Quantum Risk Analysis

An Introduction

Contents

Overview of Lectures

In this series of lectures, we aim to introduce the basic concepts of Quantum Risk Analysis, firstly through introducing the importance of analysing financial risk and the limitations of the classical processes that evaluates certain risk metrics.

We then introduce quantum methods to analyse risk, looking at the derivation of the quantum algorithm used and its application in measuring risk metrics in a two-asset portfolio example.

Finally, we look at the sources of error and uncertainties that arise in quantum risk analysis methods and how these affect the efficiency and accuracy of the processes. However, despite these we can clearly see the distinctive advantages that quantum computational methods have over classical methods, and we conclude by looking at the possibilities for future developments and uses of quantum computers in finance.

The mathematics of the quantum algorithm and the example of its application in a two-asset Treasury portfolio is largely based on the work of two renowned researchers and professionals working at IBM, Stefan Woerner and Daniel J. Egger.[1,2] They ran this algorithm on a small-scale problem using an IBM Q 5-qubit computer in 2019 in New York. This was the first time a quantum algorithm like this was run on a quantum computer to carry out a financial calculation.

Financial Definitions

Asset: An asset is any resource that has economic value belonging to and controlled by a certain individual, an institution, or a country with the expectation that it will provide future benefit.[3]

Price: The value of an asset assigned by the market.[4]

Security: A security is a financial instrument, such as an asset that can be traded. There are three different categories of securities; Equity securities which include stocks, Debt securities which include bonds and banknotes and Derivatives which include options and futures.[5]

Options: An option is a financial instrument that is based on the value of underlying securities such as stocks. They give buyers the right to buy or sell an underlying asset at a predetermined price.[6]

[1] Woerner, S., Egger, D.J. Quantum risk analysis. *npj Quantum Inf* **5,** 15 (2019). https://doi.org/10.1038/s41534-019-0130-6

[2] arXiv:1907.03044v1 **[quant-ph]**

[3] https://www.investopedia.com/terms/a/asset.asp

[4] https://financial-dictionary.thefreedictionary.com/price

[5] https://corporatefinanceinstitute.com/resources/knowledge/finance/security/

[6] https://www.investopedia.com/terms/o/option.asp

Bond: A fixed income instrument representing a loan made by an investor to a borrower.[7]

Debt: Anything (usually money) that is borrowed and that must be paid back, usually with interest.[8]

Stock options: Form of equity derivative that gives an investor the right, but not the obligation, to buy or sell a stock at an agreed-upon price and date.[9]

Financial portfolio: A collection of financial investments such as stocks, bonds, commodities etc.[10]

Treasury bill: A treasury bill is a short-financial instrument that is issued by a government.[11]

Maturity dates: The date on which an issuer or borrower of a loan or bond must repay the principal amount and the interest back to the holder or investor. (i.e., it is the lifespan of a security).[12]

Face value: This is the monetary value of a financial instrument when it is released at the time of maturity.[13]

Yield curve: A yield curve is a graphical representation of the yields available for bonds that have the same credit quality but that have different maturity dates. The slope of the graph provides us with information on future interest rate changes.[14]

Interest rate: The amount that a lender charges a borrower which is a percentage of the original amount that is loaned.[15]

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[7] https://www.investopedia.com/terms/b/bond.asp

[8] https://www.investopedia.com/terms/d/debt.asp

[9] https://www.investopedia.com/terms/s/stockoption.asp

[10] https://www.investopedia.com/terms/p/portfolio.asp

[11] https://gocardless.com/guides/posts/treasury-bills-definition-and-examples/

[12] https://www.bankrate.com/glossary/m/maturity/

[13] https://www.investopedia.com/terms/f/facevalue.asp

[14] https://www.investopedia.com/terms/y/yieldcurve.asp

[15] https://www.investopedia.com/terms/i/interestrate.asp

Lecture 1

1. Introduction

In the introduction in the lecture video, we discussed the importance of physics and mathematics in financial applications. From the Black-Scholes model to the algorithms used to optimize and price stock options, physical and mathematical concepts are essential to financial modelling and analysis.

Important recent advances in technology and more specifically quantum computing have enabled banks and other financial institutions to update and enhance their models and algorithms to analyse these risks. The classical methods used in these processes rely heavily on the physics of Monte-Carlo operations, but now with a new quantum era, financial institutions can explore new methods that are not possible with traditional computers.[16]

The financial process which we focus on for this lecture is Risk profiling. This is an incredibly important aspect in the work of all those working in the financial services. It is carried out to assess the possibility of a company losing money from any investments it makes or other transactions.[17] Examples of these include credit risk which we talk about later in the lecture. Managing and analysing risk is thus extremely important to ensure that clients reach their financial goals and that economies thrive.

2. Quantum Computing

Quantum computing is a fundamental to understand and solve complex computing solutions. It uses the laws quantum mechanics which explains our natural world and uses the knowledge that we have on the properties of the subatomic world to process information and deliver solutions. This has given us new ways of addressing some problems, for example, in quantum chemistry, optimization, or machine learning.

You will be learning about trading and price optimization as well as targeting and prediction in this course, all of which have been enhanced by recent incredible advances in quantum computing. Indeed, these advances are becoming so significant and essential to financial processes that the homeland Security Research corporation has estimated that the quantum computing market will be worth of $10 billion in the next few years.[18]

How they work:

Quantum Computers process and store information in quantum form using quantum-bits, in the same way that bits are used in classical computers.[19]

[16] https://www.ibm.com/thought-leadership/institute-business-value/report/quantumleap

[17] https://www.investopedia.com/terms/f/financialrisk.asp

[18] https://www.ibm.com/thought-leadership/institute-business-value/report/quantumleap

[19] https://www.ibm.com/quantum-computing/what-is-quantum-computing/

Qubits can exist in a state known as quantum superposition. This superposition makes the qubits exponentially powerful, for example, a 2-qubit system can exist in a superposition of four states, a 3-qubit system can exist in a superposition of eight states, a 4-qubit system can be in a superposition of 16 states, and so on. As qubits are added, quantum computing capability can grow exponentially.

