c) For $\frac{dC_A}{dt} + \left(\frac{1}{\tau} + 0.5k\right)C_A = \frac{1}{\tau}C_{A0}$

$$\frac{dC_A}{dt} + \left(\frac{1+0.5k\tau}{\tau}\right)C_A = \frac{1}{\tau}C_{A0}$$

@ $t=0$, $C_A = C_{A0}$

$$\frac{dC_A}{dt} = \frac{C_{A0} - (1+0.5k\tau)C_A}{\tau}$$

$$\int_{C_A^*}^{C_A(t)} \frac{dC_A}{C_{A0} - (1+0.5k\tau)C_A} = \int_0^t \frac{dt}{\tau}$$

$$-\frac{1}{1+0.5k\tau} \ln \frac{C_{A0} - (1+0.5k\tau)C_A(t)}{C_{A0} - (1+0.5k\tau)C_A^*} = \frac{t}{\tau}$$

$$C_{A0} - (1+0.5k\tau)C_A(t) = \exp\left(-(1+0.5k\tau)\frac{t}{\tau}\right)\left(C_{A0} - (1+0.5k\tau)C_A^*\right)$$

$$\boxed{C_A(t) = \frac{C_{A0}}{1+0.5k\tau} + \left(C_A^* - \frac{C_{A0}}{1+0.5k\tau}\right)\exp\left(-\frac{(1+0.5k\tau)t}{\tau}\right)}$$

The new steady state value for C_A after power failure is C_A^*

d) For $\frac{dC_A}{dt} + \frac{1}{\tau}C_A = 0.5kC_A$

$$\frac{dC_A}{dt} = \frac{0.5k\,C_{A0}}{1+0.5k\tau} + 0.5k\left(C_A^* - \frac{C_{A0}}{1+0.5k\tau}\right)\exp\left(-\frac{(1+0.5k\tau)t}{\tau}\right)$$

Using I.F solution method

$$\frac{dY(t)}{dt} + P(t)Y = Q(t)$$

$$P(t) = \frac{1}{\tau}$$

$$Q(t) = \frac{0.5kC_{A0}}{1+0.5k\tau} + 0.5k\left(C_A^* - \frac{C_{A0}}{1+0.5k\tau}\right)\exp\left[-\frac{(1+0.5k\tau)t}{\tau}\right]$$

$$IF = \exp\left(\int P(t)dt\right) = \exp\int \frac{1}{\tau}dt = \exp\frac{t}{\tau}$$

PROBLEM 2

$$A \rightarrow B$$

$$-r_A = kC_A$$

a) Before partial power failure,

$$\frac{dN_A}{dt} = (\nu C_A)_{in} - (\nu C_A)_{out} + Vr_A$$

$$\frac{dN_B}{dt} = (\nu C_B)_{in} - (\nu C_B)_{out} + Vr_B$$

Given $r_B = -r_A = kC_A$

Assuming constant density $\nu_{in} = \nu_{out} = \nu_0$

$$\frac{dN_A}{dt} = \frac{d(VC_A)}{dt} = V\frac{dC_A}{dt} = (\nu C_A)_{in} - (\nu C_A)_{out} - VkC_A$$

$$V\frac{dC_B}{dt} = (\nu C_B)_{in} - (\nu C_B)_{out} + VkC_A$$

$$\left.\begin{aligned} \frac{dC_A}{dt} &= \frac{C_{A0} - C_A^*}{\tau} - kC_A^* = 0 \\ \frac{dC_B}{dt} &= -\frac{C_B^*}{\tau} + kC_A^* = 0 \end{aligned}\right\} t < 0$$

$$C_A^*(k\tau + 1) = C_{A0} \Rightarrow \boxed{C_A^* = \frac{C_{A0}}{k\tau + 1}}$$

$$C_B^* = kC_A^* = \boxed{\frac{k\tau C_{A0}}{k\tau + 1}}$$

b) After partial failure.

$$-r_A = 0.5kC_A$$

$$\frac{dC_A}{dt} = \frac{C_{A0}}{\tau} - \frac{C_A}{\tau} - 0.5kC_A$$

$$\boxed{\begin{aligned} \frac{dC_A}{dt} &= \frac{C_{A0}}{\tau} - C_A\left(\frac{1}{\tau} + 0.5k\right) \\ \frac{dC_B}{dt} &= -\frac{C_B}{\tau} + 0.5kC_A \end{aligned}}$$

