Lecture I213E – Class 8

Discrete Signal Processing

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JAPAN ADVANCED INSTITUTE OF SCIENCE AND TECHNOLOGY 1990

Course Materials

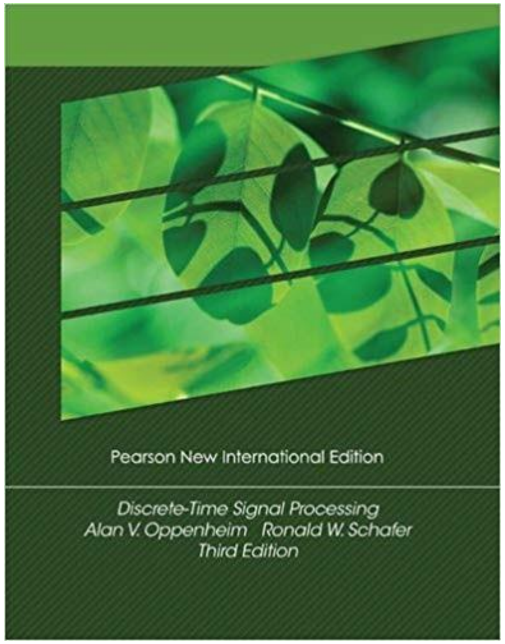
- **Materials**

→ Lecture notes will be uploaded before each lecture
https://jstorage-2018.jaist.ac.jp/s/PGXRrC7iFmN2FWo
Pass: dsp-i213e-2022
(Slide Courtesy of Prof. Nak Young Chong)

- **References**

→ Chi-Tsong Chen:
Linear System Theory and Design, 4th Ed., Oxford University Press, 2013.

→ Alan V. Oppenheim and Ronald W. Schafer:
Discrete-Time Signal Processing, 3rd Ed., Pearson New International Ed., 2013.



Related Courses & Prerequisite

- **Related Courses**
  - → I212 Analysis for Information Science
  - → I114 Fundamental Mathematics for Information Science

- **Prerequisite**
  - → None

Evaluation

Viewpoint of evaluation

→ Students are able to understand:

- Basic principles in modeling and analysis of linear time-invariant systems
- Applications of mathematical methods and tools to different signal processing problems.

Evaluation method

→ Homework, term project, midterm exam, and final exam

Evaluation criteria

→ Homework/labs (30%), term project (30%)
midterm exam (15%), and final exam (25%)

Contact

- **Lecturer**

  → Sakriani Sakti

- **TA**

  Tutorial hours & Term project

  → WANG Lijun (s2010026)

  → TANG Bowen (s2110411)

  Homework

  → PUTRI Fanda Yuliana (s2110425)

- **Contact Email**

  → dsp-i213e-2022@ml.jaist.ac.jp

Schedule

- **December 8th, 2022 – February 9th, 2023**

- **Lecture Course Term 2-2**
  - → Tuesday 9:00 — 10:40
  - → Thursday 10:50 — 12:30

- **Tutorial Hours**
  - → Tuesday 13:30-15:10

Schedule

Dec

Sun	Mon	Tue	Wed	Thu	Fri	Sat
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	31

Jan

Sun	Mon	Tue	Wed	Thu	Fri	Sat
1	2	3	4	~~5~~	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	~~24~~	25	26	27	28
29	30	31				

Feb

Sun	Mon	Tue	Wed	Thu	Fri	Sat
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28				

Lecture:
Tuesday 9:00 — 10:40
Thursday 10:50 — 12:30

Tutorial:
Tuesday 13:30 — 15:10

Midterm & final exam
Thursday 10:50 — 12:30

Course review &
term project evaluation
(on tutorial hours)

Syllabus

Class	Date	Lecture Course Tue 9:00 — 10:40 / Thr 10:50 — 12:30	Tutorial Hours Tue 13:30 — 15:10
1	12/08	Introduction to Linear Systems with Applications to Signal Processing	
2	12/13	State Space Description	○
3	12/15	Linear Algebra	
4	12/20	Quantitative Analysis (State Space Solutions) and Qualitative Analysis (Stability)	○
5	12/22	Discrete-time Signals and Systems	
X	01/05		
6	01/10	Discrete-time Fourier Analysis	
7	01/10*	Review of Discrete-time Linear Time-Invariant Signals and Systems (on Tutorial Hours)	
	01/12	Midterm Exam	
8	01/17	Sampling and Reconstruction of Analog Signals	○
9	01/19	z-Transform	
X	01/24		○
10	01/26	Discrete Fourier Transform	
11	01/31	FFT Algorithms	○
12	01/02	Implementation of Digital Filters	
13	02/07	Digital Signal Processors and Design of Digital Filters	
14	02/07*	Review of the Course and Term Project Evaluation (on Tutorial Hours)	
	02/09	Final exam	

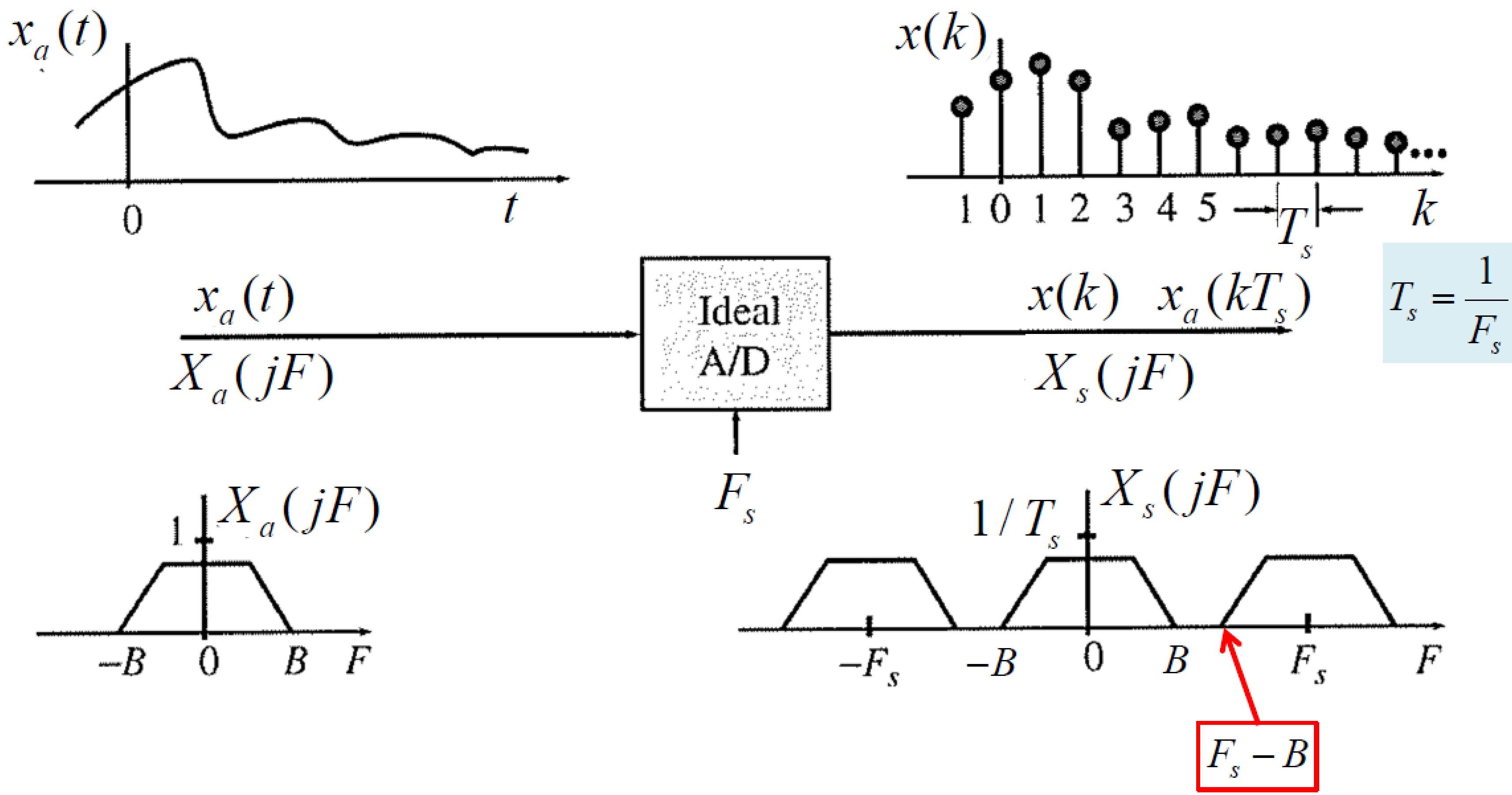
Class 8
Sampling and Reconstruction of Analog Signals

Sampling & Reconstruction

1. Given a continuous input signal $x_a(t)$ and its sampled counterpart $x(k)$, are the samples of $x(k)$ sufficient to exactly describe $x_a(t)$?

2. If so, how can $x_a(t)$ be reconstructed from $x(k)$?

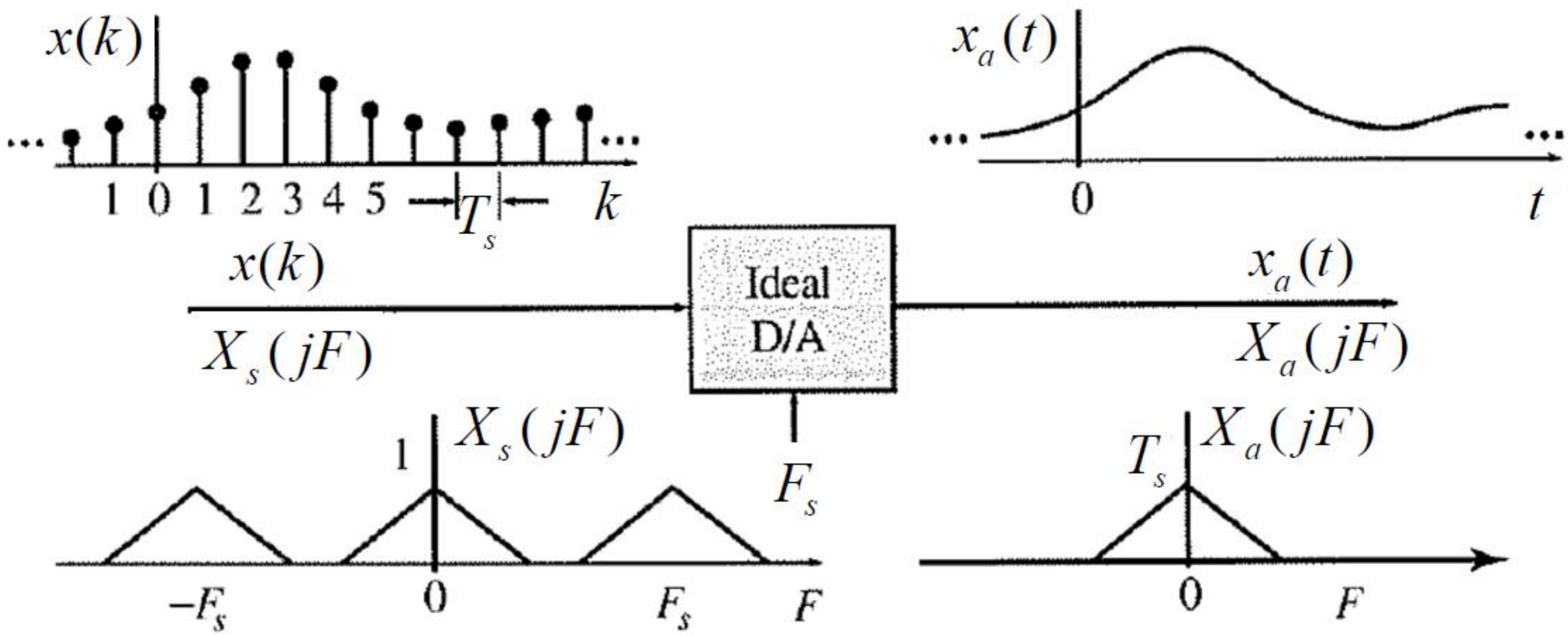
An Ideal A/D Converter



$$\boxed{F_s - B > B \quad \Rightarrow \quad F_s > 2B}$$

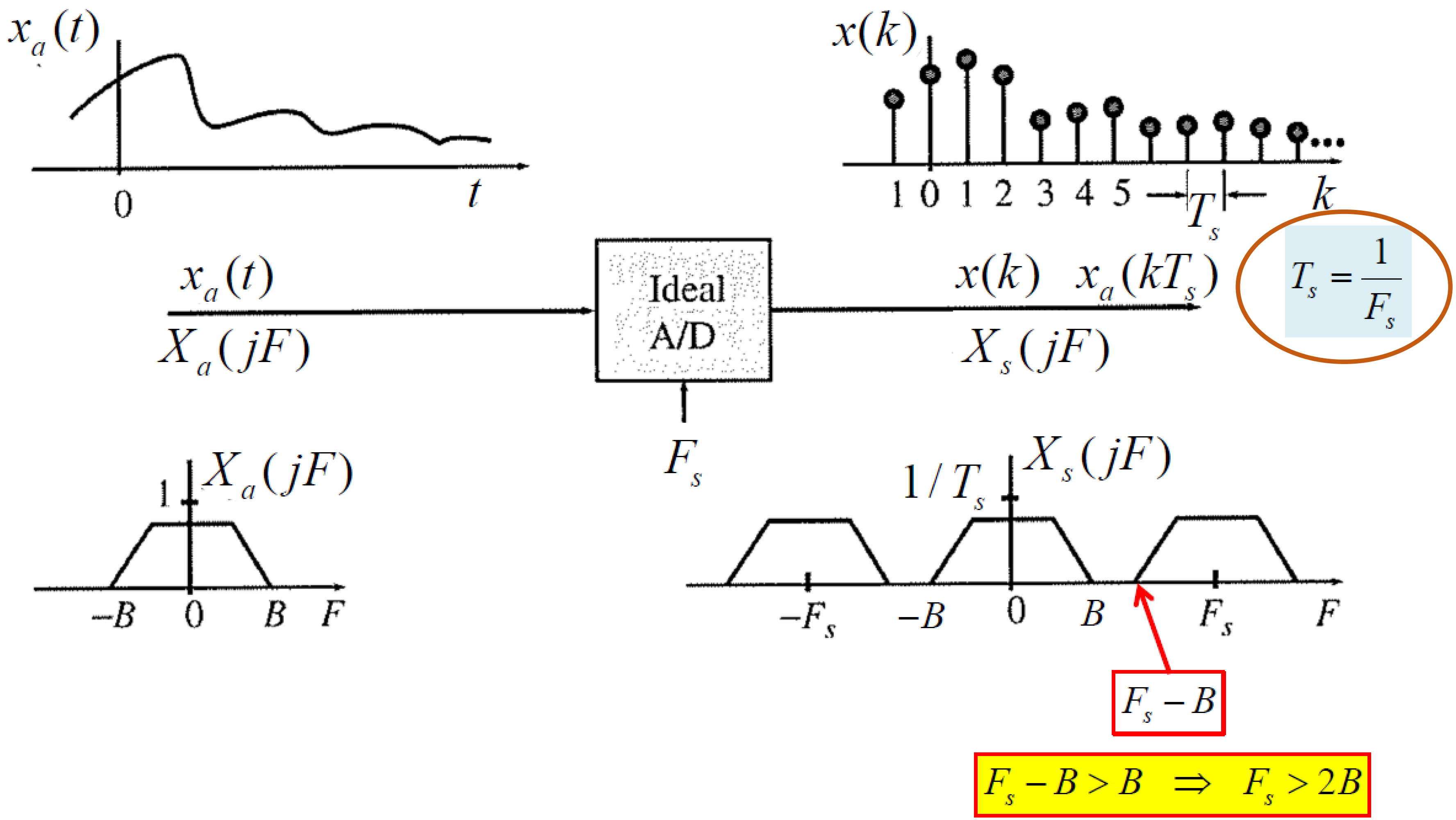
An Ideal D/A Converter

An ideal D/A converter in the time and frequency domains

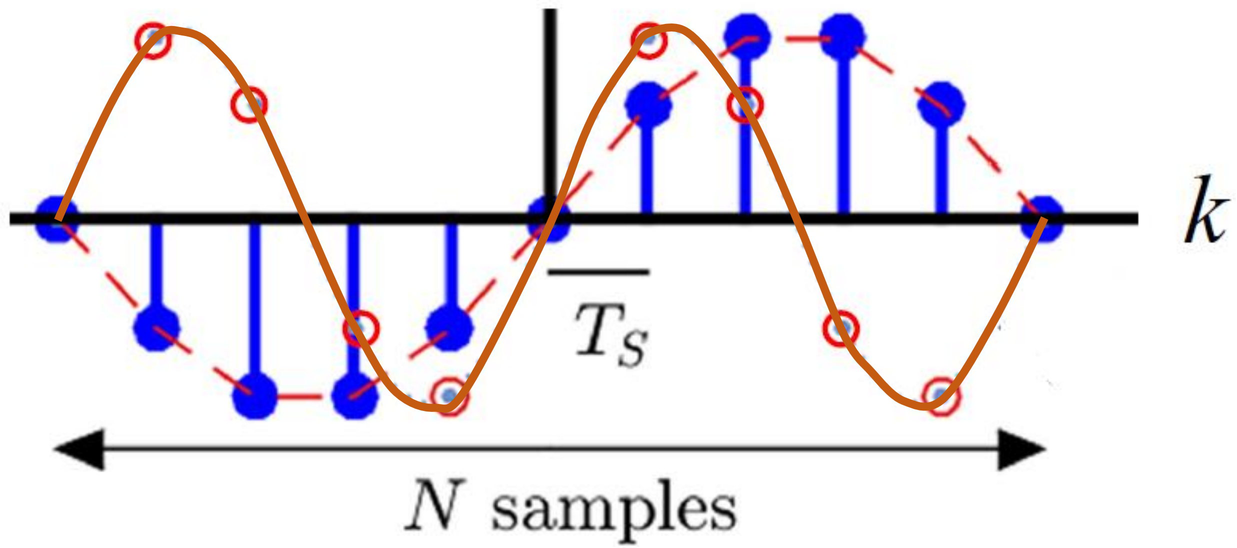


Discrete Frequency

An Ideal A/D Converter



Discrete Frequency



one full cycle

frequency resolution (the lowest frequency) $\dfrac{1}{NT_s} = \dfrac{F_s}{N} = F_s \boxed{\dfrac{1}{N}}$

discrete frequency

integer multiples of $\dfrac{1}{N}$

$0, \dfrac{1}{N}, \dfrac{2}{N}, \ldots$

Discrete Frequency

There are only N frequencies from 0 to N-1.

$$\boxed{0, \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}}, \frac{N}{N}, \frac{N+1}{N}, \ldots$$

$$\rightarrow \quad \sin 2\pi \frac{N}{N} k = \sin 2\pi k = 0$$

$$\rightarrow \quad \sin 2\pi \frac{N+1}{N} k = \sin 2\pi \left(1 + \frac{1}{N}\right) k$$

$$= \sin 2\pi k \cos 2\pi \frac{1}{N} k + \cos 2\pi k \sin 2\pi \frac{1}{N} k$$

$$= \sin 2\pi \frac{1}{N} k$$

For N samples in discrete time domain, there are N samples in the discrete frequency domain.

Discrete Frequency

Orthogonality

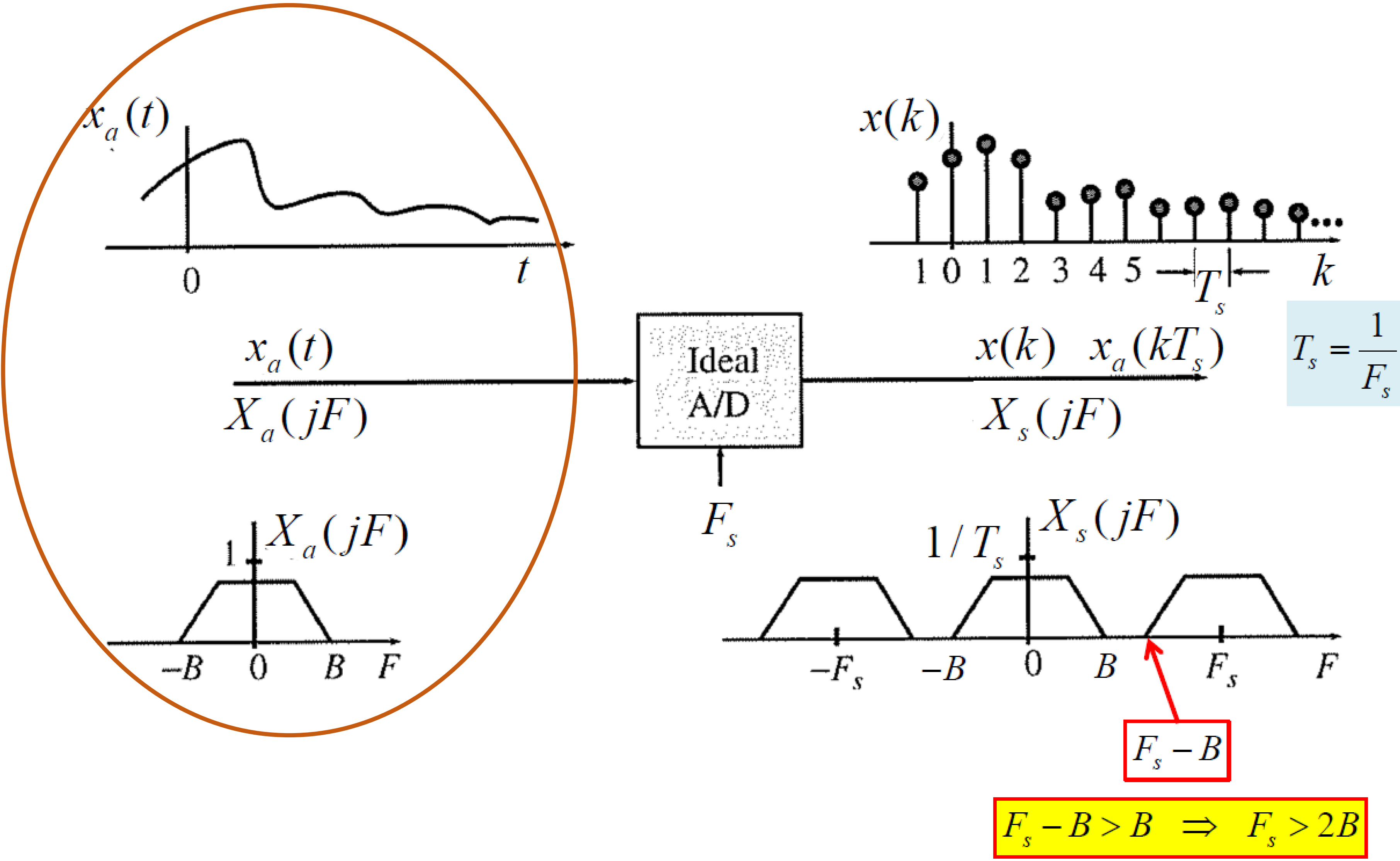
$$\boxed{0, \frac{1}{N}, \frac{2}{N}, \ldots, \frac{N-1}{N}}, \frac{N}{N}, \frac{N+1}{N}, \ldots$$

$$\rightarrow \quad \sum_{k=0}^{N-1} \sin 2\pi \frac{1}{N} k \cdot \sin 2\pi \frac{2}{N} k = 0$$