Classical supercomputers don't have the capacity or ability to solve many real-world problems. As well as this, they can only analyse problems one at time, such as an investment company wanting to balance the risk in their portfolios, which can take a long time. Quantum computers can create incredibly large multidimensional spaces in which very large problems can be presented

3. Risk Profiling

Risk profiling is an incredibly important process which all financial services must consider and employ and is usually the first step that is carried out in an investment process. [20]

It is an overall evaluation of the willingness and ability of someone to take risks. In a financial frame, this concerns the individual's assets and monetary belongings. There is a huge amount of pressure on financial institutions to perform analysis of their assets and securities to estimate any possible risks that may affect them.

Financial advisers use this process to help their clients determine their optimum levels of investment risk to meet their financial objectives and desired return on investment. When carrying out risk analysis, a risk profile is created.[21] This profile consists of three main branches, all representing different facets of risk that a client must consider when investing their money (shown in figure 1).

The first of these is the risk required, which is the amount of risk that needs to be taken on investments so that a client's goal of investment return is still achieved. The risk tolerance describes how much risk a client is willing and comfortable to take with their assets. Finally, the risk capacity is the level of investment risk that clients can afford to take.

[Figure 1]

[20] https://www.canaccordgenuity.com/wealth-management-australia/investment-management/your-risk-profile/

[21] https://www.investopedia.com/terms/r/risk-profile.asp

The physical and mathematical theories and strategies that are used to measure financial risks include using standard deviation, which provides a measure of the volatility of asset prices in comparison to their historical averages for a given time frame and market risk (also known as beta, which measures the volatility of a financial portfolio compared to the market as a whole).

Value at Risk (VaR) is a statistic used to quantify the possible extent of loss of finances in a portfolio within a certain amount of time.[22]

Capital Asset Pricing Model (CAPM) describes how volatility, or systematic risk is related to the expected profit or return obtained from an investment.[23]

4. Credit Risk Analysis

Credit risk analysis is a very important part in lending process that financial services undertake. Credit analysts determine a borrower's ability to pay back their debt which they have taken out in loans. It essentially determines how worthy they are to be given credit.[24]

Quantum computers analyse credit risk by estimating the economic capital requirement (ECR), which is the difference between the Value at risk and the expected value of a given loss distribution.[25] ECR is a very important risk metric as it describes the amount of capital, or funds that a company requires to remain at its desired solvency level, which is a business' ability to meet its financial obligations such as its long-term debts.[26]

Credit risk prediction monitoring as well as analysing the reliability of models and ensuring that efficient processes are set up to enable loans are vital in the work of banks, and these need to be as efficient as possible to maximise financial success as well as ensuring complete transparency in all their work.

As with risk profiling, there are many methods that can be used to evaluate and analyse credit risk. One method that is used is ratio analysis which uses information drawn from financial statements of a company to determine various aspects of its business, for example how profitable a company is as well as its liquidity.

[22] https://www.investopedia.com/terms/v/var.asp

[23] https://www.investopedia.com/terms/r/risk.asp

[24] https://corporatefinanceinstitute.com/resources/knowledge/credit/purpose-of-credit-risk-analysis/

[25] arXiv:1907.03044v1 [quant-ph]

[26] https://www.investopedia.com/terms/s/solvency.asp

5. Classical Methods of Risk Analysis

Monte Carlo simulations are one of the more commonly used processes to estimate risk. This computational method uses stochastic asset models to carry out analysis of the financial risks by constructing various models illustrating possible results by substituting a probability distribution instead of a factor that has uncertainty or an unknown value. These probability distributions provide a way to realistically describe the uncertainty of parameters in risk analysis.

A Monte Carlo process samples a value *x* from its probability distributions *p(x)* and then takes an average of all the samples extracted to obtain an estimation for the expectation value.[27]

The advantage of Monte Carlo methods in comparison to other techniques increases as the number of sources of uncertainty (i.e., the dimensions of the problems) increases.
During the simulation, the variables are sampled at random from the inputted probability distributions and each set of samples is called an iteration.

Monte Carlo simulations are usually the method of choice to calculate an estimation for the VaR of a portfolio.

This is done by constructing a model of the portfolio assets and total risk value for *M* number of samples with the model input parameters. To achieve a representative distribution for the portfolio risk value, we need to run multiple Monte-Carlo simulations.[28]

The width of the confidence interval, that is the probability that a parameter will lie between two values around the mean, is of order $M^{-1/2}$. The formula to calculate the confidence interval is shown in figure 2. Confidence interval, CI, is calculated by dividing the sample standard deviation s by the square root of the sample size and multiplying this value by the confidence level value.[29]

$$\overline{x} \pm z\frac{s}{\sqrt{n}}$$

[Figure 2]

Monte Carlo Methods used to evaluate credit risk produce an error on the estimation scales of order $M^{1/sqrt2}$. Thus, evaluating credit risk with MC methods requires many samples therefore making it a slow and expensive process. Statisticians have tried to improve these processes in many ways, for example by using importance sampling methods which is an approximation method instead of extracting each sample.[30] This reduces the time and cost for computers to process the samples but does not however, increase the accuracy of the risk model.

