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Access the code, data, and analysis at <tegorman13.github.io>

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ABSTRACT In project 1, we applied model-based techniques to quantify and control for the similarity between training and testing experience, which in turn enabled us to account for the difference between varied and constant training via an extended version of a similarity based generalization model. In project 2, we will go a step further, implementing a full process model capable of both 1) producing novel responses and 2) modeling behavior in both the learning and testing stages of the experiment. Project 2 also places a greater emphasis on extrapolation performance following training - as varied training has often been purported to be particularly beneficial in such situations.

KEYWORDS Learning Generalization; Function Learning; Visuomotor learning; Training Variability

Introduction

In project 1, we applied model-based techniques to quantify and control for the similarity between training and testing experience, which in turn enabled us to account for the difference between varied and constant training via an extended version of a similarity based generalization model. In project 2, we will go a step further, implementing a full process model capable of both 1) producing novel responses and 2) modeling behavior in both the learning and testing stages of the experiment. Project 2 also places a greater emphasis on extrapolation performance following training - as varied training has often been purported to be particularly beneficial in such situations. Extrapolation has long been a focus of the literature on function learning (Brehmer, 1974; Carroll, 1963). Central questions of the function learning literature have included the relative difficulties of learning various functional forms (e.g. linear vs.bilinear vs. quadratic), and the relative effectiveness of rule-based vs. association-based exemplar models vs. various hybrid models (Bott & Heit, 2004; DeLosh et al., 1997; Jones et al., 2018; Kalish et al., 2004; M. Mcdaniel et al., 2009; M. A. Mcdaniel & Busemeyer, 2005). However the issue of training variation has received surprisingly little attention in this area.

Methods

Participants

Data was collected from 647 participants (after exclusions). The results shown below consider data from subjects in our initial experiment, which consisted of 196 participants (106 constant, 90 varied). The follow-up experiments entailed minor manipulations: 1) reversing the velocity bands that were trained on vs. novel during testing; 2) providing ordinal rather than numerical feedback during training (e.g. correct, too low, too high). The data from these subsequent experiments are largely consistently with our initial results shown below.

Task

We developed a novel visuomotor extrapolation task, termed the Hit The Wall task, wherein participants learned to launch a projectile such that it hit a rectangle at the far end of the screen with an appropriate amount of force. Although the projectile had both x and y velocity components, only the x-dimension was relevant for the task. Link to task demo

Design

1) 90 training trials split evenly divided between velocity bands. Varied training with 3 velocity bands and Constant training with 1 band.

2) No-feedback testing from 3 novel extrapolation bands. 15 trials each.

3) No-feedbacd testing from the 3 bands used during the training phase (2 of which were novel for the constant group). 9 trials each.

4) Feedback testing for each of the 3 extrapolation bands. 10 trials each.

Table 1: Testing Deviation - Empirical Summary

Table 2: Summary of Deviation- Constant

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	254	148	298
350-550	Extrapolation	191	110	229
600-800	Extrapolation	150	84	184
800-1000	Trained	184	106	242
1000-1200	Extrapolation	233	157	282
1200-1400	Extrapolation	287	214	290

Table 3: Summary of Deviation- Varied

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	386	233	426
350-550	Extrapolation	285	149	340
600-800	Extrapolation	234	144	270
800-1000	Trained	221	149	248
1000-1200	Trained	208	142	226
1200-1400	Trained	242	182	235

Results

Testing Phase - No feedback.

In the first part of the testing phase, participants are tested from each of the velocity bands, and receive no feedback after each throw.

Deviation From Target Band Descriptive summaries testing deviation data are provided in Table 1 and Figure 1. To model differences in accuracy between groups, we used Bayesian mixed effects regression models to the trial level data from the testing phase. The primary model predicted the absolute deviation from the target velocity band (dist) as a function of training condition (condit), target velocity band (band), and their interaction, with random intercepts and slopes for each participant (id).

$$dist_{ij} = \beta_0 + \beta_1 \cdot condit_{ij} + \beta_2 \cdot band_{ij} + \beta_3 \cdot condit_{ij} \cdot band_{ij} + b_{0i} + b_{1i} \cdot band_{ij} + \epsilon_{ij} \tag{1}$$

