Package 'HMC'

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Title High Dimensional Mean Comparison with Projection and

Cross-Fitting

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|--|
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| Description Provides interpretable High-dimensional Mean Comparison methods (HMC). For example, users can use them to assess the difference in gene expression between two treatment groups. It is not a gene-by-gene comparison. Instead, we focus on the interplay between features and are interested in those that are predictive of the group label. The methods are valid frequentist tests and give sparse estimates indicating which features contribute to the test results. |
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```
anchored_lasso_testing
```

Anchored test for two-sample mean comparison.

Description

Anchored test for two-sample mean comparison.

Usage

```
anchored_lasso_testing(
  sample_1,
  sample_2,
  pca_method = "sparse_pca",
  mean_method = "lasso",
  num_latent_factor = 1,
  n_folds = 5,
  verbose = TRUE
)
```

Arguments

| | sample_1 | Group 1 sample. Each row is a subject and each column corresponds to a feature. | | |
|-------------------|-------------|---|--|--|
| | sample_2 | Group 2 sample. Each row is a subject and each column corresponds to a feature. | | |
| | pca_method | Methods used to estimate principle component The default is "sparse_pca", using sparse PCA from package PMA. Other choices are "dense_pca"—the regular PCA; and "hard"— hard-thresholding PCA, which also induces sparsity. | | |
| | mean_method | Methods used to estimate the mean vector. Default is sample mean "naive". There is also a hard-thresholding sparse estiamtor "hard". | | |
| num_latent_factor | | | | |
| | | The principle component that lasso coefficient anchors at. The default is $PC1 = 1$. | | |
| | n_folds | Number of splits when performing cross-fitting. The default is 5 , if computational time allows, you can try to set it to 10 . | | |
| | verbose | Print information to the console. Default is TRUE. | | |
| | | | | |

Value

```
A list of test statistics.
```

```
test_statistics
```

Test statistics. Each entry corresponds to the test result of one principle component.

 $standard_error \ \ Estimated \ standard \ error \ of \ test_statistics_before_studentization.$

test_statistics_before_studentization

Similar to test_statistics but does not have variance = 1.

split_data Intermediate quantities needed for further assessment and interpretation of the test results.

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Examples

debiased_pc_testing

Debiased one-step test for two-sample mean comparison. A small p-value tells us not only there is difference in the mean vectors, but can also indicates which principle component the difference aligns with.

Description

Debiased one-step test for two-sample mean comparison. A small p-value tells us not only there is difference in the mean vectors, but can also indicates which principle component the difference aligns with.

Usage

```
debiased_pc_testing(
  sample_1,
  sample_2 = NULL,
  pca_method = "sparse_pca",
  mean_method = "naive",
  num_latent_factor = 1,
  n_folds = 5,
  verbose = TRUE
)
```

Arguments

| sample_1 | Group 1 sample. Each row is a subject and each column corresponds to a feature. |
|-------------|--|
| sample_2 | Group 2 sample. Each row is a subject and each column corresponds to a feature. |
| pca_method | Methods used to estimate principle component The default is "sparse_pca", using sparse PCA from package PMA. Other choices are "dense_pca"—the regular PCA; and "hard"—hard-thresholding PCA, which also induces sparsity. |
| mean_method | Methods used to estimate the mean vector. Default is sample mean "naive". There is also a hard-thresholding sparse estiamtor "hard". |

num_latent_factor

Number of principle to be estimated/tested. Default is 1.

n_folds Number of splits when performing cross-fitting. The default is 5, if computa-

tional time allows, you can try to set it to 10.

verbose Print information to the console. Default is TRUE.

Value

A list of test statistics.

test_statistics

Test statistics. Each entry corresponds to the test result of one principle component

standard_error Estimated standard error of test_statistics_before_studentization.

test_statistics_before_studentization

Similar to test_statistics but does not have variance = 1.

test results.

Examples