All complex sinusoids having frequencies as integer multiples of a fundamental frequency $F_s(1/N)$ *are orthogonal to each other.*

Band-limited Signal

An Ideal A/D Converter



An Ideal A/D Converter: Analog Signal

Let $x_a(t)$ be an analog $\boxed{\textit{(absolutely integrable)}}$ signal.
Its CTFT is given by

$$X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t) e^{-j\Omega t} dt$$

where Ω is an analog frequency in radians/sec.
The inverse CTFT is given by

$$x_a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(j\Omega) e^{j\Omega t} d\Omega$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x[k] e^{-j\omega k} \qquad x[k] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega k} d\omega$$

Not Absolutely Integrable Signals

- **Fourier Transform**

$$\frac{d}{dt}x_a(t) = \frac{d}{dt}\left\{\frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega)e^{j\Omega t}d\Omega\right\}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega)\frac{d}{dt}e^{j\Omega t}d\Omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left[j\Omega X_a(j\Omega)\right]e^{j\Omega t}d\Omega$$

$$\boxed{\begin{aligned} &\frac{d}{dt}x_a(t) \leftrightarrow j\Omega X_a(j\Omega) \\ &\frac{d^n}{dt^n}x_a(t) \leftrightarrow (j\Omega)^n X_a(j\Omega) \end{aligned}}$$

Not Absolutely Integrable Signals

- **Example: Constant Function**

$$x_a(t) = \delta(t) \qquad \boxed{\text{Impulse in time}}$$

$$X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t) e^{-j\Omega t} dt = \int_{-\infty}^{\infty} \delta(t) e^{-j\Omega t} dt = \int_{-\infty}^{\infty} e^{-j\Omega t} \delta(t) dt$$

$$= e^{-j\Omega t}\Big|_{t=0} = e^0 = 1 \qquad \boxed{\text{Constant in frequency}}$$

$$x_a(t) = a \qquad \boxed{\text{Constant in time}}$$

$$x_a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_a(j\Omega) e^{j\Omega t} d\Omega \qquad \boxed{\text{Impulse in frequency}}$$

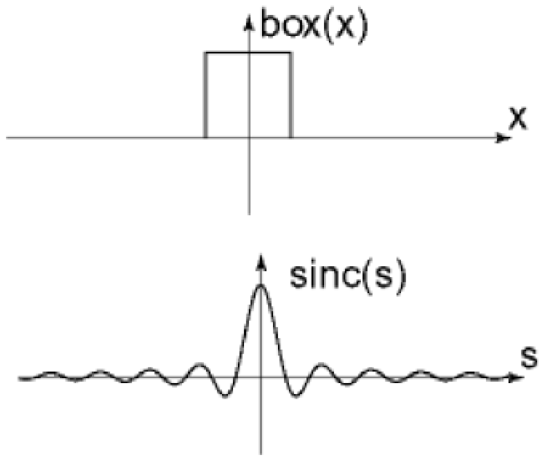
"guess" $$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[2\pi a \delta(\Omega)\right] e^{j\Omega t} d\Omega = \frac{2\pi a}{2\pi} \int_{-\infty}^{\infty} \delta(\Omega) e^{j\Omega t} d\Omega$$

$$= \frac{2\pi a}{2\pi} \cdot 1 = a \qquad \boxed{x_a(t) = a \Leftrightarrow X_a(j\Omega) = 2\pi a \delta(\Omega)}$$

Time-Frequency Duality

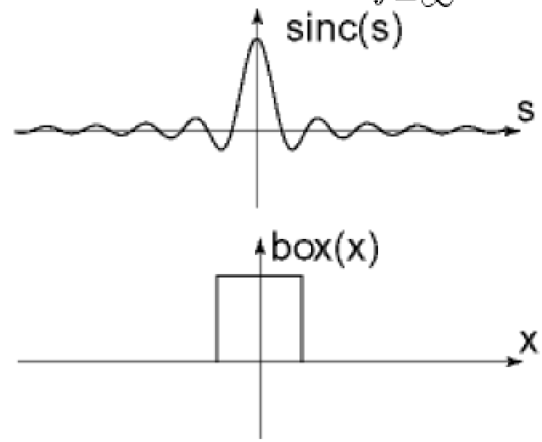
Time-domain

$f(x)$



Frequency-domain

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi s x} dx$$



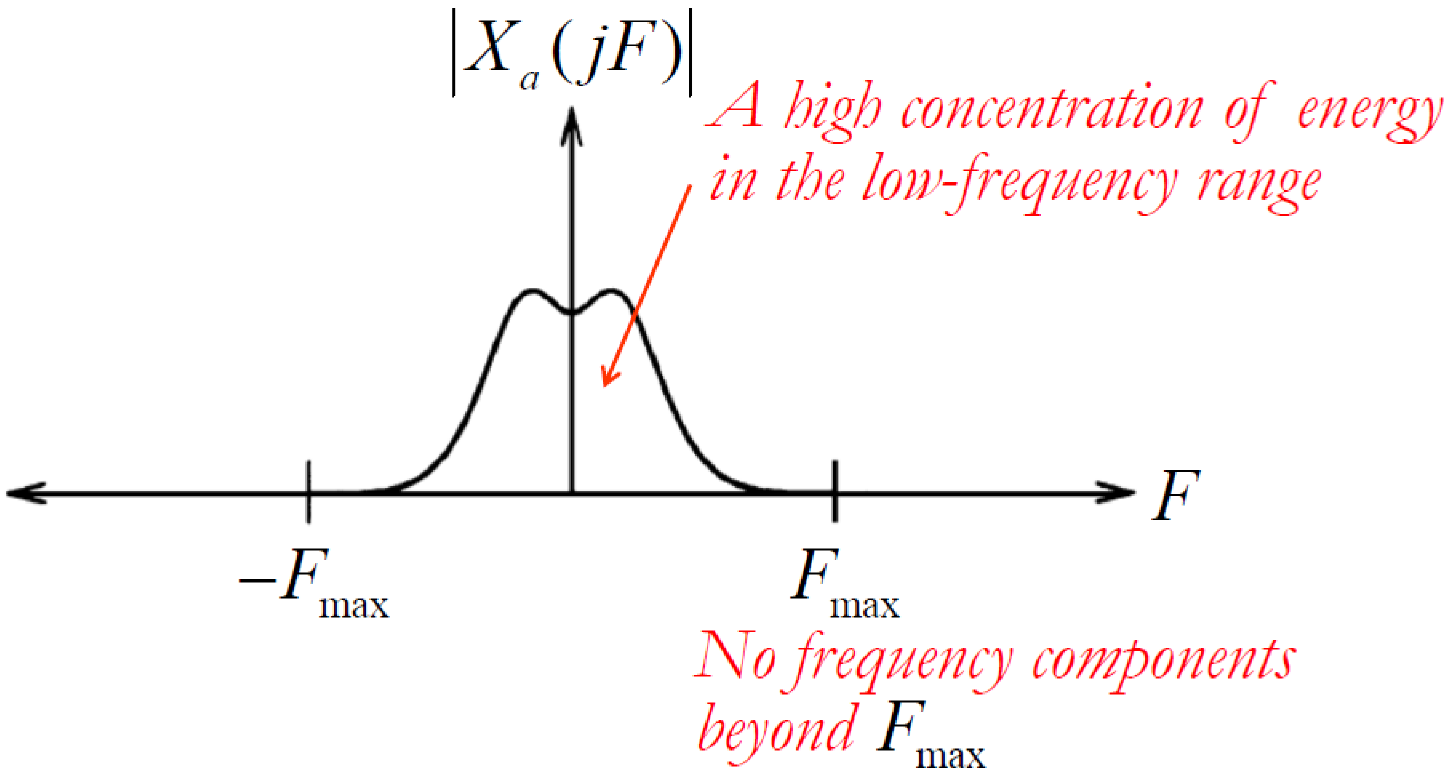
Band-limited signals have infinite time duration and time-limited signals have infinite bandwidth.

Band-limited Signal

- **Band-limited Signal**
  - → Signals that are restricted to a fixed frequency range
  - → Most signals in communication are band limited.
  - → For example, the human voice can typically produce a sound between 80Hz to about 6KHz, so we can say the human voice is a band limited signal limited between 80Hz and 6KHz

- **Digital Signal Processing: A/D Converter**
  - → It is vital to know the band limits of the input signal as the sampling frequency

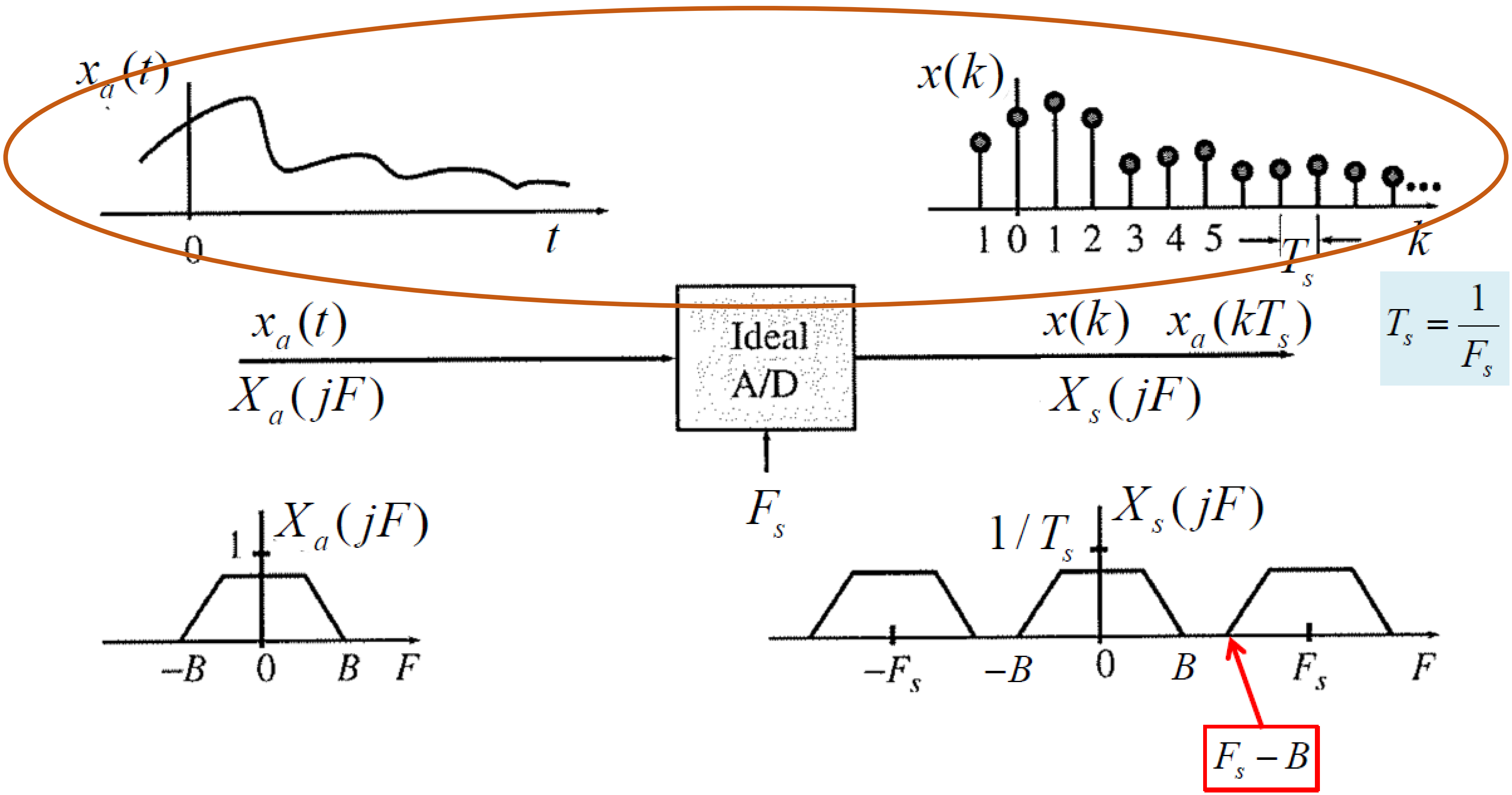
Band-limited Signal



$$X_a(jF) = \int_{-\infty}^{\infty} x_a(t) e^{-j2\pi Ft} dt$$

Sampling

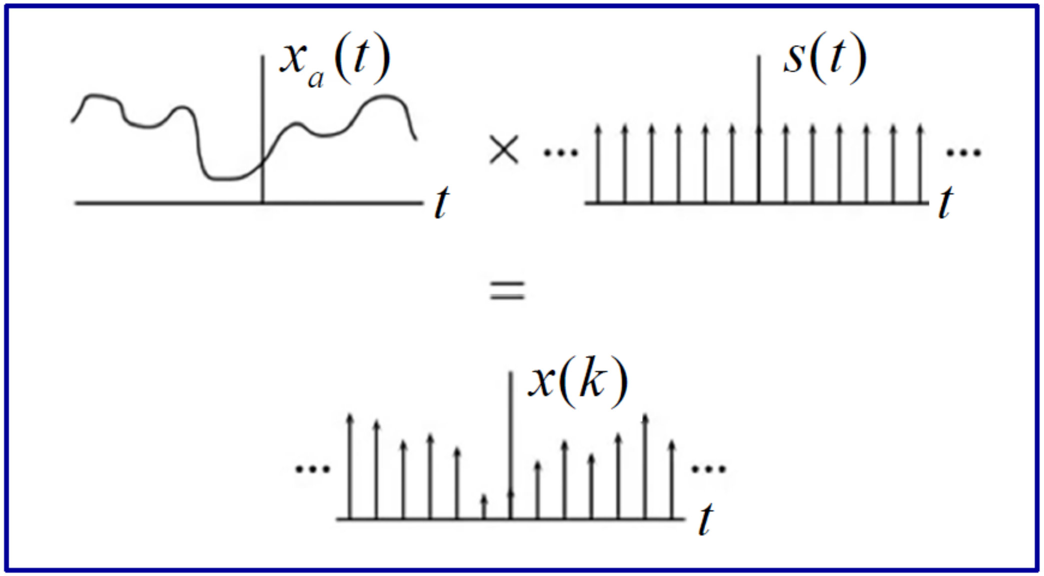
An Ideal A/D Converter



$$\boxed{F_s - B > B \quad \Rightarrow \quad F_s > 2B}$$

Sampling in Time-Domain

- **Sampling: Multiplication with Diract Comb / Impulse Train**



The ideal sampler $s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$

$$x(k) = x_a(t)s(t) = \sum_{k=-\infty}^{\infty} x_a(t)\delta(t - kT_s)$$

$$= \sum_{k=-\infty}^{\infty} x_a(kT_s)\delta(t - kT_s)$$

Fourier Series Representation

- **Fourier Series of Diract Comb / Impulse Train**

$$s(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s) = \sum_{n=-\infty}^{\infty} C_n e^{jn\Omega_0 t}, \qquad \Omega_0 = \frac{2\pi}{T_s}$$

$$C_n = \frac{1}{T_s}\int_{<T_s>} s(t) e^{-jn\Omega_0 t} dt = \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} \delta(t) e^{-jn\Omega_0 t} dt = \frac{1}{T_s}$$

$$s(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T_s} e^{jn\Omega_0 t}$$

$$x(k) = x_a(t)s(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T_s} x_a(t) e^{jn\Omega_0 t}$$

Fourier Transform

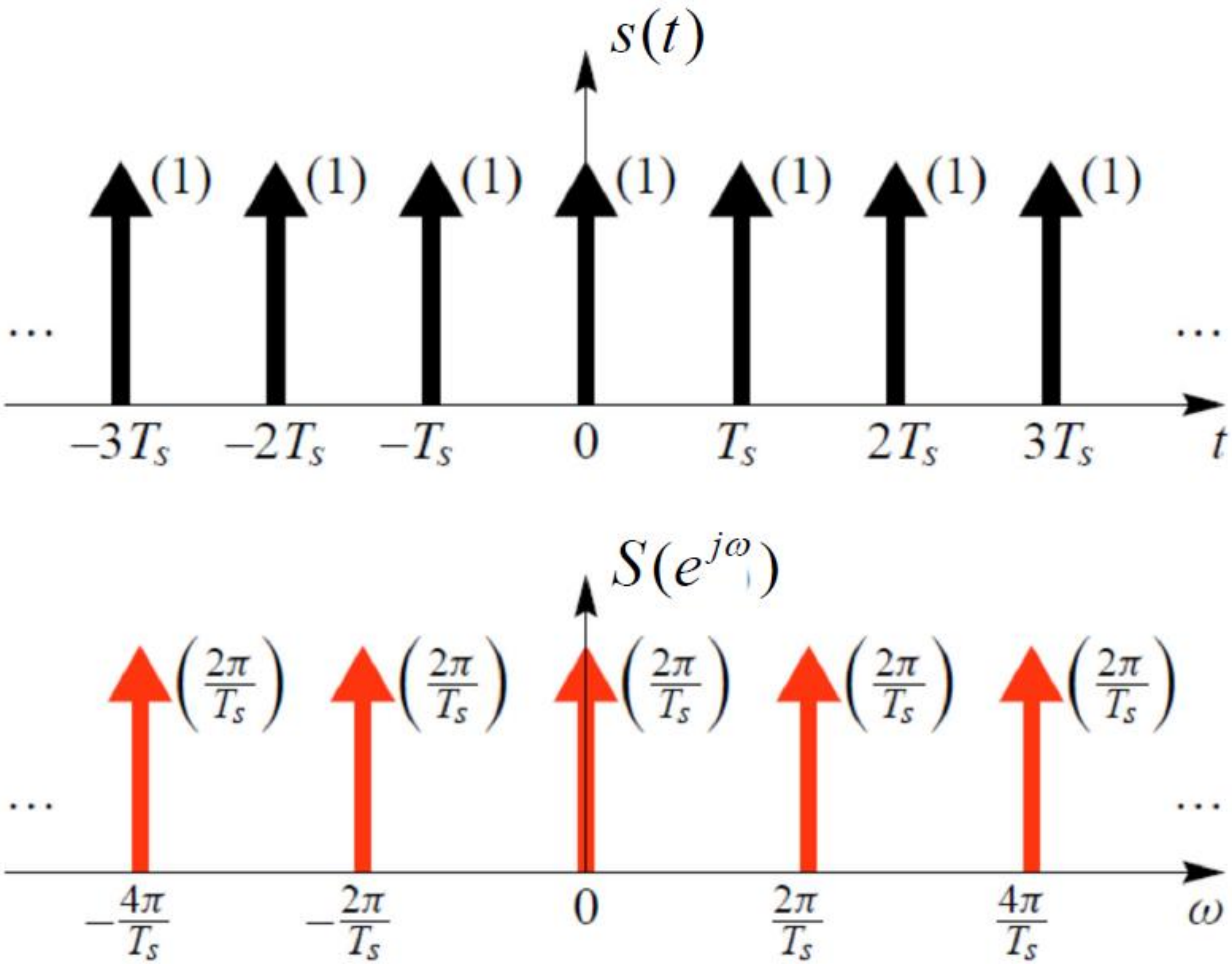
- **Fourier Transform of Diract Comb / Impulse Train**

$$e^{j2\pi F_s t} \rightleftharpoons \delta(F - F_s)$$

$$\sum_{k=-\infty}^{\infty} \delta(t - kT_s) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} e^{jn2\pi \frac{t}{T_s}} \rightleftharpoons \frac{1}{T_s} \sum_{n=-\infty}^{\infty} \delta\left(F - \frac{n}{T_s}\right)$$

Fourier Transform

- **Fourier Transform of Diract Comb / Impulse Train**



Fourier Transform

Fourier Transform of Multiplication

The Fourier transform of the product is the convolution (in the frequency domain) of the individual Fourier transforms.

$$X_s(jF) = X_a(jF) * S(jF)$$
$$= X_a(jF) * \left[\sum_{k=-\infty}^{\infty} F_s \delta(F - kF_s) \right]$$
$$= F_s \sum_{k=-\infty}^{\infty} X_a\left(j(F - kF_s) \right)$$

Sampling period

$$T_s = \frac{1}{F_s}$$

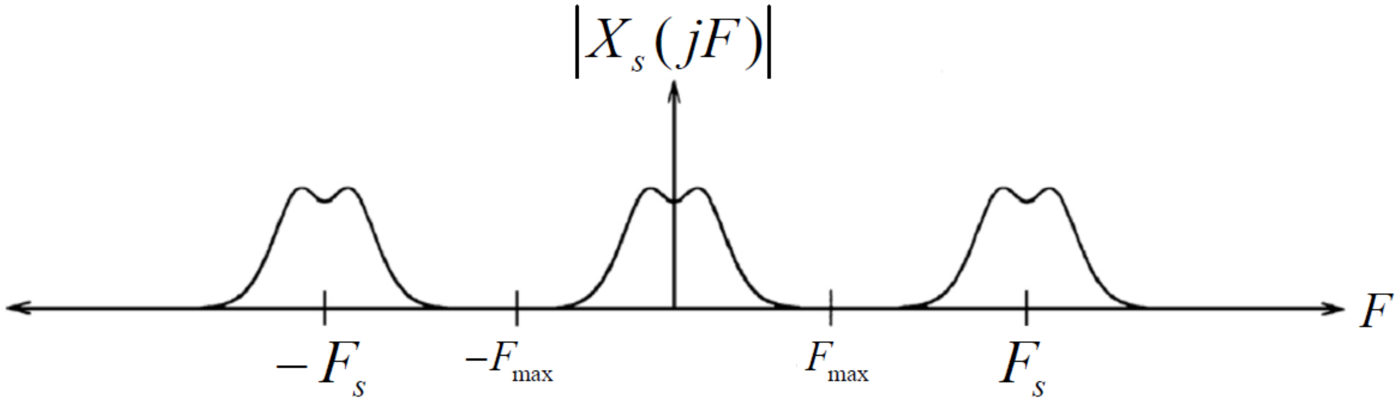
Convolution of a general function $g(x)$ *with a delta function* $\delta(x-a)$

$$\delta(x-a) * g(x) = \int_{-\infty}^{\infty} \delta(\overline{x} - a) g(x - \overline{x}) d\overline{x} = g(x-a)$$

"Sifting property"

Fourier Transform

▪ Fourier Transform of Sampled Signals



Periodic in frequency with period F_s

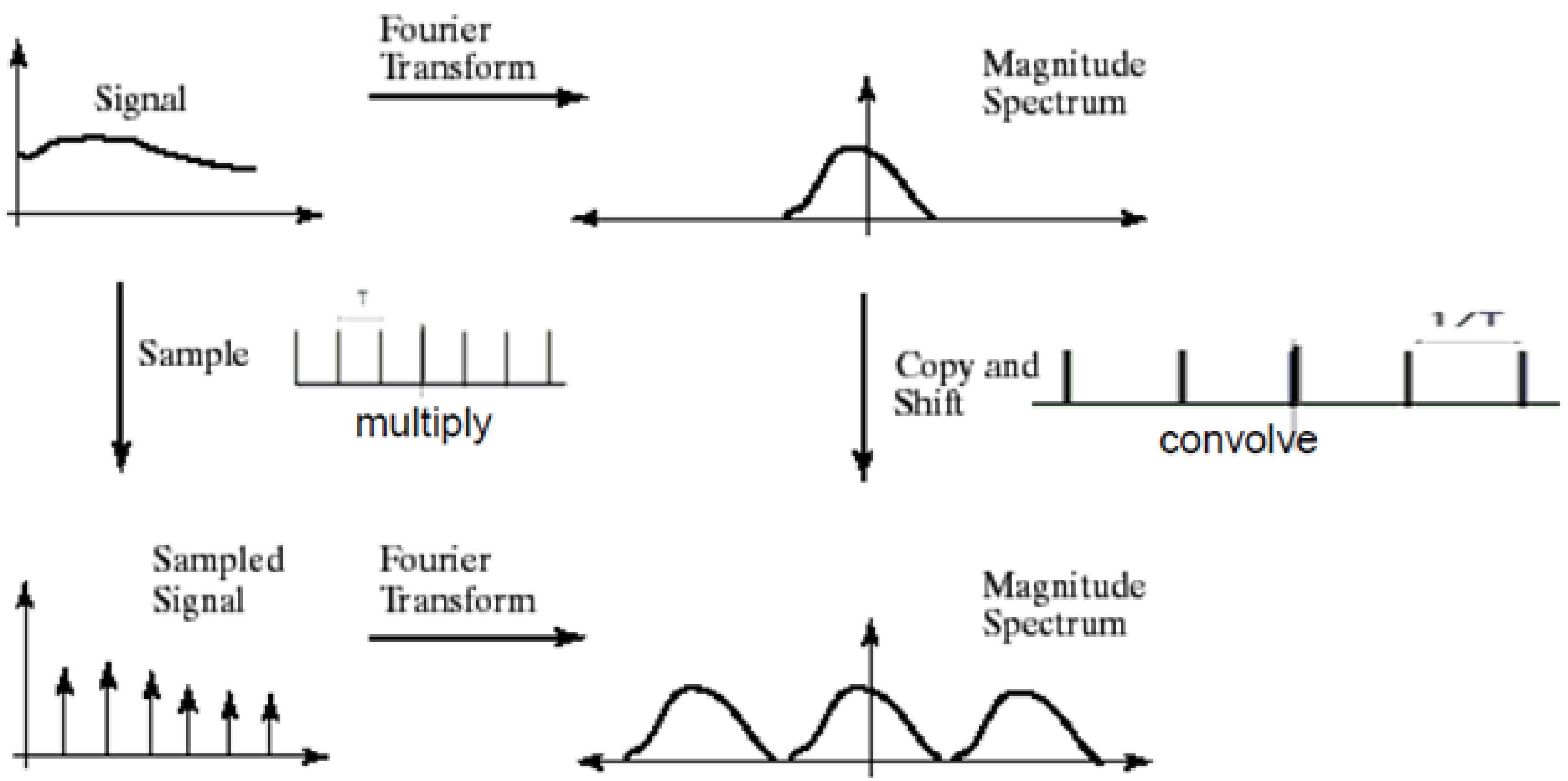
Sampling period $T_s = \dfrac{1}{F_s}$

A small sampling period (equivalent to a high sampling frequency) yields spectra replicated far apart from each other.