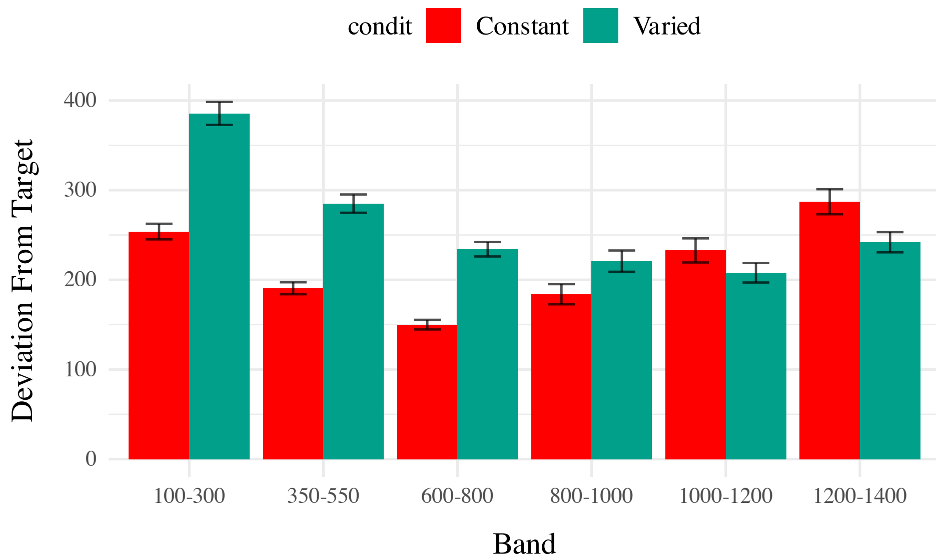


Figure 1: E1. Deviations from target band during testing without feedback stage.

Table 4: Experiment 1. Bayesian Mixed Model predicting absolute deviation as a function of condition (Constant vs. Varied) and Velocity Band

Table 5: Coefficients

Term	Estimate	95% CrI Lower	95% CrI Upper	pd
Intercept	205.09	136.86	274.06	1.00
conditVaried	157.44	60.53	254.90	1.00
Band	0.01	-0.07	0.08	0.57
condit*Band	-0.16	-0.26	-0.06	1.00

contrast	Band	value	lower	upper	pd
Constant - Varied	100	-141.49	-229.2	-53.83	1.00
Constant - Varied	350	-101.79	-165.6	-36.32	1.00
Constant - Varied	600	-62.02	-106.2	-14.77	1.00
Constant - Varied	800	-30.11	-65.1	6.98	0.94
Constant - Varied	1000	2.05	-33.5	38.41	0.54
Constant - Varied	1200	33.96	-11.9	81.01	0.92

The model predicting absolute deviation (dist) showed clear effects of both training condition and target velocity band (Table X). Overall, the varied training group showed a larger deviation relative to the constant training group (β = 157.44, 95% CI [60.53, 254.9]). Deviation also depended on target velocity band, with lower bands showing less deviation. See Table 4 for full model output.

Discrimination between bands In addition to accuracy/deviation, we also assessed the ability of participants to reliably discriminate between the velocity bands (i.e. responding differently when prompted for band 600-800 than when prompted for band 150-350). Table 6 shows descriptive statistics of this measure, and Figure 1 visualizes the full distributions of throws for each combination of condition and velocity band. To quantify discrimination, we again fit Bayesian Mixed Models as above, but this time the dependent variable was the raw x velocity generated by participants on each testing trial.

$$vx_{ij} = \beta_0 + \beta_1 \cdot condit_{ij} + \beta_2 \cdot bandInt_{ij} + \beta_3 \cdot condit_{ij} \cdot bandInt_{ij} + b_{0i} + b_{1i} \cdot bandInt_{ij} + \epsilon_{ij} \quad (2)$$

