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Assignment 1
ME714: Computer Integrated Manufacturing

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1 Question 1

No, matrix multiplication is NOT commutative, ie., given 2 matrices P & Q, $PQ \neq QP$.

For the given triangle, the vertices (in Cartesian coordinates) are given by:

$$A \equiv (3, 0)$$
$$B \equiv (0, 0)$$
$$C \equiv (0, 3)$$

In homogeneous coordinates, the vertices can be represented as a matrix $V = \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 3 & 1 \end{bmatrix}$

The transformations T & R can be represented in homogeneous coordinates as follows:

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

$$R = \begin{bmatrix} cos(\theta) & sin(\theta) & 0 \\ -sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} cos(-90^\circ) & sin(-90^\circ) & 0 \\ -sin(-90^\circ) & cos(-90^\circ) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Translation followed by Rotation:

The new vertices $V' = \begin{bmatrix} A' \\ B' \\ C' \end{bmatrix}$ are given by $V' = VTR$, where the overall transformation $T' = TR$.

Thus, $T' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix}$

Now, $V' = VT' = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -6 & 1 \\ 0 & -3 & 1 \\ 3 & -3 & 1 \end{bmatrix} = \begin{bmatrix} A' \\ B' \\ C' \end{bmatrix}$

Thus, the new vertices have the following Cartesian coordinates: $A' \equiv (0, -6), B' \equiv (0, -3), C' \equiv (3, -3)$

Rotation followed by Translation:

The new vertices $V'' = \begin{bmatrix} A'' \\ B'' \\ C'' \end{bmatrix}$ are given by $V'' = VRT$, where the overall transformation $T'' = RT$.

Thus, $T'' = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

Now, $V'' = VT'' = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -3 & 1 \\ 3 & 0 & 1 \\ 6 & 0 & 1 \end{bmatrix} = \begin{bmatrix} A'' \\ B'' \\ C'' \end{bmatrix}$

Thus, the new vertices have the following Cartesian coordinates: $A'' \equiv (3, -3), B'' \equiv (3, 0), C'' \equiv (6, 0)$

From the above calculations, clearly $TR \neq RT$, which justifies that matrix multiplication is NOT commutative.

