfplanes(vector<line2> in,
    vector<vec2>& out) {
    sort(all(in), [](line2 a, line2 b) {
        return (a.v.angleCmp(b.v) ? :
            a.c*b.v.len() - b.c*a.v.len()) < 0;
    }); // 82fb

    int a = 0, b = 0, n = sz(in);
    vector<line2> dq(n+5);
    out.resize(n+5);
    dq[0] = in[0];

    rep(i, 1, n+1) {
        if (i == n) in.pb(dq[a]);
        if (!in[i].v.angleCmp(in[i-1].v)) continue;
        while (a < b && in[i].side(out[b-1]) > 0)
            b--;
        while (i!=n && a<b && in[i].side(out[a])>0)
            a++;
        if (in[i].intersect(dq[b], out[b]))
            dq[++b] = in[i];
    } // b9ba

    out.resize(b);
    out.erase(out.begin(), out.begin()+a);
    return b-a > 2;
} // f334
```

geometry/line2.h

9207

```
#include "vec2.h"

// 2D line/halfplane structure
// PARTIALLY TESTED

// Base class of versions for ints and doubles
template<class T, class P, class S>
struct bline2 {
    // For lines: v * point == c
    // For halfplanes: v * point <= c
    // (i.e. normal vector points outside)
    P v; // Normal vector [A; B]
    T c; // Offset (C parameter of equation)
    DBP(v, c);

    // Line through 2 points; normal vector
    // points to the right of ab vector
    static S through(P a, P b) {
        return { (a-b).perp(), a.cross(b) };
    } // 9ac7

    // Parallel line through point
    static S parallel(P a, S b) {
        return { b.v, b.v.dot(a) };
    } // 8e1c

    // Perpendicular line through point
    static S perp(P a, S b) {
        return { b.v.perp(), b.v.cross(a) };
    } // 7b75

    // Distance from point to line
    double distTo(P a) {
        return fabs(v.dot(a)-c) / v.len();
    } // 79e6
}; // ee4f

// Version for integer coordinates (long long)
struct line2i : bline2<ll, vec2i, line2i> {
    line2i() : bline2({}, 0) {}
    line2i(vec2i a, ll b) : bline2{a, b} {}

    // Returns 0 if point a lies on the line,
    // 1 if on side where normal vector points,
```

```
// -1 if on the other side.
int side(vec2i a) {
    ll d = v.dot(a);
    return (d > c) - (d < c);
} // 18a7
}; // fc9c

// Version for double coordinates
// Requires cmp() from template
struct line2d : bline2<double, vec2d, line2d> {
    line2d() : bline2({}, 0) {}
    line2d(vec2d a, double b) : bline2{a, b} {}

    // Returns 0 if point a lies on the line,
    // 1 if on side where normal vector points,
    // -1 if on the other side.
    int side(vec2d a) { return cmp(v.dot(a), c); }

    // Intersect this line with line a, returns
    // true on success (false if parallel).
    // Intersection point is saved to 'out'.
    bool intersect(line2d a, vec2d& out) {
        double d = v.cross(a.v);
        if (!cmp(d, 0)) return 0;
        out = (v*a.c - a.v*c).perp() / d;
        return 1;
    } // 2e68
}; // ab54
```

using line2 = line2d;

geometry/rmst.h

476a

```
#include "../structures/find_union.h"

// Rectilinear Minimum Spanning Tree
// (MST in Manhattan metric); time: O(n lg n)
// Returns MST weight. Outputs spanning tree
// to G, vertex indices match point indices.
// Edge in G is pair (target, weight).
ll rmst(vector<Pii>& points,
    vector<vector<Pii>& G) {
    int n = sz(points);
    vector<pair<int, Pii>> edges;
    vector<Pii> close;
    Vi ord(n, merged(n);
    iota(all(ord), 0);

    function<void(int,int)> octant =
        [&](int begin, int end) {
            if (begin+1 >= end) return;
            int mid = (begin+end) / 2;
            octant(begin, mid);
            octant(mid, end);

            int j = mid;
            Pii best = {INT_MAX, -1};
            merged.clear();

            rep(i, begin, mid) {
                int v = ord[i];
                Pii p = points[v];

                while (j < end) {
                    int e = ord[j];
                    Pii q = points[e];
                    if (q.x-q.y > p.x-p.y) break;
                    best = min(best, make_pair(q.x+q.y, e));
                    merged.pb(e);
                    j++;
                } // 8576

                if (best.y != -1) {
                    int alt = best.x-p.x-p.y;
```

```
if (alt < close[v].x)
    close[v] = {alt, best.y};
} // 4208
merged.pb(v);
} // f3ff

while (j < end) merged.pb(ord[j++]);
copy(all(merged), ord.begin()+begin);
}; // a4e1

rep(i, 0, 4) {
    rep(j, 0, 2) {
        sort(all(ord), [&](int l, int r) {
            return points[l] < points[r];
        }); // fe33
        close.assign(n, {INT_MAX, -1});
        octant(0, n);
        rep(k, 0, n) {
            Pii p = close[k];
            if (p.y != -1) edges.pb({p.x, {k, p.y}});
            points[k].x += -1;
        } // 1c1d
    } // 9b38
    each(p, points) p = {p.y, -p.x};
} // d06f

ll sum = 0;
FAU fau(n);
sort(all(edges));
G.assign(n, {});

each(e, edges) if (fau.join(e.y.x, e.y.y)) {
    sum += e.x;
    G[e.y.x].pb({e.y.y, e.x});
    G[e.y.y].pb({e.y.x, e.x});
} // b04a
return sum;
} // f586
```

geometry/segment2.h

6504

```
#include "vec2.h"

// 2D segment structure; NOT HEAVILY TESTED

// Base class of versions for ints and doubles
template<class P, class S> struct bseg2 {
    P a, b; // Endpoints

    // Distance from segment to point
    double distTo(P p) const {
        if ((p-a).dot(b-a) < 0) return (p-a).len();
        if ((p-b).dot(a-b) < 0) return (p-b).len();
        return double(abs((p-a).cross(b-a)))
            / (b-a).len();
    } // 62a2
}; // 85bc

// Version for integer coordinates (long long)
struct seg2i : bseg2<vec2i, seg2i> {
    seg2i() {}
    seg2i(vec2i c, vec2i d) : bseg2{c, d} {}

    // Check if segment contains point p
    bool contains(vec2i p) {
        return (a-p).dot(b-p) <= 0 &&
            (a-p).cross(b-p) == 0;
    } // c598

    // Compare distance to p with sqrt(d2)
    // -1 if smaller, 0 if equal, 1 if greater
    int cmpDistTo(vec2i p, ll d2) const {
        if ((p-a).dot(b-a) < 0) {
            ll l = (p-a).len2();
```

```
return (l > d2) - (l < d2);
} // dia6
if ((p-b).dot(a-b) < 0) {
    ll l = (p-b).len2();
    return (l > d2) - (l < d2);
} // 9e65

ll c = abs((p-a).cross(b-a));
d2 *= (b-a).len2();
return (c*c > d2) - (c*c < d2);
} // 726d
}; // 4df2

// Version for double coordinates
// Requires cmp() from template
struct seg2d : bseg2<vec2d, seg2d> {
    seg2d() {}
    seg2d(vec2d c, vec2d d) : bseg2{c, d} {}

    bool contains(vec2d p) {
        return cmp((a-p).dot(b-p), 0) <= 0 &&
            cmp((a-p).cross(b-p), 0) == 0;
    } // b507
}; // 2036

using seg2 = seg2d;
```

geometry/vec2.h

6e47

```
// 2D point/vector structure; PARTIALLY TESTED

// Base class of versions for ints and doubles
template<class T, class S> struct bvec2 {
    T x, y;
    S operator+(S r) const { return{x+r.x, y+r.y}; }
    S operator-(S r) const { return{x-r.x, y-r.y}; }
    S operator*(T r) const { return {x*r, y*r}; }
    S operator/(T r) const { return {x/r, y/r}; }

    T dot(S r) const { return x*r.x + y*r.y; }
    T cross(S r) const { return x*r.y - y*r.x; }
    T len2() const { return x*x + y*y; }
    double len() const { return hypot(x, y); }
    S perp() const { return {-y, x}; } // CCW

    pair<T, T> yx() const { return {y, x}; }

    double angle() const { //[0;2*PI] CCW from OX
        double a = atan2(y, x);
        return (a < 0 ? a+2*M_PI : a);
    } // 7095
}; // 17ed

// Version for integer coordinates (long long)
struct vec2i : bvec2<ll, vec2i> {
    vec2i() : bvec2{0, 0} {}
    vec2i(ll a, ll b) : bvec2{a, b} {}

    bool upper() const { return (y ? x) >= 0; }

    int angleCmp(vec2i r) const {
        ll c = cross(r);
        return r.upper() - upper() ? : (c<0) - (c>0);
    } // b35f

    // Compare by angle, length if angles equal
    bool operator<(vec2i r) const {
        return (angleCmp(r) ? :
            len2() - r.len2()) < 0;
    } // 6f78

    bool operator==(vec2i r) const {
        return x == r.x && y == r.y;
    } // 136e
}; // d3f4
```

```
// Version for double coordinates
// Requires cmp() from template
struct vec2d : bvec2<double, vec2d> {
    vec2d() : bvec2{0, 0} {}
    vec2d(double a, double b) : bvec2{a, b} {}

    bool upper() const {
        return (cmp(y, 0) ? : cmp(x, 0)) >= 0;
    } // 086c

    int angleCmp(vec2d r) const {
        return r.upper() - upper() ? :
            cmp(0, cross(r));
    } // 12f3

    // Compare by angle, length if angles equal
    bool operator<(vec2d r) const {
        return (angleCmp(r) ? :
            cmp(len2(), r.len2())) < 0;
    } // f3d7

    bool operator==(vec2d r) const {
        return !cmp(x, r.x) && !cmp(y, r.y);
    } // 81cd

    vec2d unit() const { return *this / len(); }

    vec2d rotate(double a) const { // CCW
        return {x*cos(a) - y*sin(a),
            x*sin(a) + y*cos(a)}; // 1890
    } // 97e3
}; // 08e9

using vec2d = vec2d;
```

graphs/2sat.h 2443

```
// 2-SAT solver; time: O(n+m), space: O(n+m)
// Variables are indexed from 1 and
// negative indices represent negations!
// Usage: SAT2 sat(variable_count);
// (add constraints...)
// bool solution_found = sat.solve();
// sat[i] = value of i-th variable, 0 or 1
// (also indexed from 1!)
// (internally: positive = i*2-1, neg. = i*2-2)
struct SAT2 : Vi {
    vector<Vi> G;
    Vi order, flags;

    // Init n variables, you can add more later
    SAT2(int n = 0) : G(n*2) {}

    // Add new var and return its index
    int addVar() {
        G.resize(sz(G)+2); return sz(G)/2;
    } // 98f3

    // Add (i => j) constraint
    void imply(int i, int j) {
        i = i*2 ^ i >> 31;
        j = j*2 ^ j >> 31;
        G[--i].pb(--j); G[j^1].pb(i^1);
    } // 8e25

    // Add (i v j) constraint
    void either(int i, int j) { imply(-i, j); }

    // Constraint at most one true variable
    void atMostOne(Vi& vars) {
        int y, x = addVar();
        each(i, vars) {
            imply(x, y = addVar());
            imply(i, -x); imply(i, x = y);
        } // 24aa
```

```
} // 3ed7

// Solve and save assignments in `values`
bool solve() { // O(n+m), Kosaraju is used
    assign(sz(G)/2+1, -1);
    flags.assign(sz(G), 0);
    rep(i, 0, sz(G)) dfs(i);
    while (!order.empty()) {
        if (!propag(order.back()^1, 1)) return 0;
        order.pop_back();
    } // 5594
    return 1;
} // 1e58

void dfs(int i) {
    if (flags[i]) return;
    flags[i] = 1;
    each(e, G[i]) dfs(e);
    order.pb(i);
} // d076

bool propag(int i, bool first) {
    if (!flags[i]) return 1;
    flags[i] = 0;
    if (at(i/2+1) >= 0) return first;
    at(i/2+1) = i&1;
    each(e, G[i]) if (!propag(e, 0)) return 0;
    return 1;
} // 4c1b
}; // d74c
```

graphs/bellman_ineq.h cd51

```
struct Ineq {
    ll a, b, c; // a - b >= c
}; // 663a

// Solve system of inequalities of form a-b>=c
// using Bellman-Ford; time: O(n*m)
bool solveIneq(vector<Ineq>& edges,
    vector<ll>& vars) {
    rep(i, 0, sz(vars)) each(e, edges)
        vars[e.b] = min(vars[e.b], vars[e.a]-e.c);
    each(e, edges)
        if (vars[e.a]-e.c < vars[e.b]) return 0;
    return 1;
} // 241e
```

graphs/biconnected.h 2b9e

```
// Biconnected components; time: O(n+m)
// Usage: Biconnected bi(graph);
// bi[v] = indices of components containing v
// bi.verts[i] = vertices of i-th component
// bi.edges[i] = edges of i-th component
// Bridges <=> components with 2 vertices
// Articulation points <=> vertices that belong
// to > 1 component
// Isolated vertex <=> empty component list
struct Biconnected : vector<Vi> {
    vector<Vi> verts;
    vector<vector<Pii>> edges;
    vector<Pii> S;

    Biconnected() {}

    Biconnected(vector<Vi>& G) : S(sz(G)) {
        resize(sz(G));
        rep(i, 0, sz(G)) S[i].x ? : dfs(G, i, -1);
        rep(c, 0, sz(verts)) each(v, verts[c])
            at(v).pb(c);
    } // cfce

    int dfs(vector<Vi>& G, int v, int p) {
```

```
int low = S[v].x = sz(S)-1;
S.pb({v, -1});

each(e, G[v]) if (e != p) {
    if (S[e].x < S[v].x) S.pb({v, e});
    low = min(low, S[e].x ? : dfs(G, e, v));
} // 446d

if (p+1 && low >= S[p].x) {
    verts.pb({p}); edges.pb({});
    rep(i, S[v].x, sz(S)) {
        if (S[i].y == -1)
            verts.back().pb(S[i].x);
        else
            edges.back().pb(S[i]);
    } // 4fab
    S.resize(S[v].x);
} // 6d66

return low;
} // 7fcc
}; // 4fa4
```

graphs/boski_matching.h dbd2

```
// Bosek's algorithm for partially online
// bipartite maximum matching - white vertices
// are fixed, black vertices are added
// one by one; time: O(E*sqrt(V))
// Usage: Matching match(num_white);
// match[v] = index of black vertex matched to
// white vertex v or -1 if unmatched
// match.add(indices_of_white_neighbours);
// Black vertices are indexed in order they
// were added, the first black vertex is 0.
struct Matching : Vi {
    vector<Vi> adj;
    Vi rank, low, pos, vis, seen;
    int k{0};
```

```
// Initialize structure for n white vertices
Matching(int n = 0) : Vi(n, -1), rank(n) {}

// Add new black vertex with its neighbours
// given by `vec`. Returns true if maximum
// matching is increased by 1.
bool add(Vi vec) {
    adj.pb(move(vec));
    low.pb(0); pos.pb(0); vis.pb(0);
    if (!adj.back().empty()) {
        int i = k;
        nxt:
        seen.clear();
        if (dfs(sz(adj)-1, ++k-i)) return 1;
        each(v, seen) each(e, adj[v])
            if (rank[e] < 1e9 && vis[at(e)] < k)
                goto nxt;
        each(v, seen) each(w, adj[v])
            rank[w] = low[v] = 1e9;
    } // 6aec
    return 0;
} // d2a7
```

```
bool dfs(int v, int g) {
    if (vis[v] < k) vis[v] = k, seen.pb(v);
    while (low[v] < g) {
        int e = adj[v][pos[v]];
        if (at(e) != v && low[v] == rank[e]) {
            rank[e]++;
            if (at(e) == -1 || dfs(at(e), rank[e]))
                return at(e) = v, 1;
        } else if (++pos[v] == sz(adj[v])) {
```

```
pos[v] = 0; low[v]++;
    } // e532
    } // 3d88
    return 0;
} // 8561
}; // aac1

graphs/bridges_online.h 4124

// Dynamic 2-edge connectivity queries
// Usage: Bridges bridges(vertex_count);
// - bridges.addEdge(u, v); - add edge (u, v)
// - bridges.cc[v] = connected component ID
// - bridges.bi(v) = 2-edge connected comp ID
struct Bridges {
    vector<Vi> G; // Spanning forest
    Vi cc, size, par, bp, seen;
    int cnt{0};

    // Initialize structure for n vertices; O(n)
    Bridges(int n = 0) : G(n), cc(n), size(n, 1),
        par(n, -1), bp(n, -1),
        seen(n) {
        iota(all(cc), 0);
    } // ed70
```

```
// Add edge (u, v); time: amortized O(lg n)
void addEdge(int u, int v) {
    if (cc[u] == cc[v]) {
        int r = lca(u, v);
        for (int x : {u, v})
            while ((x = root(x)) != r)
                x = bp[bi(x)] = par[x];
    } else {
        G[u].pb(v); G[v].pb(u);
        if (size[cc[u]] > size[cc[v]]) swap(u, v);
        size[cc[v]] += size[cc[u]];
        dfs(u, v);
    } // abc7
} // a6fd
```

```
// Get 2-edge connected component ID
int bi(int v) { // amortized time: < O(lg n)
    return bp[v] + 1 ? bp[v] = bi(bp[v]) : v;
} // 3206
```

```
int root(int v) {
    return par[v] == -1 || bi(par[v]) != bi(v)
        ? v : par[v] = root(par[v]);
} // 2d27
```

```
void dfs(int v, int p) {
    cc[v] = cc[par[v] = p];
    each(e, G[v]) if (e != p) dfs(e, v);
} // 85f5
```

```
int lca(int u, int v) { // Don't use this!
    for (cnt++; swap(u, v)) if (u != -1) {
        if (seen[u = root(u)] == cnt) return u;
        seen[u] = cnt; u = par[u];
    } // afed
} // 7f56
}; // bd70
```

graphs/dense_dfs.h 4fbd

```
#include "../math/bit_matrix.h"

// DFS over bit-packed adjacency matrix
// G = NxN adjacency matrix of graph
// G(i,j) <=> (i,j) is edge
// V = 1xN matrix containing unvisited vertices
// V(0,i) <=> i-th vertex is not visited
// Total DFS time: O(n^2/64)
```



```
struct DenseDFS {
    BitMatrix G, V; // space: O(n^2/64)

    // Initialize structure for n vertices
    DenseDFS(int n = 0) : G(n, n), V(1, n) {
        reset();
    } // 79e4

    // Mark all vertices as unvisited
    void reset() { each(x, V.M) x = -1; }

    // Get/set visited flag for i-th vertex
    void setVisited(int i) { V.set(0, i, 0); }
    bool isVisited(int i) { return !V(0, i); }

    // DFS step: func is called on each unvisited
    // neighbour of i. You need to manually call
    // setVisited(child) to mark it visited
    // or this function will call the callback
    // with the same vertex again.
    template<class T>
    void step(int i, T func) {
        ull* E = G.row(i);
        for (int w = 0; w < G.stride; w++) {
            ull x = E[w] & V.row(0)[w];
            if (x) func((w < 6) | __builtin_ctzll(x));
            else w++;
        } // 4c0a
    } // f045
}; // 8ed8
```

graphs/dominators.h119f

```
// Tarjan's algorithm for finding dominators
// in directed acyclic graph; time: O(m log n)
// Returns array of immediate dominators idom.
// idom[root] = root
// idom[v] = -1 if v is unreachable from root
Vi dominators(const vector<Vi>& G, int root) {
    int n = sz(G);
    vector<Vi> in(n), bucket(n);
    Vi pre(n, -1), anc(n, -1), par(n), best(n);
    Vi ord, idom(n, -1), sdom(n, n), rdom(n);

    function<void(int, int)> dfs =
        [&](int v, int p) {
            if (pre[v] == -1) {
                par[v] = p;
                pre[v] = sz(ord);
                ord.pb(v);
                each(e, G[v]) in[e].pb(v), dfs(e, v);
            } // 1182
        }; // ffd2

    function<Pii(int)> find = [&](int v) {
        if (anc[v] == -1) return mp(best[v], v);
        int b; tie(b, anc[v]) = find(anc[v]);
        if (sdom[b] < sdom[best[v]]) best[v] = b;
        return mp(best[v], anc[v]);
    }; // c07b

    rdom[root] = idom[root] = root;
    iota(all(best), 0);
    dfs(root, -1);

    rep(i, 0, n) {
        int v = ord[n-i-1], b = pre[v];
        each(e, in[v])
            b = min(b, pre[e] < pre[v] ? pre[e] :
                sdom[find(e).x]);
        each(u, bucket[v]) rdom[u] = find(u).x;
        sdom[best[v] = v] = b;
        anc[v] = par[v];
    }
```