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[27] https://www.palisade.com/risk/monte_carlo_simulation.asp

[28] https://www.nature.com/articles/s41534-019-0130-6

[29] https://www.investopedia.com/terms/c/confidenceinterval.asp

[30] https://towardsdatascience.com/importance-sampling-introduction-e76b2c32e744

Lecture 2

6. Quantum Risk Analysis

Research has shown that quantum computers are able to sample data with a quadratic speedup for the complexity of the algorithms. This means that if say a classical computer needs N^2 separate steps to process m inputs, a quantum computer would only require N steps for a number m inputs. So, the quadratic term describes the relationship between the time complexities between classical and quantum computers. As well as this, every time one qubit is added, the power of the quantum computers approximately doubles.[31]

The quantum algorithm that we will be focusing on is called Quantum Amplitude Estimation (QAE). This is an algorithm that can measure and evaluate risk much more effectively than Monte Carlo Methods. It can price assets and estimate various risk measures such as Value at Risk which we have already encountered, as well as the Conditional Value at Risk metric (CVaR). This is derived from the VaR metric and is also known as the expected shortfall.

CVaR describes the amount of "tail risk"- the probability of loss occurring due to a rare event predicted by a probability distribution and is therefore more susceptible to extreme events in the end of the probability distributions.[32]

These QAE algorithms are done on a gate-based quantum computer and are used to give an estimation for an unknown parameter.

Convergence rate of the algorithm and circuit depth:

The circuit depth is the length of the longest path from the input of the circuit all the way to the output, so evidently something that we aim to minimise to reduce the length of time of operation and thus reduce costs.[33]

The shortest possible circuit depth which grows polynomially as the number of qubits increases, gives us a rate of convergence of order $M^{-2/3}$, with M being the sample number. This has been proven to already be speedier than Monte Carlo simulations which have a convergence rate of $O(M^{-1/2})$. The optimum rate of convergence of the QAE algorithm is $O(M^{-1})$, which is nearly reached when the circuit depth of the quantum computers is allowed to grow at a faster rate. Thus, the quadratic speed up.[34]

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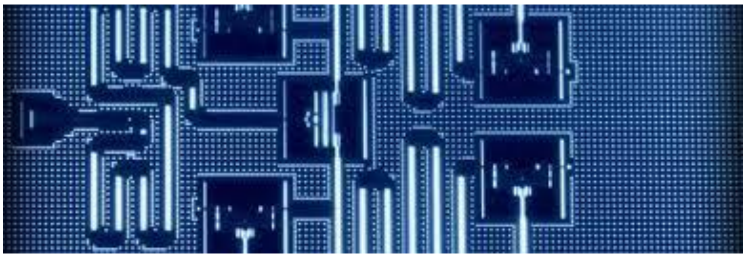
[31] https://www.ibm.com/thought-leadership/institute-business-value/report/exploring-quantum-financial#

[32] https://www.investopedia.com/terms/c/conditional_value_at_risk.asp

[33] https://quantumcomputing.stackexchange.com/questions/5769/how-to-calculate-circuit-depth-properly

[34] https://www.nature.com/articles/s41534-019-0130-6

In this lecture we will be exploring the use of this algorithm to calculate financial risk in the context of a two-asset portfolio that consists of government debt with different maturity dates, and we will show that the algorithm does indeed give us a better convergence rate that Monte Carlo simulations provide.



[Figure 3- Circuit in an IBM Q Computer]

7. Applying the Algorithm

(i) Let us set an operator A acting on $n + 1$ qubits, shown in figure 4.

$$\mathcal{A}|0\rangle_{n+1} = \sqrt{1-a}|\psi_0\rangle_n |0\rangle + \sqrt{a}|\psi_1\rangle_n |1\rangle$$ [Figure 4]

Quantum amplitude estimation aims to find the amplitude a of a state acted on by this operator.

Amplitude estimation also uses m sampling qubits as well as Quantum Phase Estimation to build algorithms by estimating the phase of a unitary operator.

After applying the inverse Quantum Fourier Transform, the qubit states are measured resulting in an integer $y \in \{0, ..., M - 1\}$.

This is classically mapped to the estimator:

$$\tilde{a} = \sin^2(y\pi/M)$$

[Figure 5]

Where y is in the range from 0 to $M\text{-}1$ and M is the number of samples. Also, $M=2^m$.

This estimator $\tilde{a}$ then satisfies

$$|a - \tilde{a}| \leq \frac{\pi}{M} + \frac{\pi^2}{M^2} = O\left(M^{-1}\right)$$

[Figure 6]

Which has a minimum probability of $\frac{8}{\pi^2}$. Figure 6 illustrates the quadratic speedup that quantum algorithms have in comparison to Monte Carlo methods which have a convergence rate of order $O(M^{-1/2})$.

The estimation error given by Figure 6 also depends on the exact result of the amplitude of the state a. If a is close to 0 or 1 the constant in the error bound becomes very small. When computing VaR_α, we want to find the minimal threshold such that the estimated probability is larger than or equal to $|a\text{-}\tilde{a}|$.

We can then represent a quantum state of a random variable *X* and then estimate its value through amplitude estimation. We map *X* in the interval from 0 to *N*-1, where *N* (number of samples is equal to 2^n.

X will then be represented by the state created by an operator *R:*

$$\mathcal{R}|0\rangle_n = |\psi\rangle_n = \sum_{i=0}^{N-1} \sqrt{p_i}|i\rangle_n \quad \text{with} \quad \sum_{i=0}^{N-1} p_i = 1$$

[Figure 7]

(P_i represents the probability of measuring the state *i* where *i* is one of the possible values for *X).*

Once we have done this, we define a function *f* ranging from 0 to *N*-1 with its corresponding operator *F* that acts on an ancilla qubit as is shown in figure 8.

$$F : |i\rangle_n |0\rangle \mapsto |i\rangle_n \left(\sqrt{1 - f(i)}\,|0\rangle + \sqrt{f(i)}\,|1\rangle\right)$$

[Figure 8]

For all $i \in \{0, \ldots, N-1\}$. Applying *F* to $|\psi\rangle_n |0\rangle$ yields:

$$\sum_{i=0}^{N-1} \sqrt{1 - f(i)}\sqrt{p_i}|i\rangle_n |0\rangle + \sum_{i=0}^{N-1} \sqrt{f(i)}\sqrt{p_i}|i\rangle_n |1\rangle$$

[Figure 9]

We can then use this state with amplitude estimation to measure the probability of obtaining 1 in the last qubit. This is equal to:[35]

$$\textstyle\sum_{i=0}^{N-1} p_i f(i) = \mathbb{E}[f(X)]$$ [Figure 10]

We will then see how AE can then be used to approximate the expected value of a random variable.