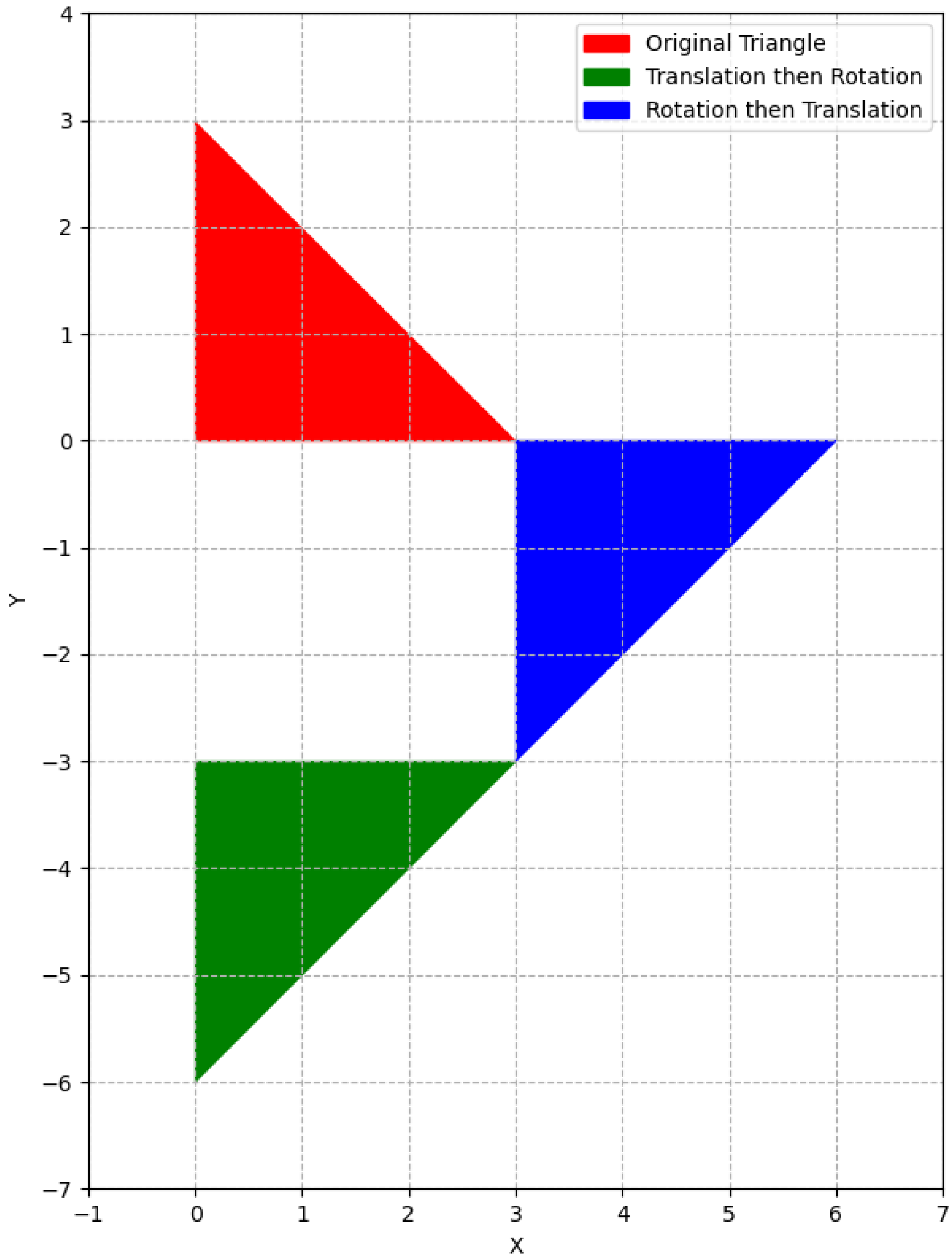

Figure 1: Plot for the Original & Translated Triangles

2 Question 2

Implicit Equation of the the Ellipse is given by:

$$\frac{(x-5)^2}{9} + \frac{(y-3)^2}{1} = 1$$

Parametric Equation of the Ellipse is given by:

$$x = 5 + 3\ cos(t)$$

$$y = 3 + sin(t)$$

such that $t \in [0, 2\pi]$ *(t represents the anticlockwise angle w.r.t the +ve x-axis)*

3 Question 3

The implicit equation of the ellipse is mentioned in Question 2. Now, considering the line between the center of the ellipse $O \equiv (5, 3)$ & Q, the equation of OQ is given by ($\theta = 30°$):

$$(y - 3) = (x - 5)\ tan(\theta)$$

Let $X = x - 5$ & $Y = y - 3$. Thus, $OQ \equiv Y = X\ tan(30°)$ & equation of ellipse becomes $\frac{X^2}{9} + \frac{Y^2}{1} = 1$.

Point Q lies on the ellipse also & hence satisfies the equation of ellipse. Substituting $Y = Xtan(30°)$ in $\frac{X^2}{9} + \frac{Y^2}{1} = 1$, we get:

$$X = \pm\frac{3}{\sqrt{(1 + 9\ tan(30°)^2)}}$$

$$\Rightarrow X = 1.5\ \ (we\ choose\ positive\ value\ since\ Q\ lies\ to\ the\ right\ of\ center)$$

$$\Rightarrow x = 6.5$$

Substituting $x = 6.5$ in $(y - 3) = (x - 5)\ tan(30°)$, we get $y = \frac{6+\sqrt{3}}{2} = 3.86$.

Thus $Q \equiv (6.5, 3.86)$

4 Question 4

Given,

$$P_1 \equiv (3, 1)$$

$$P_2 \equiv (8, 3)$$

$$P_3 \equiv (6, 3)$$

We need to determine if the following points lie inside, outside or on the ellipse mentioned in Question 2. This can be done by substituting the value of the coordinates for each of the points in the LHS of the Implicit Equation & check if the value is less than, greater than or equal to 1.

$$LHS < 1 \Rightarrow Inside\ the\ ellipse$$

$$LHS > 1 \Rightarrow Outside\ the\ ellipse$$

$$LHS = 1 \Rightarrow On\ the\ ellipse$$

Substituting P_1, we get $LHS = \frac{(3-5)^2}{9} + \frac{(1-3)^2}{1} = \frac{40}{9} > 1$
Substituting P_2, we get $LHS = \frac{(8-5)^2}{9} + \frac{(3-3)^2}{1} = 1$
Substituting P_3, we get $LHS = \frac{(6-5)^2}{9} + \frac{(3-3)^2}{1} = \frac{1}{9} < 1$

Thus,

- P_1 lies **outside** the ellipse
- P_2 lies **on** the ellipse
- P_3 lies **inside** the ellipse