The sampling operation has left the original input spectrum intact, merely replicating it periodically in the frequency domain with a spacing of F_s.

Fourier Transform

- **Fourier Transform of Sampled Signals**



Mathematical Proof

Let $x_a(t)$ be an analog (*absolutely integrable*) signal.

Its CTFT is given by

$$X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t)e^{-j\Omega t}dt = \int_{-\infty}^{\infty} x_a(t)e^{-j2\pi Ft}dt$$

The inverse CTFT is given by

$$x_a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega)e^{j\Omega t}d\Omega = \int_{-\infty}^{\infty} X_a(jF)e^{j2\pi Ft}dF$$

$$\Downarrow$$

We now sample $x_a(t)$ at *sampling interval* T_s seconds apart to obtain the discrete-time signal $x(k)$.

$$\boxed{x(k) \equiv x_a(kT_s) = \int_{-\infty}^{\infty} X_a(jF)e^{j2\pi kF/F_s}dF} \quad (1)$$

Its DTFT is given by

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} x(k)e^{-j\omega k} = \sum_{k=-\infty}^{\infty} x(k)e^{-j2\pi fk}$$

$$\boxed{\omega = 2\pi f}$$

The inverse DTFT is given by

$$x(k) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega k}d\omega = \boxed{\int_{-1/2}^{1/2} X(e^{jf})e^{j2\pi fk}df} \quad (2)$$

Mathematical Proof

(1) $\boxed{x(k) \equiv x_a(kT_s) = \int_{-\infty}^{\infty} X_a(jF) e^{j2\pi kF/F_s} dF}$

(2) $x(k) = \dfrac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega k} d\omega = \boxed{\int_{-1/2}^{1/2} X(e^{jf}) e^{j2\pi fk} df}$

$$\int_{-1/2}^{1/2} X(e^{jf}) e^{j2\pi fk} df = \int_{-\infty}^{\infty} X_a(jF) e^{j2\pi kF/F_s} dF$$

$\boxed{f = \dfrac{F}{F_s}}$ *Periodic sampling*

$$\frac{1}{F_s}\int_{-F_s/2}^{F_s/2} X_s(jF) e^{j2\pi kF/F_s} dF = \boxed{\int_{-\infty}^{\infty} X_a(jF) e^{j2\pi kF/F_s} dF} \quad (3)$$

The integration range can be divided into an infinite number of intervals of width F_s

$$\int_{-\infty}^{\infty} X_a(jF) e^{j2\pi kF/F_s} dF = \sum_{l=-\infty}^{\infty} \int_{(l-1/2)F_s}^{(l+1/2)F_s} X_a(jF) e^{j2\pi kF/F_s} dF$$

Mathematical Proof

$$\int_{-\infty}^{\infty} X_a(jF)e^{j2\pi kF/F_s}dF = \sum_{l=-\infty}^{\infty}\int_{(l-1/2)F_s}^{(l+1/2)F_s} X_a(jF)e^{j2\pi kF/F_s}dF$$

$$X_a(jF) \quad \overset{identical}{\Leftrightarrow} \quad X_a[j(F-lF_s)]$$

$$\left(l-\frac{1}{2}\right)F_s \le F \le \left(l+\frac{1}{2}\right)F_s \qquad -F_s/2 \le F \le F_s/2$$

$$\sum_{l=-\infty}^{\infty}\int_{(l-1/2)F_s}^{(l+1/2)F_s} X_a(jF)e^{j2\pi kF/F_s}dF = \sum_{l=-\infty}^{\infty}\int_{-F_s/2}^{F_s/2} X_a\left[j(F-lF_s)\right]e^{j2\pi kF/F_s}dF$$

$$= \int_{-F_s/2}^{F_s/2}\left[\sum_{l=-\infty}^{\infty} X_a\left[j(F-lF_s)\right]\right]e^{j2\pi kF/F_s}dF$$

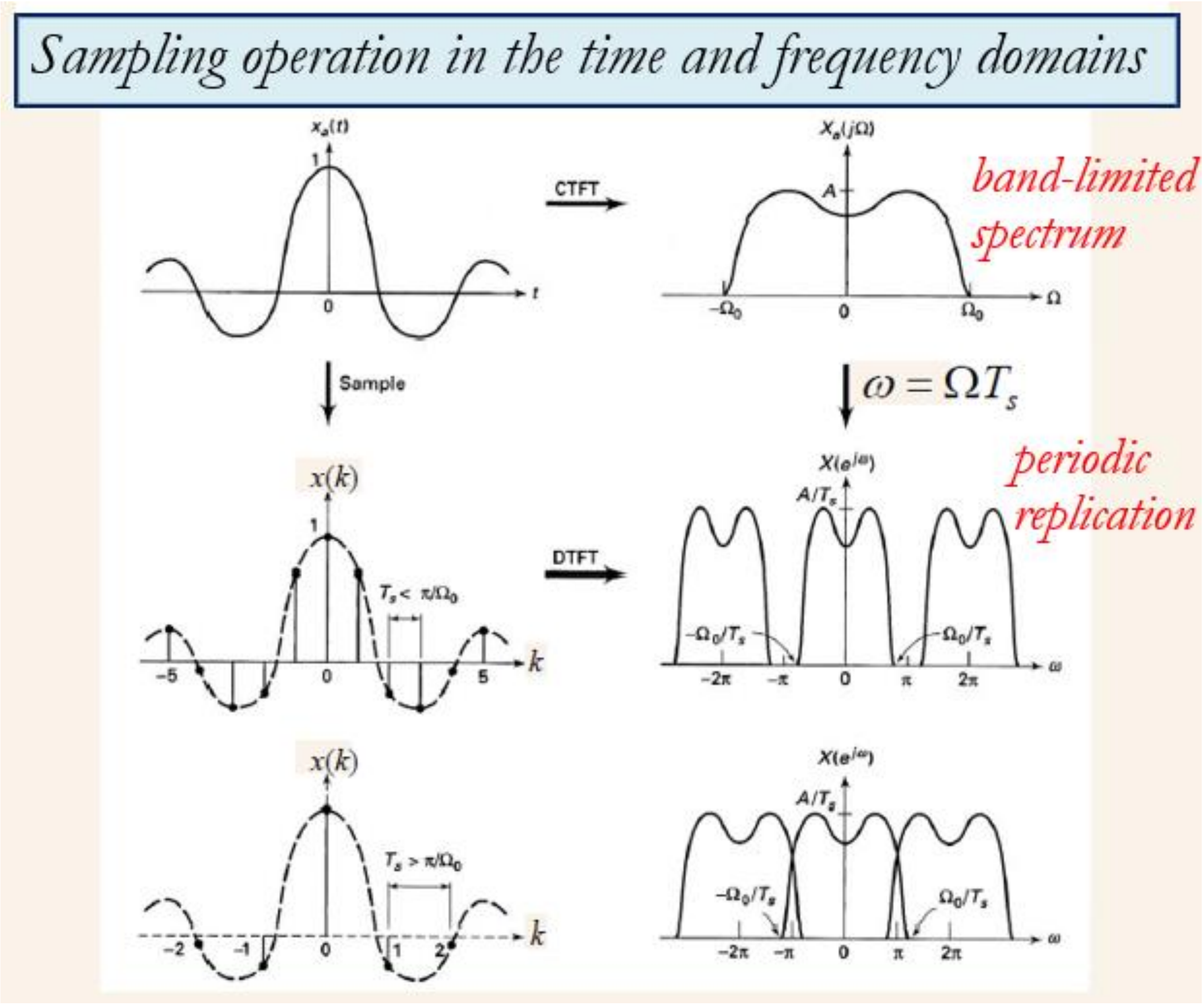
(3) $$\frac{1}{F_s}\int_{-F_s/2}^{F_s/2} X_s(jF)e^{j2\pi kF/F_s}dF = \boxed{\int_{-\infty}^{\infty} X_a(jF)e^{j2\pi kF/F_s}dF}$$

$$X_s(jF) = F_s\sum_{l=-\infty}^{\infty} X_a\left[j(F-lF_s)\right], \quad X_s(jf) = F_s\sum_{l=-\infty}^{\infty} X_a\left[j(f-l)F_s\right]$$

the spectrum of the discrete-time signal   *the spectrum of the analog signal*

A periodic repetition of the scaled spectrum $F_s X_a(jF)$ *with period* F_s.

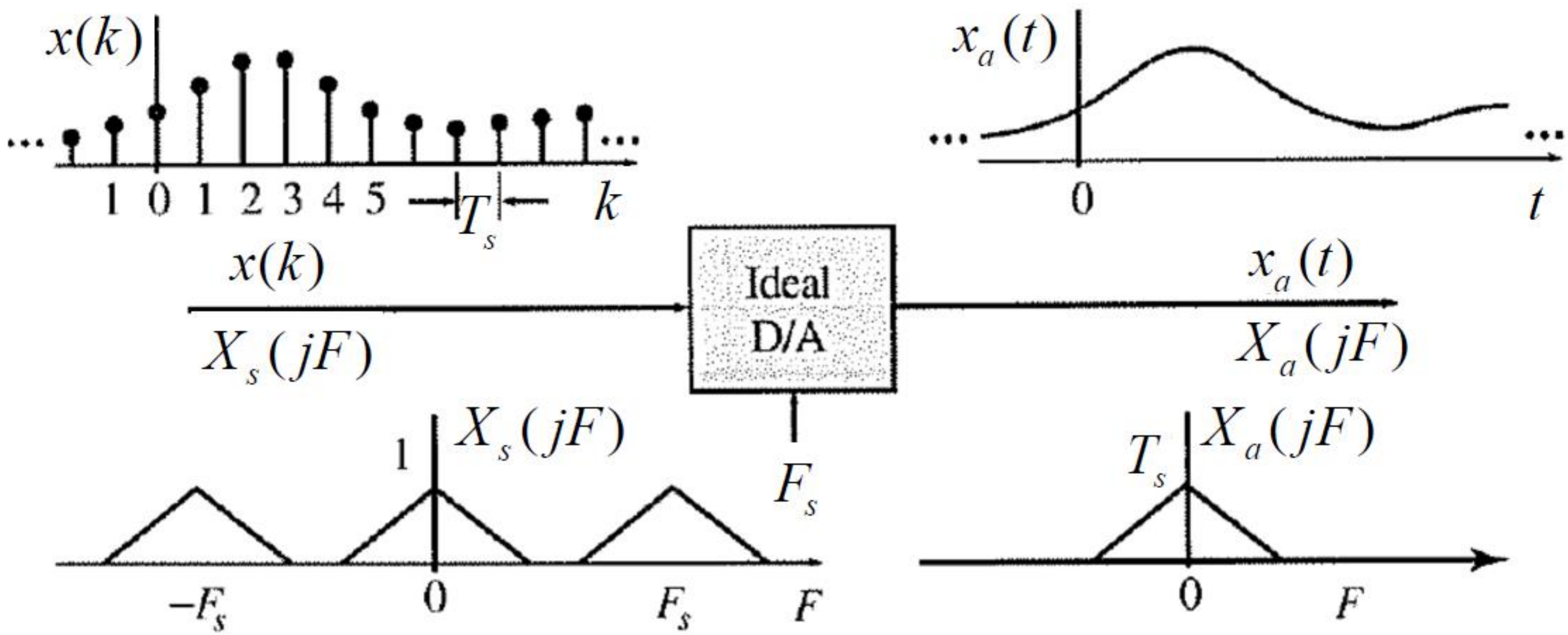
Mathematical Proof



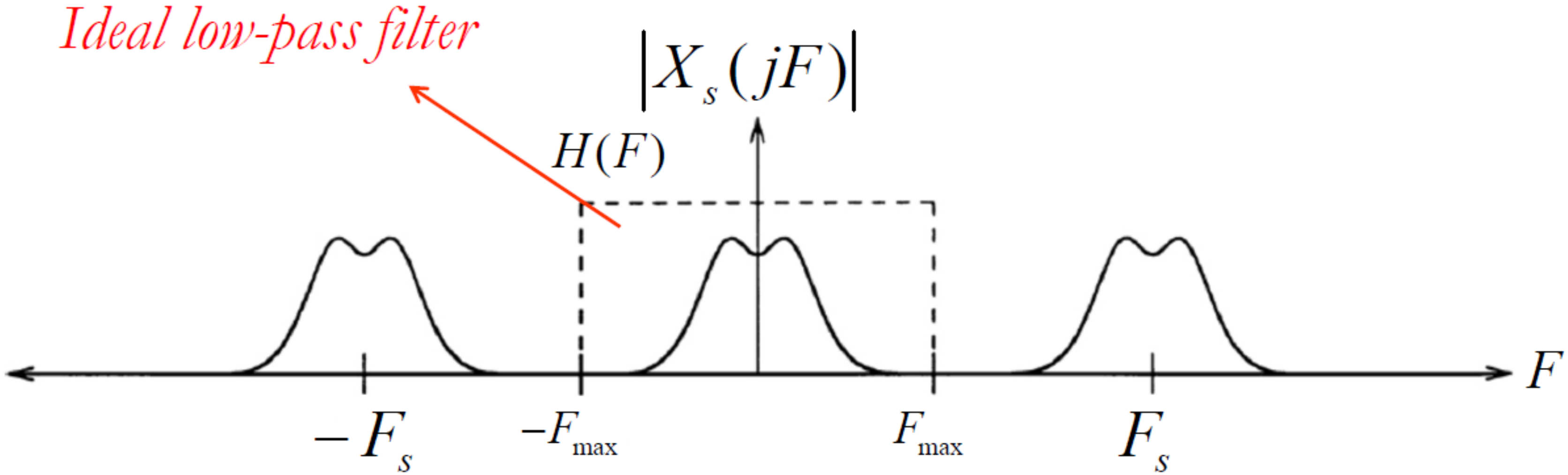
Signal Reconstruction

An Ideal D/A Converter

An ideal D/A converter in the time and frequency domains



Low-Pass Filter



$$X_s(jF) = \underbrace{X_a(jF)}_{\text{the baseband spectrum}} + \underbrace{X_{high}(jF)}_{\text{the high-frequency components consisting of the remaining replicated versions of } X_a(jF)}$$

the baseband spectrum

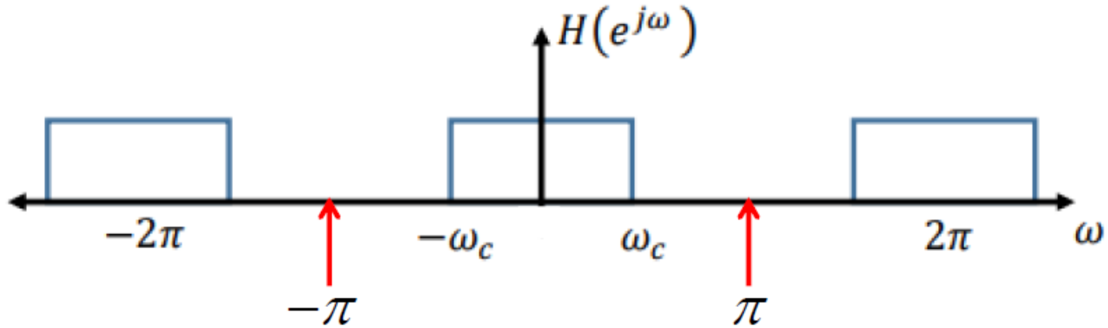
the high-frequency components consisting of the remaining replicated versions of $X_a(jF)$

Exact signal reconstruction from sampled data:
Discard the replicated spectra $X_{high}(jF)$, *leaving only* $X_a(jF)$.

the spectrum of the signal we seek to recover

Ideal Low-Pass Filter: Rectangular Signal

Ideal Filters



$$h_L[k] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})e^{j\omega k}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega k}\,d\omega = \frac{1}{2\pi}\frac{1}{jk}\left[e^{j\omega_c k} - e^{-j\omega_c k}\right] = \frac{1}{2\pi}\frac{1}{jk}\left[2j\sin\omega_c k\right]$$

$$= \frac{1}{\pi k}\sin\omega_c k = \frac{\omega_c}{\pi}\boxed{\frac{\sin\omega_c k}{\omega_c k}}$$

$$\boxed{\mathrm{sinc}(\omega_c k)}$$

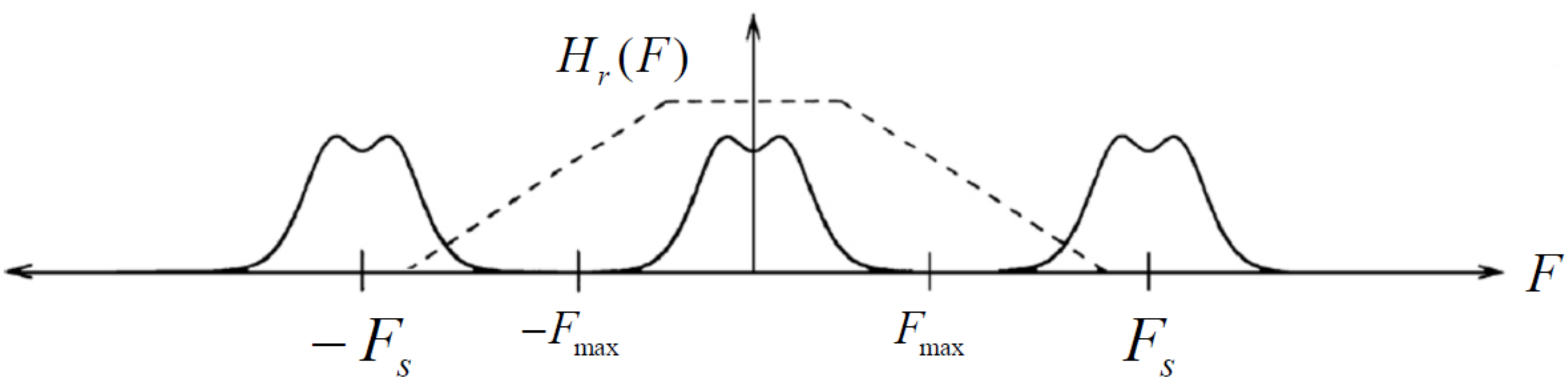
Ideal Low-Pass Filter

An infinite-order interpolation

$$x_a(t) = \text{sinc}(t) * x(k) = \int_{-\infty}^{\infty} \text{sinc}(\tau)x(t-\tau)d\tau$$

Multiplication in the frequency domain is identical to convolution in the time domain.

