Persistent Segment trees

Special class

Surya Kiran Adury • Jan 13, 2021

Surya Kiran Adury

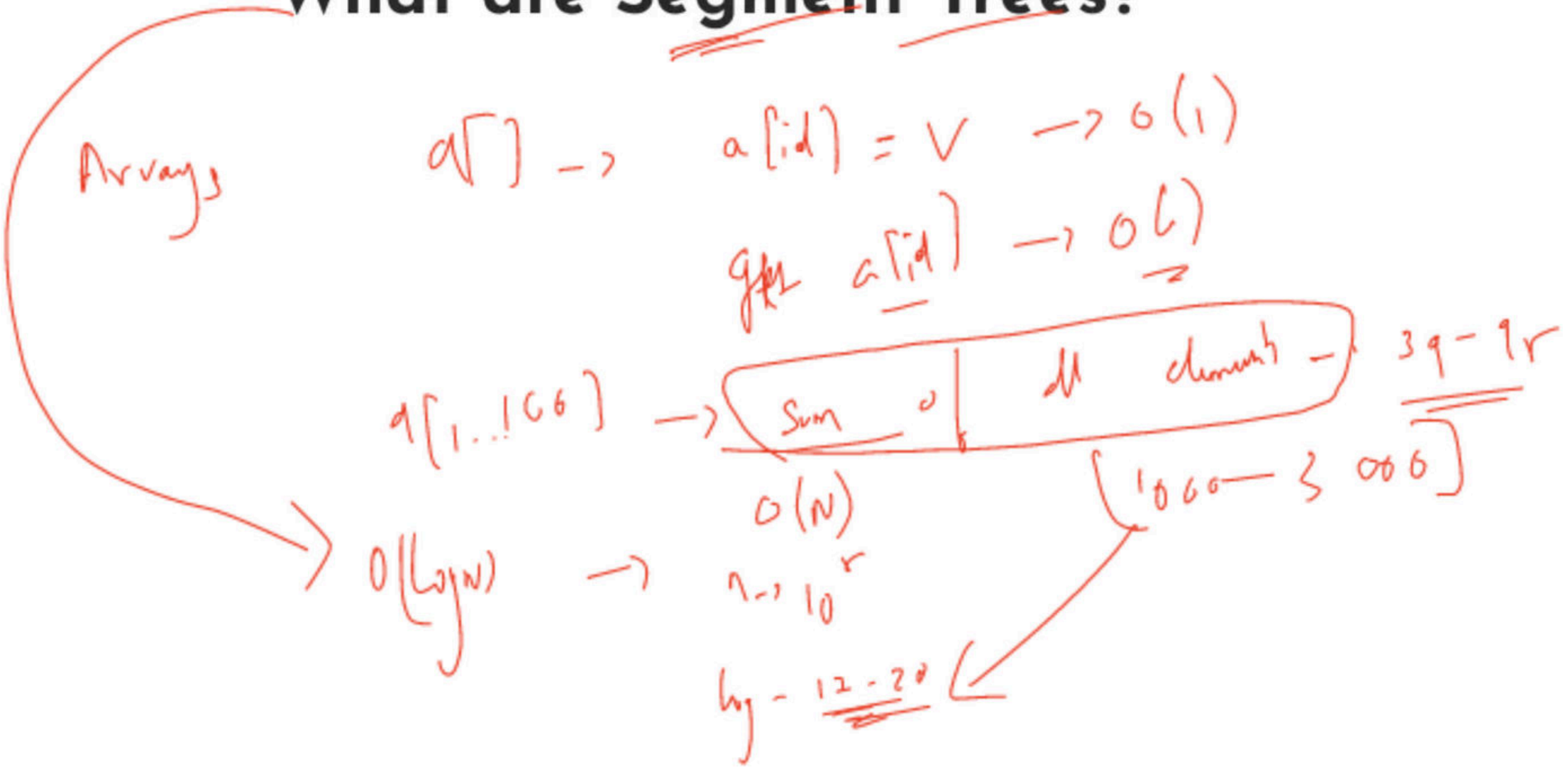
- ICPC WF - 2014, 15
- Google MTV 2017-20
- Google LON 2015-17
- B.Tech in ECE from IIT Roorkee


Question
What are we doing?

