

# A Hands-On Introduction to the GraphBLAS

<http://graphblas.org>

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Brought to you by the “GraphBLAS C Specification Gang” (of Five):  
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... and a special thank you to **Tim Davis (Texas A&M)** for GraphBLAS support in SuiteSparse

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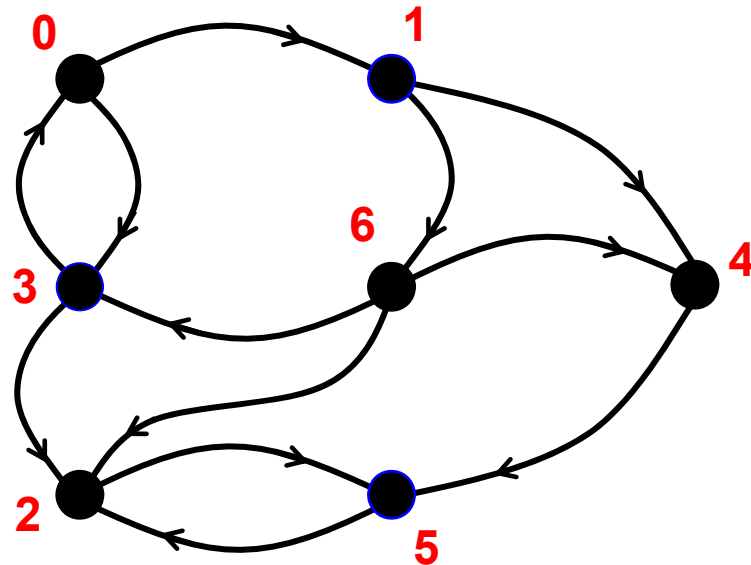
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# Outline

- ➡ • Graphs and Linear Algebra
  - The GraphBLAS C API and Adjacency Matrices
  - GraphBLAS Operations
  - Breadth-First Traversal
  - Connected Components

# Understanding relationships between items

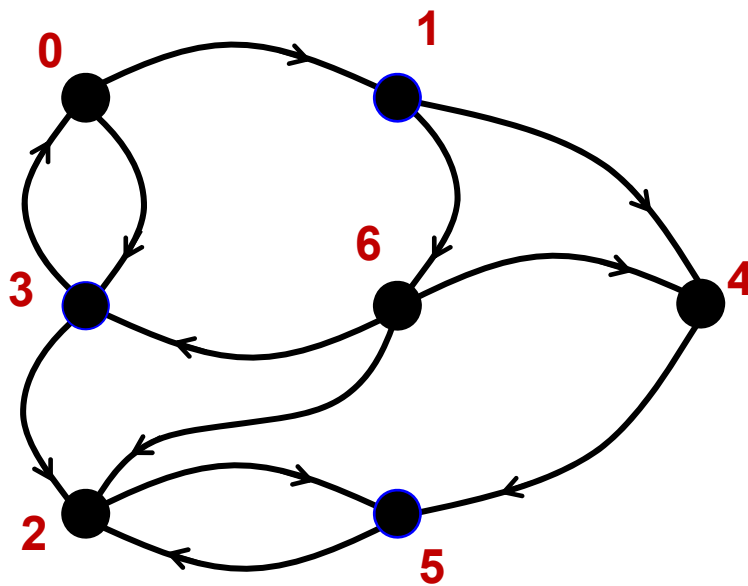
- Graph: A visual representation of a set of vertices and the connections between them (edges).



- Graph: Two sets, one for the vertices ( $v$ ) and one for the edges ( $e$ )  
 $v \in [0, 1, 2, 3, 4, 5, 6]$   
 $e \in [(0,1), (0,3), (1,4), (1,6), (2,5), (3,0), (3,2), (4,5), (5,2), (6,2), (6,3), (6,4)]$

# A graph as a matrix

- Adjacency Matrix: A square matrix (usually sparse) where rows and columns are labeled by vertices and non-empty values are edges from a row vertex to a column vertex

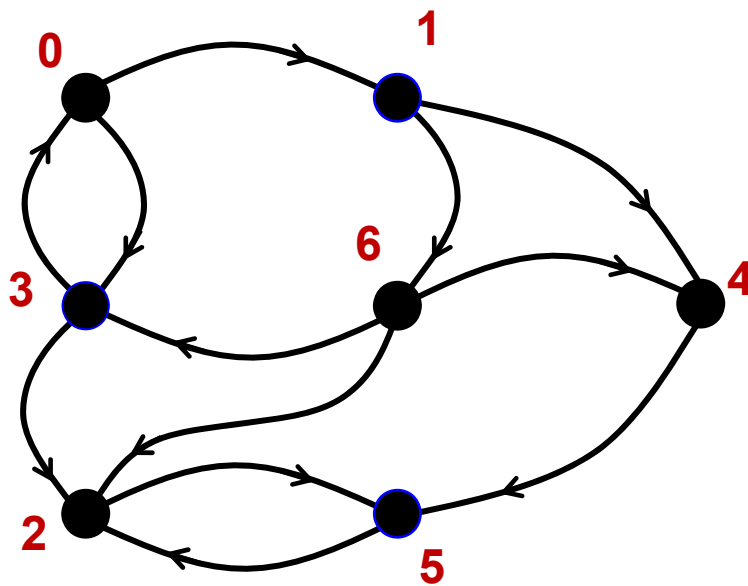


$$A = \begin{matrix} & \begin{matrix} \text{To vertex} \\ \text{(columns)} \end{matrix} \\ \begin{matrix} \text{From} \\ \text{vertex} \\ \text{(rows)} \end{matrix} & \begin{pmatrix} - & \star & - & \star & - & - & - \\ - & - & - & - & \star & - & \star \\ - & - & - & - & - & \star & - \\ \star & - & \star & - & - & - & - \\ - & - & - & - & - & \star & - \\ - & - & \star & - & - & - & - \\ - & - & \star & \star & \star & - & - \end{pmatrix} \end{pmatrix}$$

By using a matrix, I can turn algorithms working with graphs into linear algebra.

# A Directed graph as a matrix

- Adjacency Matrix: A square matrix (usually sparse) where rows and columns are labeled by vertices and non-empty values are edges from a row vertex to a column vertex

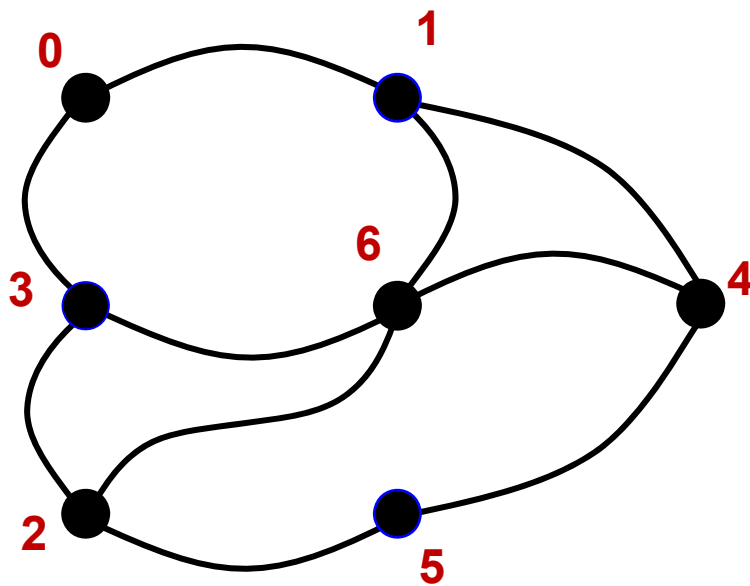


$$A = \begin{matrix} & \begin{matrix} \text{To vertex} \\ \text{(columns)} \end{matrix} \\ \begin{matrix} \text{From} \\ \text{vertex} \\ \text{(rows)} \end{matrix} & \begin{pmatrix} - & \star & - & \star & - & - & - \\ - & - & - & - & \star & - & \star \\ - & - & - & - & - & \star & - \\ \star & - & \star & - & - & - & - \\ - & - & - & - & - & \star & - \\ - & - & \star & - & - & - & - \\ - & - & \star & \star & \star & - & - \end{pmatrix} \end{pmatrix}$$

This is a directed graph  
(the edges have arrows)

# An Undirected graph as a matrix

- Adjacency Matrix: A square matrix (usually sparse) where rows and columns are labeled by vertices and non-empty values are edges from a row vertex to a column vertex

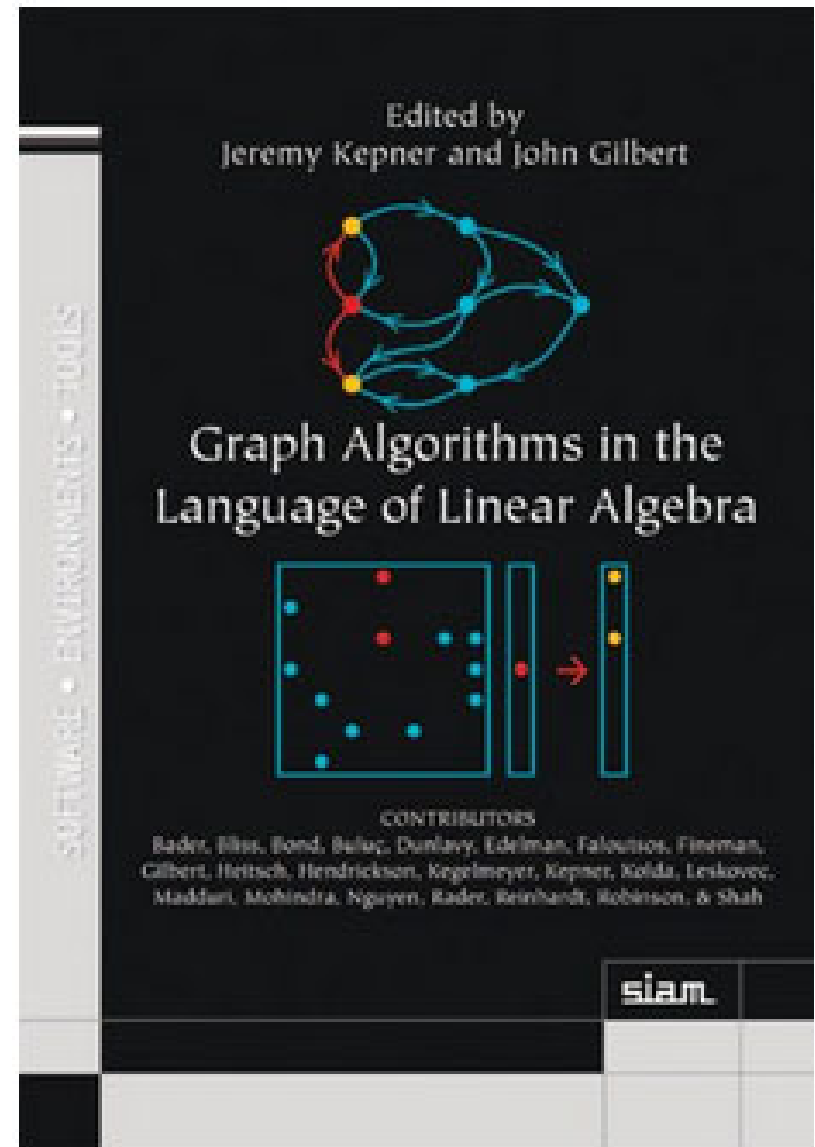


$$A = \begin{matrix} & \begin{matrix} \text{To vertex} \\ \text{(columns)} \end{matrix} \\ \begin{matrix} \text{From} \\ \text{vertex} \\ \text{(rows)} \end{matrix} & \begin{pmatrix} - & \star & - & \star & - & - & - \\ \star & - & - & - & \star & - & \star \\ - & - & - & \star & - & \star & \star \\ \star & - & \star & - & - & - & \star \\ - & \star & - & - & - & \star & \star \\ - & - & \star & - & \star & - & - \\ - & \star & \star & \star & \star & - & - \end{pmatrix} \end{pmatrix}$$

This is an undirected graph (no arrows on the edges)  
and the Adjacency matrix is symmetric

# Graph Algorithms and Linear Algebra

- Most common graph algorithms can be represented in terms of linear algebra.
  - This is a mature field ... it even has a book.
- Benefits of graphs as linear algebra
  - Well suited to memory hierarchies of modern microprocessors
  - Can utilize decades of experience in distributed/parallel computing from linear algebra in supercomputing.
  - Easier to understand ... for some people.





# How do linear algebra people write software?

- They do so in terms of the BLAS:
  - The **B**asic **L**inear **A**lgebra **S**ubprograms: low-level building blocks from which any linear algebra algorithm can be written

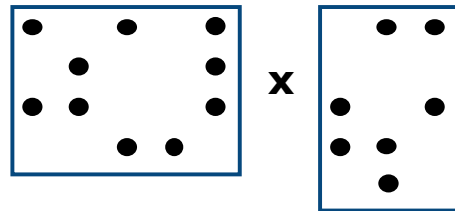
BLAS 1	Vector/vector	Lawson, Hanson, Kincaid and Krogh, 1979	LINPACK
BLAS 2	Matrix/vector	Dongarra, Du Croz, Hammarling and Hanson, 1988	LINPACK on vector machines
BLAS 3	Matrix/matrix	Dongarra, Du Croz, Hammarling and Hanson, 1990	LAPACK on cache based machines

- The BLAS supports a separation of concerns:
  - HW/SW optimization experts tuned the BLAS for specific platforms.
  - Linear algebra experts built software on top of the BLAS ... high performance “for free”.
- It is difficult to over-state the impact of the BLAS ... they revolutionized the practice of computational linear algebra.

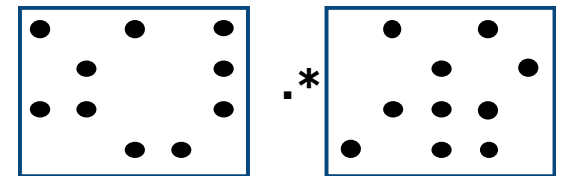
# GraphBLAS: building blocks for graphs as linear algebra

- Basic objects
  - Matrix, vector, algebraic structures, and "control objects"
- Fundamental operations over these objects

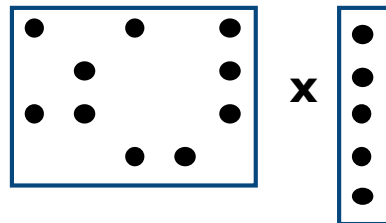
Matrix multiplication



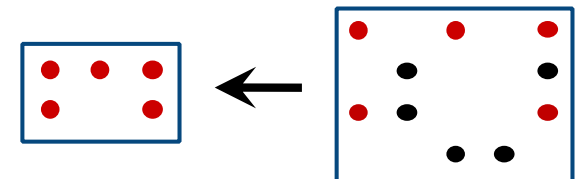
Element-wise operations  
(eWiseAdd, eWiseMult)



Matrix-vector multiplication  
(vxm, mxv)



Extract (and Assign)  
submatrices



...plus reductions, transpose, and application of a function to each element of a matrix or vector

# GraphBLAS References

## Mathematical Foundations of the GraphBLAS

Jeremy Kepner (MIT Lincoln Laboratory Supercomputing Center), Peter Aaltonen (Indiana University), David Bader (Georgia Institute of Technology), Aydın Buluç (Lawrence Berkeley National Laboratory), Franz Franchetti (Carnegie Mellon University), John Gilbert (University of California, Santa Barbara), Dylan Hutchison (University of Washington), Manoj Kumar (IBM), Andrew Lumsdaine (Indiana University), Henning Meyerhenke (Karlsruhe Institute of Technology), Scott McMillan (CMU Software Engineering Institute), Jose Moreira (IBM), John D. Owens (University of California, Davis), Carl Yang (University of California, Davis), Marcin Zalewski (Indiana University), Timothy Mattson (Intel)

IEEE HPEC 2016

## Design of the GraphBLAS API for C

Aydın Buluç<sup>†</sup>, Tim Mattson<sup>‡</sup>, Scott McMillan<sup>§</sup>, José Moreira<sup>¶</sup>, Carl Yang<sup>\*,†</sup>

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<sup>\*</sup>*Electrical and Computer Engineering Department, University of California, Davis, USA*

IEEE HPEC 2017

The official GraphBLAS C spec can be found at: [www.graphblas.org](http://www.graphblas.org)

# GraphBLAS Implementations

- SuiteSparse library (Texas A&M): First fully conforming GraphBLAS release.
  - <http://faculty.cse.tamu.edu/davis/suitesparse.html>
- GraphBLAS C (IBM): the second fully conforming release,
  - <https://github.com/IBM/ibmgraphblas>
- GBTL: GraphBLAS Template Library (CMU/SEI): Pushing GraphBLAS into C++
  - <https://github.com/cmu-sei/gbtl>
- GraphBLAST: A C++ implementations for GraphBLAS for GPUs (UC Davis),
  - <https://github.com/gunrock/graphblast>
- Python bindings:
  - PyGraphBLAS: A Python Wrapper around SuiteSparse GraphBLAS
    - <https://github.com/michelp>
  - PyGB: A Python Wrapper around GBTL (UW/PNNL/CMU)
    - <https://github.com/jessecoleman/gbtl-python-binding>
- pggraphblas: A PostgreSQL wrapper around Suite Sparse GraphBLAS
  - <https://github.com/michelp>
- Matlab and Julia wrappers around SuiteSparse GraphBLAS
  - <https://aldenmath.com>

# GraphBLAS Implementations

- SuiteSparse library (Texas A&M): First fully conforming GraphBLAS release.
  - <http://faculty.cse.tamu.edu/davis/suitesparse.html>
- GraphBLAS C (IBM): the second fully conforming release,
  - <https://github.com/IBM/ibmgraphblas>
- GBTL: GraphBLAS Template Library (CMU/SEI): Pushing GraphBLAS into C++
  - <https://github.com/cmu-sei/gbtl>
- GraphBLAST: A C++ implementations for GraphBLAS for GPUs (UC Davis),
  - <https://github.com/gunrock/graphblast>
- Python bindings:



# LAGraph: A curated collection of high level Graph Algorithms

Graph Algorithms built on top of the GraphBLAS.

