PnS

① Discrete Distribution:

① Conditional Probability $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$$P(B) = P(S_1)P\left(\frac{B}{S_1}\right) + P(S_2)P\left(\frac{B}{S_2}\right) \cdots$$

② Probability Mass function $P(X=x)$

③ Probability Distribution function $F_X(x) = P(X \leq x)$

④ Expected Value

$$\mathcal{E}(H(x)) = \mathcal{E}H(x)P(X=x)$$
$$\mathcal{E}(k) = k$$
$$\mathcal{E}(X+Y) = \mathcal{E}(X) + \mathcal{E}(Y)$$
$$\mathcal{E}(kX) = k\mathcal{E}(X)$$

⑤ Variance $= \mathcal{E}((X-\mu)^2) = V(X) = \sigma^2 = \mathcal{E}(X-\mu)^2$
$$= \mathcal{E}(X-\mu)^2 P(X=x)$$

$$Sd = \sqrt{var(X)}$$
$$\sigma^2 = \mathcal{E}(X^2) - \mathcal{E}(X)^2$$

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6) Moment generating function

$$M_X(t) = E(e^{tx}) = \Sigma e^{tx} f(x) = \Sigma e^{tx} p(x)$$

$$E(X) = \frac{dM_X(t)}{dt} \quad t=0$$

$$E(X^2) = \frac{d^2M_X(t)}{dt^2} \quad t=0$$

$$E(X^k) = \frac{d^kM_X(t)}{dt^k} \quad k=0$$

$M_{\alpha}t = \alpha$

$M_{\beta}t = \beta$

$M_{\gamma}t = \gamma$

$M_{\alpha+\beta+\gamma}t = \alpha\beta\gamma$

7) Bernoulli Trial.

- Has only two outcomes
- $P(X=x) = p^x(1-p)^{1-x}$
- $var = pq = \sigma^2$
- $E = p$

8) Binomial Distribution

- Bernoulli Trial with n trials
- $b(x,n,p) = {}^nC_x p^x q^{n-x}$
- $E(X) = np$
- $var = npq = \sigma^2$
- $B(x,n,p) = P(X \le x)$

(9) Hypergeometric Distribution

$$h(x, n, a, N) = \frac{{}^{a}C_{x} \; {}^{N-a}C_{n-x}}{{}^{N}C_{n}}$$

x → Defective Selⁿ, n → Total Selⁿ, a → Total defⁿ, N → Total Numbers

We choose n, but total a are defective. We need x to be defective

$$E(X) = \frac{na}{N}$$

$$V(X) = \left(\frac{N-n}{N-1}\right)\frac{na}{N}\left(1-\frac{a}{N}\right)$$

(10) Chebyshev's Theorem

$$P\left(\left|\frac{X}{n} - \mu\right| \geq k\sigma\right) \leq \frac{1}{k^2}$$

$$P(|X-\mu| < k\sigma) \geq 1 - 1/k^2$$

(10) Negative Binomial

$$nb(x, r, p) = {}^{x+r-1}C_{r-1}\, p^r q^x$$

x → no of successes

r → no. of failures before that

$$E(X) = \frac{r(1-p)}{p}$$

$$V(X) = \frac{r(1-p)}{p^2}$$

(11) Geometric Distribution

$$nb(x, 1, p) = p(1-p)^{x-1}$$

-ve binomial but we take trials rather than no. of failures

$$E(X) = \frac{1}{p} \qquad V(X) = \frac{1-p}{p^2}$$

$$M_X(t) = \frac{pe^t}{1-qe^t}$$

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(12) Poisson Distribution

$$p(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!} = f(x,\lambda)$$

$$\mu = E(X) = \lambda$$

$$V(X) = \lambda$$

$$M_X t = e^{\lambda e^{t} - \lambda}$$

(13) Poisson Appnx.

