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5.2.)

b.) $\mathbb{P}(Cavity) = \big(P(cavity = true) \, ; \, P(cavity = false) \big)$

$= \big(P(cavity) \, ; \, P(\neg cavity) \big)$

$= \big(0{,}108 + 0{,}012 + 0{,}072 + 0{,}08 \, ;$

$0{,}016 + 0{,}064 + 0{,}144 + 0{,}576 \big)$

$= \underline{\underline{(0{,}2 \, ; \, 0{,}8)}}$

c.) $\mathbb{P}(Toothache \mid cavity) = \big(P(toothache \mid cavity) \, ;$

$P(\neg toothache \mid cavity) \big)$

$= \left(\dfrac{P(toothache \wedge cavity)}{P(cavity)} \, ; \, P\dfrac{(\neg toothache \wedge cavity)}{P(cavity)} \right)$

$= \left(\dfrac{0{,}108 + 0{,}012}{0{,}2} \, ; \, \dfrac{0{,}072 + 0.008}{0{,}2} \right)$

$= \underline{\underline{(0{,}6 \, ; \, 0{,}4)}}$

d.) $\mathbb{P}(Cavity \mid toothache \vee catch)$

$= \big(P(cavity \mid \underbrace{toothache \vee catch}_{=: E}) \, , \, P(\neg cavity \mid toothache \vee catch) \big)$

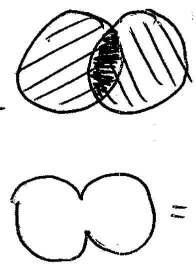
$\ulcorner P(E) = P(toothache) + P(catch) - P(toothache \wedge catch)$

$= 0{,}108 + 0{,}012 + 0{,}016 + 0{,}064$

$+ 0{,}108 + 0{,}016 + 0{,}072 + 0{,}144$

$- (0{,}108 + 0{,}016$

$= 0{,}416$



t c

$= t + c - (t \wedge c)$ $\lrcorner$

$= \left(\dfrac{P(cavity \wedge E)}{P(E)} \, ; \, \dfrac{P(\neg cavity \wedge E)}{P(E)} \right)$

$= \left(\dfrac{0{,}108 + 0{,}012 + 0{,}072}{0{,}416} \, ; \, \dfrac{0{,}016 + 0{,}144 + 0{,}064}{0{,}616} \right)$

$\approx (0{,}462;\ 0{,}538)$

	t		$\neg t$	
	cat	$\neg$cat	cat	$\neg$cat
cav				
$\neg$cav				

$P(\text{cavity} \wedge (\text{toothache} \vee \text{catch}))$

$= P(\text{cav.} \wedge \text{toothache} \vee \text{cav.} \wedge \text{catch})$

Exercise 5.3.)

$P(B_1 \vee \ldots \vee B_n) = 1$

$P(A) = P(A \wedge [B_1 \vee \ldots \vee B_n])$

$= P([A \wedge B_1] \vee \ldots \vee [A \wedge B_n])$

$= P(A \wedge B_1) + P(A \wedge B_2) + \ldots + P(A \wedge B_n)$

(*) Prop.: If $P(x_i \wedge x_j) = 0$ for $i \neq j$ then
$P(x_1 \vee \ldots \vee x_k) = P(x_1) + \ldots + P(x_k)$

Proof: by induction on k

IB: $k = 1$

IS: $P(x_1 \vee \ldots \vee x_k \vee x_{k+1}) = P([x_1 \vee \ldots \vee x_k] \vee x_{k+1})$

$= P(x_1 \vee \ldots \vee x_k) + P(x_{k+1}) - \underbrace{P([x_1 \vee \ldots \vee x_k] \wedge x_{k+1})}_{(**)\ =0}$

$= P(x_1) + \ldots + P(x_k) + P(x_{k+1})$

(**) To show: $P([x_1 \vee \ldots \vee x_k] \wedge x_l) = 0$ if $l > k$

Proof: by induction on k

IB: $k = 1$ ✓

IS: $P([x_1 \vee \ldots \vee x_k \vee x_{k+1}] \wedge x_l)$ $(l > k+1)$

$= P([[x_1 \vee \ldots \vee x_k] \wedge x_l] \vee [x_{k+1} \wedge x_l])$

$= \underbrace{P([x_1 \vee \ldots \vee x_k] \wedge x_l)}_{\overset{IH}{=} 0} + \underbrace{P(x_{k+1} \wedge x_l)}_{=0}$

$- \underbrace{P([x_1 \vee \ldots \vee x_k] \wedge x_l \wedge x_{k+1} \wedge x_l)}_{=0}$

$\underline{\underline{= 0}}$ $\square$

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$$\overset{\otimes}{=} P(A \wedge B_1) + P(A \wedge B_2) + \ldots + P(A \wedge B_n)$$
$$= P(A \mid B_1)\, P(B_1) + \ldots + P(A \mid B_n) \cdot P(B_n) \quad \square$$

Ex 5.4.)

a.)