[For more information on Quantum Phase Estimation please refer to: https://qiskit.org/textbook/ch-algorithms/quantum-phase-estimation.html]

[Equations drawn from the work of Daniel Egger and Stefan Woerner; (https://www.nature.com/articles/s41534-019-0130-6, https://arxiv.org/pdf/1907.03044.pdf)]

35 https://www.nature.com/articles/s41534-019-0130-6

(ii) Measuring VaR:

Amplitude Estimation can be used to calculate the approximate expected value of a random variable (figure 10) as well as its variance (figure 11):

By choosing $f(i) = i/(N-1)$ we estimate the expected value $\mathbb{E}[\frac{X}{N-1}]$ [Figure 10]

and

$$\mathrm{Var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

[Figure 11]

We can use the same methods to calculate the VaR and CVaR. The Value at Risk is the minimum value of x between 0 and N-1 such that we have satisfied the condition shown in figure 12, where the probability that X is less than or equal to x is greater than or equal to (1-α).

$\mathbb{P}[\mathrm{X} \leq x] \geq (1-\alpha)$ [Figure 12]

To calculate an estimation for the value at risk on a quantum computer we define a function $f_l(i)=1$ if $i \leq 0$ and $f_l(i)= 0$ otherwise, where l is again in the range from 0 to N-1.

We can then apply the operator F acting on $|\psi\rangle_n |0\rangle$ which gives us the expression shown in figure 13.

$$\sum_{i=l+1}^{N-1} \sqrt{p_i}|i\rangle_n |0\rangle + \sum_{i=0}^{l} \sqrt{p_i}|i\rangle_n |1\rangle$$

[Figure 13]

The probability of measuring the qubit $|1\rangle$ for the last qubit is equal to figure 14,

$\sum_{i=0}^{l} p_i = \mathbb{P}[X \leq l]$ [Figure 14]

Therefore, if we then apply a bisection search (a method employed to find the roots of a continuous distribution function) over l we can find the smallest value l_α such that

$\mathbb{P}[X \leq l_\alpha] \geq 1 - \alpha$ [Figure 15]

In at most n steps. The smallest level l_α is equal to $\mathrm{VaR}_\alpha(X)$.

This estimation for the value at risk value has accuracy of $O(M^{-1})$ thus a quadratic speed up to the Monte Carlo methods.

Conditional value at risk is similarly estimated but is the conditional expectation of X which is restricted between $\{0\ldots, l_\alpha\}$. Where l_α is computed as before.[36]

[Please do refer to the Qiskit coding examples for more information on Credit Risk Examples: https://qiskit.org/documentation/finance/tutorials/09_credit_risk_analysis.html]

(iii) Applying the Algorithm to a two asset-portfolio example:

Let us now apply the algorithm that we have seen to calculate daily risk in a portfolio which consists of 1-year and 2-year Treasury bills with face values V_{F1} and V_{F2} respectively.

We apply the amplitude estimation to evaluate Value at Risk (VaR) and we define the worth of the portfolio by the equation shown in figure 16,

$$V(r_1, r_2) = \frac{V_{F_1}}{1 + r_1} + \sum_{i=1}^{4} \frac{r_c V_{F_2}}{(1 + r_2/2)^i} + \frac{V_{F_2}}{(1 + r_2/2)^4},$$

[Figure 16]

where r_c is the annual coupon rate. This is calculated by dividing the interest payment that the shareholder receives every year by the face value of the bond. r_1, r_2 give us the yield to maturity (i.e., the percentage rate of return for a bond) of the 1-year and 2-year bills respectively.

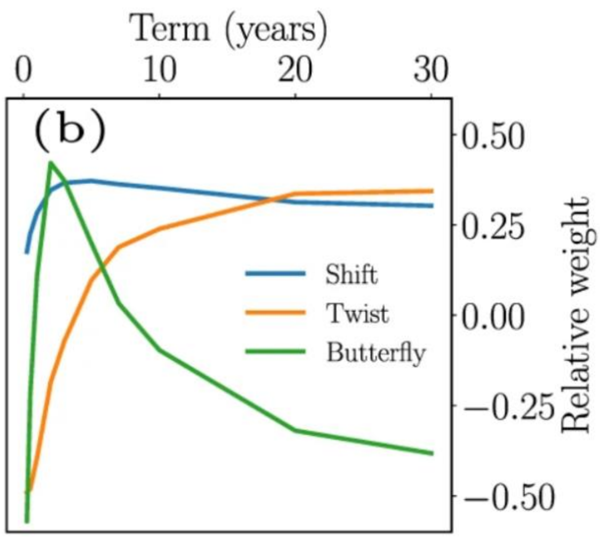
To model the uncertainty that occurs in r_1 and r_2 we use the Constant Maturity Treasury (CMT) rates. These are calculated from the predicted values of a US Treasury which are based on recent values of US Treasuries and are interpolated from the Treasury yield curve.[37]

Then, to calculate the risks of the portfolio we must observe by how much the CMT rate differs each day. However, as theses variations in the rate agree with predictions and the differences are highly correlated, we do not need to model all of them.