If $n \to \infty, p \to 0$ such that $np = \lambda =$ constant

$$P(X=x) \approx b(x, n, \lambda \Delta t) = f(x, \lambda t)$$

$$np = n\lambda\Delta t = \lambda t$$

3) Continuous Probability

① $P(a \le x \le b) = \int_a^b f(x)\,dx$

② $P(X=a) = 0$

③ $P(a \le x \le b) = P(a < x < b)$

④ $\int_{-\infty}^{\infty} f(x)\,dx = 1$

⑤ For cdf take cases less than 0 and greater than max

⑥ $\text{Mean} = \int_{-\infty}^{\infty} x f(x)\,dx$

⑦ $M_X(t)$

→ $M_X(0) = \underline{\underline{01}}$ for sure else invalid

→ $M_X(t) = e^{bt} M_X(at)$ $\quad \underline{Y = aX + b}$

$M_\alpha t = \alpha$

$M_\beta t = \beta$

$M_{\alpha+\beta} t = \alpha\beta$

⑧ Uniform distribution

$$f(x) = \begin{cases} \dfrac{1}{\beta - \alpha} & \alpha < x < \beta \\ 0 & \text{otherwise} \end{cases}$$

$E(X) = \dfrac{\alpha + \beta}{2}$ $\qquad V(X) = \dfrac{(\beta - \alpha)^2}{12}$

$$F(x) = \begin{cases} 0 & x < \alpha \\ \dfrac{x - \alpha}{\beta - \alpha} & \alpha < x < \beta \\ 1 & x > \beta \end{cases}$$

* Percentile

$$p = F(\eta(p)) = \int_{-\infty}^{\eta(p)} f(y)\,dy$$

↳ X Value p → Percentile

(I) Normal Distribution

- $f(x, \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $\quad -\infty < x < \infty$

- Bell Shaped
- Points of Inflection $\mu - \sigma$ and $\mu + \sigma$
- Distribution centered at mean

$E(X) = \mu$

$V(X) = \sigma^2$

$M_X t = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$ (a μ t)

(II) Standard Normal Distribution

$\mu = 0$

$\sigma = 1$

$P(a \le X \le b)$

$P\left(\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right)$

↳ easy to find out

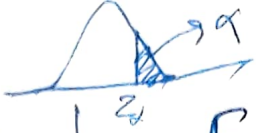
$F(z) = P(Z \le z)$

$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-t^2/2}\,dt$

$F(z) + F(-z) = 1$

$F(a) - F(-a) = 2F(a) - 1 = P(-a \le Z \le a)$

NOTE



$P(Z \le Z_\alpha) = 1-\alpha$

$Z_{1-\alpha} = -Z_\alpha$

$P(Z \le 1.01)$

$F(1.01) = 0.8438$

$Z_{1-0.8438} = 1.01$

$P(Z \le Z_\alpha) = 1-\alpha$

$F(X) = 1-\alpha$

$X = Z_\alpha$

Normal approx to binomial

$$Z = \frac{X-\mu}{\sigma} = \frac{X-np}{\sqrt{npq}}$$

Since the step which is $\frac{1}{\sqrt{npq}}$ is very less → continuous

(12) Gamma Distribution

$$f(x) = \begin{cases} \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

- $\Gamma(\alpha) = (\alpha-1)!$
- $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$
- $\Gamma(1/2) = \sqrt{\pi}$

Mean $= \alpha\beta$

Variance $= \sigma^2 = \alpha\beta^2$

$E(X^2) = \alpha(\alpha+1)\beta^2$

$MGF = (1-\beta t)^{-\alpha} \quad t < \frac{1}{\beta}$

For Gamma Use table

$F\left(\frac{x}{\beta}, \alpha\right)$

(13) Exponential Distribution

$$f(x) = \begin{cases} \lambda e^{-\lambda t} & x \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

$1/\beta = \lambda$

$\alpha = 1$

$E(X) = 1/\lambda$

$V(X) = 1/\lambda^2$

$M_X(t) = \frac{\lambda}{\lambda - t}$

$$CDF = \begin{cases} 1 - e^{-\lambda t} & t > 0 \\ 0 \end{cases}$$

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(14) Chi-squared distribution.

