AI Ü 7

Ex 4,31

$KB \models \alpha$ iff for all I, if $I \models KB$ then $I \models \alpha$

, iff $I \not\models KB \cup \{\neg\alpha\}$

iff $KB \cup \{\neg\alpha\}$ is unsat.

iff $KB \cup \{\neg\alpha\} \models FALSE$

iff $KB \cup \{\neg\alpha\} \rightarrow [\,]$

a.)

$\{\} \models (P \supset (Q \supset P))$

$\{\} \cup \{\neg(P \supset (Q \supset P))\}$

1.) Eliminate $\supset$ and $\equiv$

$\supset \;\downarrow\; \neg\alpha \vee \beta$

$\equiv \;\downarrow\; \alpha \supset \beta \wedge \beta \supset \alpha$

$\hat{=} \neg(\neg P \vee (\neg Q \vee P))$

2.) Push $\neg$ „inwards"

$\equiv (\neg\neg P \wedge \neg(\neg Q \vee P))$

$\equiv (\neg\neg P \wedge (\neg\neg Q \wedge \neg P))$

3.) Distribut $\vee$ over $\wedge$

$(\alpha \wedge \beta) \vee \gamma \equiv (\alpha \vee \gamma) \wedge (\beta \vee \gamma)$

4.) Simplify

$\equiv P \wedge Q \wedge \neg P$

$\hat{=} \{[P], [Q], [\sim P]\}$

$[P] \quad [Q] \quad [\sim P]$

$[\,]$

b.)

$\{(P \supset (Q \supset R))\} \models ((P \supset Q) \supset (P \supset R))$

$\{(P \supset (Q \supset R))\} \cup \{\neg((P \supset Q) \supset (P \supset R))\}$

$\hat{=} \; (P \supset (Q \supset R)) \wedge \neg((P \supset Q) \supset (P \supset R))$

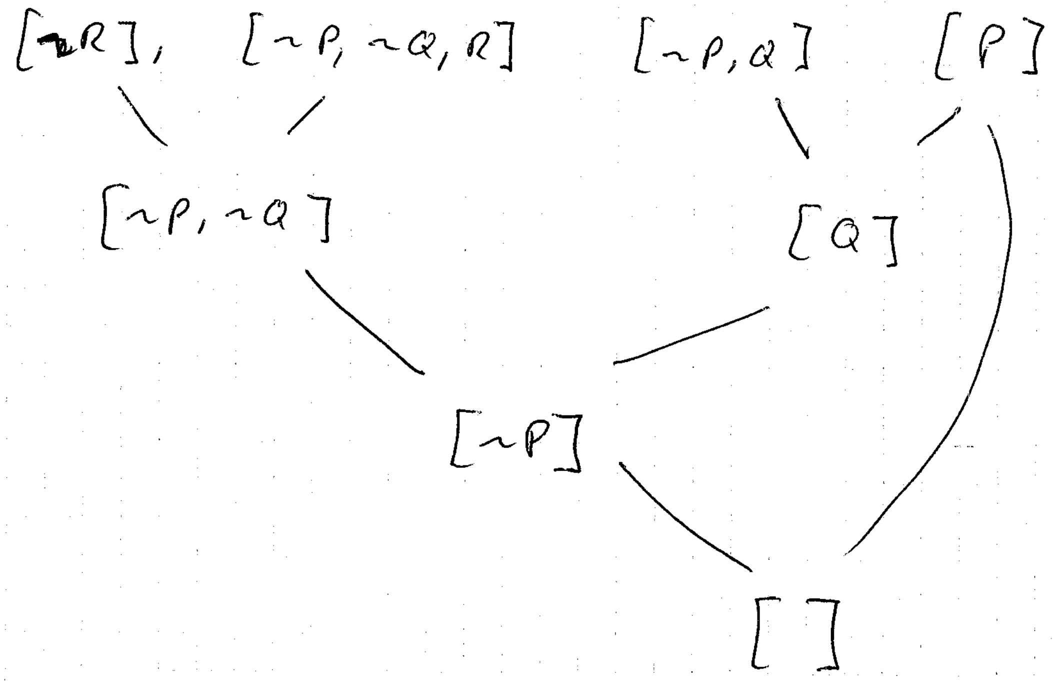
$\overset{1.)}{\equiv} (\neg P \vee (\neg Q \vee R)) \wedge \neg((\neg P \vee Q) \vee (\neg P \vee R))$

$\overset{2.)}{\equiv} (\neg P \vee (\neg Q \vee R)) \wedge \neg((\neg\neg P \wedge \neg Q) \vee (\neg P \vee R))$

$\overset{2.)}{\equiv} (\neg P \vee (\neg Q \vee R)) \wedge ((\neg\neg\neg P \vee \neg\neg Q) \wedge \neg\neg P \wedge \neg R)$

$\overset{4.)}{\equiv} (\neg P \vee \neg Q \vee R) \wedge (\neg P \vee Q) \wedge P \wedge \neg R$

$\hat{=} \; \{[\sim P, \sim Q, R], [\sim P, Q], [P], [\sim R]\}$



c.)

