Predictability in a Dynamic Model of Game Learning

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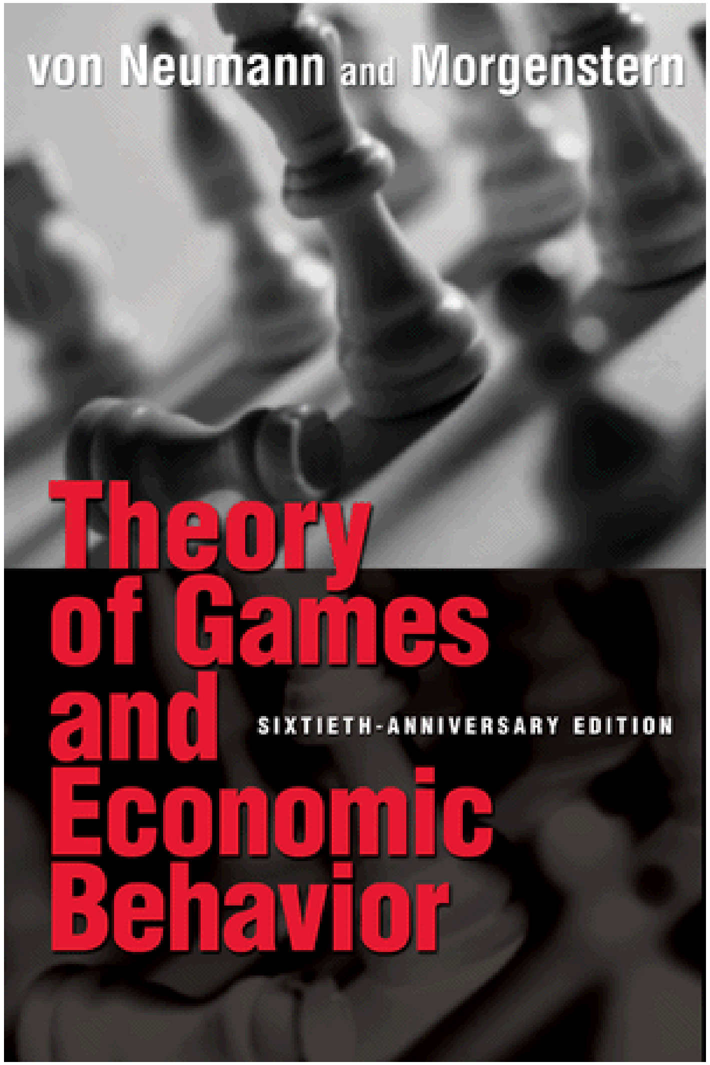
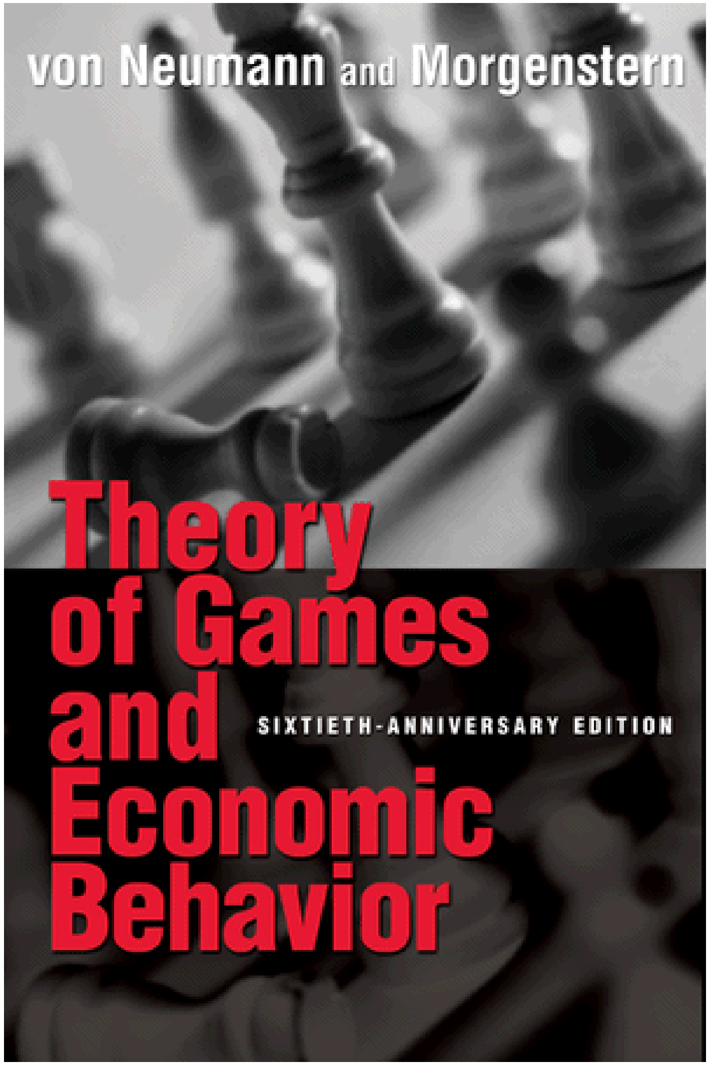