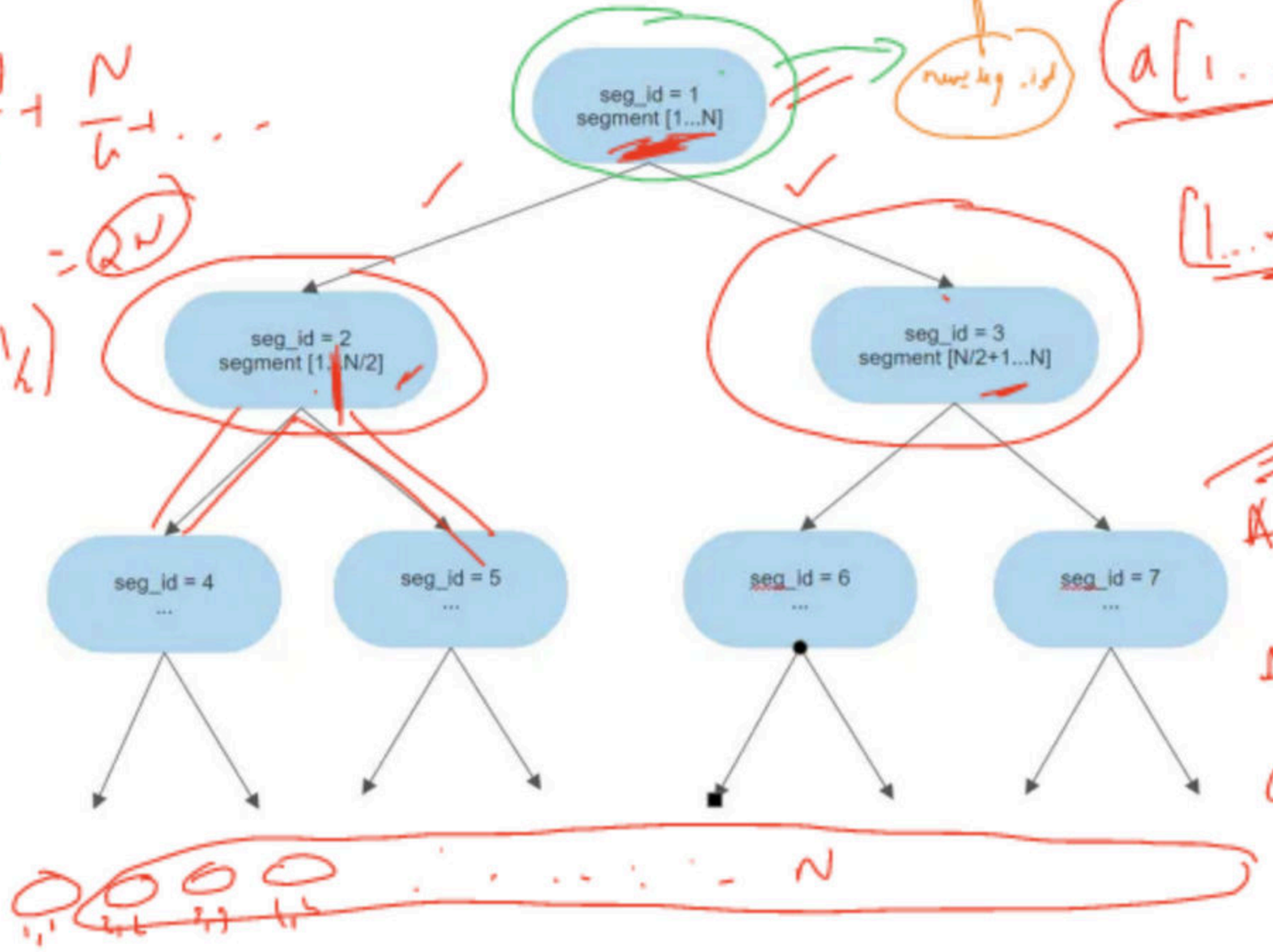
Objective

1. Class 1
   a. Fenwick Trees
2. Class 2
   a. Segment Trees
3. **Class 3**
   a. **Persistent Segment Trees**
4. Class 4
   a. Competitive programming problems

What are Segment Trees?

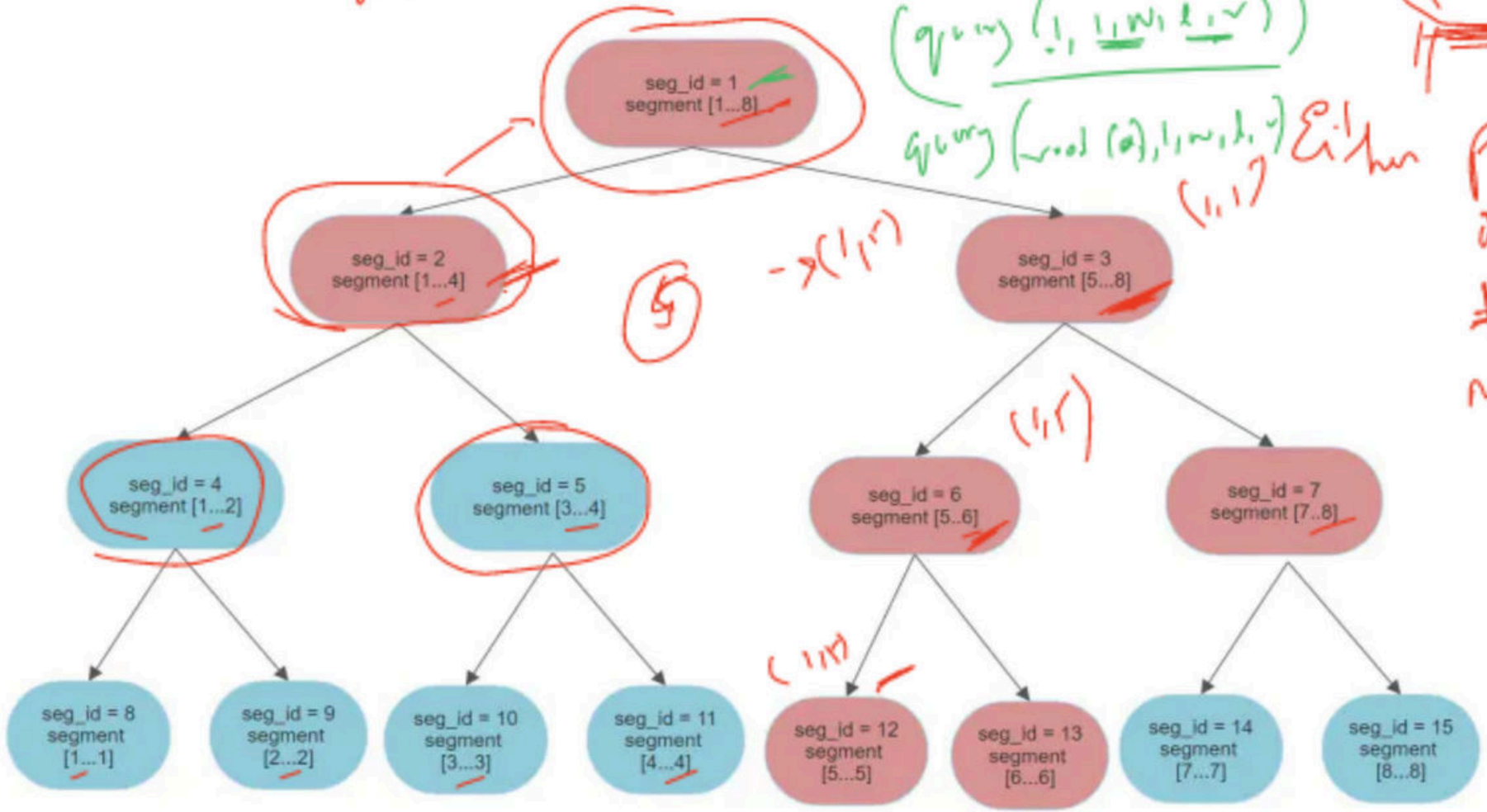


What are Segment Trees?