## LAGraph: A Community Effort to Collect Graph Algorithms Built on Top of the GraphBLAS

Tim Mattson<sup>‡</sup>, Timothy A. Davis<sup>◊</sup>, Manoj Kumar<sup>¶</sup>, Aydın Buluç<sup>†</sup>, Scott McMillan<sup>§</sup>, José Moreira<sup>¶</sup>, Carl Yang<sup>\*,†</sup>

<sup>‡</sup>Intel Corporation <sup>†</sup>Computational Research Division, Lawrence Berkeley National Laboratory

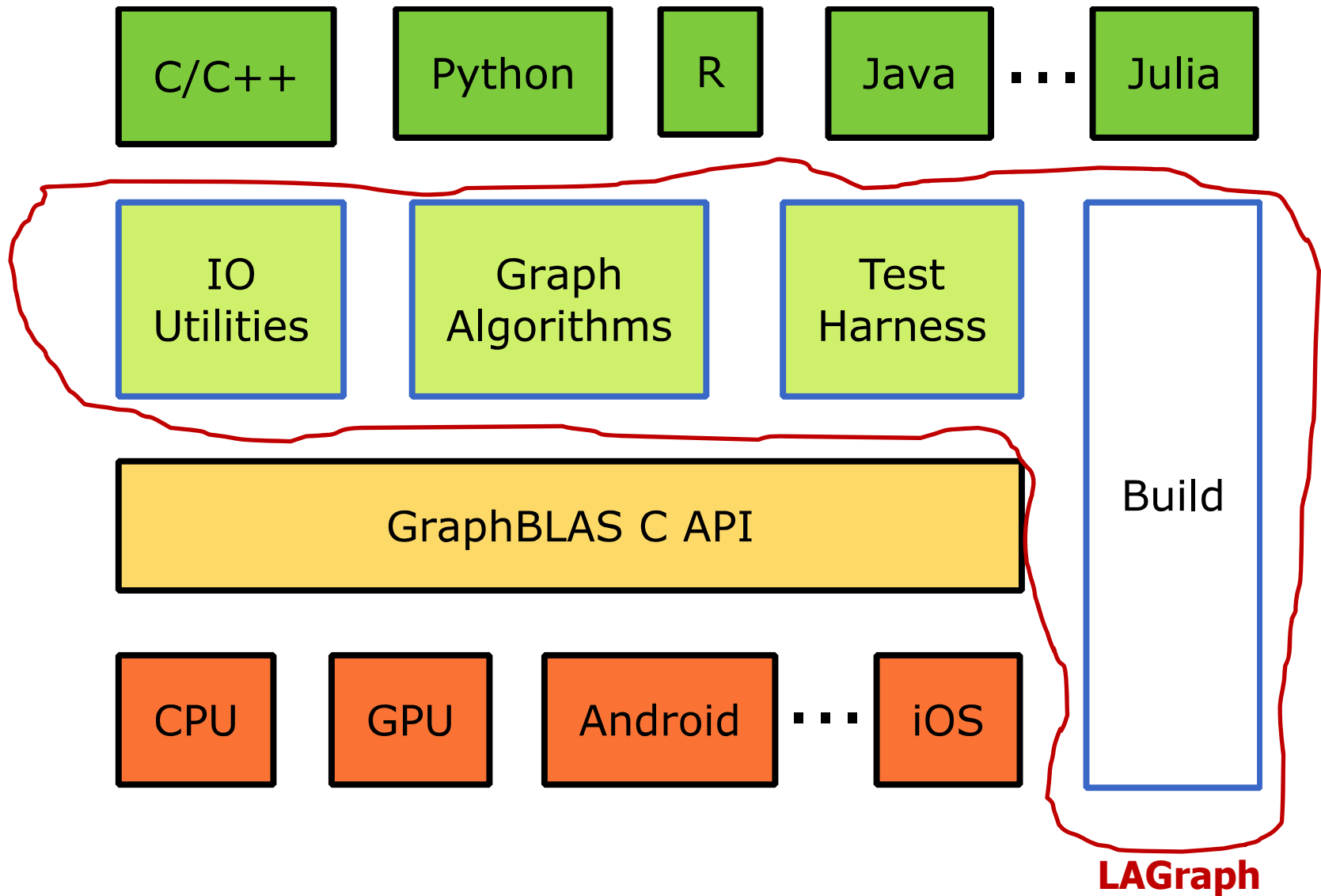
<sup>◊</sup>Texas A&M University <sup>¶</sup>IBM Corporation <sup>§</sup>Software Engineering Institute, Carnegie Mellon University

<sup>\*</sup>Electrical and Computer Engineering Department, University of California, Davis

GrAPL 2019

The LAGraph project's official launch was at  
GrAPL'2019 in May'2019

# LAGraph Effort



# Exercise 1: Build an LAGraph program

- Clone our git repository
- Includes the following components
  - Exercises and solutions
  - SuiteSparse library, binaries for Linux and OSX
  - LAGraph library, binaries for Linux and OSX
- Load software onto your system, make sure you can build and run our test program

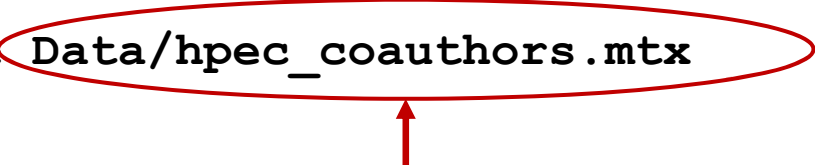
```
$ git clone https://github.com/tgmattso/GraphBLAS.git
```

```
$ cd GraphBLAS/src
```

```
$ git checkout hpec19
```

```
$ make AnalyzeGraph.exe
```

```
$ ./AnalyzeGraph.exe Data/hpec_coauthors.mtx
```



Graph of coauthors: vertices are authors, edges connect authors who've published HPEC papers together



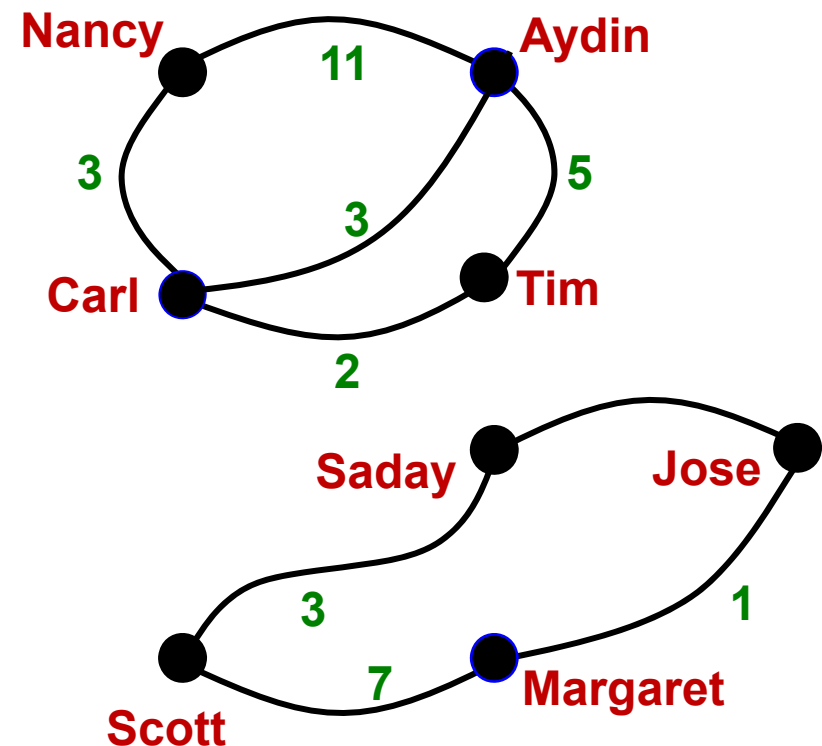
# Solution to Exercise 1:

- If all goes well, output should look like this:

```
$ ./AnalyzeGraph.exe Data/hpec_coauthors.mtx
*** Step 1: loading input graph: Data/hpec_coauthors.mtx
*** Step 1: Elapsed time: 0.00531851 sec
*** Step 2: compute some basic statistics
*** Step 2: Elapsed time: 0.00181371 sec
Num nodes: 1747
Num edges: 10072
Avg degree: 5.765312
Max degree: 461
Min degree: 1
Node with max degree (target ID): 800
*** Step 3: Running LAGraph's connected components (LACC) algorithm.
*** Step 3: Elapsed time: 0.00678869 sec
Number of connected components: 246
ID for component containing target ID 800: 0
*** Step 4: Find all the nodes from the target ID's cluster.
*** Step 4: Elapsed time: 0.000114254 sec
Cluster mask nvals (after masking): 822
Component size: 822
*** Step 5: extract and perform PageRank on the target component.
*** Step 5: Elapsed time: 0.0198434 sec
Author with the highest rank: 800 (0.019374)
Author with the smallest rank: 1094 (0.000214)
...
```

# HPEC Authors Dataset and Connected Components

- Data represents all pairs of HPEC authors that have coauthored papers. The edge value represents how many papers the pair have coauthored.
- File: src/Data/hpec\_authors.mtx
- Graph (undirected):
  - 1,747 vertices (unique authors)
  - 10,072 edges (coauthor count)
- Data directory contains index tables containing the mapping between vertex ID and author name, the raw publication data, and python scripts to perform various queries
- Connected components: a subgraph of vertices connected to each other by a path, but not to vertices outside the subgraph



# Outline

- Graphs and Linear Algebra
- ➔ • The GraphBLAS C API and Adjacency Matrices
- GraphBLAS Operations
- Breadth-First Traversal
- Connected Components

# GraphBLAS C API

- A binding of the GraphBLAS math to the C programming language.
- Requires C99 extended with function polymorphism based on static-types and number-of-parameters.
  - All modern C compilers in common use today support these extensions
- Basic include file with function prototypes, types, and constants
  - `#include <GraphBLAS.h>`
- Includes a few types and opaque objects (e.g. matrices and vectors) to give implementations maximum flexibility
  - `GrB_Index` → An integer type used to set dimensions and index into arrays
  - `GrB_Matrix` → A 2D sparse array, row indices, column indices and values
  - `GrB_Vector` → A 1D sparse array
  - ... plus additional opaque objects we'll describe later (descriptors, semirings, binary operators, and unary operators)

# GraphBLAS C API: Basic definitions

- **Opaque object:** An object manipulated strictly through the GraphBLAS API whose implementation is not defined by the GraphBLAS specification.
- **Transparent object:** an object whose structure is fully exposed to the programmer. E.g.: an array of tuples <i, j, value>
- **Method:** Any C function that manipulates a GraphBLAS opaque object.
- **Domain:** the set of available values used for the elements of matrices, the elements of vectors, and when defining operators.
  - Examples are `GrB_UINT64`, `GrB_INT32`, `GrB_BOOL`, `GrB_FP32`
- **Operation:** a method that corresponds to an operation defined in the GraphBLAS math spec. <http://www.mit.edu/~kepner/GraphBLAS/GraphBLAS-Math-release.pdf>
  - Examples: matrix multiply, matrix-vector multiply, reduction, apply

# Code from our first example (EXERPTS)

```
#include <stdio.h>
#include "GraphBLAS.h"
#include "LAGraph.h"
#include "tutorial_utils.h"

int main(int argc, char** argv) {
    LAGraph_init();

    FILE *fd = fopen(argv[1], "r");
    GrB_Matrix graph = NULL;
    if (GrB_SUCCESS != LAGraph_mmread(&graph, fd))
        exit(-1);

    GrB_Index nrows, ncols, nvals;
    GrB_Matrix_nrows(&nrows, graph);
    GrB_Matrix_ncols(&ncols, graph);
    GrB_Matrix_nvals(&nvals, graph);

    printf("Num nodes: %ld\n", nrows);
    printf("Num edges: %ld\n", nvals);

    // Cleanup
    GrB_free(&A);
    LAGraph_finalize();
}
```

Initialize a LAGraph and GraphBLAS

Create and load a matrix object with data from a file.

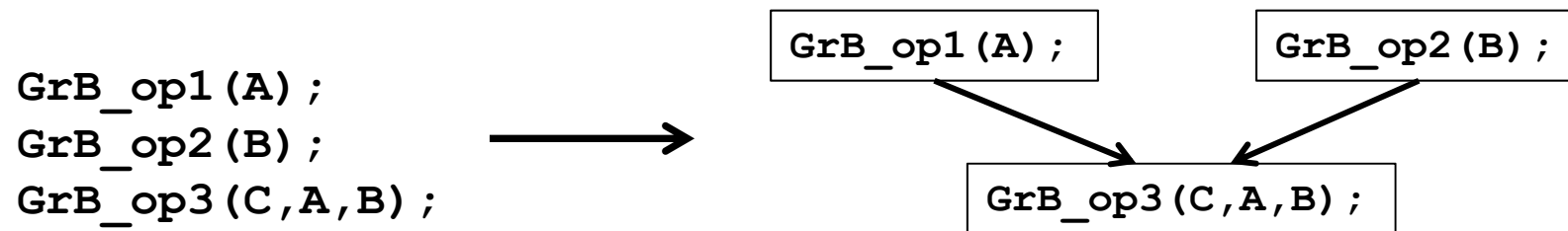
Query the matrix for the dimensions and number of defined (stored) values

Free memory used for our matrix

Close the context, release resources

# Execution modes

- A GraphBLAS program defines a DAG of operations.
- Objects are defined by the sequence of GraphBLAS method calls, but the value of the object is not assured until a GraphBLAS method queries its state.
- This gives an implementation flexibility to optimize the execution (fusing methods, replacing method sequences by more efficient ones, etc.)



- An execution of a GraphBLAS program defines a context for the library.
- The execution runs in one of two modes:
  - Blocking mode ... executes methods in program order with each method completing before the next is called
  - Non-Blocking mode ... methods launched in order. Complete in any order consistent with the DAG. Objects do not exit in fully defined state until queried.
- Most implementations only support blocking mode.

# Predefined low-level types

- Predefined types used to define domains in GraphBLAS

GrB_Type values	C type	domain
GrB_BOOL	bool	{false, true}
GrB_INT8	int8_t	$\mathbb{Z} \cap [-2^7, 2^7)$
GrB_UINT8	uint8_t	$\mathbb{Z} \cap [0, 2^8)$
GrB_INT16	int16_t	$\mathbb{Z} \cap [-2^{15}, 2^{15})$
GrB_UINT16	uint16_t	$\mathbb{Z} \cap [0, 2^{16})$
GrB_INT32	int32_t	$\mathbb{Z} \cap [-2^{31}, 2^{31})$
GrB_UINT32	uint32_t	$\mathbb{Z} \cap [0, 2^{32})$
GrB_INT64	int64_t	$\mathbb{Z} \cap [-2^{63}, 2^{63})$
GrB_UINT64	uint64_t	$\mathbb{Z} \cap [0, 2^{64})$
GrB_FP32	float	IEEE 754 binary32
GrB_FP64	double	IEEE 754 binary64



## Exercise 2: Build a GraphBLAS program

- Build your first GraphBLAS program
- Build and run our BuildGraph.exe program

```
$ cd GraphBLAS/src
```

```
$ make BuildGraph.exe
```

```
$ ./BuildGraph.exe
```

- If all goes well, your output should look like this:

```
$ ./BuildGraph.exe
```

```
Matrix: GRAPH =
```

```
[ -, -, -]
```

```
[ -, -, 4]
```

```
[ -, -, -]
```

# Code from our BuildGraph exercise

```
#include <stdio.h>
#include <assert.h>
#include <GraphBLAS.h>
#include "tutorial_utils.h"

int main(int argc, char** argv) {
    GrB_init(GrB_BLOCKING);
    GrB_Index const NUM_NODES = 3;
    GrB_Matrix graph;
    GrB_Matrix_new(&graph, GrB_UINT64,
                  NUM_NODES, NUM_NODES);
    GrB_Matrix_setElement(graph, 4, 1, 2);

    pretty_print_matrix_UINT64(graph, "GRAPH");
    GrB_Index nvals;
    GrB_Matrix_nvals(&nvals, graph);
    assert(nvals == 1);

    // Cleanup
    GrB_free(&graph);
    GrB_finalize();
}
```

Initialize a context in BLOCKING mode

GrB\_Index used for matrix dimension

Create a matrix object of order  
NUM\_NODES and domain UINT64

Store the value 4 in element (1,2)

Our own "pretty print" routine  
(not part of GraphBLAS)

Query the matrix for the number of  
defined (stored) values and check for  
correctness

Free memory used for our matrix

Close the context, release resources

## Exercise 3: Adjacency matrix

- Draw a simple graph with 3 to 5 nodes.
- Write a program to create the adjacency matrix.
  - Use BuildGraph.c as an example.
- Output the result and verify that your adjacency graph is correct.
- You will need the following types and methods from the GraphBLAS
  - `GrB_Index, GrB_Matrix`
  - `GrB_init(); GrB_finalize();`
  - `GrB_Matrix_new(&graph, GrB_domain, Nrows, Ncols);`
  - `GrB_Matrix_setElement(graph, value, from_node, to_node);`
  - `GrB_Matrix_nvals(&nvals, graph);`
  - `GrB_free(&graph);`
- Hint: Save time and minimize typing
  - Copy BuildGraph.c into another file and modify it to build your adjacency matrix program.
  - Edit the makefile and add your new source file to the list in the definition of SOURCES. Then you can just type “make” to build your program.

## Exercise 3: Adjacency matrix

- Draw a simple graph with 3 to 5 nodes.
- Write a program to create the adjacency matrix.
  - Use BuildGraph.c as an example.
- Output the result and verify that your adjacency graph is correct.
- You will need the following types and methods from the GraphBLAS

A quick API note ... Opaque objects are passed around through a handle (e.g. graph).

