Homework 4

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4.1)

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 2 & 3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

* Calculate $\Psi(s) = [sI - A]^{-1}$

+ Method 1 : Taking the inverse

$$(sI - A)^{-1} = \begin{bmatrix} s+1 & 0 \\ 2 & s-3 \end{bmatrix}^{-1} = \frac{1}{(s+1)(s-3)} \begin{bmatrix} s-3 & 0 \\ -2 & s+1 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{-2}{(s+1)(s+3)} & \frac{1}{s-3} \end{bmatrix}$$

+ Method 2: Using Theorem 5

$$(A - \lambda I) = \begin{bmatrix} -1-\lambda & 0 \\ 2 & 3-\lambda \end{bmatrix}$$

$$\Delta(\lambda) = \det(A - \lambda I) = (-1-\lambda)(3-\lambda) = 0 \Leftrightarrow \begin{cases} \lambda_1 = -1 \\ \lambda_2 = 3 \end{cases}$$

Define $h(\lambda) = \beta_0 + \beta_1 \lambda$, $f(\lambda) = (s-\lambda)^{-1}$

$$f(\lambda) = h(\lambda) \Leftrightarrow \begin{cases} f(-1) = h(-1) \\ f(3) = h(3) \end{cases}$$

$$\Leftrightarrow \begin{cases} (s+1)^{-1} = \beta_0 - \beta_1 \\ (s-3)^{-1} = \beta_0 + 3\beta_1 \end{cases}$$

$$\Leftrightarrow \begin{cases} (s+1)^{-1} - (s-3)^{-1} = -4\beta_1 \\ (s+1)^{-1} = \beta_0 - \beta_1 \end{cases}$$

$$\Leftrightarrow \begin{cases} \beta_0 = (2s-5)/2(s+1)(s-3) \\ \beta_1 = 1/2(s+1)(s-3) \end{cases}$$

$$h(A) = \beta_0 I + \beta_1 A$$

$$= \begin{bmatrix} (2s-5)/2(s+1)(s-3) & 0 \\ 0 & (2s-5)/2(s+1)(s-3) \end{bmatrix} + \begin{bmatrix} -1/2(s+1)(s-3) & 0 \\ -2/(s+1)(s-3) & 3/2(s+1)(s-3) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{-2}{(s+1)(s-3)} & \frac{1}{s-3} \end{bmatrix}$$

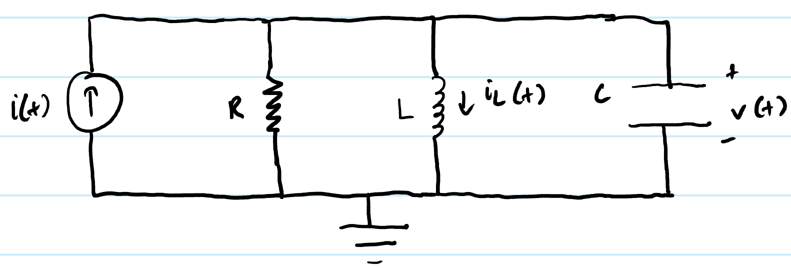
$$= \begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{-2}{(s+1)(s-3)} & \frac{1}{s-3} \end{bmatrix}$$

* Calculate $\varphi(t) = L^{-1}[sI - A]^{-1}$

Using the Laplace pairs table, we have:

$$\varphi(t) = L^{-1}[sI - A]^{-1} = L^{-1}\left\{ \begin{bmatrix} \frac{1}{s+1} & 0 \\ \frac{-2}{(s+1)(s-3)} & \frac{1}{s-3} \end{bmatrix} \right\} = \begin{bmatrix} e^{-t} & 0 \\ \frac{e^{-t} - e^{3t}}{2} & e^{3t} \end{bmatrix}$$

4.2)



+ $i_R = \frac{v(t)}{R}$

* Kirchoff's current law:

$$i(t) = i_R + i_L(t) + i_C$$

* State equation:

+ $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix}$

+ $\dot{x}_1 = \frac{i_C}{C} = \frac{1}{C}\left(i(t) - \frac{v(t)}{R} - i_L(t) \right) = -\frac{1}{RC} x_1 - \frac{1}{C} x_2 + \frac{1}{C} i(t)$

+ $\dot{x}_2 = \frac{v(t)}{L} = \frac{1}{L} x_1$

+ $\dot{x} = Ax + Bi(t) = \begin{bmatrix} -\frac{1}{RC} & -\frac{1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} i(t)$

* Output equation for $v(t)$:

$$y = v(t) = Cx + Di(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} i(t)$$

* Output equation for $i_L(t)$:

$$y = i_L(t) = Cx + Di(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} v(t) \\ i_L(t) \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} i(t)$$

4.3)

$$\dot{x} = Ax \quad \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} x$$

$$\dot{x} = Ax \quad \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} x$$

- $\det(\lambda I - A) = \det \begin{bmatrix} -1-\lambda & 0 & 1 \\ 0 & -\lambda & 1 \\ 0 & 0 & -\lambda \end{bmatrix} = -\lambda^2(\lambda+1) = 0 \Leftrightarrow \left[\begin{array}{l} \lambda_1 = 0 \text{ with muliplicity } 2 \\ \lambda_1 = -1 \end{array} \right.$

- Find the minimal polynomial $\Psi(\lambda)$:

+ $\Psi(\lambda) = -\lambda(\lambda+1) \Rightarrow \Psi(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow \Psi(\lambda) = \Delta(\lambda) = -\lambda^2(\lambda+1)$

- $\lambda_1 = 0$ is a repeated zero eigenvalue $\Rightarrow$ The system is unstable

4.4) $y(s) = s^3 + 11s^2 + 38.36s + 41.8$

By the Routh-Hurwitz Condition, we have the following table

s_3	1	38.56
s_2	11	41.8
s_1	$(11 \times 38.56 - 41.8 \times 1)/11 = 34.56$	
s_0	$(34.56 \times 41.8 - 11 \times 0)/34.56 = 41.8$	

No sign change in the first column $\Rightarrow$ No root in the right half plane