```
        bucket[ord[sdom[v]]].pb(v);
    } // 3888

    each(v, ord) idom[v] = (rdom[v] == v ?
        ord[sdom[v]] : idom[rdom[v]]);
    return idom;
} // b527

graphs/edmonds_karp.he8cc

using flow_t = int;
constexpr flow_t INF = 1e9+10;

// Edmonds-Karp algorithm for finding
// maximum flow in graph; time: O(V*E^2)
// NOT HEAVILY TESTED
struct MaxFlow {
    struct Edge {
        int dst, inv;
        flow_t flow, cap;
    }; // a53c

    vector<vector<Edge>> G;
    vector<flow_t> add;
    Vi prev;

    // Initialize for n vertices
    MaxFlow(int n = 0) : G(n) {}

    // Add new vertex
    int addVert() {
        G.emplace_back(); return sz(G)-1;
    } // 907a

    // Add edge between u and v with capacity cap
    // and reverse capacity rcap
    void addEdge(int u, int v,
        flow_t cap, flow_t rcap = 0) {
        G[u].pb({ v, sz(G[v]), 0, cap });
        G[v].pb({ u, sz(G[u])-1, 0, rcap });
    } // 0f10

    // Compute maximum flow from src to dst.
    // Flow values can be found in edges,
    // vertices with `add` >= 0 belong to
    // cut component containing `s`.
    flow_t maxFlow(int src, int dst) {
        flow_t i, m, f = 0;
        each(v, G) each(e, v) e.flow = 0;

    nxt:
        queue<int> Q;
        Q.push(src);
        prev.assign(sz(G), -1);
        add.assign(sz(G), -1);
        add[src] = INF;

        while (!Q.empty()) {
            m = add[i = Q.front()];
            Q.pop();

            if (i == dst) {
                while (i != src) {
                    auto& e = G[i][prev[i]];
                    e.flow += m;
                    G[i] = e.dst; [e.inv].flow += m;
                } // 1f86
                f += m;
                goto nxt;
            } // 43a2

            each(e, G[i])
                if (add[e.dst] < 0 && e.flow < e.cap) {
                    Q.push(e.dst);
                    prev[e.dst] = e.inv;
                }
```

```
                add[e.dst] = min(m, e.cap-e.flow);
            } // 4cdb
        } // 887e

        return f;
    } // cec0

    // Get if v belongs to cut component with src
    bool cutSide(int v) {
        return add[v] >= 0;
    } // 41e9
}; // 8673
```

graphs/gomory_hu.ha520

```
#include "edmonds_karp.h"
#include "push_relabel.h" // if you need

struct Edge {
    int a, b; // vertices
    flow_t w; // weight
}; // c331

// Build Gomory-Hu tree; time: O(n*maxflow)
// Gomory-Hu tree encodes minimum cuts between
// all pairs of vertices: mincut for u and v
// is equal to minimum on path from u and v
// in Gomory-Hu tree. n is vertex count.
// Returns vector of Gomory-Hu tree edges.
vector<Edge> gomoryHu(vector<Edge>& edges,
    int n) {
    MaxFlow flow(n);
    each(e, edges) flow.addEdge(e.a, e.b, e.w, e.w);

    vector<Edge> ret(n-1);
    rep(i, 1, n) ret[i-1] = {i, 0, 0};

    rep(i, 1, n) {
        ret[i-1].w = flow.maxFlow(i, ret[i-1].b);
        rep(j, i+1, n)
            if (ret[j-1].b == ret[i-1].b &&
                flow.cutSide(j)) ret[j-1].b = i;
    } // 5ae4

    return ret;
} // afdb
```

graphs/kth_shortest.h1f40

```
constexpr ll INF = 1e18;

// Eppstein's k-th shortest path algorithm;
// time and space: O((m+k) log (m+k))
struct Eppstein {
    using T = ll; // Type for edge weights
    using Edge = pair<int, T>;

    struct Node {
        int E[2] = {}, s{0};
        Edge x;
    }; // 013b

    T shortest; // Shortest path length
    priority_queue<pair<T, int>> Q;
    vector<Node> P[1];
    Vi h;

    // Initialize shortest path structure for
    // weighted graph G, source s and target t;
    // time: O(m log m)
    Eppstein(vector<vector<Edge>>& G,
        int s, int t) {
        int n = sz(G);
        vector<vector<Edge>> H(n);
        rep(i, 0, n) each(e, G[i]) H[e.x].pb({i, e.y});
    }
```

```
    Vi ord, par(n, -1);
    vector<T> d(n, -INF);
    Q.push({d[t] = 0, t});

    while (!Q.empty()) {
        auto v = Q.top();
        Q.pop();
        if (d[v.y] == v.x) {
            ord.pb(v.y);
            each(e, H[v.y]) if (v.x-e.y > d[e.x]) {
                Q.push({d[e.x] = v.x-e.y, e.x});
                par[e.x] = v.y;
            } // 5895
        } // 1b62
    } // 1a6d

    if ((shortest = -d[s]) >= INF) return;
    h.resize(n);

    each(v, ord) {
        int p = par[v];
        if (p+1) h[v] = h[p];
        each(e, G[v]) if (d[e.x] > -INF) {
            T k = e.y - d[e.x] + d[v];
            if (k || e.x != p)
                h[v] = push(h[v], {e.x, k});
            else
                p = -1;
        } // 5e05
    } // 31b9

    P[0].x.x = s;
    Q.push({0, 0});
} // e00e

int push(int t, Edge x) {
    P.pb(P[t]);
    if (!P[t] = sz(P)-1).s || P[t].x.y >= x.y)
        swap(x, P[t].x);
    if (P[t].s) {
        int i = P[t].E[0], j = P[t].E[1];
        int d = P[i].s > P[j].s;
        int k = push(d ? j : i, x);
        P[t].E[d] = k; // Don't inline k!
    } // 10e1
    P[t].s++;
    return t;
} // a2dc

// Get next shortest path length,
// the first call returns shortest path.
// Returns -1 if there's no more paths;
// time: O(log k), where k is total count
// of nextPath calls.
ll nextPath() {
    if (Q.empty()) return -1;
    auto v = Q.top();
    Q.pop();
    for (int i : P[v.y].E) if (i)
        Q.push({ v.x-P[i].x.y+P[v.y].x.y, i });
    int t = h[P[v.y].x.x];
    if (t) Q.push({ v.x - P[t].x.y, t });
    return shortest - v.x;
} // 08af
}; // 5326
```

graphs/matroids.h55ef

```
// Find largest subset S of [n] such that
// S is independent in both matroid A and B.
// A and B are given by their oracles,
// see example implementations below.
```

```

// Returns vector V such that V[i] = 1 iff
// i-th element is included in found set;
// time: O(r^2*init + r^2*n*add),
// where r is max independent set,
// `init` is max time of oracles init
// and `add` is max time of oracles canAdd.
template<class T, class U>
vector<bool> intersectMatroids(T& A, U& B,
                             int n) {
    vector<bool> ans(n);
    bool ok = 1;

    A.init(ans);
    B.init(ans);
    rep(i, 0, n) if (A.canAdd(i) && B.canAdd(i))
        ans[i] = 1, A.init(ans), B.init(ans);

    while (ok) {
        vector<Vi> G(n);
        vector<bool> good(n);
        queue<int> que;
        Vi prev(n, -1);

        A.init(ans);
        B.init(ans);
        ok = 0;

        rep(i, 0, n) if (!ans[i]) {
            if (A.canAdd(i)) que.push(i), prev[i] = -2;
            good[i] = B.canAdd(i);
        } // 9581

        rep(i, 0, n) if (ans[i]) {
            ans[i] = 0;
            A.init(ans);
            B.init(ans);
            rep(j, 0, n) if (i != j && !ans[j]) {
                if (A.canAdd(j)) G[i].pb(j);
                if (B.canAdd(j)) G[j].pb(i);
            } // bd2a
            ans[i] = 1;
        } // bf3e

        while (!que.empty()) {
            int i = que.front();
            que.pop();

            if (good[i]) {
                ans[i] = 1;
                while (prev[i] >= 0) {
                    ans[i = prev[i]] = 0;
                    ans[i = prev[i]] = 1;
                } // 51c8
                ok = 1;
                break;
            } // 384b

            each(j, G[i]) if (prev[j] == -1)
                que.push(j), prev[j] = i;
        } // 6eb6
    } // e092

    return ans;
} // ae8e

// Matroid where each element has color
// and set is independent iff for each color c
// #elements of color c <= maxAllowed[c].
struct LimOracle {
    Vi color; // color[i] = color of i-th element
    Vi maxAllowed; // Limits for colors
    Vi tmp;

    // Init oracle for independent set S; O(n)

```

```

void init(vector<bool>& S) {
    tmp = maxAllowed;
    rep(i, 0, sz(S)) tmp[color[i]] -= S[i];
} // 4dfb

// Check if S+{k} is independent; time: O(1)
bool canAdd(int k) {
    return tmp[color[k]] > 0;
} // e312
} // 7b5d

// Graphic matroid - each element is edge,
// set is independent iff subgraph is acyclic.
struct GraphOracle {
    vector<Pii> elems; // Ground set: graph edges
    int n; // Number of vertices, indexed [0;n-1]
    Vi par;

    int find(int i) {
        return par[i] == -1 ? i
            : par[i] = find(par[i]);
    } // b8b7

    // Init oracle for independent set S; ~O(n)
    void init(vector<bool>& S) {
        par.assign(n, -1);
        rep(i, 0, sz(S)) if (S[i])
            par[find(elems[i].x)] = find(elems[i].y);
    } // 1827

    // Check if S+{k} is independent; time: ~O(1)
    bool canAdd(int k) {
        return
            find(elems[k].x) != find(elems[k].y);
    } // 8ca4
} // c506

// Co-graphic matroid - each element is edge,
// set is independent iff after removing edges
// from graph number of connected components
// doesn't change.
struct CographOracle {
    vector<Pii> elems; // Ground set: graph edges
    int n; // Number of vertices, indexed [0;n-1]
    vector<Vi> G;
    Vi pre, low;
    int cnt;

    int dfs(int v, int p) {
        pre[v] = low[v] = ++cnt;
        each(e, G[v]) if (e != p)
            low[v] = min(low[v], pre[e] ? dfs(e, v));
        return low[v];
    } // 9d30

    // Init oracle for independent set S; O(n)
    void init(vector<bool>& S) {
        G.assign(n, {});
        pre.assign(n, 0);
        low.resize(n);
        cnt = 0;
        rep(i, 0, sz(S)) if (!S[i]) {
            Pii e = elems[i];
            G[e.x].pb(e.y);
            G[e.y].pb(e.x);
        } // 79a1
        rep(v, 0, n) if (!pre[v]) dfs(v, -1);
    } // 1200

    // Check if S+{k} is independent; time: O(1)
    bool canAdd(int k) {
        Pii e = elems[k];
        return max(pre[e.x], pre[e.y])

```

```

        != max(low[e.x], low[e.y]);
    } // 2550
}; // a5cc

// Matroid equivalent to linear space with XOR
struct XorOracle {
    vector<ll> elems; // Ground set: numbers
    vector<ll> base;

    // Init for independent set S; O(n+r^2)
    void init(vector<bool>& S) {
        base.assign(63, 0);
        rep(i, 0, sz(S)) if (S[i]) {
            ll e = elems[i];
            rep(j, 0, sz(base)) if ((e >> j) & 1) {
                if (!base[j]) {
                    base[j] = e;
                    break;
                } // 1df5
                e ^= base[j];
            } // 8495
        } // 655e
    } // b68c

    // Check if S+{k} is independent; time: O(r)
    bool canAdd(int k) {
        ll e = elems[k];
        rep(i, 0, sz(base)) if ((e >> i) & 1) {
            if (!base[i]) return 1;
            e ^= base[i];
        } // 49d1
        return 0;
    } // 66ff
}; // 4af3

```

graphs/push_relabel.h

2a0c

```

using flow_t = int;
constexpr flow_t INF = 1e9+10;

// Push-relabel algorithm for maximum flow;
// O(V^2*sqrt(E)), but very fast in practice.
// Preflow is not converted to flow!
// UNTESTED
struct MaxFlow {
    struct Edge {
        int to, inv;
        flow_t rem, cap;
    }; // bc77

    vector<basic_string<Edge>> G;
    vector<flow_t> extra;
    Vi hei, arc, prv, nxt, act, bot;
    queue<int> Q;
    int n, high, cut, work;

    // Initialize for n vertices
    MaxFlow(int k = 0) : G(k) {}

    // Add new vertex
    int addVert() {
        G.emplace_back();
        return sz(G)-1;
    } // 907a

    // Add edge between u and v with
    // capacity cap and reverse capacity rcap
    void addEdge(int u, int v,
                flow_t cap, flow_t rcap = 0) {
        G[u].pb({v, sz(G[v]), 0, cap});
        G[v].pb({u, sz(G[u])-1, 0, rcap});
    } // 0f10

    void raise(int v, int h) {

```

```

        if (hei[v] < n)
            prv[nxt[prv[v]] = nxt[v]] = prv[v];
        if ((hei[v] = h) < n) {
            if (extra[v] > 0) {
                bot[v] = act[h], act[h] = v;
                high = max(high, h);
            } // 0ae9
            cut = max(cut, h);
            nxt[v] = nxt[prv[v] = h += n];
            prv[nxt[nxt[h] = v]] = v;
        } // 9bcf
    } // 1c35

    void global(int t) {
        hei.assign(n, n);
        act.assign(n, -1);
        iota(all(prv), 0);
        iota(all(nxt), 0);
        hei[t] = high = cut = work = 0;
        for (Q.push(t); !Q.empty(); Q.pop()) {
            int v = Q.front();
            each(e, G[v])
                if (hei[e.to] == n && G[e.to][e.inv].rem)
                    Q.push(e.to), raise(e.to, hei[v]+1);
        } // 4edc
    } // d78f

    void push(int v, Edge& e) {
        auto f = min(extra[v], e.rem);
        if (f > 0) {
            if (!extra[e.to]) {
                bot[e.to] = act[hei[e.to]];
                act[hei[e.to]] = e.to;
            } // 9d90
            e.rem -= f; G[e.to][e.inv].rem += f;
            extra[v] -= f; extra[e.to] += f;
        } // a86a
    } // 94aa

    void discharge(int v) {
        int h = n;

        auto go = [&](int a, int b) {
            rep(i, a, b) {
                auto& e = G[v][i];
                if (e.rem) {
                    if (hei[v] == hei[e.to]+1) {
                        push(v, e);
                        if (extra[v] <= 0)
                            return arc[v] = i, 0;
                    } else h = min(h, hei[e.to]+1);
                } // 8adf
            } // 6369
            return 1;
        } // 0671

        if (go(arc[v], sz(G[v])) && go(0, arc[v])) {
            int k = hei[v] + n;
            if (nxt[k] == prv[k]) {
                rep(j, k, cut+n+1)
                    while (nxt[j] < n) raise(nxt[j], n);
                cut = k-n-1;
            } else raise(v, h), work++;
        } // dcb2
    } // 4d2f

    // Compute maximum flow from src to dst
    flow_t maxFlow(int src, int dst) {
        extra.assign(n = sz(G), 0);
        hei.assign(n, 0);
        arc.assign(n, 0);

```

```

    prv.resize(n*2);
    nxt.resize(n*2);
    act.resize(n);
    bot.resize(n);
    each(v, G) each(e, v) e.rem = e.cap;

    extra[dst] = -(extra[src] = INF);
    each(e, G[src]) push(src, e);
    global(dst);

    for (; high; high--)
        while (act[high] != -1) {
            int v = act[high];
            act[high] = bot[v];
            if (hei[v] == high) {
                discharge(v);
                if (work > 4*n) global(dst);
            } // 21cf
        } // b0c8

    return extra[dst] + INF;
} // 8e2f

// Get if v belongs to cut component with src
bool cutSide(int v) { return hei[v] >= n; }
}; // 44ea

```

graphs/scc.h 1c43

```

// Tarjan's SCC algorithm; time: O(n+m)
// Usage: SCC scc(graph);
// scc[v] = index of SCC for vertex v
// scc.comps[i] = vertices of i-th SCC
// Components are in reversed topological order
struct SCC : Vi {
    vector<Vi> comps;
    Vi S;

    SCC() {}

    SCC(vector<Vi>& G) : Vi(sz(G), -1), S(sz(G)) {
        rep(i, 0, sz(G)) if (!S[i]) dfs(G, i);
    } // f0fa

    int dfs(vector<Vi>& G, int v) {
        int low = S[v] = sz(S);
        S.pb(v);

        each(e, G[v]) if (at(e) < 0)
            low = min(low, S[e] ? : dfs(G, e));

        if (low == S[v]) {
            comps.pb({});
            rep(i, S[v], sz(S)) {
                at(S[i]) = sz(comps) - 1;
                comps.back().pb(S[i]);
            } // 8ed0
            S.resize(S[v]);
        } // ecc7

        return low;
    } // f3c6
}; // 215e

```

graphs/turbo_matching.h d400

```

// Find maximum bipartite matching; time: ?
// G must be bipartite graph!
// Returns matching size (edge count).
// match[v] = vert matched to v or -1
int matching(vector<Vi>& G, Vi& match) {
    vector<bool> seen;
    int n = 0, k = 1;
    match.assign(sz(G), -1);

    function<int(int)> dfs = [&](int i) {

```

```

        if (seen[i]) return 0;
        seen[i] = 1;
        each(e, G[i]) {
            if (match[e] < 0 || dfs(match[e])) {
                match[i] = e; match[e] = i;
                return 1;
            } // 893d
        } // 9532
        return 0;
    }; // d332

    while (k) {
        seen.assign(sz(G), 0);
        k = 0;
        rep(i, 0, sz(G)) if (match[i] < 0)
            k += dfs(i);
        n += k;
    } // 1128
    return n;
} // 0d38

```

```

// Convert maximum matching to vertex cover
// time: O(n+m)
Vi vertexCover(vector<Vi>& G, Vi& match) {
    Vi ret, col(sz(G)), seen(sz(G));

```

```

    function<void(int, int)> dfs =
        [&](int i, int c) {
            if (col[i]) return;
            col[i] = c + 1;
            each(e, G[i]) dfs(e, !c);
        }; // 1f1b

    function<void(int)> aug = [&](int i) {
        if (seen[i] || col[i] != 1) return;
        seen[i] = 1;
        each(e, G[i]) seen[e] = 1, aug(match[e]);
    }; // 2465

    rep(i, 0, sz(G)) dfs(i, 0);
    rep(i, 0, sz(G)) if (match[i] < 0) aug(i);
    rep(i, 0, sz(G))
        if (seen[i] == col[i] - 1) ret.pb(i);
    return ret;
} // 6f72

```

graphs/weighted_matching.h 8264

```

// Minimum cost bipartite matching; O(n^2*m)
// Input is n x m cost matrix, where n <= m.
// Returns matching weight.
// L[i] = right vertex matched to i-th left
// R[i] = left vertex matched to i-th right
ll hungarian(const vector<vector<ll>>& cost,
              Vi& L, Vi& R) {
    if (cost.empty())
        return L.clear(), R.clear(), 0;
    int b, c = 0, n = sz(cost), m = sz(cost[0]);
    assert(n <= m);

    vector<ll> x(n), y(m + 1);
    L.assign(n, -1);
    R.assign(m + 1, -1);

```

```

    rep(i, 0, n) {
        vector<ll> sla(m, INT64_MAX);
        Vi vis(m + 1), prv(m, -1);
        for (R[b = m] = i; R[b] + 1; b = c) {
            int a = R[b];
            ll d = INT64_MAX;
            vis[b] = 1;
            rep(j, 0, m) if (!vis[j]) {
                ll cur = cost[a][j] - x[a] - y[j];

```

