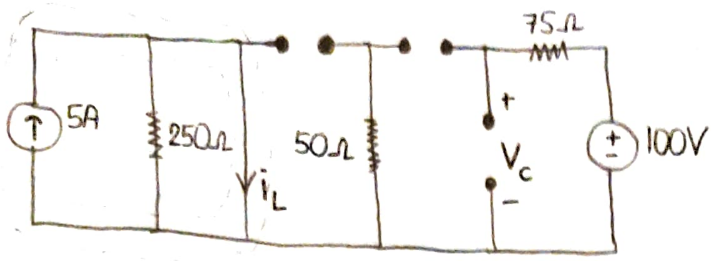
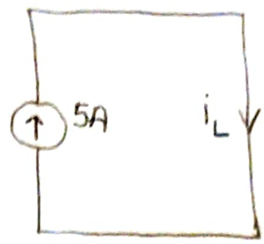
(8.11)

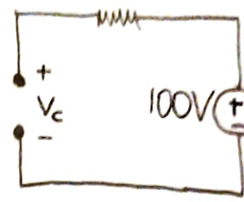
for $t = 0^-$



$\rightarrow$

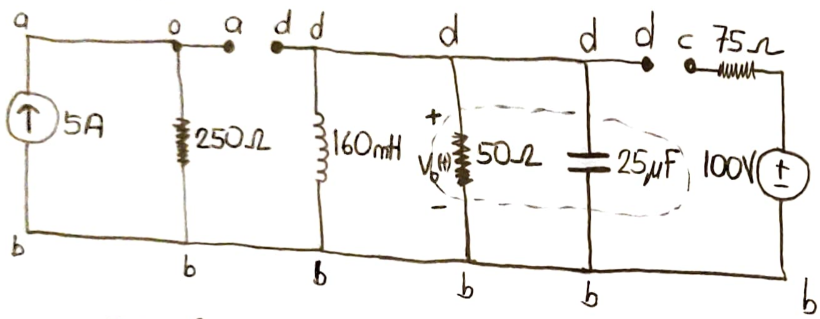


, hence $i_L(0^-) = 5A$



, hence $V_c(0^-) = 100V$

for $t = 0^+$



$$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(160 \times 10^{-3}).(25 \times 10^{-6})}} = 500 \, rad/s$$

$$a = \frac{1}{2RC} = \frac{1}{2.(50).(25 \times 10^{-6})} = 400 \, rad/s$$

$\rightarrow w_0^2 > a^2$, hence this circuit is underdamped

underdamped voltage eq

$$V_o(t) = e^{-at}\left[B_1 \cos(w_d t) + B_2 \sin(w_d t)\right]$$

$$w_d = \sqrt{(w_0^2 - a^2)} = \sqrt{500^2 - 400^2} = 300 \, rad/s$$

$$V_o(0) = e^{-a.0}\left[B_1 \cos(0) + B_2 \sin(0)\right] = B_1 = 100V$$

$$\frac{dV_o(0)}{dt} = \frac{1}{C} \cdot \left(-I_0 - \frac{V_o}{R}\right) = \frac{1}{25 \times 10^{-6}} \cdot \left(-5 - \frac{100}{50}\right) = -280{,}000 \, V/s$$

from Ertan teacher's slide

$$\frac{dV_o(t)}{dt} = -a.e^{-at}\left[B_1 \cos(w_d t) + B_2 \sin(w_d t)\right] + e^{-at}\left[-w_d . B_1 \sin(w_d t) + w_d B_2 \cos(w_d t)\right]$$

$$\frac{dV_o(0)}{dt} = -a . B_1 + w_d . B_2 = -280{,}000$$

$$= -400 . 100 + 300 B_2 = -280{,}000$$

, $B_2 = -800V$

$a = 400$ rad/s

$w_d = 300$ rad/s

$B_1 = 100V$

$B_2 = -800V$

Equation $\Rightarrow V_o(t) = e^{-400t}\left[100 \cos(300t) - 800 \sin(300t)\right]$

for $t \geq 0$

8.27

$L = 25\times10^{-3}\,H$

$C = 62.5\times10^{-6}\,F$

$R = 12.5\,\Omega$

$I_f = 2A$

$V_0 = 50V$

$$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(25\times10^{-3}).(62.5\times10^{-6})}} = 800 \ rad/s$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2.(12.5).(62.5\times10^{-6})} = 640 \ rad/s$$