CHE [illegible]28 HOMEWORK

PROBLEM 1

a) $$\frac{d}{dt}\left[m\left(\hat{U}+\frac{v^2}{2}+gz\right)\right]_{sys} = \left[\dot{m}\left(\hat{H}+\frac{v^2}{2}+gz\right)\right]_{in} - \left[\dot{m}\left(\hat{H}+\frac{v^2}{2}+gz\right)\right]_{out}$$

$$+ \frac{dQ}{dt} + \frac{dW_{non\text{-}flow}}{dt} + \frac{dE_{gen}}{dt}$$

Assumptions: Zero energy generation

Constant-density

Zero kinetic energy effect

Zero potential energy effect

Zero non work flow

Negligible PV, Constant density

Simplifies to: $\rho V_0 \hat{C}_p \frac{dT}{dt} = \rho \dot{v}_0 \hat{C}_p (T_{in} - T_{out}) + \frac{dQ}{dt}$

$$\frac{dT}{dt} = \frac{T_{in} - T_{out}}{\tau} + \frac{1}{\rho \hat{C}_p V_0}\frac{dQ}{dt} \quad \text{when } \tau = \frac{V_0}{\dot{v}_0}$$

For:

$0 < t < t_1$ $(T < T_{on})$, $\frac{dQ}{dt} = 0 \Rightarrow \frac{dT}{dt} = \frac{T_0}{\tau} - \frac{T}{\tau}$

$t \geq t_1$ $(T_{off} < T < T_{on}) \Rightarrow \frac{dQ}{dt} = Q$

$$\frac{dT}{dt} = \frac{T_0}{\tau} + \frac{Q}{\rho \hat{C}_p V_0} - \frac{T}{\tau}$$

b) Idealized temperature as a function of time before refrigeration unit is turned on

$0 < t < t_1$ $(T < T_{on} \Rightarrow \frac{dT}{dt} = \frac{T_0 - T}{\tau}$

$$\int_{T^\circ}^{T(t) \leq T_{on}} \frac{1}{T_0 - T}\, dT = \int_0^{t \leq t_{on}} \frac{1}{\tau}\, dt$$

$$-\ln \frac{T_0 - T(t)}{T_0 - T^\circ} = \frac{t}{\tau}$$

$$\boxed{T(t) = T_0 - (T_0 - T^\circ)\exp(-t/\tau)}$$

c) Time at which refrigeration unit is turned on.

$$\boxed{t_{on} = \tau \ln \frac{T_0 - T^*}{T_0 - T_{on}}}$$

d) Water temperature in the reservoir as a function of time after refrigeration unit is turned on. $(t \geq t_1)$

$$\frac{dT}{dt} = \frac{T_0}{\tau} - \frac{T}{\tau} + \frac{Q}{\rho \hat{C}_p V_0} = \Omega - \frac{T}{\tau}$$

let $\Omega = \frac{Q}{\rho \hat{C}_p V_0} + \frac{T_0}{\tau}$

$$\frac{dT}{dt} = \frac{\Omega\tau - T}{\tau}$$

$$\int_{T_{on}}^{T(t) \leq T_{off}} \frac{dT}{\Omega\tau - T} = \int_{t_{on}}^{t_1 \leq t} \frac{dt}{\tau}$$

$$-\ln \frac{\Omega\tau - T(t)}{\Omega\tau - T_{on}} = \frac{t - t_{on}}{\tau}$$

$$T(t) = \Omega t + (T_{on} - \Omega t) \exp\left[-\frac{(t - t_{on})}{\tau}\right]$$

e) Time at which it is turned off

$$\boxed{t_{off} = t_{on} + \tau \ln \frac{\Omega\tau - T_{on}}{\Omega\tau - T_{off}}}$$