```

                if (cur < sla[j])
                    sla[j] = cur, prv[j] = b;
                if (sla[j] < d) d = sla[j], c = j;
            } // 6717
            rep(j, 0, m + 1) {
                if (vis[j]) x[R[j]] += d, y[j] -= d;
                else sla[j] -= d;
            } // 8bb3
        } // 01c6
        while (b - m) c = b, R[c] = R[b = prv[b]];
    } // 71f5

    rep(j, 0, m) if (R[j] + 1) L[R[j]] = j;
    R.resize(m);
    return -y[m];
} // 349d

```

math/berlekamp_massey.h c714

```

constexpr int MOD = 1e9 + 7;

ll modInv(ll a, ll m) { // a^(-1) mod m
    if (a == 1) return 1;
    return ((a - modInv(m % a, a)) * m + 1) / a;
} // c437

// Find shortest linear recurrence that matches
// given starting terms of recurrence; O(n^2)
// Returns vector C such that for each i >= |C|
// A[i] = sum A[i-j-1]*C[j] for j = 0..|C|-1
// UNTESTED
vector<ll> massey(vector<ll>& A) {
    int n = sz(A), len = 0, k = 0;
    ll s = 1;
    vector<ll> B(n), C(n), tmp;
    B[0] = C[0] = 1;

    rep(i, 0, n) {
        ll d = 0;
        k++;
        rep(j, 0, len + 1)
            d = (d + C[j] * A[i - j]) % MOD;

        if (d) {
            ll q = d * modInv(s, MOD) % MOD;
            tmp = C;

```

```

            rep(j, k, n)
                C[j] = (C[j] - q * B[j - k]) % MOD;

            if (len + 2 <= i) {
                B.swap(tmp);
                len = i - len + 1;
                s = d + (d < 0) * MOD;
                k = 0;
            } // c350
        } // 79c7
    } // f70c

    C.resize(len + 1);
    C.erase(C.begin());
    each(x, C) x = (MOD - x) % MOD;
    return C;
} // 4a08

```

math/bit_gauss.h 13eb

```

constexpr int MAX_COLS = 2048;

// Solve system of linear equations over Z_2
// time: O(n^2*m/W), where W is word size
// - A - extended matrix, rows are equations,
//       columns are variables,
//       m-th column is equation result
//       (A[i][j] - i-th row and j-th column)

```

```

// - ans - output for variables values
// - m - variable count
// Returns 0 if no solutions found, 1 if one,
// 2 if more than 1 solution exist.
int bitGauss(vector<bitset<MAX_COLS>>& A,
             vector<bool>& ans, int m) {
    Vi col;
    ans.assign(m, 0);

    rep(i, 0, sz(A)) {
        int c = int(A[i]._Find_first());
        if (c >= m) {
            if (c == m) return 0;
            continue;
        } // a6bb

        rep(k, i + 1, sz(A)) if (A[k][c]) A[k] ^= A[i];
        swap(A[i], A[sz(col)]);
        col.pb(c);
    } // a953

```

```

    for (int i = sz(col); i--; ) if (A[i][m]) {
        ans[col[i]] = 1;
        rep(k, 0, i) if (A[k][col[i]]) A[k][m].flip();
    } // 4ca1

```

```

    return sz(col) < m ? 2 : 1;
} // 996e

```

math/bit_matrix.h 2e3f

```

using ull = uint64_t;

// Matrix over Z_2 (bits and xor)
// TODO: arithmetic operations
struct BitMatrix {
    vector<ull> M;
    int rows, cols, stride;

    // Create matrix with n rows and m columns
    BitMatrix(int n = 0, int m = 0) {
        rows = n; cols = m;
        stride = (m + 63) / 64;
        M.resize(n * stride);
    } // 7ef0

    // Get pointer to bit-packed data of i-th row
    ull* row(int i) { return M[i * stride]; }

    // Get value in i-th row and j-th column
    bool operator()(int i, int j) {
        return (row(i)[j / 64] >> (j % 64)) & 1;
    } // 28bd

    // Set value in i-th row and j-th column
    void set(int i, int j, bool val) {
        ull &w = row(i)[j / 64], m = 1ull << (j % 64);
        if (val) w |= m;
        else w &= ~m;
    } // 98a8
}; // 4df7

```

math/crt.h 8a85

```

using Pll = pair<ll, ll>;

ll egcd(ll a, ll b, ll& x, ll& y) {
    if (!a) return x = 0, y = 1, b;
    ll d = egcd(b % a, a, y, x);
    x -= b / a * y;
    return d;
} // 23c8

// Chinese Remainder Theorem; time: O(lg lcm)
// Solves x = a.x (mod a.y), x = b.x (mod b.y)
// Returns pair (x mod lcm, lcm(a.y, b.y))

```

```
// or (-1, -1) if there's no solution.
// WARNING: a.x and b.x are assumed to be
// in [0;a.y) and [0;b.y) respectively.
// Works properly if lcm(a.y, b.y) < 2^63.
Pll crt(Pll a, Pll b) {
    if (a.y < b.y) swap(a, b);
    ll x, y, g = egcd(a.y, b.y, x, y);
    ll c = b.x-a.x, d = b.y/g, p = a.y*d;
    if (c % g) return {-1, -1};
    ll s = (a.x + c/g*x % d * a.y) % p;
    return {s < 0 ? s+p : s, p};
} // 35a8
```

math/fft_complex.h

31ee

```
using dbl = double;
using cmpl = complex<dbl>;

// Default std::complex multiplication is slow.
// You can use this to achieve small speedup.
cmpl operator*(cmpl a, cmpl b) {
    dbl ax = real(a), ay = imag(a);
    dbl bx = real(b), by = imag(b);
    return {ax*bx-ay*by, ax*by+ay*bx};
} // 3b78

cmpl operator==(cmpl &a, cmpl b) {return a==b;}

// Compute DFT over complex numbers; O(n lg n)
// Input size must be power of 2!
void fft(vector<cmpl>& a) {
    static vector<cmpl> w(2, 1);
    int n = sz(a);

    for (int k = sz(w); k < n; k *= 2) {
        w.resize(n);
        rep(i, 0, k) w[k+i] = exp(cmpl(0, M_PI*i/k));
    } // 92a9

    Vi rev(n);
    rep(i, 0, n) rev[i] = (rev[i/2] | i%2*n) / 2;
    rep(i, 0, n) if(i < rev[i]) swap(a[i], a[rev[i]]);

    for (int k = 1; k < n; k *= 2) {
        for (int i = 0; i < n; i += k*2) rep(j, 0, k) {
            auto d = a[i+j+k] * w[j+k];
            a[i+j+k] = a[i+j] - d;
            a[i+j] += d;
        } // b389
    } // 84bf
} // adf8
```

```
// Convolve complex-valued a and b,
// store result in a; time: O(n lg n), 3x FFT
void convolve(vector<cmpl>& a, vector<cmpl> b) {
    int len = max(sz(a) + sz(b) - 1, 0);
    int n = 1 << (32 - __builtin_clz(len));
    a.resize(n); b.resize(n);
    fft(a); fft(b);
    rep(i, 0, n) a[i] *= b[i] / dbl(n);
    reverse(a.begin()+1, a.end());
    fft(a);
    a.resize(len);
} // 7987
```

```
// Convolve real-valued a and b, returns result
// time: O(n lg n), 2x FFT
// Rounding to integers is safe as long as
// (max_coeff^2)*n*log_2(n) < 9*10^14
// (in practice 10^16 or higher).
vector<dbl> convolve(vector<dbl>& a,
    vector<dbl>& b) {
    int len = max(sz(a) + sz(b) - 1, 0);
```

```
int n = 1 << (32 - __builtin_clz(len));
vector<cmpl> in(n), out(n);
rep(i, 0, sz(a)) in[i].real(a[i]);
rep(i, 0, sz(b)) in[i].imag(b[i]);

fft(in);
each(x, in) x *= x;
rep(i, 0, n) out[i] = in[-i&(n-1)]-conj(in[i]);
fft(out);

vector<dbl> ret(len);
rep(i, 0, len) ret[i] = imag(out[i]) / (n*4);
return ret;
} // 19ed
```

constexpr ll MOD = 1e9+7;

```
// High precision convolution of integer-valued
// a and b mod MOD; time: O(n lg n), 4x FFT
// Input is expected to be in range [0;MOD)!
// Rounding is safe if MOD*n*log_2(n) < 9*10^14
// (in practice 10^16 or higher).
vector<ll> convMod(vector<ll>& a,
    vector<ll>& b) {
    vector<ll> ret(sz(a) + sz(b) - 1);
    int n = 1 << (32 - __builtin_clz(sz(ret)));
    ll cut = ll(sqrt(MOD))+1;

    vector<cmpl> c(n), d(n), g(n), f(n);

    rep(i, 0, sz(a))
        c[i] = {dbl(a[i]/cut), dbl(a[i]%cut)};
    rep(i, 0, sz(b))
        d[i] = {dbl(b[i]/cut), dbl(b[i]%cut)};

    fft(c); fft(d);

    rep(i, 0, n) {
        int j = -i & (n-1);
        f[j] = (c[i]+conj(c[j])) * d[i] / (n*2.0);
        g[j] = (c[i]-conj(c[j])) * d[i] / cmpl(0, n*2);
    } // e877

    fft(f); fft(g);

    rep(i, 0, sz(ret)) {
        ll t = llround(real(f[i])) % MOD * cut;
        t += llround(imag(f[i]));
        t = (t + llround(real(g[i]))) % MOD * cut;
        t = (t + llround(imag(g[i]))) % MOD;
        ret[i] = (t < 0 ? t+MOD : t);
    } // e75d

    return ret;
} // 513f
```

math/fft_mod.h

17aa

```
// Number Theoretic Transform (NTT)
// For functions below you can choose 2 params:
// 1. M - prime modulus that MUST BE of form
//    a*2^k+1, computation is done in Z_M
// 2. R - generator of Z_M

// Modulus often seen on Codeforces:
// M = (119<<23)+1, R = 62; M is 998244353

// Parameters for ll computation with CRT:
// M = (479<<21)+1, R = 62; M is > 10^9
// M = (483<<21)+1, R = 62; M is > 10^9

ll modPow(ll a, ll e, ll m) {
    ll t = 1 % m;
    while (e) {
        if (e % 2) t = t*a % m;
```

```
    e /= 2; a = a*a % m;
    } // 66ca
    return t;
} // 1973

// Compute DFT over Z_M with generator R.
// Input size must be power of 2; O(n lg n)
// Input is expected to be in range [0;MOD)!
// dit == true <=> inverse transform * 2^n
// (without normalization)
template<ll M, ll R, bool dit>
void ntt(vector<ll>& a) {
    static vector<ll> w(2, 1);
    int n = sz(a);

    for (int k = sz(w); k < n; k *= 2) {
        w.resize(n, 1);
        ll c = modPow(R, M/2/k, M);
        if (dit) c = modPow(c, M-2, M);
        rep(i, k+1, k*2) w[i] = w[i-1]*c % M;
    } // 0d98

    for (int t = 1; t < n; t *= 2) {
        int k = (dit ? t : n/t/2);
        for (int i = 0; i < n; i += k*2) rep(j, 0, k) {
            ll &c = a[i+j], &d = a[i+j+k];
            ll e = w[j+k], f = d;
            d = (dit ? c - (f*f*e%M) : (c-f)*e % M);
            if (d < 0) d += M;
            if ((c += f) >= M) c -= M;
        } // e4a6
    } // 8d38
} // 01f5

// Convolve a and b mod M (R is generator),
// store result in a; time: O(n lg n), 3x NTT
// Input is expected to be in range [0;MOD)!
template<ll M = (119<<23)+1, ll R = 62>
void convolve(vector<ll>& a, vector<ll> b) {
    int len = max(sz(a) + sz(b) - 1, 0);
    int n = 1 << (32 - __builtin_clz(len));
    ll t = modPow(n, M-2, M);
    a.resize(n); b.resize(n);
    ntt<M, R, 0>(a); ntt<M, R, 0>(b);
    rep(i, 0, n) a[i] = a[i]*b[i] % M * t % M;
    ntt<M, R, 1>(a);
    a.resize(len);
} // 4b4d

ll egcd(ll a, ll b, ll& x, ll& y) {
    if (!a) return x=0, y=1, b;
    ll d = egcd(b%a, a, y, x);
    x -= b/a*y;
    return d;
} // 23c8
```

```
// Convolve a and b with 64-bit output,
// store result in a; time: O(n lg n), 6x NTT
// Input is expected to be non-negative!
void convLong(vector<ll>& a, vector<ll> b) {
    const ll M1 = (479<<21)+1, M2 = (483<<21)+1;
    const ll MX = M1*M2, R = 62;

    vector<ll> c = a, d = b;
    each(k, a) k %= M1; each(k, b) k %= M1;
    each(k, c) k %= M2; each(k, d) k %= M2;
```

```
convolve<M1, R>(a, b);
convolve<M2, R>(c, d);

ll x, y; egcd(M1, M2, x, y);

rep(i, 0, sz(a)) {
```

```
    a[i] += (c[i]-a[i])*x % M2 * M1;
    if ((a[i] %= MX) < 0) a[i] += MX;
} // 2279
} // ef93
```

math/fwht.h

3e6f

```
// Fast Walsh-Hadamard Transform; O(n lg n)
// Input must be power of 2!
// Uncommented version is for XOR.
// OR version is equivalent to sum-over-subsets
// (Zeta transform, inverse is Moebius).
// AND version is same as sum-over-supersets.
// TESTED ON RANDS
template<bool inv, class T>
void fwht(vector<T>& b) {
    for (int s = 1; s < sz(b); s *= 2) {
        for (int i = 0; i < sz(b); i += s*2) {
            rep(j, i, i+s) {
                auto &x = b[j], &y = b[j+s];
                tie(x, y) =
                    mp(x+y, x-y); //XOR
                // inv ? mp(x-y, y) : mp(x+y, y); //AND
                // inv ? mp(x, y-x) : mp(x, x+y); //OR
            } // eea9
        } // a3d5
    } // 95ed

    // ONLY FOR XOR:
    if (inv) each(e, b) e /= sz(b);
} // 0779
```

```
// Compute convolution of a and b such that
// ans[i#j] += a[i]*b[j], where # is OR, AND
// or XOR, depending on FWHT version.
// Stores result in a; time: O(n lg n)
// Both arrays must be of same size = 2^n!
template<class T>
void bitConv(vector<T>& a, vector<T> b) {
    fwht<0>(a);
    fwht<0>(b);
    rep(i, 0, sz(a)) a[i] *= b[i];
    fwht<1>(a);
} // 7b82
```

math/gauss.h

7f0b

```
// Solve system of linear equations; O(n^2*m)
// - A - extended matrix, rows are equations,
//    columns are variables,
//    m-th column is equation result
//    (A[i][j] - i-th row and j-th column)
// - ans - output for variables values
// - m - variable count
// Returns 0 if no solutions found, 1 if one,
// 2 if more than 1 solution exist.
int gauss(vector<vector<double>>& A,
    vector<double>& ans, int m) {
    Vi col;
    ans.assign(m, 0);

    rep(i, 0, sz(A)) {
        int c = 0;
        while (c <= m && !cmp(A[i][c], 0)) c++;
        // For Zp:
        //while (c <= m && !A[i][c].x) c++;

        if (c >= m) {
            if (c == m) return 0;
            continue;
        } // a6bb

        rep(k, i+1, sz(A)) {
```



```
auto mult = A[k][c] / A[i][c];
rep(j, 0, m+1) A[k][j] -= A[i][j]*mult;
} // 8dd5

swap(A[i], A[sz(col)]);
col.pb(c);
} // ea2c

for (int i = sz(col); i--;) {
    ans[col[i]] = A[i][m] / A[i][col[i]];
    rep(k, 0, i)
        A[k][m] -= ans[col[i]] * A[k][col[i]];
} // 31b9

return sz(col) < m ? 2 : 1;
} // 0b76
```

math/matrix.h

9bf7

```
#include "modular.h"

// UNTESTED
using Row = vector<Zp>;
using Matrix = vector<Row>;

// Create n x n identity matrix
Matrix ident(int n) {
    Matrix ret(n, Row(n));
    rep(i, 0, n) ret[i][i] = 1;
    return ret;
} // ad1d

// Add matrices
Matrix& operator+=(Matrix& l, const Matrix& r){
    rep(i, 0, sz(l)) rep(k, 0, sz(l[0]))
        l[i][k] += r[i][k];
    return l;
} // b6bf

Matrix operator+(Matrix l, const Matrix& r) {
    return l += r;
} // d9b3

// Subtract matrices
Matrix& operator-=(Matrix& l, const Matrix& r){
    rep(i, 0, sz(l)) rep(k, 0, sz(l[0]))
        l[i][k] -= r[i][k];
    return l;
} // 90a1

Matrix operator-(Matrix l, const Matrix& r) {
    return l -= r;
} // dc4f

// Multiply matrices
Matrix operator*(const Matrix& l,
    const Matrix& r) {
    Matrix ret(sz(l), Row(sz(r[0])));
    rep(i, 0, sz(l)) rep(j, 0, sz(r[0]))
        rep(k, 0, sz(r))
            ret[i][j] += l[i][k] * r[k][j];
    return ret;
} // 52ca

Matrix& operator**=(Matrix& l, const Matrix& r){
    return l = l*r;
} // da8a

// Square matrix power; time: O(n^3 * lg e)
Matrix pow(Matrix a, ll e) {
    Matrix t = ident(sz(a));
    while (e) {
        if (e % 2) t *= a;
        e /= 2; a *= a;
    } // 4400
    return t;
} // 65ea
```

```
// Transpose matrix
Matrix transpose(const Matrix& m) {
    Matrix ret(sz(m[0]), Row(sz(m)));
    rep(i, 0, sz(m)) rep(j, 0, sz(m[0]))
        ret[j][i] = m[i][j];
    return ret;
} // 5650

// Transform matrix to echelon form
// and compute its determinant sign and rank.
int echelon(Matrix& A, int& sign) { // O(n^3)
    int rank = 0;
    sign = 1;
    rep(c, 0, sz(A[0])) {
        if (rank >= sz(A)) break;
        rep(i, rank+1, sz(A)) if (A[i][c].x) {
            swap(A[i], A[rank]);
            sign *= -1;
            break;
        } // f98a
        if (A[rank][c].x) {
            rep(i, rank+1, sz(A)) {
                auto mult = A[i][c] / A[rank][c];
                rep(j, 0, sz(A[0]))
                    A[i][j] -= A[rank][j]*mult;
            } // f519
            rank++;
        } // 4cd8
    } // 36e9
    return rank;
} // 6882

// Compute matrix rank; time: O(n^3)
#define rank_rank_
int rank(Matrix A) {
    int s; return echelon(A, s);
} // c599
```

```
// Compute square matrix determinant; O(n^3)
Zp det(Matrix A) {
    int s; echelon(A, s);
    Zp ret = s;
    rep(i, 0, sz(A)) ret *= A[i][i];
    return ret;
} // b252

// Invert square matrix if possible; O(n^3)
// Returns true if matrix is invertible.
bool invert(Matrix& A) {
    int s, n = sz(A);
    rep(i, 0, n) A[i].resize(n*2), A[i][n+i] = 1;
    echelon(A, s);
    for (int i = n; i--;) {
        if (!A[i][i].x) return 0;
        auto mult = A[i][i].inv();
        each(k, A[i]) k *= mult;
        rep(k, 0, i) rep(j, 0, n)
            A[k][n+j] -= A[i][n+j]*A[k][i];
    } // 1e97
    each(r, A) r.erase(r.begin(), r.begin()+n);
    return 1;
} // 65b9
```

math/miller_rabin.h

7005

```
#include "modular64.h"

// Miller-Rabin primality test
// time O(k*lg^2 n), where k = number of bases
// Deterministic for p <= 10^9
```

```
// constexpr ll BASES[] = {
//     336781006125, 9639812373923155
// }; // d41d