* $w_0^2 > \alpha^2$, hence circuit $\Rightarrow$ underdamped (needed α, w_d, B_1, B_2)

underdumped current eq

$$i_L(t) = e^{-\alpha t}\left[B_1.\cos(w_d t) + B_2.\sin(w_d t)\right] + I_f$$

* $w_d = \sqrt{w_0^2 - \alpha^2} = \sqrt{(800)^2 - (600)^2} = 480$ rad/s

$i_L(0) = 1A$

$$i_L(0) = e^{-\alpha 0}\left[B_1.\cos(w_d.0) + B_2.\sin(w_d.0)\right] + I_f = 1A$$

$i_L(0) = B_1 + 2A = 1A$, $B_1 = -1A$

* $B_1 = -1A$ $\quad w_d = 480 \ rad/s$

$B_2 = 2.833A$ $\quad \alpha = 640 \ rad/s$

$$\frac{di_L(t)}{dt} = -\alpha.e^{-\alpha t}\left[B_1.\cos(w_d t) + B_2.\sin(w_d t)\right] + e^{-\alpha t}\left[(-w_d).B_1.\sin(w_d t) + (w_d).B_2.\cos(w_d t)\right]$$

$\dfrac{di_L(0^+)}{dt} = -\alpha.B_1 + w_d.B_2$, $\quad 2000 = -640.(-1) + 480.B_2$, $B_2 = 2.833A$

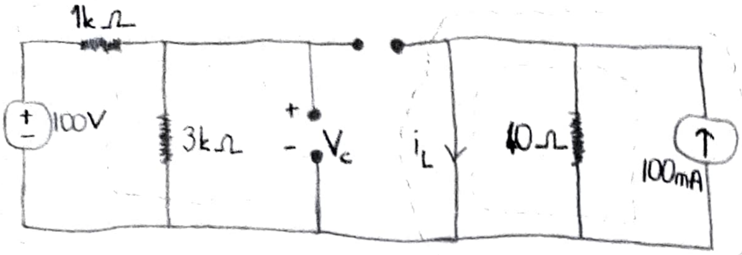
$V(t) = L\dfrac{di}{dt}$, $V(0^+) = V_0 = 50V$

$$\frac{di_L(0^+)}{dt} = \frac{V_0}{L} = \frac{50}{25\times10^{-3}} = 2000 \ A/s$$

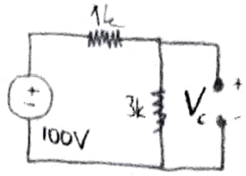
$$i_L(t) = e^{-640t}\left[(-1).\cos(480t) + (2.833).\sin(480t)\right] + 2 \quad \text{for } t \geq 0$$

8.35

for $t=0^-$



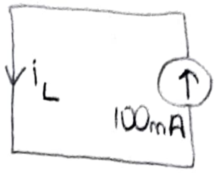
$\rightarrow$



$* \; V_c(0^-) = 100 \cdot \frac{3}{1+3}$

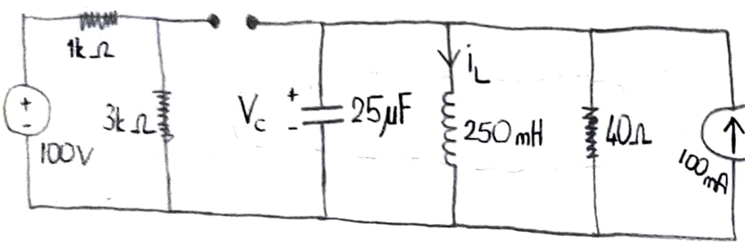
$* \; V_c(0^-) = 75V$

$\rightarrow$



$* \; i_L(0^-) = 100mA$

for $t=0^+$



$$* \; w_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(250\times10^{-3}).(25\times10^{-6})}} = 400 \text{ rad/s}$$

$$* \; \alpha = \frac{1}{2RC} = \frac{1}{2.40.(25\times10^{-6})} = 500 \text{ rad/s}$$

