

Show $IP, E, \oplus K, S, P$ are linear?

1.) IP

$$IP(a_1, a_2, \dots, a_{64}) = (a_{58}, a_{50}, \dots, a_7)$$

$$IP(b_1, b_2, \dots, b_{64}) = (b_{58}, b_{50}, \dots, b_7)$$

$$\begin{aligned} IP(a \oplus b) &= (a_{58} \oplus b_{58}, a_{50} \oplus b_{50}, \dots, a_7 \oplus b_7) \\ &= IP(a) \oplus IP(b) \Rightarrow IP \text{ linear!} \end{aligned}$$

2.) E :

$$E(a_1, a_2, \dots, a_{32}) = (a_{32}, a_1, a_2, \dots, a_1)$$

$$E(b_1, b_2, \dots, b_{32}) = (b_{32}, b_1, b_2, \dots, b_1)$$

$$\begin{aligned} E(a \oplus b) &= (a_{32} \oplus b_{32}, \dots, b_1 \oplus a_1) \\ &= E(a) \oplus E(b) \\ &\Rightarrow \text{linear!} \end{aligned}$$

3.)

$$\oplus K: (a \oplus k) \oplus (b \oplus k) = a \oplus k \oplus b \oplus k = a \oplus b \neq (a \oplus b) \oplus k$$

\Rightarrow linear!

4.)

$$S: S_7(\underbrace{000000}_{\text{column}}) \oplus S_7(000001)$$

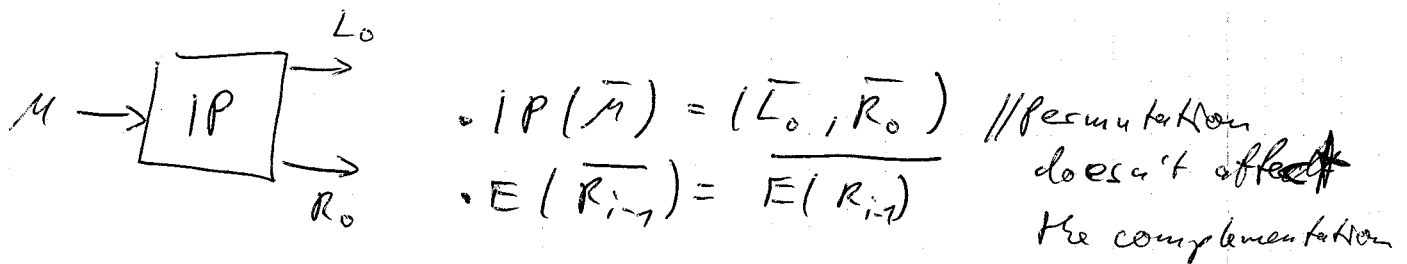
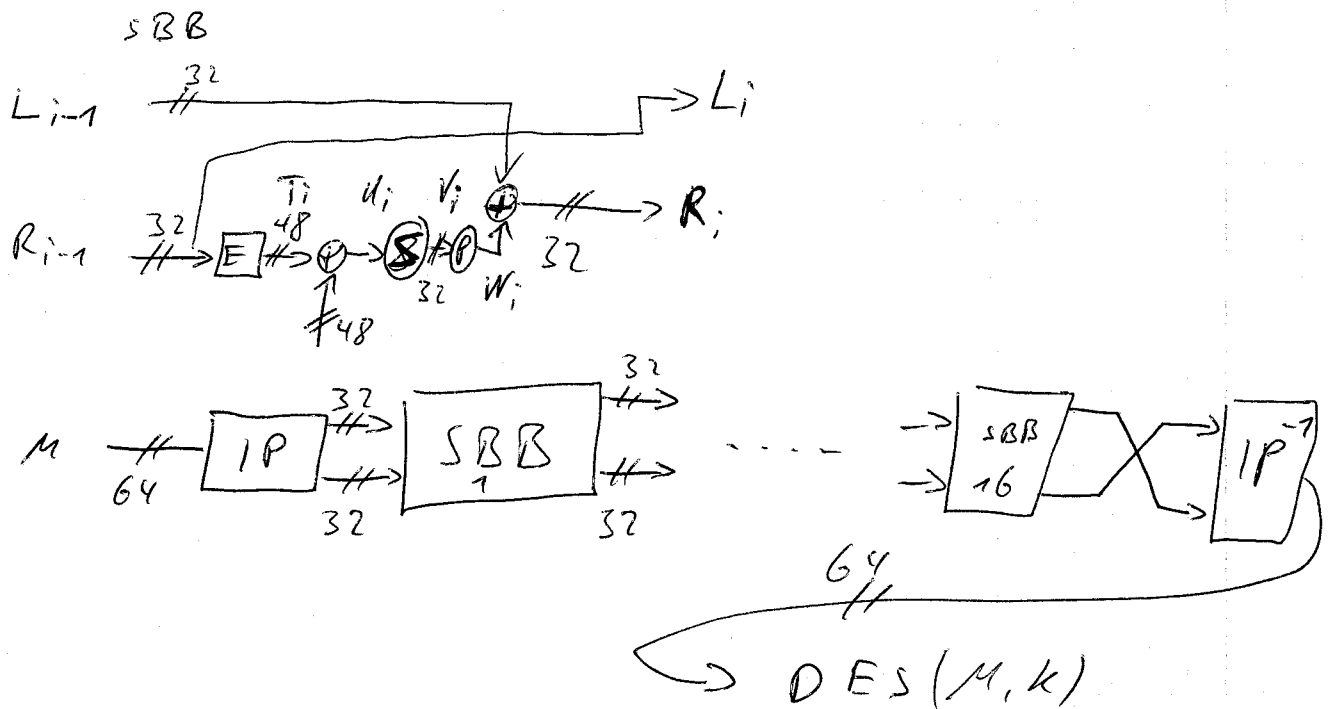
$$= 1110 \oplus 0100 = 1010 \leftarrow \neq$$