// Deterministic for p <= 2^64
constexpr ll BASES[] = {
    2,325,9375,28178,450775,9780504,1795265022
}; // b8e0

bool isPrime(ll p) {
    if (p <= 2) return p == 2;
    if (p%2 == 0) return 0;

    ll d = p-1, t = 0;
    while (d%2 == 0) d /= 2, t++;

    each(a, BASES) if (a%p) {
        // ll a = rand() % (p-1) + 1;
        ll b = modPow(a%p, d, p);
        if (b == 1 || b == p-1) continue;
        rep(i, 1, t)
            if ((b = modMul(b, b, p)) == p-1) break;
        if (b != p-1) return 0;
    } // 9342

    return 1;
} // bec2
```

math/modinv_precompute.h

2882

```
constexpr ll MOD = 234567899;
vector<ll> modInv(MOD); // You can lower size

// Precompute modular inverses; time: O(MOD)
void initModInv() {
    modInv[1] = 1;
    rep(i, 2, sz(modInv)) modInv[i] =
        (MOD - (MOD/i) * modInv[MOD%i]) % MOD;
} // 22c1
```

math/modular.h

71e5

```
// Modulus often seen on Codeforces:
constexpr int MOD = 998244353;
// Some big prime: 15*(1<27)+1 ~ 2*10^9

ll modInv(ll a, ll m) { // a^(-1) mod m
    if (a == 1) return 1;
    return ((a - modInv(m%a, a))*m + 1) / a;
} // c437

ll modPow(ll a, ll e, ll m) { // a^e mod m
    ll t = 1 % m;
    while (e) {
        if (e % 2) t = t*a % m;
        e /= 2; a = a*a % m;
    } // 66ca
    return t;
} // 1973

// Wrapper for modular arithmetic
struct Zp {
    ll x; // Contained value, in range [0;MOD-1]
    Zp() : x(0) {}
    Zp(ll a) : x(a%MOD) { if (x < 0) x += MOD; }

    #define OP(c,d) Zp& operator c##=(Zp r) { \
        x = x d; return *this; } \
    Zp operator c(Zp r) const { \
        Zp t = *this; return t c##= r; } // e986

    OP(+, +r.x - MOD*(x+r.x >= MOD));
    OP(-, -r.x + MOD*(x-r.x < 0));
    OP(*, *r.x % MOD);
    OP(/, *r.inv().x % MOD);
    Zp operator-()
```

```
    const { Zp t; t.x = MOD-x; return t; }

    // For composite modulus use modInv, not pow
    Zp inv() const { return pow(MOD-2); }
    Zp pow(ll e) const { return modPow(x,e,MOD); }
    void print() { cerr << x; } // For deb()
}; // 407f

// Extended Euclidean Algorithm
ll egcd(ll a, ll b, ll& x, ll& y) {
    if (!a) return x=0, y=1, b;
    ll d = egcd(b%a, a, y, x);
    x -= b/a*y;
    return d;
} // 23c8
```

math/modular64.h

4b73

```
// Modular arithmetic for modulus < 2^62

ll modAdd(ll x, ll y, ll m) {
    x += y;
    return x < m ? x : x-m;
} // b653

ll modSub(ll x, ll y, ll m) {
    x -= y;
    return x >= 0 ? x : x+m;
} // b073

// About 4x slower than normal modulo
ll modMul(ll a, ll b, ll m) {
    ll c = ll((long double)a * b / m);
    ll r = (a*b - c*m) % m;
    return r < 0 ? r+m : r;
} // 1815

ll modPow(ll x, ll e, ll m) {
    ll t = 1;
    while (e) {
        if (e & 1) t = modMul(t, x, m);
        e >>= 1;
        x = modMul(x, x, m);
    } // bd61
    return t;
} // c8ba
```

math/modular_generator.h

845b

```
#include "modular.h" // modPow

// Get unique prime factors of n; O(sqrt n)
vector<ll> factorize(ll n) {
    vector<ll> fac;
    for (ll i = 2; i*i <= n; i++) {
        if (n%i == 0) {
            while (n%i == 0) n /= i;
            fac.pb(i);
        } // 6069
    } // a0cc
    if (n > 1) fac.pb(n);
    return fac;
} // 4a2a

// Find smallest primitive root mod n;
// time: O(sqrt(n) + g*log^2 n)
// Returns -1 if generator doesn't exist.
// For n <= 10^7 smallest generator is <= 115.
// You can use faster factorization algorithm
// to get rid of sqrt(n). [UNTESTED]
ll generator(ll n) {
    if (n <= 1 || (n > 4 && n%4 == 0)) return -1;

    vector<ll> fac = factorize(n);
    if (sz(fac) > (fac[0] == 2)+1) return -1;
```

```
ll phi = n;
each(p, fac) phi = phi / p * (p-1);
fac = factorize(phi);

for (ll g = 1; g++; if (__gcd(g, n) == 1) {
    each(f, fac) if (modPow(g, phi/f, n) == 1)
        goto nxt;
    return g;
nxt;;
} // db24
} // 7641
```

math/modular_log.h

ac62

```
#include "modular.h" // modInv

// Baby-step giant-step algorithm; O(sqrt(p))
// Finds smallest x such that a^x = b (mod p)
// or returns -1 if there's no solution.
ll dlog(ll a, ll b, ll p) {
    int m = int(min(ll(sqrt(p))+2, p-1));
    unordered_map<ll, int> small;
    ll t = 1;

    rep(i, 0, m) {
        int& k = small[t];
        if (!k) k = i+1;
        t = t*a % p;
    } // f1d0

    t = modInv(t, p);

    rep(i, 0, m) {
        int j = small[b];
        if (j) return i*ll(m) + j - 1;
        b = b*t % p;
    } // c7ed

    return -1;
} // 5c26
```

math/modular_sqrt.h

db16

```
#include "modular.h" // modPow

// Tonelli-Shanks algorithm for modular sqrt
// modulo prime; O(lg^2 p), O(lg p) for most p
// Returns -1 if root doesn't exists or else
// returns square root x (the other one is -x).
// UNTESTED
ll modSqrt(ll a, ll p) {
    a %= p;
    if (a < 0) a += p;
    if (a <= 1) return a;
    if (modPow(a, p/2, p) != 1) return -1;
    if (p%4 == 3) return modPow(a, p/4+1, p);

    ll s = p-1, n = 2;
    int r = 0, j;
    while (s%2 == 0) s /= 2, r++;
    while (modPow(n, p/2, p) != p-1) n++;

    ll x = modPow(a, (s+1)/2, p);
    ll b = modPow(a, s, p), g = modPow(n, s, p);

    for (; r = j) {
        ll t = b;
        for (j = 0; j < r && t != 1; j++)
            t = t*t % p;
        if (!j) return x;
        ll gs = modPow(g, 1LL << (r-j-1), p);
        g = gs*gs % p;
        x = x*gs % p;
        b = b*g % p;
```

```
} // f83f
} // 7a97

math/montgomery.h a4ba

#include "modular.h" // modInv

// Montgomery modular multiplication
// MOD < MG_MULT, gcd(MG_MULT, MOD) must be 1
// Don't use if modulo is constexpr; UNTESTED

constexpr ll MG_SHIFT = 32;
constexpr ll MG_MULT = 1LL << MG_SHIFT;
constexpr ll MG_MASK = MG_MULT - 1;
const ll MG_INV = MG_MULT-modInv(MOD, MG_MULT);

// Convert to Montgomery form
ll MG(ll x) { return (x*MG_MULT) % MOD; }

// Montgomery reduction
// redc(mg * mg) = Montgomery-form product
ll redc(ll x) {
    ll q = (x * MG_INV) & MG_MASK;
    x = (x + q*MOD) >> MG_SHIFT;
    return (x >= MOD ? x-MOD : x);
} // d0f5
```

math/number.h

474f

```
// Nimbers are defined as sizes of Nim heaps.
// Operations on numbers are defined as:
// a+b = mex({a'+b : a' < a} u {a+b' : b' < b})
// ab = mex({a'b'+ab'+a'b' : a' < a, b' < b})
// Nimbers smaller than M = 2^2^k form a field.
// Addition is equivalent to xor, meanwhile
// multiplication can be evaluated
// in O(lg^2 M) after precomputing.

using ull = uint64_t;
ull nbuf[64][64]; // Nim-products for 2^i * 2^j

// Multiply nimbers; time: O(lg^2 M)
// WARNING: Call initNimMul() before using.
ull nimMul(ull a, ull b) {
    ull ret = 0;
    for (ull s = a; s; s &= (s-1))
        for (ull t = b; t; t &= (t-1))
            ret ^= nbuf[__builtin_ctzll(s)]
                [__builtin_ctzll(t)];
    return ret;
} // 25be

// Initialize nim-products lookup table
void initNimMul() {
    rep(i, 0, 64)
        nbuf[i][0] = nbuf[0][i] = 1ull << i;
    rep(b, 1, 64) rep(a, 1, b+1) {
        int i = 1 << (63 - __builtin_clzll(a));
        int j = 1 << (63 - __builtin_clzll(b));
        ull t = nbuf[a-i][b-j];
        if (i < j)
            t = nimMul(t, 1ull << i) << j;
        else
            t = nimMul(t, 1ull << (i-1)) ^ (t << i);
        nbuf[a][b] = nbuf[b][a] = t;
    } // ca24
} // 1811

// Compute a^e under nim arithmetic; O(lg^3 M)
// WARNING: Call initNimMul() before using.
ull nimPow(ull a, ull e) {
    ull t = 1;
    while (e) {
        if (e % 2) t = nimMul(t, a);
```

```
        e /= 2; a = nimMul(a, a);
    } // da53
    return t;
} // c06c

// Compute inverse of a in 2^64 nim-field;
// time: O(lg^3 M)
// WARNING: Call initNimMul() before using.
ull nimInv(ull a) {
    return nimPow(a, ull(-2));
} // c6d9

// If you need to multiply many nimbers by
// the same value you can use this to speedup.
// WARNING: Call initNimMul() before using.
struct NimMult {
    ull M[64] = {0};

    // Initialize lookup; time: O(lg^2 M)
    NimMult(ull a) {
        for (ull t=a; t; t &= (t-1)) rep(i, 0, 64)
            M[i] ^= nbuf[__builtin_ctzll(t)][i];
    } // ea88

    // Multiply by b; time: O(lg M)
    ull operator()(ull b) {
        ull ret = 0;
        for (ull t = b; t; t &= (t-1))
            ret ^= M[__builtin_ctzll(t)];
        return ret;
    } // e480
}; // 1b80
```

math/phi_large.h

8703

```
#include "pollard_rho.h"

// Compute Euler's totient of large numbers
// time: O(n^(1/4)) <- factorization
ll phi(ll n) {
    each(p, factorize(n)) n = n / p.x * (p.x-1);
    return n;
} // 798e

math/phi_precompute.h 728b

Vi phi(1e7+1);

// Precompute Euler's totients; time: O(n lg n)
void calcPhi() {
    iota(all(phi), 0);
    rep(i, 2, sz(phi)) if (phi[i] == i)
        for (int j = i; j < sz(phi); j += i)
            phi[j] = phi[j] / i * (i-1);
} // 3c65
```

math/phi_prefix_sum.h

a9e0

```
#include "phi_precompute.h"

vector<ll> phiSum; // [k] = sum from 0 to k-1

// Precompute Euler's totient prefix sums
// for small values; time: O(n lg n)
void calcPhiSum() {
    calcPhi();
    phiSum.resize(sz(phi)+1);
    rep(i, 0, sz(phi))
        phiSum[i+1] = phiSum[i] + phi[i];
} // 3855

// Get prefix sum of phi(0) + ... + phi(n-1).
// WARNING: Call calcPhiSum first!
// For n > 4*10^9, answer will overflow.
// If you wish to get answer mod M use
// commented lines.
```

```
ll getPhiSum(ll n) { // time: O(n^(2/3))
    static unordered_map<ll, ll> big;
    if (n < sz(phiSum)) return phiSum[n];
    if (big.count(--n)) return big[n];

    ll ret = n*(n+1)/2;
    // ll ret = (n%2 ? n%M * ((n+1)/2 % M)
    //           : n/2%M * (n%M+1) % M;

    for (ll s, i = 2; i <= n; i = s+1) {
        s = n / (n/i);
        ret -= (s-i+1) * getPhiSum(n/i+1);
        // ret -= (s-i+1)%M * getPhiSum(n/i+1) % M;
    } // e792

    // ret = (ret%M + M) % M;
    return big[n] = ret;
} // 820b
```

math/pi_large.h

fcbd

```
constexpr int MAX_P = 1e7;
vector<ll> pis, prl;

// Precompute prime counting function
// for small values; time: O(n lg lg n)
void initPi() {
    pis.assign(MAX_P+1, 1);
    pis[0] = pis[1] = 0;

    for (int i = 2; i*i <= MAX_P; i++)
        if (pis[i])
            for (int j = i*i; j <= MAX_P; j += i)
                pis[j] = 0;

    rep(i, 1, sz(pis)) {
        if (pis[i]) prl.pb(i);
        pis[i] += pis[i-1];
    } // 0672
} // 6d92

ll partial(ll x, ll a) {
    static vector<unordered_map<ll, ll>> big;
    big.resize(sz(prl));
    if (!a) return (x+1) / 2;
    if (big[a].count(x)) return big[a][x];
    ll ret = partial(x, a-1)
        - partial(x / prl[a], a-1);
    return big[a][x] = ret;
} // 774f

// Count number of primes <= x;
// time: O(n^(2/3) * log(n)^(1/3))
// Set MAX_P to be > sqrt(x) and call initPi
// before using!
ll pi(ll x) {
    static unordered_map<ll, ll> big;
    if (x < sz(pis)) return pis[x];
    if (big.count(x)) return big[x];

    ll a = 0;
    while (prl[a]*prl[a]*prl[a]*prl[a] < x) a++;

    ll ret = 0, b = --a;

    while (++b < sz(prl) && prl[b]*prl[b] < x) {
        ll w = x / prl[b];
        ret -= pi(w);
        for (ll j = b; prl[j]*prl[j] <= w; j++)
            ret -= pi(w / prl[j]) - j;
    } // a584

    ret += partial(x, a) + (b+a-1)*(b-a)/2;
    return big[x] = ret;
} // eald
```

math/pi_large_precomp.h

7fc0

```
#include "sieve.h"

// Count primes in given interval
// using precomputed table.
// Set MAX_P to sqrt(MAX_N) and run sieve()!
// Precomputed table will contain N_BUCKETS
// elements - check source size limit.

constexpr ll MAX_N = 1e11+1;
constexpr ll N_BUCKETS = 10000;
constexpr ll BUCKET_SIZE = (MAX_N/N_BUCKETS)+1;
constexpr ll precomputed[] = { /* ... */ };

ll sieveRange(ll from, ll to) {
    bitset<BUCKET_SIZE> elems;
    from = max(from, 2LL);
    to = max(from, to);
    each(p, primesList) {
        ll c = max((from+p-1) / p, 2LL);
        for (ll i = c*p; i < to; i += p)
            elems.set(i-from);
    } // a29f
    return to-from-elems.count();
} // c646

// Run once on local computer to precompute
// table. Takes about 10 minutes for n = 1e11.
// Sanity check (for default params):
// 664579, 606028, 587253, 575795, ...
void localPrecompute() {
    for (ll i = 0; i < MAX_N; i += BUCKET_SIZE) {
        ll to = min(i+BUCKET_SIZE, MAX_N);
        cout << sieveRange(i, to) << ' ' << flush;
    } // f6a7
    cout << endl;
} // 2b1e

// Count primes in [from;to) using table.
// O(N_BUCKETS + BUCKET_SIZE*lg lg n + sqrt(n))
ll countPrimes(ll from, ll to) {
    ll bFrom = from/BUCKET_SIZE+1,
        bTo = to/BUCKET_SIZE;
    if (bFrom > bTo) return sieveRange(from, to);
    ll ret = accumulate(precomputed+bFrom,
        precomputed+bTo, 0);
    ret += sieveRange(from, bFrom*BUCKET_SIZE);
    ret += sieveRange(bTo*BUCKET_SIZE, to);
    return ret;
} // cced
```

math/pollard_rho.h

ef01

```
#include "modular64.h"
#include "miller_rabin.h"

using Factor = pair<ll, int>;

void rho(vector<ll>& out, ll n) {
    if (n <= 1) return;
    if (isPrime(n)) out.pb(n);
    else if (n%2 == 0) rho(out, 2), rho(out, n/2);
    else for (ll a = 2; a++) {
        ll x = 2, y = 2, d = 1;
        while (d == 1) {
            x = modAdd(modMul(x, x, n), a, n);
            y = modAdd(modMul(y, y, n), a, n);
            y = modAdd(modMul(y, y, n), a, n);
            d = __gcd(abs(x-y), n);
        } // 3378
        if (d != n) return rho(out, d), rho(out, n/d);
    } // 047e
```

```
// ba89
// Pollard's rho factorization algorithm
// Las Vegas version; time: n^(1/4)
// Returns pairs (prime, power), sorted
vector<Factor> factorize(ll n) {
    vector<Factor> ret;
    vector<ll> raw;
    rho(raw, n);
    sort(all(raw));
    each(f, raw) {
        if (ret.empty() || ret.back().x != f)
            ret.pb({f, 1});
        else
            ret.back().y++;
    } // 2ab1
    return ret;
} // 471c
```

math/polynomial.h

c486

```
#include "modular.h"
#include "fft_mod.h"

// UNTESTED
using Poly = vector<Zp>;

// Cut off trailing zeroes; time: O(n)
void norm(Poly& P) {
    while (!P.empty() && !P.back().x)
        P.pop_back();
} // 8a8a

// Evaluate polynomial at x; time: O(n)
Zp eval(const Poly& P, Zp x) {
    Zp n = 0, y = 1;
    each(a, P) n += a*y, y *= x;
    return n;
} // d865

// Add polynomial; time: O(n)
Poly& operator+=(Poly& l, const Poly& r) {
    l.resize(max(sz(l), sz(r)));
    rep(i, 0, sz(r)) l[i] += r[i];
    norm(l);
    return l;
} // 656e
Poly operator+(Poly l, const Poly& r) {
    return l += r;
} // d9b3

// Subtract polynomial; time: O(n)
Poly& operator-=(Poly& l, const Poly& r) {
    l.resize(max(sz(l), sz(r)));
    rep(i, 0, sz(r)) l[i] -= r[i];
    norm(l);
    return l;
} // c68b
Poly operator-(Poly l, const Poly& r) {
    return l -= r;
} // dc4f

// Multiply by polynomial; time: O(n lg n)
Poly& operator*=(Poly& l, const Poly& r) {
    if (min(sz(l), sz(r)) < 50) {
        // Naive multiplication
        Poly P(sz(l)+sz(r));
        rep(i, 0, sz(l)) rep(j, 0, sz(r))
            P[i+j] += l[i]*r[j];
        l.swap(P);
    } else {
        // FFT multiplication
        vector<ll> a, b;
```

```
each(k, l) a.pb(k.x);
each(k, r) b.pb(k.x);
// Choose appropriate convolution method,
// see fft_mod.h and fft_complex.h
convolve<MOD, 62>(a, b);
l.assign(all(a));
} // f730
norm(l);
return l;
} // 28de
Poly operator*(Poly l, const Poly& r) {
    return l *= r;
} // 2de3

// Derivate polynomial; time: O(n)
Poly derivate(Poly P) {
    if (!P.empty()) {
        rep(i, 1, sz(P)) P[i-1] = P[i]*i;
        P.pop_back();
    } // bd78
    return P;
} // c6c5

// Integrate polynomial; time: O(n)
Poly integrate(Poly P) {
    if (!P.empty()) {
        P.pb(0);
        for (int i = sz(P); --i;) P[i] = P[i-1]/i;
        P[0] = 0;
    } // eec1
    return P;
} // e2f3

// Compute inverse series mod x^n; O(n lg n)
Poly invert(const Poly& P, int n) {
    assert(!P.empty() && P[0].x != 0);
    Poly tmp, ret = {P[0].inv()};

    for (int i = 1; i < n; i += 2) {
        tmp.clear();
        rep(j, 0, min(i*2, sz(P))) tmp.pb(-P[j]);
        tmp *= ret;
        tmp[0] += 2;
        ret *= tmp;
        ret.resize(i*2);
    } // 139b

    ret.resize(n);
    return ret;
} // 4d3e

// Floor division by polynomial; O(n lg n)
Poly operator/(Poly l, Poly r) {
    norm(l); norm(r);
    int d = sz(l)-sz(r)+1;
    if (d <= 0) return {};
    reverse(all(l));
    reverse(all(r));
    l.resize(d);
    l *= invert(r, d);
    l.resize(d);
    reverse(all(l));
    return l;
} // d4a4
Poly& operator/=(Poly& l, const Poly& r) {
    return l = l/r;
} // e12e