$\{(Q \supset P), (Q \supset \neg P)\} \models \neg Q$

$\{(Q \supset P), (Q \supset \neg P)\} \cup \{\neg\neg Q\}$

$\hat{=} \; (Q \supset P) \wedge (Q \supset \neg P) \wedge \neg\neg Q$

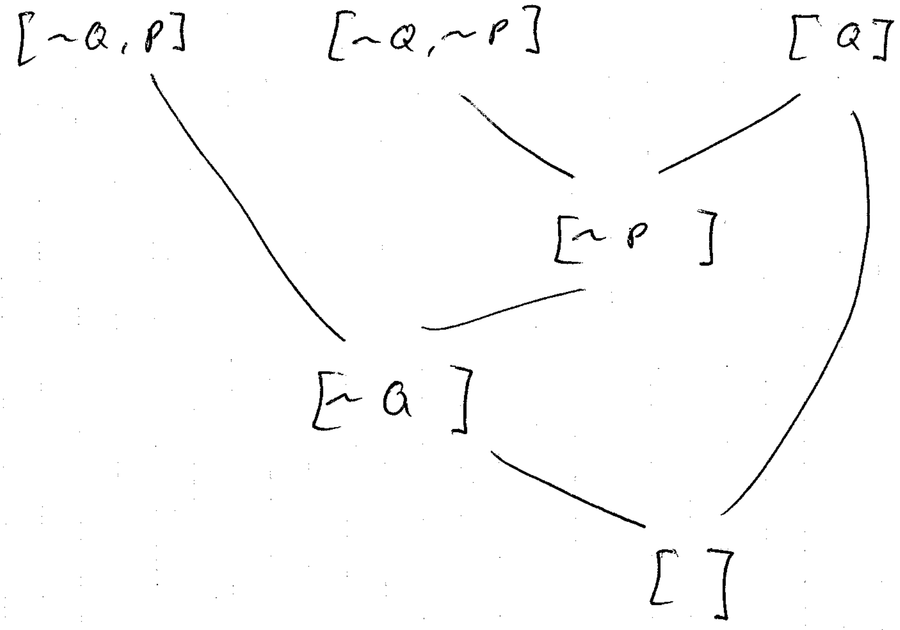
$\overset{1.)}{\equiv} (\neg Q \vee P) \wedge (\neg Q \vee \neg P) \wedge \neg\neg Q$

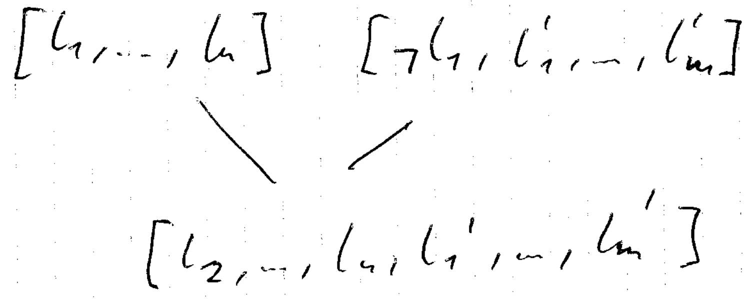
$\overset{4.)}{\equiv} (\neg Q \vee P) \wedge (\neg Q \vee \neg P) \wedge Q$

$\hat{=} \; \{[\sim Q, P], [\sim Q, \sim P], [Q]\}$

14. DEZ. 2011

AI Ü 7





<u>Ex 4.4.)</u>

a.) $\{\exists x\, P(x), \exists x\, Q(x)\} \models \exists x\, [P(x) \wedge Q(x)]$

