Exercise 2.4 - Solution

Consider the "**water-jug puzzle**":

> There is a 3-liter water jug and a 4-liter water jug. At the beginning, both are empty. At the end, the 4-liter jug shall contain exactly 2 liter. A jug can be emptied or filled with water (completely). Water can be poured from one jug into the other. This must be done exactly until one jug is empty or full.

(a) Formalize this as a search problem where the costs of all actions are 1.

initial state: $(0, 0)$

actions:
$e3 : (x, y) \to (0, y)$ for $x \neq 0$
$e4 : (x, y) \to (x, 0)$ for $y \neq 0$
$f3 : (x, y) \to (3, y)$ for $x \neq 3$
$f4 : (x, y) \to (x, 4)$ for $y \neq 4$
$p3 : (x, y) \to (x - z, y + z)$ for $x \neq 0$ and $y \neq 4$ where $z = min\{x, 4 - y\}$
$p4 : (x, y) \to (x + z, y - z)$ for $x \neq 3$ and $y \neq 0$ where $z = min\{3 - x, y\}$

state space: (set of all states reachable from the initial state)
$\{0, 3\} \times \{0, 1, 2, 3, 4\} \cup \{1, 2\} \times \{0, 4\}$
$= \{0, 1, 2, 3\} \times \{0, 1, 2, 3, 4\} \backslash \{1, 2\} \times \{1, 2, 3\}$

goal test: (x, y) is a goal iff $y = 2$

path cost: length of path

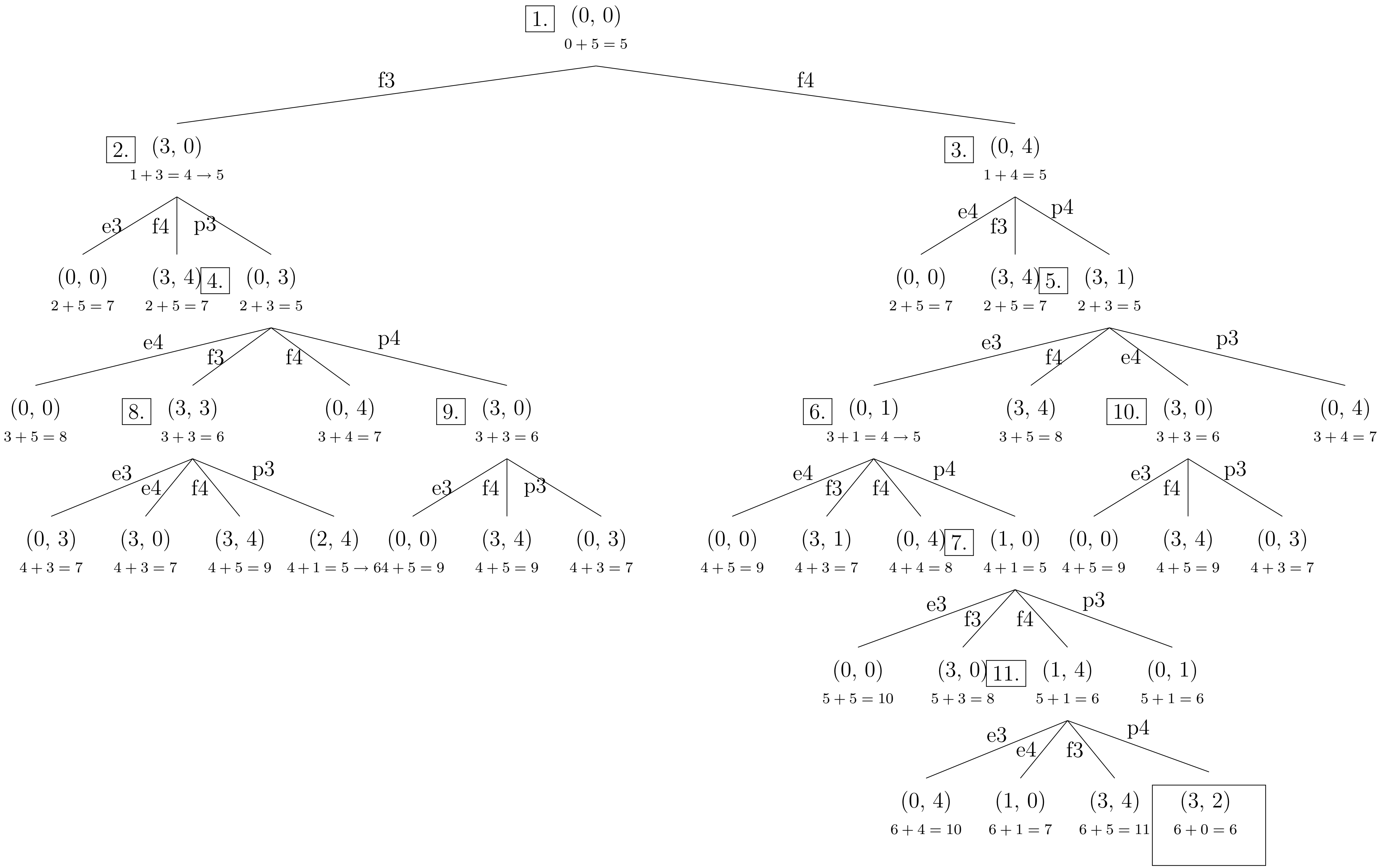
(b) The following is true for that problem in every state except goal states:

> If the 3-liter jug is full, then at least 3 steps are necessary to reach a goal state. If the 3-liter jug is empty and the 4-liter jug contains x liter, then at least x steps are necessary to reach a goal state. If both jugs are full or both jugs are empty, then at least 5 steps are necessary to reach a goal state.

Use this (and only this) information to find an admissible heuristic that is as good as possible.

state $[n]$	$h(n)$
$(x, 2)$	0
$(0, 0)$	5
$(3, 4)$	5
$(3, y)$, $y \notin \{2, 4\}$	3
$(0, y)$, $y \notin \{0, 2\}$	y
else	1

(c) Solve the problem with A* search using your heuristic and draw the A* search tree. Label each node with the corresponding state and the estimated cost of the cheapest solution path through it. Additionally, mark in the tree the order of the expansion of the nodes.



1. (0, 0)
0 + 5 = 5
f3
f4
2. (3, 0)
1 + 3 = 4 → 5
3. (0, 4)
1 + 4 = 5
e3
f4
p3
e4
f3
p4
(0, 0)
2 + 5 = 7
(3, 4)
2 + 5 = 7
4. (0, 3)
2 + 3 = 5
(0, 0)
2 + 5 = 7
(3, 4)
2 + 5 = 7
5. (3, 1)
2 + 3 = 5
e4
f3
f4
p4
e3
f4
e4
p3
(0, 0)
3 + 5 = 8
8. (3, 3)
3 + 3 = 6
(0, 4)
3 + 4 = 7
9. (3, 0)
3 + 3 = 6
6. (0, 1)
3 + 1 = 4 → 5
(3, 4)
3 + 5 = 8
10. (3, 0)
3 + 3 = 6
(0, 4)
3 + 4 = 7
e3
e4
f4
p3
e3
f4
p3
e4
f3
f4
p4
e3
f4
p3
(0, 3)
4 + 3 = 7
(3, 0)
4 + 3 = 7
(3, 4)
4 + 5 = 9
(2, 4)
4 + 1 = 5 → 6
(0, 0)
4 + 5 = 9
(3, 4)
4 + 5 = 9
(0, 3)
4 + 3 = 7
(0, 0)
4 + 5 = 9
(3, 1)
4 + 3 = 7
(0, 4)
4 + 4 = 8
7. (1, 0)
4 + 1 = 5
(0, 0)
4 + 5 = 9
(3, 4)
4 + 5 = 9
(0, 3)
4 + 3 = 7
e3
f3
f4
p3
(0, 0)
5 + 5 = 10
(3, 0)
5 + 3 = 8
11. (1, 4)
5 + 1 = 6
(0, 1)
5 + 1 = 6
e3
e4
f3
p4
(0, 4)
6 + 4 = 10
(1, 0)
6 + 1 = 7
(3, 4)
6 + 5 = 11
(3, 2)
6 + 0 = 6