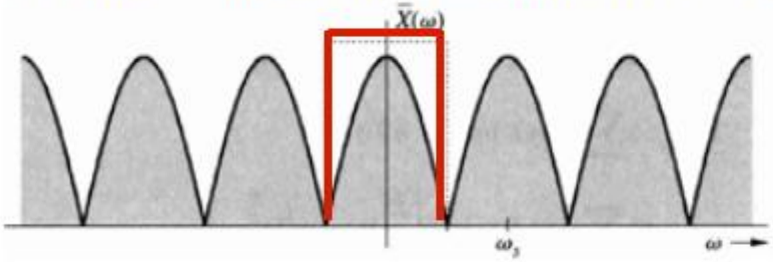
$$\left|X_s(jF)\right|$$

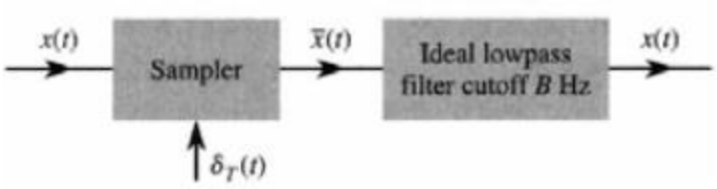


Nonideal reconstruction

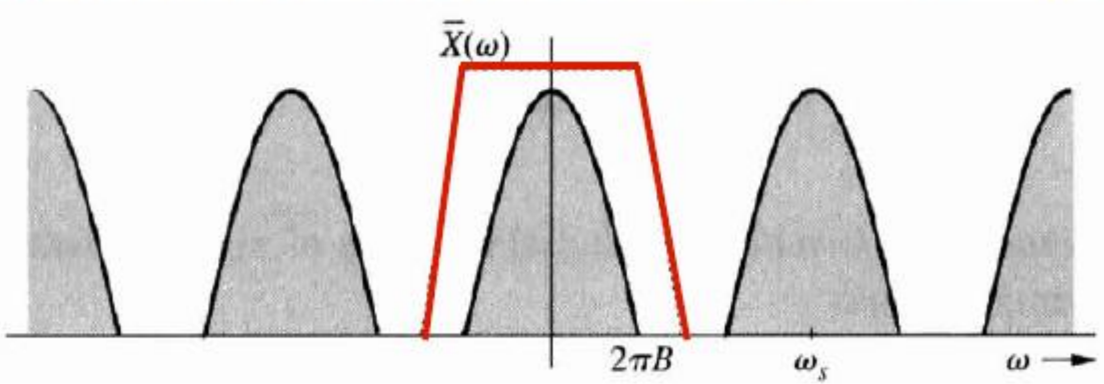
Practical Low-Pass Filter

- Ideal reconstruction system is therefore:

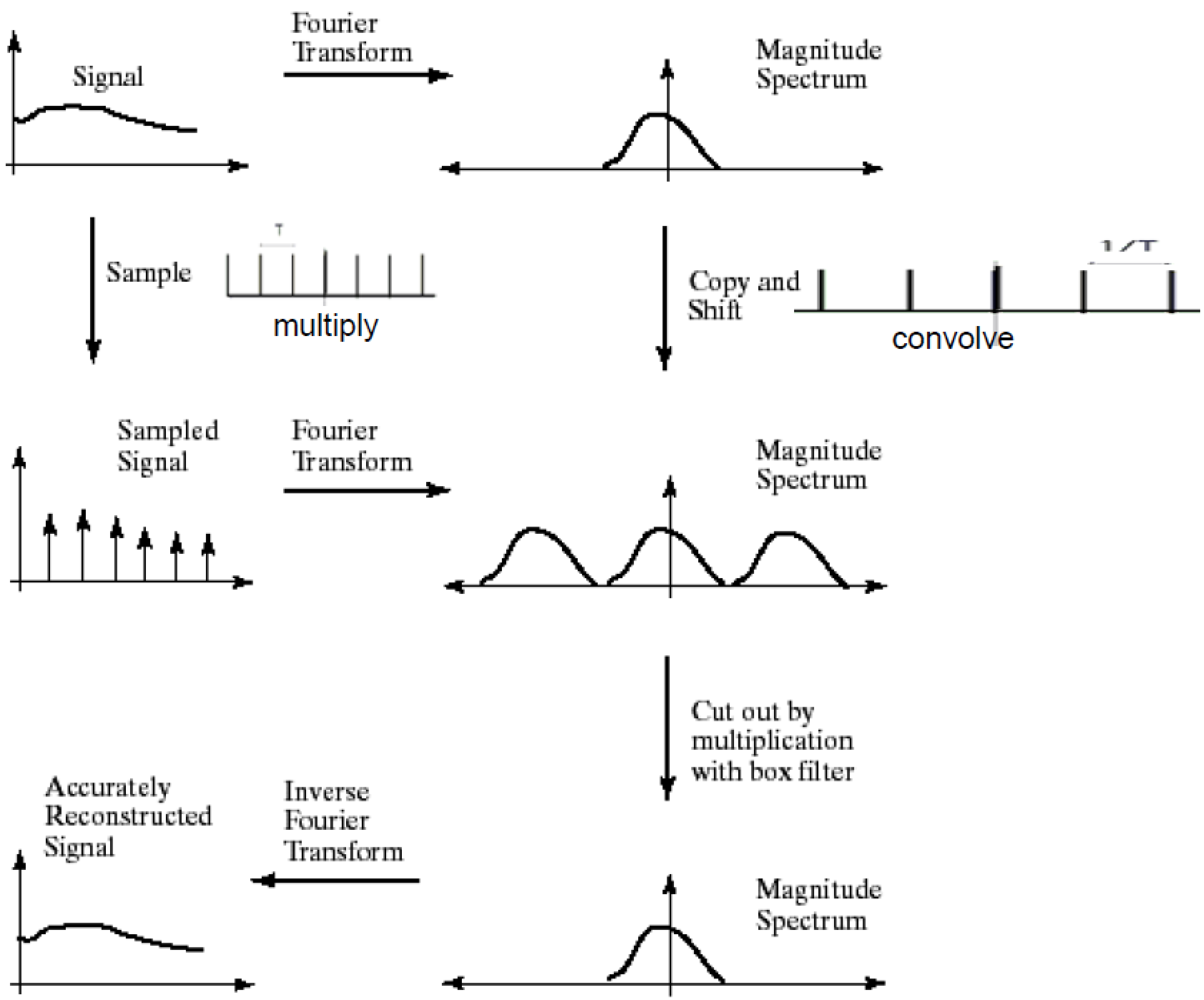




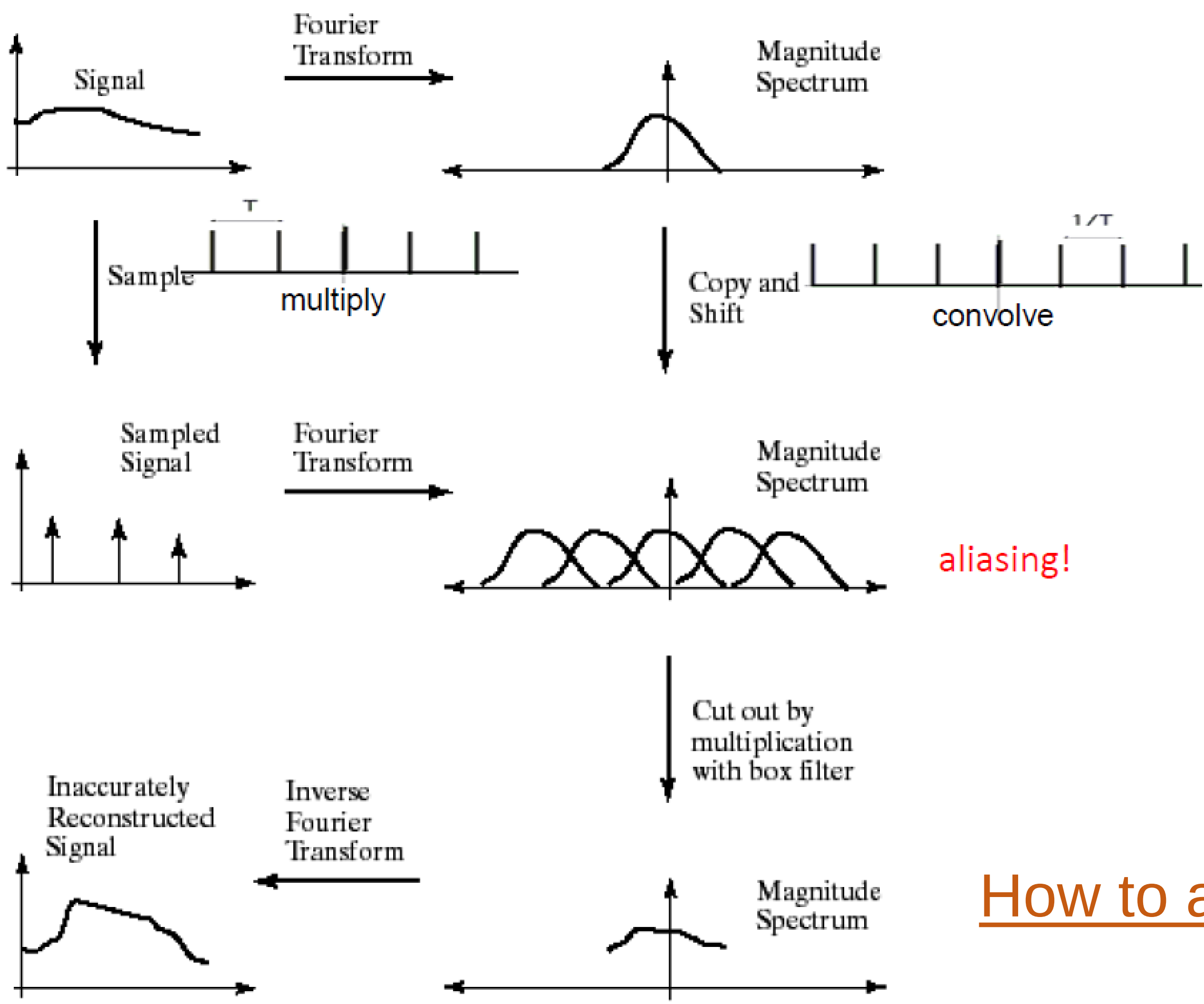
- In practise, we normally sample at higher frequency than Nyquist rate:



Sampling & Reconstruction Process



Sampling & Reconstruction Process



How to avoid aliasing?

Sampling Principle

Sampling Principle

Nyquist Theorem

A *band-limited* signal $x_a(t)$ with bandwidth F_0 can be *reconstructed* from its sample values $x(k) = x_a(kT_s)$ if the *sampling frequency* $F_s = 1/T_s$ is greater than twice the bandwidth F_0 of $x_a(t)$.

$$F_s > 2F_0$$

Otherwise *aliasing* would result in $x(k)$. The sample rate of $2F_0$ for an analog band-limited signal is called the *Nyquist rate*.

After $x_a(t)$ *is sampled, the highest analog frequency that* $x(k)$ *represents is* $F_s / 2\,Hz$ (*or* $\omega = \pi$).

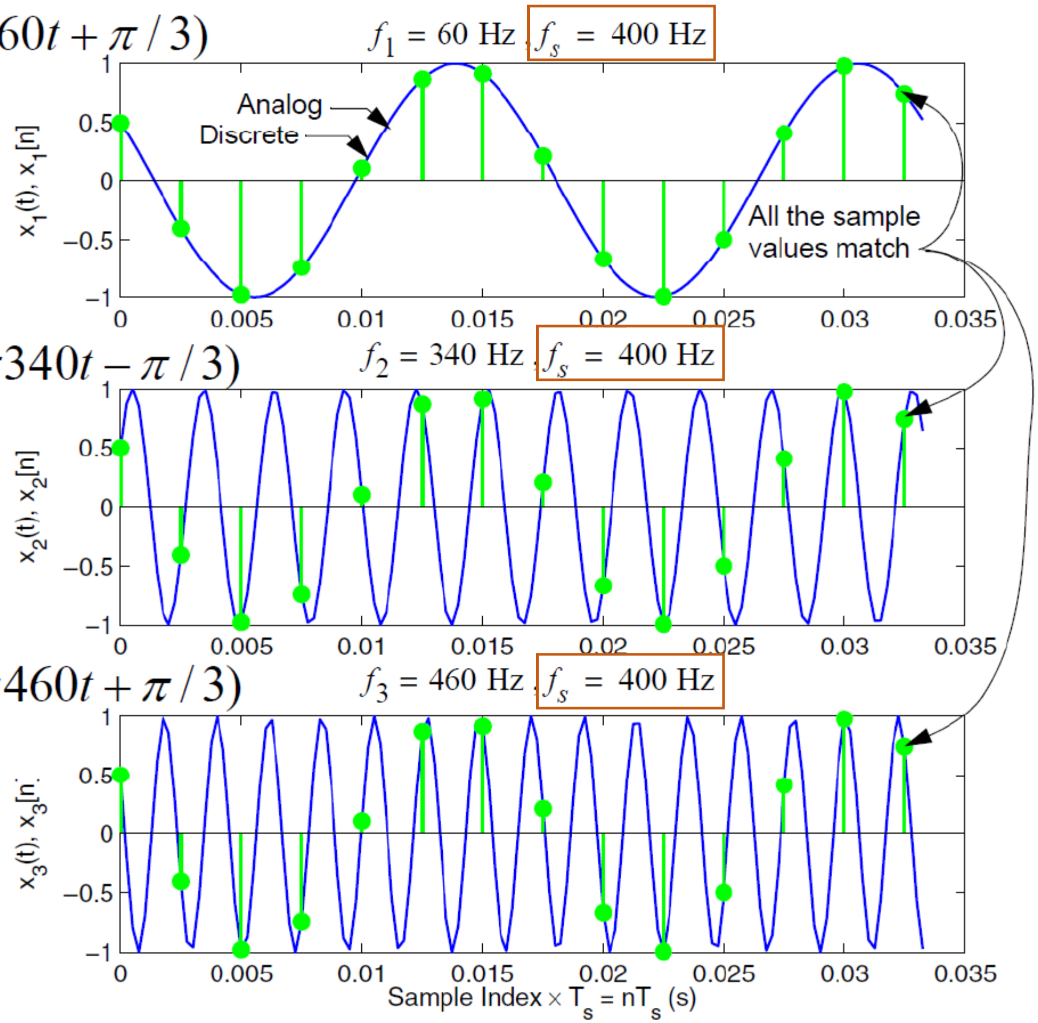
Sampling Principle

▪ Example:

$$x_1(t) = \cos(2\pi 60t + \pi/3)$$

$$x_2(t) = \cos(2\pi 340t - \pi/3)$$

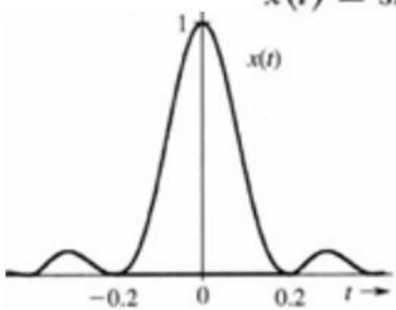
$$x_3(t) = \cos(2\pi 460t + \pi/3)$$



Oversampling vs Undersampling

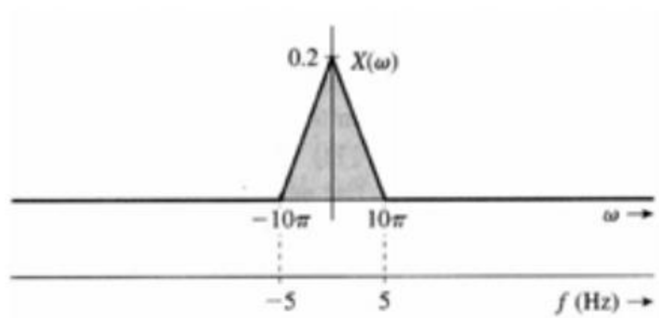
- What are the effects of sampling a signal at, above, and below the Nyquist rate? Consider a signal bandlimited to 5Hz:

$x(t) = \mathrm{sinc}^2(5\pi t)$



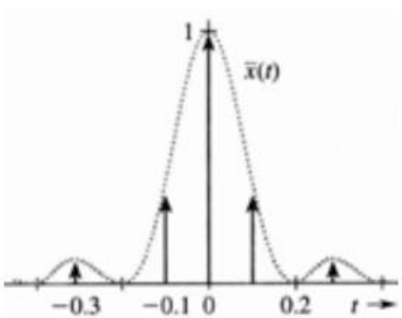
$\Leftrightarrow$

$X(\omega) = 0.2\,\Delta(\omega/20\pi)$

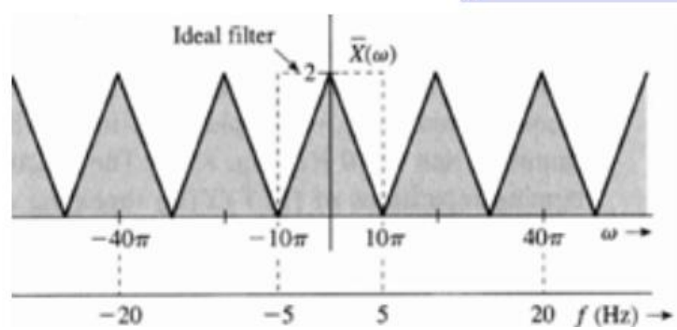


- Sampling at Nyquist rate of 10Hz give:

perfect reconstruction possible



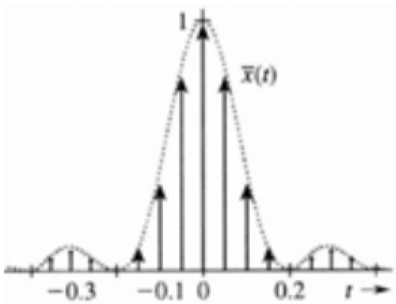
$\Leftrightarrow$



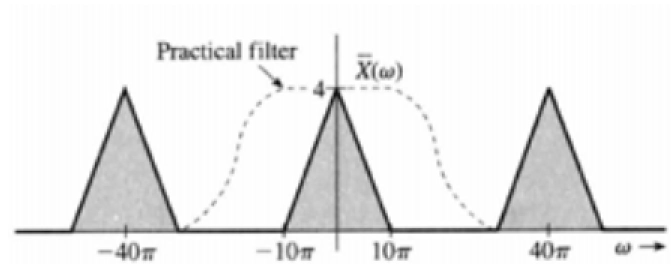
Oversampling vs Undersampling

- Sampling at higher than Nyquist rate at 20Hz makes reconstruction much easier.

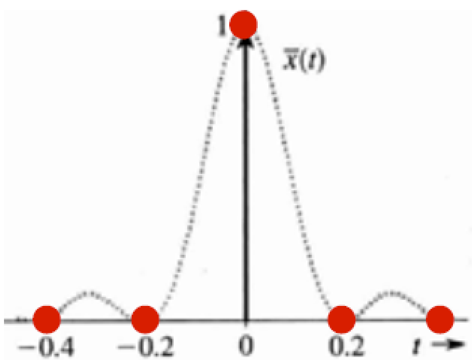
perfect reconstruction practical



$\Leftrightarrow$

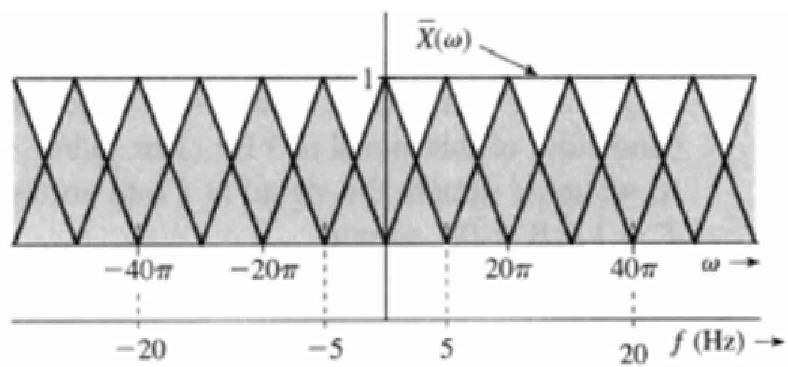


- Sampling below Nyquist rate at 5Hz corrupts the signal.



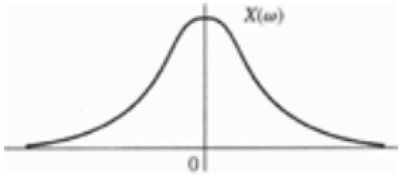
ALIASING

$\Leftrightarrow$

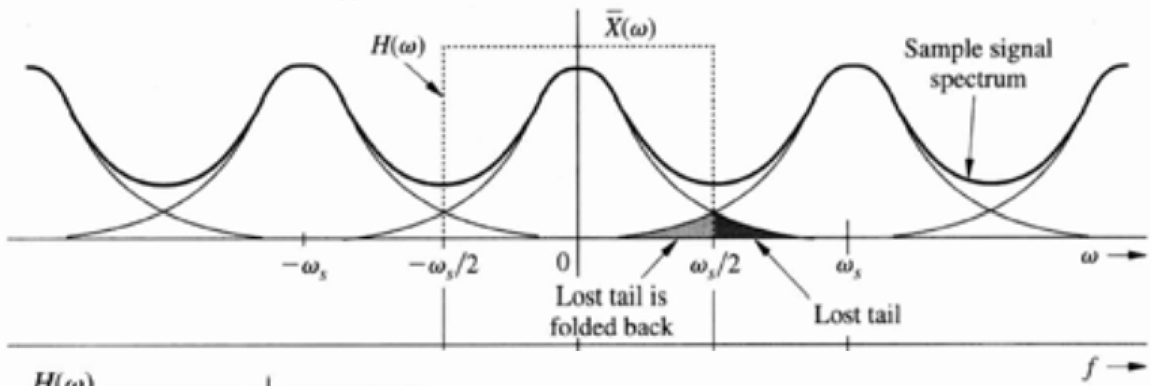


Aliasing

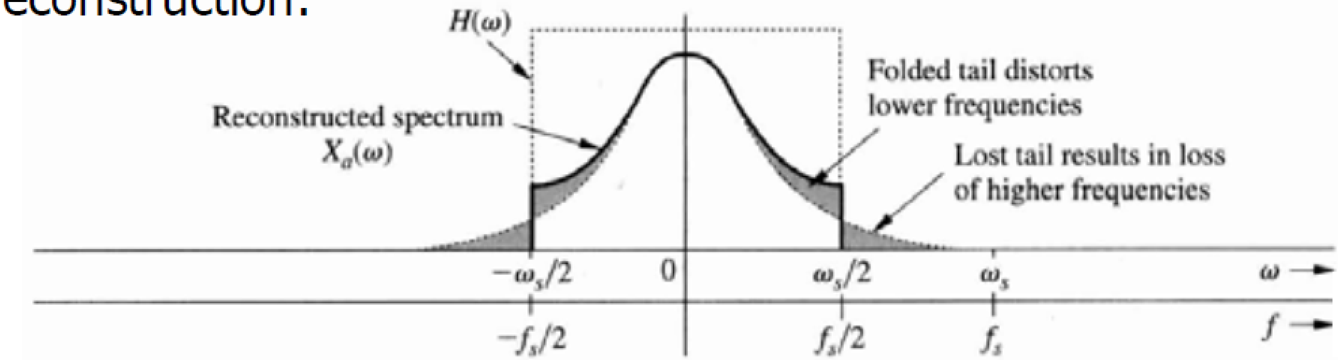
- To avoid corruption of signal after sampling, one must ensure that the signal being sampled at fs is bandlimited to a frequency B, where B < fs/2.
- Consider this signal spectrum:



- After sampling:



- After reconstruction:



Example (1)

Consider the analog signal $\boxed{\omega = 2\pi f}$

$$x_a(t) = 4 + 2\cos\left(150\pi t + \frac{\pi}{3}\right) + 4\sin(350\pi t)$$

The highest freq. $\boxed{F_0 = 175Hz}$

sampled at $F_s = 200 \quad sam/\text{sec}.$ $\boxed{< 2F_0}$

$$x(k) = x_a(kT_s) = x_a\left(k\frac{1}{F_s}\right) = x_a(0.005k)$$

$$= 4 + 2\cos\left(0.75\pi k + \frac{\pi}{3}\right) + 4\sin(1.75\pi k)$$

Outside the primary interval of $-\pi \le \omega \le \pi$

$$\boxed{\begin{aligned} x(k) &= 4 + 2\cos\left(0.75\pi k + \frac{\pi}{3}\right) + 4\sin(1.75\pi k - 2\pi k) \\ &= 4 + 2\cos\left(0.75\pi k + \frac{\pi}{3}\right) - 4\sin(0.25\pi k) \end{aligned}}$$