$P(Q_1 \mid W) = 0{,}95 \qquad i = 1,2,3$

$P(Q_1 \mid \neg W) = 0{,}3$

$P(Q_2 \mid \neg W) = 0{,}5$

$P(Q_3 \mid \neg W) = 0{,}1$

$P(W) = 4/5 = 0{,}8$

b.)

$$P(W \mid Q_1) = \frac{P(W \wedge Q_1)}{P(Q_1)} = \frac{P(Q_1 \mid W)\, P(W)}{P(Q_1)}$$
$$= \frac{P(Q_1 \mid W) \cdot P(W)}{P(Q_1 \mid W) \cdot P(W) + P(Q \mid \neg W) \cdot P(\neg W)}$$
$$= \frac{0{,}95 + 0{,}8}{0{,}95 \cdot 0{,}8 + 0{,}3 \cdot (1 - 0{,}8)}$$
$$= \frac{0{,}76}{0{,}82} \approx \underline{\underline{0{,}927}}$$

$$P(Q_1, Q_2 \mid \neg W) \overset{*}{=} P(Q_1 \mid \neg W) \cdot P(Q_2 \mid \neg W) = 0{,}3 \cdot 0{,}5 = \underline{\underline{0{,}15}}$$

Q_i are independent given W

$\otimes$ A, B cond. indep. C: $P(A \mid B, C) = P(A \mid C)$

$$P(A, B \mid C) = \frac{P(A, B, C)}{P(C)} = \frac{P(A, B, C)}{P(B, C)} \cdot \frac{P(B, C)}{P(C)}$$
$$= P(A \mid B, C) \cdot P(B \mid C)$$
$$\overset{!}{=} P(A \mid C) \cdot P(B \mid C)$$

$P(Q_3 | Q_1, Q_2, w) = P(Q_3 | w) = \underline{\underline{0{,}95}}$

c.)

$P(w | Q_1, Q_2, \neg Q_3) = \alpha \cdot P(w) \cdot P(Q_1 | w)$
$\cdot P(Q_2 | w) \cdot \underbrace{P(\neg Q_3 | w)}_{= 1 - P(Q_3 | w)}$

$= \alpha \cdot 0{,}8 \cdot 0{,}95 \cdot 0{,}95 \cdot 0{,}05$
$= 0{,}0361 \cdot \alpha$

$P(\neg w | Q_1, Q_2, \neg Q_3) = \alpha \cdot P(\neg w) \cdot P(Q_1 | \neg w) \cdot P(Q_2 | \neg w)$
$\cdot \underbrace{P(\neg Q_3 | \neg w)}_{= 1 - P(Q_3 | \neg w)}$

$= \alpha \cdot (1 - 0{,}8) \cdot 0{,}3 \cdot 0{,}5 \cdot (1 - 0{,}1)$
$= 0{,}027\, \alpha \approx \underline{\underline{0{,}572}}$

$1 \overset{!}{=} \alpha (0{,}0361 + 0{,}027) = \alpha \cdot 0{,}0631$
$\Rightarrow \alpha \approx 15{,}848$

d.)

$P(w | Q_1, \neg Q_2, \neg Q_3) = \beta \cdot P(w) \cdot P(Q_1 | w) \cdot P(\neg Q_2 | w)$
$\cdot P(\neg Q_3 | w)$
$= \beta \cdot 0{,}8 \cdot 0{,}95 \cdot 0{,}05 \cdot 0{,}05$

$P(\neg w | Q_1, \neg Q_2, \neg Q_3) = \beta \cdot P(\neg w) \cdot P(Q_1 | \neg w)$
$\cdot P(\neg Q_2 | \neg w) \cdot P(\neg Q_3 | \neg w)$
$= \beta \cdot 0{,}2 \cdot 0{,}3 \cdot 0{,}5 \cdot 0{,}9 = 0{,}027 \cdot \beta$

$\Rightarrow \beta \approx 34{,}602$
$\Rightarrow P(w | Q_1, \neg Q_2, \neg Q_3) \approx \underline{\underline{0{,}066}}$

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Ex 5.4. e)

Conditional independence