- `GrB_Index, GrB_Matrix`
- `GrB_init(); GrB_finalize();`
- `GrB_Matrix_new(&graph, GrB_domain, Nrows, Ncols);`
- `GrB_Matrix_setElement(graph, value, from_node, to_node);`
- `GrB_Matrix_nvals(&nvals, graph);`
- `GrB_free(&graph);`

When the handle itself changes, we pass by address (i.e. with a &).

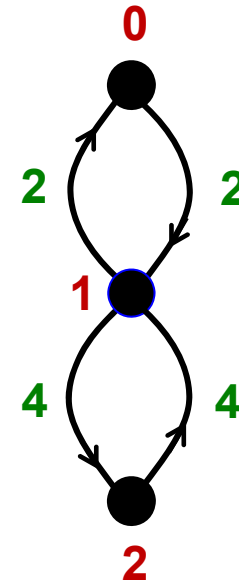
When the object referenced by the handle is manipulated but the handle doesn't change, we pass by value (i.e. without the &).

- Hint: Save time and minimize typing
  - Copy BuildGraph.c into another file and modify it to build your adjacency matrix program.
  - Edit the makefile and add your new source file to the list in the definition of SOURCES. Then you can just type “make” to build your program.

# Solution to Exercise 3

```
...  
GrB_init(GrB_BLOCKING);  
  
GrB_Index const NUM_NODES = 3;  
GrB_Matrix graph;  
GrB_Matrix_new(&graph, GrB_UINT64,  
               NUM_NODES, NUM_NODES);  
  
GrB_Matrix_setElement(graph, 4, 1, 2);  
GrB_Matrix_setElement(graph, 4, 2, 1);  
GrB_Matrix_setElement(graph, 2, 0, 1);  
GrB_Matrix_setElement(graph, 2, 1, 0);  
  
pretty_print_matrix_UINT64(graph, "Graph");  
  
GrB_free(&graph);  
GrB_finalize();
```

Our three node graph  
with edge weights:



Matrix: Graph:

[	-	,	2	,	-]
[	2	,	-	,	4]
[	-	,	4	,	-]

# Building matrices

- Building a matrix one edge at a time is awkward.
- It is often more convenient to do it from vectors defining the indices and values for non-empty elements of the sparse matrix

```
GrB_Info GrB_Matrix_build( GrB_Matrix      C,  
                           const GrB_Index  *row_indices,  
                           const GrB_Index  *col_indices,  
                           const <type>     *values,  
                           GrB_Index        n,  
                           const GrB_BinaryOp dup) ;
```

- **row\_indices**, **col\_indices**, and **values** are transparent arrays.
- **<type>** is a C type consistent with the domain of the matrix
- **n** is the number of entries in the sparse matrix
- **dup** is an associative, commutative function to apply to the values should duplicate locations be specified.
  - Typically use one of the GraphBLAS predefined operators

# Building matrices

- Building a matrix one edge at a time is awkward.
- It is often more convenient to do it from vectors defining the indices and values for non-empty elements of the sparse matrix

**GrB\_Info** GrB\_Matrix\_build( GrB\_Matrix C,  
const GrB\_Index \*row\_indices,  
const GrB\_Index \*col\_indices,  
const <type> \*values,  
GrB\_Index n,  
const GrB\_BinaryOp dup) ;

Return values:

- GrB\_SUCCESS if everything worked
- Other values for problems with input arguments, memory issues, internal errors or other problems.

- **row\_indices**, **col\_indices**, and **values** are transparent arrays.
- **<type>** is a C type consistent with the domain of the matrix
- **n** is the number of entries in the sparse matrix
- **dup** is an associative, commutative function to apply to the values should duplicate locations be specified.
  - Typically use one of the GraphBLAS predefined operators

# GraphBLAS predefined operators

- A subset of operators from Table 2.3 of the GraphBLAS specification

Identifier	Domains	Description	
GrB_LOR	$\text{bool} \times \text{bool} \rightarrow \text{bool}$	$f(x,y) = x \vee y$	Logical OR
GrB_LAND	$\text{bool} \times \text{bool} \rightarrow \text{bool}$	$f(x,y) = x \wedge y$	Logical AND
GrB_EQ_T	$T \times T \rightarrow \text{bool}$	$f(x,y) = (x==y)$	Equal
GrB_MIN_T	$T \times T \rightarrow T$	$f(x,y) = (x < y) ? x : y$	minimum
GrB_MAX_T	$T \times T \rightarrow T$	$f(x,y) = (x > y) ? x : y$	maximum
GrB_PLUS_T	$T \times T \rightarrow T$	$f(x,y) = x + y$	addition
GrB_TIMES_T	$T \times T \rightarrow T$	$f(x,y) = x * y$	multiplication
GrB_FIRST_T	$T \times T \rightarrow T$	$f(x,y) = x$	First argument
GrB_SECOND_T	$T \times T \rightarrow T$	$f(x,y) = y$	Second argument

Where  $T$  is a suffix indicating type and includes `FP32`, `FP64`, `INT32`, `UINT32`, `BOOL`

Note: `GrB_FIRST` and `GrB_SECOND` are not commutative operators

This is a subset of the defined types and operators. See table 2.3 for the full list.



# C code fragment using GrB\_Matrix\_build

```
GrB_Index const NUM_NODES = 3;
GrB_Index const NUM_EDGES = 4;

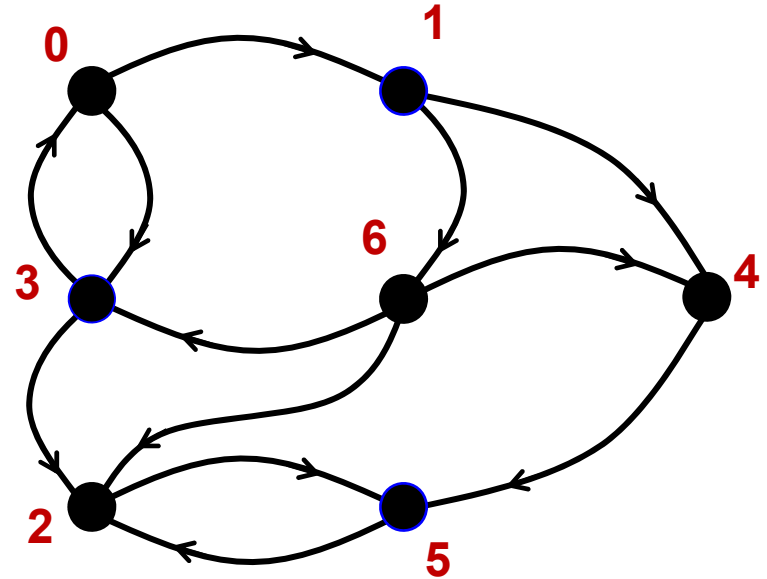
GrB_Index row_indices[] = {0, 1, 1, 2};
GrB_Index col_indices[] = {1, 0, 2, 1};
bool values[] = {true, true, true, true};
GrB_Matrix graph;

GrB_Matrix_new(&graph, GrB_BOOL, NUM_NODES, NUM_NODES);

GrB_Matrix_build(graph,
                 row_indices, col_indices, (bool*)values,
                 NUM_EDGES, GrB_LOR);
```

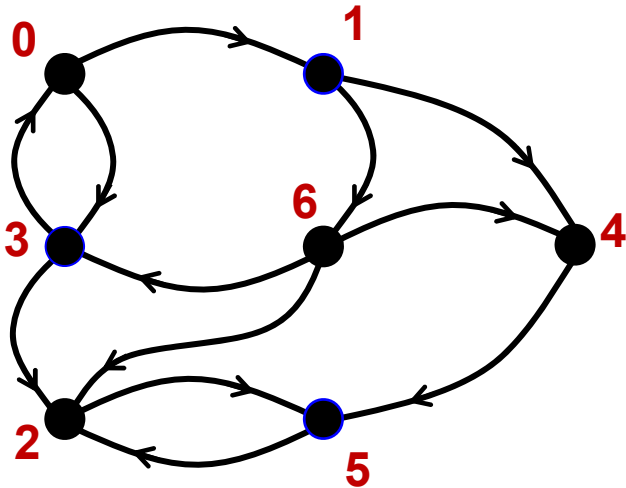
## Exercise 4: Adjacency matrix

- Write a program to create the adjacency matrix for the GraphBLAS “logo” graph using row, column and value arrays.



- You will need the following types and methods from the GraphBLAS
  - `GrB_Index, GrB_Matrix`
  - `GrB_init(); GrB_finalize();`
  - `GrB_Matrix_new(&graph, GrB_domain, Nrows, Ncols);`
  - `GrB_Matrix_build(graph, row_indices, col_indices, values, NUM_EDGES, dup);`
  - `GrB_Matrix_nvals(&nvals, graph);`
  - `GrB_free(&graph);`

# Summary of solution to exercise 4



Matrix: Graph =

[	-	,	1	,	-	,	1	,	-	,	-	,	-	]
[	-	,	-	,	-	,	-	,	1	,	-	,	1	]
[	-	,	-	,	-	,	-	,	-	,	1	,	-	]
[	1	,	-	,	1	,	-	,	-	,	-	,	-	]
[	-	,	-	,	-	,	-	,	-	,	1	,	-	]
[	-	,	-	,	1	,	-	,	-	,	-	,	-	]
[	-	,	-	,	1	,	1	,	1	,	-	,	-	]

```
GrB_Index const NUM_NODES = 7;
```

```
GrB_Index const NUM_EDGES = 12;
```

```
GrB_Index row_indices[] = {0, 0, 1, 1, 2, 3, 3, 4, 5, 6, 6, 6};
```

```
GrB_Index col_indices[] = {1, 3, 4, 6, 5, 0, 2, 5, 2, 2, 3, 4};
```

```
bool values[] = {true, true, true, true, true, true,
                 true, true, true, true, true, true};
```


```
GrB_Matrix graph;
```

```
GrB_Matrix_new(&graph, GrB_BOOL, NUM_NODES, NUM_NODES);
```

```
GrB_Matrix_build(graph, row_indices, col_indices, (bool*)values,
                 NUM_EDGES, GrB_LOR);
```

```
pretty_print_matrix_UINT64(graph, "Graph");
```

# Outline

- Graphs and Linear Algebra
- The GraphBLAS C API and Adjacency Matrices
-  • GraphBLAS Operations
- Breadth-First Traversal
- Connected Components

# GraphBLAS Operations (from the Math Spec\*)

Operation name	Mathematical description
mxm	$\mathbf{C} \odot= \mathbf{A} \oplus . \otimes \mathbf{B}$
mxv	$\mathbf{w} \odot= \mathbf{A} \oplus . \otimes \mathbf{v}$
vxm	$\mathbf{w}^T \odot= \mathbf{v}^T \oplus . \otimes \mathbf{A}$
eWiseMult	$\mathbf{C} \odot= \mathbf{A} \otimes \mathbf{B}$
	$\mathbf{w} \odot= \mathbf{u} \otimes \mathbf{v}$
eWiseAdd	$\mathbf{C} \odot= \mathbf{A} \oplus \mathbf{B}$
	$\mathbf{w} \odot= \mathbf{u} \oplus \mathbf{v}$
reduce (row)	$\mathbf{w} \odot= \bigoplus_j \mathbf{A}(:, j)$
apply	$\mathbf{C} \odot= F_u(\mathbf{A})$
	$\mathbf{w} \odot= F_u(\mathbf{u})$
transpose	$\mathbf{C} \odot= \mathbf{A}^T$
extract	$\mathbf{C} \odot= \mathbf{A}(\mathbf{i}, \mathbf{j})$
	$\mathbf{w} \odot= \mathbf{u}(\mathbf{i})$
assign	$\mathbf{C}(\mathbf{i}, \mathbf{j}) \odot= \mathbf{A}$
	$\mathbf{w}(\mathbf{i}) \odot= \mathbf{u}$

We use  $\odot$ ,  $\oplus$ , and  $\otimes$  since later on we'll manipulate the algebraic structure to generalize them to other operations.

GrB\_mxv()

$$w \odot = A \oplus . \otimes u$$

Multiply a matrix times a vector to produce a vector

$$w(i) = w(i) \odot \sum_{k=0}^N A(i, k) \otimes u(k)$$

$$w \in S^M \quad u \in S^N \quad A \in S^{M \times N}$$

Definitions:

- $S$  is the domain of the objects  $w$ ,  $u$ , and  $A$
- $\odot$  is an optional accumulation operator (a binary operator)
- $\otimes$  and  $\oplus$  are multiplication and addition (or generalizations thereof)
- $\Sigma$  uses the  $\oplus$  operator

GrB\_mxv()

$$w \odot = A \oplus . \otimes u$$

Multiply a matrix times a vector to produce a vector

$$w(i) = w(i) \odot \sum_{k \in \text{ind}(A(i,:)) \cap \text{ind}(u)} A(i, k) \otimes u(k)$$

The summation is over the intersection of the existing elements in the  $i^{\text{th}}$  row of  $A$  with  $u$  ... which avoids exposing how empty elements (i.e. "zeros") are represented. This becomes important when we change the semiring between operations

$$w \in S^M \quad u \in S^N \quad A \in S^{M \times N}$$

Definitions:

- $S$  is the domain of the objects  $w$ ,  $u$ , and  $A$
- $\odot$  is an optional accumulation operator (a binary operator)
- $\otimes$  and  $\oplus$  are multiplication and addition (or generalizations thereof)
- $\Sigma$  uses the  $\oplus$  operator
- $\text{ind}(u)$  returns the indices of the stored values of  $u$

## GrB\_mxv()

$$w \odot = A \oplus \cdot \otimes u$$

- Compute the product of a GraphBLAS sparse matrix with a GraphBLAS vector.
- Returns error codes of type GrB\_info. See the spec for details.

```
GrB_Info GrB_mxv(GrB_Vector          w,  
                  const GrB_Vector    mask,  
                  const GrB_BinaryOp  accum,  
                  const GrB_Semiring  op,  
                  const GrB_Matrix    A,  
                  const GrB_Vector    u,  
                  const GrB_Descriptor desc);
```



## GrB\_mxv()

$$w \otimes = A \oplus \otimes u$$

- Compute the product of a GraphBLAS sparse matrix with a GraphBLAS vector.
- Returns error codes of type GrB\_info. See the spec for details.


```
GrB_Info GrB_mxv(GrB_Vector w,  
                 const GrB_Vector mask → GrB_NULL,  
                 const GrB_BinaryOp accum → GrB_NULL,  
                 const GrB_Semiring op,  
                 const GrB_Matrix A,  
                 const GrB_Vector u,  
                 const GrB_Descriptor desc → GrB_NULL)
```

**Let's ignore mask, accum and desc for now and use default values (indicated by GrB\_NULL)**

## GrB\_mxv()

$$w \otimes = A \oplus \cdot \otimes u$$

- Compute the product of a GraphBLAS sparse matrix with a GraphBLAS vector.
- Returns error codes of type GrB\_info. See the spec for details.

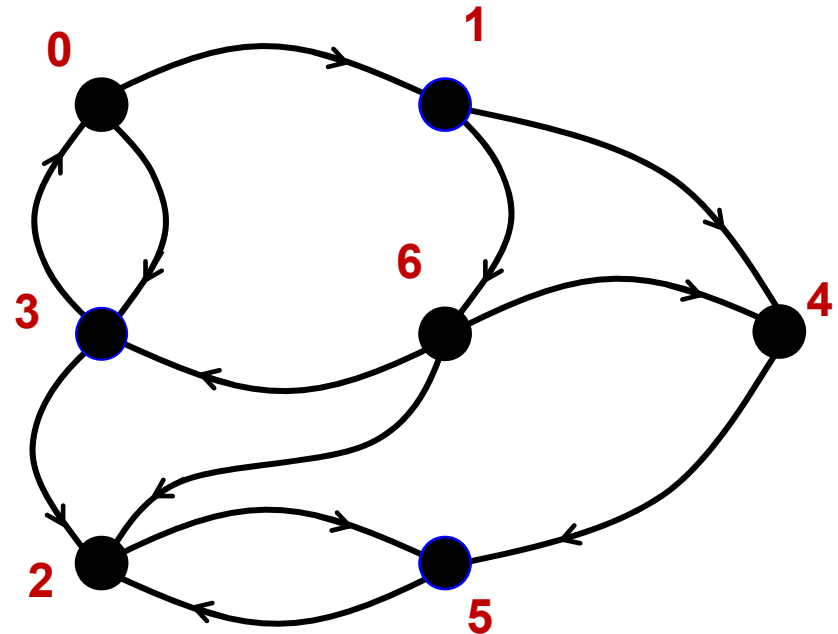
```
GrB_Info GrB_mxv(GrB_Vector w,  
                  const GrB_Vector mask → GrB_NULL,  
                  const GrB_BinaryOp accum → GrB_NULL,  
                  const GrB_Semiring op,   
                  const GrB_Matrix A,  
                  const GrB_Vector u,  
                  const GrB_Descriptor desc → GrB_NULL)
```

Op defines the algebraic structure, a semiring in this case. This gives us  $\otimes$  and  $\oplus$  and the identity for  $\oplus$ . We'll say much more about this later. For our first exercises with bool objects, we'll use a built-in SuiteSparse semiring `GxB_LOR_LAND_BOOL`.