```
sample_size_1 <- sample_size_2 <- 300</pre>
true_mean_1 <- matrix(c(rep(1, 10), rep(0, 90)), ncol = 1)
true_mean_2 <- matrix(c(rep(1.5, 10), rep(0, 90)), ncol = 1)
pc1 <- c(rep(1, 10), rep(0, 90))
pc1 \leftarrow pc1/norm(pc1, type = '2')
simulation_covariance <- 10 * pc1 %*% t(pc1)</pre>
simulation_covariance <- simulation_covariance + diag(1, 100)</pre>
sample_1 <- data.frame(MASS::mvrnorm(sample_size_1,</pre>
                                 mu = true_mean_1,
                                 Sigma = simulation_covariance))
 sample_2 <- data.frame(MASS::mvrnorm(sample_size_2,</pre>
                                 mu = true_mean_2,
                                 Sigma = simulation_covariance))
 result <- debiased_pc_testing(sample_1, sample_2)</pre>
 result$test_statistics
 ##these are test statistics. Each one of them corresponds to one PC.
summarize\_pc\_name(result, \ latent\_fator\_index = 1) \ \#shows \ which \ features \ contribute \ to \ PC1
 extract_pc(result) # extract the estimated leading PCs.
```

```
estimate_nuisance_parameter_lasso
```

The function for nuisance parameter estimation in anchored_lasso_testing().

Description

The function for nuisance parameter estimation in anchored_lasso_testing().

Usage

```
estimate_nuisance_parameter_lasso(
  nuisance_sample_1,
  nuisance_sample_2,
  pca_method = "sparse_pca",
  mean_method = "lasso",
  num_latent_factor = 1,
  local_environment = local_environment,
  verbose = TRUE
)
```

Arguments

nuisance_sample_1

Group 1 sample. Each row is a subject and each column corresponds to a feature.

nuisance_sample_2

Group 2 sample. Each row is a subject and each column corresponds to a feature.

pca_method

Methods used to estimate principle component The default is "sparse_pca", using sparse PCA from package PMA. Other choices are "dense_pca"—the regular PCA; and "hard"— hard-thresholding PCA, which also induces sparsity.

mean_method

Methods used to estimate the discriminant direction. Default is logistic Lasso "lasso".

num_latent_factor

The principle component that lasso coefficient anchors at. The default is PC1 = 1

local_environment

A environment for hyperparameters shared between folds.

verbose

Print information to the console. Default is TRUE.

Value

A list of estimated nuisance quantities.

```
estimate_leading_pc
```

Leading principle components

estimate_mean_1

Sample mean for group 1

estimate_mean_2

Sample mean for group 1

estimate_lasso_beta

Logistic Lasso regression coefficients.

 ${\tt estimate_projection_direction}$

Anchored projection direction. It is similar to PC1 when signal is weak but similar to estimate_optimal_direction when the signal is moderately large.

 ${\tt estimate_optimal_direction}$

Discriminant direction.

6 estimate_nuisance_pc

Description

The function for nuisance parameter estimation in simple_pc_testing() and debiased_pc_testing().

Usage

```
estimate_nuisance_pc(
  nuisance_sample_1,
  nuisance_sample_2 = NULL,
  pca_method = "sparse_pca",
  mean_method = "naive",
  num_latent_factor = 1,
  local_environment = NA
)
```

Arguments

```
nuisance_sample_1
```

Group 1 sample. Each row is a subject and each column corresponds to a feature.

nuisance_sample_2

Group 2 sample. Each row is a subject and each column corresponds to a feature.

pca_method

mean_method

Methods used to estimate principle component The default is "sparse_pca", using sparse PCA from package PMA. Other choices are "dense_pca"—the regular

PCA; and "hard"— hard-thresholding PCA, which also induces sparsity.

Methods used to estimate the mean vector. Default is sample mean "naive". There is also a hard-thresholding sparse estiamtor "hard".

num_latent_factor

Number of principle to be estimated/tested. Default is 1.

local_environment

A environment for hyperparameters shared between folds.

Value

```
A list of estimated nuisance quantities.
```

Noise variance, I need this to construct block-diagonal estimates of the covariance matrix.