"…our theory is thoroughly static … a static theory deals with equilibria.

The essential characteristic of an equilibrium is that it has no tendency to change"

- pp 44-45

Image credit: Princeton University Press



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(or *Guess 2/3 the Average*)

Pick a number between 0 – 100

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- k(0) players pick at random, average of 50
  - k(1) players assume everyone is k(0), guess $2/3 \times 50 = 33$
    - k(2) players assume everyone is k(1), guess $2/3 \times 33 = 22$
      - ...
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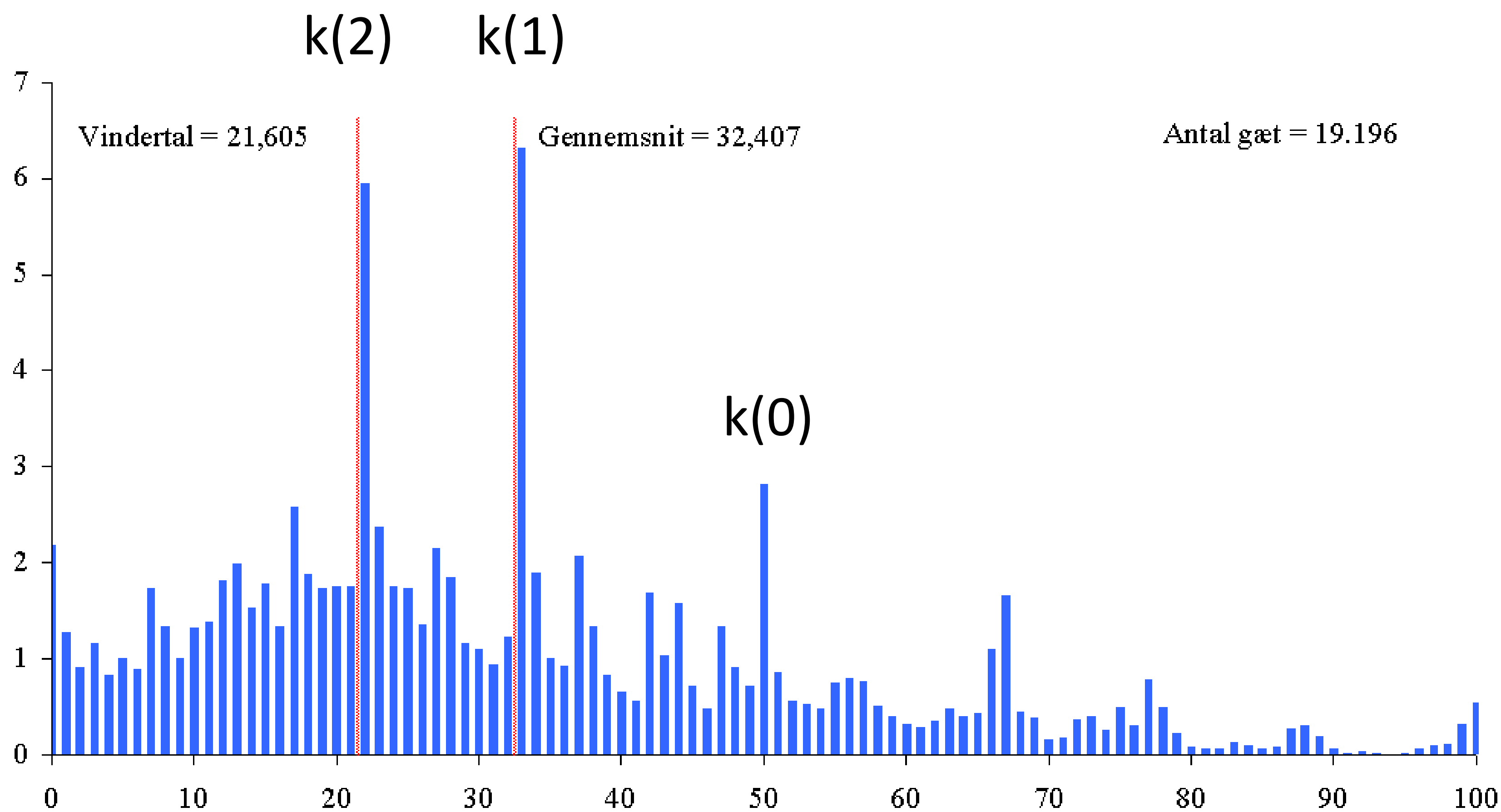
Classical game theory result
(Nash Equilibrium)