Multiplying through by IF & rearranging:

$$\exp\left(\frac{t}{\tau}\right)\left(\frac{dC_R}{dt} + \frac{C_R}{\tau}\right) = \frac{d(C_R \exp(t/\tau))}{dt}$$

$$= \frac{0.5kC_{A0}\exp(t/\tau)}{1+0.5k\tau} + 0.5k\left(C_A^{\circ} - \frac{C_{A0}}{1+0.5k\tau}\right)\exp\left[\frac{t}{\tau} - \frac{(1+0.5k\tau)t}{\tau}\right]$$

$$= \frac{0.5kC_{A0}\exp(t/\tau)}{1+0.5k\tau} + 0.5k\left(C_A^{\circ} - \frac{C_{A0}}{1+0.5k\tau}\right)\exp(-0.5kt)$$

Seperating variables and integrating both sides

$$\int_{C_R^{\circ}}^{C_R\exp(t/\tau)} d\left[C_R\exp(t/\tau)\right] = \int_0^t \left[\frac{0.5kC_{A0}\exp(t/\tau)}{1+0.5k\tau}\right] dt$$

$$+ \int_0^t 0.5k\left(C_A^{\circ} - \frac{C_{A0}}{1+0.5k\tau}\right)\exp(-0.5kt)\, dt$$

$$C_R\exp(t/\tau) - C_R^{\circ} = \frac{0.5k\tau C_{A0}}{1+0.5k\tau}\left[\exp\left(\frac{t}{\tau}\right) - 1\right]$$

$$- \left(C_A^{\circ} - \frac{C_{A0}}{1+0.5k\tau}\right)\left[\exp(-0.5kt) - 1\right]$$

$$C_R\exp(t/\tau) = C_R^{\circ} - \frac{0.5k\tau C_{A0}}{1+0.5k\tau} + C_A^{\circ} - \frac{C_{A0}}{1+0.5k\tau} + \frac{0.5k\tau C_{A0}\exp\left(\frac{t}{\tau}\right)}{1+0.5k\tau}$$

$$- \left(C_A^{\circ} - \frac{C_{A0}}{1+0.5k\tau}\right)\exp(-0.5kt)$$

$$C_R\exp(t/\tau) = \underbrace{C_R^{\circ} + C_A^{\circ} - C_{A0}}_{0} + \frac{0.5k\tau C_{A0}\exp\left(\frac{t}{\tau}\right)}{1+0.5k\tau}$$

$$+ \left(\frac{C_{A0}}{1+0.5k\tau} - C_A^{\circ}\right)\exp(-0.5kt)$$

$$C_R(t) = \frac{0.5k\tau C_{A0}\exp\left(\frac{t}{\tau} - \frac{t}{\tau}\right)}{1+0.5k\tau} + \left(\frac{C_{A0}}{1+0.5k\tau} - C_A^{\circ}\right)\exp\left(-0.5kt - \frac{t}{\tau}\right)$$

$$\boxed{C_R(t) = \frac{0.5k\tau C_{A0}}{1+0.5k\tau} + \left(\frac{C_{A0}}{1+0.5k\tau} - C_A^{\circ}\right)\exp\left(-(1+0.5k\tau)\frac{t}{\tau}\right)}$$