$$f(x) = \begin{cases} \frac{1}{\beta^{\alpha}\Gamma\alpha} x^{\alpha-1} e^{-x/\beta} \\ 0 \end{cases} = \begin{cases} \frac{1}{2^{n/2}\Gamma n/2} x^{n/2-1} e^{-x/2} \\ 0 \end{cases}$$

$$\text{Mean} = \mu = v$$

$$\text{Variance} = \sigma^2 = 2v$$

$$MGF = \frac{1}{(1-2t)^{v/2}}$$

(15) Log-normal distribution

$$g(y) = \frac{1}{\sqrt{2\pi}\sigma y} e^{-\frac{1}{2}\left(\frac{\ln y - \mu}{\sigma}\right)^2}$$

$$E(x) = e^{\mu + \frac{\sigma^2}{2}} \qquad V(x) = e^{2\mu+\sigma^2} \cdot (e^{\sigma^2} - 1)$$

(16) Transformation of Variables

$$g(y) = f(x) \left(\frac{dx}{dy}\right)$$

③ Joint Distribution

① $p(x,y) = P(X=x \text{ and } Y=y)$

such that $p(x,y) \geq 0$ and $\sum_x \sum_y p(x,y) = 1$

② Marginal Probability mass function

$$P_X(x) = \sum_{y:p(x,y)>0} p(x,y) = \int_{-\infty}^{\infty} f(x,y)\,dy \qquad -\infty < x < \infty$$

$$P_Y(y) = \sum_{x:p(x,y)>0} p(x,y) = \int_{-\infty}^{\infty} f(x,y)\,dx \qquad -\infty < y < \infty$$

Also in cont. probability

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$$

③ Independent Variables

$$p(x,y) = p_X(x)\,p_Y(y) \qquad \text{Discrete}$$

$$f(x,y) = h_X(x)\,h_Y(y) \qquad \text{Continuous}$$

④ More than 2 random variables

$$p(x_1, x_2 \ldots x_n) = P(X_1 = x_1, X_2 = x_2 \ldots X_n = x_n)$$

$$P(a_1 \leq X_1 \leq b_1 \ldots a_n \leq X_n \leq b_n) = \int_{a_1}^{b_1} \cdots \int_{a_n}^{b_n} f(x_1 \ldots x_n)\,dx_n \ldots dx_1$$

⑤ Conditional Probability

$$f_{Y|X}(y|x) = \frac{f(x,y)}{h_X(x)}$$

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6) Expected Value

$$E(h(x,y)) = \begin{cases} \sum_x \sum_y h(x,y) \cdot p(x,y) & \text{Discrete} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x,y) \cdot f(x,y)\, dx\, dy & \text{Continuous} \end{cases}$$

7) Covariance

$$E[(X-\mu_x)(Y-\mu_y)] = \sum_x \sum_y (X-\mu_x)(Y-\mu_y)\, p(x,y) \quad \text{Discrete}$$

$$= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (X-\mu_x)(Y-\mu_y) f(x,y)\, dy\, dx \quad \text{Continuous}$$

$$Cov(X,Y) = E[XY] - \underline{\underline{\mu_x \mu_y}}$$

8) Correlation Coefficient

$$\rho_{xy} = \frac{Cov(X,Y)}{\sigma_x \sigma_y}$$

$$Corr(aX+b, cY+d) = \underline{\underline{Corr(X,Y)}}$$

9) Statistics

$$\bar{x} = \frac{x_1 + x_2}{2}$$

$$s^2 = \underline{\underline{\Sigma(X-\bar{X})^2}}$$

(10) Distribution of Sample Mean

(1) $E(\bar{X}) = \mu_{\bar{X}} = \mu$

(2) $V(\bar{X}) = \sigma_{\bar{X}}^2 = \frac{\sigma^2}{n}$ $\qquad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