```
evaluate_influence_function_multi_factor

Calculate the test statistics on the left-out samples. Called in debi-
ased_pc_testing().
```

Description

Calculate the test statistics on the left-out samples. Called in debiased_pc_testing().

Usage

```
evaluate_influence_function_multi_factor(
  cross_fitting_sample_1,
  cross_fitting_sample_2 = NULL,
  nuisance_collection,
  num_latent_factor = 1
)
```

Arguments

```
cross_fitting_sample_1
```

Group 1 sample. Each row is a subject and each column corresponds to a feature.

cross_fitting_sample_2

Group 2 sample. Each row is a subject and each column corresponds to a feature.

nuisance_collection

A collection of nuisance quantities estimated using "nuisance" samples. It is the output of estimate_nuisance_pc().

num_latent_factor

Number of principle components to be considered.

Value

```
evaluate_pca_lasso_plug_in
```

Calculate the test statistics on the left-out samples. Called in anchored_lasso_testing().

Description

Calculate the test statistics on the left-out samples. Called in anchored_lasso_testing().

Usage

```
evaluate_pca_lasso_plug_in(
  cross_fitting_sample_1,
  cross_fitting_sample_2,
  nuisance_collection,
  mean_method = "lasso"
)
```

Arguments

```
cross_fitting_sample_1
```

Group 1 sample. Each row is a subject and each column corresponds to a feature.

cross_fitting_sample_2

Group 2 sample. Each row is a subject and each column corresponds to a feature.

nuisance_collection

A collection of nuisance quantities estimated using "nuisance" samples. It is the output of estimate nuisance pc()

output of estimate_nuisance_pc().

 ${\tt mean_method}$

Methods used to estimate the discriminant direction. Default is logistic Lasso "lasso".

Value

```
A list of test statistics.
```

evaluate_pca_plug_in 9

Description

Calculate the test statistics on the left-out samples. Called in simple_pc_testing().

Usage

```
evaluate_pca_plug_in(
  cross_fitting_sample_1,
  cross_fitting_sample_2 = NULL,
  nuisance_collection
)
```

Arguments

```
cross_fitting_sample_1
Group 1 sample. Each row is a subject and each column corresponds to a feature.

cross_fitting_sample_2
Group 2 sample. Each row is a subject and each column corresponds to a feature.
```

nuisance_collection

A collection of nuisance quantities estimated using "nuisance" samples. It is the output of estimate_nuisance_pc().

Value

```
A list of test statistics.

influence_each_subject_1

Statistics for sample 1.

influence_each_subject_2

Statistics for sample 2.
```

extract_lasso_coef

Extract the lasso estimate from the output of anchored_lasso_testing().

Description

Extract the lasso estimate from the output of anchored_lasso_testing().

Usage

```
extract_lasso_coef(testing_result)
```

Arguments

testing_result The output/test result list from anchored_lasso_testing().

index_spliter

Value

A list, whose elements are the estimated discriminant directions for each split—the length of the output list is the same as n_folds.

The discriminant vectors for each split.

| extract_pc | Extract | the | principle | components | from | the | output | of | sim- |
|------------|----------|--------|-------------|----------------|-------|-----|--------|----|------|
| | ple_pc_i | testin | g() and deb | iased_pc_testi | ng(). | | | | |

Description

Extract the principle components from the output of simple_pc_testing() and debiased_pc_testing().

Usage

```
extract_pc(testing_result)
```

Arguments

testing_result The output/test result list from simple_pc_testing() or debiased_pc_testing().

Value

A list, whose elements are the estimated PC for each split—the length of the output list is the same as n_folds.

The PC vectors for each split.

| index_spliter Split the sample index into n_folds many groups so that we can perform cross-fitting | index_spliter | Split the sample index into n_folds many groups so that we can perform cross-fitting |
|--|---------------|--|
|--|---------------|--|

Description

Split the sample index into n_folds many groups so that we can perform cross-fitting

Usage

```
index_spliter(array, n_folds = 5)
```

Arguments

Sample index. Usually just an array from 1 to the number of samples in one

group.

n_folds Number of splits

Value

A list indicates the sample indices in each split.

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simple_pc_testing

Simple plug-in test for two-sample mean comparison.

Description

Simple plug-in test for two-sample mean comparison.

Usage

