Physical World 2D Report

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2D Project

Systems World

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Question 2.

Our group used Excel's matrix multiplication (=MMULT) of the entries of error and the k coefficients to get the linear combination of calculated v(n) for every order controller.

We decided to use minimum chi-square in assessing the goodness of fit between the set of calculated values of v(n) and actual values of v(n) given in the question.

Pearson's Chi-Square Test:

$X^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$

where

X^2 *= Pearson's cumulative test statistic*

O_i *= calculated v(n)*

E_i *= actual v(n) given*

n *= number of cells in the table*

A lower X^2 *value indicates that the calculated v(n) is closer to the actual v(n) given. Therefore, our objective is to minimize the chi-square value (*X^2*) for each order controller.*

Using Excel Solver, we set the chi-square value (X^2) as the objective function to a minimum and set the unknown k coefficients (k_0, k_1,...,k_9) as the changing variable.

Result:

[CHART]

Order	X^2
0	20.26071659
1	3.089836451
2	0.006754156
3	0.006716346
4	0.006493217
5	0.005102069
6	0.003122692
7	0.002727727
8	0.000374947
9	3.57577E-10

The next step is to analyze the result obtained. Since 10 data set is given, the degree of freedom (N-1) is 9. A chi-square distribution with 9 degrees of freedom for a lower one-sided test at significance level $\alpha = 0.001$ (extreme) has a critical value of 1.152.

From the table of values, we can see that the minimized chi-square values (X^2) of 2^{nd} order onwards do not exceed this critical value and hence satisfy the condition to be considered "correct". However, it is also apparent, from the table of values and the graph, that X^2 already approaches 0 at 2^{nd}order and rate of change of X^2 from 2^{nd} order onwards to the 9^{th} order is negligible.

Therefore, we conclude that the team from last year's 2D project used controller of 2^{nd} order.

This approach is still reasonable even if we have more data points. In fact, minimized chi-square test becomes more relevant with increasing data points. The plot of X^2 vs order would give data points forming the full similarly converging curve. With more data points, there is increased precision in assessing the data point which chi-square value (X^2) first converges to approximately 0. Therefore, the chi-square test is a reliable tool for assessing the goodness of fit between observed and expected values.

The k coefficients in the 2^{nd} order controller is:

k_0	0.107782668
k_1	0.546854887
k_2	0.769143476

k_0	0.107782668

We have observed that the coefficients of k_0 and k_1 are the same for 2^{nd} and 3^{rd} order controller, with a different coefficient of k_2 because the 3^{rd} order controller has an additional k_3 coefficient. The unchanged values of k_0 and k_1 suggest that the 2^{nd} order controller is indeed sufficient and these k coefficients are "best" or true.

Therefore, we conclude that the sufficient 2^{nd} order controller was used and the corresponding k coefficients are correct.

Bibliography

Appendix C Critical Values for the Chi-Squared Distribution. (2015). *Beyond Basic Statistics*,167-167.

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