If we have $T_0 = \sum_i X_i \rightarrow$ $E(T_0) = n\mu$

$V(T_0) = n\sigma^2$

$\sigma_{T_0} = \sqrt{n}\,\sigma$

(11) For ~~standard~~ normal, we can approximate a standard distribution with $E(\bar{X})$ and $\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}}$

if $\underline{\underline{n \geq 30}}$

(12) Linear Combinations

$$Y = aX_1 + bX_2 + cX_3 \cdots$$

$$E(Y) = aE(X_1) + bE(X_2) + cE(X_3)$$

$$V(Y) = a^2V(X_1) + b^2V(X_2) + c^2V(X_3)$$ } if X_i are independent

$$V(Y) = \sum_i \sum_j a_i \, Cov(X_i X_j)$$

(4) Point Estimation

- Random sample from a normal distribution with μ & σ

$\hat{\mu} = \bar{X} \rightarrow$ MVUE for μ

Minimum Variance Unbiased Estimator

$$\text{bias} = E(\hat{\theta}) - \theta$$

If bias $= 0 \rightarrow$ unbiased

- $$\hat{\sigma}^2 = S^2 = \frac{\Sigma(x-\bar{x})^2}{n-1}$$

Only in Point Estimation

1. Normal Distribution $\rightarrow$ $\bar{X} \rightarrow$ Mean
2. Cauchy Distribution $\rightarrow$ $\tilde{X} \rightarrow$ Median
3. Uniform distribution $\rightarrow$ $X_e \rightarrow$ (Max + Min/2)
4. Use trimmed mean if none of these 3 situations $\rightarrow \bar{X}_{tr(10)}$

Discard smallest and largest 10% of space then average.

- Standard error of an estimator $\hat{\theta}$ is its standard deviation

$$\sigma_{\hat{\theta}} = \sqrt{V(\hat{\theta})}$$

Methods of Point Estimation

① Method of Moments $\quad E(X^k) = \frac{\Sigma X^k}{n} = \int x^k f(x)\,dx$

Use this to find out

② Method of Max Likelihood Estimation

Find fn function, differentiate → Equate to 0

↳ Likelihood $L = f(X) = f(x_1) f(x_2) \cdots f(x_n)$

$\frac{dL}{dP} = 0 \Rightarrow$ Get answer

⑤ Confidence Intervals

① Mostly will be 95% or 99% accurate

$$P(-1.96 < Z < 1.96) = 0.95$$

→ 95%

→ $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$

∴ We get Interval

$$\left(\bar{X} - 1.96\frac{\sigma}{\sqrt{n}}, \bar{X} + 1.96\frac{\sigma}{\sqrt{n}}\right)$$

② $100(1-\alpha)\%$ Confidence Level

$$\left(\bar{X} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right)$$

For 95% $Z_{\alpha/2} = Z_{0.025} = 1.95$

↳ If $P(Z > x) = 1 - 0.025$

$Z_\alpha = x$

Width of Interval $= 2 Z_{critical} \frac{\sigma}{\sqrt{n}}$

$$n = \left(\frac{2 Z_{\alpha/2} \sigma}{w} \right)^2$$

If $n > 40$ we use S instead of σ

$$\bar{x} \pm Z_{\alpha/2} \frac{S}{\sqrt{n}}$$

We cannot find n using S since we need for S. We divide range by 4 to get approx S

NOTE: If $np \geq 10$, $nq \geq 10$ Binomial $\rightarrow$ Normal Distribution

(3) One sided confidence intervals

$$\mu < \bar{x} + Z_\alpha \frac{S}{\sqrt{n}}$$

$$\mu > \bar{x} - Z_\alpha \frac{S}{\sqrt{n}}$$

If $n < 40$ and we have S, we use t distribution:

(4) t distributions

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$ with $n-1$ degrees of freedom

t values:

$t_{\alpha, v}$ — [redacted]