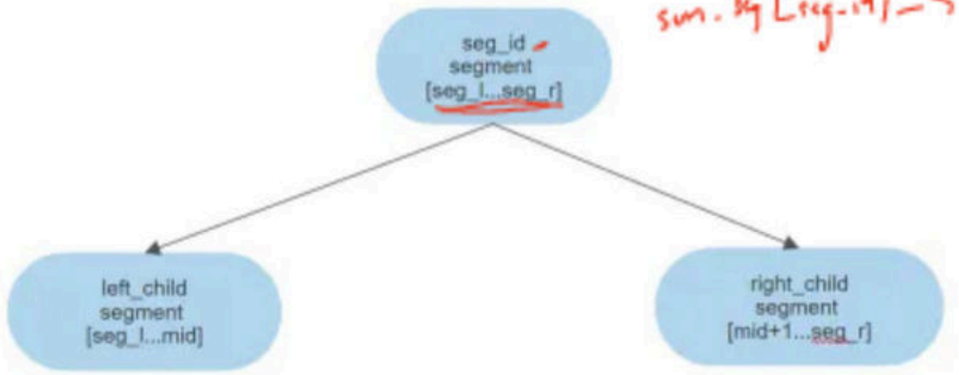


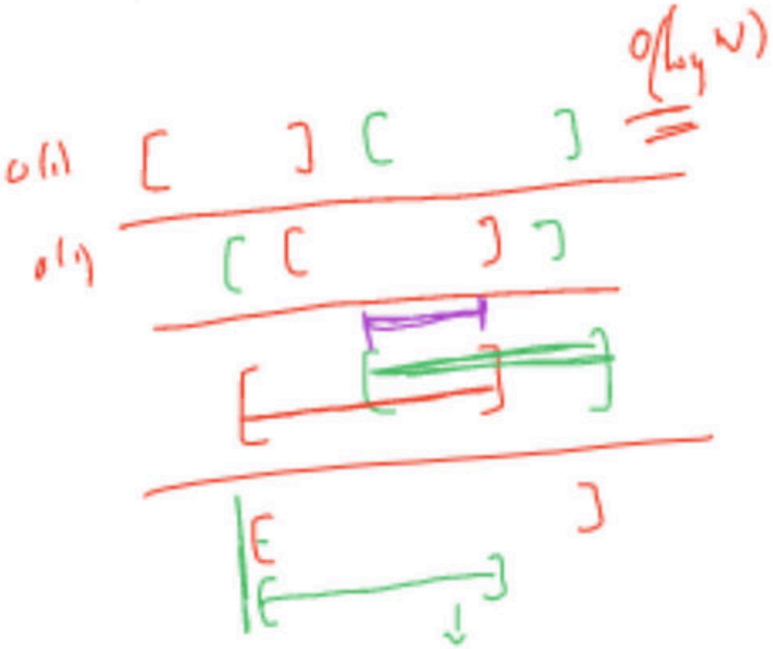
Query a Range

Query a Range



Query a Range





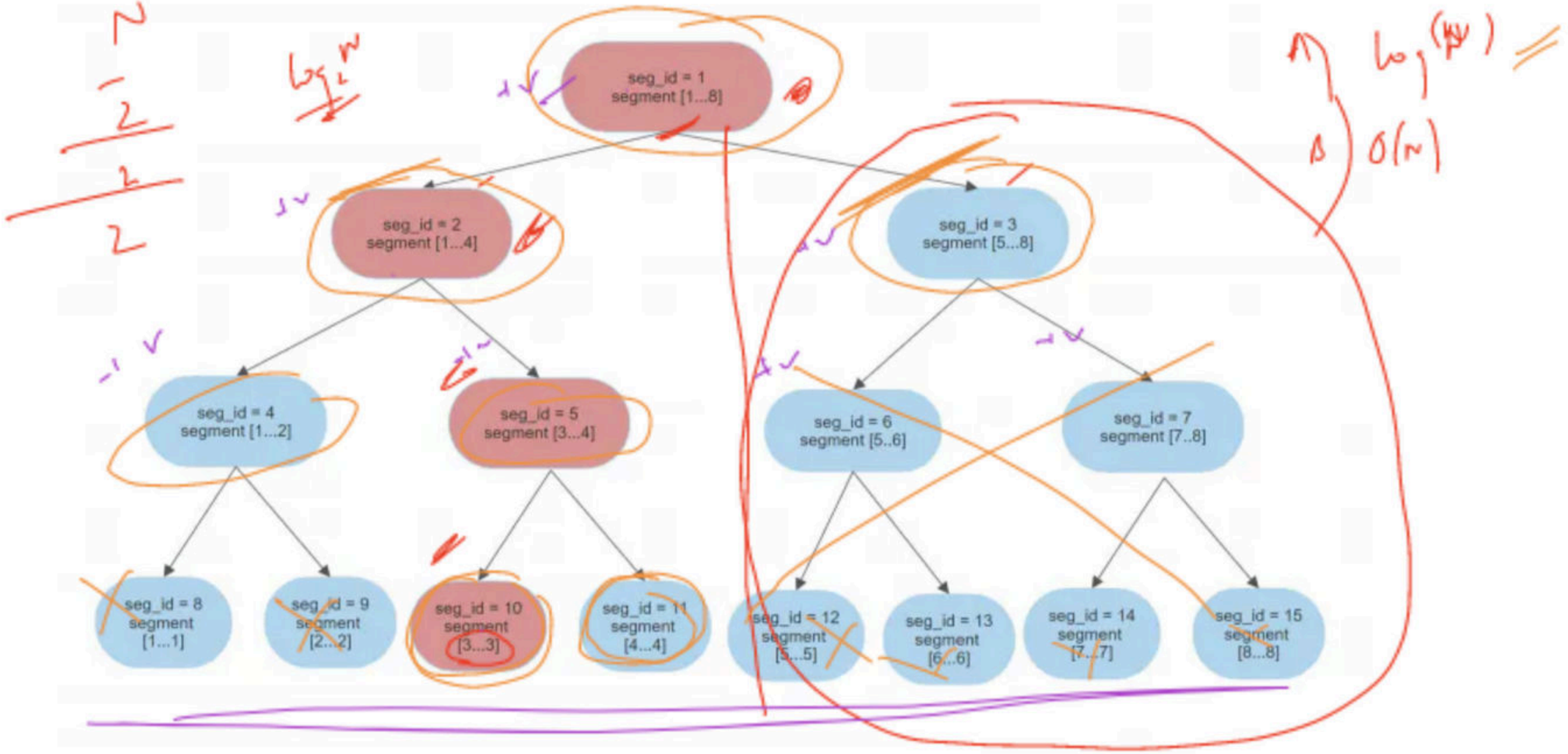
```
int sum_seg[1...S]; // sum_seg[seg_id] stores sum of the elements of
                    // the subarray a[seg_l, seg_r].
int a[N];           // array over which segment tree is built.

// Returns the sum of elements of overlap of the subarray a[query_l, query_r]
// and the subarray a[seg_l, seg_r].
int query(int seg_id, int seg_l, int seg_r, int query_l, int query_r) {
    if (query_l > seg_r || query_r < seg_l) {
        return 0; // No overlap between current seg_id and query.
    }
    if (quer_l <= seg_l && seg_r <= query_r) {
        return sum_seg[seg_id]; // Full overlap of current seg_id and query.
    }
    // We are left with partial overlap.
    // So we pass on to both children and return sum of values returned by them.
    int left_child = seg_id * 2;
    int left_l = seg_l;
    int left_ r = (seg_l + seg_r) / 2;

    int right_child = seg_id * 2 + 1;
    int right_l = left_r + 1;
    int right_r = seg_r;

    return query(left_child, left_l, left_r, query_l, query_r)
         + query(right_child, right_l, right_r, query_l, query_r);
}
```