$\rightarrow w_o < \alpha$, hence the circuit is overdamped

overdamped current and voltage eq

$$V_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$i_L(t) = C(s_1 . A_1 e^{s_1 t} + s_2 A_2 e^{s_2 t})$$

! $i(t) = C \cdot \frac{dV(t)}{dt}$

$* \; S_1 = -200$

$* \; S_2 = -800$

$* \; A_1 = -8$

$* \; A_2 = 12$

$$S_1 = -\alpha + \sqrt{\alpha^2 - w_o^2} = -500 + \sqrt{(500)^2 - (400)^2} = -200$$

$$S_2 = -\alpha - \sqrt{\alpha^2 - w_o^2} = -500 - \sqrt{(500)^2 - (400)^2} = -800$$

$$A_1 = \frac{1}{s_1 - s_2} \cdot \left[\frac{dV(0^+)}{dt} - s_2 V(0^+)\right] = \frac{1}{-200+800} \cdot \left[-8000 + 800.4\right] = -8$$

$$A_2 = \frac{1}{S_2 - S_1} \cdot \left[\frac{dV(0^+)}{dt} - s_1 V(0^+)\right] = \frac{1}{-800+200} \left[-8000 + 200.4\right] = 12$$

$$\frac{dV(0^+)}{dt} = \frac{1}{C} \cdot \left(-I_o - \frac{V_o}{R}\right) = \frac{1}{(25\times10^{-6})} \cdot \left(-0.1 - \frac{4}{40}\right) = -8000$$

$$V_o = V(0^+) = i.R = (100\times10^{-3}).40 = 4V$$

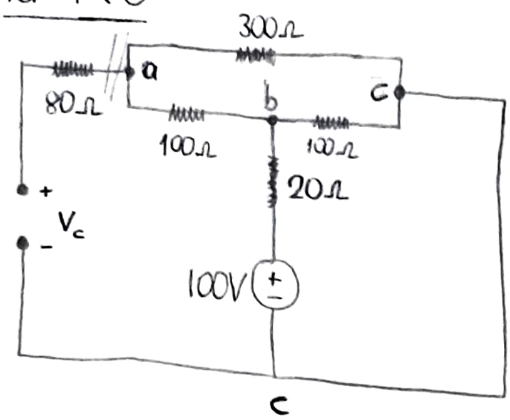
$$I_o = 100mA$$

$$V_c(t) = -8e^{-200t} + 12e^{-800t}$$

$$i_L(t) = (25\times10^{-6}).(-8e^{-200t} + 12e^{-800t})$$

8.47

for $t<0$



node a

$$* \quad \frac{V_a - V_b}{100} + \frac{V_a - V_c}{300} = 0$$

$$* \quad 4V_a - 3V_b - V_c = 0 \quad \ldots (1)$$

node b

$$* \quad \frac{V_b - V_a}{100} + \frac{V_b - V_c}{100} + \frac{V_b - V_c - 100}{100} = 0$$

$$* \quad -V_a + 7V_b - 6V_c = 500 \quad \ldots (2)$$

node c

$$* \quad \frac{V_c - V_a}{300} + \frac{V_c - V_b}{100} + \frac{V_c - V_b + 100}{20} = 0$$

$$* \quad -V_a - 18V_b + 19V_c = -1500 \quad \ldots (3)$$

eq

$$V_a = 60 + V_c$$

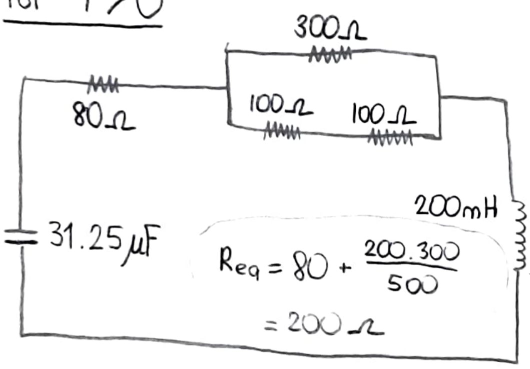
$$V_b = 80 + V_c$$

$$V_c = V_c$$

$$V_c(0^-) = V_c - V_a = -60V$$

$$i_0(0^-) = \frac{V_c - V_b + 100}{20} = 1A$$

for $t>0$



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(200\times10^{-3}).(31.25\times10^{-6})}} = 400 \text{ rad/s}$$