- $\hookrightarrow$ v df
- $\hookrightarrow$ Area to right of $t_{\alpha/2}$

We get

$$\left(\bar{x} - t_{\alpha/2, n-1} \cdot \frac{s}{\sqrt{n}},\ \bar{x} + t_{\alpha/2, n-1} \cdot \frac{s}{\sqrt{n}}\right) \leftarrow CI$$

Upper/lower Bound

$\hookrightarrow \bar{x} \pm t_{\alpha, n-1} \cdot \frac{s}{\sqrt{n}}$

Prediction interval

$\hookrightarrow \bar{x} \pm t_{\alpha/2, n-1} s \sqrt{\frac{n+1}{n}}$

$100(1-\alpha)\%$

(5) Confidence Intervals for variance and standard deviation

Let $X_1 \ldots X_n$ be a random sample from a normal distribution with μ, σ^2

$$\frac{(n-1)S^2}{\sigma^2} = \frac{\Sigma(X_i - \bar{X})^2}{\sigma^2}$$ has a chi-squared pdf with

n-1 df

$$\chi^2_{1-\alpha/2, n-1} < \frac{(n-1)S^2}{\sigma^2} < \chi^2_{\alpha/2, n-1}$$ for $100(1-\alpha)\%$

$$\boxed{\frac{(n-1)S^2}{\chi^2_{\frac{\alpha}{2}, n-1}} < \sigma^2 < \frac{(n-1)S^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}}$$

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NOTE:-

~~If n > 40, we use z~~

① Normal is given and σ is given → we use z

② Normal is given, σ is not given, but $n > 40$, we use z with s

③ Normal is given, σ is not given, s is given, $n < 40$, we use t distribution

s → Sample sd

$\bar{x}$ → Sample Mean

σ → Population sd

μ → Population Mean

⑥ Hypotheses

Ⓐ Null hypothesis → H_0 → Claim initially assumed to be true.

Alternative hypothesis → H_a → Assertion contradictory to H_0

$H_a : \theta > \theta_0$ where $H_0 : \theta \leq \theta_0$

$H_a : \theta < \theta_0$ where $H_0 : \theta \geq \theta_0$

$H_a : \theta \neq \theta_0$

Test statistic → function of sample data on which decision (reject H_0 or not) is based

Rejection region → set of all test statistic values for which H_0 will be rejected

(B) Type I error → Rejecting the null hypothesis H_0 when it is true
Type II error → Not rejecting H_0 when H_0 is false

(C) Find P(Type 1 error) and P(Type 2 error) using given data and rejection region

(D) $\alpha \longrightarrow$ small, $\beta \rightarrow$ increase reduce rejection region

(E) Level α Tests about a mean

(a) Normal Population with known mean

Null Hypothesis is $H_0 : \mu = \mu_0$

Test Statistic $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

Alternative Hypothesis	Rejection Region
$H_a : \mu > \mu_0$	$z \geq z_\alpha$
$H_a : \mu < \mu_0$	$z \leq -z_\alpha$
$H_a : \mu \neq \mu_0$	$z \geq z_{\alpha/2}$ or $z \leq -z_{\alpha/2}$

If z satisfies Rejection region, then H_0 is wrong

(b) β and Sample size distribution

↳ H_0 is not rejected when $\mu = \mu'$

$P\left(\bar{X} < \mu_0 + z_\alpha \dfrac{\sigma}{\sqrt{n}} \text{ when } \mu = \mu'\right)$

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H_a $\quad \mu > \mu_0 \qquad \phi\left(z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$

$\mu < \mu_0 \qquad 1 - \phi\left(-z_\alpha + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$

$\mu \neq \mu_0 \qquad \phi\left(z_{\alpha/2} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right) - \phi\left(-z_{\alpha/2} + \frac{\mu_0 - \mu'}{\sigma/\sqrt{n}}\right)$

$$n = \left[\frac{\sigma(z_\alpha + z_\beta)}{\mu_0 - \mu'}\right]^2 \quad \text{One tailed}$$

$$\left[\frac{\sigma(z_{\alpha/2} + z_\beta)}{\mu_0 - \mu'}\right]^2 \quad \text{Two tailed}$$