Using a method called principal component analysis we see that the first three principal components make up for over 95% of variance that occurs in the model. These are called Shift, Twist, and Butterfly. Thus, to model the portfolio we look at the distributions of these three components.

[36] https://www.nature.com/articles/s41534-019-0130-6

[37] https://www.investopedia.com/terms/c/constantmaturity.asp



[Figure 17]

This is to our advantage as it reduces the number of resources that are required by the quantum algorithm.

The graph in figure 17 illustrates the three principal components, Shift, Twist and Butterfly expressed in terms of the constant maturity treasury rates.

In order to calculate the daily risk in the portfolio we set $r_i = r_{i,0} + \delta_{ri}$ for i=1,2 where the first term, $r_{i,0}$ is the maturity yield observed on the day and the variable r_i is derived from the distribution of the changes in CMT rates.

Principal component analysis is then carried out for δ_{r1} and δ_{r2} where we only keep components of S (Shift) and T (Twist) and show their relation to for δ_{r1} and δ_{r2}, shown in figure 18.

$$\begin{pmatrix} \delta r_1 \\ \delta r_2 \end{pmatrix} = W \begin{pmatrix} S \\ T \end{pmatrix}$$

[Figure 18]

From calculations we see that the correlation coefficient shift and twist is -1%.

We can then model the uncertainty in the quantum computer by using three qubits, q_0, q_1 and q_2 to represent the distribution of S, and two qubits, q_3 and q_4 to represent the distribution of T. (Note in this example we only analyse the first two principal components, but all three would be retained when the algorithm is run on real quantum hardware for larger portfolios).

We can encode these probability distributions with the states shown in figures 19 & 20, where S can take 8 different values and T can take 4.

$$|\psi_S\rangle = \sum_{i=0}^{7} \sqrt{p_{i,S}}|i\rangle_3$$

[Figure 19]

$$|\psi_T\rangle = \sum_{i=0}^{3} \sqrt{p_{i,T}}|i\rangle_2$$

[Figure 20]

Applying affine mappings to represent S and T in terms of variables x and y to denote the integer values of S and T, we see that the qubits represent integers using binary encoding. The portfolio risk value is then calculated.

To investigate and evaluate the accuracy of the model and the convergence rate we can assume an ideal quantum computer, i.e., one that does not produce any uncertainty or error in the calculations, in simulating the two-asset portfolio example.

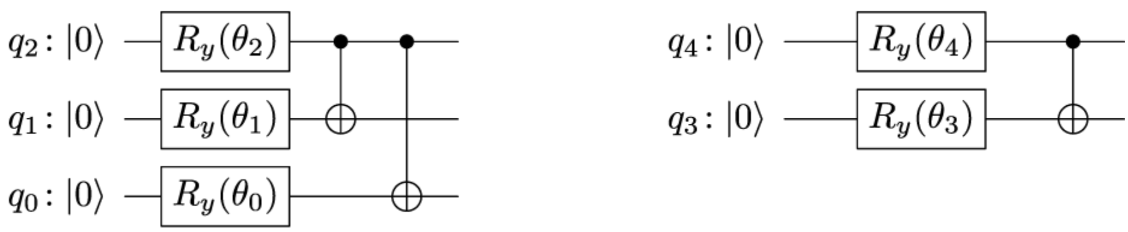
Simulations of this algorithm showed that if we have a number N samples = 2^n, we need $n+12$ qubits to calculate expected values of VaR, and $n+13$ qubits for CVaR.

Five of the qubits are needed to show the distribution of the changing interest rates, one of the qubits is needed to create the state as was shown in Figure 8 and six ancillas are required to create a controlled Q operator.

[This section has again been based on the work of Egger and Woerner, IBM researchers][38].

(iv) The Quantum Circuit

Now we can simulate the shift and twist distributions on the quantum computer using the circuits shown in figure 21,



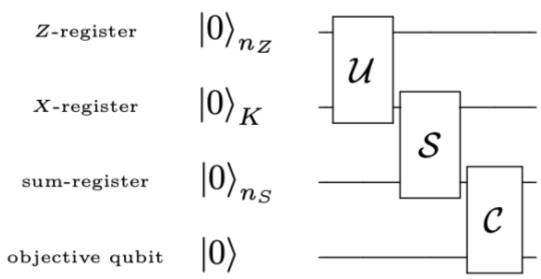
[Figure 21]

These circuits consist of CNOT gates and the qubits q_0 to q_4 are used to model the uncertainty in the simulation for both distributions S and T can be seen. Using this circuit, we can apply the operator F (figure 8) to create the quantum state.

Figure 22 shows another quantum circuit that can be used to evaluate the distribution function of the quantum operator acting on $n+1$ qubits in three steps, each step corresponding to a quantum operator. The first one, U, loads the uncertainty model, the second one S, evaluates the total loss into a quantum register that contains n_s qubits and finally the last operator, C, switches a target qubit to if the total loss is less than or equal to the given level l that is used to find VaR. Thus, we have $A=CSU$. [39]