Example (1)

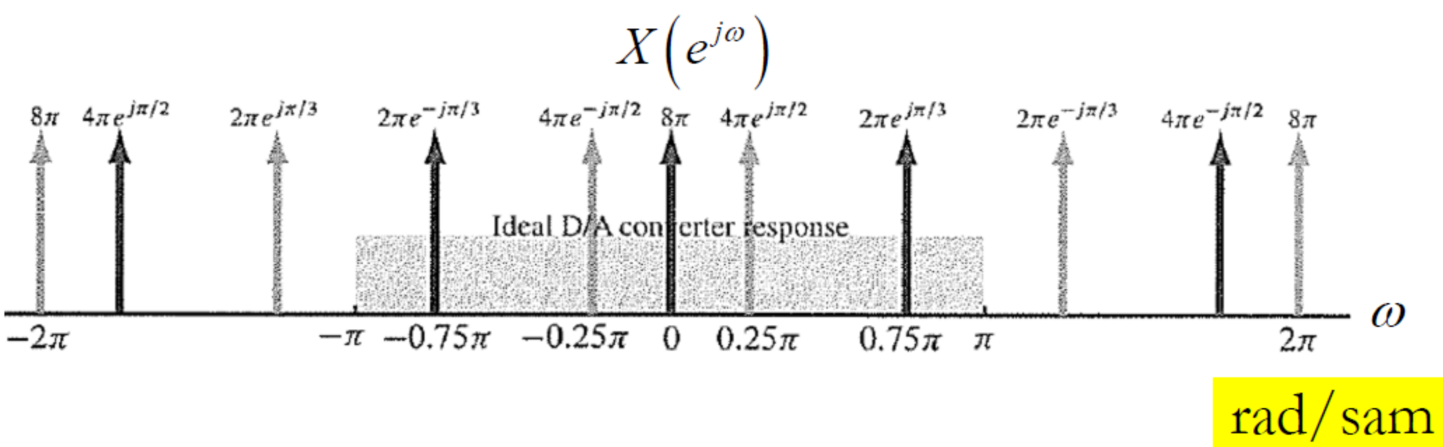
Using Euler's identity,

$$x(k) = 4 + e^{j\frac{\pi}{3}} e^{j0.75\pi k} + e^{-j\frac{\pi}{3}} e^{-j0.75\pi k} + 2je^{j0.25\pi k} - 2je^{-j0.25\pi k}$$

DTFT

$$X\left(e^{j\omega}\right) = 8\pi\delta(\omega) + 2\pi e^{j\frac{\pi}{3}}\delta(\omega - 0.75\pi) + 2\pi e^{-j\frac{\pi}{3}}\delta(\omega + 0.75\pi)$$

$$+ j4\pi\delta(\omega - 0.25\pi) - j4\pi\delta(\omega + 0.25\pi), \quad -\pi \le \omega \le \pi$$



Example (2):

Example $x_a(t) = e^{-1000|t|}$

$$X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t) e^{-j\Omega t} dt = \int_{-\infty}^{0} e^{1000t} e^{-j\Omega t} dt + \int_{0}^{\infty} e^{-1000t} e^{-j\Omega t} dt$$

$$= \frac{0.002}{1 + \left(\frac{\Omega}{1000}\right)^2}$$

Using the approximation $e^{-5} \approx 0$,

$x_a(t)$ *can be approximated by a finite-duration signal over* $-0.005 \le t \le 0.005$.

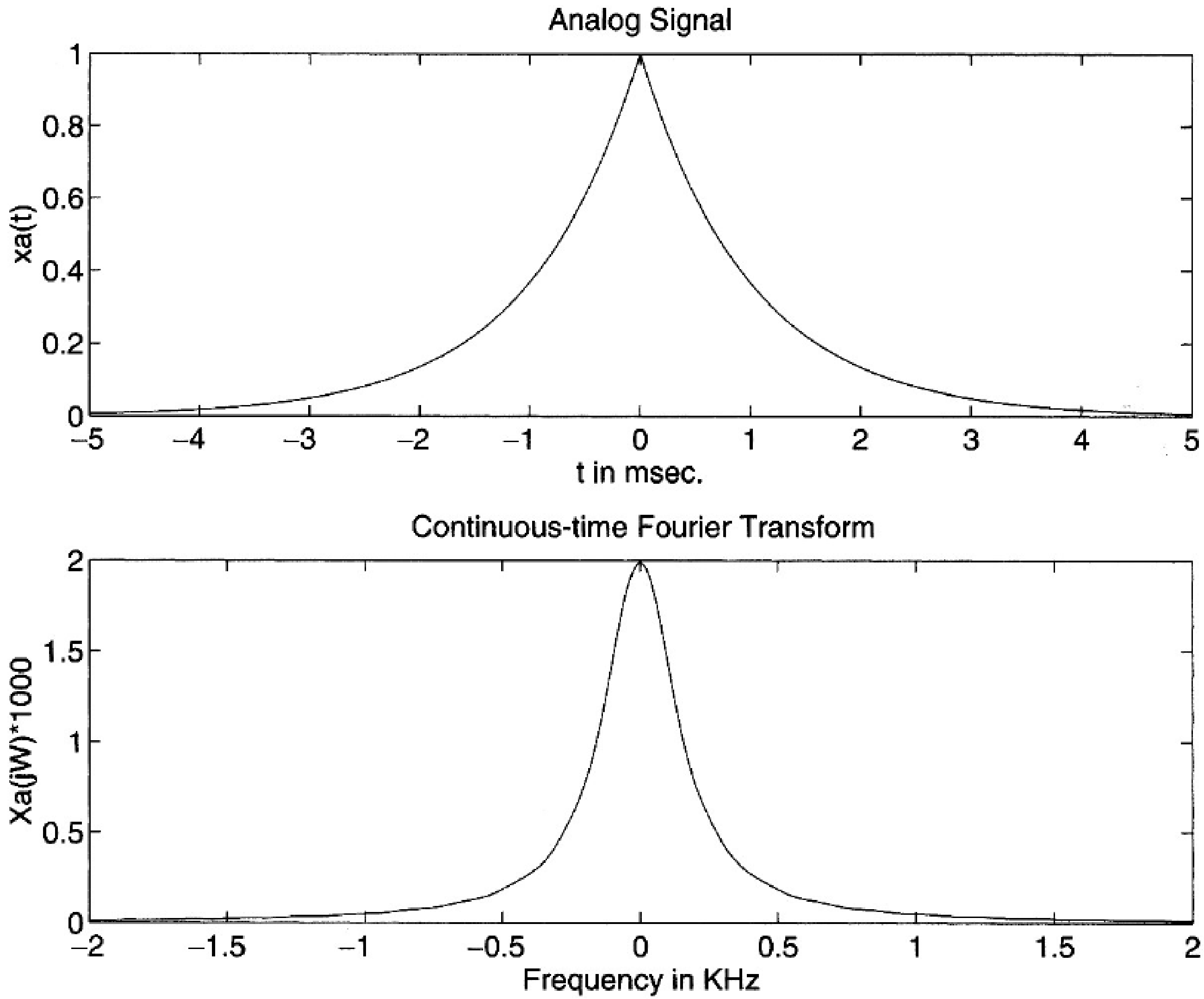
$$X_a(j\Omega) \approx 0 \;\; \textit{for} \;\; \Omega \ge 2\pi\underbrace{(2000)}_{\textit{bandwidth}}$$

$$\Delta t = 5 \times 10^{-5} \ll \frac{1}{2(2000)} = 25 \times 10^{-5}$$

Example (2): MATLAB

```
% Analog Signal
>> Dt = 0.00005; t = -0.005:Dt:0.005; xa = exp(-1000*abs(t))
% CTFT
>> Wmax = 2*pi*2000; L = 500; i = 0:1:L; W = i*Wmax/L;
>> Xa = xa * exp(-j*t'*W) * Dt; Xa = real(Xa);
>> W = [-fliplr(W), W(2:501)]; % Omega from –Wmax to Wmax
>> Xa = [fliplr(Xa), Xa(2:501)]; % Xa over –Wmax to Wmax interval
>> subplot(2,1,1); plot(t*1000,xa);
>> xlabel('t in msec.'); ylabel('xa(t)')
>> title('Analog Signal')
>> subplot(2,1,2); plot(W/(2*pi*1000),Xa*1000);
>> xlabel('Frequency in KHz'); ylabel('Xa(jW)*1000')
>> title('Continuous-time Fourier Transform')
```

Example (2): MATLAB



Lab #8.1 (1 pt.): Due Jan 24

Study the effect of sampling on the frequency-domain quantities:

$x_a(t) = e^{-1000|t|}$

a. Sample $x_a(t)$ at $F_s = 5000$ *sam/sec* to obtain $x_1(k)$. Determine and plot $X_1(e^{j\omega})$.

b. Sample $x_a(t)$ at $F_s = 1000$ *sam/sec* to obtain $x_2(k)$. Determine and plot $X_2(e^{j\omega})$.

c. Sample $x_a(t)$ at $F_s =$ X000*sam/sec* to obtain $x_3(k)$. Determine and plot $X_3(e^{j\omega})$.

X=A+1

Use Your ID: sGFEDCBA

Signal Reconstruction with Sampling Principle

Reconstruction

If we sample band-limited $x_a(t)$ above its Nyquist rate, then we can reconstruct $x_a(t)$ from its samples $x(k)$.

1. The samples are converted into a weighted impulse train.

$$\sum_{k=-\infty}^{\infty} x(k)\delta(t-kT_s) = \cdots + x(-1)\delta(k+T_s) + x(0)\delta(t) + x(1)\delta(k-T_s) + \cdots$$

2. Then, the impulse train is filtered through an ideal analog lowpass filter band-limited to the $[-F_s/2, F_s/2]$ band.

Reconstruction

$x(k)$ → Impulse train conversion → Ideal lowpass filter → $x_a(t)$

This two-step procedure can be described mathematically using an interpolating formula:

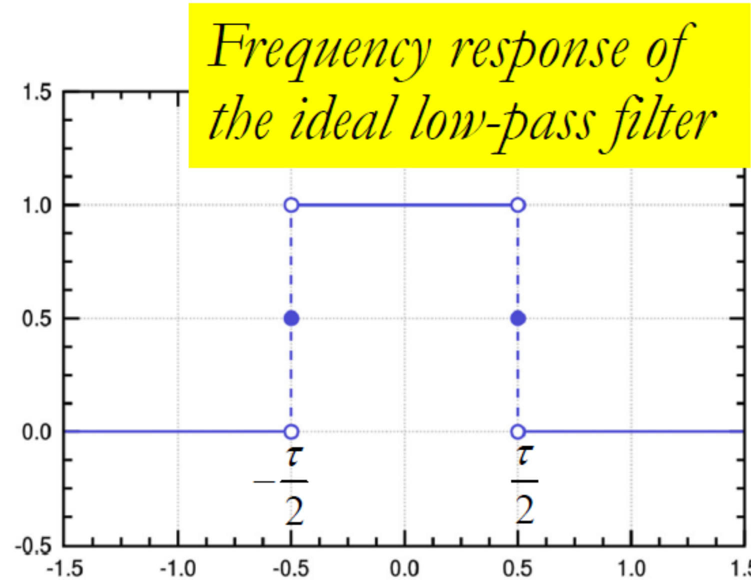
$$x_a(t) = \sum_{k=-\infty}^{\infty} x(k)\,\mathrm{sinc}[F_s(t - kT_s)]$$

where $\mathrm{sinc}(x) = \dfrac{\sin \pi x}{\pi x}$ is an interpolating function.

Ideal Low-pass Filter

Rectangular Function

$$rect(t) = \begin{cases} 0 & |t| > \frac{1}{2} \\ \frac{1}{2} & |t| = \frac{1}{2} \\ 1 & |t| < \frac{1}{2} \end{cases}$$



$$\int_{-\infty}^{\infty} rect(t)\cdot e^{-j\Omega t}dt = \int_{-\tau/2}^{\tau/2} e^{-j\Omega t}dt = \frac{1}{-j\Omega}e^{-j\Omega t}\Big|_{-\tau/2}^{\tau/2} = \frac{1}{-j\Omega}\left[e^{-j\Omega\tau/2} - e^{j\Omega\tau/2}\right]$$

$$= \frac{2}{\Omega}\left[\frac{e^{j\Omega\tau/2} - e^{-j\Omega\tau/2}}{2j}\right] = \frac{\sin(\Omega\tau/2)}{\Omega/2} = \tau\frac{\sin(\Omega\tau/2)}{\Omega\tau/2}$$

The ideal low-pass filter impulse response extends from $-\infty$ to ∞.

Sinc Function

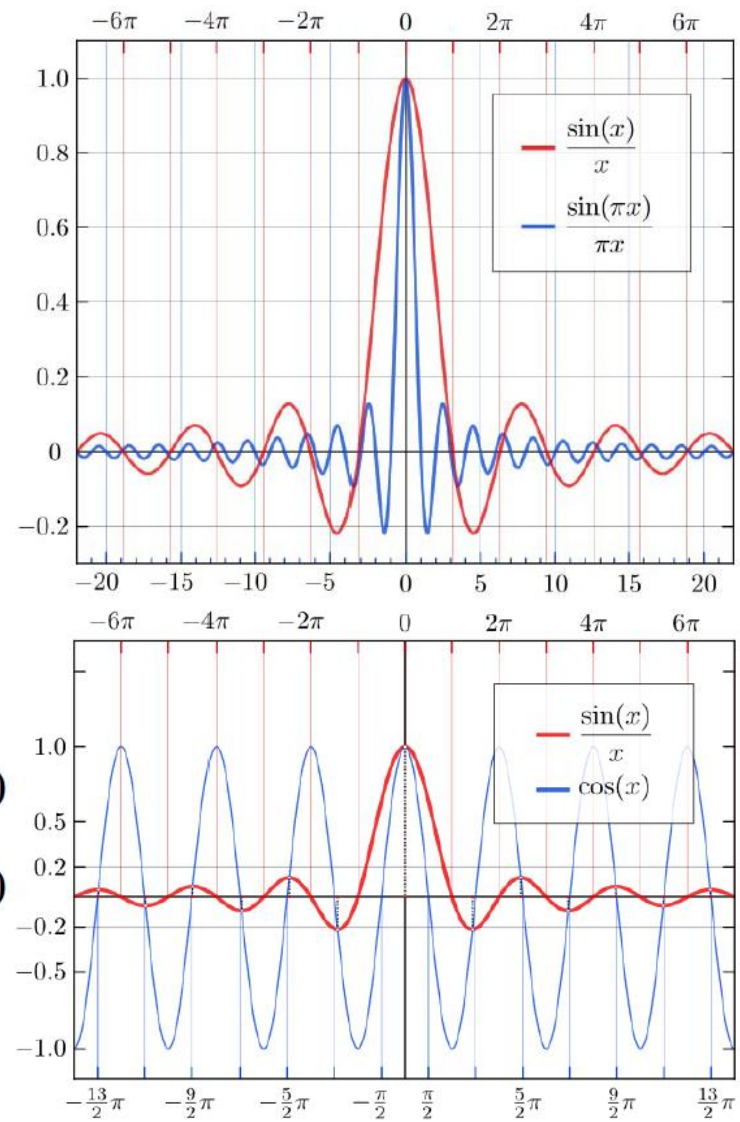
Sinc (Sine Cardinal)

$$\mathrm{sinc}(x) = \begin{cases} 1 & x = 0 \\ \dfrac{\sin x}{x} & x \neq 0 \end{cases}$$

unnormalized

$$\mathrm{sinc}(\pi x) = \begin{cases} 1 & x = 0 \\ \dfrac{\sin \pi x}{\pi x} & x \neq 0 \end{cases}$$

normalized



Duality Property

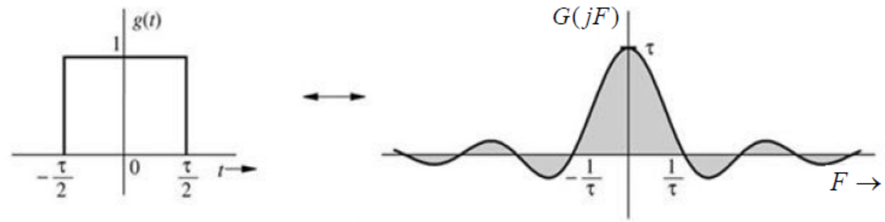
Duality Property *The shape of the signal in the time domain and the shape of the spectrum can be interchanged.*

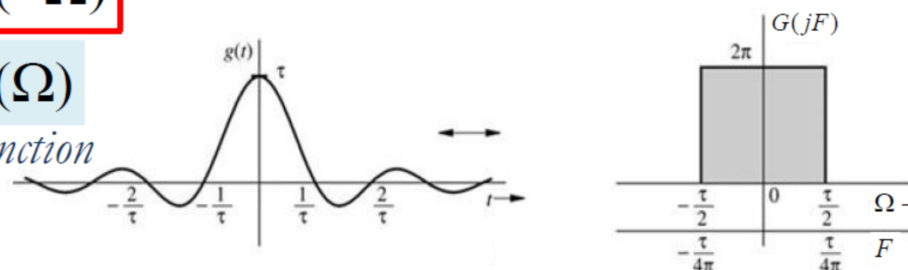
$$g(t) \xleftrightarrow{F.T.} G(j\Omega)$$

$$G(jt) \xleftrightarrow{F.T.} 2\pi g(-\Omega)$$

$$G(jt) \xleftrightarrow{F.T.} 2\pi g(\Omega)$$

an even function

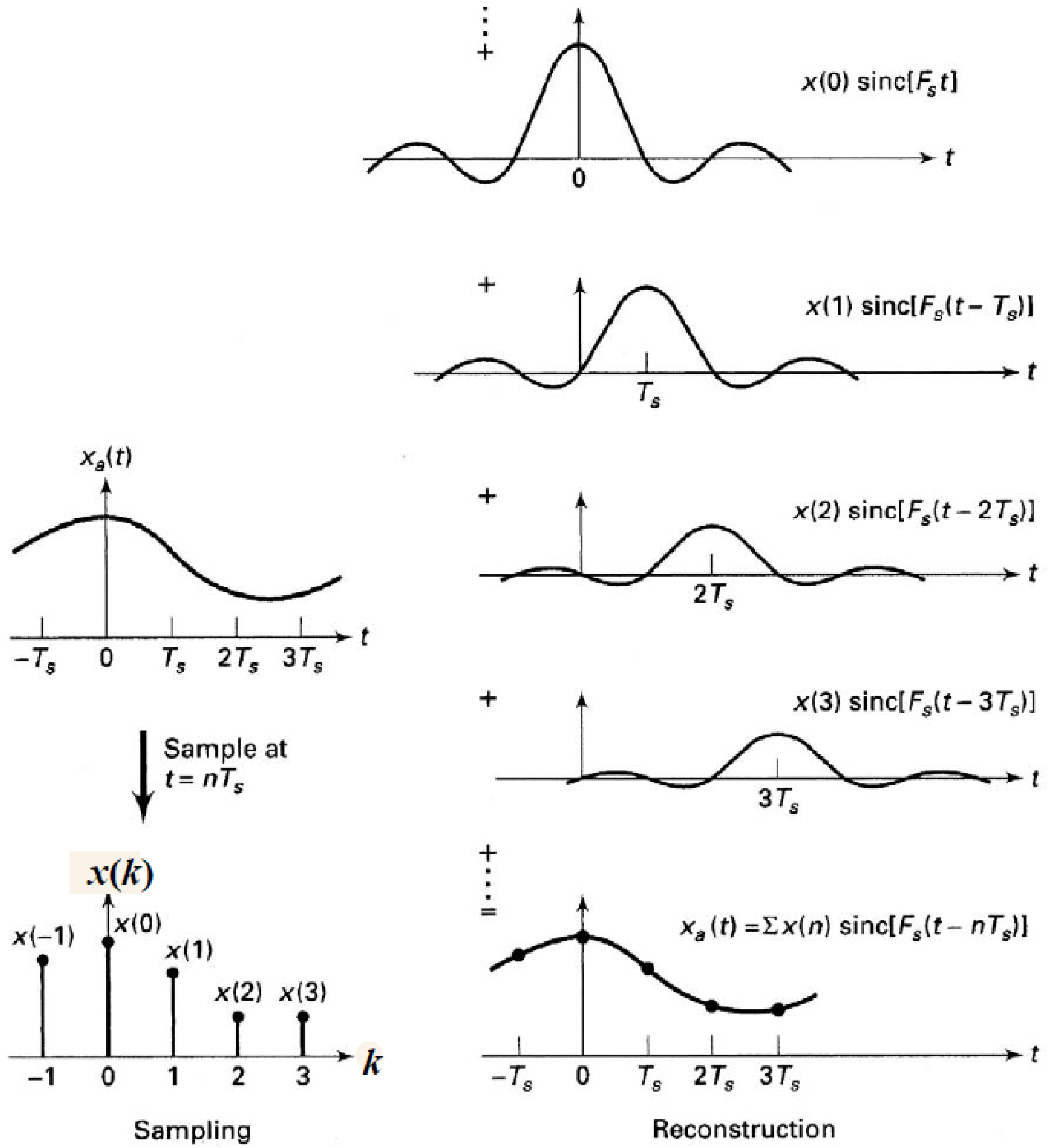




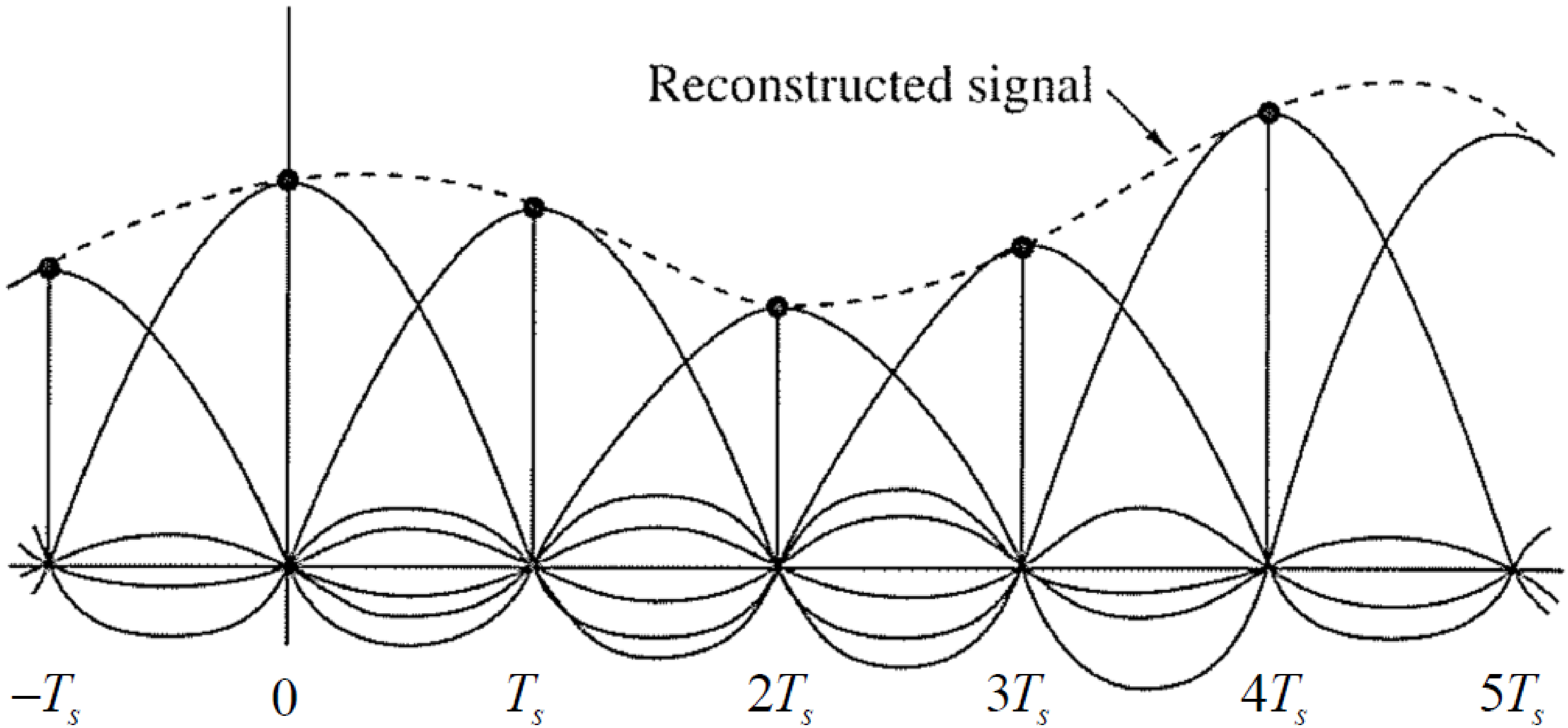
$$g(t) = \frac{1}{2\pi}\int_{-\tau/2}^{\tau/2} 2\pi e^{j\Omega t} d\Omega = \frac{1}{jt} e^{j\Omega t}\Big|_{-\tau/2}^{\tau/2} = \frac{1}{jt}\left[e^{j(\tau/2)t} - e^{-j(\tau/2)t}\right]$$

$$= \frac{2}{t}\left[\frac{e^{j(\tau/2)t} - e^{-j(\tau/2)t}}{2j}\right] = \frac{1}{t/2}\sin(\tau t/2) = \tau\frac{\sin(\tau t/2)}{\tau t/2}$$