Real world results from a Danish newspaper competition involving 19,192 people



Experience Weighted Attraction

A dynamic game learning model

- Two players learning to play a game with outcomes a_{ij}
- Probabilities of playing a strategy i updated each round

$$x_i(t+1) = \frac{x_i(t)^{1-\lambda} \exp\left[\beta \sum_k a_{ik} \bar{x}_k(t)\right]}{\sum_j x_j(t)^{1-\lambda} \exp\left[\beta \sum_k a_{jk} \bar{x}_k(t)\right]}$$

Two parameters:

- λ – *memory loss* parameter
- β – *intensity of selection* parameter

Define a k(1) player as one using this map.

k-Level Extension

Allow players to employ '*p*-beauty contest' reasoning:

- A k(*n*) player assumes their opponent will update their strategy according to a k(*n* - 1) rule

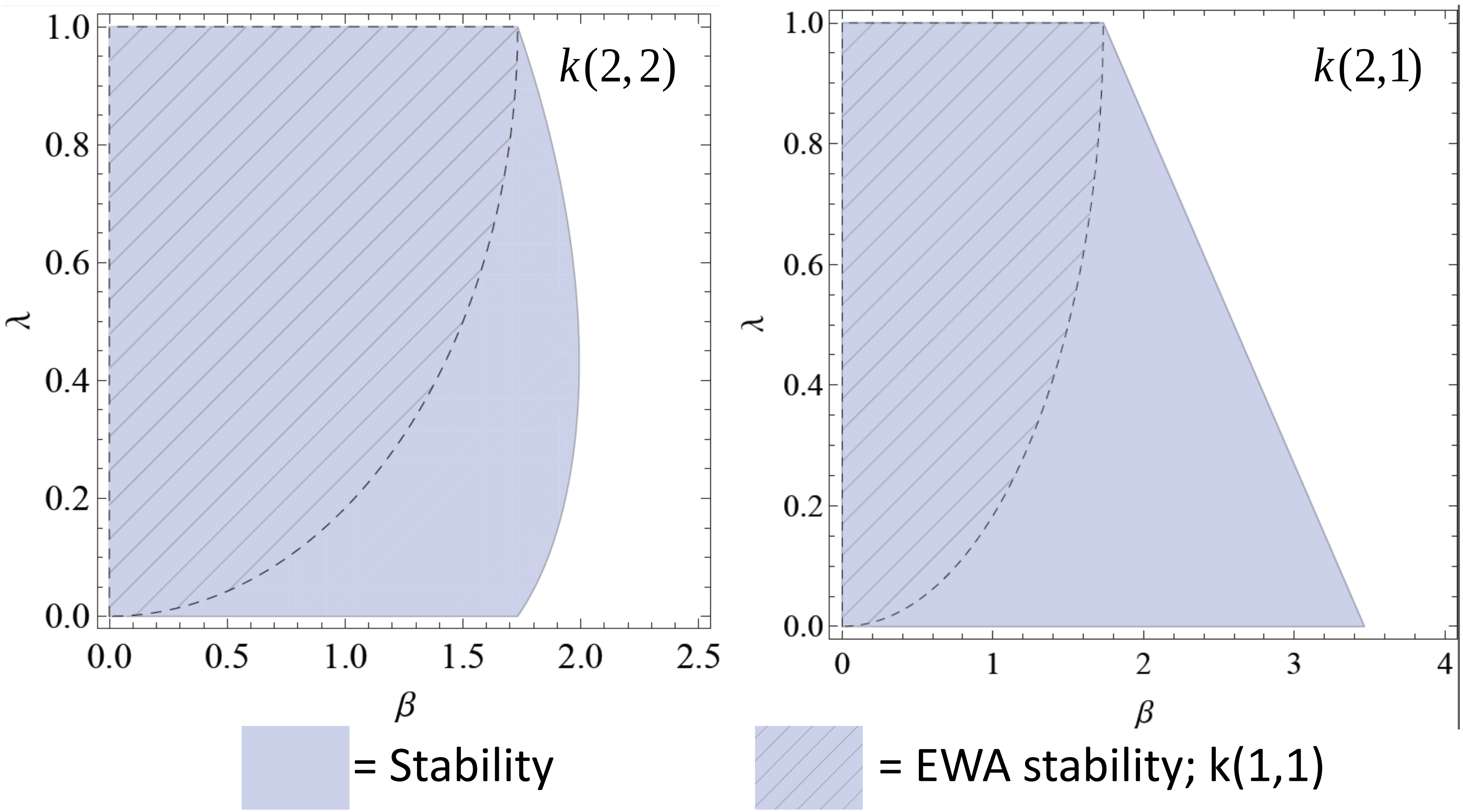
$$x_i^{(n)}(t+1) = \frac{1}{\mathcal{Z}^{(n)}} \cdot x_i^{(n)}(t)^{1-\lambda} \cdot \exp\left\{ \beta \sum_{j=1}^{N} \pi_{ij} \bar{x}_j^{(n-1)}(t+1) \right\}$$