// Compute modulo by polynomial; O(n lg n)
Poly operator%(const Poly& l, const Poly& r) {
    return l - r*(l/r);
} // 4fc8
```

```
Poly& operator%=(Poly& l, const Poly& r) {
    return l -= r*(l/r);
} // 80bb

// Evaluate polynomial P in given points;
// time: O(n lg^2 n)
Poly eval(const Poly& P, Poly points) {
    int len = 1;
    while (len < sz(points)) len *= 2;

    vector<Poly> tree(len*2, {1});
    rep(i, 0, sz(points))
        tree[len+i] = {-points[i], 1};

    for (int i = len; --i;)
        tree[i] = tree[i*2] * tree[i*2+1];

    tree[0] = P;
    rep(i, 1, len*2)
        tree[i] = tree[i/2] % tree[i];

    rep(i, 0, sz(points)) {
        auto& vec = tree[len+i];
        points[i] = vec.empty() ? 0 : vec[0];
    } // clc2
    return points;
} // 69b0

// Given n points (x, f(x)) compute n-1-degree
// polynomial f that passes through them;
// time: O(n lg^2 n)
// For O(n^2) version see polynomial_interp.h.
Poly interpolate(const vector<pair<Zp, Zp>>& P) {
    int len = 1;
    while (len < sz(P)) len *= 2;

    vector<Poly> mult(len*2, {1}), tree(len*2);
    rep(i, 0, sz(P))
        mult[len+i] = {-P[i].x, 1};

    for (int i = len; --i;)
        mult[i] = mult[i*2] * mult[i*2+1];

    tree[0] = derivate(mult[1]);
    rep(i, 1, len*2)
        tree[i] = tree[i/2] % mult[i];

    rep(i, 0, sz(P))
        tree[len+i][0] = P[i].y / tree[len+i][0];

    for (int i = len; --i;)
        tree[i] = tree[i*2]*mult[i*2+1]
            + mult[i*2]*tree[i*2+1];

    return tree[1];
} // b706
```

math/polynomial_interp.h

a4cc

```
// Interpolate set of points (i, vec[i])
// and return it evaluated at x; time: O(n)
template<class T>
T polyExtend(vector<T>& vec, T x) {
    int n = sz(vec);
    vector<T> fac(n, 1), suf(n, 1);

    rep(i, 1, n) fac[i] = fac[i-1] * i;
    for (int i=n; --i;) suf[i-1] = suf[i]*(x-i);

    T pref = 1, ret = 0;
    rep(i, 0, n) {
        T d = fac[i] * fac[n-i-1] * ((n-i)%2*2-1);
        ret += vec[i] * suf[i] * pref / d;
        pref *= x-i;
    } // 681d
    return ret;
} // dd92
```

```
// Given n points (x, f(x)) compute n-1-degree
// polynomial f that passes through them;
// time: O(n^2)
// For O(n lg^2 n) version see polynomial.h
template<class T>
vector<T> polyInterp(vector<pair<T, T>> P) {
    int n = sz(P);
    vector<T> ret(n), tmp(n);
    T last = 0;
    tmp[0] = 1;

    rep(k, 0, n-1) rep(i, k+1, n)
        P[i].y = (P[i].y-P[k].y) / (P[i].x-P[k].x);

    rep(k, 0, n) rep(i, 0, n) {
        ret[i] += P[k].y * tmp[i];
        swap(last, tmp[i]);
        tmp[i] -= last * P[k].x;
    } // aflc
    return ret;
} // 7c2c
```

math/sieve.h 3f3d

```
constexpr int MAX_P = 1e6;
bitset<MAX_P+1> primes;
Vi primesList;

// Erathostenes sieve; time: O(n lg lg n)
void sieve() {
    primes.set();
    primes.reset(0);
    primes.reset(1);

    for (int i = 2; i*i <= MAX_P; i++)
        if (primes[i])
            for (int j = i*i; j <= MAX_P; j += i)
                primes.reset(j);

    rep(i, 0, MAX_P+1) if (primes[i])
        primesList.pb(i);
} // d5ca
```

math/sieve_factors.h 312d

```
constexpr int MAX_P = 1e6;
Vi factor(MAX_P+1);

// Erathostenes sieve with saving smallest
// factor for each number; time: O(n lg lg n)
void sieve() {
    for (int i = 2; i*i <= MAX_P; i++)
        if (!factor[i])
            for (int j = i*i; j <= MAX_P; j += i)
                if (!factor[j])
                    factor[j] = i;

    rep(i, 0, MAX_P+1) if (!factor[i]) factor[i] = i;
} // 82b6

// Factorize n <= MAX_P; time: O(lg n)
// Returns pairs (prime, power), sorted
vector<Pii> factorize(ll n) {
    vector<Pii> ret;
    while (n > 1) {
        int f = factor[n];
        if (ret.empty() || ret.back().x != f)
            ret.pb({f, 1});
        else
            ret.back().y++;
        n /= f;
    } // 664c
    return ret;
}
```

math/sieve_segmented.h 849b

```
// bb65
constexpr int MAX_P = 1e9;
bitset<MAX_P/2+1> primes; // Only odd numbers

// Cache-friendly Erathostenes sieve
// ~1.5s on Intel Core i5 for MAX_P = 10^9
// Memory usage: MAX_P/16 bytes
void sieve() {
    constexpr int SEG_SIZE = 1<<18;
    int pSqrt = int(sqrt(MAX_P)+0.5);
    vector<Pii> dels;
    primes.set();
    primes.reset(0);

    for (int i = 3; i <= pSqrt; i += 2) {
        if (primes[i/2]) {
            int j;
            for (j = i*i; j <= pSqrt; j += i*2)
                primes.reset(j/2);
            dels.pb({i, j/2});
        } // 9e62
    } // ff49

    for (int seg = pSqrt/2;
        seg <= sz(primes); seg += SEG_SIZE) {
        int lim = min(seg+SEG_SIZE, sz(primes));
        each(d, dels) for (; d.y < lim; d.y += d.x)
            primes.reset(d.y);
    } // 97ae
} // 6456

bool isPrime(int x) {
    return x == 2 || (x%2 && primes[x/2]);
} // 422c
```

structures/bitset_plus.h 6737

```
// Undocumented std::bitset features:
// - _Find_first() - returns first bit = 1 or N
// - _Find_next(i) - returns first bit = 1
// after i-th bit
// or N if not found

// Bitwise operations for vector<bool>
// UNTESTED

#define OP(x) vector<bool>& operator x##=( \
    vector<bool>& l, const vector<bool>& r) { \
    assert(sz(l) == sz(r)); \
    auto a = l.begin(); auto b = r.begin(); \
    while (a<l.end()) *a._M_p++ x##= *b._M_p++; \
    return l; } // fl64

OP(&)OP(!)OP(~)
```

structures/fenwick_tree.h b467

```
// Fenwick tree (BIT tree); space: O(n)
// Default version: prefix sums
struct Fenwick {
    using T = int;
    T ID = 0;
    T f(T a, T b) { return a+b; }

    vector<T> s;
    Fenwick(int n = 0) : s(n, ID) {}

    // A[i] = f(A[i], v); time: O(lg n)
    void modify(int i, T v) {
        for (; i < sz(s); i |= i+1) s[i] = f(s[i], v);
    } // a047

    // Get f(A[0], ..., A[i-1]); time: O(lg n)
    T query(int i) {
```

```
T v = ID;
    for (; i > 0; i &= i-1) v = f(v, s[i-1]);
    return v;
} // 9810

// Find smallest i such that
// f(A[0], ..., A[i-1]) >= val; time: O(lg n)
// Prefixes must have non-decreasing values.
int lowerBound(T val) {
    if (val <= ID) return 0;
    int i = -1, mask = 1;
    while (mask <= sz(s)) mask *= 2;
    T off = ID;

    while (mask /= 2) {
        int k = mask+i;
        if (k < sz(s)) {
            T x = f(off, s[k]);
            if (val > x) i=k, off=x;
        } // de7f
    } // 929c
    return i+2;
} // 4be9
} // 1470
```

structures/fenwick_tree_2d.h 4945

```
// Fenwick tree 2D (BIT tree 2D); space: O(n*m)
// Default version: prefix sums 2D
// Change s to hashmap for O(q lg^2 n) memory
struct Fenwick2D {
    using T = int;
    T ID = 0;
    T f(T a, T b) { return a+b; }

    vector<T> s;
    int w, h;

    Fenwick2D(int n = 0, int m = 0)
        : s(n*m, ID), w(n), h(m) {}

    // A[i,j] = f(A[i,j], v); time: O(lg^2 n)
    void modify(int i, int j, T v) {
        for (; i < w; i |= i+1)
            for (int k = j; k < h; k |= k+1)
                s[i*h+k] = f(s[i*h+k], v);
    } // d46b

    // Query prefix; time: O(lg^2 n)
    T query(int i, int j) {
        T v = ID;
        for (; i > 0; i &= i-1)
            for (int k = j; k > 0; k &= k-1)
                v = f(v, s[i*h+k-1]);
        return v;
    } // 08cf
} // 36b4
```

structures/find_union.h b4de

```
// Disjoint set data structure; space: O(n)
// Operations work in amortized O(alfa(n))
struct FAU {
    Vi G;
    FAU(int n = 0) : G(n, -1) {}

    // Get size of set containing i
    int size(int i) { return -G[find(i)]; }

    // Find representative of set containing i
    int find(int i) {
        return G[i] < 0 ? i : G[i] = find(G[i]);
    } // 5bc1

    // Union sets containing i and j
```

```
bool join(int i, int j) {
    i = find(i); j = find(j);
    if (i == j) return 0;
    if (G[i] > G[j]) swap(i, j);
    G[i] += G[j]; G[j] = i;
    return 1;
} // c721
}; // 62a4
```

structures/hull_offline.h 3030

```
constexpr ll INF = 2e18;
// constexpr double INF = 1e30;
// constexpr double EPS = 1e-9;

// MAX of linear functions; space: O(n)
// Use if you add lines in increasing `a` order
// Default uncommented version is for int64
struct Hull {
    using T = ll; // Or change to double

    struct Line {
        T a, b, end;
        T intersect(const Line& r) const {
            // Version for double:
            //if (r.a-a < EPS) return b>r.b?INF:-INF;
            //return (b-r.b) / (r.a-a);
            if (a==r.a) return b > r.b ? INF : -INF;
            ll u = b-r.b, d = r.a-a;
            return u/d + ((u^d) >= 0 || !(u%d));
        } // f27f
    }; // 10dc

    vector<Line> S;
    Hull() { S.pb({0, -INF, INF}); }

    // Insert f(x) = ax+b; time: amortized O(1)
    void push(T a, T b) {
        Line l(a, b, INF);
        while (true) {
            T e = S.back().end=S.back().intersect(l);
            if (sz(S) < 2 || S[sz(S)-2].end < e)
                break;
            S.pop_back();
        } // 044f
        S.pb(l);
    } // 978e

    // Query max(f(x) for each f); time: O(lg n)
    T query(T x) {
        auto t = *upper_bound(all(S), x,
            [](int l, const Line& r) {
                return l < r.end;
            }); // de77
        return t.a*x + t.b;
    } // b8de
}; // 1d64
```

structures/hull_online.h 2a7b

```
constexpr ll INF = 2e18;

// MAX of linear functions online; space: O(n)
struct Hull {
    static bool modeQ; // Toggles operator< mode

    struct Line {
        mutable ll a, b, end;

        ll intersect(const Line& r) const {
            if (a==r.a) return b > r.b ? INF : -INF;
            ll u = b-r.b, d = r.a-a;
            return u/d + ((u^d) >= 0 || !(u%d));
        } // f27f
    };
```



```

    bool operator<(const Line& r) const {
        return modeQ ? end < r.end : a < r.a;
    } // cfab
}; // 6046

multiset<Line> S;
Hull() { S.insert({ 0, -INF, INF }); }

// Updates segment end
bool update(multiset<Line>::iterator it) {
    auto cur = it++; cur->end = INF;
    if (it == S.end()) return false;
    cur->end = cur->intersect(*it);
    return cur->end >= it->end;
} // 63b8

// Insert f(x) = ax+b; time: O(lg n)
void insert(ll a, ll b) {
    auto it = S.insert({ a, b, INF });
    while (update(it)) it = --S.erase(++it);
    rep(i, 0, 2)
        while (it != S.begin() && update(--it))
            update(it = --S.erase(++it));
} // 4f69

// Query max(f(x) for each f): time: O(lg n)
ll query(ll x) {
    modeQ = 1;
    auto l = *S.upper_bound({ 0, 0, x });
    modeQ = 0;
    return l.a*x + l.b;
} // 7533
}; // 037e

bool Hull::modeQ = false;

structures/max_queue.h 3e9e

// Queue with max query on contained elements
struct MaxQueue {
    using T = int;
    deque<T> Q, M;

    // Add v to the back; time: amortized O(1)
    void push(T v) {
        while (!M.empty() && M.back() < v)
            M.pop_back();
        M.pb(v); Q.pb(v);
    } // 57a2

    // Pop from the front; time: O(1)
    void pop() {
        if (M.front() == Q.front()) M.pop_front();
        Q.pop_front();
    } // 101c

    // Get max element value; time: O(1)
    T max() const { return M.front(); }
}; // b6c4

structures/pairing_heap.h b2a7

// Pairing heap implementation; space O(n)
// Elements are stored in vector for faster
// allocation. It's MINIMUM queue.
// Allows to merge heaps in O(1)
template<class T, class Cmp = less<T>>
struct PHeap {
    struct Node {
        T val;
        int child{-1}, next{-1}, prev{-1};

        Node(T x = T()) : val(x) {}
    }; // 11ee

```

```

    using Vnode = vector<Node>;
    Vnode& M;
    int root{-1};

    int unlink(int& i) {
        if (i >= 0) M[i].prev = -1;
        int x = i; i = -1;
        return x;
    } // d9f6

    void link(int host, int& i, int val) {
        if (i >= 0) M[i].prev = -1;
        i = val;
        if (i >= 0) M[i].prev = host;
    } // 47d5

    int merge(int l, int r) {
        if (l < 0) return r;
        if (r < 0) return l;
        if (Cmp()(M[l].val, M[r].val)) swap(l, r);

        link(l, M[l].next, unlink(M[r].child));
        link(r, M[r].child, l);
        return r;
    } // fc42

    int mergePairs(int v) {
        if (v < 0 || M[v].next < 0) return v;
        int v2 = unlink(M[v].next);
        int v3 = unlink(M[v2].next);
        return merge(merge(v, v2), mergePairs(v3));
    } // 2eea

    // ---

    // Initialize heap with given node storage
    // Just declare 1 Vnode and pass it to heaps
    PHeap(Vnode& mem) : M(mem) {}

    // Add given key to heap, returns index; O(1)
    int push(const T& x) {
        int index = sz(M);
        M.emplace_back(x);
        root = merge(root, index);
        return index;
    } // e744

    // Change key of i to smaller value; O(1)
    void decrease(int i, T val) {
        assert(!Cmp()(M[i].val, val));
        M[i].val = val;

        int prev = M[i].prev;
        if (prev < 0) return;

        auto& p = M[prev];
        link(prev, (p.child == i ? p.child
            : p.next), unlink(M[i].next));

        root = merge(root, i);
    } // 1a67

    bool empty() { return root < 0; }
    const T& top() { return M[root].val; }

    // Merge with other heap. Must use same vec.
    void merge(PHeap& r) { // time: O(1)
        assert(&M == &r.M);
        root = merge(root, r.root); r.root = -1;
    } // 9623

    // Remove min element; time: O(lg n)
    void pop() {
        root = mergePairs(unlink(M[root].child));
    } // 5b13

```

```

}; // 09f3

structures/rmq.h 4551

// Range Minimum Query; space: O(n lg n)
struct RMQ {
    using T = int;
    T ID = INT_MAX;
    T f(T a, T b) { return min(a, b); }

    vector<vector<T>> s;

    // Initialize RMQ structure; time: O(n lg n)
    RMQ(const vector<T>& vec = {}) {
        s = {vec};
        for (int h = 1; h <= sz(vec); h *= 2) {
            s.emplace_back();
            auto& prev = s[sz(s)-2];
            rep(i, 0, sz(vec)-h*2+1)
                s.back().pb(f(prev[i], prev[i+h]));
        } // b5ff
    } // cf67

    // Query f(s[b], ... ,s[e-1]); time: O(1)
    T query(int b, int e) {
        if (b >= e) return ID;
        int k = 31 - __builtin_clz(e-b);
        return f(s[k][b], s[k][e - (1<<k)]);
    } // caaa
}; // e694

structures/segtree_config.h 7ef8

// Segment tree configurations to be used
// in segtree_general and segtree_persistent.
// See comments in TREE_PLUS version
// to understand how to create custom ones.
// Capabilities notation: (update; query)

#if TREE_PLUS // (+; sum, max, max count)
// time: O(lg n) [UNTESTED]
using T = int; // Data type for update
// operations (lazy tag)
T ID = 0; // Neutral value for
// updates and lazy tags

// This structure keeps aggregated data
struct Agg {
    // Aggregated data: sum, max, max count
    // Default values should be neutral
    // values, i.e. "aggregate over empty set"
    T sum{0}, vMax{INT_MIN}, nMax{0};

    // Initialize as leaf (single value)
    void leaf() { sum = vMax = 0; nMax = 1; }

    // Combine data with aggregated data
    // from node to the right
    void merge(const Agg& r) {
        if (vMax < r.vMax) nMax = r.nMax;
        else if (vMax == r.vMax) nMax += r.nMax;
        vMax = max(vMax, r.vMax);
        sum += r.sum;
    } // 8850

    // Apply update provided in `x`:
    // - update aggregated data
    // - update lazy tag `lazy`
    // - `size` is amount of elements
    // - return 0 if update should branch
    // (to be used in "segment tree beats")
    // - if you change value of `x` changed
    // value will be passed to next node
    // to the right during updates

```

```

    bool apply(T& lazy, T& x, int size) {
        lazy += x;
        sum += x*size;
        vMax += x;
        return 1;
    } // 4858
    }; // 9bf5
    #elif TREE_MAX // (max; max, max count)
    // time: O(lg n) [UNTESTED]
    using T = int;
    T ID = INT_MIN;

    struct Agg {
        // Aggregated data: max value, max count
        T vMax{INT_MIN}, nMax{0};
        void leaf() { vMax = 0; nMax = 1; }

        void merge(const Agg& r) {
            if (vMax < r.vMax) nMax = r.nMax;
            else if (vMax == r.vMax) nMax += r.nMax;
            vMax = max(vMax, r.vMax);
        } // f56b

        bool apply(T& lazy, T& x, int size) {
            if (vMax <= x) nMax = size;
            lazy = max(lazy, x);
            vMax = max(vMax, x);
            return 1;
        } // 8bd5
    }; // 15b6
    #elif TREE_SET // (=; sum, max, max count)
    // time: O(lg n) [UNTESTED]
    // Set ID to some unused value.
    using T = int;
    T ID = INT_MIN;

    struct Agg {
        // Aggregated data: sum, max, max count
        T sum{0}, vMax{INT_MIN}, nMax{0};
        void leaf() { sum = vMax = 0; nMax = 1; }

        void merge(const Agg& r) {
            if (vMax < r.vMax) nMax = r.nMax;
            else if (vMax == r.vMax) nMax += r.nMax;
            vMax = max(vMax, r.vMax);
            sum += r.sum;
        } // 8850

        bool apply(T& lazy, T& x, int size) {
            lazy = x;
            sum = x*size;
            vMax = x;
            nMax = size;
            return 1;
        } // 845b
    }; // 7488
    #elif TREE_BEATS // (+, min; sum, max)
    // time: amortized O(lg n) if not using +
    // amortized O(lg^2 n) if using +
    // Lazy tag is pair (add, min).
    // To add x: run update with {x, INT_MAX},
    // to min x: run update with {0, x}.
    // When both parts are provided addition
    // is applied first, then minimum.
    using T = Pii;
    T ID = {0, INT_MAX};

    struct Agg {
        // Aggregated data: max value, max count,
        // second max value, sum
        int vMax{INT_MIN}, nMax{0}, max2{INT_MIN};
        int sum{0};
    }

```