$$S_7(000000 \oplus 000001) = S_7(000001) = \underline{0100}$$

\Rightarrow non-linear!

S.1 \Rightarrow Linear since IP is linear

a) Show that $DES(M, k) = \overline{DES(\bar{M}, \bar{k})}$



// The double bfts are complemented

$$\bar{T}_i \oplus \bar{K}_i = T_i \oplus K_i$$

$\Rightarrow S(U_i)$ doesn't change

$\Rightarrow P(V_i)$ - " -

$$W_i \oplus \bar{L}_{i-1} = \bar{R}_i$$

$$L_i \Rightarrow \bar{R}_{i-1} = \bar{L}_i \Rightarrow SBB(\bar{R}_i, L_i) = \overline{SBB(R_i, L_i)}$$

This is done for 16 iterations

$$\Rightarrow IP^{-1}(R_{16}, L_{16}) = \overline{DES(\bar{M}, \bar{k})} = \overline{DES(M, k)}$$

$$\Rightarrow \overline{\text{DES}(\bar{m}, \bar{k})} = \text{DES}(m, k)$$

b.) In a brute force attack, the amount of calculations is halved.

Ex 22 Linear Feedback Shift Register (LFSR) based stream cipher

Message $m = m_1, m_2, \dots, m_L \in \mathbb{F}_2^L$

key $k = k_1, \dots, k_n \in \mathbb{F}_2^L \quad n < L$

Keystream $z = z_1, \dots, z_L$

$$z_i = k_i \quad 1 \leq i \leq n$$

$$z_i = \sum_{j=1}^n s_j z_j \pmod{2} \quad n \leq i \leq L$$

$$c_i = m_i \oplus z_i \quad 1 \leq i \leq L$$

a.) Decryption

$$m_i = c_i \oplus z_i \Rightarrow \text{Encryption} = \text{Decryption}$$

b.)

$$k = 0, \dots, 0 \Rightarrow z_i = 0 \quad 1 \leq i \leq n$$

$$z_i = 0 \quad n < i \leq L$$

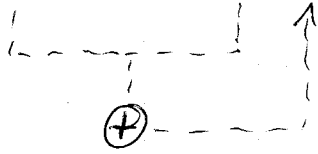
$$\Rightarrow c_i = m_i$$

\Rightarrow plaintext is not encrypted

c.) $n=1, s_1=s_4=1, s_2=s_3=0, L=20$

$$K = 0110$$

z_1	z_2	z_3	z_4	z_5	z_6	7	8	9	10	11	12	13	14	15	16
0	1	1	0	0	1	0	0	0	0	1	1	1	0	1	0



77 18 15 20

1 1 0 0