Update an Index

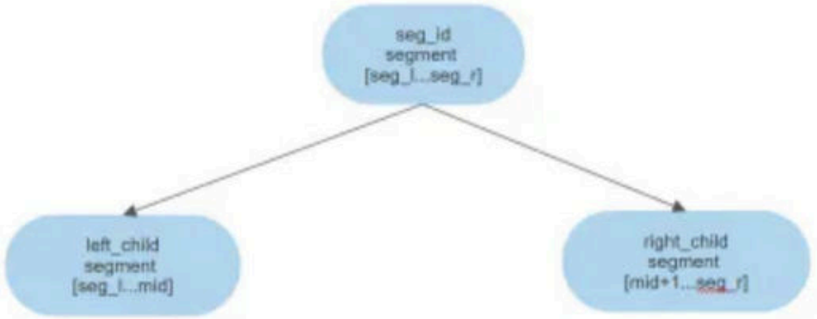
$A[1 \ldots N]$

$Seg[1 \ldots N]$

Update an Index



Update an Index




```
// Index update, adding "v" to a[id].
// Updates all segments to imitate adding "v" to a[id].
void index_update(int seg_id, int seg_l, int seg_r, int id, int v) {
    if (seg_l > id || seg_r < id) return; // No overlap.
    sum_seg[seg_id] += v;
    if (seg_l == seg_r) {
        return; // We are at leaf node.
    }
    int left_child = seg_id * 2;
    int left_l = seg_l;
    int left_ r = (seg_l + seg_r) / 2;

    int right_child = seg_id * 2 + 1;
    int right_l = left_r + 1;
    int right_r = seg_r;
    update(left_child, left_l, left_r, id, v);
    update(right_child, right_l, right_r, id, v);
}
```

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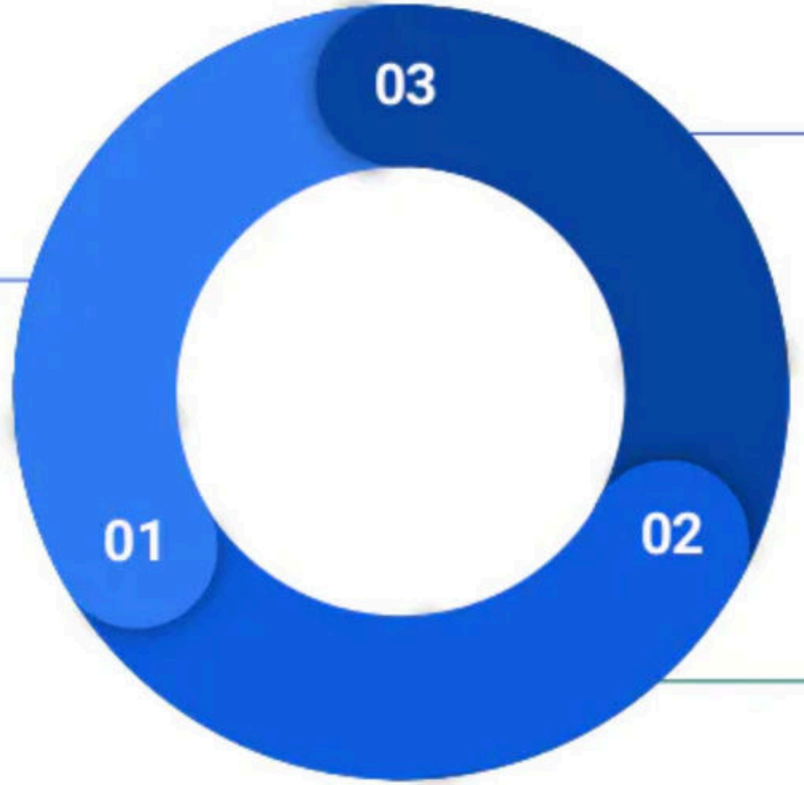