```

void leaf() { sum = vMax = 0; nMax = 1; }

void merge(const Agg& r) {
    if (r.vMax > vMax) {
        max2 = vMax;
        vMax = r.vMax;
        nMax = r.nMax;
    } else if (r.vMax == vMax) {
        nMax += r.nMax;
    } else if (r.vMax > max2) {
        max2 = r.vMax;
    } // b074
    max2 = max(max2, r.max2);
    sum += r.sum;
} // 3124

bool apply(T& lazy, T& x, int size) {
    if (max2 != INT_MIN && max2+x.x >= x.y)
        return 0;

    lazy.x += x.x;
    sum += x.x*size;
    vMax += x.x;
    if (max2 != INT_MIN) max2 += x.x;

    if (x.y < vMax) {
        sum -= (vMax-x.y) * nMax;
        vMax = x.y;
    } // 7025
    lazy.y = vMax;
    return 1;
} // fe0c
}; // 2924
#endif

```

structures/segtree_general.h 725a

```

// Highly configurable statically allocated
// (interval; interval) segment tree;
// space: O(n) [UNTESTED]
struct SegTree {
    // Choose/write configuration
    #include "segtree_config.h"

    // Root node is 1, left is i*2, right i*2+1
    vector<Agg> agg; // Aggregated data for nodes
    vector<T> lazy; // Lazy tags for nodes
    int len[1]; // Number of leaves

    // Initialize tree for n elements; time: O(n)
    SegTree(int n = 0) {
        while (len < n) len *= 2;
        agg.resize(len*2);
        lazy.resize(len*2, ID);
        rep(i, 0, n) agg[len+i].leaf();
        for (int i = len; --i;)
            (agg[i] = agg[i*2]).merge(agg[i*2+1]);
    } // 4417

    void push(int i, int s) {
        if (lazy[i] != ID) {
            agg[i*2].apply(lazy[i*2], lazy[i], s/2);
            agg[i*2+1].apply(lazy[i*2+1],
                            lazy[i], s/2);
            lazy[i] = ID;
        } // 3ba9
    } // 5d19

    // Modify interval [vb;ve) with val; O(lg n)
    T update(int vb, int ve, T val, int i = 1,
            int b = 0, int e = -1) {
        if (e < 0) e = len;
        if (vb >= e || b >= ve) return val;

```

```

        if (b >= vb && e <= ve &&
            agg[i].apply(lazy[i], val, e-b))
            return val;

        int m = (b+e) / 2;
        push(i, e-b);
        val = update(vb, ve, val, i*2, b, m);
        val = update(vb, ve, val, i*2+1, m, e);
        (agg[i] = agg[i*2]).merge(agg[i*2+1]);
        return val;
    } // aa8e

    // Query interval [vb;ve); time: O(lg n)
    Agg query(int vb, int ve, int i = 1,
            int b = 0, int e = -1) {
        if (e < 0) e = len;
        if (vb >= e || b >= ve) return {};
        if (b >= vb && e <= ve) return agg[i];

        int m = (b+e) / 2;
        push(i, e-b);
        Agg t = query(vb, ve, i*2, b, m);
        t.merge(query(vb, ve, i*2+1, m, e));
        return t;
    } // la1e
}; // db5c

```

structures/segtree_persist.h dcfc

```

// Highly configurable (interval; interval)
// persistent segment tree;
// space: O(queries lg n) [UNTESTED]
// First tree version number is 0.
struct SegTree {
    // Choose/write configuration
    #include "segtree_config.h"

    vector<Agg> agg; // Aggregated data for nodes
    vector<T> lazy; // Lazy tags for nodes
    vector<bool> cow; // Copy children on push?
    Vi L, R; // Children links
    int len[1]; // Number of leaves

    // Initialize tree for n elements; O(lg n)
    SegTree(int n = 0) {
        int k = 1;
        while (len < n) len *= 2, k++;

        agg.resize(k);
        lazy.resize(k, ID);
        cow.resize(k, 1);
        L.resize(k);
        R.resize(k);
        agg[--k].leaf();

        while (k-- > 0) {
            (agg[k] = agg[k+1]).merge(agg[k+1]);
            L[k] = R[k] = k+1;
        } // 211f
    } // 83cf

    // New version from version `i`; time: O(1)
    // First version number is 0.
    int fork(int i) {
        L.pb(L[i]); R.pb(R[i]); cow.pb(cow[i] = 1);
        agg.pb(agg[i]); lazy.pb(lazy[i]);
        return sz(L)-1;
    } // a21b

    void push(int i, int s, bool w) {
        bool has = (lazy[i] != ID);
        if ((has || w) && cow[i]) {
            int a = fork(L[i]), b = fork(R[i]);
            L[i] = a; R[i] = b; cow[i] = 0;

```

```

        } // la3e
        if (has) {
            agg[L[i]].apply(lazy[L[i]], lazy[i], s/2);
            agg[R[i]].apply(lazy[R[i]], lazy[i], s/2);
            lazy[i] = ID;
        } // eca6
    } // 9f84

    // Modify interval [vb;ve) with val
    // in tree version `i`; time: O(lg n)
    T update(int i, int vb, int ve, T val,
            int b = 0, int e = -1) {
        if (e < 0) e = len;
        if (vb >= e || b >= ve) return val;

        if (b >= vb && e <= ve &&
            agg[i].apply(lazy[i], val, e-b))
            return val;

        int m = (b+e) / 2;
        push(i, e-b, 1);
        val = update(L[i], vb, ve, val, b, m);
        val = update(R[i], vb, ve, val, m, e);
        (agg[i] = agg[L[i]]).merge(agg[R[i]]);
        return val;
    } // 776e

    // Query interval [vb;ve)
    // in tree version `i`; time: O(lg n)
    Agg query(int i, int vb, int ve,
            int b = 0, int e = -1) {
        if (e < 0) e = len;
        if (vb >= e || b >= ve) return {};
        if (b >= vb && e <= ve) return agg[i];

        int m = (b+e) / 2;
        push(i, e-b, 0);
        Agg t = query(L[i], vb, ve, b, m);
        t.merge(query(R[i], vb, ve, m, e));
        return t;
    } // abf4
}; // 8a44

```

structures/segtree_point.h 2b0f

```

// Segment tree (point, interval)
// Configure by modifying:
// - T - stored data type
// - ID - neutral element for query operation
// - f(a, b) - combine results
struct SegTree {
    using T = int;
    T ID = INT_MIN;
    T f(T a, T b) { return max(a, b); }

    vector<T> V;
    int len;

    // Initialize tree for n elements; time: O(n)
    SegTree(int n = 0, T def = 0) {
        for (len = 1; len < n; len *= 2);
        V.resize(len*2, ID);
        rep(i, 0, n) V[len+i] = def;
        for (int i = len; --i;)
            V[i] = f(V[i*2], V[i*2+1]);
    } // 459e

    // Set element `i` to `val`; time: O(lg n)
    void set(int i, T val) {
        V[i += len] = val;
        while (i /= 2)
            V[i] = f(V[i*2], V[i*2+1]);
    } // 4bcd

    // Query interval [b;e); time: O(lg n)
    T query(int b, int e) {
        b += len; e += len-1;
        if (b > e) return ID;
        if (b == e) return V[b];
        T x = f(V[b], V[e]);

        while (b/2 < e/2) {
            if (~b&1) x = f(x, V[b^1]);
            if (e&1) x = f(x, V[e^1]);
            b /= 2; e /= 2;
        } // 444a
        return x;
    } // de36
}; // d571

```

structures/treap.h

bbb4

```

// "Set" of implicit keyed treaps; space: O(n)
// Nodes are keyed by their indices in array
// of all nodes. Treap key is key of its root.
// "Node x" means "node with key x".
// "Treap x" means "treap with key x".
// Key -1 is "null".
// Put any additional data in Node struct.
struct Treap {
    struct Node {
        // E[0] = left child, E[1] = right child
        // weight = node random weight (for treap)
        // size = subtree size, par = parent node
        int E[2] = {-1, -1}, weight{rand()};
        int size[1], par{-1};
        bool flip{0}; // Is interval reversed?
    }; // c082

    vector<Node> G; // Array of all nodes

    // Initialize structure for n nodes
    // with keys 0, ..., n-1; time: O(n)
    // Each node is separate treap,
    // use join() to make sequence.
    Treap(int n = 0) : G(n) {}

    // Create new treap (a single node),
    // returns its key; time: O(1)
    int make() {
        G.emplace_back(); return sz(G)-1;
    } // 907a

    // Get size of node x subtree. x can be -1.
    int size(int x) { // time: O(1)
        return (x >= 0 ? G[x].size : 0);
    } // 81cf

    // Propagate down data (flip flag etc).
    // x can be -1; time: O(1)
    void push(int x) {
        if (x >= 0 && G[x].flip) {
            G[x].flip = 0;
            swap(G[x].E[0], G[x].E[1]);
            each(e, G[x].E) if (e >= 0) G[e].flip ^= 1;
        } // ed19

    // Update aggregates of node x.
    // x can be -1; time: O(1)
    void update(int x) {
        if (x >= 0) {
            int& s = G[x].size = 1;
            G[x].par = -1;
            each(e, G[x].E) if (e >= 0) {
                s += G[e].size;
                G[e].par = x;
            }

```

```

    } // f7a7
    } // + any other aggregates
    } // 46a3

    // Split treap x into treaps l and r
    // such that l contains first i elements
    // and r the remaining ones.
    // x, l, r can be -1; time: ~O(lg n)
    void split(int x, int& l, int& r, int i) {
        push(x); l = r = -1;
        if (x < 0) return;
        int key = size(G[x].E[0]);
        if (i <= key) {
            split(G[x].E[0], l, G[x].E[0], i);
            r = x;
        } else {
            split(G[x].E[1], G[x].E[1], r, i-key-1);
            l = x;
        } // fe19
        update(x);
    } // 8211

    // Join treaps l and r into one treap
    // such that elements of l are before
    // elements of r. Returns new treap.
    // l, r and returned value can be -1.
    int join(int l, int r) { // time: ~O(lg n)
        push(l); push(r);
        if (l < 0 || r < 0) return max(l, r);

        if (G[l].weight < G[r].weight) {
            G[l].E[1] = join(G[l].E[1], r);
            update(l);
            return l;
        } // 18c7

        G[r].E[0] = join(l, G[r].E[0]);
        update(r);
        return r;
    } // b559

    // Find i-th node in treap x.
    // Returns its key or -1 if not found.
    // x can be -1; time: ~O(lg n)
    int find(int x, int i) {
        while (x >= 0) {
            push(x);
            int key = size(G[x].E[0]);
            if (key == i) return x;
            x = G[x].E[key < i];
            if (key < i) i -= key+1;
        } // 054c
        return -1;
    } // 0b9b

    // Get key of treap containing node x
    // (key of treap root). x can be -1.
    int root(int x) { // time: ~O(lg n)
        while (G[x].par >= 0) x = G[x].par;
        return x;
    } // be8b

    // Get position of node x in its treap.
    // x is assumed to NOT be -1; time: ~O(lg n)
    int index(int x) {
        int p, i = size(G[x].E[G[x].flip]);
        while ((p = G[x].par) >= 0) {
            if (G[p].E[1] == x) i+=size(G[p].E[0])+1;
            if (G[p].flip) i = G[p].size-i-1;
            x = p;
        } // 3f81
        return i;
    }

```

```

    } // ddad

    // Reverse interval [l;r) in treap x.
    // Returns new key of treap; time: ~O(lg n)
    int reverse(int x, int l, int r) {
        int a, b, c;
        split(x, b, c, r);
        split(b, a, b, l);
        if (b >= 0) G[b].flip ^= 1;
        return join(join(a, b), c);
    } // e418
    } // 6a6c

```

structures/wavelet_tree.h 69f4

```

// Wavelet tree ("merge-sort tree over values")
// Each node represent interval of values.
// seq[1] = original sequence
// seq[i] = subsequence with values
//          represented by i-th node
// left[i][j] = how many values in seq[0:j)
//             go to left subtree
struct WaveletTree {
    vector<Vi> seq, left;
    int len;

    // Build wavelet tree for sequence `elems`;
    // time and space: O((n+maxVal) log maxVal)
    // Values are expected to be in [0;maxVal).
    WaveletTree(const Vi& elems, int maxVal) {
        for (len = 1; len < maxVal; len *= 2);
        seq.resize(len*2);
        left.resize(len*2);
        seq[1] = elems;
        build(1, 0, len);
    } // a5e9

    void build(int i, int b, int e) {
        if (i >= len) return;
        int m = (b+e) / 2;
        left[i].pb(0);
        each(x, seq[i]) {
            left[i].pb(left[i].back() + (x < m));
            seq[i*2 + (x >= m)].pb(x);
        } // ac25
        build(i*2, b, m);
        build(i*2+1, m, e);
    } // 8153

    // Find k-th smallest element in [begin;end)
    // [begin;end); time: O(log maxVal)
    int kth(int begin, int end, int k, int i=1) {
        if (i >= len) return seq[i][0];
        int x = left[i][begin], y = left[i][end];
        if (k < y-x) return kth(x, y, k, i*2);
        return kth(begin-x, end-y, k-y+x, i*2+1);
    } // 7861

    // Count number of elements >= vb and < ve
    // in [begin;end); time: O(log maxVal)
    int count(int begin, int end, int vb, int ve,
              int i = 1, int b = 0, int e = -1) {
        if (e < 0) e = len;
        if (b >= ve || vb >= e) return 0;
        if (b >= vb && e <= ve) return end-begin;
        int m = (b+e) / 2;
        int x = left[i][begin], y = left[i][end];
        return count(x, y, vb, ve, i*2, b, m) +
            count(begin-x, end-y, vb, ve, i*2+1, m, e);
    } // 71cf
    } // dd48

```

structures/ext/hash_table.h 2d30

```

#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
// gp_hash_table<K, V> = faster unordered_set

// Anti-anti-hash
const size_t HXOR = mt19937_64(time(0))();
template<class T> struct SafeHash {
    size_t operator() (const T& x) const {
        return hash<T>() (x ^ T(HXOR));
    } // 3a78
}; // 7d0e

```

structures/ext/rope.h 051f

```

#include <ext/rope>
using namespace __gnu_cxx;
// rope<T> = implicit cartesian tree

```

structures/ext/tree.h a3bc

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/tree_policy.hpp>
using namespace __gnu_pbds;

template<class T, class Cmp = less<T>>
using ordered_set = tree<
    T, null_type, Cmp, rb_tree_tag,
    tree_order_statistics_node_update
>;

```

```

// Standard set functions and:
// t.order_of_key(key) - index of first >= key
// t.find_by_order(i) - find i-th element
// t1.join(t2) - assuming t1<t2 merge t2 to t1

```

structures/ext/trie.h 5cc2

```

#include <ext/pb_ds/assoc_container.hpp>
#include <ext/pb_ds/trie_policy.hpp>
using namespace __gnu_pbds;

using pref_trie = trie<
    string, null_type,
    trie_string_access_traits<>, pat_trie_tag,
    trie_prefix_search_node_update
>;

```

text/aho_corasick.h 697d

```

constexpr char AMIN = 'a'; // Smallest letter
constexpr int ALPHA = 26; // Alphabet size

// Aho-Corasick algorithm for linear-time
// multiple pattern matching.
// Add patterns using add(), then call build().
struct Aho {
    vector<array<int, ALPHA>> nxt{1};
    Vi suf = {-1}, accLink = {-1};
    vector<Vi> accept{1};

    // Add string with given ID to structure
    // Returns index of accepting node
    int add(const string& str, int id) {
        int i = 0;
        each(c, str) {
            if (!nxt[i][c-AMIN]) {
                nxt[i][c-AMIN] = sz(nxt);
                nxt.pb({}); suf.pb(-1);
                accLink.pb(1); accept.pb({});
            } // 5ead
            i = nxt[i][c-AMIN];
        } // ace9
        accept[i].pb(id);
        return i;
    }

```

```

    } // 27c8

    // Build automata; time: O(V*ALPHA)
    void build() {
        queue<int> que;
        each(e, nxt[0]) if (e) {
            suf[e] = 0; que.push(e);
        } // c34d
        while (!que.empty()) {
            int i = que.front(), s = suf[i], j = 0;
            que.pop();
            each(e, nxt[i]) {
                if (e) que.push(e);
                (e ? suf[e] : e) = nxt[s][j++];
            } // 8521
            accLink[i] = (accept[s].empty() ?
                accLink[s] : s);
        } // 1e8a
    } // 2561

    // Append `c` to state `i`
    int next(int i, char c) {
        return nxt[i][c-AMIN];
    } // 6bb7

    // Call `f` for each pattern accepted
    // when in state `i` with its ID as argument.
    // Return true from `f` to terminate early.
    // Calls are in decreasing length order.
    template<class F> void accepted(int i, F f) {
        while (i != -1) {
            each(a, accept[i]) if (f(a)) return;
            i = accLink[i];
        } // c175
    } // 1f0d
}; // 2768

```

text/kmp.h 5729

```

// Computes prefsuf array; time: O(n)
// ps[i] = max prefsuf of [0;i); ps[0] := -1
template<class T> Vi kmp(const T& str) {
    Vi ps; ps.pb(-1);
    each(x, str) {
        int k = ps.back();
        while (k >= 0 && str[k] != x) k = ps[k];
        ps.pb(k+1);
    } // 05aa
    return ps;
} // 8a6c

```

```

// Finds occurrences of pat in vec; time: O(n)
// Returns starting indices of matches.
template<class T>
Vi match(const T& str, T pat) {
    int n = sz(pat);
    pat.pb(-1); // SET TO SOME UNUSED CHARACTER
    pat.insert(pat.end(), all(str));
    Vi ret, ps = kmp(pat);
    rep(i, 0, sz(ps)) {
        if (ps[i] == n) ret.pb(i-2*n-1);
    } // ale9
    return ret;
} // f986

```

text/kmr.h ee8c

```

// KMR algorithm for O(1) lexicographical
// comparison of substrings.
struct KMR {
    vector<Vi> ids;

    KMR() {}