$$\alpha = R/2L = \frac{200}{2.(200\times10^{-3})} = 500 \text{ rad/s}$$

$\rightarrow \omega_0^2 < \alpha^2$, hence circuit is overdamped

overdamped voltage eq

$$V_0(t) = L.\frac{di_0(t)}{dt}$$

$$i_0(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$$\frac{di_0(t)}{dt} = s_1.A_1 e^{s_1 t} + s_2 A_2 e^{s_2 t}$$

KVL

$$* \quad V_C(0) + V_R(0) + V_L(0) = 0$$

$$* \quad 60 + (i_0(0).R) + L.\frac{di(0)}{dt} = 0$$

$$* \quad \frac{di(0)}{dt} = -\frac{140}{200\times(10^{-3})} = -700$$

$S_1 = -200$ rad/s

$S_2 = -800$ rad/s

$A_1 = \frac{1}{6}$

$A_2 = \frac{5}{6}$

$$* \quad S_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -500 + \sqrt{500^2 - 400^2} = -200 \text{ rad/s}$$

$$* \quad S_2 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} = -500 - \sqrt{500^2 - 400^2} = -800 \text{ rad/s}$$

$$i_0(0) = A_1 + A_2 = 1A \quad \ldots (1)$$

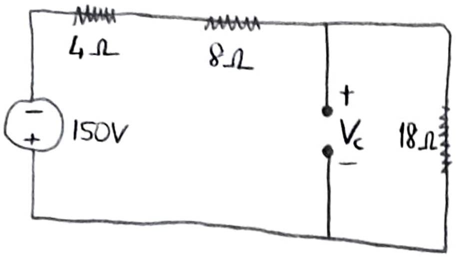
$$\frac{di_0(0)}{dt} = +200A_1 + 800A_2 = +700 \quad (2)$$

$$600A_2 = 500$$

$$V(t) = (200\times10^{-3})\left[\left(-200.\frac{1}{6}e^{-200t}\right) + \left(-800.\frac{5}{6}e^{-800t}\right)\right]$$

8.54

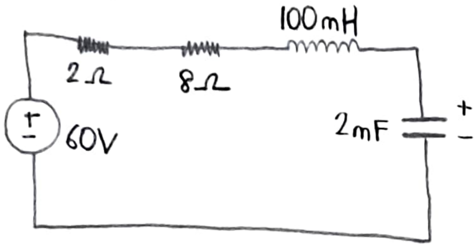
for $t<0$



$$* \; i_L(0^-) = \frac{V}{R} = \frac{-150}{30} = -5A$$

$$* \; V_c(0^-) = -150 \cdot \frac{18}{30} = -90V$$

for $t>0$

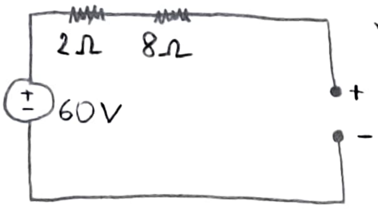


$$* \; w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(100\times10^{-3}).(2\times10^{-3})}} = 70.71\, rad/s$$

$$* \; a = \frac{R}{2L} = \frac{10}{2.(0.1)} = 50\, rad/s$$

$\rightarrow w_0^2 > a^2$, hence circuit is underdamped

for $t \rightarrow \infty$



$$* \; V_c(\infty) = V_f = 60V$$

$a = 50\, rad/s$

$B_1 = -150V$

$B_2 = -200V$

$w_d = 50\, rad/s$

underdamped in series RLC eq

$$V_c(t) = V(t) + V_f$$

$$V(t) = e^{-at}\left[B_1\cos(w_d t) + B_2\sin(w_d t)\right]$$

final $\Rightarrow V_c(t) = e^{-50t}\left[-150\cos(50t) - 200\sin(50t)\right] + 60$