$$n \in \{2, 3, 4 \ldots\}$$

- k(1) is updated as before.

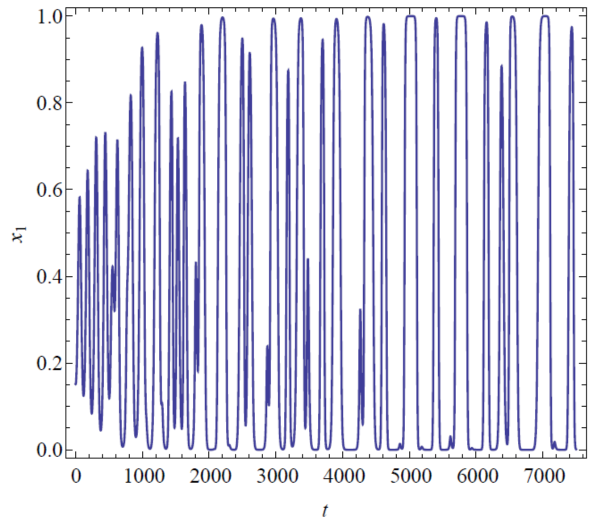
Comparative behaviour:

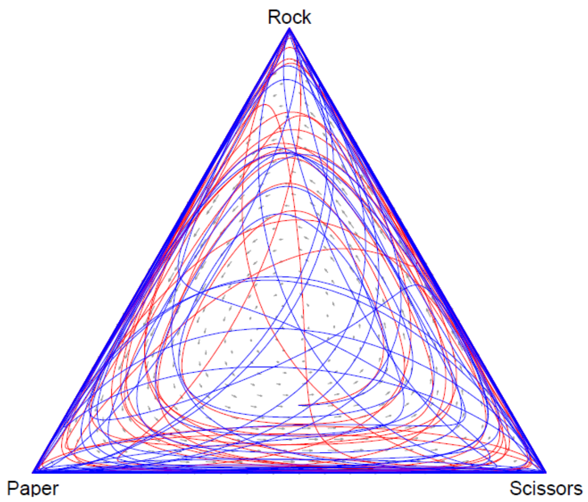
Rock Paper Scissors



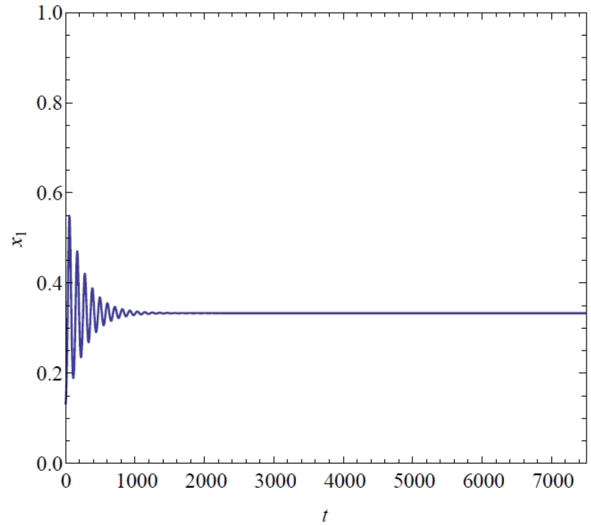
Comparative behaviour:
Asymmetric Rock Paper Scissors

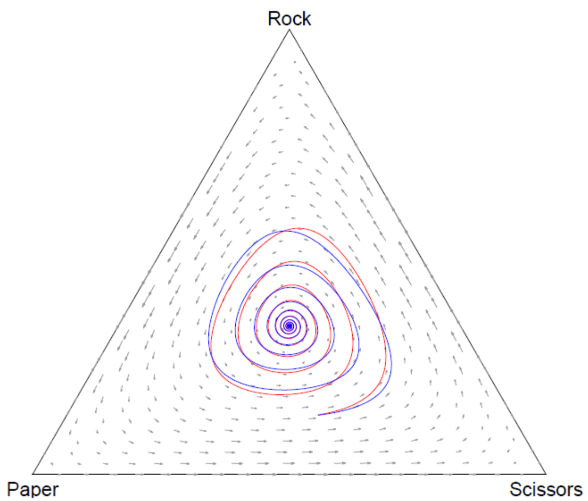
k(1,1):
Chaotic





k(2,2):
Stable





Comparative behaviour:

LARGE RANDOM GAMES

Player has payoffs

$$a_{ij}$$

Opponent has payoffs

$$b_{ij}$$

Drawn from a multivariate normal distribution and correlated such that

$$\mathbb{E}[a_{ij}b_{ji}] = \Gamma$$

$$\Gamma = \begin{cases} -1 & \text{- payoffs anticorrelated} \\ 0 & \text{- payoffs uncorrelated} \\ +1 & \text{- payoffs correlated} \end{cases}$$

Comparative behaviour:
LARGE RANDOM GAMES

- Characterise predictability of long term dynamics by largest Lyapunov exponent λ_L:

$\lambda_L<0$: Predictable
e.g. fixed points,
limit cycles.

$\lambda_L>0$: Unpredictable
e.g. chaos.