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Week 18-19	Number Theory and Interview Questions
Week 20-22	Recursion and DP Concepts and Handpicked Problem Solving
Week 23-27	Discrete Mathematics in C++- Concept to Problems
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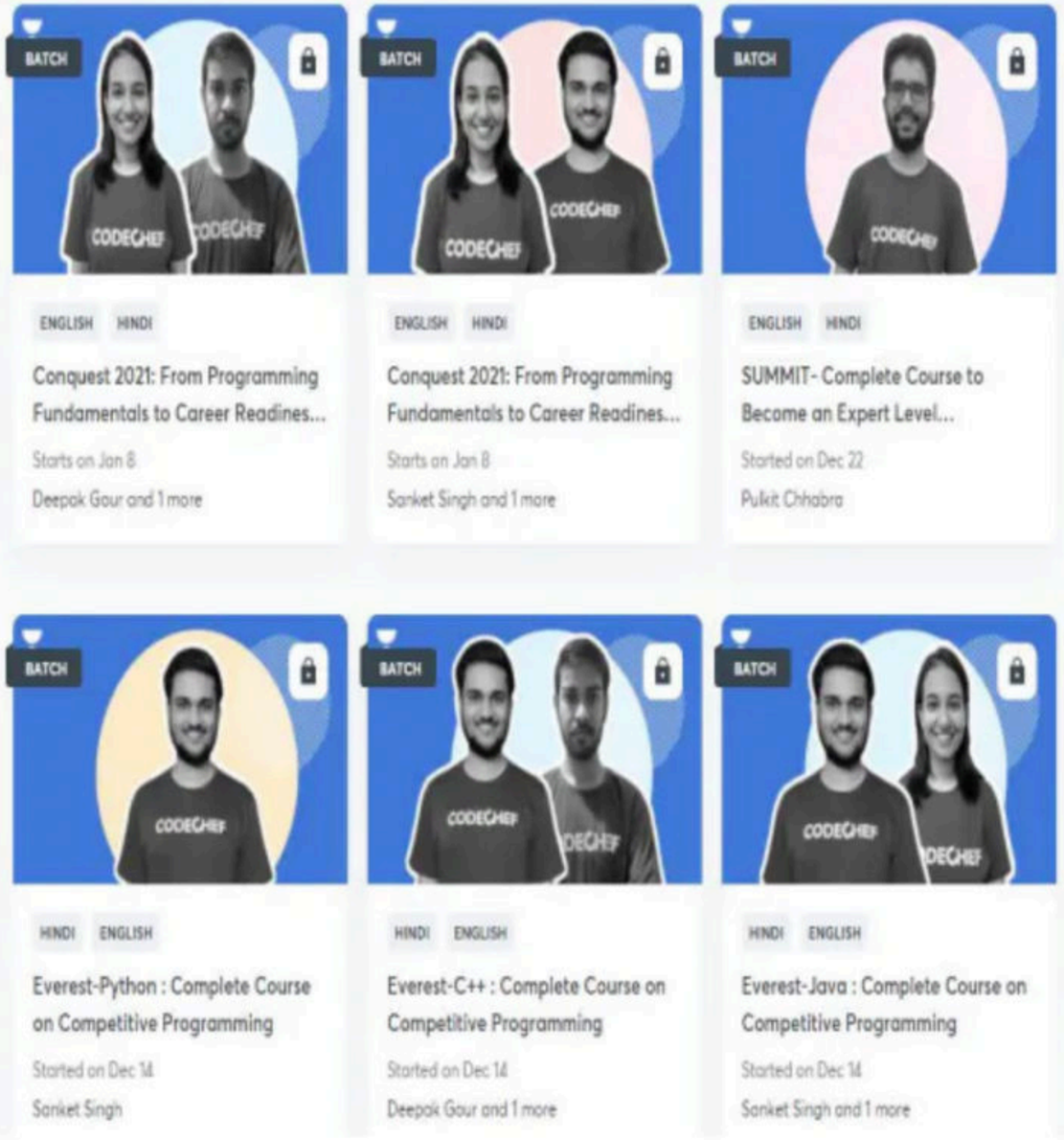

Educators

Tanuj Khattar

ACM ICPC World Finalist - 2017, 2018. Indian IOI Team Trainer 2016-2018. Worked @ Google, Facebook, HFT. Quantum Computing Enthusiast.

Sanket Singh

Software Development Engineer @ LinkedIn | Former SDE @ Interviewbit | Google Summer of Code 2019 @ Harvard University | Former Intern @ISRO

Pulkit Chhabra

Codeforces: 2246 | Codechef: 2416 | Former SDE Intern @CodeNation | Former Intern @HackerRank

Riya Bansal

Software Engineer at Flipkart | Former SDE and Instructor @ InterviewBit | Google Women TechMakers Scholar 2018

Triveni Mahatha

Qualified ICPC 2016 World Final. Won multiple Codechef Long Challenges (India). ICPC Onsite Regionals' Problem setter and Judge. IIT Kanpur.

Deepak Gour

ICPC World Finalist 2020 | Former Instructor @InterviewBit | Software Engineer at AppDynamics

Educators

Himanshu Singh

World Finalist ICPC 2020, Winner Techgig Code Gladiators 2020, Winner TCC '19, 2020 CSE Graduate from IIT BHU, Works at Nutanix

Nishchay Manwani

Hey I am Nishchay Manwani from CSE, IIT Guwahati and I'm a Seven star on Codechef and International Grandmaster on Codeforces.

Murugappan S

Software engineer at Google. Have won many programming contests. Max Rating of 2192 in codeforces and 2201 in codechef.

Vivek Chauhan

Codechef: 7 stars (2612) India Rank 6, Codeforces: MASTER (2279), Won Codechef Long Challenges(India), TCO20 Southern Asia Runner up

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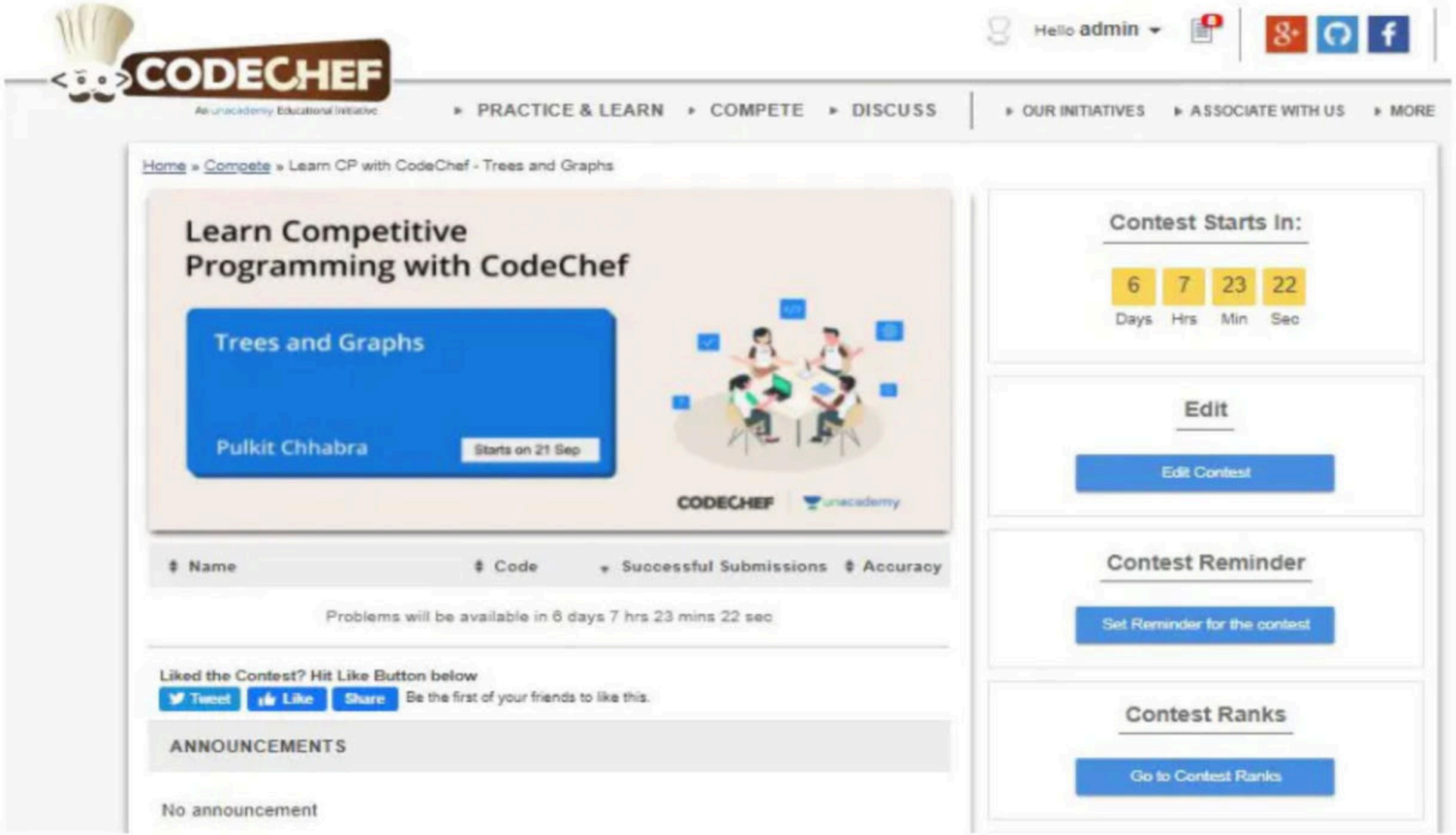