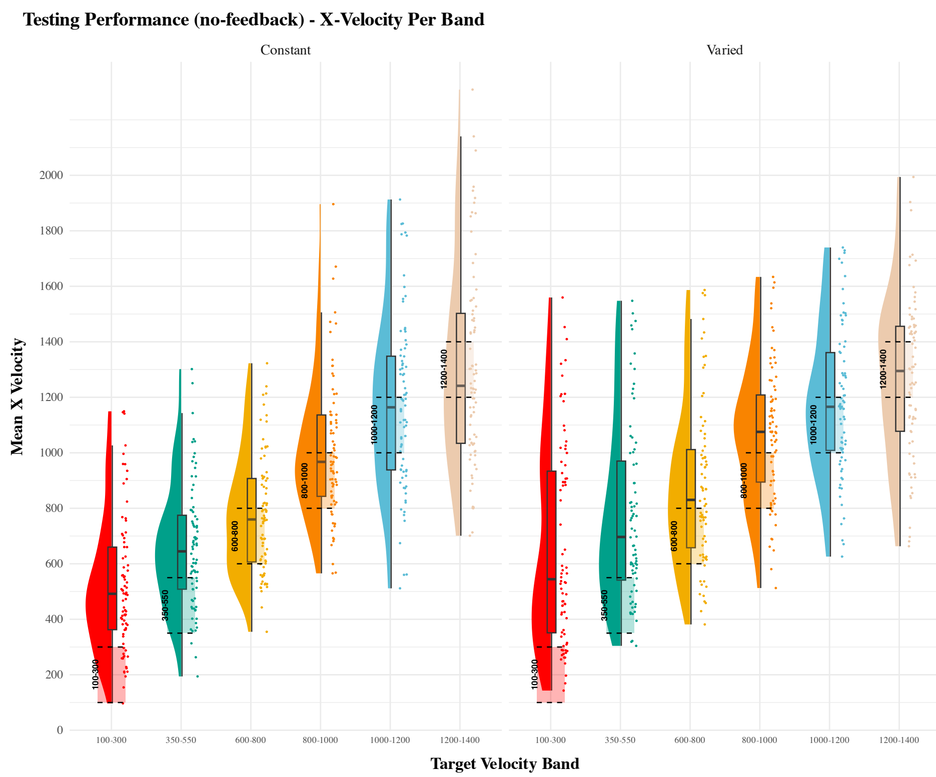


Figure 2: E1 testing x velocities. Translucent bands with dash lines indicate the correct range for each velocity band.

Table 6: Testing vx - Empirical Summary

Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	524	448	327
350-550	Extrapolation	659	624	303
600-800	Extrapolation	770	724	300
800-1000	Trained	1001	940	357
1000-1200	Extrapolation	1167	1104	430
1200-1400	Extrapolation	1283	1225	483
Band	Band Type	Mean	Median	Sd
100-300	Extrapolation	664	533	448
350-550	Extrapolation	768	677	402
600-800	Extrapolation	876	813	390
800-1000	Trained	1064	1029	370
1000-1200	Trained	1180	1179	372
1200-1400	Trained	1265	1249	412

Table 7: Experiment 1. Bayesian Mixed Model Predicting Vx as a function of condition (Constant vs. Varied) and Velocity Band

Table 8: Fit to all 6 bands

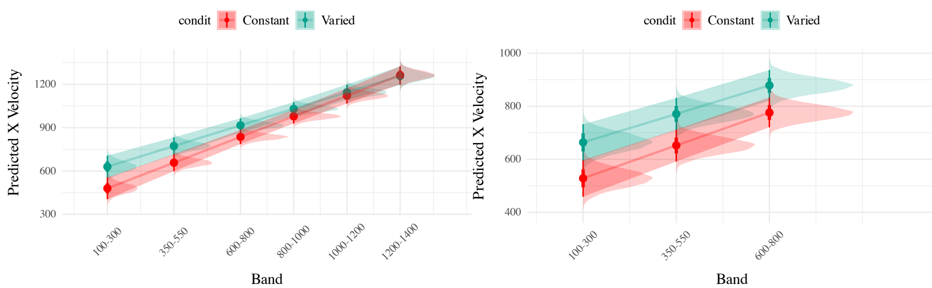
Term	Estimate	95% CrI Lower	95% CrI Upper	pd
Intercept	408.55	327.00	490.61	1.00
conditVaried	164.05	45.50	278.85	1.00
Band	0.71	0.62	0.80	1.00
condit*Band	-0.14	-0.26	-0.01	0.98

Table 9: Fit to 3 extrapolation bands

Term	Estimate	95% CrI Lower	95% CrI Upper	pd
Intercept	478.47	404.00	551.45	1.00
conditVaried	142.04	37.17	247.59	1.00
Band	0.50	0.42	0.57	1.00
condit*Band	-0.07	-0.17	0.04	0.89