(C) Large Sample Test with S given, use $Z = \frac{\bar{X} - \mu}{S/\sqrt{n}}$

(D) When n is small & S is given, use t distribution $T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$

$t_{\alpha, n-1}$

df

$H_0: \mu = \mu_0 \qquad H_a: \mu > \mu_0 \qquad t \geq t_{\alpha, n-1}$ (upper-tailed)

$t = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \qquad H_a: \mu < \mu_0 \qquad t \leq -t_{\alpha, n-1}$ (lower-tailed)

$H_a: \mu \neq \mu_0 \qquad t \geq t_{\alpha/2, n-1}$ or $t \leq -t_{\alpha/2, n-1}$ (two-tailed)

(F) Tests concerning Population Proportion

$$Z = \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$$

	Rejection Region	
$H_a: p > p_0$	$Z \geq Z_\alpha$	$np_0 \geq 10$
$H_a: p < p_0$	$Z \leq -Z_\alpha$	$n(1-p_0) \geq 10$
$H_a: p \neq p_0$	$Z \geq Z_{\alpha/2}$ or $Z \leq -Z_{\alpha/2}$	

$$E(Z) = \frac{p' - p_0}{\sqrt{p_0(1-p_0)/n}} \qquad V(Z) = \frac{p'(1-p')n}{p_0(1-p_0)/n}$$

(G) β for Population proportion

$H_a: p > p_0$ $\quad \phi\left[\frac{p_0 - p' + Z_\alpha\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right]$

$p < p_0$ $\quad 1 - \phi\left[\frac{p_0 - p' - Z_\alpha\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right]$

$p \neq p_0$ $\quad \phi\left[\frac{p_0 - p' + Z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right]$

(H) Concept of P Value

→ Probability calculated assuming H_0 is true
→ Not Probability that H_0 is true
→ It's not error Probability

Reject if $P \leq \alpha$
Accept if $P > \alpha$

→ $1 - \phi(z)$ Upper tailed z test
$\phi(z)$ lower tailed z test
$2(1 - \phi(z))$ two tailed test

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7) Linear Regression

ⓐ $Y = \beta_0 + \beta_1 x + \epsilon$

$E(\epsilon) = 0 \quad V(\epsilon) = \sigma^2$

ⓑ $f(b_0, b_1) = \sum_i \left[y_i - (b_0 - b_1 x_i)\right]^2$

ⓒ $b_1 = \hat{\beta}_1 = \dfrac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \dfrac{S_{xy}}{S_{xx}}$

$S_{xy} = \sum x_i y_i - \dfrac{(\sum x_i)(\sum y_i)}{n}$

$S_{xx} = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n}$

$b_0 = \hat{\beta}_0 = \dfrac{\sum y_i - \hat{\beta}_1 \sum x_i}{n} = \bar{y} - \hat{\beta}_1 \bar{x}$

Residuals $\longrightarrow y_i - \hat{y}_i$

ⓓ $SSE = \sum (y_i - \hat{y}_i)^2 = \sum (y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i))^2$

$\sigma^2 = s^2 = \dfrac{SSE}{n-2} = \dfrac{\sum (y_i - \hat{y}_i)^2}{n-2}$

$SSE = \sum y_i^2 - \beta_0 \sum y_i - \beta_1 \sum x_i y_i$

Coefficient of Determination

$$r^2 = 1 - \frac{SSE}{SST}$$

$$SST = S_{yy} = \Sigma(y_i - \bar{y})^2$$
$$= \Sigma y_i^2 - \frac{(\Sigma y_i)^2}{n}$$

$$SSR = \Sigma(\hat{y}_i - \bar{y})^2 = SST - SSE$$

Normal Equations

$$n b_0 + \Sigma x_i b_1 = \Sigma y_i$$

$$\Sigma x_i b_0 + (\Sigma x_i^2) b_1 = \Sigma x_i y_i$$