```
simple_pc_testing(
  sample_1,
  sample_2 = NULL,
  pca_method = "sparse_pca",
  mean_method = "naive",
  num_latent_factor = 1,
  n_folds = 5,
  verbose = TRUE
)
```

Arguments

| sample_1 | Group 1 sample. Each row is a subject and each column corresponds to a feature. | | | | |
|-------------------|--|--|--|--|--|
| sample_2 | Group 2 sample. Each row is a subject and each column corresponds to a feature. | | | | |
| pca_method | Methods used to estimate principle component The default is "sparse_pca", using sparse PCA from package PMA. Other choices are "dense_pca"—the regular PCA; and "hard"—hard-thresholding PCA, which also induces sparsity. | | | | |
| mean_method | Methods used to estimate the mean vector. Default is sample mean "naive". There is also a hard-thresholding sparse estiamtor "hard". | | | | |
| num_latent_factor | | | | | |
| | Number of principle to be estimated/tested. Default is 1. | | | | |
| n_folds | Number of splits when performing cross-fitting. The default is 5, if computational time allows, you can try to set it to 10. | | | | |
| verbose | Print information to the console. Default is TRUE. | | | | |

Value

A list of test statistics.

```
test_statistics
```

Test statistics. Each entry corresponds to the test result of one principle component.

 $standard_error \ \ Estimated \ standard \ error \ of \ test_statistics_before_studentization.$

test_statistics_before_studentization

Similar to test_statistics but does not have variance = 1.

test results.

Examples

```
sample_size_1 <- sample_size_2 <- 300</pre>
true_mean_1 \leftarrow matrix(c(rep(1, 10), rep(0, 90)), ncol = 1)
true_mean_2 \leftarrow matrix(c(rep(1.5, 10), rep(0, 90)), ncol = 1)
pc1 <- c(rep(1, 10), rep(0, 90))
pc1 <- pc1/norm(pc1, type = '2')
simulation_covariance <- 10 * pc1 %*% t(pc1)</pre>
simulation_covariance <- simulation_covariance + diag(1, 100)</pre>
sample_1 <- data.frame(MASS::mvrnorm(sample_size_1,</pre>
                                       mu = true_mean_1,
                                       Sigma = simulation_covariance))
sample_2 <- data.frame(MASS::mvrnorm(sample_size_2,</pre>
                                       mu = true_mean_2,
                                       Sigma = simulation_covariance))
result <- simple_pc_testing(sample_1, sample_2)</pre>
result$test_statistics
##these are test statistics. Each one of them corresponds to one PC.
summarize_pc_name(result, latent_fator_index = 1) #shows which features contribute to PC1
extract_pc(result) # extract the estimated leading PCs.
```

summarize_feature_name

Summarize the features (e.g. genes) that contribute to the test result, i.e. those features consistently show up in Lasso vectors.

Description

Summarize the features (e.g. genes) that contribute to the test result, i.e. those features consistently show up in Lasso vectors.

Usage

```
summarize_feature_name(testing_result, method = "majority voting")
```

Arguments

testing_result The output/test result list from anchored_lasso_testing().

method

How to combine the feature list across different splits. Default is 'majority voting'—features that show up more than 50% of the splits are considered active/useful. It can be 'union'—all the features pooled together; or 'intersection'—only include features showing up in all splits.

Value

A list of names of features (your very original input data need to have column names!) that contribute to the test result. An empty list means there is barely any difference between the two groups.

Feature names that consistently showing up in the discriminant vectors.

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summarize_pc_name

Summarize the features (e.g. genes) that contribute to the test result, i.e. those features consistently show up in the sparse principle components.

Description

Summarize the features (e.g. genes) that contribute to the test result, i.e. those features consistently show up in the sparse principle components.

Usage

```
summarize_pc_name(
  testing_result,
  latent_fator_index = 1,
  method = "majority voting"
)
```

Arguments

 $testing_result \ \ The \ output/test \ result \ list \ from \ simple_pc_testing() \ or \ debiased_pc_testing().$ $latent_fator_index$

Which principle component should the algorithm summarize? Default is PC1.

method

How to combine the feature list across different splits. Default is 'majority voting'—features that show up more than 50% of the splits are considered active/useful. It can be 'union'—all the features pooled together; or 'intersection'—only include features showing up in all splits.

Value

A list of names of features (your very original input data need to have column names!) that contribute to the test result.

Feature names that consistently showing up in the estimated PC vectors.

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