$$w_d = \sqrt{w_0^2 - a^2} = \sqrt{(70.71)^2 - (50)^2} = 50\, rad/s$$

$$V_c(0) = B_1 + V_f \Rightarrow -90 = B_1 + 60$$

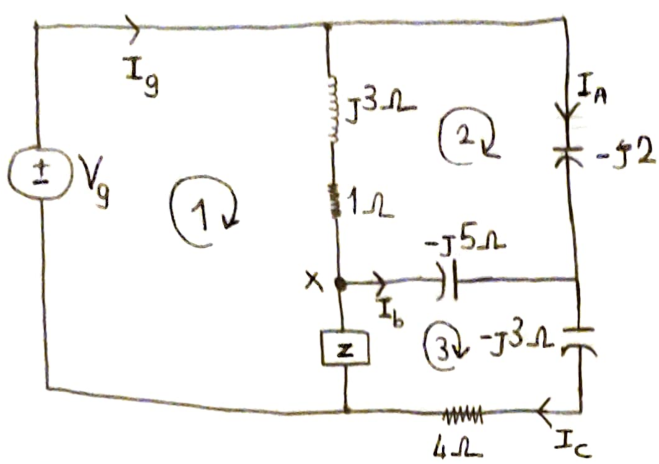
$$\frac{dV(0^+)}{dt} = \frac{i_c(0)}{C} = \frac{-5}{2\times10^{-3}} = -2500$$

$$\frac{dV_c(t)}{t} = -a.e^{-at}\left[B_1\cos(w_d t) + B_2\sin(w_d t)\right] + e^{-at}\left[-w_d.B_1\sin(w_d t) + w_d.B_2\cos(w_d t)\right]$$

$$\frac{dV_c(0)}{t} = -aB_1 + w_d B_2 = -2500$$

$$= -50B_1 + 50B_2 = -2500$$

(9.32)



$* \; I_a = 5e^{j90} A = 5\angle 90^\circ = j5$

$* \; V_g = 25e^{j0} = 25V$

mesh 1

$$* \; \overset{\nearrow 25V}{V_g} + (I_g - \overset{\nearrow 5\angle 90^\circ}{I_a}).(1+j3) + (I_g - I_c).Z = 0 \;\ldots\; (1)$$

mesh 2

$$* \; (I_a - I_g).(1+j3) + \overbrace{(I_a).(-j2)}^{(j5).(-j2)=10V} + (-I_B).(-j5) = 0 \;\ldots\; (2)$$

mesh 3

$$* \; (I_c - I_g).Z + (I_b).(-j5) + (I_c).(4-j3) = 0 \;\ldots\; (3)$$

(1)+(2)

$$* \; 25 + (I_g - I_c).Z + 10 + (-I_B).(-j5) = 0 \;\ldots\; (4)$$

(3)+(4)

$$* \; 35 + (I_c).(4-j3) = 0$$

$$* \; I_c = -\frac{35}{4-j3} = 2.4 + j1.8A$$

$$* \; I_B = I_c - I_a = (2.4+j1.8) - j5 = 2.4 - j3.2A$$

$I_B = 2.4 - j3.2A$

$Z = 1.42 - j1.88$

(1)+(3)