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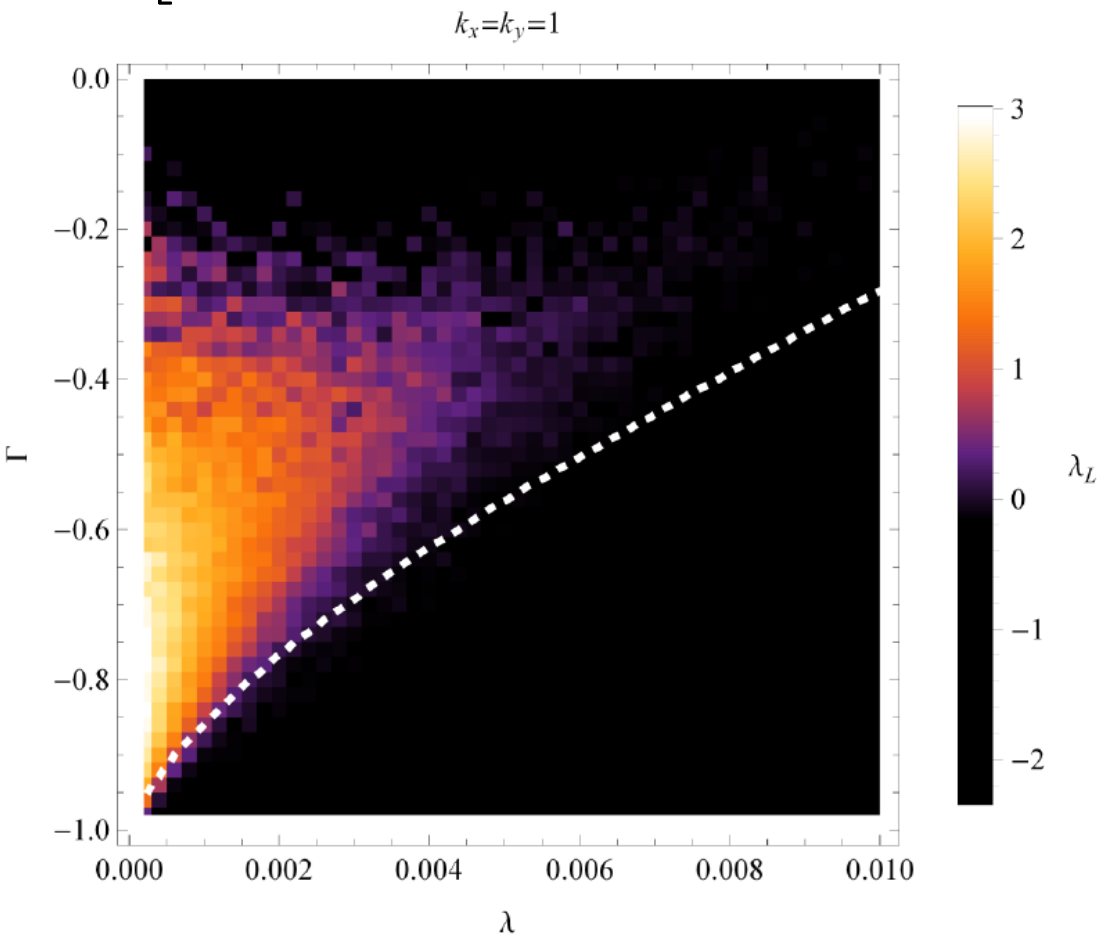