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Segment trees → Query → Range

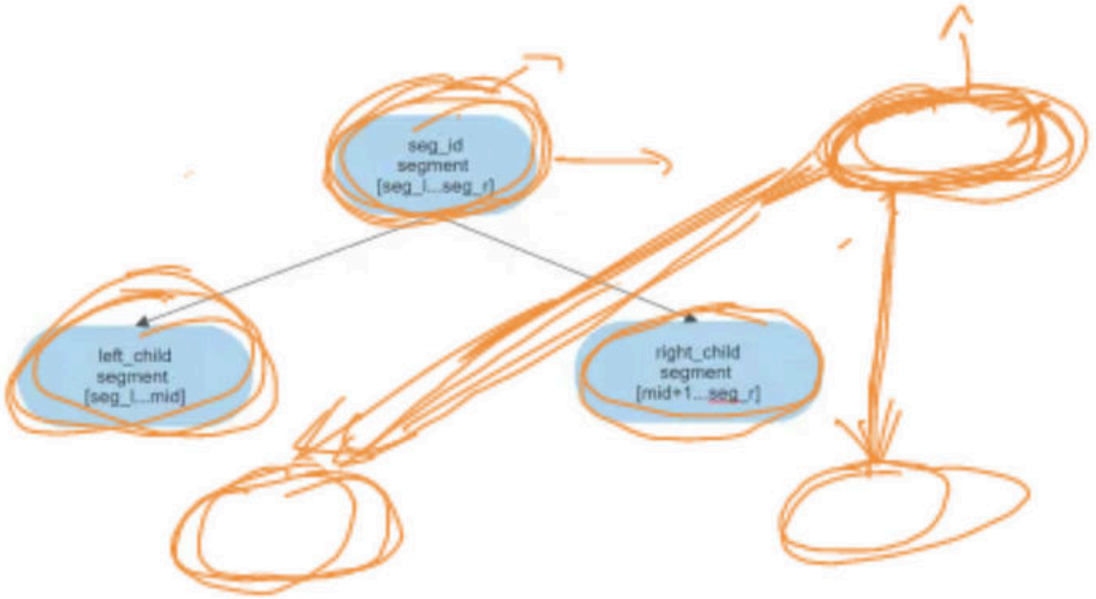
Updating → Index

What are persistent Segment Trees?

Range Minimum Query ⟶ (query_l, query_r)

Maximum

Updating an index



A) O(N)

B) O(log N)

```
int left_child[1...S];
int right_child[1...S];
int sum_seg[1...S];
int seg_cnt;

int copy(int seg_id) {
    seg_cnt++;
    sum_seg[seg_cnt] = sum_seg[seg_id];
    left_child[seg_cnt] = left_child[seg_id];
    right_child[seg_cnt] = right_child[seg_id];
    return seg_cnt;
}

int update(int seg_id, int seg_l, int seg_r, int id, int v) {
    int new_seg_id = copy(seg_id);

    if (id < seg_l || seg_r < id) return new_seg_id; // No overlap.

    seg_sum[new_seg_id] += v;
    if (seg_l == seg_r) return new_seg_id; // We are at leaf node.

    int left_l = seg_l;
    int left_r = (seg_l + seg_r) / 2;

    int right_l = left_r + 1;
    int right_r = seg_r;

    left_child[new_seg_id] = update(left_child[seg_id], left_l, left_r, id, v);
    right_child[new_seg_id] = update(right_child[seg_id], right_l, right_r, id, v);

    return new_seg_id;
}
```

Problem 1

A[1 . . . N]

Day 1

Day 2

Day 3

Day d

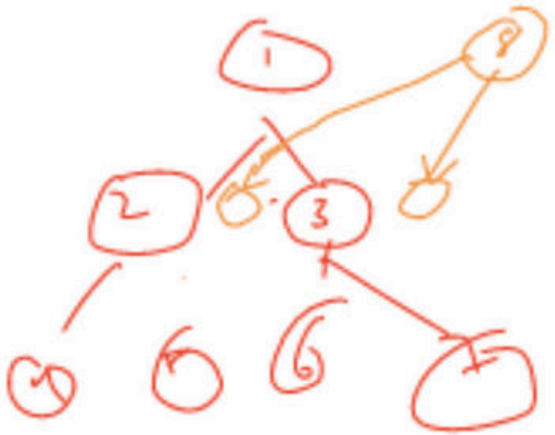
Day current day

operation → id, +v

(l, v), d (ok current day)

int root_id[1 . . . D]

root_id[1, 8, x . . .]



