Systems World 2D Question 3
F06 Team 06

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1 Part (a)

The χ^2 goodness of fit test will be used as a continuation of Question 2. To formulate this as an optimisation problem, the objective would be to:

1. Maximise the number of zero coefficients (k_i)
2. Minimise the χ^2 value of the m^{th} order controller.

In other words, as two conflicting objective functions will have to be optimised, a compromise has to be found based on a maximum predetermined value of χ^2. This value of χ^2 will correspond to a significance level α that is acceptable for the goodness of fit, where $\alpha = P(\text{Reject } H_0 | H_0 \text{ is true})$.

For the null hypothesis: $\{H_0$: The m^{th} order controller fits the measured data$\}$, α would be set to 0 in the ideal case. However, it is a mathematical fallacy to set $\alpha = 0$ as it is always possible to find an exact solution solution for 10 unknowns (k_i values) given a 9^{th} order controller (which gives a system of 10 equations). Hence, prior to analysing the data, the significance level α is set to 0.001. In order to minimise the number of non-zero coefficients (k_i), we introduce a binary switch x_i.

Modifying the objective function in question 2, we have:

$$\min \sum_{n=0}^{9} \left(\sum_{i=0}^{9} \frac{(k_i x_i e(n-i) - v(n))^2}{v(n)} \right) + \sum_{i=0}^{9} x_i$$

Constraints:

$$x_{1,2,3\ldots n} \ \epsilon \ \{0, 1\}$$

$$(\chi^2 \text{ test p-value}) \ < 0.001$$

Decision Variables:

$$x_{1,2,3\ldots n} \ , \ k_{1,2,3\ldots n}$$

Parameters:

$$e(n) \ , \ v(n); \ \forall n \in [1, \ 9]$$

2 Part (b)

From question 2, a chi-square distribution with 9 degrees of freedom for a lower one-sided test at significance level $\alpha = 0.001$ has a critical value of 1.152. Therefore, a constraint that has to be set is to have the χ^2 value to be less than or equal to the critical value 1.152. This constraint is equivalent of setting a constraint on the p-value < 0.001, and sets the ceiling or upper bound for the objective function.

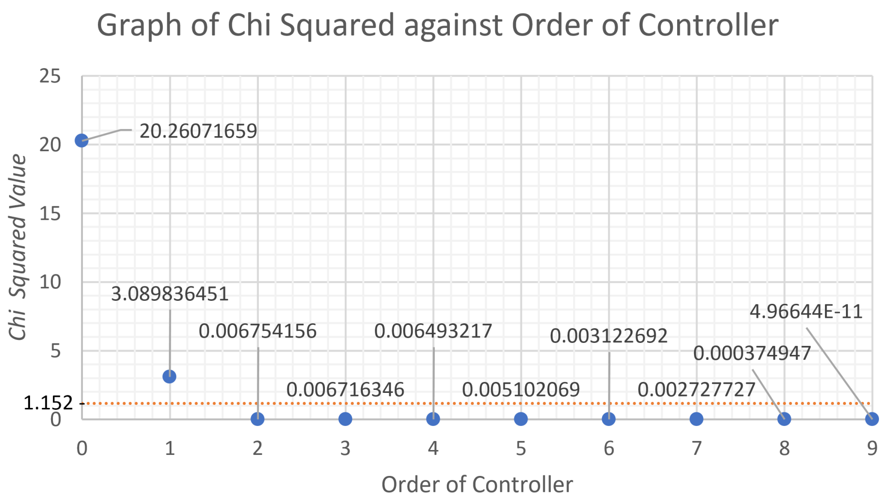

Figure 1: Graph of χ^2 values against Order of Controller

Figure 1 above shows the minimum attainable values of χ^2 for each order m. It can be observed that the order $m = 2$ satisfies the condition $\chi^2 < 1.152$ as shown by the dotted line.

From the objective function defined in part (a), an additional binary switch multiplier was included as a decision variable. This results in 10 χ^2 values from each order multiplied by the binary switch. Since we may only choose one value that corresponds to the smallest order controller, we set another constraint: $\sum_i x_i = 1$. Hence, only one chi-square value out of the ten data points, corresponding to the smallest order controller, will appear to be non-zero. Since Excel Solver requires the objective function to be a formula, we simply set it as the sum of chi-square values with the binary switch on. The objective function simply becomes $\sum_i \chi_i^2 x_i$

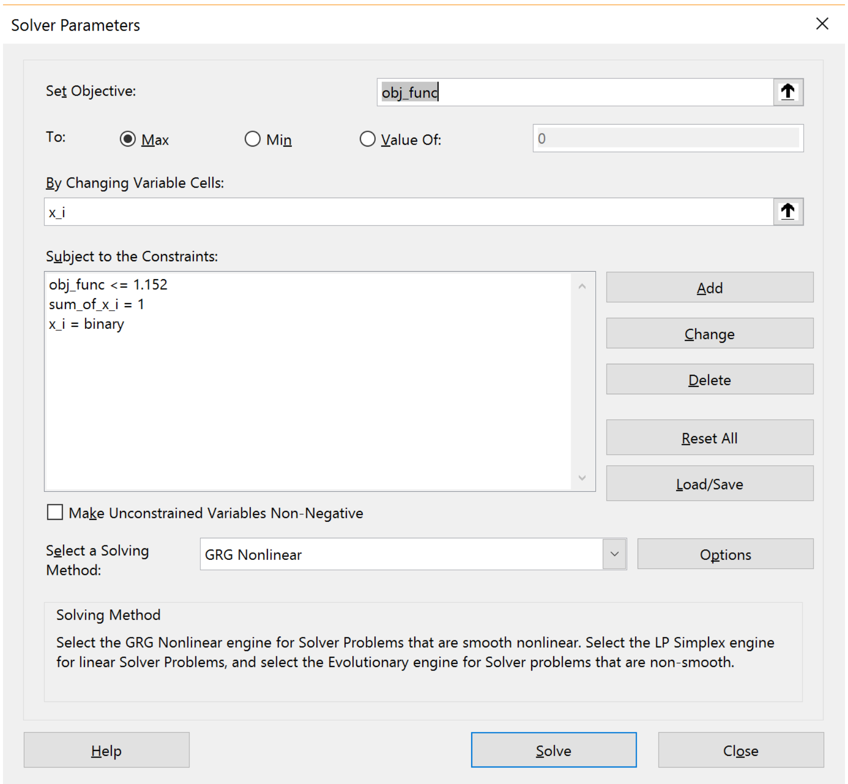

Figure 2: Constraints to find the objective function in Excel Solver

order	*chi square*	x_i	*chi square* * x_i
0	20.260717	0	0
1	3.0898365	0	0
2	0.0067542	1	0.006754156
3	0.0067163	0	0
4	0.0064932	0	0
5	0.0051021	0	0
6	0.0031227	0	0
7	0.0027277	0	0
8	0.0003749	0	0
9	4.966E-11	0	0

Figure 3: Screenshot of Calculations in Microsoft Excel

Since we have considered all possible orders $m \in [0,\ 9]$ in maximising the objective function (m cannot be larger than 9 as there is insufficient data), the only controller that satisfies the constraints is of order $m = 2 \Rightarrow H_0$ cannot be rejected.