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$$I = \langle D, \phi \rangle$$

universe/domain

non-empty set

interpretation function:

n-ary predicate symbol P

$\phi(P) \subseteq D \times \dots \times D$ n-ary relationship over D

n-ary function symbol f:

$\phi(f) \in \{ \underbrace{D \times \dots \times D}_{n} \to D \}$

n-ary fct. over D

prop. case:

$\phi(P) \in \{TRUE, FALSE\}$ for all prop. vars P

$I \models P(t_1, \dots, t_n)$ iff. $(d_1, \dots, d_n) \in \phi(P)$ and $d_i = I\,\|t_i\|$

Interpretation $\to I \overset{\text{satisfies}}{\models} \alpha \land \beta$ iff $I \models \alpha$ and $I \models \beta$

entailment $S \models \alpha$ iff for all interpretations I, if $I \models S$ then $I \models \alpha$

sentence

α is unsatisfiable iff for no I, $I \models \alpha$

α is valid. iff for all I, $I \models \alpha$

Ex 4.1.)

a.) α is valid

iff for all $I \models \alpha$ (def. of validity)
iff for all I with $I \models TRUE$ also $I \models \alpha$
iff $TRUE \models \alpha$

e.g. $TRUE = p \vee \neg p$
$FALSE = p \wedge \neg p$

b.) Suppose $FALSE \not\models \alpha$ for arbitrary sentence α
Then there is at least one interpretation I s.th. $\underbrace{I \models FALSE}_{\lightning}$ but $I \not\models \alpha$

since no interpretation satisfies FALSE, i.e. $\{I \mid I \models FALSE\} = \emptyset$

c.) $\alpha \supset \beta$ is valid
iff for all interpretations I, $I \models \alpha \supset \beta$
iff ——— " ———, $I \models \neg\alpha \vee \beta$
iff ——— " ———, $I \models \neg\alpha$ or $I \models \beta$
iff ——— " ———, $I \not\models \alpha$ or $I \models \beta$
iff ——— " ———, $I \models \alpha$ and $I \models \beta$
or $I \not\models \alpha$ and $I \models \beta$
or $I \not\models \alpha$ and $I \not\models \beta$
where $I \models \alpha$ then $I \models \beta$

iff $\alpha \models \beta$

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α	β	$\supset$
0	0	1
0	1	1
1	0	0
1	1	1

$\Rightarrow$ „$\neg\alpha \vee \beta$"

d.)
$\alpha \equiv \beta$ is valid
iff $(\alpha \supset \beta) \wedge (\beta \supset \alpha)$ is valid
iff $(\neg\alpha \vee \beta) \wedge (\neg\beta \vee \alpha)$ is valid
iff for all I: $I \models (\neg\alpha \vee \beta) \wedge (\neg\beta \vee \alpha)$
iff —— " ——: $I \models \neg\alpha \vee \beta$ and $I \models \neg\beta \vee \alpha$
iff —— " ——: $(I \not\models \alpha$ or $I \models \beta)$ and $(I \not\models \beta$ or $I \models \alpha)$
iff for all I: $I \not\models \alpha$ and $I \not\models \beta$ or $I \models \alpha$ and $I \models \beta$
iff —— " ——: $I \models \alpha$ iff $I \models \beta$
iff —— " ——: $\underbrace{\{I \mid I \models \alpha\}}_{= Mod(\alpha)} = \underbrace{\{I \mid I \models \beta\}}_{= Mod(\beta)}$
iff α is equivalent to β

e.)
$\alpha \wedge \neg\beta$ is unsat.
iff no I exists with $I \models \alpha \wedge \neg\beta$
iff for all I: $I \not\models \alpha \wedge \neg\beta$
iff —— " ——: $I \not\models \alpha$ or $I \not\models \neg\beta$
$I \models \beta$
iff —— " ——: $I \models \alpha \supset \beta$
iff $\alpha \models \beta$ (from c)

Ex 4.2.)

(1) Daughter (NJ, $\overset{\text{function}}{\downarrow}$parents(MM))

(2) $\neg$ Sister (NJ, NN)

(3) $\forall x \, \forall y \, [\, \text{Daughter}(x,y) \equiv \text{Female}(x) \wedge y = \text{parents}(x) \wedge x \neq y \,]$

(4) $\forall x \, \forall y \, [\, \text{Sister}(x,y) \equiv \text{Female}(x) \wedge \text{parents}(x) = \text{parents}(y) \wedge x \neq y \,]$

KB = { (1), (2), (3), (4) }   Knowledge Base

Claim: KB $\models$ MM = NJ

Let $I = \langle D, \phi \rangle$ be an interpretation with $I \models KB$. To show $I \models MM = NJ$

Let $a = \phi(NJ)$, $b = \phi(MM)$, $c = I \| \text{parents}(MM) \|$

$= \phi(\text{parents})(\phi(MM))$

$= \phi(\text{parents})(b)$ (5)

with: $I \models (1)$ have $\langle a, c \rangle \in \phi(\text{Daughter})$ (6)

$I \models (2)$ have $\langle a, b \rangle \notin \phi(\text{Sister})$ (7)

$I \models (3)$ have $\langle d, d' \rangle \in \phi(\text{Daughter})$ iff $d \in \phi(\text{Female})$, $d' = \phi(\text{Parents})(d)$, $d \neq d'$

$\overset{(6)}{\Longrightarrow}$ $a \in \phi(\text{Female})$, $c = \phi(\text{parents})(a)$, $a \neq c$ (8)

$I \models (4)$ $\langle d, d' \rangle \in \phi(\text{Sister})$ iff $d \in \phi(\text{Female})$, $\phi(\text{parents})(d) = \phi(\text{Parents})(d')$, $d \neq d'$

$\overset{(7)}{\Longrightarrow}$ $\underbrace{a \in \phi(\text{Female})}_{(*)}$ or $\underbrace{\phi(\text{Parents})(a) \neq \phi(\text{parents})(d)}_{(**)}$

or $a = b$

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$\circledast$ impossible because of (8)

$(**)$ impossible $\phi(\text{parents})(a) \overset{(8)}{=} c = \phi(\text{parents})(c)$

$\Rightarrow$ $\underline{\underline{a = b}}$ $\Rightarrow$ $I \models NJ = MM$

$\square$

$\Rightarrow$ $\underline{\underline{\phi(NJ) = \phi(MM)}}$