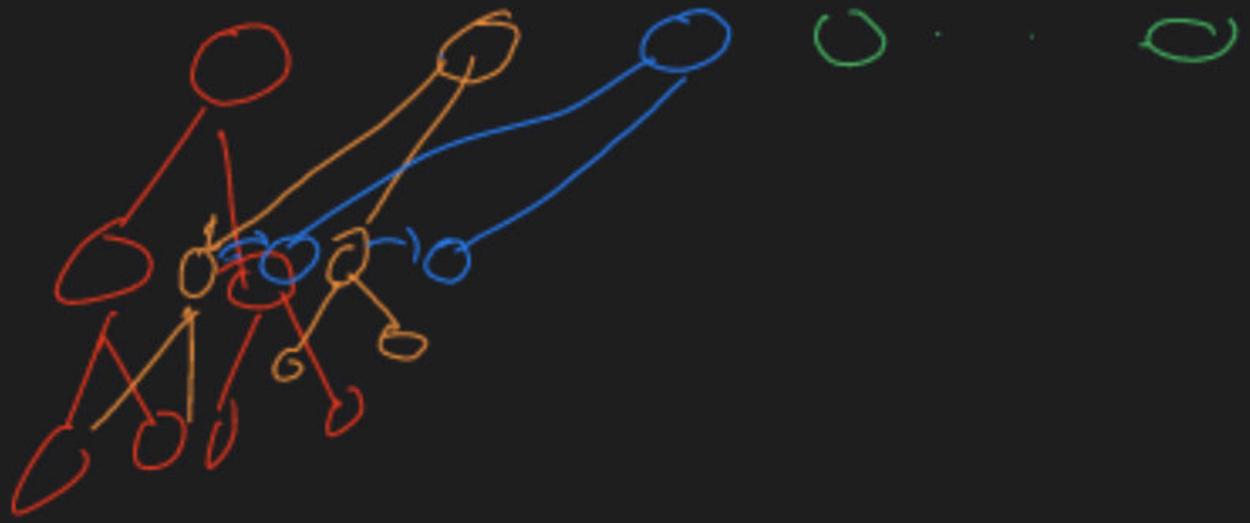
query (root_id[d], l, r, 1, N)

root_id(current day) = update (root_id[prev], 1, N, id, v)

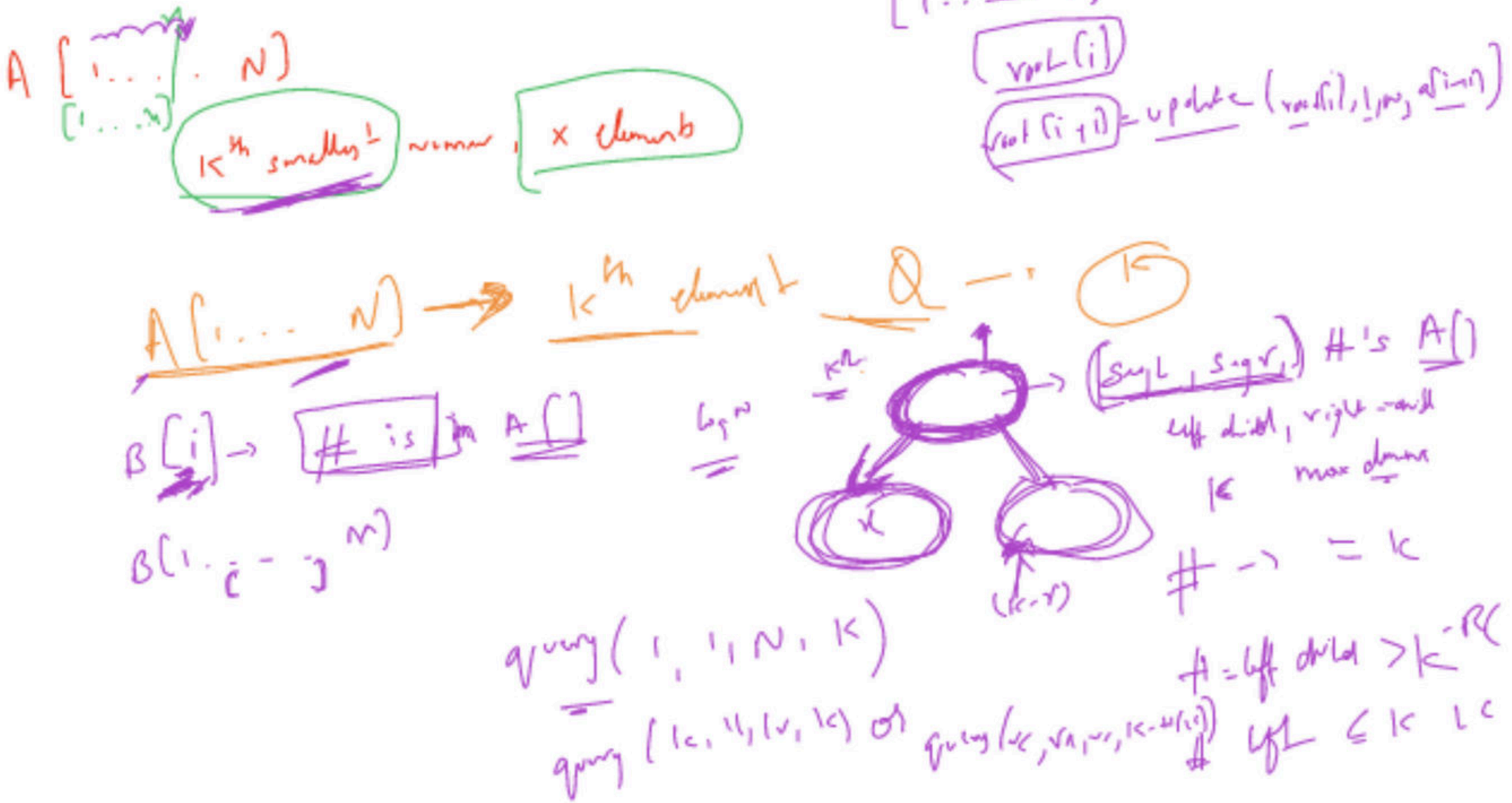
root_id[1 ... B]

root[current_day] = update(root[current_day-1], 1, N, id, v);

cout << query(root[d], 1, N, d, v) << endl;