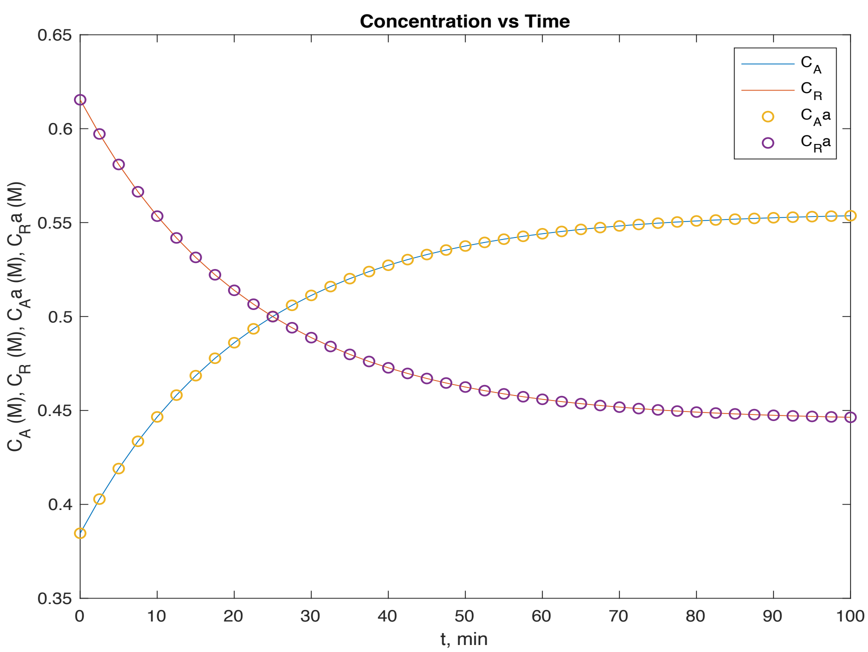
HOMEWORK 1
PROBLEM 1

```
function dT = HW1file2(t,T)

global rho Cp Vo vo tau Tzero To Ton Toff Qo Omega ton toff k

if (k==0)
    dT = (To - T)/tau;
end

if (T > Ton) | (k==1)
    dT = Omega - T/tau;
    k = 1;
end

if (T < Toff)
    dT = (To - T)/tau;
    k = 0;
end
```

```
function y = HW1file3ana(tt)

global rho Cp Vo vo tau Tzero To Ton Toff Qo Omega ton toff k

if tt > toff
    a = fix(tt/toff);
    tt = tt - a*toff;
end

if tt < ton
    y = To + (Tzero - To)*exp(-tt/tau);
else
    y = Omega*tau + (Ton - Omega*tau)*exp(-(tt-ton)/tau);
end
```

```
clc
clf
clear
global rho Cp Vo vo tau Tzero To Ton Toff Qo Omega ton toff k

k = 0;      Vo = 4.0;    vo = 0.50;  tau = Vo/vo;rho = 1.0;  Cp = 1.0;
Tzero = 8.0;      To = 18.0;  Ton = 12.0;      Toff = 8.0; Qo = -6.0;

Omega = To/tau + Qo/(rho*Cp*Vo);
ton = tau*log((Tzero - To)/(Ton - To));
toff = ton + tau*log((Omega*tau - Ton)/(Omega*tau - Toff));
t0 = 0;
tf = 25.0;
tspan = [t0 tf];
```

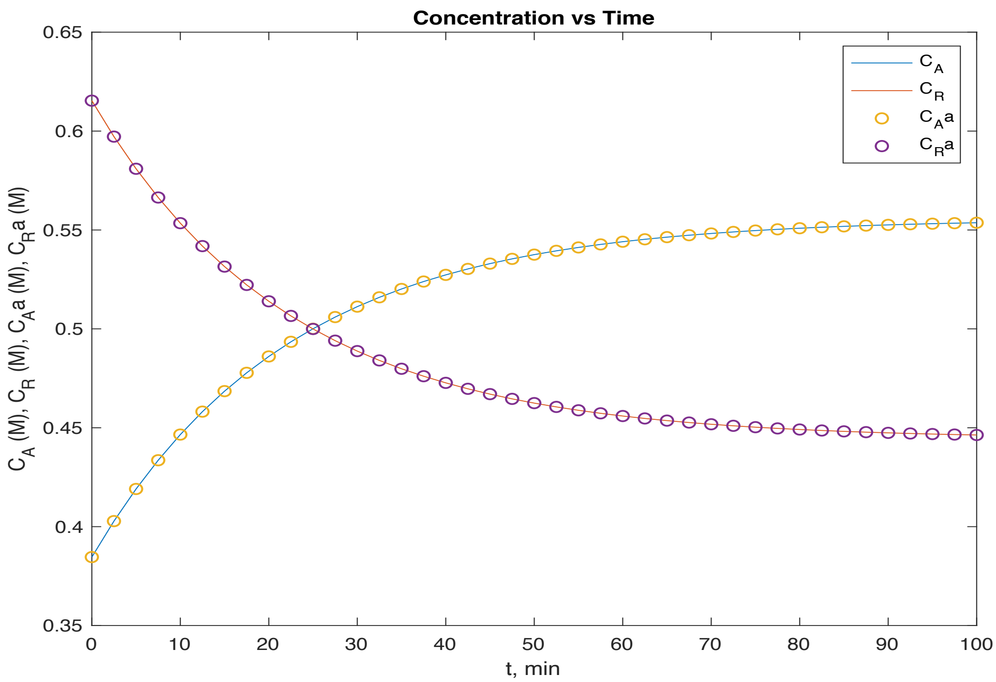
```
[t, T] = ode45('HW1file2', tspan, Tzero);     % for "ode45" solver

  for i = 1:45
      Ta(i) = HW1file3ana(t(i));
  end

% Plot the results on the same figure.
  plot (t, T(:), '-', t, Ta(:), 'o'), title ('Temperature vs time'), ...
       legend ('T', 'Ta'), xlabel ('t, min'), ylabel ('T (oC), Ta (oC)');
```