## Exercise 5: Matrix Vector Multiplication

- Use the adjacency matrix from exercise 4 and a vector with a single value to select one of the nodes in the graph.
- Find the product  $mxv$ , print the result, and interpret its meaning.
- In addition to those from Exercise 4, you'll need the functions:

```
- GrB_Vector result, vec;  
- GrB_Index NODE;  
- GrB_Vector_new(&vec, GrB_BOOL, NUM_NODES);  
- GrB_Vector_setElement(vec, true, NODE);  
- pretty_print_vector_UINT64(vec, "Input node");  
- GrB_mxv(result, GrB_NULL, GrB_NULL,  
          GxB_LOR_LAND_BOOL, graph, vec, GrB_NULL);
```

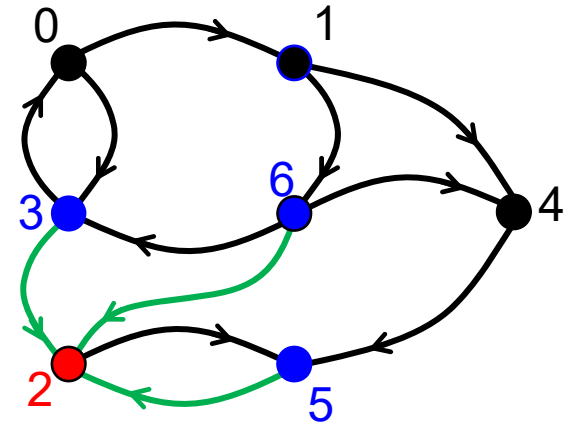


# Solution to exercise 5

```
pretty_print_matrix_UINT64(graph, "GRAPH");

// Build a vector with one node set.
GrB_Index const NODE = 2;
GrB_Vector vec, result;
GrB_Vector_new(&result, GrB_BOOL, NUM_NODES);
GrB_Vector_new(&vec, GrB_BOOL, NUM_NODES);
GrB_Vector_setElement(vec, true, NODE);
pretty_print_vector_UINT64(vec, "Target node");

GrB_mxxv(result, GrB_NULL, GrB_NULL,
          GxB_LOR_LAND_BOOL, graph, vec, GrB_NULL);
pretty_print_vector_UINT64(result, "sources");
```



```
Matrix: GRAPH =
[ -, 1, -, 1, -, -, -]
[ -, -, -, -, 1, -, 1]
[ -, -, -, -, -, 1, -]
[ 1, -, 1, -, -, -, -]
[ -, -, -, -, -, 1, -]
[ -, -, 1, -, -, -, -]
[ -, -, 1, 1, 1, -, -]
Vector: Target node =
[ -, -, 1, -, -, -, -]
Vector: sources =
[ -, -, -, 1, -, 1, 1]
```

The stored elements of the adjacency matrix,  $a(i,j)$  indicate an edge from vertex  $i$  to vertex  $j$

So the matrix vector product scans over a row (from) to find when an edge lands at the destination

# Finding neighbors

- A more common operation is to input a vector selecting a source and find all the neighbors one hop away from that vertex.
- Using `mxv()`, how would you do this?

# Finding neighbors

- A more common operation is to input a vector selecting a source and find all the neighbors one hop away from that vertex.
- Using `mxv()`, how would you do this?
  - The adjacency matrix elements indicate edges
    - from a vertex (row index)
    - to another vertex (columns index)
  - Then the transpose of the adjacency matrix indicates edges
    - To a vertex (row index)
    - From other vertices (column index)
- Therefore, we can find the neighbors of a vertex (marked by the non-empty elements of  $v$ )

$$\text{Neighbors} = A^T \oplus \cdot \otimes v$$

- The GraphBLAS defines a transpose operation, but given how often you need to do a transpose, there must be a better way

# Changing the behavior of a GraphBLAS operation

- Most GraphBLAS operations take an argument that is an opaque object called a “descriptor”. You declare a descriptor called “desc” and create it as follows:

```
GrB_Descriptor desc;  
GrB_Descriptor_new (&desc);
```

- The descriptor controls the behavior of the method and how objects are handled inside the method.
- The descriptor controls:
  - Do you *transpose input matrices*? (GrB\_TRAN)
  - Does the computation *replace existing values in the output object* or combine with them? (GrB\_REPLACE)
  - Take the *structural complement of the mask* object (swap empty/false  $\leftrightarrow$  filled/true values in a sparse object). (GrB\_SCMP)

....To be discussed later

# Using Descriptors

- A descriptor is an opaque object so you set its values with a GraphBLAS method.
- A descriptor *field* selects the object it impacts:
  - **GrB\_OUTP**: The output GraphBLAS object
  - **GrB\_INP0**: The first input GraphBLAS object (matrix or vector)
  - **GrB\_INP1**: The second input GraphBLAS object (matrix or vector)
  - **GrB\_MASK**: The GraphBLAS mask object (described later).
- A descriptor *value* describes the action to be taken.
- For example, to transpose the first input matrix, you'd call the operation and pass in the following descriptor

```
GrB_Descriptor desc;  
GrB_Descriptor_new(&desc) ;  
GrB_Descriptor_set(desc, GrB_INP0, GrB_TRAN) ;
```



## Exercise 6: Matrix Vector Multiplication

- Modify your program from exercise 5 to multiply by the transpose of the adjacency matrix.
- Verify that you can use that to find the one-hop neighbors of any vertex

```
- GrB_Vector result, vec;  
- GrB_Index NODE;  
- GrB_Vector_new(&vec, GrB_BOOL, NUM_NODES);  
- GrB_Vector_setElement(vec, true, NODE);  
- pretty_print_vector_UINT64(vec, "Input node");  
- GrB_Descriptor desc;  
- GrB_Descriptor_new(&desc);  
- GrB_Descriptor_set(desc, FIELD, VALUE)  
- GrB_m xv(result, GrB_NULL, GrB_NULL,  
            GxB_LOR_LAND_BOOL, graph, vec, desc);
```

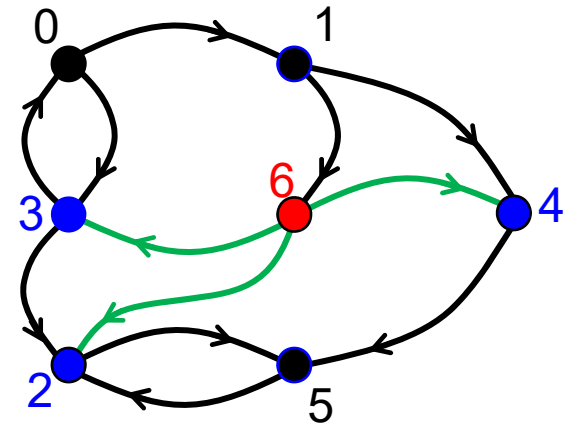
```
FIELD: GrB_INP0, GrB_INP1, GrB_OUTP, GrB_MASK  
VALUE: GrB_TRAN, GrB_REPLACE, GrB_SCMP
```

# Solution to exercise 6

```
// Build a vector with one node set.  
GrB_Index const SRC_NODE = 6;  
GrB_Vector vec;  
GrB_Vector_new(&vec, GrB_BOOL, NUM_NODES);  
GrB_Vector_setElement(vec, true, SRC_NODE);
```

```
GrB_Descriptor desc;  
GrB_Descriptor_new(&desc);  
GrB_Descriptor_set(desc, GrB_INP0, GrB_TRAN);
```

```
pretty_print_vector_UINT64(vec, "source node");  
GrB_mxv(vec, GrB_NULL, GrB_NULL,  
        GxB_LOR_LAND_BOOL, graph, vec, desc);  
pretty_print_vector_UINT64(vec, "neighbors");
```



```
Vector: source node =  
[ -, -, -, -, -, -, 1]  
Vector: neighbors =  
[ -, -, 1, 1, 1, -, -]  
GrB_mxv test passed.
```

The transposed matrix vector product scans over a columns (to) to find edges that start at the source node.

## GrB\_mxv()

$$W \otimes = A \oplus \otimes u$$

- Compute the product of a GraphBLAS Sparse Matrix with a GraphBLAS vector.
- Returns error codes of type GrB\_info. See the spec for details.

```
GrB_Info GrB_mxv(GrB_Vector w,  
                 const GrB_Vector mask → GrB_NULL,  
                 const GrB_BinaryOp accum → GrB_NULL,  
                 const GrB_Semiring op,  
                 const GrB_Matrix A,  
                 const GrB_Vector u,  
                 const GrB_Descriptor desc → GrB_NULL)
```

It's time to explain semirings in GraphBLAS operations

# Algebraic Semirings

- Semiring: An Algebraic structure that generalizes real arithmetic by replacing  $(+,*)$  with binary operations  $(Op1, Op2)$ 
  - $Op1$  and  $Op2$  have identity elements sometimes called 0 and 1
  - $Op1$  and  $Op2$  are associative.
  - $Op1$  is commutative,  $Op2$  distributes over  $Op1$  from both left and right
  - The  $Op1$  identity is an  $Op2$  annihilator.

# Algebraic Semirings

- Semiring: An Algebraic structure that generalizes real arithmetic by replacing  $(+, *)$  with binary operations  $(Op1, Op2)$ 
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  - The  $Op1$  identity is an  $Op2$  annihilator.

$(R, +, *, 0, 1)$ Real Field	Standard operations in linear algebra
---------------------------------	---------------------------------------

Notation:  $(R, +, *, 0, 1)$

Scalar type

$Op1$

$Op2$

Identity  $Op1$

Identity  $Op2$

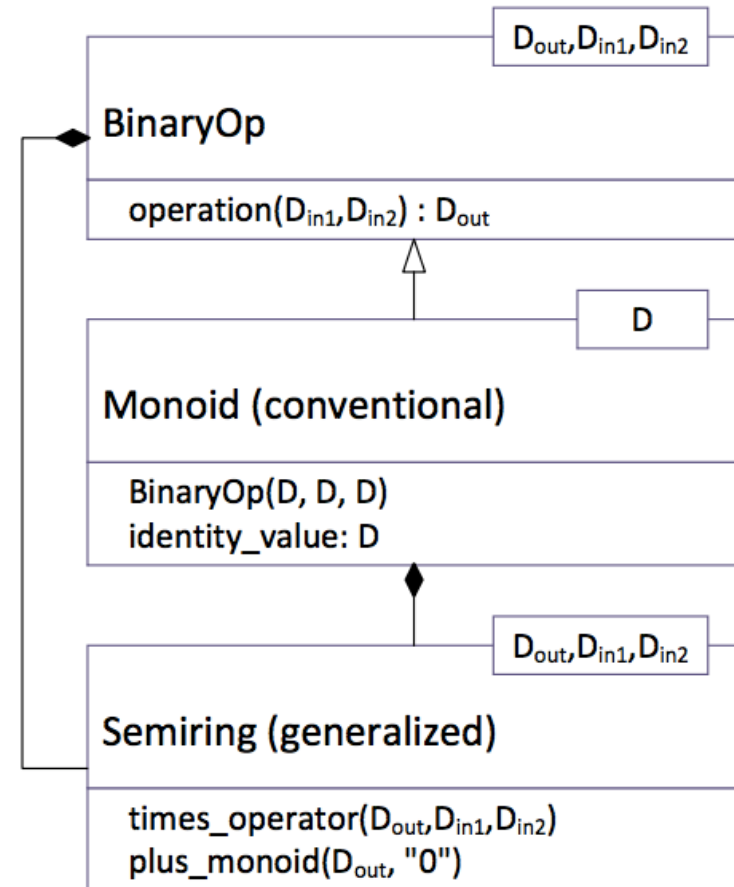
# Algebraic Semirings

- Semiring: An Algebraic structure that generalizes real arithmetic by replacing  $(+,*)$  with binary operations  $(Op1, Op2)$ 
  - $Op1$  and  $Op2$  have identity elements sometimes called 0 and 1
  - $Op1$  and  $Op2$  are associative.
  - $Op1$  is commutative,  $Op2$  distributes over  $Op1$  from both left and right
  - The  $Op1$  identity is an  $Op2$  annihilator.

$(\mathbb{R}, +, *, 0, 1)$ Real Field	Standard operations in linear algebra
$(\mathbb{R} \cup \{\infty\}, \min, +, \infty, 0)$ Tropical semiring	Shortest path algorithms
$(\{0,1\},  , \&, 0, 1)$ Boolean Semiring	Graph traversal algorithms
$(\mathbb{R} \cup \{\infty\}, \min, *, \infty, 1)$	Selecting a subgraph or contracting nodes to form a quotient graph.

# Algebraic structures in the GraphBLAS: Semirings and Monoids

- The GraphBLAS semiring defines:
  - A set of allowed values (the domain)
  - Two commutative operators called addition and multiplication
  - An additive identity (called 0) that is the annihilator over multiplication.
- A Monoid is used in defining a semiring:
  - Monoid: A domain, an associative binary operator and an identity corresponding to that operator



Hierarchy of algebraic object classes showing relationships between the various domains and the operators.

# Building Semirings in the GraphBLAS

- First you build the monoid (M) for a particular domain, D, the “addition” operator, and its identity:

$$M = \langle D, \oplus, 0 \rangle$$

- Then define the semiring (S) in terms of the Monoid and the multiplications operator:

$$S = \langle D_{out}, D_{in1}, D_{in2}, M, \otimes \rangle$$

- The domains must be consistent:

$$\otimes: D_{in1} \times D_{in2} \rightarrow D_{out}$$

$$\oplus: D_{out} \times D_{out} \rightarrow D_{out}$$

$$0 \in D_{out}$$



# Building Semirings in the GraphBLAS

- First you build the monoid (M) for the “addition” and its identity:

```
GrB_Info GrB_Monoid_new( GrB_Monoid      *monoid,  
                          GrB_BinaryOp    binary_op,  
                          <type>         identity);
```

- Where the type must be consistent with that of the binary operator which is either a built-in operator (Spec. Table 2.3) or a user-defined operator (not covered here)
- Example:

```
GrB_Monoid UInt64Plus ;  
GrB_Monoid_new(&UInt64Plus, GrB_PLUS_UINT64, 0 ul);
```

# Building Semirings in the GraphBLAS

- Then you build the semiring pairing a monoid (“add”) with a binary operator (“mul”) :

```
GrB_Info GrB_Semiring_new( GrB_Semiring    *semiring,  
                           GrB_Monoid      add_op,  
                           GrB_BinaryOp    mul_op) ;
```

- The monoid’s identity *should* be the binary operator’s annihilator (not enforced).

- Example using the monoid from the previous page:

```
GrB_Semiring UInt64Arith;  
GrB_Semiring_new(&UInt64Arith, UInt64Plus, GrB_TIMES_UINT64) ;
```

# Common Semirings


semiring	Domain	Add	Add-identity	multiply
Boolean	GrB_BOOL	GrB_LOR	false	GrB_LAND
Int32 arithmetic	GrB_INT32	GrB_PLUS_INT32	0	GrB_TIMES_INT32
FP32 arithmetic	GrB_FP32	GrB_PLUS_FP32	0.0f	GrB_TIMES_FP32
Max_second	GrB_FP32	GrB_MAX_FP32	-INFINITY	GrB_SECOND_FP32

## Exercise 7: Changing semirings

- Up to this point, we've used a built-in Boolean semiring that is included with SuiteSparse (`GxB_LOR LAND_BOOL`).
- Pick any of the past exercises and experiment with different semi-rings.
  - `GrB_Monoid UInt64Plus;`
  - `GrB_Monoid_new(&UInt64Plus, GrB_PLUS_UINT64, 0ul);`
  - `GrB_Semiring UInt64Arith;`
  - `GrB_Semiring_new(&UInt64Arith, UInt64Plus, GrB_TIMES_UINT64);`

To save some time, we will  
skip this exercise

# Outline

- Graphs and Linear Algebra
- The GraphBLAS C API and Adjacency Matrices
- GraphBLAS Operations
-  • Breadth-First Traversal
- Connected Components

# Breadth First Traversal

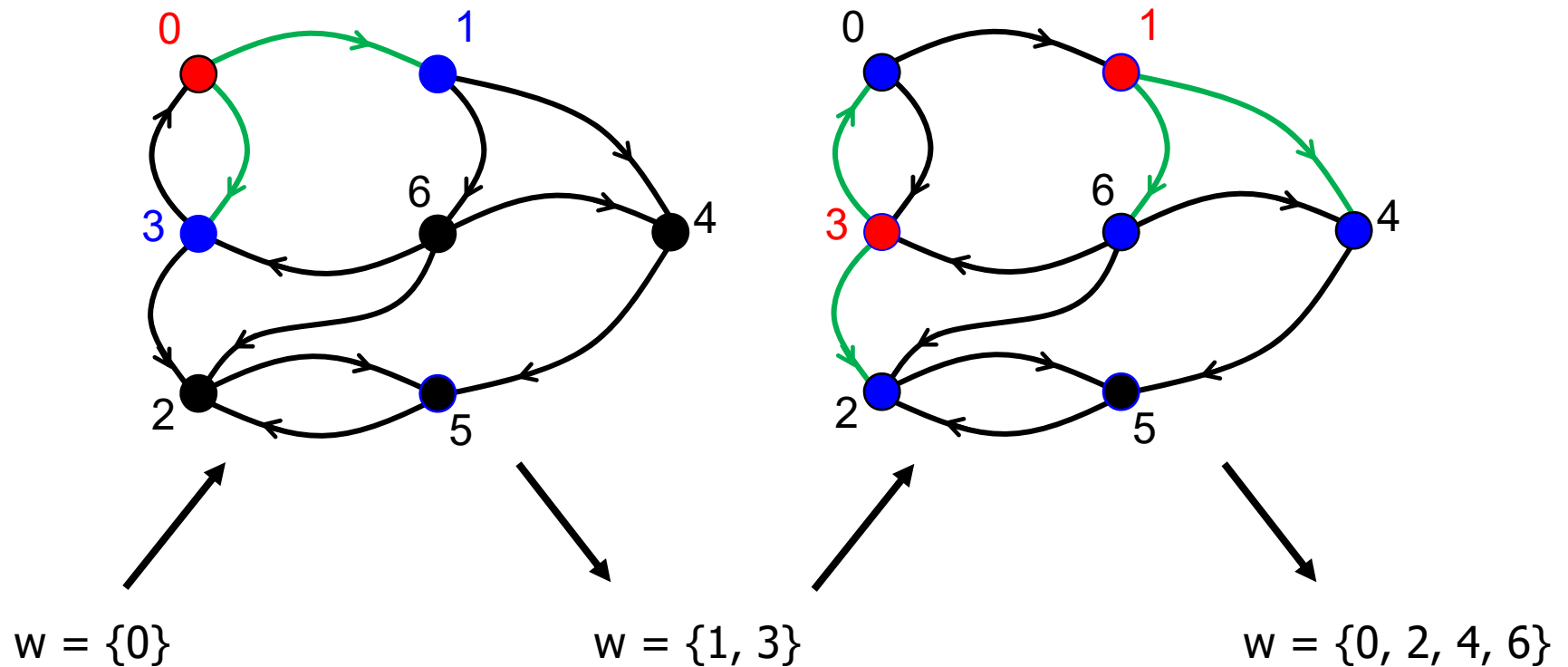
- The Breadth First Traversal:
  - Start from one or more initial vertices
  - Visit all accessible one hop neighbors,
  - Visit all accessible unique two hop neighbors,
  - Continue until no more unique vertices to visit
  - Note: keep track of vertices visited so you don't visit the same vertex more than once
- Breadth first traversal is a common pattern used in a range of graph algorithms
  - Build a spanning tree that contains all vertices and minimal number of edges
  - Search for accessible vertices with certain properties.
  - Find shortest paths between vertices.
  - Other more advanced algorithms such as maxflow and betweenness centrality

# Our Breadth First Traversal plan

- We will build up this algorithm using the GraphBLAS through a series of exercises:
  - Wavefronts and how to move from one wavefront to the next.
  - Iteration across wavefronts
  - Track which vertices have been visited
  - Avoid revisiting vertices
  - Construct the Level Breadth first traversal algorithm

# Wavefronts

- A subset of vertices accessed at one stage in a breadth first search pattern ... for example ....
  - “You tell two friends and they tell two friends...”