```

```
// Initialize structure; time: O(n lg^2 n)
// You can change str type to Vi freely.
KMR(const string& str) {
    ids.clear();
    ids.pb(Vi(all(str)));

    for (int h = 1; h <= sz(str); h *= 2) {
        vector<pair<Pii, int>> tmp;

        rep(j, 0, sz(str)) {
            int a = ids.back()[j], b = -1;
            if (j+h < sz(str)) b = ids.back()[j+h];
            tmp.pb({ {a, b}, j });
        } // a210

        sort(all(tmp));
        ids.emplace_back(sz(tmp));

        int n = 0;
        rep(j, 0, sz(tmp)) {
            if (j > 0 && tmp[j-1].x != tmp[j].x)
                n++;
            ids.back()[tmp[j].y] = n;
        } // bd2e
    } // 969a
} // a40e

// Get representative of [begin;end); O(1)
Pii get(int begin, int end) {
    if (begin >= end) return {0, 0};
    int k = 31 - __builtin_clz(end-begin);
    return {ids[k][begin], ids[k][end-(1<<k)]};
} // 85f3

// Compare [b1;e1) with [b2;e2); O(1)
// Returns -1 if <, 0 if ==, 1 if >
int cmp(int b1, int e1, int b2, int e2) {
    int l1 = e1-b1, l2 = e2-b2;
    int l = min(l1, l2);
    Pii x = get(b1, b1+l), y = get(b2, b2+l);

    if (x == y) return (l1 > l2) - (l1 < l2);
    return (x > y) - (x < y);
} // bf42

// Compute suffix array of string; O(n)
Vi sufArray() {
    Vi sufs(sz(ids.back()));
    rep(i, 0, sz(ids.back()))
        sufs[ids.back()[i]] = i;
    return sufs;
} // d98d
}; // 457e
```

text/lcp.h 0c65

```
// Compute Longest Common Prefix array for
// given string and it's suffix array; O(n)
// In order to compute suffix array use kmr.h
// or suffix_array_linear.h
template<class T>
Vi lcpArray(const T& str, const Vi& sufs) {
    int n = sz(str), k = 0;
    Vi pos(n), lcp(n-1);
    rep(i, 0, n) pos[sufs[i]] = i;
    rep(i, 0, n) {
        if (pos[i] < n-1) {
            int j = sufs[pos[i]+1];
            while (i+k < n && j+k < n &&
                str[i+k] == str[j+k]) k++;
            lcp[pos[i]] = k;
        } // 2cba
    }
```

```
    if (k > 0) k--;
} // 8b22
return lcp;
} // d438
```

text/lyndon_factorization.h 688c

```
// Compute Lyndon factorization for s; O(n)
// Word is simple iff it's stricly smaller
// than any of it's nontrivial suffixes.
// Lyndon factorization is division of string
// into non-increasing simple words.
// It is unique.
vector<string> duval(const string& s) {
    int n = sz(s), i = 0;
    vector<string> ret;
    while (i < n) {
        int j = i+1, k = i;
        while (j < n && s[k] <= s[j])
            k = (s[k] < s[j] ? i : k+1), j++;
        while (i <= k)
            ret.pb(s.substr(i, j-k)), i += j-k;
    } // 3f17
    return ret;
} // 0e48
```

text/main_lorentz.h 9afb

```
#include "z_function.h"

struct Sqr {
    int begin, end, len;
}; // f012

// Main-Lorentz algorithm for finding
// all squares in given word; time: O(n lg n)
// Results are in compressed form:
// (b, e, l) means that for each b <= i < e
// there is square at position i of size 2l.
// Each square is present in only one interval.
vector<Sqr> lorentz(const string& s) {
    int n = sz(s);
    if (n <= 1) return {};
    auto a = s.substr(0, n/2), b = s.substr(n/2);

    auto ans = lorentz(a);
    each(p, lorentz(b))
        ans.pb({p.begin+n/2, p.end+n/2, p.len});

    string ra(a.rbegin(), a.rend());
    string rb(b.rbegin(), b.rend());

    rep(j, 0, 2) {
        Vi z1 = prefPref(ra), z2 = prefPref(b+a);
        z1.pb(0); z2.pb(0);

        rep(c, 0, sz(a)) {
            int l = sz(a)-c;
            int x = c - min(l-1, z1[l]);
            int y = c - max(l-z2[sz(b)+c], j);
            if (x > y) continue;

            if (j)
                ans.pb({n-y-l*2, n-x-l*2+1, l});
            else
                ans.pb({x, y+1, l});
        } // 6c51

        a.swap(rb);
        b.swap(ra);
    } // b6b2

    return ans;
} // 5567
```

text/manacher.h 8680

```
// Manacher algorithm; time: O(n)
// Finds largest radiuses for palindromes:
// r[2*i] = for center at i (single letter = 1)
// r[2*i+1] = for center between i and i+1
template<class T> Vi manacher(const T& str) {
    int n = sz(str)*2, c = 0, e = 1;
    Vi r(n, 1);
    auto get = [&](int i) { return i%2 ? 0 :
        (i >= 0 && i < n ? str[i/2] : i); }; // 3d98

    rep(i, 0, n) {
        if (i < e) r[i] = min(r[c*2-i], e-i);
        while (get(i-r[i]) == get(i+r[i])) r[i]++;
        if (i+r[i] > e) c = i, e = i+r[i]-1;
    } // 0f87

    rep(i, 0, n) r[i] /= 2;
    return r;
} // a300
```

text/min_rotation.h e4d6

```
// Find lexicographically smallest
// rotation of s; time: O(n)
// Returns index where shifted word starts.
// You can use std::rotate to get the word:
// rotate(s.begin(), s.begin()+minRotation(s),
// s.end());
int minRotation(string s) {
    int a = 0, n = sz(s); s += s;
    rep(b, 0, n) rep(i, 0, n) {
        if (a+i == b || s[a+i] < s[b+i]) {
            b += max(0, i-1); break;
        } // 865b
        if (s[a+i] > s[b+i]) {
            a = b; break;
        } // 7628
    } // 40be
    return a;
} // 9ed8
```

text/palindromic_tree.h 8d2d

```
constexpr int ALPHA = 26; // Set alphabet size

// Tree of all palindromes in string,
// constructed online by appending letters.
// space: O(n*ALPHA); time: O(n)
// Code marked with [EXT] is extension for
// calculating minimal palindrome partition
// in O(n lg n). Can also be modified for
// similar dynamic programmings.
struct PalTree {
    Vi txt; // Text for which tree is built

    // Node 0 = empty palindrome (root of even)
    // Node 1 = "-1" palindrome (root of odd)
    Vi len{0, -1}; // Lengths of palindromes
    Vi link{1, 0}; // Suffix palindrome links
    // Edges to next palindromes
    vector<array<int, ALPHA>> tof{ {}, {} };
    int last{0}; // Current node (max suffix pal)

    Vi diff{0, 0}; // len[i]-len[link[i]] [EXT]
    Vi slink{0, 0}; // Serial links [EXT]
    Vi series{0, 0}; // Series DP answer [EXT]
    Vi ans{0}; // DP answer for prefix[EXT]

    int ext(int i) {
        while (len[i]+2 > sz(txt) ||
            txt[sz(txt)-len[i]-2] != txt.back())
            i = link[i];
    }
```

```
    return i;
} // d442

// Append letter from [0;ALPHA); time: O(1)
// (or O(lg n) if [EXT] is enabled)
void add(int x) {
    txt.pb(x);
    last = ext(last);

    if (!to[last][x]) {
        len.pb(len[last]+2);
        link.pb(to[ext(link[last])][x]);
        to[last][x] = sz(to);
        to.emplace_back();

        // [EXT]
        diff.pb(len.back() - len[link.back()]);
        slink.pb(diff.back() == diff[link.back()]
            ? slink[link.back()] : link.back());
        series.pb(0);
    } // 0adb
    last = to[last][x];

    // [EXT]
    ans.pb(INT_MAX);
    for (int i=last; len[i] > 0; i=slink[i]) {
        series[i] = ans[sz(ans) - len[slink[i]]
            - diff[i] - 1];
        if (diff[i] == diff[link[i]])
            series[i] = min(series[i],
                series[link[i]]);

        // If you want only even palindromes
        // set ans only for sz(txt)%2 == 0
        ans.back() = min(ans.back(), series[i]+1);
    } // ab3b
    // [/EXT]
} // 909a
}; // b3a7
```

text/suffix_array_linear.h 1341

```
#include "../util/radix_sort.h"

// KS algorithm for suffix array; time: O(n)
// Input values are assumed to be in [1;k]
Vi sufArray(Vi str, int k) {
    int n = sz(str);
    Vi suf(n);
    str.resize(n+15);

    if (n < 15) {
        iota(all(suf), 0);
        rep(j, 0, n) countSort(suf,
            [&](int i) { return str[i+n-j-1]; }, k);
        return suf;
    } // 5fcf

    // Compute triples codes
    Vi tmp, code(n+2);
    rep(i, 0, n) if (i % 3) tmp.pb(i);

    rep(j, 0, 3) countSort(tmp,
        [&](int i) { return str[i-j+2]; }, k);

    int mc = 0, j = -1;

    each(i, tmp) {
        code[i] = mc += (j == -1 ||
            str[i] != str[j] ||
            str[i+1] != str[j+1] ||
            str[i+2] != str[j+2]);
        j = i;
    } // bfdc
```



```

// Compute suffix array of 2/3
tmp.clear();
for (int i=1; i < n; i += 3) tmp.pb(code[i]);
tmp.pb(0);
for (int i=2; i < n; i += 3) tmp.pb(code[i]);
tmp = sufArray(move(tmp), mc);

// Compute partial suffix arrays
Vi third;
int th = (n+4) / 3;
if (n%3 == 1) third.pb(n-1);

rep(i, 1, sz(tmp)) {
    int e = tmp[i];
    tmp[i-1] = (e < th ? e*3+1 : (e-th)*3+2);
    code[tmp[i-1]] = i;
    if (e < th) third.pb(e*3);
} // f9f1

tmp.pop_back();
countSort(third,
    [&](int i) { return str[i]; }, k);

// Merge suffix arrays
merge(all(third), all(tmp), suf.begin(),
    [&](int l, int r) {
        while (l%3 == 0 || r%3 == 0) {
            if (str[l] != str[r])
                return str[l] < str[r];
            l++; r++;
        } // 2f8a
        return code[l] < code[r];
    }); // 4cb3

return suf;
} // 9165

// KS algorithm for suffix array; time: O(n)
Vi sufArray(const string& str) {
    return sufArray(Vi(all(str)), 255);
} // 593f

```

text/suffix_automaton.h d00d

```

constexpr char AMIN = 'a'; // Smallest letter
constexpr int ALPHA = 26; // Set alphabet size

// Suffix automaton - minimal DFA that
// recognizes all suffixes of given string
// (and encodes all substrings);
// space: O(n*ALPHA); time: O(n)
// Paths from root are equivalent to substrings
// Extensions:
// - [OCC] - count occurrences of substrings
// - [PATHS] - count paths from node
struct SufDFA {
    // State v represents endpos-equivalence
    // class that contains words of all lengths
    // between link[len[v]]+1 and len[v].
    // len[v] = longest word of equivalence class
    // link[v] = link to state of longest suffix
    // in other equivalence class
    // to[v][c] = automaton edge c from v
    Vi len{0}, link{-1};
    vector<array<int, ALPHA>> to{ {} };
    int last{0}; // Current node (whole word)

    vector<Vi> inSufs; // [OCC] Suffix-link tree
    Vi cnt{0}; // [OCC] Occurrence count
    vector<ll> paths; // [PATHS] Out-path count

    SufDFA() {}

    // Build suffix automaton for given string

```

```

// and compute extended stuff; time: O(n)
SufDFA(const string& s) {
    each(c, s) add(c);
    finish();
} // ec2e

// Append letter to the back
void add(char c) {
    int v = last, x = c-AMIN;
    last = sz(len);
    len.pb(len[v]+1);
    link.pb(0);
    to.pb({});
    cnt.pb(1); // [OCC]

    while (v != -1 && !to[v][x]) {
        to[v][x] = last;
        v = link[v];
    } // 4cfc

    if (v != -1) {
        int q = to[v][x];
        if (len[v]+1 == len[q]) {
            link[last] = q;
        } else {
            len.pb(len[v]+1);
            link.pb(link[q]);
            to.pb(to[q]);
            cnt.pb(0); // [OCC]
            link[last] = link[q] = sz(len)-1;
            while (v != -1 && to[v][x] == q) {
                to[v][x] = link[q];
                v = link[v];
            } // 784f
        } // 90aa
    } // af69
} // 345a

// Compute some additional stuff (offline)
void finish() {
    // [OCC]
    inSufs.resize(sz(len));
    rep(i, 1, sz(link)) inSufs[link[i]].pb(i);
    dfsSufs(0);
    // [OCC]

    // [PATHS]
    paths.assign(sz(len), 0);
    dfs(0);
    // [PATHS]
    // 3f75

    // Only for [OCC]
    void dfsSufs(int v) {
        each(e, inSufs[v]) {
            dfsSufs(e);
            cnt[v] += cnt[e];
        } // 2469
    } // 0c60

    // Only for [PATHS]
    void dfs(int v) {
        if (paths[v]) return;
        paths[v] = 1;
        each(e, to[v]) if (e) {
            dfs(e);
            paths[v] += paths[e];
        } // 22b3
    } // d004

    // Go using edge `c` from state `i`.
    // Returns 0 if edge doesn't exist.

```

```

int next(int i, char c) {
    return to[i][c-AMIN];
} // c363

// Get lexicographically k-th substring
// of represented string; time: O(|substr|)
// Empty string has index 0.
// Requires [PATHS] extension.
string lex(ll k) {
    string s;
    int v = 0;
    while (k--) rep(i, 0, ALPHA) {
        int e = to[v][i];
        if (e) {
            if (k < paths[e]) {
                s.pb(char(AMIN+i));
                v = e;
                break;
            } // f307
            k -= paths[e];
        } // 29be
    } // 4600
    return s;
} // e4af
}; // 7135

```

text/suffix_tree.h 40a4

```

constexpr int ALPHA = 26;

// Ukkonen's algorithm for online suffix tree
// construction; space: O(n*ALPHA); time: O(n)
// Real tree nodes are called dedicated nodes.
// "Nodes" lying on compressed edges are called
// implicit nodes and are represented
// as pairs (lower node, label index).
// Labels are represented as intervals [L;R)
// which refer to substrings [L;R) of txt.
// Leaves have labels of form [L;infinity),
// use getR to get current right endpoint.
// Suffix links are valid only for internal
// nodes (non-leaves).
struct SuffTree {
    Vi txt; // Text for which tree is built
    // to[v][c] = edge with label starting with c
    // from node v
    vector<array<int, ALPHA>> to{ {} };
    Vi L{0}, R{0}; // Parent edge label endpoints
    Vi par{0}; // Parent link
    Vi link{0}; // Suffix link
    Pii cur{0, 0}; // Current state

    // Get current right end of node label
    int getR(int i) { return min(R[i], sz(txt)); }

    // Follow edge `e` of implicit node `s`.
    // Returns (-1, -1) if there is no edge.
    Pii next(Pii s, int e) {
        if (s.y < getR(s.x))
            return txt[s.y] == e ? mp(s.x, s.y+1)
                : mp(-1, -1);

        e = to[s.x][e];
        return e ? mp(e, L[e]+1) : mp(-1, -1);
    } // f430

    // Create dedicated node for implicit node
    // and all its suffixes
    int split(Pii s) {
        if (s.y == R[s.x]) return s.x;

        int t = sz(to); to.pb({});
        to[t][txt[s.y]] = s.x;

```

```

L.pb(L[s.x]);
R.pb(L[s.x] = s.y);
par.pb(par[s.x]);
par[s.x] = to[par[t]][txt[L[t]]] = t;
link.pb(-1);

int v = link[par[t]], l = L[t] + !par[t];
while (l < R[t]) {
    v = to[v][txt[l]];
    l += getR(v) - L[v];
} // 0393

v = split({v, getR(v)-l+R[t]});
link[t] = v;
return t;
} // 10bb

// Append letter from [0;ALPHA) to the back
void add(int x) { // amortized time: O(1)
    Pii t; txt.pb(x);
    while ((t = next(cur, x)).x == -1) {
        int m = split(cur);
        to[m][x] = sz(to);
        to.pb({});
        par.pb(m);
        L.pb(sz(txt)-1);
        R.pb(INT_MAX);
        link.pb(-1);
        cur = {link[m], getR(link[m])};
        if (!m) return;
    } // 60c2
    cur = t;
} // 6f4e
}; // dbfb

```

text/z_function.h 0466

```

// Computes Z function array; time: O(n)
// zf[i] = max common prefix of str and str[i:]
template<class T> Vi prefPref(const T& str) {
    int n = sz(str), b = 0, e = 1;
    Vi zf(n);
    rep(i, 1, n) {
        if (i < e) zf[i] = min(zf[i-b], e-i);
        while (i+zf[i] < n &&
            str[zf[i]] == str[i+zf[i]]) zf[i]++;
        if (i+zf[i] > e) b = i, e = i+zf[i];
    } // e906
    zf[0] = n;
    return zf;
} // b88d

```

trees/centroid_decomp.h 5247

```

// Centroid decomposition; space: O(n lg n)
// UNTESTED
struct CentroidTree {
    // child[v] = children of v in centroid tree
    // par[v] = parent of v in centroid tree
    // (-1 for root)
    // depth[v] = depth of v in centroid tree
    // (0 for root)
    // ind[v][i] = index of vertex v in i-th
    // centroid subtree from root
    // size[v] = size of centroid subtree of v
    // subtree[v] = list of vertices
    // in centroid subtree of v
    // dists[v] = distances from v to vertices
    // in its centroid subtree
    // (in the order of subtree[v])
    // neigh[v] = neighbours of v
    // in its centroid subtree

```

```
// dir[v][i] = index of centroid neighbour
//          that is first vertex on path
//          from centroid v to i-th vertex
//          of centroid subtree
//          (-1 for centroid)
vector<Vi> child, ind, dists, subtree,
           neigh, dir;
Vi par, depth, size;
int root; // Root centroid

CentroidTree() {}

CentroidTree(vector<Vi>& G)
: child(sz(G)), ind(sz(G)), dists(sz(G)),
  subtree(sz(G)), neigh(sz(G)),
  dir(sz(G)), par(sz(G), -2),
  depth(sz(G)), size(sz(G)) {}
root = decomp(G, 0, 0);
} // 026c

void dfs(vector<Vi>& G, int v, int p) {
    size[v] = 1;
    each(e, G[v]) if (e != p && par[e] == -2)
        dfs(G, e, v), size[v] += size[e];
} // bbed

void layer(vector<Vi>& G, int v,
           int p, int c, int d) {
    ind[v].pb(sz(subtree[c]));
    subtree[c].pb(v);
    dists[c].pb(d);
    dir[c].pb(sz(neigh[c]) - 1);
    each(e, G[v]) if (e != p && par[e] == -2) {
        if (v == c) neigh[c].pb(e);
        layer(G, e, v, c, d + 1);
    } // dc82
} // 37ee

int decomp(vector<Vi>& G, int v, int d) {
    dfs(G, v, -1);
    int p = -1, s = size[v];
loop:
    each(e, G[v]) {
        if (e != p && par[e] == -2 &&
            size[e] > s/2) {
            p = v; v = e; goto loop;
        } // e0a5
    } // 3533

    par[v] = -1;
    size[v] = s;
    depth[v] = d;
    layer(G, v, -1, v, 0);

    each(e, G[v]) if (par[e] == -2) {
        int j = decomp(G, e, d + 1);
        child[v].pb(j);
        par[j] = v;
    } // 70b5
    return v;
} // 217c
} // 71d6
```

trees/centroid_offline.h ac92

```
// Helper for offline centroid decomposition
// Usage: CentroidDecomp(G);
// Constructor calls method 'process'
// for each centroid subtree.
struct CentroidDecomp {
    vector<Vi>& G; // Reference to target graph
    vector<bool> on; // Is vertex enabled?
    Vi size; // Used internally
```

```
// Run centroid decomposition for graph g
CentroidDecomp(vector<Vi>& G)
: G(g), on(sz(g), 1), size(sz(g)) {}
decomp(0);
} // 8677

// Compute subtree sizes for subtree rooted
// at v, ignoring p and disabled vertices
void computeSize(int v, int p) {
    size[v] = 1;
    each(e, G[v]) if (e != p && on[e])
        computeSize(e, v), size[v] += size[e];
} // 1c0d

void decomp(int v) {
    computeSize(v, -1);
    int p = -1, s = size[v];
loop:
    each(e, G[v]) {
        if (e != p && on[e] && size[e] > s/2) {
            p = v; v = e; goto loop;
        } // e0a5
    } // f31d
    process(v);
    on[v] = 0;
    each(e, G[v]) if (on[e]) decomp(e);
} // f170

// Process current centroid subtree:
// - v is centroid
// - boundary vertices have on[x] = 0
// Formally: Let H be subgraph induced
// on vertices such that on[v] = 1.
// Then current centroid subtree is
// connected component of H that contains v
// and v is its centroid.
void process(int v) {
    // Do your stuff here...
} // d41d
} // a923
```