PROBLEM 2

```
function dC = HW1file21(t,C)
global Cao k Vo v tau

dCa = 1/tau*(Cao) - (1/tau + 0.5*k).*C(1);
dCr = 0.5*k*C(1) - 1/tau * C(2);

dC = [dCa; dCr];


function Can = HW1file23ana(t)

global Cao k Vo v tau Castar Crstar

  Caa = (Cao/(1+0.5*k*tau))-((Cao/(1+0.5*k*tau))-Castar)*exp(-
(1+0.5*k*tau)*(t/tau));

  Cra = ((0.5*k*tau*Cao)/(1+0.5*k*tau))+((Cao/(1+0.5*k*tau))-Castar)*exp(-
(1+0.5*k*tau)*(t/tau));

  Can = [Caa Cra];



clc
clf
clear
global Cao k Vo v tau Castar Crstar

k = 0.04;                %min^-1
Vo = 1000;               %L
Cao = 1;                 %M
v = 25;                  %L/min
tau = Vo/v;

Castar = Cao./(1 + k*tau);
Crstar = k*tau *Castar;


Co = [Castar Crstar];

t0 = 0;
tf = 100;
tspan = [t0 tf];

[t, C] = ode45('HW1file21', tspan, Co);    % for "ode45" solver

for i = 1:41
    Cana(i,:) = HW1file23ana(t(i));
end

% Plot the results on the same figure.
```

```
plot (t, C(:,1), '-',t, C(:,2), '-', t, Cana(:,1),'o',t,Cana(:,2),'o'), title
('Concentration vs Time'), ...
legend ('C_A', 'C_R', 'C_Aa', 'C_Ra'), xlabel ('t, min'), ylabel ('C_A (M),
C_R (M), C_Aa (M), C_Ra (M)');
```



PROBLEM 2

```
clc
clf
clear
global rho Cp Vo vo tau Tzero To Ton Toff Qo Omega ton toff k

k = 0;
Vo = 4.0;
vo = 0.50;
tau = Vo/vo;

rho = 1.0;
Cp = 1.0;
Tzero = 8.0;
To = 18.0;

Ton = 12.0;
Toff = 8.0;
Qo = -6.0;
Omega = To/tau + Qo/(rho*Cp*Vo);

ton = tau*log((Tzero - To)/(Ton - To));
toff = ton + tau*log((Omega*tau - Ton)/(Omega*tau - Toff));
t0 = 0;
tf = 25.0;
tspan = [t0 tf];

[t, T] = ode45('HW1file2', tspan, Tzero);    % for "ode45" solver

  for i = 1:45
      Ta(i) = HW1file3ana(t(i));
  end

% Plot the results on the same figure.
  plot (t, T(:), '-', t, Ta(:), 'o'), title ('Temperature vs time'), ...
       legend ('T', 'Ta'), xlabel ('t, min'), ylabel ('T (oC), Ta (oC)');


function y = HW1file3ana(tt)

global rho Cp Vo vo tau Tzero To Ton Toff Qo Omega ton toff k

if tt > toff
    a = fix(tt/toff);
    tt = tt - a*toff;
end

if tt < ton
    y = To + (Tzero - To)*exp(-tt/tau);
else
    y = Omega*tau + (Ton - Omega*tau)*exp(-(tt-ton)/tau);
end
```

```
function dT = HW1file2(t,T)

global rho Cp Vo vo tau Tzero To Ton Toff Qo Omega ton toff k

if (k==0)
    dT = (To - T)/tau;
end

if (T > Ton) | (k==1)
    dT = Omega - T/tau;
    k = 1;
end

if (T < Toff)
    dT = (To - T)/tau;
    k = 0;
end
```