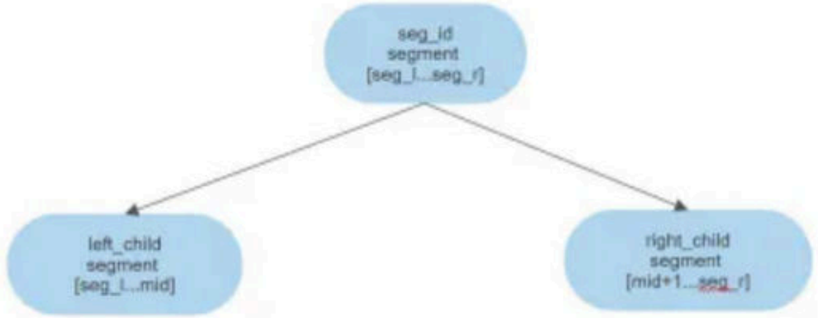
Problem 2


A [1 . . . N]
[1 . . . i]
root(i)
root(i+1) = update(root(i), ..., ...)
k^th smallest number
x elements
A[1 . . . N] → k^th element
Q →
B[i] → # is in A[]
B[1 . . . N]
log N
k
(sum, segs) # 's A[]
left child, right child
max element
# -> = k
# = left child > k - RC
# left ≤ k LC
query(1, 1, N, k)
query(lc, 1, lv, k)
query(lc, va, mv, k - u(lc))

Problem 3

Update a Range of Segment Trees
Lazy propagation

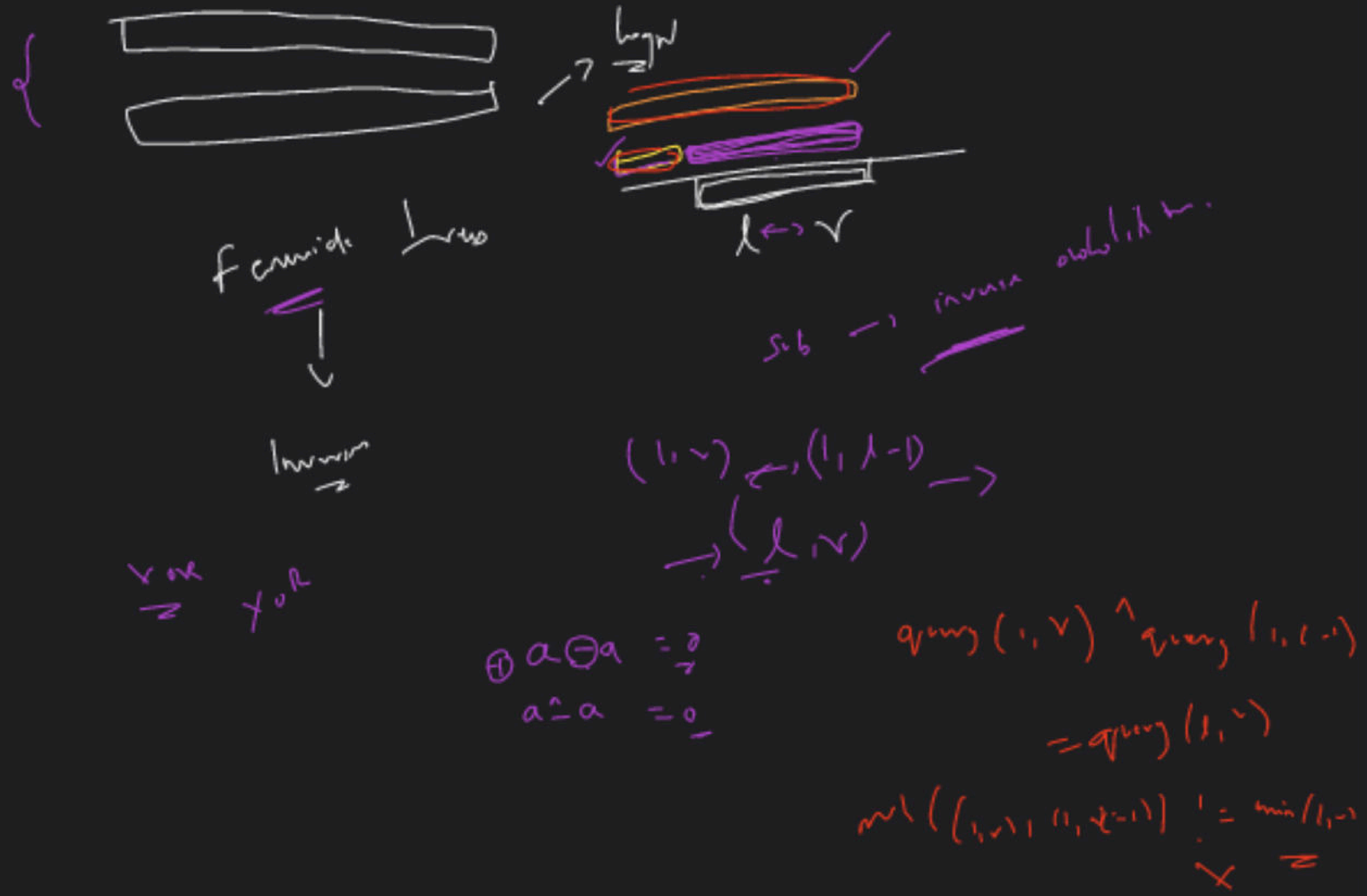
Update a Range - Lazy propagation




```
// Range update, adding "v" to each element of sub_array a[id].
// Updates all segments to imitate adding "v" to a[id].
void range_update(int seg_id, int seg_l, int seg_r, int query_l, int query_r,
                  int v) {
    if (seg_l > id || seg_r < id) return; // No overlap.
    if (query_l <= seg_l && seg_r <= query_r) {
        seg_sum[seg_id] += v * (seg_r - seg_l + 1);
        seg_add[seg_id] += v; // This is for future use, to push to children.
        return;
    }
    int left_child = seg_id * 2;
    int left_l = seg_l;
    int left_ r = (seg_l + seg_r) / 2;

    int right_child = seg_id * 2 + 1;
    int right_l = left_r + 1;
    int right_r = seg_r;
    // Push seg_add[seg_id] to left child. This is O(1).
    range_update(left_child, left_l, left_r, left_l, left_r, seg_add[seg_id]);
    // Push seg_add[seg_id] to right child. This is O(1).
    range_update(right_child, right_l, right_r, right_l, right_r, seg_add[seg_id]);
    // Regular update to left child O(logn).
    range_update(left_child, left_l, left_r, query_l, query_r, v);
    // Regular update to left child O(logn).
    range_update(right_child, right_l, right_r, query_l, query_r, v);

    // New seg_sum can be got from information of children.
    seg_sum[seg_id] = seg_sum[left_child] + seg_sum[right_child];
    // set seg_add[seg_id] back to zero since previous updates were already
    // passed on to children.
    seg_add[seg_id] = 0;
}
```



logN
Fenwick Tree
Inversion
XOR
XOR
a^a = 0
query (1, r) ^ query (1, l-1)
= query (l, r)