Question 1

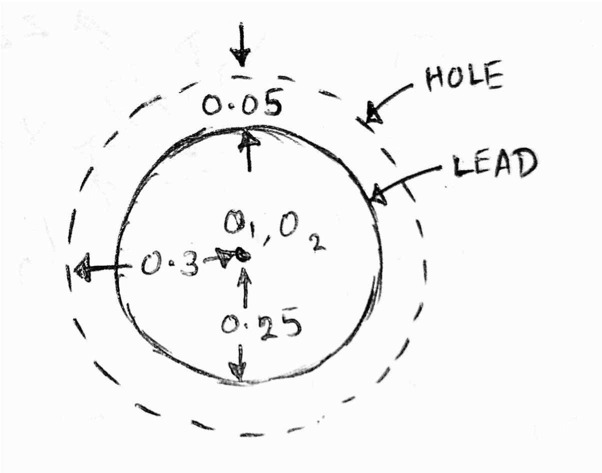
We require the range of travel for the each of the axes to be $\geq$ corresponding dimension of the PCB. Thus,

$$Range = PCB\ dimension + margin$$

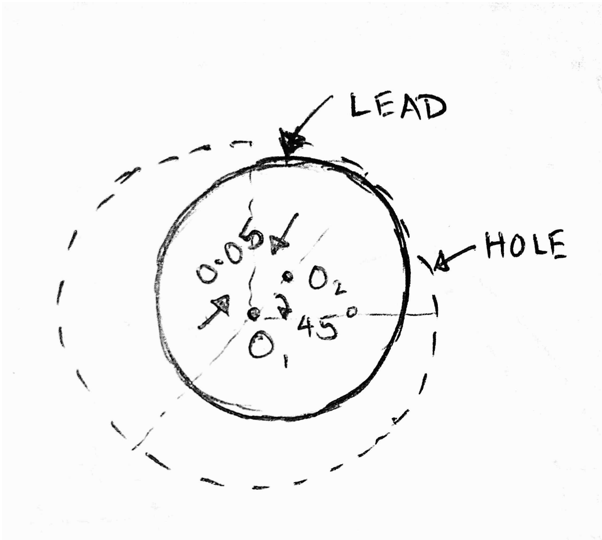
Given, the dimension of PCB $= 20\ mm$ (both axes). Let us assume a margin of $2\ mm$ on each side of the PCB for extra travel. Thus,

$$Range = 20 + (2 \cdot 2) = 24\ mm$$

Now, since both axes have the same dimensions, we demonstrate the calculations for one axis (say, X-axis) & same would hold for the other axis as well.



(a) Lead & hole during ideal fit (no offset)



(b) Maximum offset between centers of lead & hole for fit

We first calculate the accuracy required for this actuator. For this, we know that the diameter of each hole is 0.6 mm while the lead diameters are 0.5 mm. Now, we consider a case where the center of the lead is offset such that the circumference of the lead touches the circumference of the hole (as shown in Fig 1b). Clearly, the offset between the 2 centers is 0.05 mm. If the offset is more than this (in any direction), insertion won't be possible. Thus, to get this maximum offset in all possible directions, we need accuracy of each axis such that:

$$Ac \leq \frac{0.05}{\sqrt{2}} = 0.035\ mm$$

Also, we are given that $\sigma = 5\mu = 0.005\ mm$. Now, we can find the Control Resolution (CR) as follows:

$$Ac = \frac{CR}{2} + 3\sigma$$
$$\Rightarrow \frac{CR}{2} + 3 \cdot 0.005 \leq 0.035$$
$$\Rightarrow CR \leq 0.04\ mm$$

Let n be the number of bits required for representation. We have:

$$CR = \frac{Range}{2^n} \Rightarrow \frac{24}{2^n} \leq 0.04 \Rightarrow 2^n \geq 600 \Rightarrow n \geq 9.23$$

Since n should be an integer, we have $n = 10$. Thus, $CR = \frac{24}{2^{10}} = 0.023\ mm$.

Given, pitch $p = 2\ mm$, No. of sensor points per revolution $= k_g = \frac{p}{CR} = \frac{2}{0.023} = 86.96$.
But, k_g must be an integer $\Rightarrow K_g = 87$