Red=current wavefront and visited, Blue=next wavefront, Black=unvisited



## Exercise 8: Traverse the graph

- Modify your code from Exercises 5 to iterate from one wavefront to the next.
- Output each wavefront
- How long before you get a repeating pattern?
  - `GrB_Vector result, vec;`
  - `GrB_Index NODE;`
  - `GrB_Vector_new(&vec, GrB_BOOL, NUM_NODES);`
  - `GrB_Vector_setElement(vec, true, NODE);`
  - `pretty_print_vector_UINT64(vec, "Input node");`
  - `GrB_Descriptor desc;`
  - `GrB_Descriptor_new(&desc);`
  - `GrB_Descriptor_set(desc, FIELD, VALUE)`
  - `GrB_m xv(result, GrB_NULL, GrB_NULL,`  
`GxB_LOR_LAND_BOOL, graph, vec, desc);`

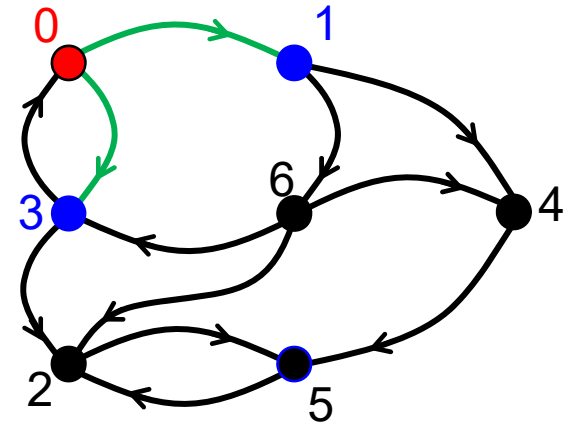
# Solution to exercise 8

```
// First wavefront has one node set.
GrB_Index const SRC_NODE = 0;
GrB_Vector w;
GrB_Vector_new(&w, GrB_BOOL, NUM_NODES);
GrB_Vector_setElement(w, true, SRC_NODE);

GrB_Descriptor desc;
GrB_Descriptor_new(&desc);
GrB_Descriptor_set(desc, GrB_INP0, GrB_TRAN);

pretty_print_vector_UINT64(w, "wavefront(src)");

for (int i = 0; i < NUM_NODES; ++i) {
    GrB_mxv(w, GrB_NULL, GrB_NULL,
            GxB_LOR_LAND_BOOL, graph, w, desc);
    pretty_print_vector_UINT64(w, "wavefront");
}
```

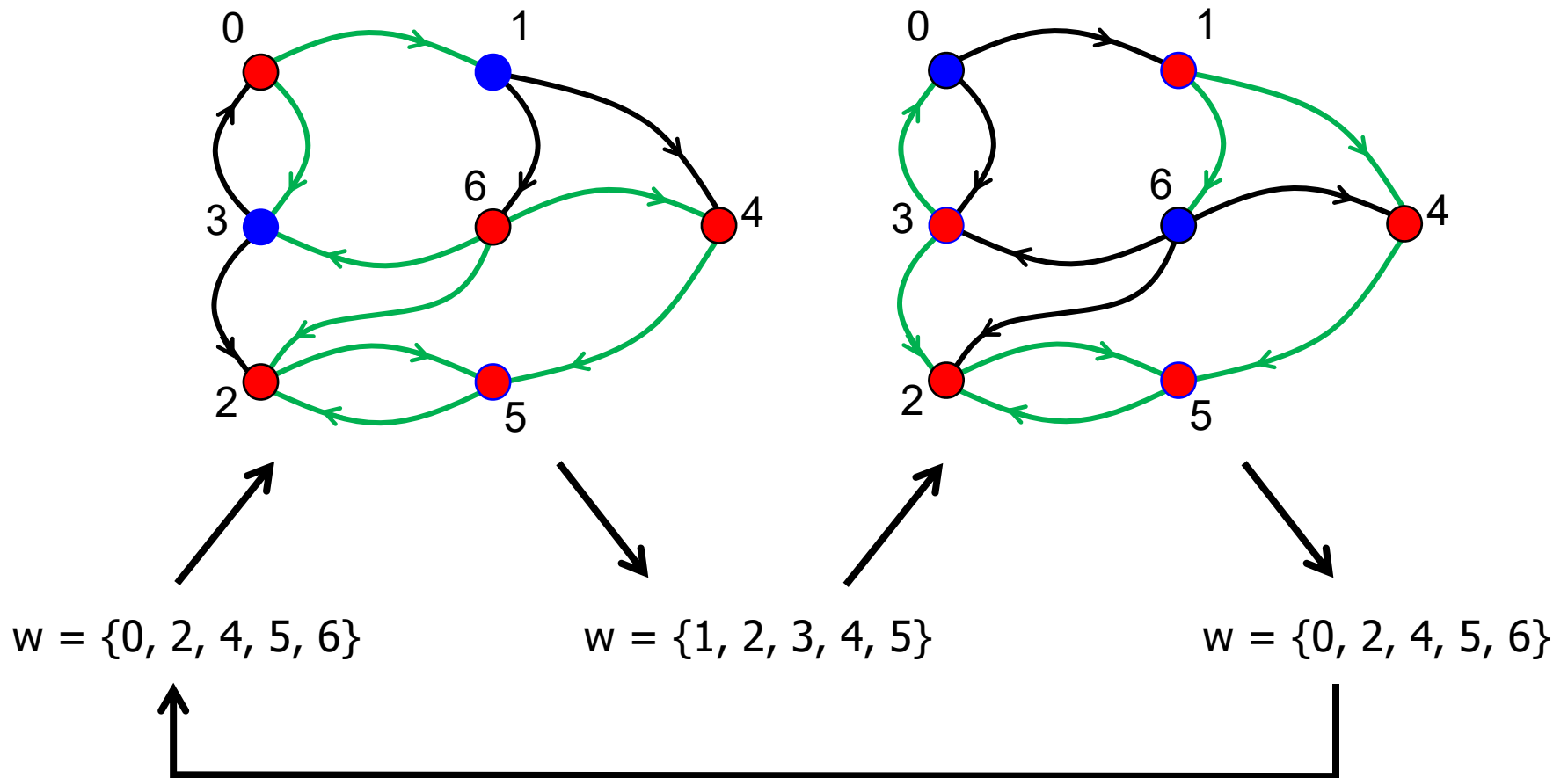


```
Vector: wavefront(src) =
[ 1, -, -, -, -, -, -]
Vector: wavefront =
[ -, 1, -, 1, -, -, -]
Vector: wavefront =
[ 1, -, 1, -, 1, -, 1]
Vector: wavefront =
[ -, 1, 1, 1, 1, 1, -]
Vector: wavefront =
[ 1, -, 1, -, 1, 1, 1]
Vector: wavefront =
[ -, 1, 1, 1, 1, 1, -]
Vector: wavefront =
[ 1, -, 1, -, 1, 1, 1]
...
```

The same container can be used for both input and output  
Starts repeating after only a few iterations. Why?

# Solution to exercise 8: wavefronts

- “We tell a bunch, and they tell bunch...(rinse and repeat)”



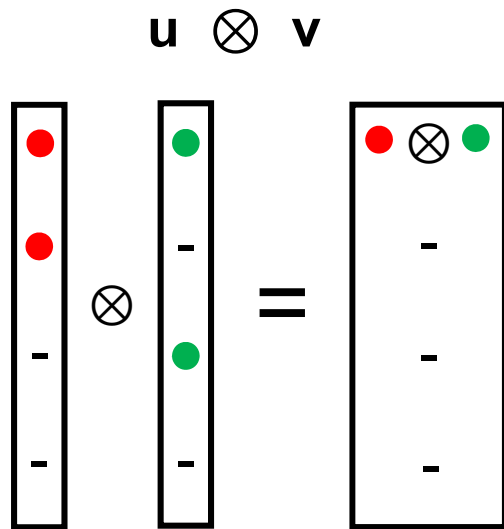
Red=current wavefront and visited, Blue=next wavefront, Black=unvisited

# Visited lists

- Breadth-first traversal requires that we only need to visit each node once.
- First step is to keep track of a visited list.
- You can do this by accumulating the wavefronts.
  - Use element-wise logical-OR.

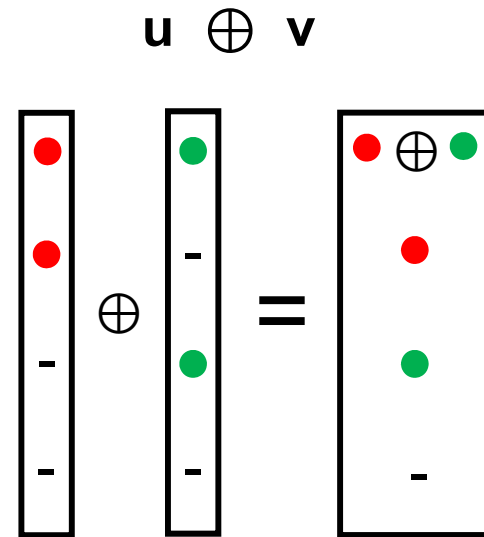
# Element-wise Operations: Mult and Add

- $\otimes$  assumes unstored values (-) are the binary operator's ***annihilator***:



Examples: (x,0), (and, false), (+,  $\infty$ )

- $\oplus$  assumes unstored values (-) are the binary operator's ***identity***:



Examples: (+,0), (or, false), (min,  $\infty$ )

**The rules for element-wise addition also apply to the accumulation operator,  $\odot$**

## GrB\_eWiseMult()

$$w \otimes = (u \otimes v)$$

- Compute the element-wise “**multiplication**” of two GraphBLAS vectors.
- Performs the specified operator (op) on the **intersection** of the sparse entries in each input vector , u and v.
  - op could be GrB\_BinaryOp, GrB\_Monoid, or GrB\_Semiring
- Returns error codes of type GrB\_info. See the spec for details.

```
GrB_Info GrB_eWiseMult( GrB_Vector      w,  
                        const GrB_Vector mask, → GrB_NULL  
                        const GrB_BinaryOp accum, → GrB_NULL  
                        const GrB_BinaryOp op,  
                        const GrB_Vector  u,  
                        const GrB_Vector  v,  
                        const GrB_Descriptor desc); → GrB_NULL
```

**Use default values for mask, accum and desc (indicated by GrB\_NULL)**

## GrB\_eWiseAdd()

$$w \otimes = (u \oplus v)$$

- Compute the element-wise “**addition**” of two GraphBLAS vectors.
- Performs the specified operator (op) on the **union** of the sparse entries in each input vector , u and v.
  - op could be GrB\_BinaryOp, GrB\_Monoid, or GrB\_Semiring
- Returns error codes of type GrB\_info. See the spec for details.

```
GrB_Info GrB_eWiseAdd( GrB_Vector          w,  
                      const GrB_Vector    mask, → GrB_NULL  
                      const GrB_BinaryOp  accum, → GrB_NULL  
                      const GrB_BinaryOp  op,  
                      const GrB_Vector    u,  
                      const GrB_Vector    v,  
                      const GrB_Descriptor desc); → GrB_NULL
```

**Use default values for mask, accum and desc (indicated by GrB\_NULL)**

## Exercise 9: Keep track of ‘visited’ nodes

- Modify code from Exercise 7 to compute the visited set as you iterate.

```
- GrB_Vector result, vec;  
- GrB_Index NODE;  
- GrB_Vector_new(&vec, GrB_BOOL, NUM_NODES);  
- GrB_Vector_setElement(vec, true, NODE);  
- pretty_print_vector_UINT64(vec, "Input node");  
- GrB_Descriptor desc;  
- GrB_Descriptor_new(&desc);  
- GrB_Descriptor_set(desc, ARG, OP)  
- GrB_eWiseAdd(vec, GrB_NULL, GrB_NULL,  
               GrB_LOR, vec, wav, GrB_NULL);  
- GrB_m xv(result, GrB_NULL, GrB_NULL,  
            GxB_LOR_LAND_BOOL, graph, vec, desc);
```



# Solution to exercise 9

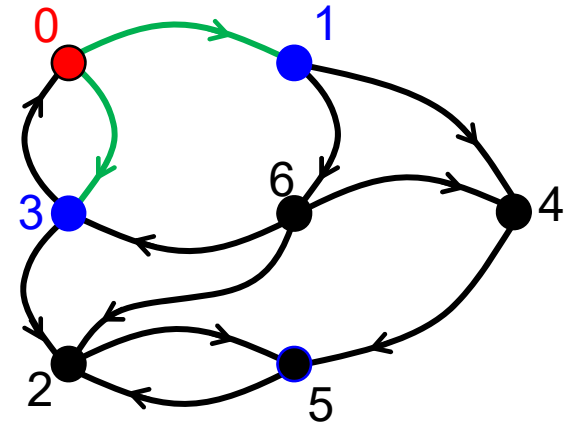
```
// First wavefront has node 0 set.
GrB_Index const SRC_NODE = 0;
GrB_Vector w, v;
GrB_Vector_new(&w, GrB_BOOL, NUM_NODES);
GrB_Vector_new(&v, GrB_BOOL, NUM_NODES);
GrB_Vector_setElement(w, true, SRC_NODE);

GrB_Descriptor desc;
GrB_Descriptor_new(&desc);
GrB_Descriptor_set(desc, GrB_INP0, GrB_TRAN);

pretty_print_vector_UINT64(w, "wavefront(src)");

for (int i=0; i<NUM_NODES; ++i) {
    GrB_eWiseAdd(v, GrB_NULL, GrB_NULL,
                GrB_LOR, v, w, GrB_NULL);

    pretty_print_vector_UINT64(v, "visited");
    GrB_mxv(w, GrB_NULL, GrB_NULL,
            GxB_LOR_LAND_BOOL, graph, w, desc);
    pretty_print_vector_UINT64(w, "wavefront");
}
```



```
Vector: wavefront(src) =
[ 1, -, -, -, -, -, -]
```

```
Vector: visited =
[ 1, -, -, -, -, -, -]
```

```
Vector: wavefront =
[ -, 1, -, 1, -, -, -]
```

```
Vector: visited =
[ 1, 1, -, 1, -, -, -]
```

```
Vector: wavefront =
[ 1, -, 1, -, 1, -, 1]
```

```
Vector: visited =
[ 1, 1, 1, 1, 1, -, 1]
```

```
Vector: wavefront =
[ -, 1, 1, 1, 1, 1, -]
```

```
Vector: visited =
[ 1, 1, 1, 1, 1, 1, 1]
```

```
Vector: wavefront =
[ 1, -, 1, -, 1, 1, 1]
```

```
Vector: visited =
[ 1, 1, 1, 1, 1, 1, 1]
```

```
...
```

# Solution to exercise 9

```
// First wavefront has node 0 set.
GrB_Index const SRC_NODE = 0;
GrB_Vector w, v;
GrB_Vector_new(&w, GrB_BOOL, NUM_NODES);
GrB_Vector_new(&v, GrB_BOOL, NUM_NODES);
GrB_Vector_setElement(w, true, SRC_NODE);

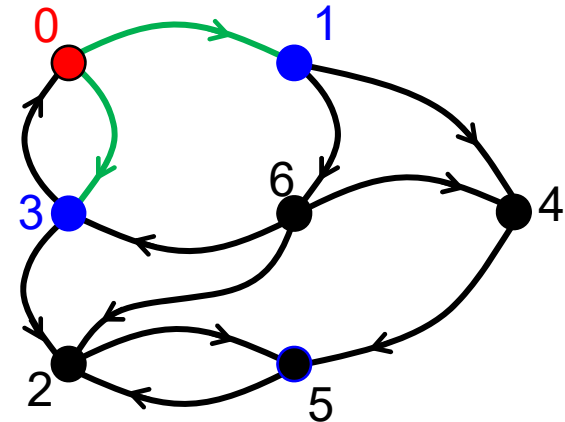
GrB_Descriptor desc;
GrB_Descriptor_new(&desc);
GrB_Descriptor_set(desc, GrB_ROW_MAJOR, GrB_BOOL);

pretty_print_vector_UINT64(w, "wavefront");

for (int i=0; i<NUM_NODES; ++i) {
    GrB_eWiseAdd(v, GrB_NULL, GrB_NULL,
                GrB_LOR, v, w, GrB_NULL);

    pretty_print_vector_UINT64(v, "visited");
    GrB_mxv(w, GrB_NULL, GrB_NULL,
            GxB_LOR_LAND_BOOL, graph, w, tran);
    pretty_print_vector_UINT64(w, "wavefront");
}
```

What should the exit condition be?



```
Vector: wavefront(src) =
[ 1, -, -, -, -, -, -]

Vector: visited =
[ 1, -, -, -, -, -, -]
Vector: wavefront =
[ -, 1, -, 1, -, -, -]

Vector: visited =
[ 1, 1, -, 1, -, -, -]
Vector: wavefront =
[ 1, -, 1, -, 1, -, 1]

Vector: visited =
[ 1, 1, 1, 1, 1, -, 1]
Vector: wavefront =
[ -, 1, 1, 1, 1, 1, -]

Vector: visited =
[ 1, 1, 1, 1, 1, 1, 1]
Vector: wavefront =
[ 1, -, 1, -, 1, 1, 1]

Vector: visited =
[ 1, 1, 1, 1, 1, 1, 1]
...
```

$$\text{GrB\_mxv}() \quad w \langle \neg m, z \rangle \otimes = (A \oplus \cdot \otimes u)$$

- ...say something
- Say something else....

```

GrB_Info GrB_mxv(GrB_Vector      w,
                  const GrB_Vector mask,
                  const GrB_BinaryOp accum → GrB_NULL,
                  const GrB_Semiring op,
                  const GrB_Matrix A,
                  const GrB_Vector u,
                  const GrB_Descriptor desc);

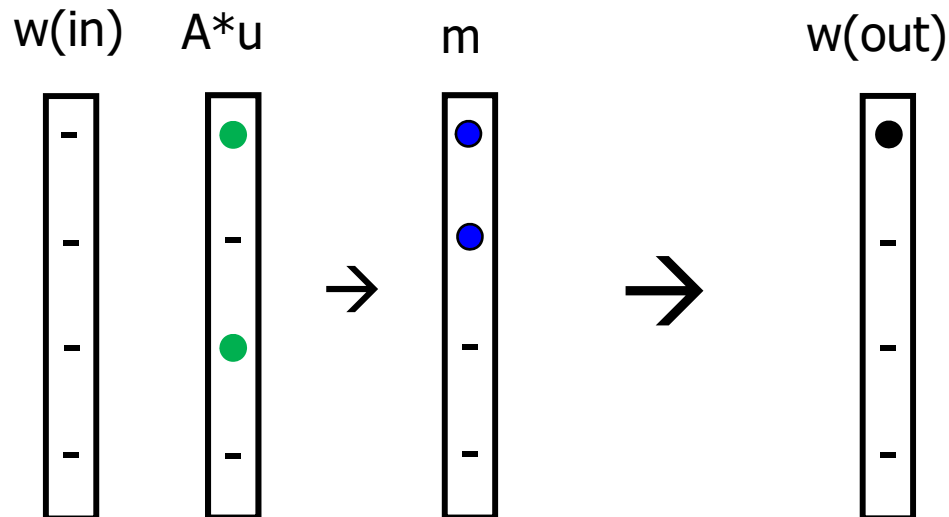
```

It's time to explain masking and REPLACE in GraphBLAS operations.