Reconstruction Signal from Its Samples

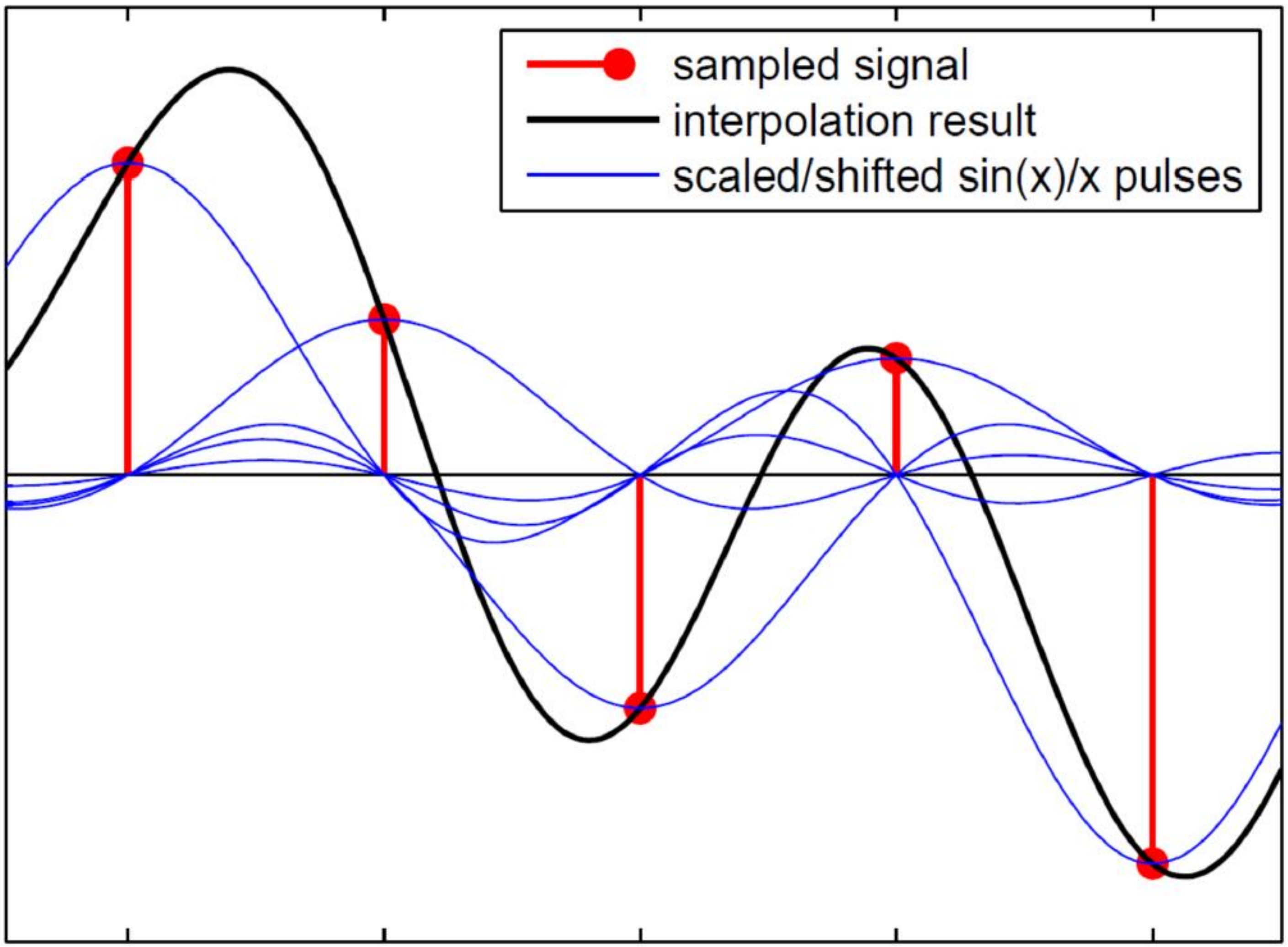


Reconstruction Signal from Its Samples



Reconstruction of a continuous-time signal using ideal interpolation

Reconstruction Signal from Its Samples



Practical D/A Converters

Practical D/A Converters

- ZOH (Zero-order Hold) Interpolation
- FOH (First-order Hold) Interpolation
- Cubic Spline Interpolation
- Langrange Interpolation

ZOH Interpolation

A given sample value is held for the sample interval until the next sample is received.

$$\hat{x}_a(t) = x(k), \quad kT_s \le k < (k+1)T_s$$

which can be obtained by filtering the impulse train through an interpolating filter of the form

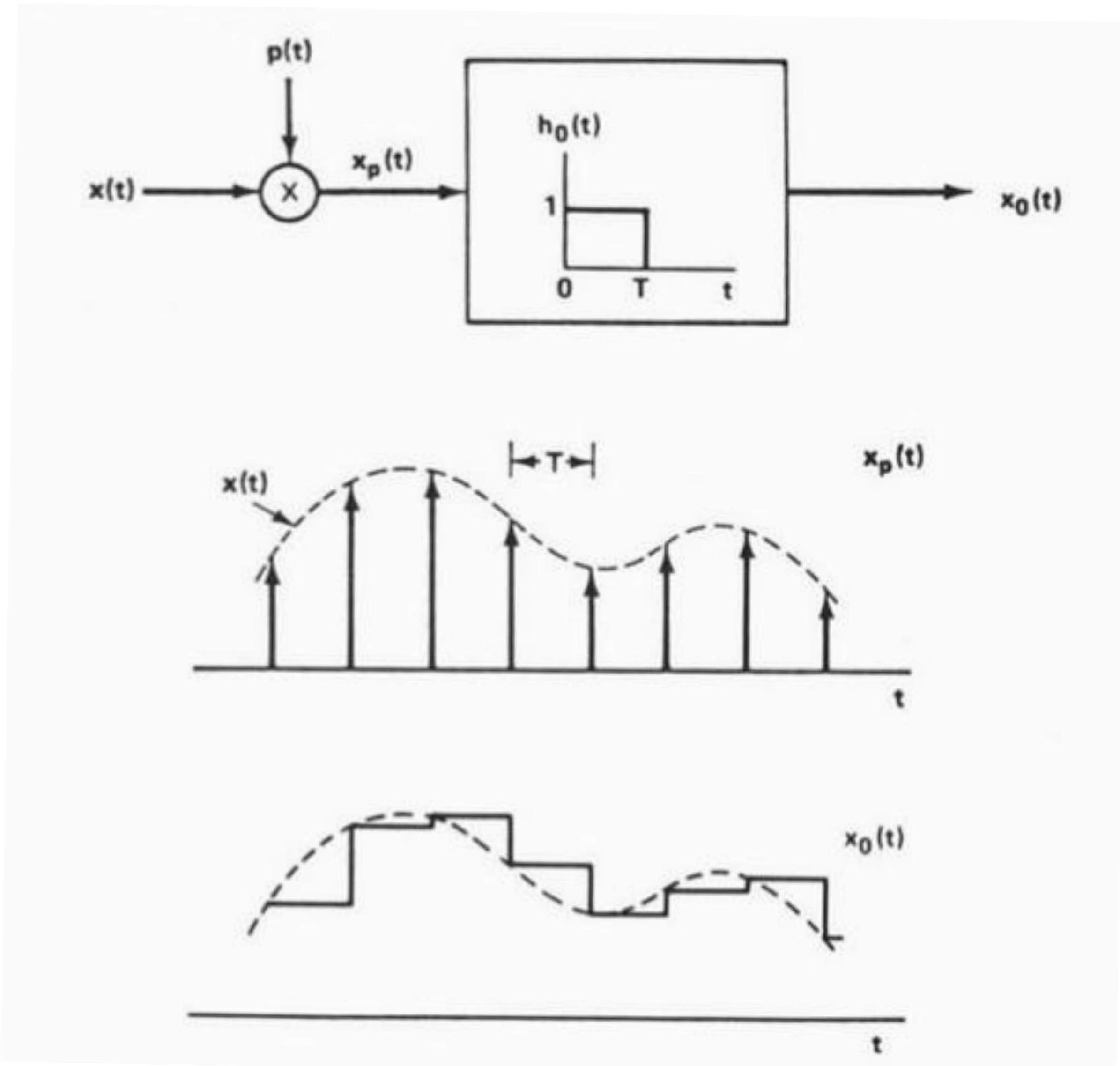
$$g_0(t) = \begin{cases} 1, & 0 \le t \le T_s \\ 0, & \text{otherwise} \end{cases}$$

A rectangular pulse

A piecewise-constant (staircase) waveform

$$x(k) \longrightarrow \boxed{\text{ZOH}} \longrightarrow \hat{x}_a(t) \longrightarrow \boxed{\text{Post-Filter}} \longrightarrow x_a(t)$$

ZOH Interpolation



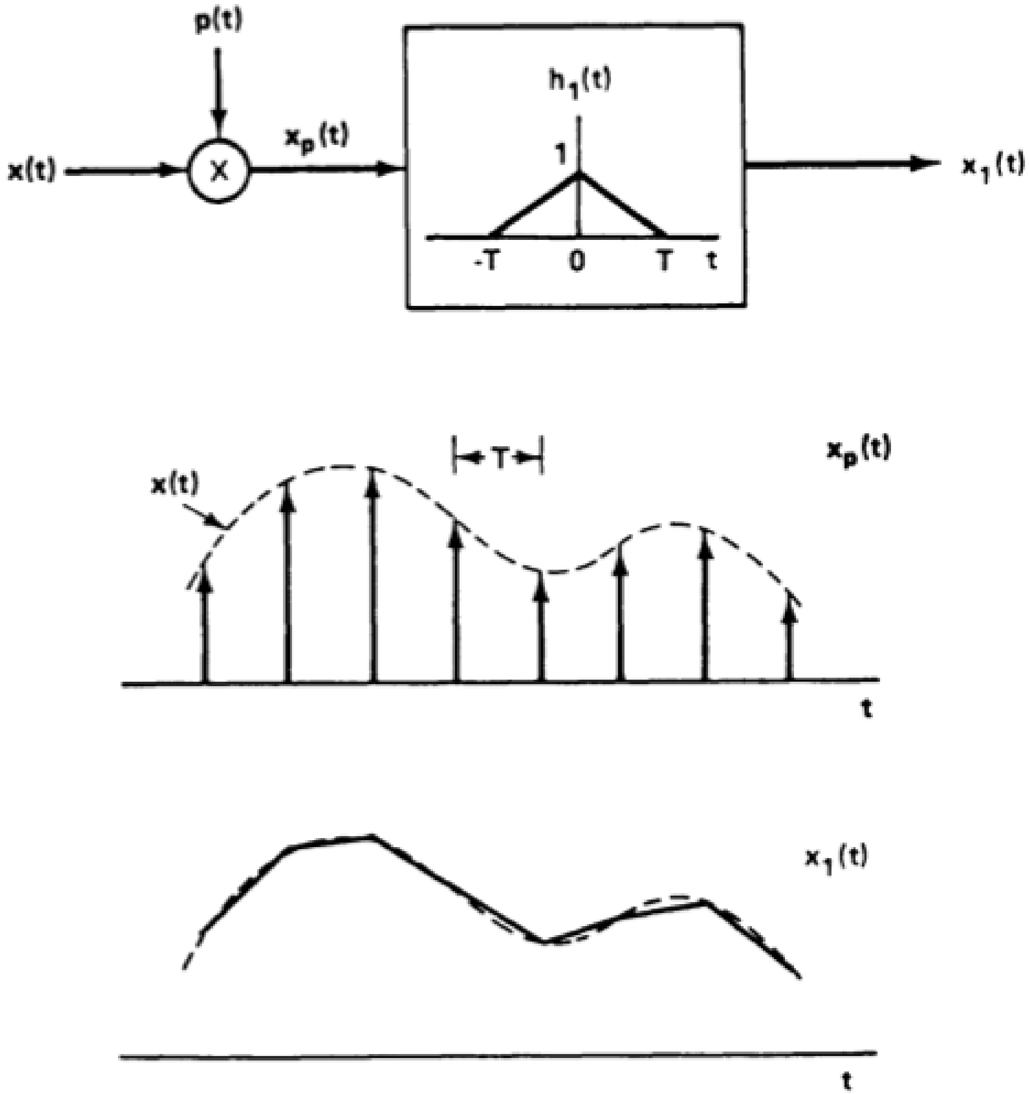
FOH Interpolation

In this case the adjacent samples are joined by straight lines. This can be obtained by filtering the impulse train through

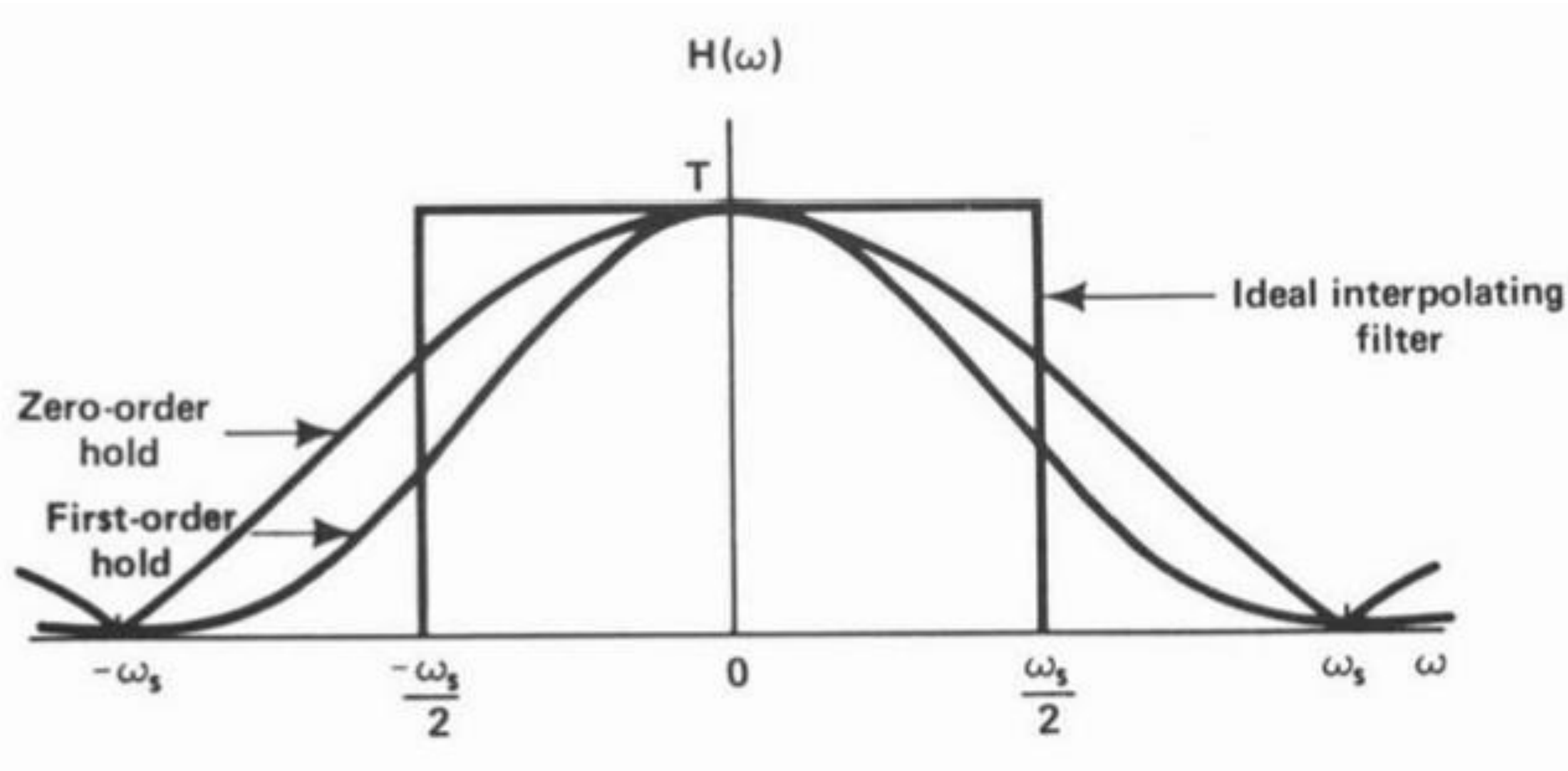
$$g_1(t) = \begin{cases} 1 + \dfrac{t}{T_s}, & 0 \le t \le T_s \\ 1 - \dfrac{t}{T_s}, & T_s \le t \le 2T_s \\ 0, & \text{otherwise} \end{cases}$$

- *Analog postfilter is required for accurate reconstruction.*
- *Can be extended to higher orders.*

FOH Interpolation



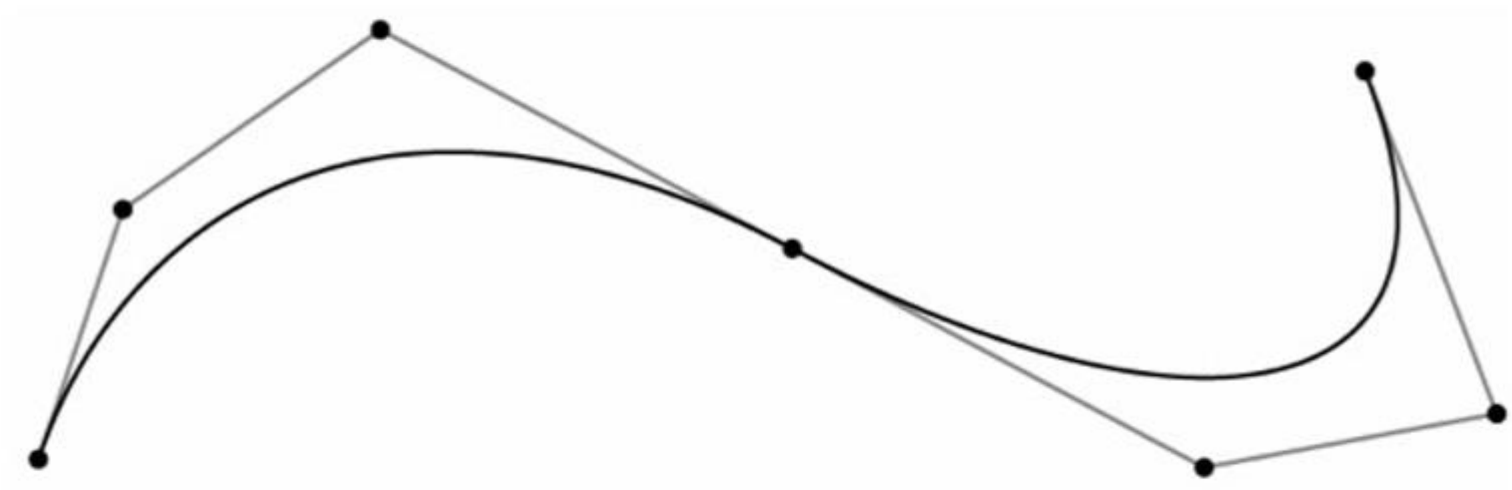
ZOH vs FOH Interpolation



Cubic Spline Interpolation

Uses spline interpolants for a smoother, but not necessarily more accurate, estimate of the analog signals between samples.

$$x_a(t) = \alpha_0(k) + \alpha_1(k)(t - kT_s) + \alpha_2(k)(t - kT_s)^2 + \alpha_3(k)(t - kT_s)^3, \quad kT_s \le k < (k+1)T_s$$



Example: Input Signal

Example $x_a(t) = e^{-1000|t|}$

$$X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t) e^{-j\Omega t} dt = \int_{-\infty}^{0} e^{1000t} e^{-j\Omega t} dt + \int_{0}^{\infty} e^{-1000t} e^{-j\Omega t} dt$$

$$= \frac{0.002}{1 + \left(\frac{\Omega}{1000} \right)^2}$$

Using the approximation $e^{-5} \approx 0$,

$x_a(t)$ *can be approximated by a finite-duration signal over* $-0.005 \le t \le 0.005$.