$$* \; 25 + (I_g - j5).(1+j3) + (2.4-j3.2).(-j5) + (2.4+j1.8)(4-j3) = 0$$

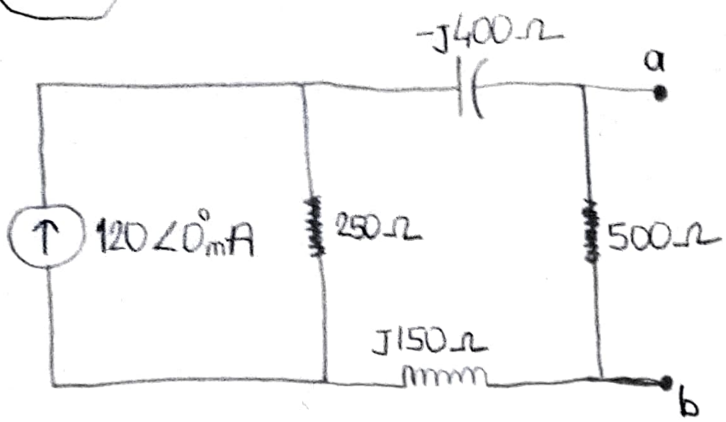
$$* \; I_g = 6.2 - j6.6A$$

solving (1) for Z

$$* \; 25 + (6.2 - j11.6)(1+j3) + (3.8 - j8.4).Z = 0$$

$$* \; Z = 1.42 - j1.88$$

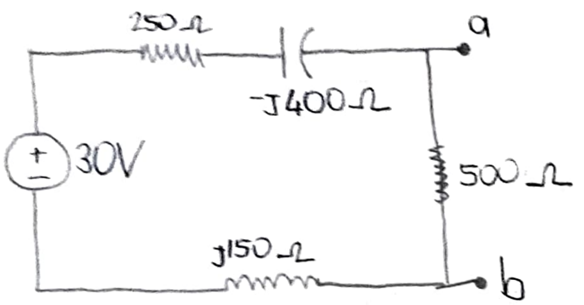
9.45



$* \; 120\angle 0^\circ = 120e^{j0} = 120mA$

$* \; V_{eq} = (120\times10^{-3}) \cdot 250$

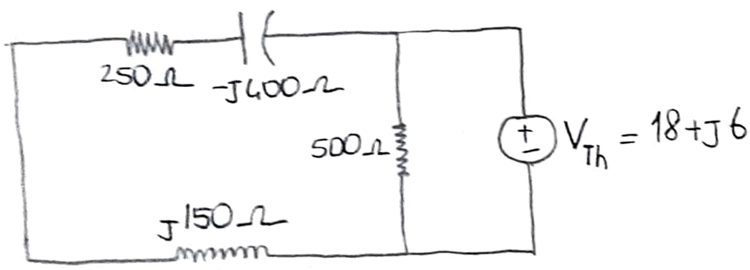
$V_{eq} = 30V$

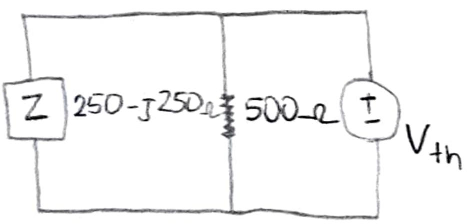


voltage division

$$V_{ab} = 30 \cdot \frac{500}{250 - j400 + 500 + j150}$$

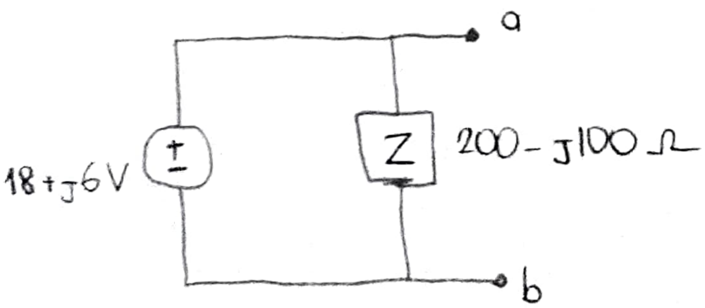
$$V_{ab} = 30 \cdot \frac{500}{750 - j250} = 30 \frac{2}{3-j} = 18 + j6$$



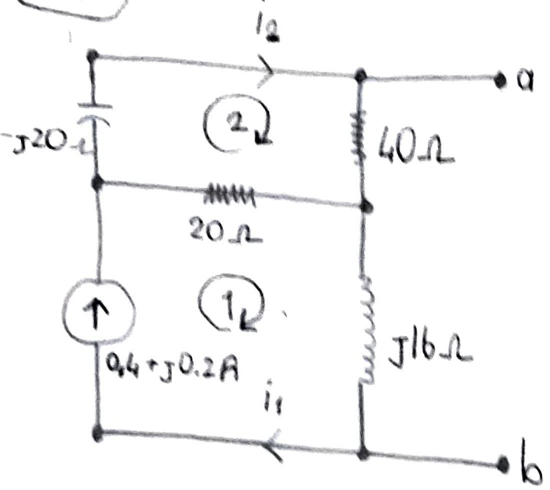


eq

$$Z_{eq} = \frac{(250 - j250) \cdot \cancel{500}^{2}}{(\cancel{750}_{3} - j\cancel{250}_{1})} = \frac{500(1-j)}{3-j} = 200 - j100$$