# Masking

- Every GraphBLAS operation that computes an opaque matrix or vector supports a “write mask”
- A mask,  $m$ , controls which elements of the output can be written:
  - Same size as output object (mask vectors or mask matrices)
  - Any location in the mask that evaluates to ‘true’ can be written in the output object

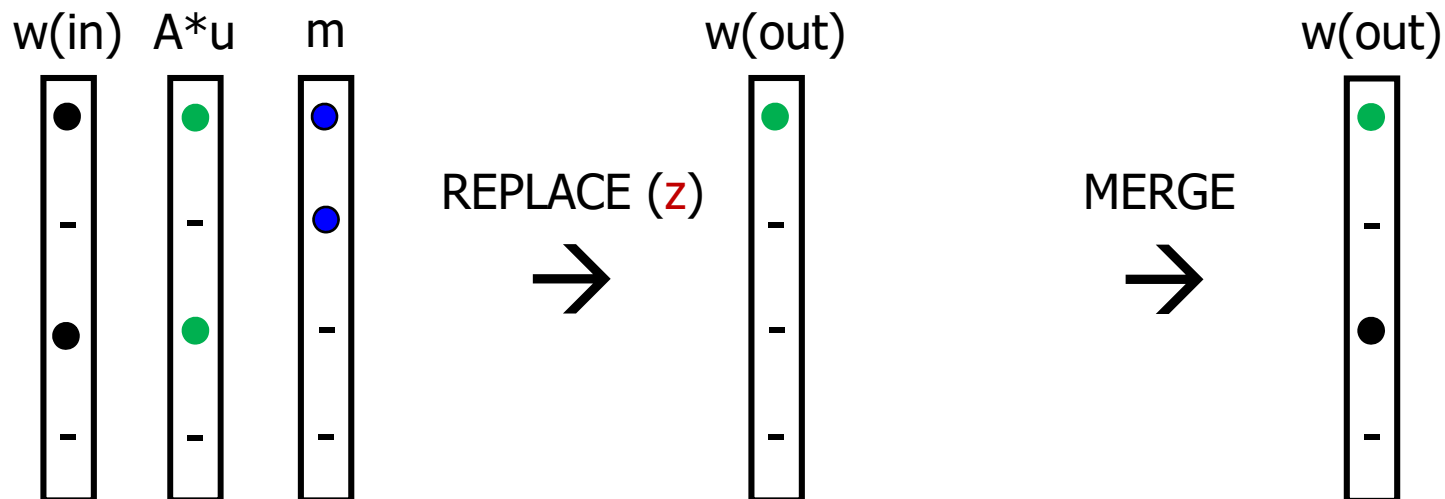
$$w\langle \mathbf{m} \rangle = (\mathbf{A} \oplus \cdot \otimes \mathbf{u})$$



# REPLACE vs. “MERGE”

- When a mask is used and the output container is not empty when the operation is called...what do you do to the “masked out” elements?
  - REPLACE (z): all unwritten locations are cleared (zeroed out).
  - MERGE: all unwritten locations are left alone.
- Behaviour defaults to MERGE; otherwise, use a descriptor:
  - GrB\_Descriptor\_set(desc, GrB\_OUTP, GrB\_REPLACE)

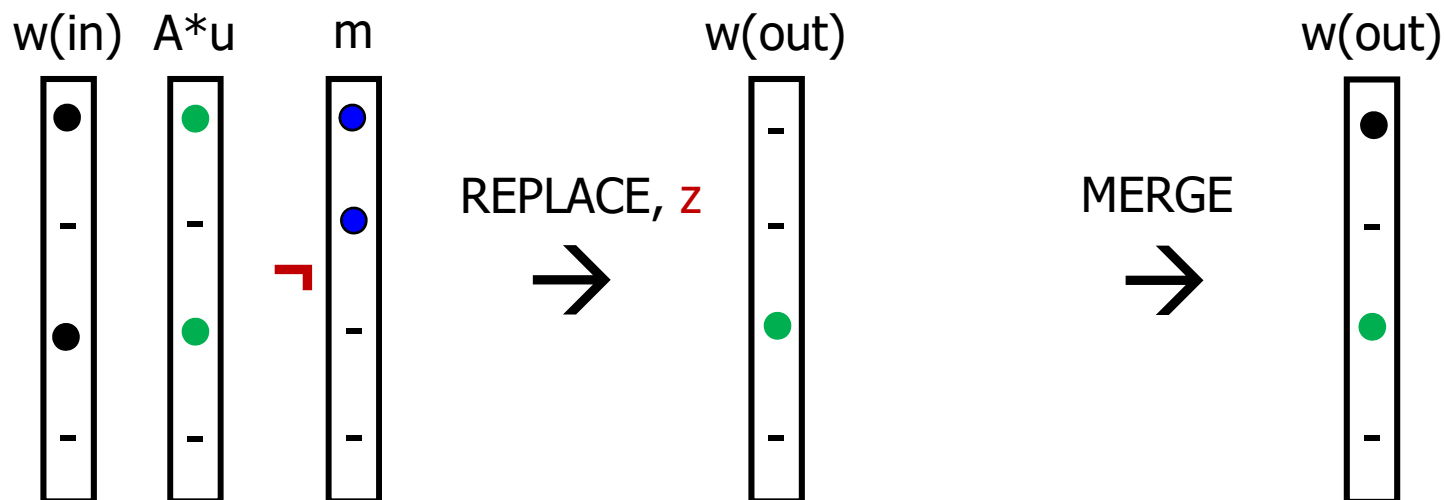
$$W\langle \mathbf{m}, \mathbf{z} \rangle = (\mathbf{A} \oplus \cdot \otimes \mathbf{u})$$



# Structural Complement (mask)

- Specified with a descriptor:
  - GrB\_Descriptor\_set(desc, GrB\_MASK, GrB\_SCMP)
- Inverts the logic of mask (write enabled on false)
- A mask, m, is interpreted as a logical ‘stencil’ that controls which elements of the output can be written:
  - Any location in the mask that evaluates to ‘true’ can be written

$$w\langle \neg m, z \rangle = (A \oplus \cdot \otimes u)$$



# Using Descriptors (summary)

- A descriptor *field* selects the object it impacts:
  - **GrB\_INP0**: The first input GraphBLAS object
  - **GrB\_INP1**: The second input GraphBLAS object
  - **GrB\_MASK**: The GraphBLAS mask object
  - **GrB\_OUTP**: The output GraphBLAS object
- Each field supports one *value* (currently):
  - **GrB\_INP0**: **GrB\_TRAN** (transpose)
  - **GrB\_INP1**: **GrB\_TRAN** (transpose)
  - **GrB\_MASK**: **GrB\_SCMP** (structural complement, **GrB\_COMP** in v1.3)
  - **GrB\_OUTP**: **GrB\_REPLACE** (clear the output before writing result)

## Exercise 10: Avoid revisiting

- Use the visited list as a mask prevent revisiting previous nodes
- Exit the loop when there is no more 'work' to be done
- You will need the following types and methods from the GraphBLAS
  - `GrB_Vector_new(&vec, GrB_BOOL, NUM_NODES);`
  - `GrB_Vector_setElement(vec, true, NODE);`
  - `GrB_eWiseAdd(vec, GrB_NULL, GrB_NULL, GrB_LOR, vec, wav, GrB_NULL);`
  - `GrB_mxv(result, GrB_NULL, GrB_NULL, GxB_LOR_LAND_BOOL, graph, vec, desc);`
  - `GrB_Descriptor desc;`
  - `GrB_Descriptor_new(&desc);`
  - `GrB_Descriptor_set(desc, FIELD, VALUE)`

**FIELD:** `GrB_INP0, GrB_INP1, GrB_OUTP, GrB_MASK`

**VALUE:** `GrB_TRAN, GrB_REPLACE, GrB_SCMP`



# Solution to exercise 10

...

```
GrB_Vector_setElement(w, true, SRC_NODE);
```

```
GrB_Descriptor desc;
```

```
GrB_Descriptor_new(&desc);
```

```
GrB_Descriptor_set(desc, GrB_INP0, GrB_TRAN);
```

```
GrB_Descriptor_set(desc, GrB_MASK, GrB_SCMP);
```

```
GrB_Descriptor_set(desc, GrB_OUTP, GrB_REPLACE);
```

```
pretty_print_vector_UINT64(w, "wavefront(src)");
```

```
GrB_Index nvals = 0;
```

```
do {
```

```
    GrB_eWiseAdd(v, GrB_NULL, GrB_NULL,
                GrB_LOR, v, w, GrB_NULL);
```

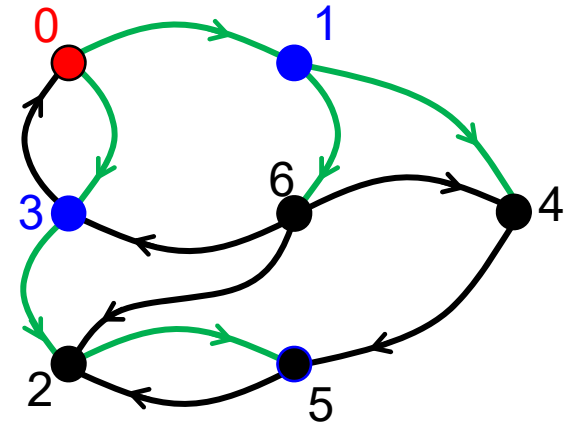
```
    pretty_print_vector_UINT64(v, "visited");
```

```
    GrB_mxv(w, v, GrB_NULL,
            GxB_LOR_LAND_BOOL, graph, w, desc);
```

```
    pretty_print_vector_UINT64(w, "wavefront");
```

```
    GrB_Vector_nvals(&nvals, w);
```

```
} while (nvals > 0);
```



```
Vector: wavefront(src) =
[ 1, -, -, -, -, -, -]
```

```
Vector: visited =
[ 1, -, -, -, -, -, -]
```

```
Vector: wavefront =
[ -, 1, -, 1, -, -, -]
```

```
Vector: visited =
[ 1, 1, -, 1, -, -, -]
```

```
Vector: wavefront =
[ -, -, 1, -, 1, -, 1]
```

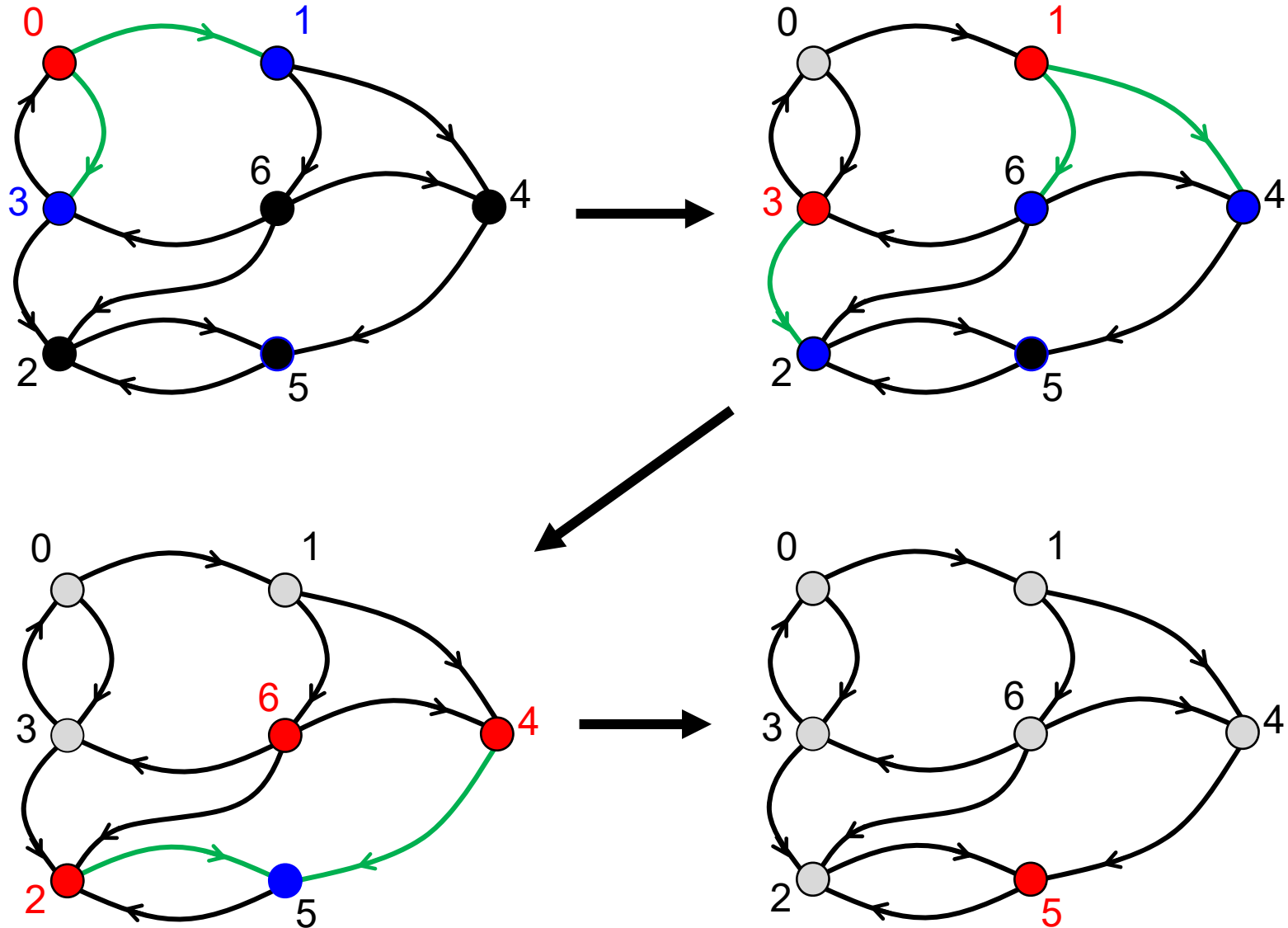
```
Vector: visited =
[ 1, 1, 1, 1, 1, -, 1]
```

```
Vector: wavefront =
[ -, -, -, -, -, 1, -]
```

```
Vector: visited =
[ 1, 1, 1, 1, 1, 1, 1]
```


```
Vector: wavefront =
[ -, -, -, -, -, -, -]
```

# Breadth-First Traversal



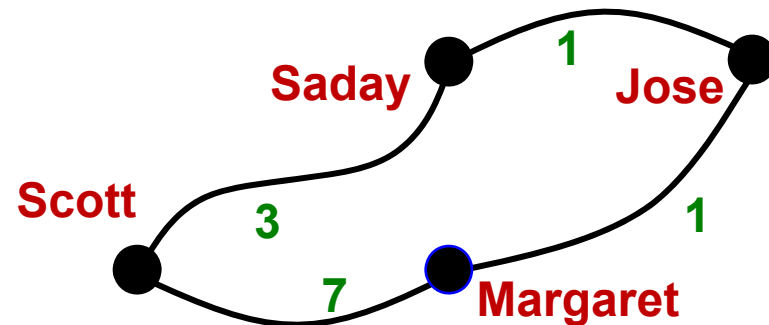
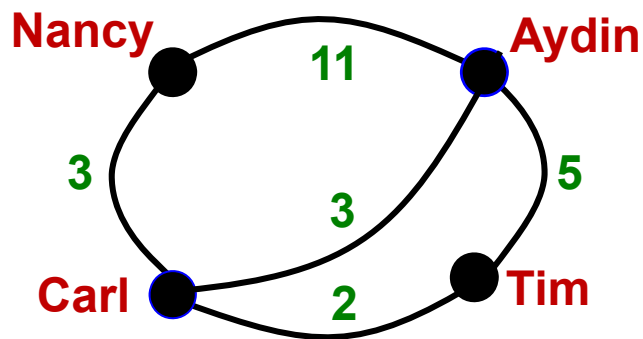
Red=current wavefront and visited, Blue=next wavefront, Gray=visited, Black=unvisited

# Outline

- Graphs and Linear Algebra
- The GraphBLAS C API and Adjacency Matrices
- GraphBLAS Operations
- Breadth-First Traversal
-  • Connected Components

# Connected Components

- Connected Components
  - Identify groups of vertices with paths to one another.
  - Identify how many of these groups (components) exist in the data.
  - Goal: **assign** all vertices within a component with the same unique ID.
- Graph
  - Consists of undirected edges
  - Note: applying this to directed graphs by converting to undirected is called “weakly connected components.”