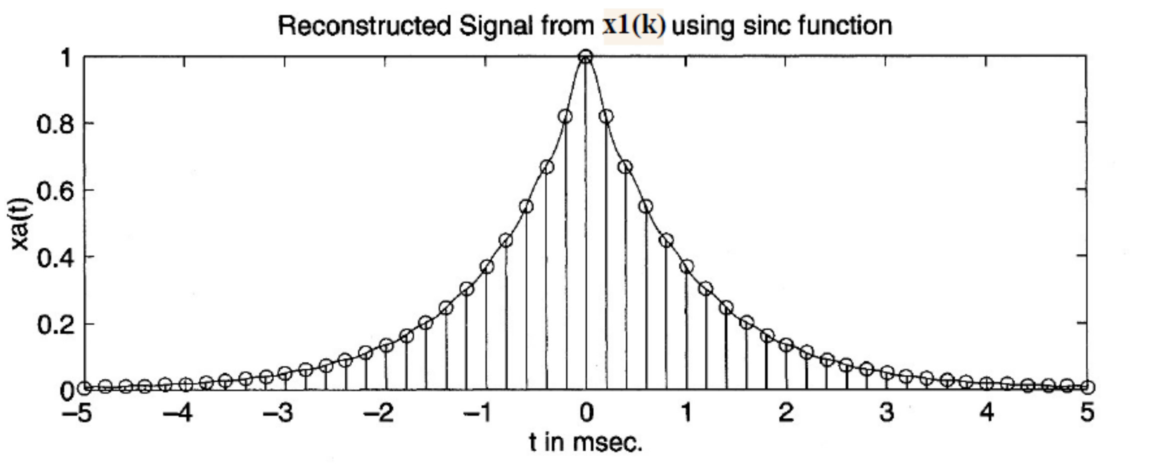
$X_a(j\Omega) \approx 0$ *for* $\Omega \ge 2\pi\underbrace{(2000)}_{\text{bandwidth}}$

$$\Delta t = 5 \times 10^{-5} \ll \frac{1}{2(2000)} = 25 \times 10^{-5}$$

Example: Reconstruction with Ideal Sinc

a. Sample $x_a(t)$ at $F_s = 5000$

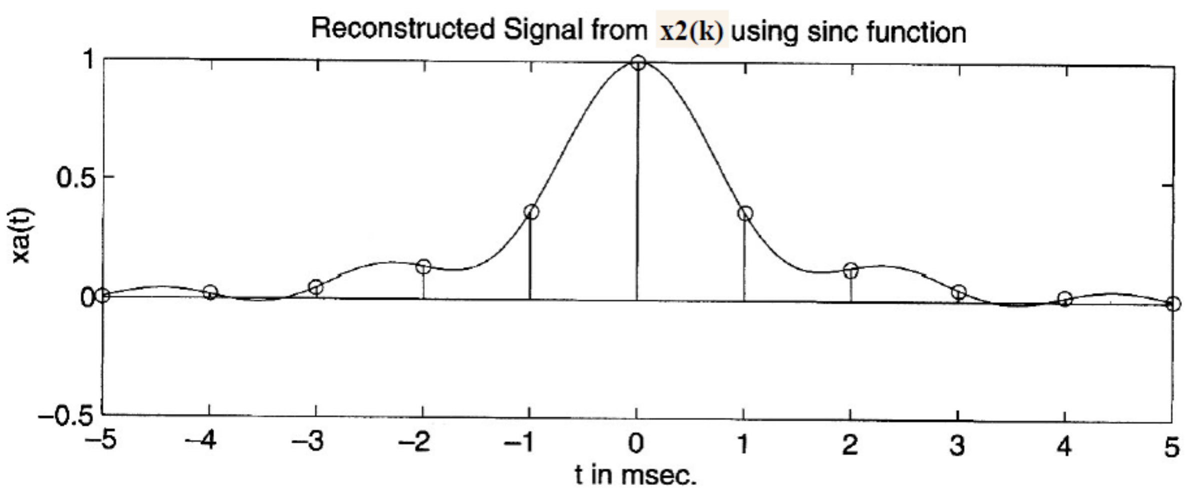
```
% Discrete-time Signal x1(k) Lab #8-a
>> Ts = 0.0002; k = -25:1:25; kTs = k*Ts; x = exp(-1000*abs(kTs));
% Analog Signal reconstruction
>> Dt = 0.00005; t = -0.005:Dt:0.005;
>> xa = x * sinc(Fs*(ones(length(k),1)*t-kTs'*ones(1,length(k))));
% check
>> error = max(abs(xa – exp(-1000*abs(t))))
error =
     0.0363
```



Example: Reconstruction with Ideal Sinc

b. Sample $x_a(t)$ at $F_s = 1000$

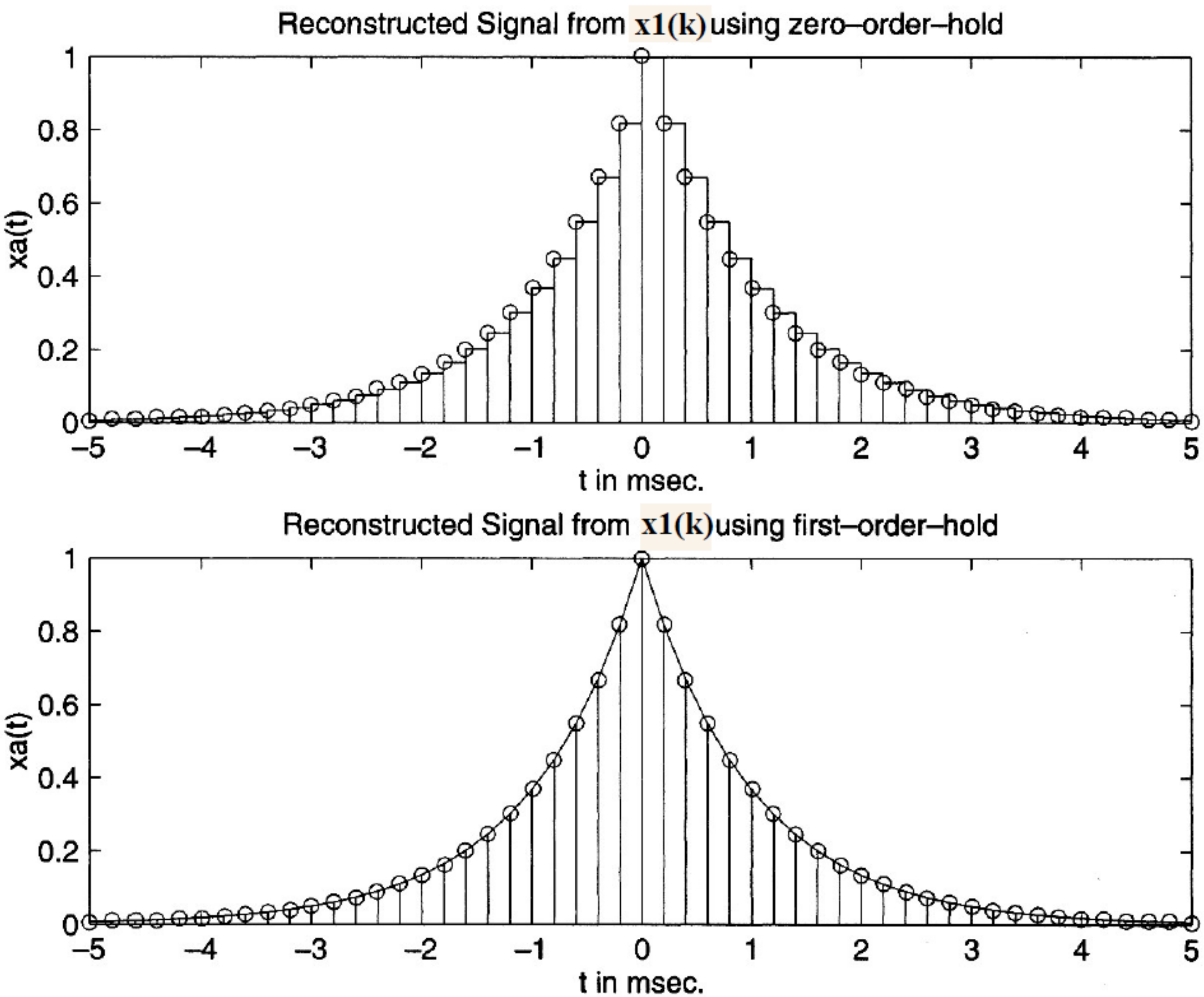
```
% Discrete-time Signal x1(k) Lab #8-b
>> Ts = 0.001; k = -5:1:5; kTs = k*Ts; x = exp(-1000*abs(kTs));
% Analog Signal reconstruction
>> Dt = 0.00005; t = -0.005:Dt:0.005;
>> xa = x * sinc(Fs*(ones(length(k),1)*t-kTs'*ones(1,length(k))));
% check
>> error = max(abs(xa – exp(-1000*abs(t))))
error =
     0.1852
```



Example: Reconstruction with ZOH & FOH

```
% Discrete-time Signal x1(k) : Ts = 0.0002
>> Ts = 0.0002; k = -25:1:25; kTs = k*Ts; x = exp(-1000*abs(kTs));
% Plots
>> subplot(2,1,1); stairs(kTs*1000,x);
>> xlabel('t in msec.'); ylabel('xa(t)')
>> title('Reconstructed Signal from x1(k) using zero-order-hold'); hold on
>> stem(k*Ts*1000,x); hold off
%
% Discrete-time Signal x2(k) : Ts = 0.001
>> Ts = 0.001; k = -5:1:5; kTs = k*Ts; x = exp(-1000*abs(kTs));
% Plots
>> subplot(2,1,2); plot(kTs*1000,x);
>> xlabel('t in msec.'); ylabel('xa(t)')
>> title('Reconstructed Signal from x2(k) using first-order-hold'); hold on
>> stem(k*Ts*1000,x); hold off
```

Example: Reconstruction with ZOH & FOH

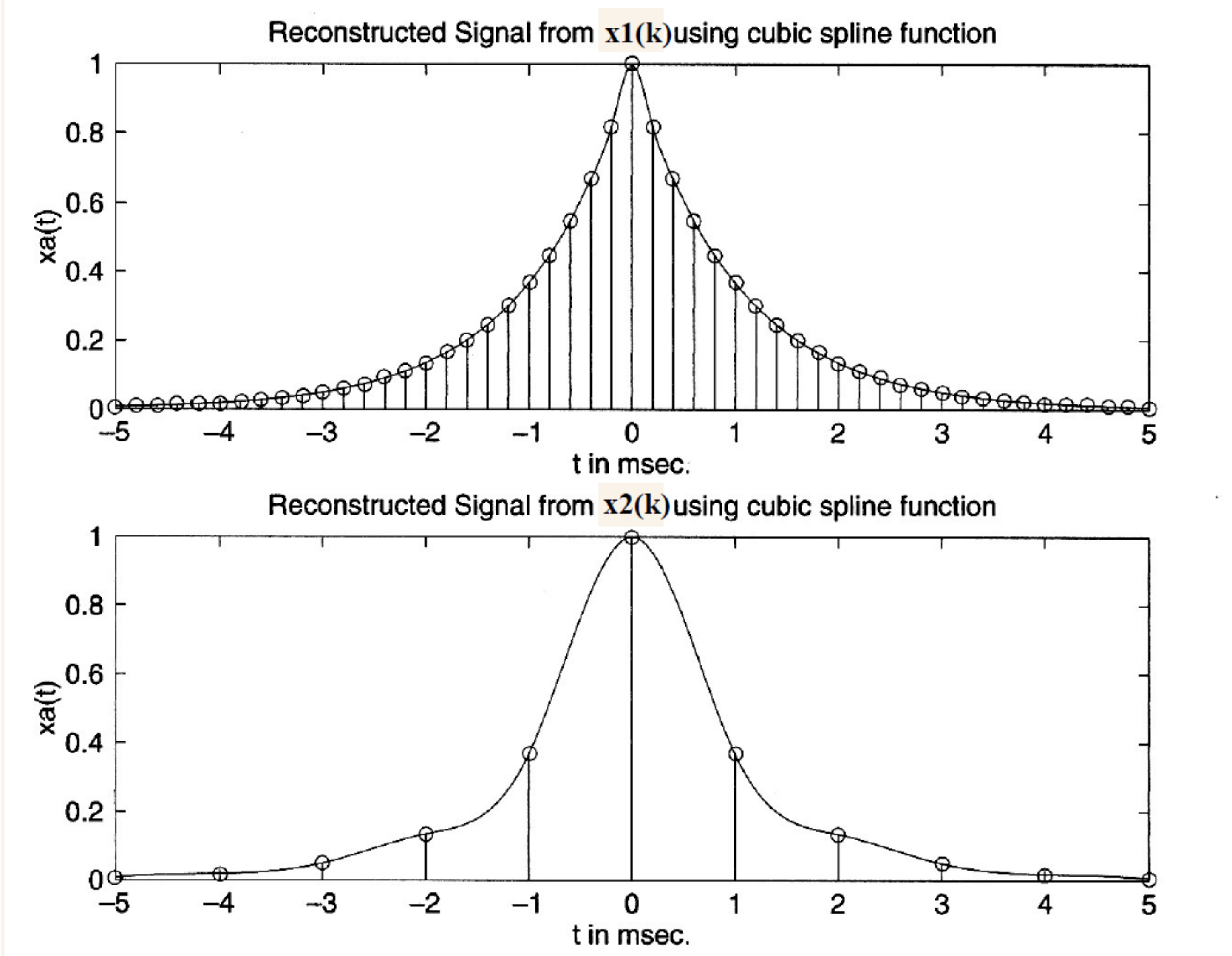


Example: Reconstruction with Cubic Spline

```
% Discrete-time Signal x1(k) : Ts = 0.0002
>> Ts = 0.0002; k = -25:1:25; kTs = k*Ts; x = exp(-1000*abs(kTs));
% Analog Signal reconstruction
>> Dt = 0.00005; t = -0.005:Dt:0.005; xa = spline(kTs,x,t);
% check
>> error = max(abs(xa-exp(-1000*abs(t))))
error = 0.0317

% Discrete-time Signal x2(k) : Ts = 0.001
>> Ts = 0.001; k = -5:1:5; kTs = k*Ts; x = exp(-1000*abs(kTs));
% Analog Signal reconstruction
>> Dt = 0.00005; t = -0.005:Dt:0.005; xa = spline(kTs,x,t);
% check
>> error = max(abs(xa - exp(-1000*abs(t))))
error = 0.1679
```

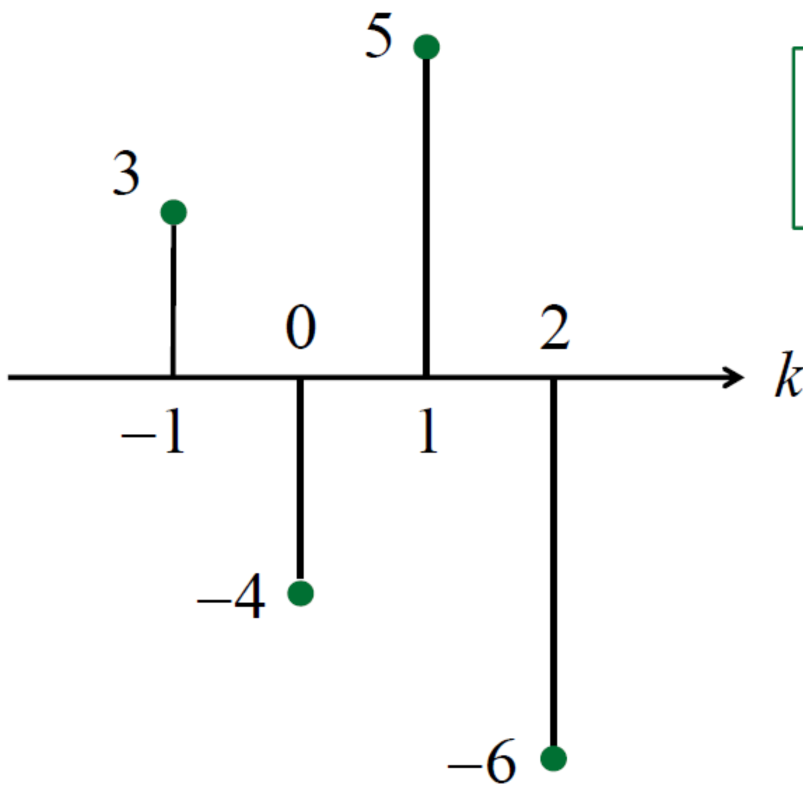
Example: Reconstruction with Cubic Spline



Lagrange Interpolation

$N\ points \rightarrow polynomial\ of\ degree\ N-1$

$$x_a(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \cdots + \alpha_{N-1} t^{N-1}$$



$x_a(-1) = 3, \quad x_a(0) = -4,$
$x_a(1) = 5, \quad x_a(2) = -6$

i	k_i	$x[k_i]$
0	−1	3
1	0	−4
2	1	5
3	2	−6

Lagrange Interpolation

Lagrange interpolation

$$x_a(t) = \sum_{j=0}^{N-1} x[k_j] L_{N-1,j}(t)$$

$$L_{N-1,j}(t) = \prod_{i=0, i \neq j}^{N-1} \frac{t - k_i}{k_j - k_i}$$

$$\begin{aligned} L_{3,0}(t) &= \frac{(t-k_1)(t-k_2)(t-k_3)}{(k_0-k_1)(k_0-k_2)(k_0-k_3)} \\ &= \frac{(t-0)(t-1)(t-2)}{(-1-0)(-1-1)(-1-2)} \\ &= \frac{t(t^2-3t+2)}{(-1)(-2)(-3)} \\ &= -\frac{1}{6}(t^3-3t^2+2t) \end{aligned}$$

$$\begin{aligned} L_{3,1}(t) &= \frac{(t-k_0)(t-k_2)(t-k_3)}{(k_1-k_0)(k_1-k_2)(k_1-k_3)} \\ &= \frac{(t+1)(t-1)(t-2)}{(0+1)(0-1)(0-2)} \\ &= \frac{(t^2-1)(t-2)}{(1)(-1)(-2)} \\ &= \frac{1}{2}(t^3-2t^2-t+2) \end{aligned}$$

Lagrange Interpolation

$$L_{3,2}(t) = \frac{(t-k_0)(t-k_1)(t-k_3)}{(k_2-k_0)(k_2-k_1)(k_2-k_3)}$$
$$= \frac{(t+1)(t-0)(t-2)}{(1+1)(1-0)(1-2)}$$
$$= \frac{t(t^2-t-2)}{(2)(1)(-1)}$$
$$= -\frac{1}{2}(t^3-t^2-2t)$$

$$L_{3,3}(t) = \frac{(t-k_0)(t-k_1)(t-k_2)}{(k_3-k_0)(k_1-k_1)(k_1-k_2)}$$
$$= \frac{(t+1)(t-0)(t-1)}{(2+1)(2-0)(2-1)}$$
$$= \frac{t(t^2-1)}{(3)(2)(1)}$$
$$= \frac{1}{6}(t^3-t)$$

$$x_a(t) = \sum_{j=0}^{N-1} x[k_j] L_{N-1,j}(t)$$
$$= x[k_0]L_{3,0}(t) + x[k_1]L_{3,1}(t) + x[k_2]L_{3,2}(t) + x[k_3]L_{3,3}(t)$$
$$= -6t^3 + 8t^2 + 7t - 4.$$

Summary

Summary

Continuous

$x_a(t)$

$$X_a(j\Omega)=\int_{-\infty}^{\infty} x_a(t)e^{-j\Omega t}dt$$

Fourier transform pair

$$x_a(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega)e^{j\Omega t}d\Omega$$

$X_a(j\Omega)$

$$X_a(j\Omega)=T_s X(e^{j\omega})$$

$$F_s > 2F_0$$

A/D

D/A

Reconstruction:

$$x_a(t)=\sum_{k=-\infty} x(k)\frac{\sin \pi[F_s(t-kT_s)]}{\pi[F_s(t-kT_s)]}$$

A/D

D/A

Sampling:

$$x(k)=x_a(kT_s)$$

$$X(e^{j\omega})=\frac{1}{T_s}\sum_{l=-\infty}^{\infty} X_a\left[j\left(\frac{\omega}{T_s}-\frac{2\pi}{T_s}l\right)\right]$$

$x(k)$

$$X(e^{j\omega})=\sum_{k=-\infty}^{\infty} x(k)e^{-j\omega k}$$

Fourier transform pair

$$x(k)=\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega k}d\omega$$

$X(e^{j\omega})$

Discrete

Continuous-time Sinusoidal Signal

continuous-time sinusoidal signals

$$x_a(t) = A\cos(\Omega t + \theta), \quad -\infty < t < \infty$$

A : the amplitude of the sinusoid

Ω : the frequency in radians per second

θ : the phase in radians

The frequency can be expressed in cycles per second or hertz (Hz)

$$F = \frac{\Omega}{2\pi}$$

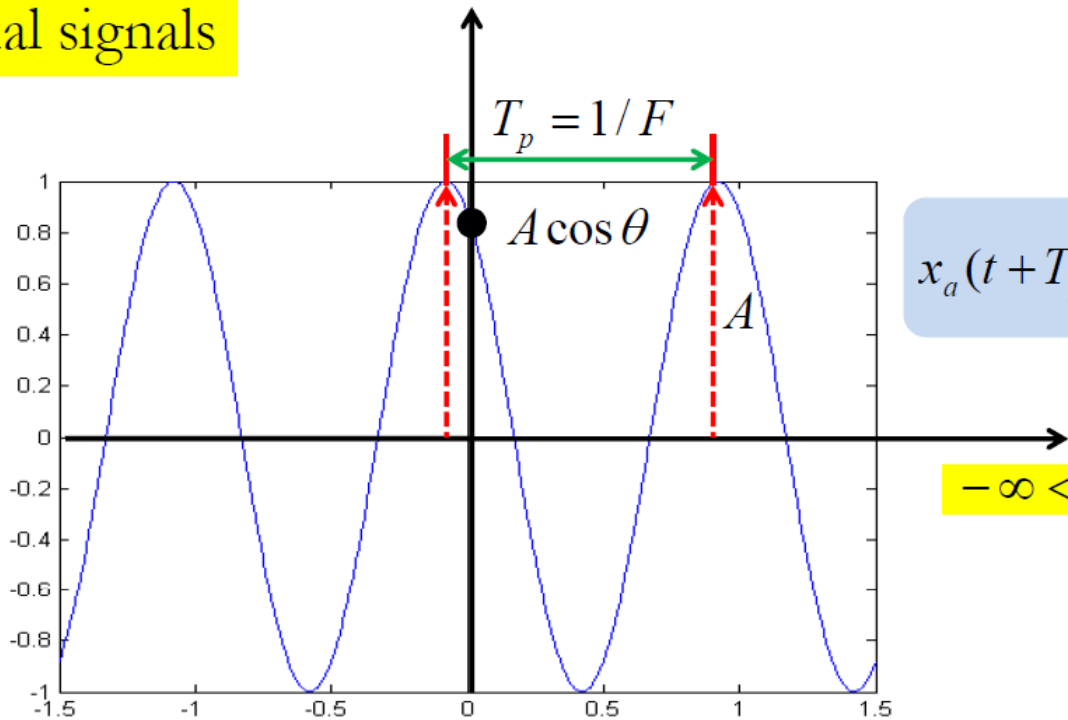
The period is defined as

$$T_p = \frac{1}{F}$$

Continuous-time Sinusoidal Signal

continuous-time sinusoidal signals

$$x_a(t) = A\cos(\Omega t + \theta)$$



$$x_a(t + T_p) = x_a(t)$$

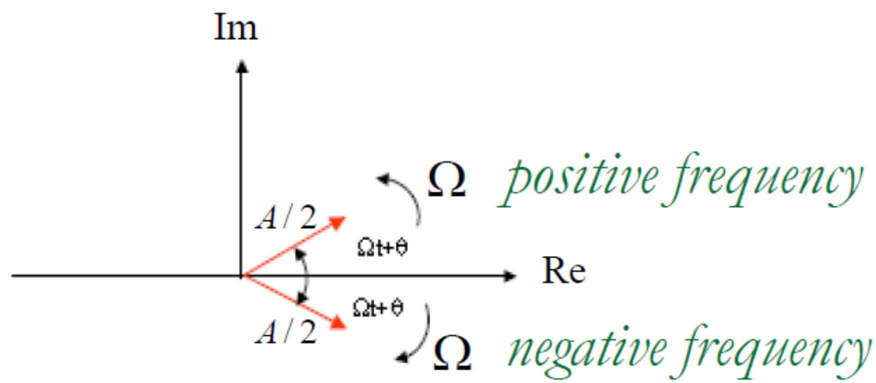
$$-\infty < t < \infty$$

Increasing the frequency F means decreasing the period.
- increasing the rate of oscillation
- more periods are included in a given time interval

Continuous-time Sinusoidal Signal

The analog signals can be expressed in terms of *complex exponentiation*.

$$x_a(t) = A\cos(\Omega t + \theta) = \frac{A}{2} e^{j(\Omega t+\theta)} + \frac{A}{2} e^{-j(\Omega t+\theta)}$$



A sinusoidal signal can be obtained by adding two equal-amplitude complex-conjugate exponential signals (sometimes called phasors).