See Table 7 for the full model results. The estimated coefficient for training condition (B = 164.05, 95% CrI [45.5, 278.85]) suggests that the varied group tends to produce harder throws than the constant group, but is not in and of itself useful for assessing discrimination. Most relevant to the issue of discrimination is the slope on Velocity Band (β = 0.71, 95% CrI [0.62, 0.8]). Although the median slope does fall underneath the ideal of value of 1, the fact that the 95% credible interval does not contain 0 provides strong evidence that participants exhibited some discrimination between bands. The estimate for the interaction between slope and condition (B = -0.14, 95% CrI [-0.26, -0.01]), suggests that the discrimination was somewhat modulated by training condition, with the varied participants showing less senitivity between vands than the constant condition. This difference is depicted visually in Figure 3.@tbl-e1-slope-quartile shows the average slope coefficients for varied and constant participants separately for each quartile. The constant participant participants appear to have larger slopes across quartiles, but the difference between conditions may be less pronounced for the top quartiles of subjects who show the strongest discrimination. Figure Figure 4 shows the distributions of slope values for each participant, and the compares the probability density of slope coefficients between training conditions. Figure 5

The second model, which focused solely on extrapolation bands, revealed similar patterns. The Velocity Band term (B = 0.5, 95% CrI [0.42, 0.57]) still demonstrates a high degree of discrimination ability. However, the posterior distribution for interaction term (β = -0.07, 95% CrI [-0.17, 0.04]) does across over 0, suggesting that the evidence for decreased discrimination ability for the varied participants is not as strong when considering only the three extrapolation bands.



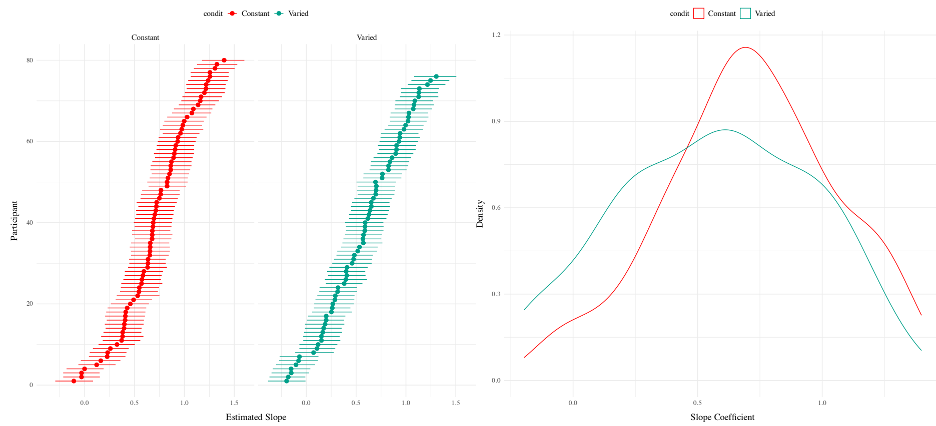
(a) Model fit to all 6 bands

(b) Model fit to only 3 extrapolation bands

Figure 3: Conditional effect of training condition and Band. Ribbons indicate 95% HDI. The steepness of the lines serves as an indicator of how well participants discriminated between velocity bands.