9.46



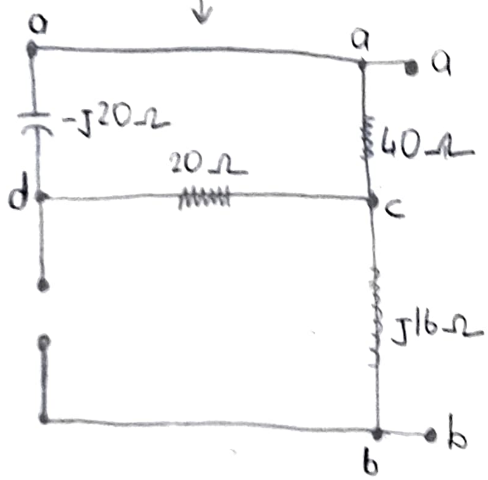
for mesh 1

* $i_1 = 0.4 + j0.2 \text{ A}$

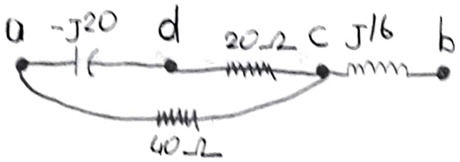
for mesh 2

* $i_2 \cdot (40) + (i_2 - i_1) \cdot 20 + i_2 \cdot (-j20) = 0$
* $40i_2 + 20i_2 - 20i_1 + i_2 \cdot (-j20) = 0$
* $-20i_1 + i_2(60 - j20) = 0$
* $i_2 = \dfrac{8+j4}{60-j20} = \dfrac{2+j}{15-j5} = 0.1 + j0.1 \text{ A}$

* $V_{Th} = V_{40} + V_{j16}$
* $V_{40} = R \cdot i_2 = 40 \cdot (0.1 + j0.1) = 4 + j4 \text{ V}$
* $V_{j16} = R \cdot i_1 = j16 \cdot (0.4 + j0.2) = -3.2 + j6.4 \text{ V}$
* $V_{Th} = 0.8 + j10.8 \text{V}$



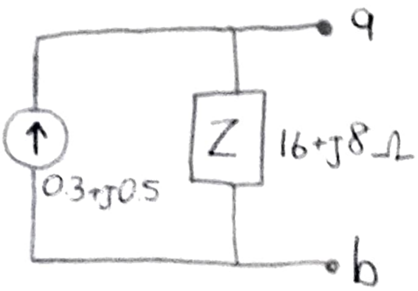
$\rightarrow$



$\rightarrow R_{eq} = \dfrac{(20-j20) \cdot 40}{60-j20} + j16$

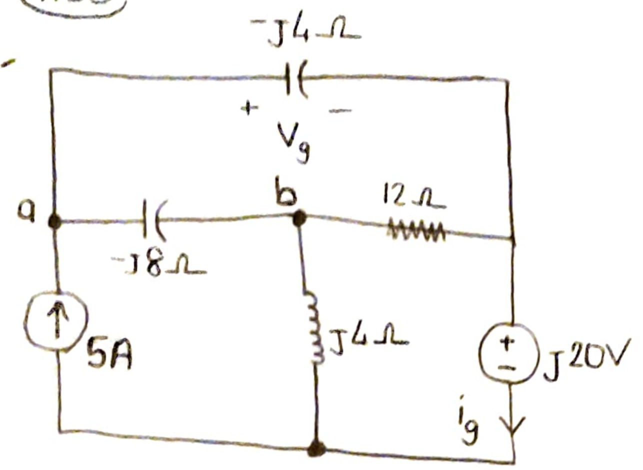
$\rightarrow R_{eq} = 16 + j8 \ \Omega$

circuit



$I_N = \dfrac{V_{Th}}{R_{eq}} = \dfrac{0.8+j10.8}{16+j8} = 0.3 + j0.5 \text{ A}$

(9.55)



<u>current source</u>

$* \; 5\angle 0^\circ = 5e^{j0} = 5A$

<u>voltage source</u>

$* \; 20\angle 90^\circ \Rightarrow a = c.\cos(\emptyset) \Rightarrow a = 20.\cos(90) = 0$