[38] https://www.nature.com/articles/s41534-019-0130-6#Sec2

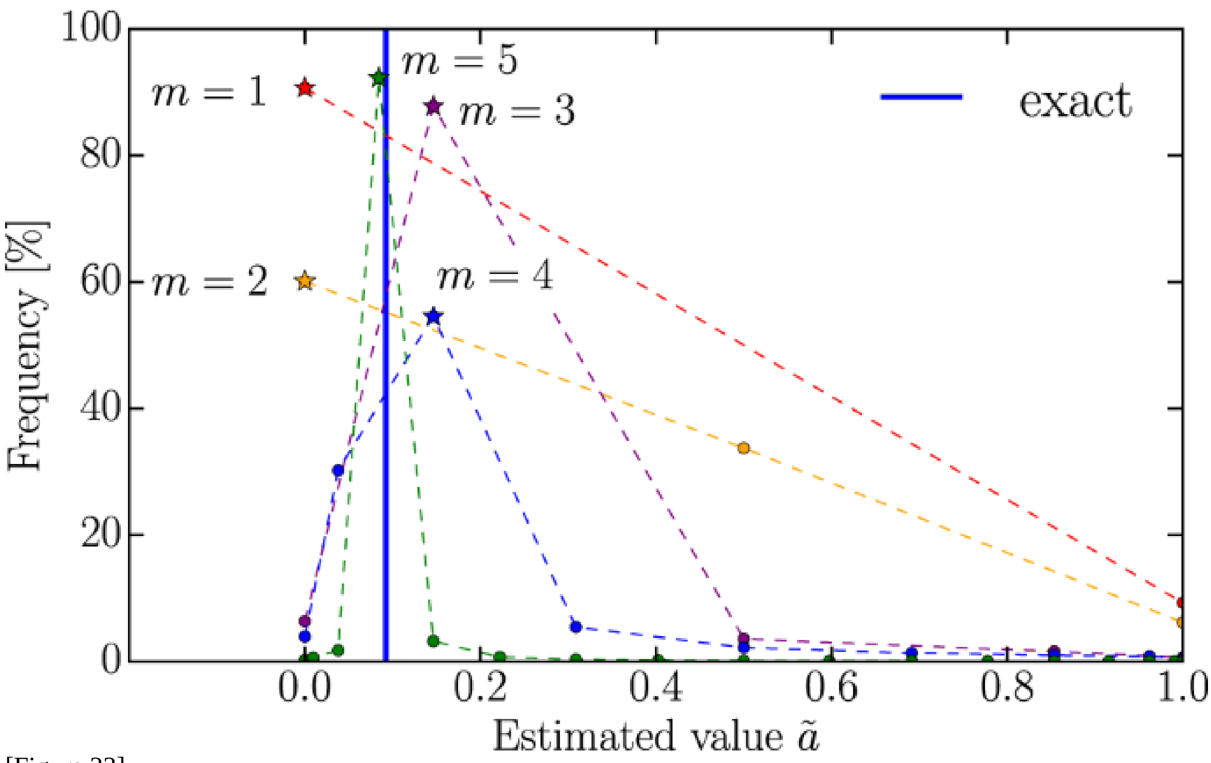
[39] https://arxiv.org/pdf/1907.03044.pdf



[Figure 22]

(v) Estimated value of ã:

Figure 23 represents the estimation of VaR when simulated through a perfect quantum computer.



[Figure 23]

The stars on the diagram indicate the most probable value of $\tilde{a}$ which we defined earlier on in the lecture. The vertical blue line shows the classical value of VaR, which is 0.093, and we can see that as the number of *m* sample qubits is increased the quantum estimation for VaR becomes closer and closer to that of the classical value.

When the sample of qubits *m* is equal to five the classical and quantum estimations differ by 9%. Every time a sample qubit is added we have that the number of CNOT gates required to calculate an estimate value of VaR doubles such that it scales with order proportional to M with an error of $O(M^{-1})$.[40]

[40] https://www.nature.com/articles/s41534-019-0130-6#Sec2

Lecture 3

8. Sources of Error

Let us investigate various sources of error and uncertainty that arise when we simulate risk using the quantum algorithm.

We can see that evaluating risk metrics for the two-asset portfolio example requires a relatively lengthy circuit. But with amplitude estimation we only need to obtain the result which has the highest probability.

In the investigations ran by IBM researchers Woerner and Egger in their paper on Quantum Risk Analysis, they conducted simulations that contained several errors on their IBM Q quantum computer to discover how many errors we can have before we cannot identify the correct state anymore. Essentially, finding the threshold for the number of errors that we can tolerate before the state is unrecognisable.

The investigation studied two different types of effects that errors had on the simulations, called the energy relaxation and cross talk, which was only that is considered for two-qubit (CNOT) gates, and that the errors that occur for two qubit gates are most of the time larger than those for single qubit gates.

Cross talk is a phenomenon which violates either of two key assumptions that go into any well-behaved quantum simulation: spatial locality and the independence of operations. Energy-relaxation is also another perturbation which results in uncertainty of results.

The order of magnitude of the single and two qubit gates was the same in the quantum algorithm that we simulated while computing risk measures. The first source of error, energy relaxation can be simulated by using a relaxation rate, gamma, such that each qubit has a probability $1 - \exp(-\gamma t)$ of relaxing to the zero state at a time t.

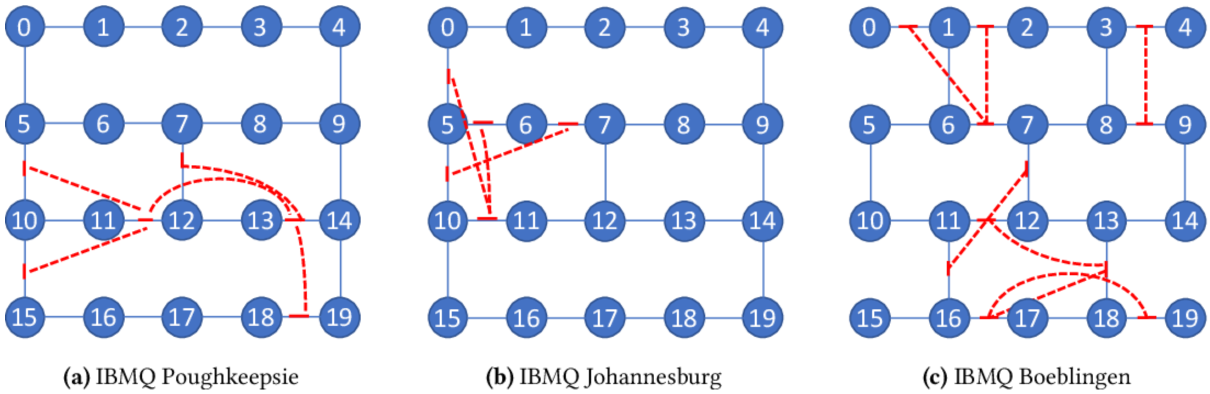
To simulate this and show this error, we first assume that the single qubit gates occur instantly and therefore have no errors associated to them. We then set the time of duration of the CNOT gates to 100 ns as well as including qubit-qubit cross talk through adding a ZZ term in the CNOT gate generator shown by the equation in figure 24.