Table 10: Slope coefficients by quartile, per condition

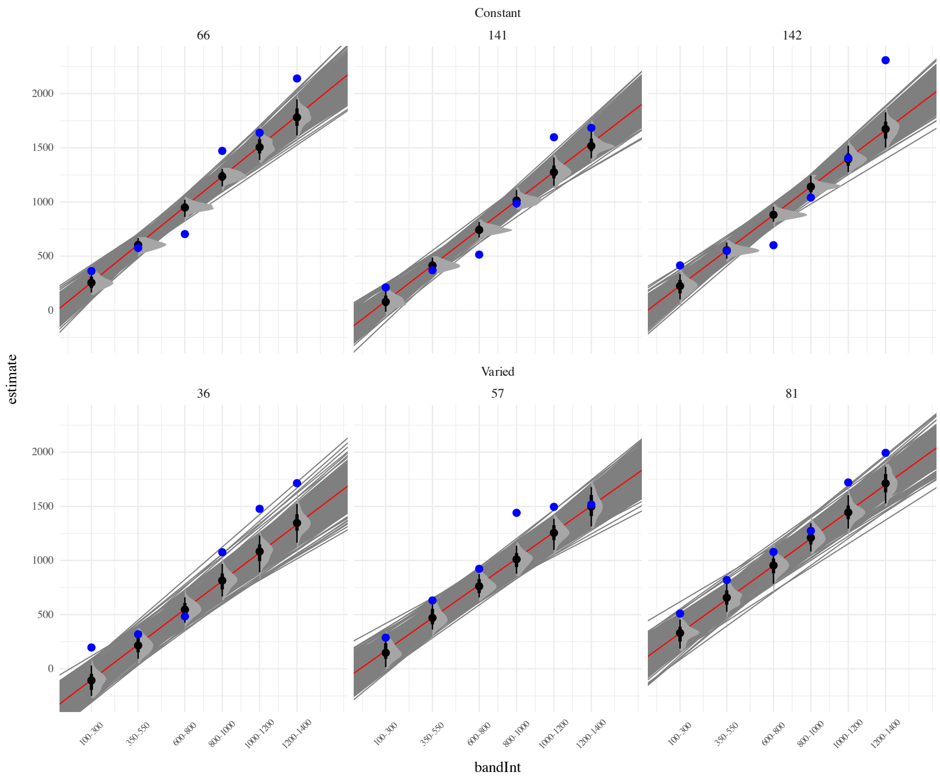
Condition	Q_0%_mean	Q_25%_mean	Q_50%_mean	Q_75%_mean	Q_100%_mean
Constant	-0.109	0.482	0.691	0.933	1.4
Varied	-0.197	0.265	0.588	0.899	1.3



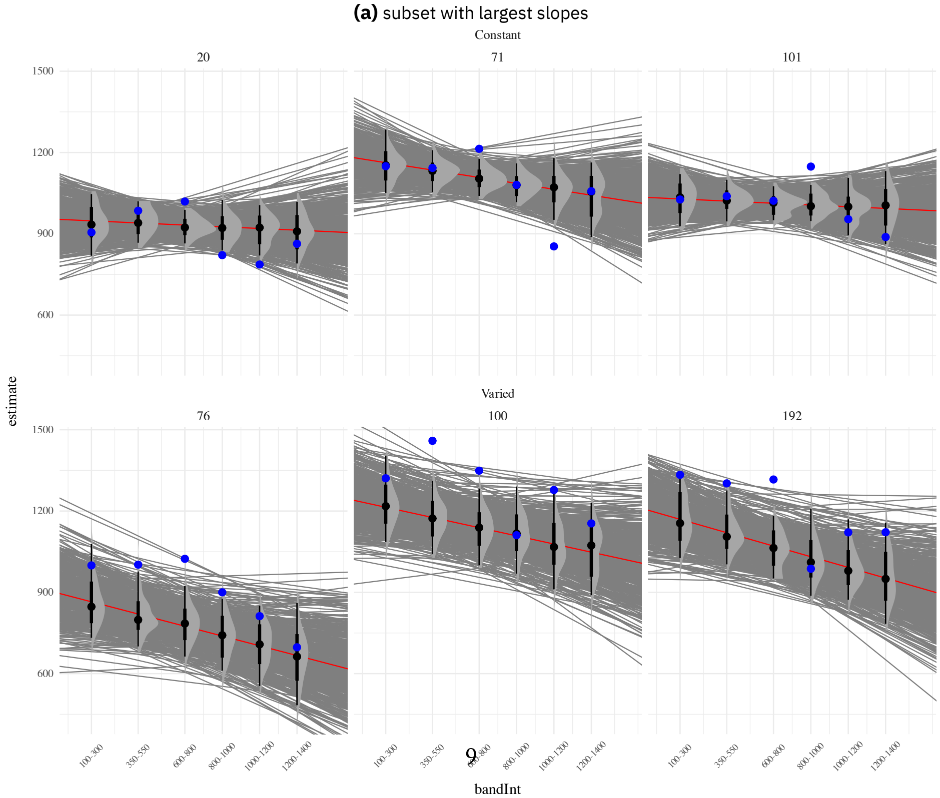
(a) Slope estimates by participant - ordered from lowest to highest within each condition.

(b) Destiny of slope coefficients by training group

Figure 4: Slope distributions between condition



(a) subset with largest slopes



(b) subset with smallest slopes

Figure 5: Subset of Varied and Constant Participants with the smallest and largest estimated slope values. Red lines represent the best fitting line for each participant, gray lines are 200 random samples from the posterior distribution. Colored points and intervals at each band represent the empirical median and 95% HDI.

Experiment 2

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