$I = \langle D, \phi \rangle$

$D = \{1, 2\}$

$\phi(P) = \{\langle 1 \rangle\}$ $\qquad$ $\phi(Q) = \{\langle 2 \rangle\}$

$I \models \exists x\, P(x)$ ✓

iff $I, v_x^d \models P(x)$ <u>for a $d \in D$</u>

iff $d \in \phi(P)$ —— " ——

$I \models \exists x\, Q(x)$ ✓

$I \models \exists x\, [P(x) \wedge Q(x)]$ ✗

iff $I, \mathcal{V}_x^d \models P(x) \wedge Q(x)$ for a $d \in D$

iff there is a $d \in D$ s.t. $d \in \phi(P)$ and $d \in \phi(Q)$

b.) $\{\forall x\, P(x) \vee \forall x\, Q(x)\} \models \forall x [P(x) \vee Q(x)]$

$\{\forall x\, P(x) \vee \forall x\, Q(x)\} \cup \{\neg \forall x\, [P(x) \vee Q(x)]\}$

$\hat{=} (\forall x\, P(x) \vee \forall x\, Q(x)) \wedge \neg \forall x [P(x) \vee Q(x)]$

$\overset{2.)}{\equiv} (\forall x\, P(x) \vee \forall x\, Q(x)) \wedge \exists x\, [\neg P(x) \wedge \neg Q(x)]$

1.) Eliminate $\supset$ and $\equiv$

2.) Push $\neg$ „inwards"

3.) Rename variables

4.) Eliminate „$\exists$"s (Skolemization)

5.) Move „$\forall$" to the left

6.) Distribute $\vee$ over $\wedge$

7.) simplify

$\overset{3.)}{\equiv} (\forall x\, P(x) \vee \forall y\, Q(y)) \wedge \exists z [\neg P(z) \wedge \neg Q(z)]$

$\overset{4.)}{\rightsquigarrow} (\forall x\, P(x) \vee \forall y\, Q(y)) \wedge \neg P(f) \wedge Q(f)$

$\forall x\, \exists y\, P(x,y)$

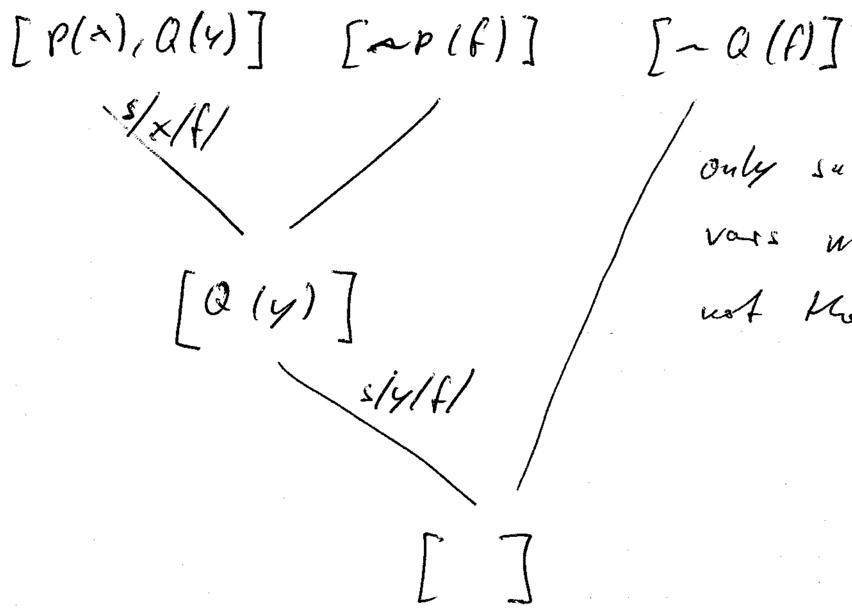
$\rightsquigarrow \forall x\, P(x, f(x))$

$\cancel{\hat{=} \{[P(x), Q(y)], [\neg P(f)], [\neg Q(f)]\}}$

$\overset{5.)}{\equiv} \forall x\, \forall y\, [(P(x) \vee Q(y)) \wedge \neg P(f) \wedge \neg Q(f)]$

$\hat{=} \{[P(x), Q(y)], [\neg P(f)], [\neg Q(f)]\}$

A1 ü 7



only substitute vars with terms! not the other way

e.) $\{ \forall x \exists y \, [(P(y) \supset P(f(x))) \wedge (P(f(x)) \supset Q(x, f(y))) \wedge (Q(x, f(x)) \vee P(y))] \} \models \forall x \exists y \, Q(x,y)$

$\{ \text{—— " ——} \} \cup \{ \neg \forall x \exists y \, Q(x,y) \}$

$\hat{=} \forall x \exists y \, [(P(y) \supset P(f(x))) \wedge (P(f(x)) \supset Q(x, f(y))) \wedge (Q(x, f(x)) \vee P(y))] \wedge \neg \forall x \exists y \, Q(x,y)$