## GrB\_assign()

- There are several variants of assign
  - Standard vector assignment
  - Standard matrix assignment

$$\mathbf{w}(\mathbf{i}) \odot = \mathbf{u}$$

$$\mathbf{C}(\mathbf{i}, \mathbf{j}) \odot = \mathbf{A}$$

- Assign a vector to the elements of column  $c_j$  of a matrix
- Assign a vector to the elements of row  $r_i$  of a matrix

$$\mathbf{C}(\mathbf{i}, c_j) \odot = \mathbf{u}$$

$$\mathbf{C}(r_i, \mathbf{j}) \odot = \mathbf{u}^T$$

- Assign a constant to a subset of a vector.
- Assign a constant to a subset of a matrix.

$$\mathbf{w}(\mathbf{i}) \odot = c$$

$$\mathbf{C}(\mathbf{i}, \mathbf{j}) \odot = c$$

## GrB\_assign() from vector

$$\mathbf{w}(i) \odot = \mathbf{u}$$

- Assign a vector to a subset of the output vector.
- Values to be assigned selected by an output index vector,  $i$

$$\begin{aligned} w(\text{indices}[j]) &= u(j), & \forall j : 0 \leq j < \text{nindices}, \\ w(\text{indices}[j]) &= w(\text{indices}[j]) \odot u(j), & \forall j : 0 \leq j < \text{nindices}. \end{aligned}$$

```
GrB_Info GrB_assign( GrB_Vector          w,  
                     const GrB_Vector    mask,  
                     const GrB_BinaryOp  accum,  
                     const GrB_Vector    u,  
                     const GrB_Index     *indices,  
                     const GrB_Index     nindices,  
                     const GrB_Descriptor desc);
```

- Use a constant **GrB\_ALL** in place of the **indices** argument to select that all elements of **u** are to be assigned in order to **w**.

## GrB\_assign() from constant $\mathbf{w}(i) \odot = c$

- Assign a constant to a subset of the output vector.
- Locations to be assigned selected by an output index vector, indices:

$$\begin{aligned} w(\text{indices}[j]) &= c, & \forall j : 0 \leq j < \text{nindices}, \\ w(\text{indices}[j]) &= w(\text{indices}[j]) \odot c, & \forall j : 0 \leq j < \text{nindices}. \end{aligned}$$

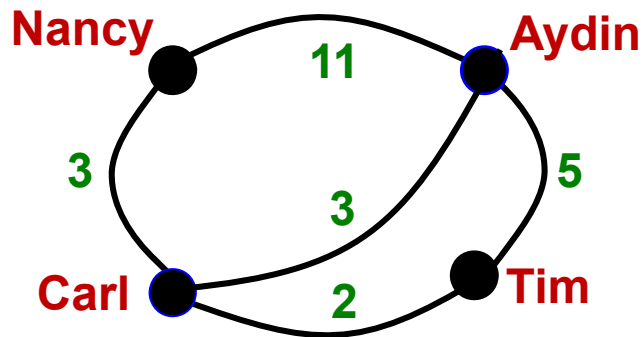
```
GrB_Info GrB_assign( GrB_Vector          w,  
                     const GrB_Vector    mask,  
                     const GrB_BinaryOp  accum,  
                     <type>              c,  
                     const GrB_Index      *indices,  
                     const GrB_Index      nindices,  
                     const GrB_Descriptor desc);
```

- Use a constant **GrB\_ALL** in place of the **indices** argument to select that all elements of **w** are to be assigned to (in order 0 to 1-nindices).

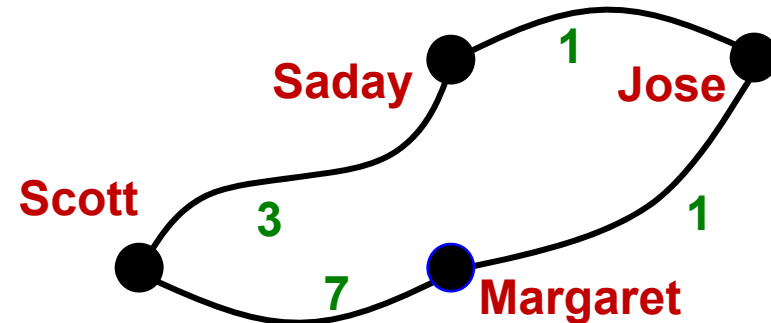
# Our Connected Components plan

- Strategy:
  - Initialize all vertex IDs to “unassigned”
  - While there are unassigned vertices:
    - Pick an unassigned vertex
    - Perform BFS marking all reachable vertices
    - Assign all reachable vertices with a unique **'component number'**.

Component '1'



Component '42'





# Our Connected Components plan

- We need an undirected graph with disconnected components to play with:

```
row_ind: {0, 0, 0, 1, 1, 2, 3, 3, 4, 4, 5, 5, 6, 7, 7, 8, 8, 8};  
col_ind: {1, 4, 8, 0, 8, 6, 5, 7, 0, 8, 3, 7, 2, 3, 5, 0, 1, 4};  
values:  {1, 2, 1, 1, 3, 4, 1, 2, 2, 5, 1, 2, 4, 2, 2, 1, 3, 5};
```

Matrix: GRAPH =

```
[ -, 1, -, -, 2, -, -, -, 1]  
[ 1, -, -, -, -, -, -, -, 3]  
[ -, -, -, -, -, -, 4, -, -]  
[ -, -, -, -, -, 1, -, 2, -]  
[ 2, -, -, -, -, -, -, -, 5]  
[ -, -, -, 1, -, -, -, 2, -]  
[ -, -, 4, -, -, -, -, -, -]  
[ -, -, -, 2, -, 2, -, -, -]  
[ 1, 3, -, -, 5, -, -, -, -]
```

# Our Connected Components plan

- We need an undirected graph with disconnected components to play with:

```
row_ind: {0, 0, 0, 1, 1, 2, 3, 3, 4, 4, 5, 5, 6, 7, 7, 8, 8, 8};
col_ind: {1, 4, 8, 0, 8, 6, 5, 7, 0, 8, 3, 7, 2, 3, 5, 0, 1, 4};
values:  {1, 2, 1, 1, 3, 4, 1, 2, 2, 5, 1, 2, 4, 2, 2, 1, 3, 5};
```

Matrix: GRAPH =

```
[ -, 1, -, -, 2, -, -, -, 1]
[ 1, -, -, -, -, -, -, -, 3]
[ -, -, -, -, -, -, 4, -, -]
[ -, -, -, -, -, 1, -, 2, -]
[ 2, -, -, -, -, -, -, -, 5]
[ -, -, -, 1, -, -, -, 2, -]
[ -, -, 4, -, -, -, -, -, -]
[ -, -, -, 2, -, 2, -, -, -]
[ 1, 3, -, -, 5, -, -, -, -]
```

How many  
components  
are there?

# Exercise 11: Connected Components

- Wrap the code from Exercise 9 in a function:

```
- GrB_Info BFS(GrB_Matrix const graph,
               GrB_Index      src_node,
               GrB_Vector      visited); // return GrB_SUCCESS
```

- Call this function to compute the membership of each connected component (CC):

- Create a vector of size NUM\_NODES to hold CC ID for each node.
- Each CC consists of all reachable (visited) nodes from a given root.
- Use the following undirected, weighted graph, **with multiple components**:

```
row_ind: {0, 0, 0, 1, 1, 2, 3, 3, 4, 4, 5, 5, 6, 7, 7, 8, 8, 8};
col_ind: {1, 4, 8, 0, 8, 6, 5, 7, 0, 8, 3, 7, 2, 3, 5, 0, 1, 4};
values:  {1, 2, 1, 1, 3, 4, 1, 2, 2, 5, 1, 2, 4, 2, 2, 1, 3, 5}; // pub. count
```

- Challenge: use GrB\_assign to assign component IDs

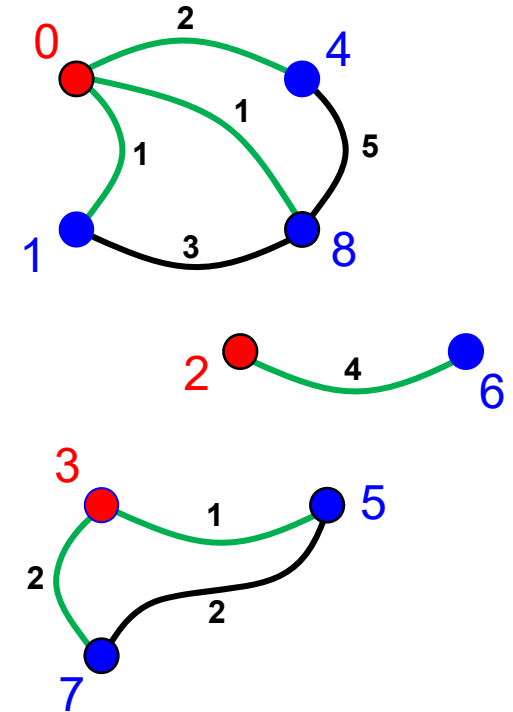
```
- GrB_Vector_new(&w, GrB_BOOL, NUM_NODES);
- GrB_Vector_setElement(w, true, SRC_NODE);
- GrB_Vector_extractElement(&s, w, index);
- pretty_print_vector_UINT64(vec, "CC IDS");
- GrB_assign(u, mask, accum, s, GrB_ALL, NUM_NODES, desc);
- GrB_mnv(w, mask, accum, GxB_LOR_LAND_BOOL, graph, w, desc);
- GrB_Vector_nvals(&nvals, w);
```

# Solution to exercise 11 (part1)

```
GrB_Info BFS(GrB_Matrix const graph,
             GrB_Index      src_node,
             GrB_Vector      v)
{
    GrB_Index num_nodes;
    GrB_Matrix_nrows(&num_nodes, graph);
    GrB_Vector w;
    GrB_Vector_new(&w, GrB_BOOL, NUM_NODES);
    GrB_Vector_setElement(w, true, src_node);
    GrB_Descriptor desc;
    ...
    GrB_Index nvals = 0;

    do {
        //GrB_eWiseAdd(v, GrB_NULL, GrB_NULL, GrB_LOR, v, w, GrB_NULL);
        GrB_assign(v, w, GrB_NULL, true, GrB_ALL, num_nodes, GrB_NULL);
        GrB_mxv(w, v, GrB_NULL, GxB_LOR_LAND_BOOL, graph, w, desc);
        GrB_Vector_nvals(&nvals, w);
    } while (nvals > 0);

    return GrB_SUCCESS;
}
```

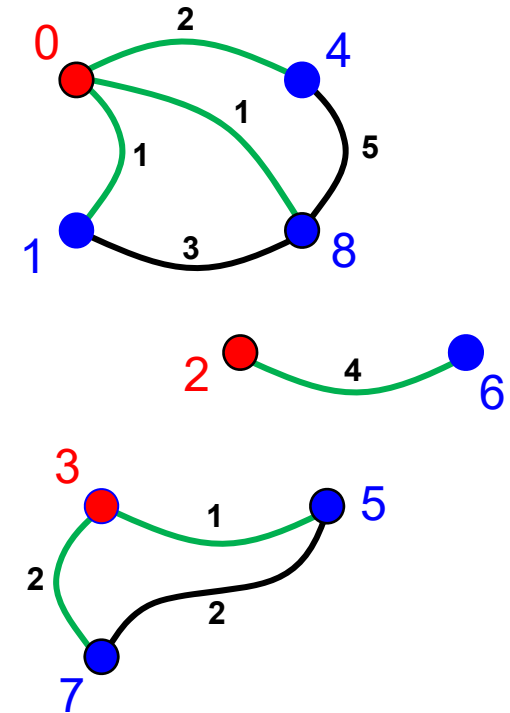


# Solution to exercise 11 (part2)

```
GrB_Index tmp = 0, num_ccs = 0;
GrB_Vector cc_ids, visited;
GrB_Vector_new(&cc_ids, GrB_UINT64, NUM_NODES);
GrB_Vector_new(&visited, GrB_BOOL, NUM_NODES);

for (GrB_Index src = 0; src < NUM_NODES; ++src) {
    // find next unassigned node
    if (GrB_NO_VALUE == GrB_VectorExtractElement(
        &tmp, cc_ids, src)) {
        BFS(graph, src, visited);
        // cc_ids[visited] = src
        GrB_assign(cc_ids, visited, GrB_NULL,
            src, GrB_ALL, NUM_NODES, GrB_NULL);
    }
    GrB_Vector_clear(visited);
}

printf("Number of connected components: %ld\n", num_ccs);
pretty_print_vector_UINT64(components, "CC IDs");
```



Output:

```
Number of connected components: 3
Vector: CC IDs =
[ 0,  0,  2,  3,  0,  3,  2,  3,  0]
```

# Putting it all together...

- Comparing with LAGraph's CC algorithm\* on HPEC dataset

```
$ Solutions/AnalyzeGraph_final.exe ../Data/hpec_coauthors.mtx
.
.
.
*** Step 3: Running Tutorial connected components algorithm.
Largest component #0 (size = 822)
*** Step 3: Elapsed time: 0.0189476 sec
Number of connected components: 246
ID for component containing target ID 800: 0
.
.
.
*** Step 3: Running LAGraph's connected components (LACC) algorithm.
*** Step 3: Elapsed time: 0.0066211 sec
Number of connected components: 246
ID for component containing target ID 800: 0
```

Platform: Dell Precision 7510, Intel Xeon CPU E3-1505M v5, 64GB RAM, VMWare 14.1.7, Ubuntu 16.0.4LTS

\*Azad, Buluç. "LACC: a linear-algebraic algorithm for finding connected components in distributed memory" (IPDPS 2019). 94

# The GraphBLAS Operations

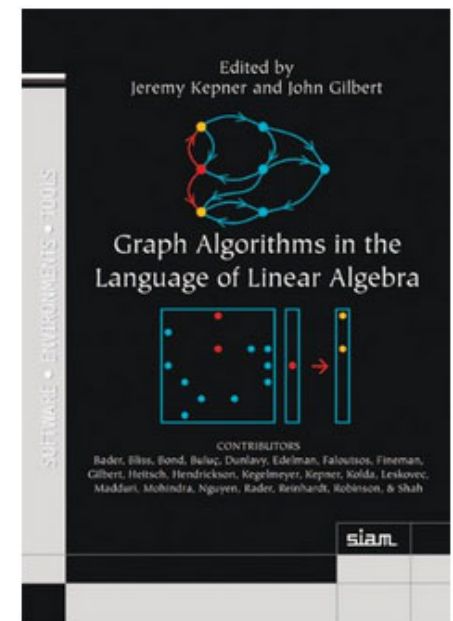
Operation Name	Mathematical Notation	
mxm	$\mathbf{C}\langle \mathbf{M}, z \rangle$	$= \mathbf{C} \odot \mathbf{A} \oplus . \otimes \mathbf{B}$
mxv	$\mathbf{w}\langle \mathbf{m}, z \rangle$	$= \mathbf{w} \odot \mathbf{A} \oplus . \otimes \mathbf{u}$
vxm	$\mathbf{w}^T \langle \mathbf{m}^T, z \rangle$	$= \mathbf{w}^T \odot \mathbf{u}^T \oplus . \otimes \mathbf{A}$
eWiseMult	$\mathbf{C}\langle \mathbf{M}, z \rangle$	$= \mathbf{C} \odot \mathbf{A} \otimes \mathbf{B}$
	$\mathbf{w}\langle \mathbf{m}, z \rangle$	$= \mathbf{w} \odot \mathbf{u} \otimes \mathbf{v}$
eWiseAdd	$\mathbf{C}\langle \mathbf{M}, z \rangle$	$= \mathbf{C} \odot \mathbf{A} \oplus \mathbf{B}$
	$\mathbf{w}\langle \mathbf{m}, z \rangle$	$= \mathbf{w} \odot \mathbf{u} \oplus \mathbf{v}$
reduce (row)	$\mathbf{w}\langle \mathbf{m}, z \rangle$	$= \mathbf{w} \odot [\oplus_j \mathbf{A}(:, j)]$
reduce (scalar)	$s$	$= s \odot [\oplus_{i,j} \mathbf{A}(i, j)]$
	$s$	$= s \odot [\oplus_i \mathbf{u}(i)]$
apply	$\mathbf{C}\langle \mathbf{M}, z \rangle$	$= \mathbf{C} \odot f_u(\mathbf{A})$
	$\mathbf{w}\langle \mathbf{m}, z \rangle$	$= \mathbf{w} \odot f_u(\mathbf{u})$
transpose	$\mathbf{C}\langle \mathbf{M}, z \rangle$	$= \mathbf{C} \odot \mathbf{A}^T$
extract	$\mathbf{C}\langle \mathbf{M}, z \rangle$	$= \mathbf{C} \odot \mathbf{A}(i, j)$
	$\mathbf{w}\langle \mathbf{m}, z \rangle$	$= \mathbf{w} \odot \mathbf{u}(i)$
assign	$\mathbf{C}\langle \mathbf{M}, z \rangle(i, j)$	$= \mathbf{C}(i, j) \odot \mathbf{A}$
	$\mathbf{w}\langle \mathbf{m}, z \rangle(i)$	$= \mathbf{w}(i) \odot \mathbf{u}$

We've covered only a small fraction of the GraphBLAS Operations

The same conventions are used across all operations so the operations we did not cover are straightforward to pick up

# Conclusion and next steps

- The GraphBLAS define a standard API for “Graph Algorithms in the Language of Linear Algebra”.
- A wide range of algorithms are variations of the basic breadth first traversal for a graph.
- To reach GraphBLAS mastery
  - Attend the Graph Architectures Programming and Learning (GrAPL) workshop at IPDPS
  - Attend GraphBLAS BoFs at HPEC and Supercomputing
  - Explore the challenge problems included with this tutorial
  - Work through the algorithms in the Graph book →





# GraphBLAS at HPEC 2019

- GraphBLAS is a community effort. Join the community:
  - Go to [graphblas.org](http://graphblas.org) and join our mailing list
- Attend the HPEC GraphBLAS Birds of a Feather (BOF) Wednesday from 6 PM to 7 PM, **Eden Vale C1/C2**.
- Please give us feedback about the tutorial by filling out the survey or sending email:
  - [timothy.g.mattson@intel.com](mailto:timothy.g.mattson@intel.com)
  - [smcmillan@sei.cmu.edu](mailto:smcmillan@sei.cmu.edu)
  - Tell us what you really liked.
  - Tell us what we should change
  - Tell us what you wish we'd covered but didn't
  - Plus anything else that might help us improve

# Appendices

- ➡ • MxM: the low-level details of the GraphBLAS operations
- Challenge Problems: Some key algorithms with the GraphBLAS
- SuiteSparse: usage notes, extensions and future plans
- Reference material

# GraphBLAS: details of operations

- When you read the GraphBLAS C API specification, the operations are described in a manner that may seem obtuse.
- The definitions, however, are presented in this way for good reasons:
  - to cover the full range of variations exposed by the various arguments and to express the operation without ever specifying the undefined elements (i.e. the “zeros” of the semiring).
  - To avoid any reference to the non-stored elements of the sparse matrix. In sparse arrays, the undefined elements are usually assumed to be the “zero of the semiring”. By defining the operations without any reference to those “un-stored values”, we can freely change the semirings between operations without having to update the un-stored elements.