$$\exp\{-i\pi(ZX + \alpha ZZ)/4\}$$

[Figure 24]

Cross resonance rates for CNOT gates are typically of order of 5MHz whereas on IBM Q computers cross talk errors occur with order -100kHz. We can therefore calculate an estimation for a value of alpha, as we defined in the second lecture, which represents the strength of the cross talk, as -2%.

The diagram shown on figure 25 [41] shows a measurement of cross talk occurring in three different IBM Q systems and thus illustrates the challenges that this type of error causes for the hardware.



[Figure 25]

The examples that we have seen in these lectures of estimating risk have been done on a small scale and so to achieve more accurate results on a wider scale we need more qubits to model realistic financial scenarios.

As well as this, the simulations for the risk analysis in the two-asset portfolio example has shown us that the circuit depth is limited on the current hardware that is available. Thus, errors on the current hardware need to be reduced and this can be done by reducing cross talk as much as possible and increasing the time for which qubits retain information, also known as coherence time.

However, quantum amplitude estimation is ongoing research and development and so we can be hopeful to see significant advantages in the hardware as well as the algorithms. Reducing error will also help shorten circuit depth required to run the simulation, thus reducing time of operation.

We can also reduce the length of the circuit depth by using a set of gates that is more versatile. For example, SWAP gates can be used in the hardware which would mean we can avoid synthesising them using CNOT gates.

As well as this, other techniques to reduce noise and error include error mitigation, which is a process by which quantum computers reduce error in the measurements by conducting more repeated experiments and processing the data after it has been through the algorithm to identify any errors.[42]

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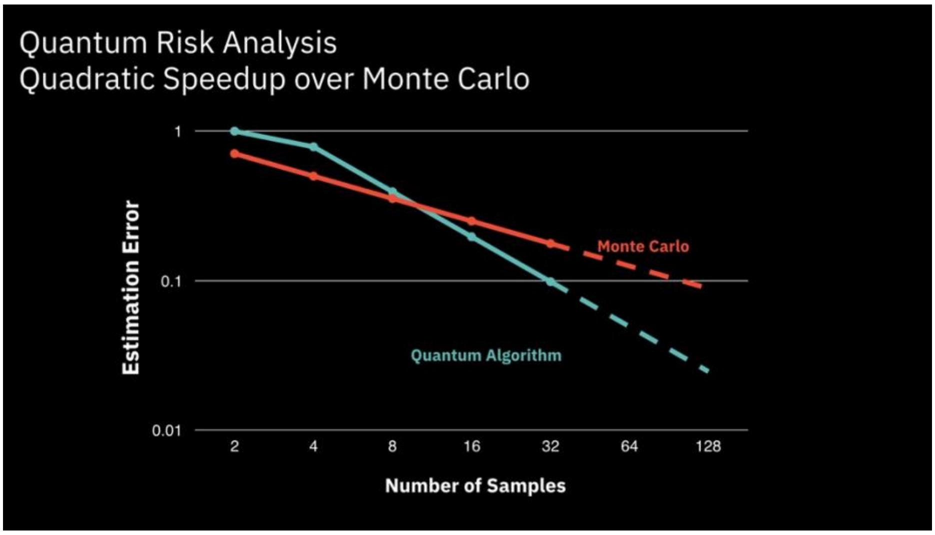
[41] https://www.semanticscholar.org/paper/Software-Mitigation-of-Crosstalk-on-Noisy-Quantum-Murali-McKay/b33e5737579a3c5483cf87d472f8a92de7dd9ca3

[42] https://www.nature.com/articles/s41534-019-0130-6#Equ13

9. Advantages of Quantum Risk Analysis

Now that we have seen how quantum algorithms estimate and analyse values to measure risk in financial portfolios, despite the errors and uncertainties that can arise in our measurements, we can clearly see all the advantages that these quantum amplitude estimation methods have over older classical methods of analysing financial risk.

The graph in figure 26 directly shows the difference in the estimation error produces for a given number of samples for quantum and the classical Monte Carlo algorithms.[43]



[Figure 26]

We can see that for a larger number of samples, quantum algorithms give much lower estimation errors, and will continue to produce more accurate results than classical algorithms as larger quantum computers are developed and are available to use.

Although in the graph of the two different algorithms we see that the quantum algorithm starts off with a larger estimation error, for number of samples *M* greater than 16 the convergence rate of the quantum algorithm wins and the error stays below that of the Monte Carlo.

The blue solid line shows the error for the experiments conducted using up to $m = 4$ evaluation qubits, i.e., $M=16$ (recall that $M=2^m$). The blue dashed line shows how the estimation error would further decrease for experiments with $m = 5$, 6 evaluation qubits, respectively.