trees/heavy_light_decomp.h 9739

```
#include "../structures/segtree_point.h"

// Heavy-Light Decomposition of tree
// with subtree query support; space: O(n)
struct HLD {
    // Subtree of v = [pos[v]; pos[v]+size[v])
    // Chain with v = [chBegin[v]; chEnd[v])
    Vi par; // Vertex parent
    Vi size; // Vertex subtree size
    Vi depth; // Vertex distance to root
    Vi pos; // Vertex position in "HLD" order
    Vi chBegin; // Begin of chain with vertex
    Vi chEnd; // End of chain with vertex
    Vi order; // "HLD" preorder of vertices
    SegTree tree; // Verts are in HLD order

    HLD() {}

    // Initialize structure for tree G
    // and given root; time: O(n lg n)
    // MODIFIES ORDER OF EDGES IN G!
    HLD(vector<Vi>& G, int root)
    : par(sz(G)), size(sz(G)),
      depth(sz(G)), pos(sz(G)),
      chBegin(sz(G)), chEnd(sz(G)) {}
    dfs(G, root, -1);
    decomp(G, root, -1, 0);
    tree = {sz(order)};
```

```
} // 8263

void dfs(vector<Vi>& G, int v, int p) {
    par[v] = p;
    size[v] = 1;
    depth[v] = p < 0 ? 0 : depth[p] + 1;

    int& fs = G[v][0];
    if (fs == p) swap(fs, G[v].back());

    each(e, G[v]) if (e != p) {
        dfs(G, e, v);
        size[v] += size[e];
        if (size[e] > size[fs]) swap(e, fs);
    } // 9872
} // 12fe

void decomp(vector<Vi>& G,
             int v, int p, int chb) {
    pos[v] = sz(order);
    chBegin[v] = chb;
    chEnd[v] = pos[v] + 1;
    order.pb(v);

    each(e, G[v]) if (e != p) {
        if (e == G[v][0]) {
            decomp(G, e, v, chb);
            chEnd[v] = chEnd[e];
        } else {
            decomp(G, e, v, sz(order));
        } // c84a
    } // f707
} // eb89

// Get root of chain containing v
int chRoot(int v) {return order[chBegin[v]];}

// Level Ancestor Query; time: O(lg n)
int laq(int v, int level) {
    while (true) {
        int k = pos[v] - depth[v] + level;
        if (k >= chBegin[v]) return order[k];
        v = par[chRoot(v)];
    } // 8c18
} // 675e

// Lowest Common Ancestor; time: O(lg n)
int lca(int a, int b) {
    while (chBegin[a] != chBegin[b]) {
        int ha = chRoot(a), hb = chRoot(b);
        if (depth[ha] > depth[hb]) a = par[ha];
        else b = par[hb];
    } // 5620
    return depth[a] < depth[b] ? a : b;
} // c168

// Call func(chBegin, chEnd) on each path
// segment; time: O(lg n * time of func)
template<class T>
void iterPath(int a, int b, T func) {
    while (chBegin[a] != chBegin[b]) {
        int ha = chRoot(a), hb = chRoot(b);
        if (depth[ha] > depth[hb]) {
            func(chBegin[a], pos[a] + 1);
            a = par[ha];
        } else {
            func(chBegin[b], pos[b] + 1);
            b = par[hb];
        } // f9a5
    } // 563c

    if (pos[a] > pos[b]) swap(a, b);
    // Remove +1 from pos[a]+1 for vertices
```

```
// queries (with +1 -> edges).
func(pos[a] + 1, pos[b] + 1);
} // 17e5

// Query path between a and b; O(lg^2 n)
SegTree::T queryPath(int a, int b) {
    auto ret = tree.ID;
    iterPath(a, b, [&](int i, int j) {
        ret = tree.f(ret, tree.query(i, j));
    }); // 1113
    return ret;
} // 1bc9

// Query subtree of v; time: O(lg n)
SegTree::T querySubtree(int v) {
    return tree.query(pos[v], pos[v] + size[v]);
} // 23db
}; // a062
```

trees/lca.h 294f

```
// LAQ and LCA using jump pointers
// space: O(n lg n)

struct LCA {
    vector<Vi> jumps;
    Vi level, pre, post;
    int cnt{0}, depth;

    LCA() {}

    // Initialize structure for tree G
    // and root r; time: O(n lg n)
    LCA(vector<Vi>& G, int root)
    : jumps(sz(G)), level(sz(G)),
      pre(sz(G)), post(sz(G)) {}
    dfs(G, root, root);
    depth = int(log2(sz(G))) + 2;
    rep(j, 0, depth) each(v, jumps)
        v.pb(jumps[v[j]][j]);
} // d6ce

void dfs(vector<Vi>& G, int v, int p) {
    level[v] = p == v ? 0 : level[p] + 1;
    jumps[v].pb(p);
    pre[v] = ++cnt;
    each(e, G[v]) if (e != p) dfs(G, e, v);
    post[v] = ++cnt;
} // e286

// Check if a is ancestor of b; time: O(1)
bool isAncestor(int a, int b) {
    return pre[a] <= pre[b] &&
           post[b] <= post[a];
} // 5514

// Lowest Common Ancestor; time: O(lg n)
int operator()(int a, int b) {
    for (int j = depth; j--;)
        if (!isAncestor(jumps[a][j], jumps[b][j]))
            a = jumps[a][j];
    return isAncestor(a, b) ? a : jumps[a][0];
} // 27d8

// Level Ancestor Query; time: O(lg n)
int laq(int a, int lvl) {
    for (int j = depth; j--;)
        if (lvl <= level[jumps[a][j]])
            a = jumps[a][j];
    return a;
} // 75b3

// Get distance from a to b; time: O(lg n)
int distance(int a, int b) {
```

```

    return level[a] + level[b] -
        level[operator()(a, b)]*2;
} // 07e0

// Get k-th vertex on path from a to b,
// a is 0, b is last; time: O(lg n)
// Returns -1 if k > distance(a, b)
int kthVertex(int a, int b, int k) {
    int c = operator()(a, b);
    if (level[a]-k >= level[c])
        return laq(a, level[a]-k);
    k += level[c]*2 - level[a];
    return (k > level[b] ? -1 : laq(b, k));
} // 46c9
}; // 2254

```

trees/lca_linear.h

7aa5

```

// LAQ and LCA using jump pointers
// with linear memory; space: O(n)
// UNTESTED

struct LCA {
    Vi par, jmp, depth, pre, post;
    int cnt{0};

    LCA() {}

    // Initialize structure for tree G
    // and root v; time: O(n lg n)
    LCA(vector<Vi>& G, int v)
        : par(sz(G), -1), jmp(sz(G), v),
          depth(sz(G)), pre(sz(G)), post(sz(G)) {
        dfs(G, v);
    } // 94cf

    void dfs(vector<Vi>& G, int v) {
        int j = jmp[v], k = jmp[j], x =
            depth[v]+depth[k] == depth[j]*2 ? k : v;
        pre[v] = ++cnt;
        each(e, G[v]) if (!pre[e]) {
            par[e] = v; jmp[e] = x;
            depth[e] = depth[v]+1;
            dfs(G, e);
        } // b123
        post[v] = ++cnt;
    } // 3280

    // Level Ancestor Query; time: O(lg n)
    int laq(int v, int d) {
        while (depth[v] > d)
            v = depth[jmp[v]] < d ? par[v] : jmp[v];
        return v;
    } // f509

    // Lowest Common Ancestor; time: O(lg n)
    int operator()(int a, int b) {
        if (depth[a] > depth[b]) swap(a, b);
        b = laq(b, depth[a]);
        while (a != b) {
            if (jmp[a] == jmp[b])
                a = par[a], b = par[b];
            else
                a = jmp[a], b = jmp[b];
        } // fe08
        return a;
    } // 25ff

    // Check if a is ancestor of b; time: O(1)
    bool isAncestor(int a, int b) {
        return pre[a] <= pre[b] &&
            post[b] <= post[a];
    } // 5514

```

```

// Get distance from a to b; time: O(lg n)
int distance(int a, int b) {
    return depth[a] + depth[b] -
        depth[operator()(a, b)]*2;
} // a340

// Get k-th vertex on path from a to b,
// a is 0, b is last; time: O(lg n)
// Returns -1 if k > distance(a, b)
int kthVertex(int a, int b, int k) {
    int c = operator()(a, b);
    if (depth[a]-k >= depth[c])
        return laq(a, depth[a]-k);
    k += depth[c]*2 - depth[a];
    return (k > depth[b] ? -1 : laq(b, k));
} // 34ed
}; // a221

```

trees/link_cut_tree.h

6bd6

```

constexpr int INF = 1e9;

// Link/cut tree; space: O(n)
// Represents forest of (un)rooted trees.
struct LinkCutTree {
    vector<array<int, 2>> child;
    Vi par, prev, flip, size;

    // Initialize structure for n vertices; O(n)
    // At first there's no edges.
    LinkCutTree(int n = 0)
        : child(n, {-1, -1}), par(n, -1),
          prev(n, -1), flip(n, -1), size(n, 1) {}

    void push(int x) {
        if (x >= 0 && flip[x]) {
            flip[x] = 0;
            swap(child[x][0], child[x][1]);
            each(e, child[x]) if (e >= 0) flip[e] ^= 1;
        } // + any other lazy path operations
    } // bae2

    void update(int x) {
        if (x >= 0) {
            size[x] = 1;
            each(e, child[x]) if (e >= 0)
                size[x] += size[e];
        } // + any other path aggregates
    } // 8ec0

    void auxLink(int p, int i, int ch) {
        child[p][i] = ch;
        if (ch >= 0) par[ch] = p;
        update(p);
    } // 0a9a

    void rot(int p, int i) {
        int x = child[p][i], g = par[x] = par[p];
        if (g >= 0) child[g][child[g][1] == p] = x;
        auxLink(p, i, child[x][!i]);
        auxLink(x, !i, p);
        swap(prev[x], prev[p]);
        update(g);
    } // 4c76

    void splay(int x) {
        while (par[x] >= 0) {
            int p = par[x], g = par[p];
            push(g); push(p); push(x);
            bool f = (child[p][1] == x);
            if (g >= 0) {
                if (child[g][f] == p) { // zig-zig
                    rot(g, f); rot(p, f);

```

```

                } else { // zig-zag
                    rot(p, f); rot(g, !f);
                } // 2ebb
            } else { // zig
                rot(p, f);
            } // f8a2
        } // 446b
        push(x);
    } // 55a7

    // After this operation x becomes the end
    // of preferred path starting in root;
    void access(int x) { // amortized O(lg n)
        while (true) {
            splay(x);
            int p = prev[x];
            if (p < 0) break;

            prev[x] = -1;
            splay(p);

            int r = child[p][1];
            if (r >= 0) swap(par[r], prev[r]);
            auxLink(p, 1, x);
        } // 2b87
    } // 30be

    // Make x root of its tree; amortized O(lg n)
    void makeRoot(int x) {
        access(x);
        int& l = child[x][0];
        if (l >= 0) {
            swap(par[l], prev[l]);
            flip[l] ^= 1;
            update(l);
            l = -1;
            update(x);
        } // 0064
    } // b246

    // Find root of tree containing x
    int find(int x) { // time: amortized O(lg n)
        access(x);
        while (child[x][0] >= 0)
            push(x = child[x][0]);
        splay(x);
        return x;
    } // d78d

    // Add edge x-y; time: amortized O(lg n)
    // Root of tree containing y becomes
    // root of new tree.
    void link(int x, int y) {
        makeRoot(x); prev[x] = y;
    } // fb4f

    // Remove edge x-y; time: amortized O(lg n)
    // x and y become roots of new trees!
    void cut(int x, int y) {
        makeRoot(x); access(y);
        par[x] = child[y][0] = -1;
        update(y);
    } // 1908

    // Get distance between x and y,
    // returns INF if x and y there's no path.
    // This operation makes x root of the tree!
    int dist(int x, int y) { // amortized O(lg n)
        makeRoot(x);
        if (find(y) != x) return INF;
        access(y);
        int t = child[y][0];

```

```

        return t >= 0 ? size[t] : 0;
    } // ae69
}; // 4480

util/arc_interval_cover.h 7507

using dbl = double;

// Find size of smallest set of points
// such that each arc contains at least one
// of them; time: O(n lg n)
int arcCover(vector<pair<dbl, dbl>>& inters,
             dbl wrap) {
    int n = sz(inters);

    rep(i, 0, n) {
        auto& e = inters[i];
        e.x = fmod(e.x, wrap);
        e.y = fmod(e.y, wrap);
        if (e.x < 0) e.x += wrap, e.y += wrap;
        if (e.x > e.y) e.x += wrap;
        inters.pb({e.x+wrap, e.y+wrap});
    } // b87d

    Vi nxt(n);
    deque<dbl> que;
    dbl r = wrap*4;
    sort(all(inters));

    for (int i = n*2-1; i--;) {
        r = min(r, inters[i].y);
        que.push_front(inters[i].x);
        while (!que.empty() && que.back() > r)
            que.pop_back();
        if (i < n) nxt[i] = i+sz(que);
    } // 5e6c

    int a = 0, b = 0;
    do {
        a = nxt[a] % n;
        b = nxt[nxt[b] % n] % n;
    } while (a != b);

    int ans = 0;
    while (b < a+n) {
        b += nxt[b % n] - b % n;
        ans++;
    } // 7350
    return ans;
} // 7871

util/bit_hacks.h 599a

// __builtin_popcount - count number of 1 bits
// __builtin_clz - count most significant 0s
// __builtin_ctz - count least significant 0s
// __builtin_ffs - like ctz, but indexed from 1
// returns 0 for 0
// For ll version add ll to name

using ull = uint64_t;

#define T64(s,up) \
    for (ull i=0; i<64; i+=s*2) \
        for (ull j = i; j < i+s; j++) { \
            ull a = (M[j] >> s) & up; \
            ull b = (M[j+s] & up) << s; \
            M[j] = (M[j] & up) | b; \
            M[j+s] = (M[j+s] & (up<<s)) | a; \
        } // a290

// Transpose 64x64 bit matrix
void transpose64(array<ull, 64>& M) {
    T64(1, 0x5555555555555555);
    T64(2, 0x3333333333333333);

```

```

T64(4, 0xF0F0F0F0F0F0F0F0);
T64(8, 0xFF00FF00FF00FF00);
T64(16, 0xFFFF0000FFFF);
T64(32, 0xFFFFFFFFLL);
} // 6889

// Lexicographically next mask with same
// amount of ones.
int nextSubset(int v) {
    int t = v | (v - 1);
    return (t + 1) | (((~t & -t) - 1) >>
        (__builtin_ctz(v) + 1));
} // 4c0c

util/bump_alloc.h 09f9

// Allocator, which doesn't free memory.

char mem[400<<20]; // Set memory limit
size_t nMem;

void* operator new(size_t n) {
    nMem += n; return &mem[nMem-n];
} // fba6

void operator delete(void*) {}

util/compress_vec.h bc5d

// Compress integers to range [0;n) while
// preserving their order; time: O(n lg n)
// Returns mapping: compressed -> original
Vi compressVec(vector<int*>& vec) {
    sort(all(vec),
        [](int* l, int* r) { return *l < *r; });
    Vi old;
    each(e, vec) {
        if (old.empty() || old.back() != *e)
            old.pb(*e);
        *e = sz(old)-1;
    } // 7eb0
    return old;
} // 2b60

util/inversion_vector.h 01f9

// Get inversion vector for sequence of
// numbers in [0;n); ret[i] = count of numbers
// greater than perm[i] to the left; O(n lg n)
Vi encodeInversions(Vi perm) {
    Vi odd, ret(sz(perm));
    int cont = 1;

    while (cont) {
        odd.assign(sz(perm)+1, 0);
        cont = 0;

        rep(i, 0, sz(perm)) {
            if (perm[i] % 2) odd[perm[i]]++;
            else ret[i] += odd[perm[i]+1];
            cont += perm[i] /= 2;
        } // 4ed0
    } // a4f0
    return ret;
} // c2e1

// Count inversions in sequence of numbers
// in [0;n); time: O(n lg n)
ll countInversions(Vi perm) {
    ll ret = 0, cont = 1;
    Vi odd;

    while (cont) {
        odd.assign(sz(perm)+1, 0);
        cont = 0;

```

```

        rep(i, 0, sz(perm)) {
            if (perm[i] % 2) odd[perm[i]]++;
            else ret += odd[perm[i]+1];
            cont += perm[i] /= 2;
        } // 916f
    } // c9b5
    return ret;
} // laaf

util/longest_inc_subseq.h 0bd1

// Longest Increasing Subsequence; O(n lg n)
int lis(const Vi& seq) {
    Vi dp(sz(seq), INT_MAX);
    each(c, seq) *lower_bound(all(dp), c) = c;
    return int(lower_bound(all(dp), INT_MAX)
        - dp.begin());
} // d0e9

util/max_rects.h 2a16

struct MaxRect {
    // begin = first column of rectangle
    // end = first column after rectangle
    // hei = height of rectangle
    // touch = columns of height hei inside
    int begin, end, hei;
    Vi touch; // sorted increasing
}; // 41fe

// Given consecutive column heights find
// all inclusion-wise maximal rectangles
// contained in "drawing" of columns; time O(n)
vector<MaxRect> getMaxRects(Vi hei) {
    hei.insert(hei.begin(), -1);
    hei.pb(-1);
    Vi reach(sz(hei), sz(hei)-1);
    vector<MaxRect> ans;

    for (int i = sz(hei)-1; --i;) {
        int j = i+1, k = i;
        while (hei[j] > hei[i]) j = reach[j];
        reach[i] = j;

        while (hei[k] > hei[i-1]) {
            ans.pb({i-1, 0, hei[k], {} });
            auto& rect = ans.back();

            while (hei[k] == rect.hei) {
                rect.touch.pb(k-1);
                k = reach[k];
            } // 6e7e
            rect.end = k-1;
        } // e03f
    } // 2796
    return ans;
} // 0e49

util/mo.h caeb

// Modified MO's queries sorting algorithm,
// slightly better results than standard.
// Allows to process q queries in O(n*sqrt(q))

struct Query {
    int begin, end;
}; // b76d

// Get point index on Hilbert curve
ll hilbert(int x, int y, int s, ll c = 0) {
    if (s <= 1) return c;
    s /= 2; c *= 4;
    if (y < s)
        return hilbert(x&(s-1), y, s, c+(x>=s)+1);

```

```

    if (x < s)
        return hilbert(2*s-y-1, s-x-1, s, c);
    return hilbert(y-s, x-s, s, c+3);
} // 0fb9

// Get good order of queries; time: O(n lg n)
Vi moOrder(vector<Query>& queries, int maxN) {
    int s = 1;
    while (s < maxN) s *= 2;

    vector<ll> ord;
    each(q, queries)
        ord.pb(hilbert(q.begin, q.end, s));

    Vi ret(sz(ord));
    iota(all(ret), 0);
    sort(all(ret), [&](int l, int r) {
        return ord[l] < ord[r];
    }); // 9aea
    return ret;
} // ecec

util/parallel_binsearch.h bc5d

// Run `n` binary searches on [b;e) parallely.
// `cmp` should be lambda with arguments:
// 1) vector<Pii>& - pairs (v, i)
//    which are queries if value for index i
//    is greater or equal to v;
//    pairs are sorted by v
// 2) vector<bool>& - output vector,
//    set true at index i if value
//    for i-th query is >= queried value
// Returns vector of found values;
// time: O((n+c) lg range), where c is cmp time
template<class T>
Vi multiBS(int b, int e, int n, T cmp) {
    vector<Pii> que(n), rng(n, {b, e});
    vector<bool> ans(n);

    rep(i, 0, n) que[i] = {(b+e)/2, i};

    for (int k = 32-__builtin_clz(e-b); k--;) {
        int last = 0, j = 0;
        cmp(que, ans);
        rep(i, 0, sz(que)) {
            Pii &q = que[i], &r = rng[q.y];
            if (q.x != last) last = q.x, j = i;
            (ans[i] ? r.x : r.y) = q.x;
            q.x = (r.x+r.y) / 2;
            if (!ans[i]) swap(que[i], que[j++]);
        } // 6c4e
    } // 622c

    Vi ret;
    each(p, rng) ret.pb(p.x);
    return ret;
} // lad7

util/radix_sort.h 6fb4

// Stable countingsort; time: O(k+sz(vec))
// See example usage in radixSort for pairs.
template<class F>
void countSort(Vi& vec, F key, int k) {
    static Vi buf, cnt;
    vec.swap(buf);
    vec.resize(sz(buf));
    cnt.assign(k+1, 0);
    each(e, buf) cnt[key(e)]++;
    rep(i, 1, k+1) cnt[i] += cnt[i-1];
    for (int i = sz(vec)-1; i >= 0; i--)
        vec[--cnt[key(buf[i])]] = buf[i];

```

```

} // cc6f

// Compute order of elems, k is max key; O(n)
Vi radixSort(const vector<Pii>& elems, int k) {
    Vi order(sz(elems));
    iota(all(order), 0);
    countSort(order,
        [&](int i) { return elems[i].y; }, k);
    countSort(order,
        [&](int i) { return elems[i].x; }, k);
    return order;
} // e8f6

```