**GrB\_mxm()**

$$\mathbf{C} = \mathbf{A} \oplus . \otimes \mathbf{B} = \mathbf{AB}$$

Matrix Multiplication ... the way we learned it in school

$$\mathbf{C}(i, j) = \bigoplus_{k=1}^l \mathbf{A}(i, k) \otimes \mathbf{B}(k, j)$$

$$\mathbf{A} : \mathbb{S}^{m \times l} \quad \mathbf{B} : \mathbb{S}^{l \times n} \quad \mathbf{C} : \mathbb{S}^{m \times n}$$

---

Matrix Multiplication ... set notation to ignore un-stored elements

$$\mathbf{C}(i, j) = \bigoplus_{k \in \mathbf{ind}(\mathbf{A}(i, :)) \cap \mathbf{ind}(\mathbf{B}(:, j))} (\mathbf{A}(i, k) \otimes \mathbf{B}(k, j))$$

With set notation, it's easier to define the operations over a matrix as the semi-ring changes

# GrB\_mxm(): Function Signature

```
GrB_Info GrB_mxm(GrB_Matrix          *C,
                  const GrB_Matrix    Mask,
                  const GrB_BinaryOp   accum,
                  const GrB_Semiring   op,
                  const GrB_Matrix     A,
                  const GrB_Matrix     B,
                  const GrB_Descriptor desc);
```

**C** (INOUT) An existing GraphBLAS matrix. On input, the matrix provides values that may be accumulated with the result of the matrix product. On output, the matrix holds the results of this operation.

**Mask** (IN) A “write” mask that controls which results from this operation are stored into the output matrix **C** (optional). If no mask is desired, **GrB\_NULL** is specified. The Mask dimensions must match those of the matrix **C** and the domain of the Mask matrix must be of type **bool** or any “built-in” GraphBLAS type.

**accum** (IN) A binary operator used for accumulating entries into existing **C** entries. For assignment rather than accumulation, **GrB\_NULL** is specified.

**op** (IN) Semiring used in the matrix-matrix multiply:  $op = \langle D_1, D_2, D_3, \oplus, \otimes, 0 \rangle$ .

**A** (IN) The GraphBLAS matrix holding the values for the left-hand matrix in the multiplication.

**B** (IN) The GraphBLAS matrix holding the values for the right-hand matrix in the multiplication.

**desc** (IN) Operation descriptor (optional). If a *default* descriptor is desired, **GrB\_NULL** should be used. Valid fields are as follows:

Argument	Field	Value	Description
<b>C</b>	<b>GrB_OUTP</b>	<b>GrB_REPLACE</b>	Output matrix <b>C</b> is cleared (all elements removed) before result is stored in it.
<b>Mask</b>	<b>GrB_MASK</b>	<b>GrB_SCOMP</b>	Use the structural complement of <b>Mask</b> .
<b>A</b>	<b>GrB_INP0</b>	<b>GrB_TRAN</b>	Use transpose of <b>A</b> for operation.
<b>B</b>	<b>GrB_INP1</b>	<b>GrB_TRAN</b>	Use transpose of <b>B</b> for operation.

# GrB\_mxm(): Function Signature

```
GrB_Info GrB_mxm(GrB_Matrix          *C,  
                  const GrB_Matrix    Mask,  
                  const GrB_BinaryOp   accum,  
                  const GrB_Semiring   op,  
                  const GrB_Matrix     A,  
                  const GrB_Matrix     B,  
                  const GrB_Descriptor desc);
```

GrB\_Info return values:

## **GrB\_SUCCESS**

Blocking mode: Operations completed successfully.  
Nonblocking mode: consistency tests passed on  
dimensions and domains for input arguments

GrB\_PANIC

Unknown Internal error

GrB\_OUTOFMEM

Not enough memory for the operation

GrB\_DIMENSION\_MISMATCH

Matrix dimensions are incompatible.

GrB\_DOMAIN\_MISMATCH

Domains of matrices are incompatible with the  
domains of the accumulator, semiring, or mask.

# Standard function behavior

- Consider the following code:

```
GrB_Descriptor_new(&desc);
GrB_Descriptor_set(desc, GrB_OUTP, GrB_REPLACE);
GrB_Descriptor_set(desc, GrB_INP0, GrB_TRANS);
GrB_mxm(&C, M, Int32Add, Int32AddMul, A, B, desc);
```

int32AddMul semiring  
int32Add accumulation

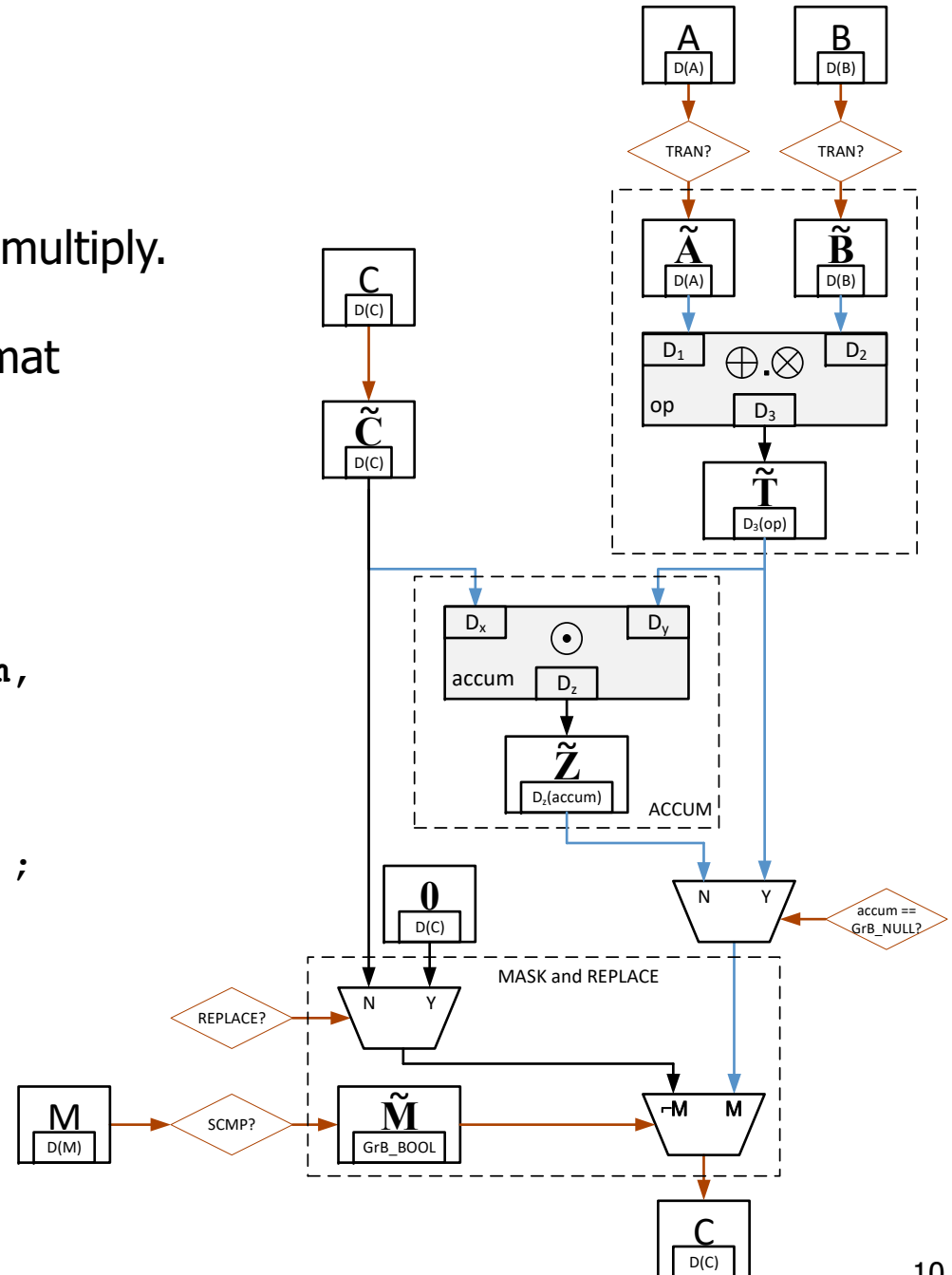
Form input operands and mask based on descriptor	$C, B, M, A \leftarrow A^T$
Test the domains and sizes for consistency.	int32, dims match
Carry out the indicated operation	$T \leftarrow A * . + B,$ $Z \leftarrow C + T$
Apply the write-mask to select output values	$Z \leftarrow Z \cap M$
Replace mode: delete elements in output object and replace with output values	$C \leftarrow Z$
Merge mode: Assign output value (i,j) to element (i,j) of output object, but leave other elements of the output object alone.	

# MXM flowchart

To understand what happens inside a graphBLAS operation, consider matrix multiply.

All the operations follow this basic format

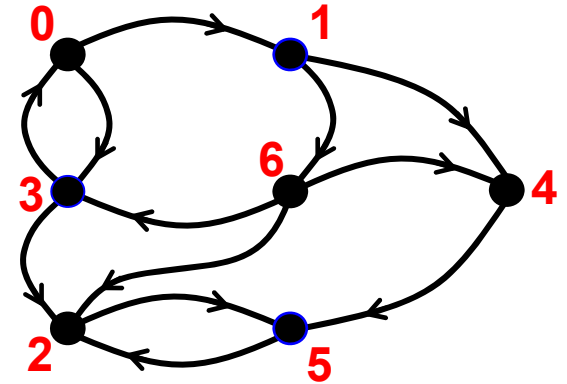
```
GrB_Info GrB_mxm(
    GrB_Matrix          C,
    const GrB_Matrix    M,
    const GrB_BinaryOp   accum,
    const GrB_Semiring   op,
    const GrB_Matrix     A,
    const GrB_Matrix     B,
    const GrB_Descriptor desc);
```





# Exercise: Matrix Matrix Multiplication

- Multiply the adjacency matrix from our “logo graph” by itself.
- Print resulting matrix and interpret the result
- Hint: Do the multiply again and compare results. Do you see the pattern?



# Appendices

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# Challenge problems

- Triangle counting
- PageRank
- Betweenness Centrality
- Maximal Independent Set

Work in Progress: We should make a slide for each problem defining the algorithm in enough detail so students can implement the GraphBLAS implementation on their own

# Counting Triangles (once) with GraphBLAS

- Given:
  - Undirected graph  $G = \{V, E\}$
  - $L$ : boolean, lower-triangular portion of adjacency matrix
- **# triangles** =  $\|L \otimes (L \oplus L^T)\|_1$ 
  - Semiring can be Plus-AND or Plus-Times
  - Element-wise multiplication is equivalent to a mask operation

```
uint64_t triangle_count(GrB_Matrix L)    // L: NxN, lower-triangular, boolean
{
    GrB_Index N;
    GrB_Matrix_nrows(&N, L);
    GrB_Matrix C;
    GrB_Matrix_new(&C, GrB_UINT64, N, N);

    GrB_mxm(C, L, GrB_NULL, GrB_UInt64AddMul, L, L, GrB_TB); // C<L> = L * L^T

    uint64_t count;
    GrB_reduce(&count, GrB_NULL, GrB_UInt64Add, C, GrB_NULL); // 1-norm of C
    return count;
}
```

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# SuiteSparse:GraphBLAS

- Full implementation of GraphBLAS Specification written by Tim Davis, Texas A&M University
- Easy-to-read User Guide with lots of examples
- Already in Ubuntu, Debian, Mac HomeBrew, ...
- Most operations just as fast as MATLAB (like  $C=A*B$ )
- `assign` and `setElement` can be 1000x faster (or more!) than MATLAB, by exploiting non-blocking mode
- V2.1: matrices by-row and by-column; by-row is often faster than by-column when  $A(i,j)$  is the edge  $(i,j)$ . Compile with `-DBYROW` or use `GxB_set(...)`
- Graph algorithms in GraphBLAS typically faster than novice-level graph algorithm without GraphBLAS, and easier to write
- <http://faculty.cse.tamu.edu/davis/GraphBLAS>

# SuiteSparse:GraphBLAS extensions

- MATLAB-like colon notation for GrB\_assign, extract
- unary operators ONE, ABS, LNOT\_[type]
- ISEQ, ISNE, ISLT, ... return same type as inputs (e.g. PLUS monoid cannot be combined with Boolean EQ, but PLUS-ISEQ can, to count the number of equal pairs)
- query: size of type, type of matrix, ...
- GxB\_select: like MATLAB  $L = \text{tril}(A, k)$ ,  $d = \text{diag}(A)$ , ...
- GxB\_get/set: to change matrix format (by row, by col, hypersparse)
- 44 built-in monoids
- 960 built-in semirings (like GxB\_LOR\_LAND\_BOOL)
- GxB\_resize: change size of matrix or vector
- GxB\_subassign: variation of GrB\_assign
- GxB\_kron: Kronecker product
- Thread-safe if called by user application threads, in parallel

# SuiteSparse:GraphBLAS future

- Multicore parallelism via OpenMP
- Variable-sized types (imagine matrix of matrices, or a matrix of arbitrary-sized integers with 10's or 1000's of digits)
- Solvers:  $Ax=b$  over a group (double, GF(2), ...)
- Better performance: e.g. many monoids could terminate quickly:
  - OR ( $x_1, x_2, x_3, \dots$ ) becomes true as soon as any  $x_i = \text{true}$
  - also for AND, and reduction ops FIRST and SECOND
- Iterators for algorithms like depth-first-search
- Reduction to vector or scalar: could also return the index for some operators (MAX, MIN, FIRST, SECOND): argmin, argmax
- Pretty-print methods
- Serialization to/from a binary string: for binary file I/O, or sending/receiving a GrB\_Matrix in an MPI message; with compression
- Priority queue: a GrB\_Vector acting like a heap
- Concatenate: like  $C=[A;B]$  in MATLAB
- Interface to MATLAB, Julia, Python, ...
- Faster  $C=A*B$  for user-defined types and operators



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# Full set of GraphBLAS opaque objects

Table 2.1: GraphBLAS opaque objects and their types.

GrB_Object types	Description
GrB_Type	User-defined scalar type.
GrB_UnaryOp	Unary operator, built-in or associated with a single-argument C function.
GrB_BinaryOp	Binary operator, built-in or associated with a two-argument C function.
GrB_Monoid	Monoid algebraic structure.
GrB_Semiring	A GraphBLAS semiring algebraic structure.
GrB_Matrix	Two-dimensional collection of elements; typically sparse.
GrB_Vector	One-dimensional collection of elements.
GrB_Descriptor	Descriptor object, used to modify behavior of methods.

# Error codes returned by GraphBLAS methods

## API Errors

Error code	Description
GrB_UNINITIALIZED_OBJECT	A GraphBLAS object is passed to a method before <code>new</code> was called on it.
GrB_NULL_POINTER	A <code>NULL</code> is passed for a pointer parameter.
GrB_INVALID_VALUE	Miscellaneous incorrect values.
GrB_INVALID_INDEX	Indices passed are larger than dimensions of the matrix or vector being accessed.
GrB_DOMAIN_MISMATCH	A mismatch between domains of collections and operations when user-defined domains are in use.
GrB_DIMENSION_MISMATCH	Operations on matrices and vectors with incompatible dimensions.
GrB_OUTPUT_NOT_EMPTY	An attempt was made to build a matrix or vector using an output object that already contains valid tuples (elements).
GrB_NO_VALUE	A location in a matrix or vector is being accessed that has no stored value at the specified location.

# Error codes returned by GraphBLAS methods

## Execution Errors

Error code	Description
GrB_OUT_OF_MEMORY	Not enough memory for operations.
GrB_INSUFFICIENT_SPACE	The array provided is not large enough to hold output.
GrB_INVALID_OBJECT	One of the opaque GraphBLAS objects (input or output) is in an invalid state caused by a previous execution error.
GrB_INDEX_OUT_OF_BOUNDS	Reference to a vector or matrix element that is outside the defined dimensions of the object.
GrB_PANIC	Unknown internal error.