AI Ü3

Ex 2.2.)

Given: $h(n) \leq c(n,a,n') + h(n')$ for all n

~~TO show~~: $h(n_0) \leq h^*(n_0)$ for all n_0

Proof: (1.) Let $n_0 \xrightarrow{a_1} n_1 \xrightarrow{a_2} n_2 \longrightarrow \ldots \xrightarrow{a_l} n_l$

be the cheapest path from n_0 to a goal, $l \geq 0$

(i.) $h(n_l) = 0$, (since n_l is a goal)

(ii.) $h^*(n_0) = \sum_{i=1}^{l} c(n_{i-1}, a_i, n_i)$

$$\geq \sum_{i=1}^{l} \left(h(n_{i-1}) - h(n_i) \right)$$

$c(n_{i-1}, a_i, n_i) \geq h(n_{i-1}) - h(n_i)$ by assumption

$$= \sum_{j=0}^{l-1} h(n_j) - \sum_{i=1}^{l} h(n_i) = h(n_0) - h(n_l) = h(n_0)$$

(2.) No path from n_0 to goal exists.

$h^*(n_0) = \min\{c \mid c = \text{cost of the path from } n_0 \text{ to a goal.}\}$

$= \min\{\}$

$= \infty \geq h(n_0)$

$\square$

Example

$$n \xrightarrow{3} n' \xrightarrow{6} n''$$

$h(n) = 8$ $\qquad$ $h(n') = 4$ $\qquad$ $h(n'') = 0$

$$\left.\begin{aligned} h^*(n) &= 9 \geq h(n) = 8 \\ h^*(n') &= 6 \geq h(n') = 4 \\ h^*(n'') &= 0 \geq h(n'') = 0 \end{aligned}\right\} \text{ h is admissible}$$

but:

$$h(n) = 8 > 7 = 3 + 4 = c(n, a, n') + h(n') \quad ⚡$$

$$h(n) \leq c(n, a, n') + h(n') \qquad (*)$$

In general: If n' is a child of n on a cheapest path from n to a goal and following inequality holds:

$$h^*(n') - h(n') > h^*(n) - h(n)$$

then h is <u>not</u> consistent.

Proof: $h(n) > \underbrace{h^*(n) - h^*(n')}_{= c(n, a, n')} + h(n')$ $\qquad (*)$

$\square$

Ex 2.3.) h_1 = misplaced-tiles heuristic
h_2 = Manhattan-distance heuristic
h_3 = Gaschnig's heuristic

Example:

1

6	2	8
	3	5
4	1	7

2

6	2	3
8	~~3~~	5
4	1	7

3

1	2	3
8		5
4	6	7

AI Ü3

16. NOV. 2011

state

④

1	2	3
8		4
7	6	5

① h_1: 7

h_2: $3+2+3+1+3+2+3 = 17$

h_3: 9

h_1^*: 19

② h_1: 5

h_2: $3+3+1+3+2 = 12$

h_3: 7

h_1^*: ∞

③ h_1: 3

h_2: $3+1+2=6$

h_3: 4

h^*: 12

④ $h_1 = h_2 = h_3 = 0$

h is „better" than h' iff $h' \leq h$
(i.e. $h'(n) \leq h(n) \leq h^*(n)$ for all n)

<u>known:</u> $h_1 \leq h_2$ misplaced tiles at least distance 1 to their correct location

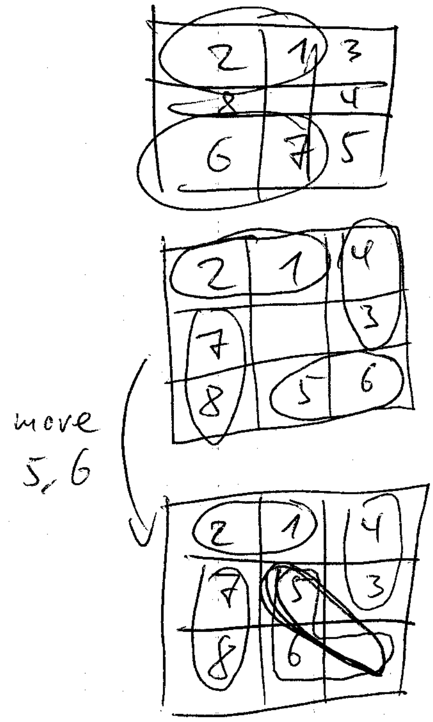
<u>also:</u> $h_1 \leq h_3$ (math. proof below)

because one Gaschnig's move brings at most one tile to its correct location.

<u>but:</u> $h_2 \not\leq h_3$ and $h_3 \not\leq h_2$

counter example above

counter examples by interchanging two or four pairs of adjacent tiles (and maybe moving the blank)



	h_1	h_2	h_3	h^*
	4	4	6	16
	8	8	12	24
	7	8	10	26

$0 \,\hat{=}\,$ blank

$\underset{2}{(0\ 8\ 3)}\ \underset{3}{(1\ 6)}\ \underset{0}{(2)}\ \underset{4}{(4\ 7\ 5)}$ state ①

Geschnig-moves: $2+3+4=9$

$(0)(1\ 6)(2)(3)(4\ 7\ 5)(8)$ state ②

$(0)(1)(2)(3)(4\ 7\ 5)(6)(8)$ ③

$(0)(1)(2) \ldots (8)$ ④

Proof: $h_1 \leq h_3$

$\pi(n) \,\hat{=}\,$ permutation representing node n (blank denoted by 0)

$\pi = \text{cyc}\ \gamma_1 \ldots \gamma_l \,\hat{=}\,$ cyclic decomposition of π

$|\gamma_l| \,\hat{=}\,$ length of cycle γ_l

$\#_3(\gamma_l) \,\hat{=}\,$ number of geschnig-moves

1.6. NOV. 2011

A1 Ü3

needed for ~~cy~~ cycle γ

$h_3(u) \hat{=}$ — " — node u

$h_3(u) = \#_3(\gamma_1) + \dots + \#_3(\gamma_l)$ where $\pi(u) =_{cyc} \gamma_1 \dots \gamma_l$

$$\#_3(\gamma) = \begin{cases} 0 & \text{if } |\gamma| = 1 \\ |\gamma|-1 & \text{if } |\gamma| > 1 \text{ and } 0 \in \gamma \\ |\gamma|+1 & \text{if } |\gamma| > 1 \text{ and } 0 \notin \gamma \end{cases}$$

$\#_1(\gamma) \hat{=}$ number of misplaced tiles in γ

$h_1(u) \hat{=}$ — " — node u

$h_1(u) = \#_1(\gamma_1) + \dots + \#_1(\gamma_l)$ where $\pi(u) =_{cyc} \gamma_1 \dots \gamma_l$

$$\#_1(\gamma) = \begin{cases} 0 & \text{if } |\gamma| = 1 \\ |\gamma|-1 & \text{if } |\gamma| > 1,\ 0 \in \gamma \\ |\gamma| & \text{if } |\gamma| > 1,\ 0 \notin \gamma \end{cases}$$

$\Rightarrow \#_1(\gamma) \leq \#_3(\gamma) \Rightarrow h_1(u) \leq h_3(u)$

$h_3(u) = h_1(u) + \left| \{ \gamma_i \mid |\gamma_i| > 1,\ 0 \notin \gamma_i \} \right|$

3