abbreviate $P(X_1 = x_1 \land \dots \land X_n = x_n)$ as $P(x_1, \dots, x_n)$

$$P(x_1, \dots, x_n) = P(x_n | x_{n-1}, \dots, x_1) \cdot P(x_{n-1}, \dots, x_1)$$

$$P(x_1, \dots, x_n) = \prod_{i=1}^{n} P(x_i | x_{i-1}, \dots, x_1)$$

a Belief-network is a correct representation of the full-joint prob. distr. if

$$\mathbb{P}(X_i | X_{i-1}, \dots, X_1) = \mathbb{P}(X_i | Parents(X_i))$$

and ~~Pa~~ $Parents(X_i) \subseteq \{X_{i-1}, \dots, X_1\}$

Exercise 6.1.)

a.) $P(W \land \neg L \land R \land S = spring)$

$= P(W | \neg L \land R \land S = spring) \cdot P(\neg L | R \land S = spring)$

$\cdot P(R | S = spring) \cdot P(S = spring)$

$= P(W | \neg L \land R) \cdot \underbrace{P(\neg L | S = spring)}_{(1 - P(L | spring))} \cdot P(R | S = spring)$

$\cdot P(S = spring)$

$= 0{,}95 \cdot (1 - 0{,}15) \cdot 0{,}45 \cdot 0{,}25$

$\approx \underline{\underline{0{,}0908}}$

b.) $P(S = winter | \neg R \land \neg L) = \dfrac{P(S = winter \land \neg R \land \neg L)}{P(\neg R \land \neg L)}$

$(*) = \dfrac{P(\neg R | S = winter \land \neg L) \cdot P(\neg L | S = winter) \cdot P(S = winter)}{P(\neg R \land \neg L)}$

$$P(\neg R \land \neg L) = \sum_{s \in S} P(\neg R \land \neg L \land S = s)$$

$$= \sum_{s \in S} P(\neg R | \neg L \land S = s) \cdot P(\neg L | S = s) \cdot P(S = s)$$

$$= \sum_{s \in S} P(\neg R | S=s)\, P(\neg L | S=s) \cdot P(S=s)$$

$$\begin{aligned}
= \; & (1-0{,}45)(1-0{,}15)(0{,}25) && \text{(spring)} \\
& + (1-0{,}15)(1-0{,}3) \cdot 0{,}25 && \text{(summer)} \\
& + (1-0{,}35)(1-0{,}05) \cdot 0{,}25 && \text{(autumn)} \\
& + (1-0{,}2)(1-0{,}00) \cdot 0{,}25 && \text{(winter)} \\
= \; & \underline{\underline{0{,}62}}
\end{aligned}$$

$$(*) = \frac{P(\neg R | S=\text{winter}) \cdot P(\neg L | S=\text{winter}) \cdot P(S=\text{winter})}{P(\neg R \wedge \neg L)}$$

$$= \frac{(1-0{,}2)(1-0{,}0)\,0{,}25}{0{,}62} = \frac{0{,}2}{0{,}62} \approx 0{,}3226$$

c.)

$$P(R | W \wedge S=\text{summer}) = \frac{P(R \wedge W \wedge S=\text{summer})}{P(W \wedge S=\text{summer})}$$

$$= \frac{P(R \wedge W \wedge S=\text{summer} \wedge L) + P(R \wedge W \wedge S=\text{summer} \wedge \neg L)}{\underbrace{\sum_{\substack{r \in R \\ l \in L}} P(W \wedge S=\text{summer} \wedge R=r \wedge L=l)}_{=\alpha}}$$

$$= \frac{1}{\alpha} \cdot \Big[P(W | R \wedge L) \cdot P(R | S=\text{summer}) \cdot P(L | S=\text{summer}) \cdot P(S=\text{summer})$$
$$+ P(W | R \wedge \neg L) \cdot P(R | S=\text{summer}) \cdot P(\neg L | S=\text{summer}) \cdot P(S=\text{summer}) \Big]$$

$$= \frac{1}{\alpha} \cdot \left[0{,}95 \cdot 0{,}15 \cdot 0{,}3 \cdot 0{,}25 + 0{,}95 \cdot 0{,}15 \cdot (1-0{,}3)\, 0{,}25 \right]$$

$$(*) = \frac{1}{\alpha} \cdot 0{,}35625$$

$$\alpha = \sum_{\substack{r \in R \\ l \in L}} P(W \wedge S=\text{summer} \wedge R=r \wedge L=l)$$

$$= P(W | R \wedge L)\, P(R | S=\text{summer})\, P(L | S=\text{summer})\, P(S=\text{summer})$$
$$+ P(W | R \wedge \neg L) \cdot P(R | S=\text{summer})\, P(\neg L | S=\text{summer})\, P(S=\text{summer})$$

$+ P(W \mid \neg R \wedge L)\, P(\neg R \mid S=summer)\, P(L \mid S=summer)\, P(S=summer)$

$+ P(W \mid \neg R \wedge \neg L)\, P(\neg R \mid S=summer)\, P(\neg L \mid S=summer)\, P(S=summer)$

$= (0{,}1004375)$

(*) $\approx \underline{\underline{0{,}3547}}$

Exercise 6.2.1

d-separation

set E separates sets X, Y iff every undirected path from X to Y is blocked by E.

(i.e. there exists a node Z on the path s.t.

① $Z \in E$ $X \to Z \to Y$

② $Z \in E$ $X \leftarrow Z \to Y$

③ $Z \notin E$ $X \to Z \leftarrow Y$

$succ(Z) \notin E$

a.) $\boxed{U} \to \underline{\underline{W}} \leftarrow \boxed{V}$ ③

$\boxed{U} \to W \to \underline{\underline{Y}} \leftarrow X \leftarrow \boxed{V}$ ③ ✓

b.) $\boxed{U \to W} \leftarrow V \to \boxed{X \to Z}$ no condition applies

$\boxed{U \to W} \to Y \leftarrow \boxed{X \to Z}$ — " — ✗

c.) $\boxed{U \to W} \leftarrow V \to \boxed{X \to Z}$ ②

$\boxed{U \to W} \to Y \leftarrow \boxed{X \to Z}$ ③ ✓

d.) $\boxed{W} \rightarrow Y \leftarrow \boxed{X}$ condition ③ applies

$\boxed{W} \leftarrow Z \quad V \rightarrow \boxed{X}$ no condition applies ×

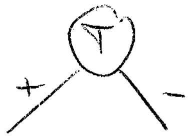
e.) $\boxed{U} \rightarrow \underline{\underline{W}} \leftarrow \boxed{Y}$ condition ③ applies

$\boxed{U} \rightarrow W \rightarrow Y \leftarrow X \leftarrow \boxed{V}$ — " — ✓

f.) $\boxed{U} \rightarrow W \rightarrow Y \leftarrow X \rightarrow \boxed{Z}$ ③

$\boxed{U} \rightarrow \underline{\underline{W}} \leftarrow V \rightarrow X \rightarrow \boxed{Z}$ ③ ✓

Ex 6.3.)



$$I(P(v_1), \dots, P(v_n)) = \sum_{i=1}^{n} -P(v_i) \log_2 (P(v_i))$$

$$\text{Remainder}(A) = \sum_{i=1}^{n} \frac{p_i + n_i}{p+n} \cdot I\left(\frac{p_i}{p_i+n_i}, \frac{n_i}{p_i+n_i}\right)$$

$$\text{Gain}(A) = I\left(\frac{p}{p+n}; \frac{n}{p+n}\right) - \text{Remainder}(A)$$