$\Rightarrow b = c.\sin(\emptyset) \Rightarrow b = 20.\sin(90) = 20$

$* \; j20$

<u>for node a</u>

$$* \; \frac{V_a - V_b}{-j8} + \frac{V_a - j20}{-j4} - 5 = 0 \quad (x(-j8))$$

$$* \; V_a - V_b + 2V_a \cancel{-j40} + \cancel{j40} = 0$$

$$* \; 3V_a - V_b = 0 \;\ldots\ldots (1)$$

<u>for node b</u>

$$* \; \frac{V_b - V_a}{\underset{(-3)}{-j8}} + \frac{V_b}{\underset{(6)}{j4}} + \frac{V_b - j20}{\underset{(j2)}{12}} = 0$$

$$* \; -3V_b + 3V_a + 6V_b + V_b.(j2) + 40 = 0$$

$$* \; +3V_a + V_b(3 + j2) = -40 \;\ldots\ldots (2)$$

<u>(2) - (1)</u>

$$* \; \cancel{3V_a} + V_b(3+j2) - 40 \cancel{-3V_a} + V_b = 0$$

$$* \; V_b(4 + j2) = 40$$

$$* \; V_b = \frac{40}{4+j2} = 8 - j4 \text{ V}$$

<u>from (1)</u>

$$* \; 3V_a - V_b = 0$$

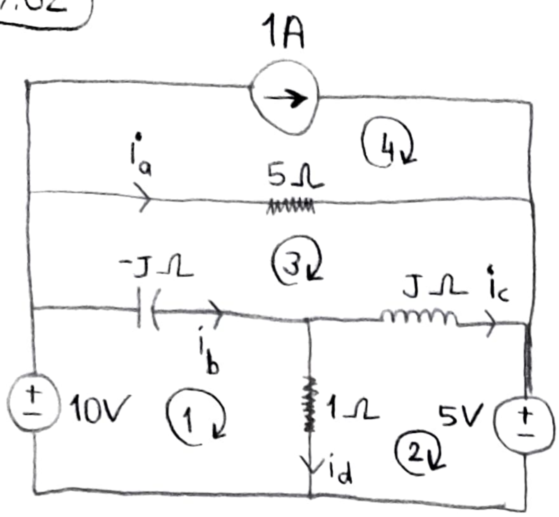
$$* \; V_a = \frac{V_b}{3} = \frac{8-j4}{3} \text{ V}$$

$$* \; V_g = V_a - j20$$

$$= \frac{8-j4}{3} - \frac{j60}{3}$$

$$\boxed{V_g = \frac{8-j64}{3} \text{ V}}$$

9.62



for mesh 1

* $i_b.(-J) + i_d.(1) - 10 = 0$
* $i_b.(-J) + i_d = +10$ 1

for mesh 2

* $-i_d.(1) + i_c.(J) + 5 = 0$
* $i_c.(J) - i_d = -5$ 2

for mesh 3

* $i_a.5 - i_c.(J) - i_b.(-J) = 0$
* $5i_a - i_c.(J) + i_b(J) = 0$ 3

for mesh 4

* $(1 - i_a).5 = 0$
* $i_a = 1A$ //

* $i_a = 1A$
  $i_b = 5+J10$ A
  $i_c = 5+J5$ A
  $i_d = J5$ A

* $i_c.J = i_d - 5$ (from 2)
* $i_c.J = i_b.J + 5$ (from 3)

from circuit

$i_b = i_c + i_d$

* $(i_c + i_d).(-J) + i_d = 10$
* $-i_c.(J) - i_d(J) + i_d = 10$
* $5 - \cancel{i_d} - i_d(J) + \cancel{i_d} = 10$

i_d

* $-i_d(J) = 5$
* $i_d = J5A$ //

i_b

* $i_b.(-J) = 10 - J5$
* $i_b = \frac{10 - J5}{-J}$
* $i_b = 5 + J10$ A

i_c

* $i_c J = J5 - 5$
* $i_c = \frac{J5-5}{J}$
* $i_c = 5+J5$ A