Continuous-time Sinusoidal Signal

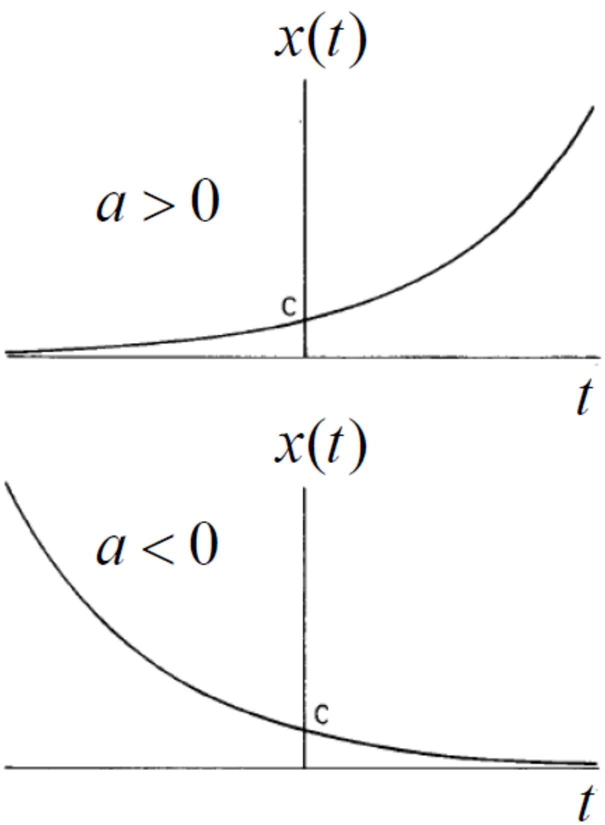
Relationship between a time shift and a change in phase

$$A\cos[\Omega(t+t_0)] = A\cos[\Omega t + \Omega t_0]$$
$$A\cos[\Omega(t+t_0)+\theta] = A\cos[\Omega t + \Omega t_0 + \theta]$$

Time Shift $\Leftrightarrow$ Phase Change

Continuous-time Real Exponential Signal

$$x(t) = Ce^{at}$$



$$Ce^{a(t+t_0)} = Ce^{at_0}e^{at}$$

Time Shift $\Leftrightarrow$ Scale Change

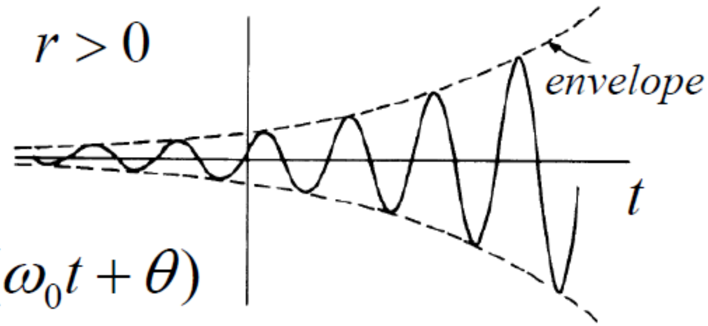
Continuous-time Complex Exponential Signal

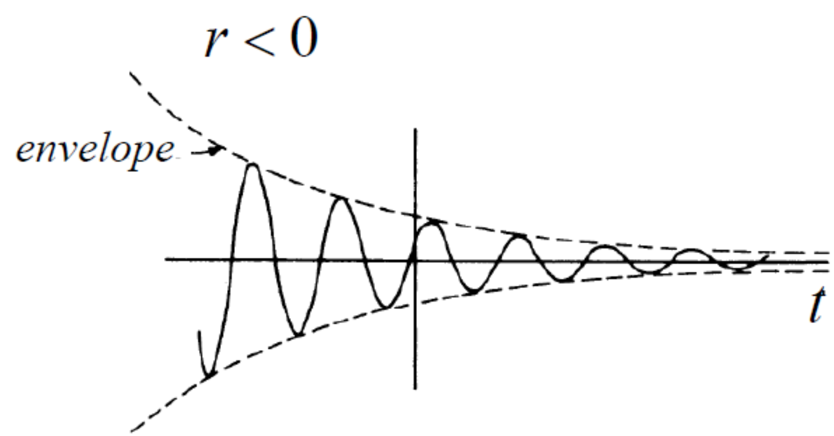
continuous-time
complex exponential signals

$$x(t) = Ce^{at}$$

$$C = |C|e^{j\theta}, \quad a = r + j\omega_0$$

$$x(t) = |C|e^{j\theta}e^{(r+j\omega_0)t}$$
$$= |C|e^{rt}e^{j(\omega_0 t+\theta)}$$
$$= |C|e^{rt}\cos(\omega_0 t + \theta) + j|C|e^{rt}\sin(\omega_0 t + \theta)$$





Discrete-time Siunusoidal Signal

$$x(k) = A\cos(\omega k + \theta), \quad -\infty < k < \infty$$

ω : the frequency in radians per sample

θ : the phase in radians

The frequency can be expressed in cycles per sample,

$$f = \frac{\omega}{2\pi}$$

and the signals is

$$x(k) = A\cos(2\pi f k + \theta), \quad -\infty < k < \infty$$

Discrete-time Sinusoidal Signal

Relationship between a time shift and a change in phase

$$A\cos[\omega(k+k_0)] = A\cos[\omega k + \omega k_0]$$

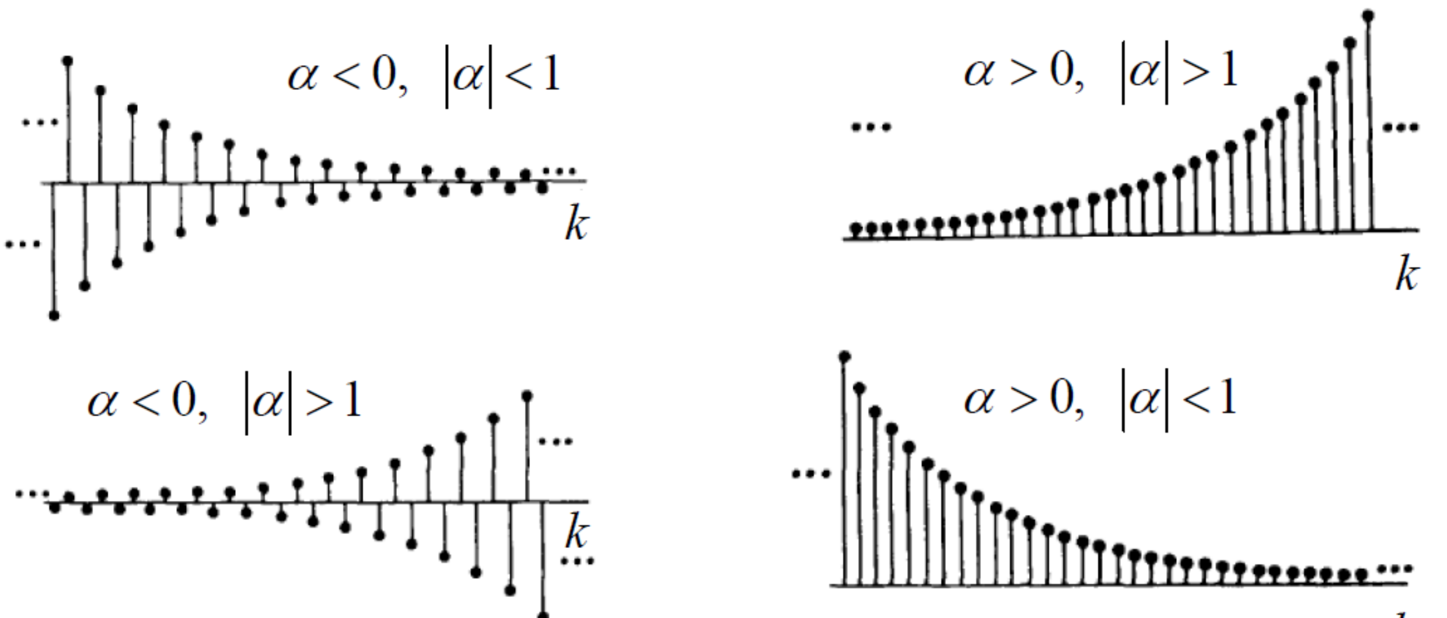
Time Shift $\Longrightarrow$ Phase Change

$$A\cos[\omega(k+k_0)] \overset{?}{=} A\cos[\omega k + \theta]$$

Time Shift $\overset{?}{\Longleftarrow}$ Phase Change

Discrete-time Real Exponential Signal

$$x(t) = C\alpha^k$$



Discrete-time Complex Exponential Signal

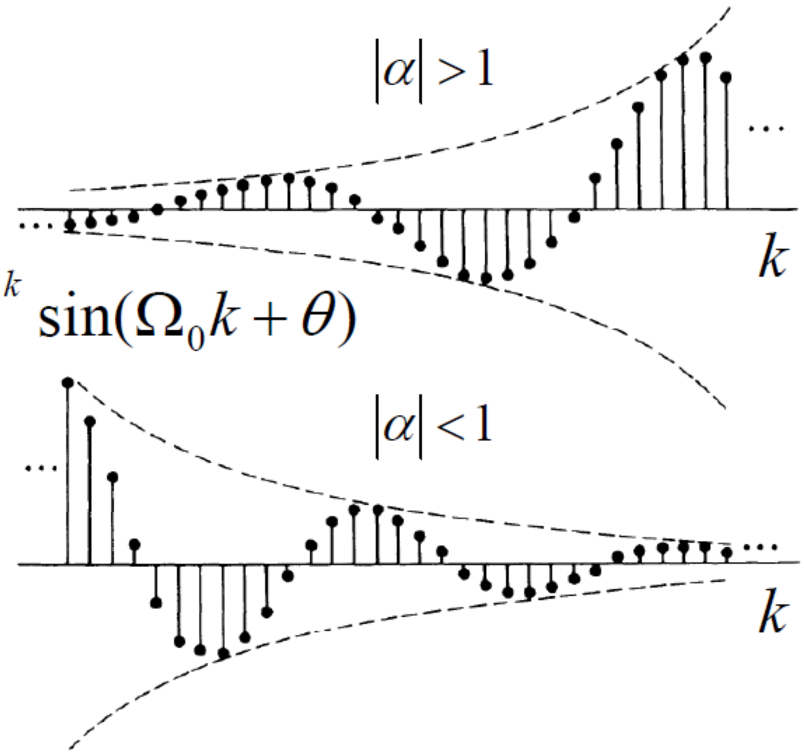
$$x(k) = C\alpha^k$$

$$C = |C|e^{j\theta}, \quad \alpha = |\alpha|e^{j\Omega_0}$$

$$x(t) = |C|e^{j\theta}\left(|\alpha|e^{j\Omega_0}\right)^k$$

$$= |C||\alpha|^k e^{j(\Omega_0 k+\theta)}$$

$$= |C||\alpha|^k \cos(\Omega_0 k+\theta) + j|C||\alpha|^k \sin(\Omega_0 k+\theta)$$



Continuous vs Discrete

Continuous time:

$$x_1(t) = A\cos\Omega_1 + \theta$$
$$x_2(t) = A\cos\Omega_2 + \theta$$

If $\Omega_2 \neq \Omega_1$

Then $x_2(t) \neq x_1(t)$

Discrete time:

$$x_1[k] = A\cos\left[\omega_1 k + \theta\right]$$
$$x_2[k] = A\cos\left[\omega_2 k + \theta\right]$$

If $\omega_2 = \omega_1 + 2m\pi$

Then $x_2[k] = x_1[k]$

For continuous-time sinusoids

$$-\infty < F < \infty$$

$$-\infty < \Omega < \infty$$

For discrete-time sinusoids

$$-\frac{1}{2} < f < \frac{1}{2}$$

$$-\pi < \omega < \pi$$

The frequency of the continuous-time sinusoid when sampled at a rate $F_s = 1/T_s$ must fall in the range

$$-\frac{1}{2T_s} = -\frac{F_s}{2} \leq F \leq \frac{F_s}{2} = \frac{1}{2T_s}$$

$$-\frac{\pi}{T_s} = -\pi F_s \leq \Omega \leq \pi F_s = \frac{\pi}{T_s}$$

$$F_{\max} = \frac{F_s}{2} = \frac{1}{2T_s}$$

$$\Omega_{\max} = \pi F_s = \frac{\pi}{T_s}$$

Range of Frequency

$\Omega = 2\pi F$

Ω : radians/sec

F : Hz

$-\infty < \Omega < \infty$

$-\infty < F < \infty$

$\omega = \Omega T_s, f = F / F_s$

$\Omega = \omega / T_s, F = f \cdot F_s$

$\omega = 2\pi f$

ω : radians/sample

f : cycles/sample

$-\pi \le \omega \le \pi$

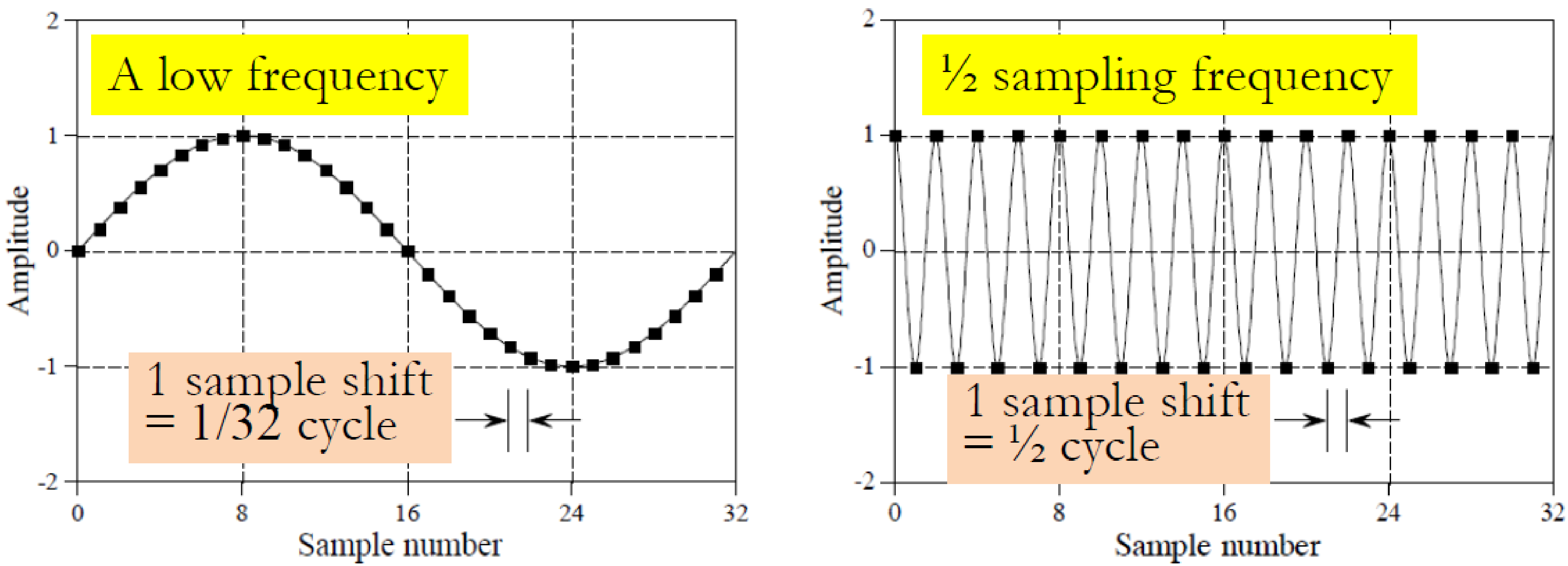
$-\frac{1}{2} \le f \le \frac{1}{2}$

$-\pi / T_s \le \Omega \le \pi / T_s$

$-F_s / 2 \le F \le F_s / 2$

Relationship between Sample & Phase

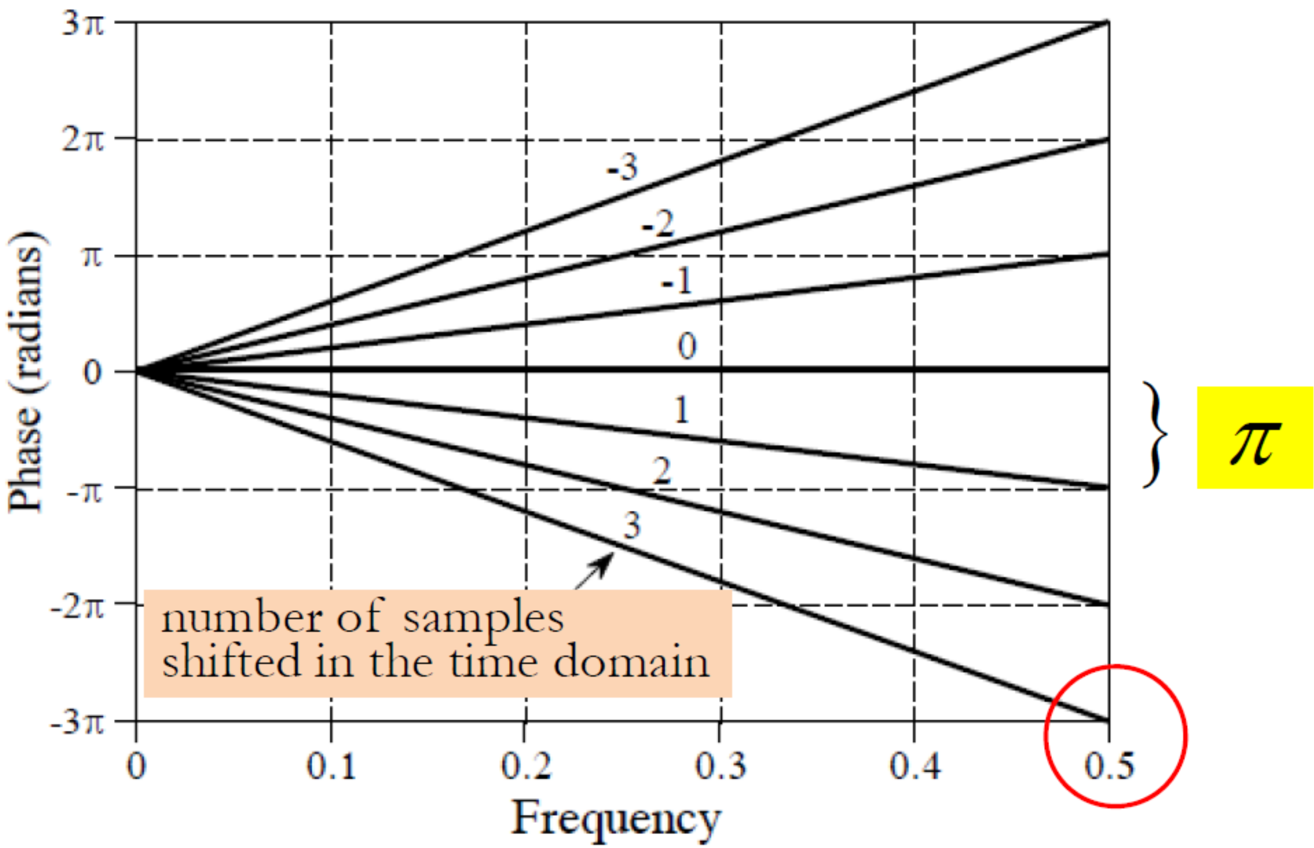
A shift in the waveform changes the phase more at *high frequencies* than at *low frequencies*.



Relationship between Sample & Phase

A waveform symmetrical around sample zero: *a zero phase*

How the phase of this signal changes when it is shifted left or right?



Sampling of Analog Signals

The discrete-time signal $x(k)$ is obtained by "*taking samples*" of the analog signal $x_a(t)$ every T_s seconds.

$$x(k) = x_a(kT_s), \quad -\infty \le k \le \infty$$

The time interval T_s between successive samples is called the *sampling period* or *sample interval*.

The *sampling rate* (samples per second) or the *sampling frequency* (hertz) is found as

$$F_s = \frac{1}{T_s}$$

The relationship between the time variables t of continuous-time signals and k of discrete-time signals is

$$t = kT_s = \frac{k}{F_s}$$

Sampling of Analog Signals

Consider an analog sinusoidal signal of the form

$$x_a(t) = A\cos(2\pi Ft + \theta)$$

Sampling frequency is

$$F_s = 1/T_s,$$

so that

$$\begin{aligned} x_a(kT_s) \equiv x(k) &= A\cos(2\pi FkT_s + \theta) \\ &= A\cos\left(\frac{2\pi kF}{F_s} + \theta\right) \\ &= A\cos(\omega k + \theta) \\ &= A\cos(2\pi fk + \theta) \end{aligned}$$

Relative or normalized frequency $f = \dfrac{F}{F_s}$

equivalently,

$$\omega = \frac{2\pi F}{F_s} = \frac{\Omega}{F_s} = \Omega T_s$$

Example:

Consider the analog signal

$$x_a(t) = 3\cos 100\pi t$$

Determine the minimum sampling rate required to avoid aliasing.

$$\Omega = 100\pi \rightarrow F = 50Hz$$

$$F_s = 2F = 100Hz$$

$$x(k) = x_a(kT_s) = 3\cos\frac{100\pi}{100}k = 3\cos\pi k$$

If $F_s = 200Hz$, what is the discrete-time signal obtained after sampling?

$$x(k) = 3\cos\frac{100\pi}{200}k = 3\cos\frac{\pi}{2}k$$

Homework #8.2 (1 pt.): Due Jan 24

Consider the analog signal

$$x_a(t) = 3\cos 50\pi t + 10\sin Z00\pi t - \cos 100\pi t$$

Z=A+1

(a) What are the frequencies present in the signal?
(b) What is the Nyquist rate for this signal?
(c) What is the discrete signal obtained after sampling at the Nyquist rate?

Use Your ID: sGFEDCBA

Thank you

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