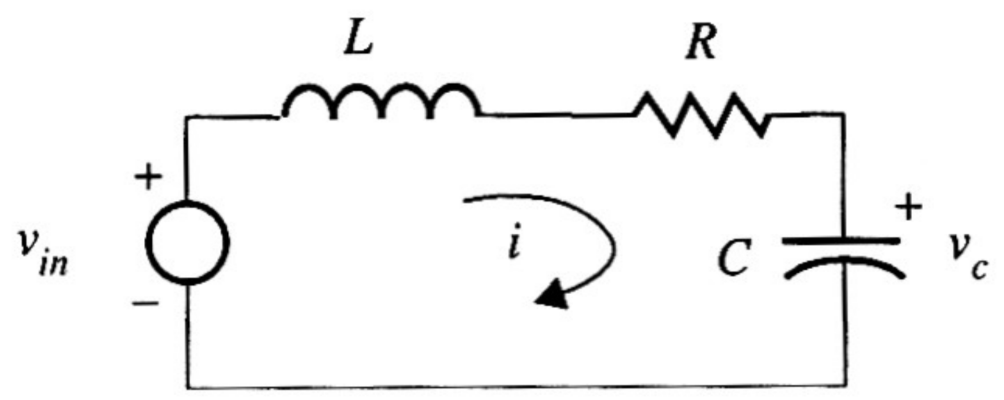
Example : RLC Series Circuit



v_{in} = *input voltage (volts)*

L = *inductance* (H)

R = *resistance* (Ω)

C = *capacitance* (F)

v_c = *voltage across the capacitor*

i = *current (amps)*

Using Kirchhoff's voltage law, we obtain

$$L\frac{di}{dt} + Ri + v_c = v_{in}, \tag{1}$$

where

$$v_c = \frac{1}{C}\int i dt. \tag{2}$$

We define the state variables as

$$x_1 = v_c,$$
$$x_2 = i.$$

Then taking the time derivative of x_1 *and using Eq.* (2) *yields*

$$\dot{x}_1 = \frac{1}{C} x_2. \tag{3}$$

Also, taking the time derivative of x_2 *and using Eq.* (1) *yields*

$$\dot{x}_2 = -\frac{R}{L} x_2 - \frac{1}{L} x_1 + \frac{1}{L} v_{in}. \tag{4}$$

We can write Eqs. (3) *and* (4) *in matrix form as*

$$\dot{x} = Ax + Bu$$

where $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} v_c \\ i \end{bmatrix}, \quad u = v_{in},$

and

$$A = \begin{bmatrix} 0 & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R}{L} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}.$$

With $R = 10\Omega, L = 0.2H,$ *and* $C = 0.0015F$, *we have*

$$\dot{x} = \begin{bmatrix} 0 & 666.6 \\ -5 & -50 \end{bmatrix} x + \begin{bmatrix} 0 \\ 5 \end{bmatrix} u.$$

If we measure v_c, *then we have*

$$y = Cx + Du,$$

where $C = [1 \quad 0], \qquad D = [0].$

We can also compute the transfer function as

$$\frac{\hat{v}_c(s)}{\hat{v}_{in}(s)} = \hat{G}(s) = C(sI - A)^{-1}B + D.$$

So this case (where we can measure v_c*), we have*

$$\hat{G}(s) = \frac{1}{LC}\frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}}.$$

On the other hand, if we can measure i *instead of* v_c*, we have*

$$y = Cx + Du,$$

where $C = [0 \;\; 1]$, *and* $D = [0]$.

In this case, the transfer function is $\hat{G}(s) = \dfrac{1}{L}\dfrac{s}{s^2 + \frac{R}{L}s + \frac{1}{LC}}.$

Lab #2 (1 Pt.):

1. Write a MATLAB script to simulate the step response of the RLC circuit.

2. Plot the time history of the voltage v_c.

3. Plot the time history of the current i.