Now, we use this value of k_g to compute all the other values accordingly as follows:

$$Minimum\ number\ of\ step\ angles = 87$$

$$CR = \frac{p}{k_g} = \frac{2}{87} = 0.0229\ mm$$

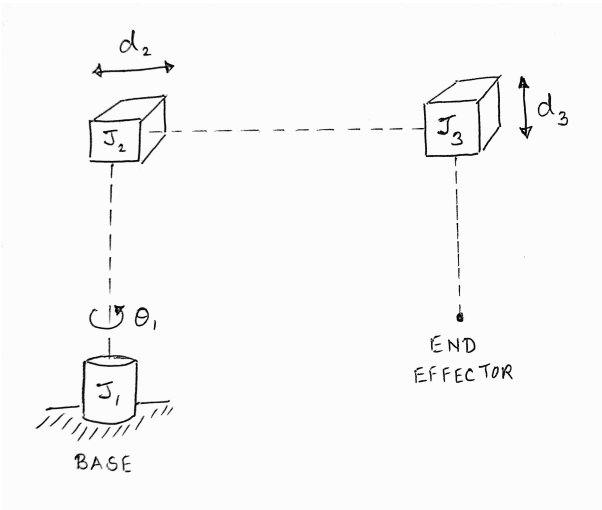
$$No.\ of\ controller\ bits = 10$$

$$Range = CR \cdot 2^n = 0.0229 \cdot 2^{10} = 23.54\ mm$$

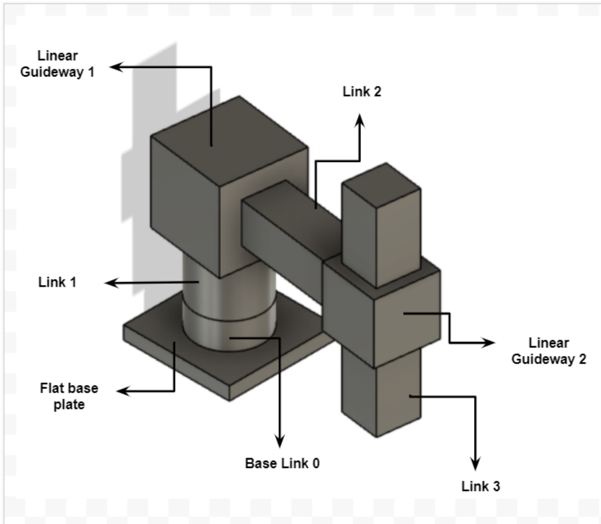
$$Accuracy = \frac{CR}{2} + 3\sigma = \frac{0.0229}{2} + 3 \cdot 0.005 = 0.0264\ mm$$

Question 2

The schematic of the robot, along with its annotated 3D isometric view (not to scale) are shown in Figure **??**. Clearly, the robot has a *TLL* configuration. As per the requirement, the end effector is attached to the linear link which moves up & down such that the robot does not collide with previously inserted components.



(a) Schematic for Robot



(b) Annotated 3D Isometric View of the Robot

The joint parameters for the robot are as follows:

Link	**a**	**d**	$\boldsymbol{\alpha}$	$\boldsymbol{\theta}$
1	0	$d_1 = 20mm$	$-90°$	$\theta_1^* \in [0, 360°)$
2	0	$d_2^* \in [0, 34mm]$	$-90°$	0
3	0	$d_3^* \in [0, 20mm]$	0	0

The dimensions of the various parts are as follows:

- The flat base plate has dimensions 5 mm $\times$ 5 mm
- The diameter of the cylindrical Base Link 0 & 1 are 4 mm
- The lengths of Links 1, 2 & 3 are mentioned in the table above
- The cross sectional dimensions of Links 2 & 3 are 4 mm $\times$ 4 mm.
- The outer dimensions of the cubical linear guideways are 5 mm $\times$ 5 mm $\times$ 5 mm, while the square holes are of dimensions 4.05 mm $\times$ 4.05 mm.

We assume that the base of the robot would be placed at the **upper left corner of the PCB**, i.e., at coordinates $(-12.5, 12.5)$ w.r.t the global reference frame attached to the center of the PCB. Since the X & Y-axis ranges of the robot should be 24 mm (as assumed in Question 1) & the horizontal Link 2 must allow the robot to reach the lower right corner of the PCB as well, we propose that the maximum length of Link 2 should be $= 24\sqrt{2}\ mm \approx 34\ mm$.

The D-H transformation matrices for adjacent coordinate systems i & $i-1$ are given by:

$$
{}^{i}A_{i-1} = \begin{bmatrix} cos\theta_i & -cos\alpha_i sin\theta_i & sin\alpha_i sin\theta_i & a_i cos\theta_i \\ sin\theta_i & cos\alpha_i cos\theta_i & -sin\alpha_i cos\theta_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Substituting the join parameters, we obtain the following transformation matrices:

$$
{}^{1}A_{0} = \begin{bmatrix} cos\theta_1 & 0 & -sin\theta_1 & 0 \\ sin\theta_1 & 0 & cos\theta_1 & 0 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}A_{1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{3}A_{2} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Based on the design parameters of the robot, the transformation matrix that connects the base reference frame on the robot to the end effector reference frame is given by:

$$
{}^{3}A_{0} = {}^{3}A_{2} \cdot {}^{2}A_{1} \cdot {}^{1}A_{0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} cos\theta_1 & 0 & -sin\theta_1 & 0 \\ sin\theta_1 & 0 & cos\theta_1 & 0 \\ 0 & -1 & 0 & 20 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\Rightarrow {}^{3}A_{0} = \begin{bmatrix} cos\theta_1 & 0 & -sin\theta_1 & 0 \\ 0 & -1 & 0 & 20 \\ -sin\theta_1 & 0 & -cos\theta_1 & d_2 + d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Now, to reach the center of the hole at the lower left corner of the PCB with coordinates $\equiv (-8, -8)$, the joint parameters have to be as follows:

Link	a	d	α	θ
1	0	$20\ mm$	$-90°$	θ_1^*
2	0	$20.98\ mm$	$-90°$	0
3	0	$20\ mm$	0	0

Question 3

Let the notation for the machines & parts be as follows:

Machine in Shop	Notation
Saw	M1
3-Axis CNC Vertical Milling Machine	M2
Drill	M3
Lathe	M4
Polishing Wheel	M5
Spray-Painting Machine	M6

Part to be Machined	Notation
Flat Base Plate	P1
Base Link 0	P2
Link 1	P3
Linear Guideway for Link 2	P4
Link 2	P5
Linear Guideway for Link 3	P6
Link 3	P7

Following are the requirements specified for the various parts:

- All the surfaces having relative motion are required to have a smooth surface ($R_a < 20\ nm$)
- Rest of the external surfaces will be spray-painted to prevent corrosion
- All the parts will require some drilling operation to accommodate actuators, electronic components etc.
- Raw material is available in the form of sheets, rectangular blocks or cylindrical rods of stainless steel

Based on these, we prepare a process plan involving a part-machine incidence matrix as shown below:

	Parts to be Machined						
Machines	**P1**	**P2**	**P3**	**P4**	**P5**	**P6**	**P7**
M1	1	0	0	1	1	1	1
M2	1	0	0	1	1	1	1
M3	1	1	1	1	1	1	1
M4	0	1	1	0	0	0	0
M5	0	1	1	1	1	1	1
M6	1	1	1	1	1	1	1

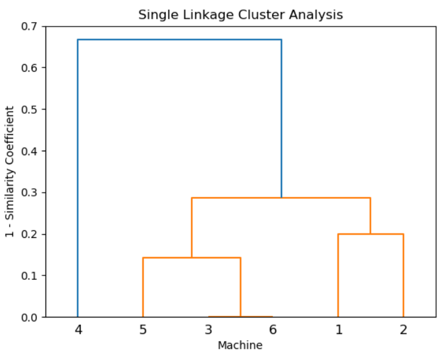
Question 4

The code for generic Single Linkage Cluster Analysis algorithm to generate the required dendrogram & the possible cell designs based on a *part-machine matrix* is given below:

```python
import matplotlib.pyplot as plt
from scipy.cluster.hierarchy import dendrogram, linkage
from sklearn.cluster import AgglomerativeClustering
import numpy as np

def SLCA(part_machine_matrix_file="part_machine.csv", save_as="dendrogram.png"):
    X = np.genfromtxt(part_machine_matrix_file, delimiter=",")
    # X is an (m * n) matrix of 1"s & 0"s
    # m = No. of Machines
    # n = No. of Parts

    Z = linkage(X, "single", "jaccard")
    _ = dendrogram(Z, labels=np.arange(1, X.shape[0]+1))
    plt.title("Single Linkage Cluster Analysis")
    plt.xlabel("Machine")
    plt.ylabel("1 - Similarity Coefficient")
    plt.savefig(save_as)
    plt.show()

    for i in range(1, X.shape[0] + 1):
        clustering = AgglomerativeClustering(n_clusters=i, affinity="jaccard", linkage="single")
        clustering = clustering.fit(X)
        print(f"Number of clusters: {i}\tCluster Assignments: {clustering.labels_}")
```

Here, the part-machine matrix is given as input to the function `SLCA(...)` in the form of a CSV file, with m rows (corresponding to the m machines) & n columns (corresponding to the n parts to be machined).

Using the code above, we generate the possible cell designs for our process plan (part-machine incidence matrix) derived in the previous question. The dendrogram for the given part-machine matrix, as generated by the code is mentioned below:



No. of Cells	Cell Configuration
1	(M1, M2, M3, M4, M5, M6)
2	(M1, M2, M3, M5, M6), (M4)
3	(M1, M2), (M3, M5, M6), (M4)
4	(M1), (M2), (M3, M5, M6), (M4)
5	(M1), (M2), (M3, M6), (M4), (M5)
6	(M1), (M2), (M3), (M4), (M5), (M6)

Question 5

To be submitted later.