Investigation has shown us that while the convergence rate for a Monte Carlo simulation is proportional to the inverse square root of the number of samples, the convergence rate of the quantum amplitude estimation algorithm is proportional to the inverse of the number of samples M, thus showing the quadratic speed up with respect to the Monte Carlo method.[44]

[43] https://www.ibm.com/blogs/research/2019/03/quantum-risk-analysis/

[44] https://www.nature.com/articles/s41534-019-0130-6

As well as this, with Monte Carlo simulations, computing risk assessments for large portfolios can be a very long task, often taking several hours to complete, and in the worst case, several days. With the quadratic speed up that comes with quantum computers, the calculation times reduce drastically, giving us huge potential for the future to assess much larger portfolios as the quantum hardware develops in the coming years.

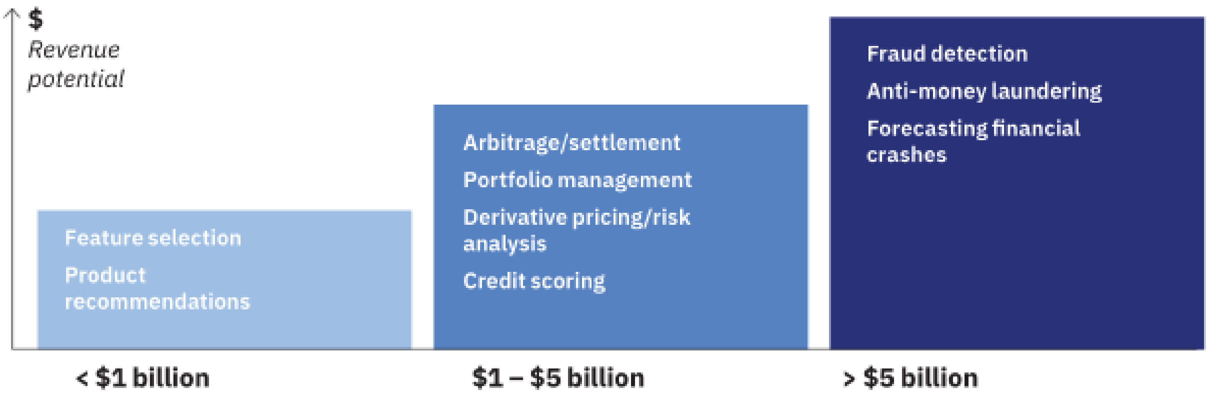
Apart from the quadratic speedup of quantum algorithm, in comparison to normal computers quantum computers can hold data of many magnitudes larger than traditional computers. This is because adding one qubit doubles the power of a quantum computer. If we wanted to double the power of a classical computer, we would have to double the number of transistors working.

Thus, we can see that with the development of quantum computers we have a promising future for the enhancement of risk analysis processes.

10. Conclusions

Throughout the lectures we have seen the benefits that quantum computing brings when evaluating and analysing risk metrics for financial portfolios. Not only do they give us more accurate methods of risk analysis than classical methods, but quantum computers also bring more advantages to the financial world.

The graph in figure 27 shows only a few of the opportunities to enhance processes and benefit financial services activities that quantum can bring and the revenue potential that it can bring to companies that implement quantum processes.



[Figure 27]

This graph shows the financial benefits Quantum computing will have in enhancing investment gains, such as with reducing capital requirements as well giving way to new investment opportunities and improving the identification and management of risk and compliance. We can clearly see the financial benefits that quantum computers will bring to financial services activities.[45]

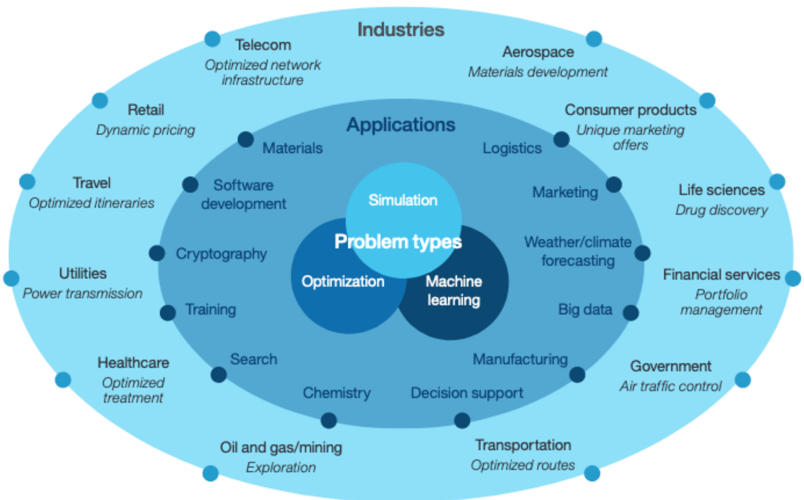
Although we have seen that there is still a long way to go in terms of hardware development and working to reduce errors, the success of IBM's algorithm shows huge potential for quantum computing in risk analysis.

The increasing pressure put on firms to balance and manage risk especially in times where markets can be extremely volatile and unpredictable as well as the extremely complex nature of risk calculations that need to be performed make it difficult to properly manage the risks on assets and trades.

We have seen that the classical methods, such as Monte Carlo simulations are hugely limited the scaling of the estimation error. As more and more risk regulations come into place into finance, quantum computers will be able to speed up the simulations and enable analysts to analyse many more risk scenarios in a shorter space of time and with increased accuracy in the measurements.

Thus, it is important for banks and other financial institutions to begin transitioning to quantum computational methods as soon as possible, as those left behind on classical computing will be at a significant disadvantage as quantum hardware continues to develop.

Figure 28 illustrates some of the industries and applications that quantum computers could provide new and more efficient solutions to.[46]



[Figure 28]

Although it won't replace classical computers, quantum innovation represents a new computing paradigm as quantum can work with classical computers to solve complex problem that were not possible beforehand.

[45] https://www.ibm.com/thought-leadership/institute-business-value/report/exploring-quantum-financial#

[46] https://www.ibm.com/thought-leadership/institute-business-value/report/quantumleap#

End of lecture notes