$\overset{1.)}{\equiv} \forall x \exists y \, [(\neg P(y) \vee P(f(x))) \wedge (\neg P(f(x)) \vee Q(x, f(y))) \wedge (Q(x, f(x)) \vee P(y))] \wedge \neg \forall x \exists y \, Q(x,y)$

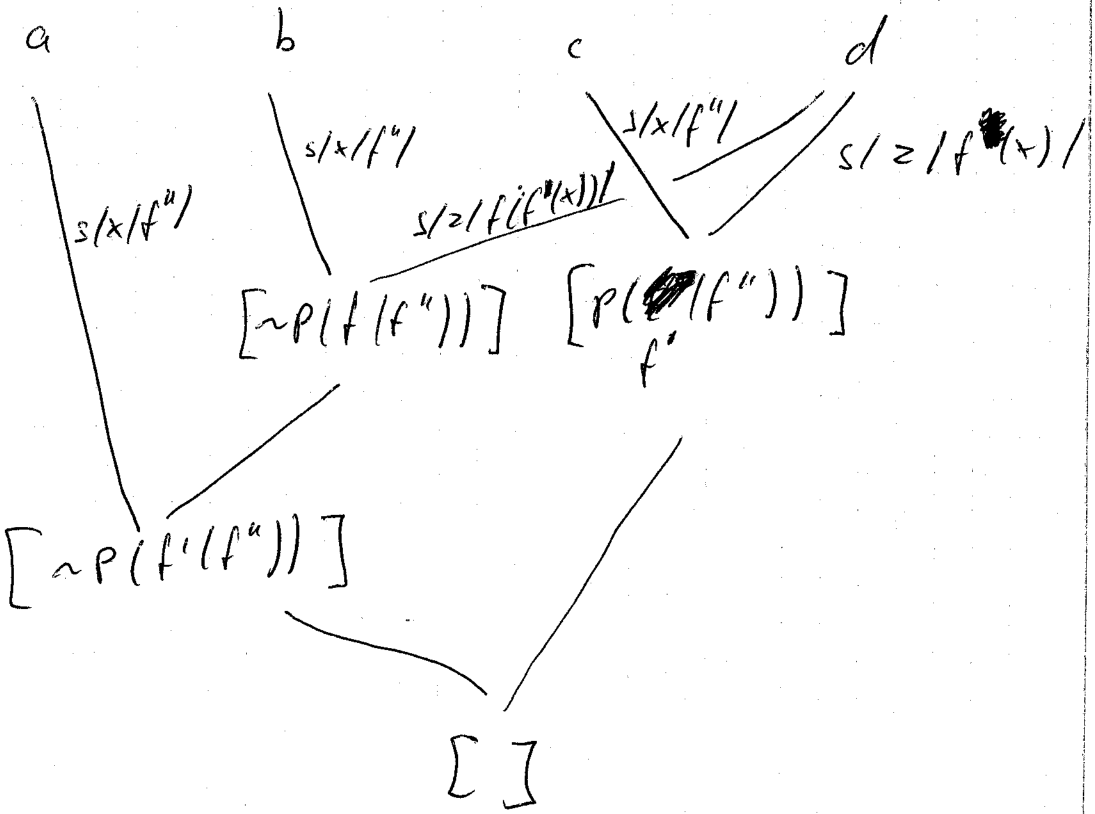
$\overset{2.)}{\equiv}$ —— " — $\wedge \exists x \forall y \, \neg Q(x,y)$

$\overset{2.)}{\equiv}$ —— " — $\wedge \exists u \forall z \, \neg Q(u,z)$

$\overset{4.)}{\rightsquigarrow} \forall x \, [(\neg P(f'(x)) \vee P(f(x))) \wedge (\neg P(f(x)) \vee Q(x, f(f'(x)))) \wedge (Q(x, f(x)) \vee P(f'(x)))] \wedge \forall z \, \neg Q(f'', z)$

$\overset{5.)}{\equiv} \forall x \forall z \, [(\neg P(f'(x)) \vee P(f(x))) \wedge (\neg P(f(x)) \vee Q(x, f(f'(x)))) \wedge (Q(x, f(x)) \vee P(f'(x))) \wedge \neg Q(f'', z)]$

$\hat{=} \{ \overset{a}{[\neg P(f'(x)), P(f(x))]}, \overset{b}{[\neg P(f(x)), Q(x, f(f'(x)))]}, {}_{c}[Q(x, f(x)), P(f'(x))], {}_{d}[\neg Q(f'', z)] \}$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