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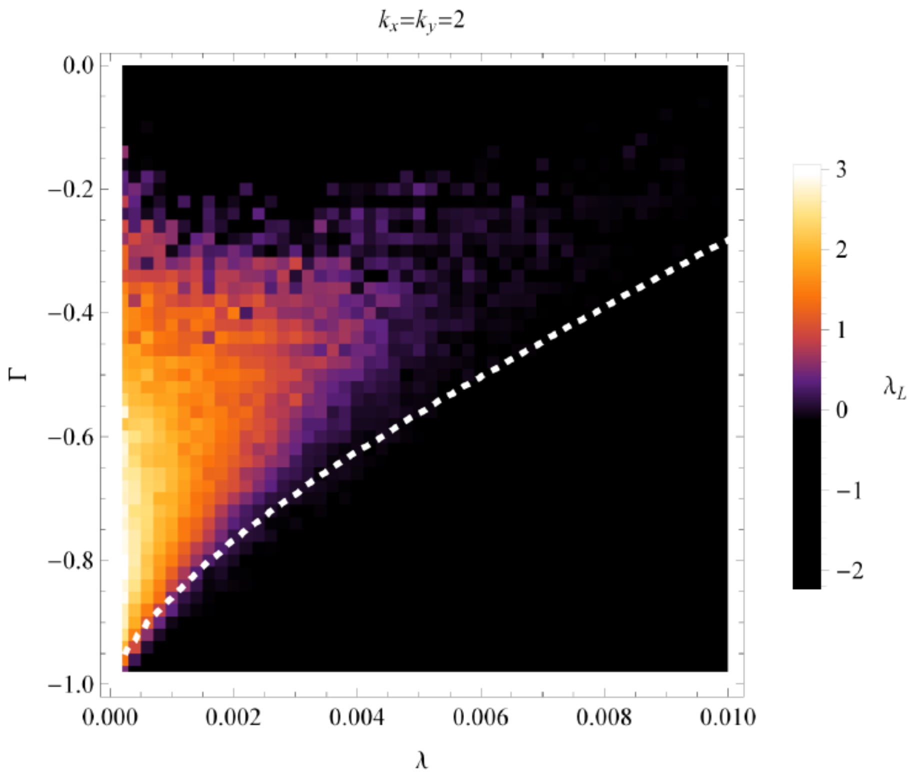
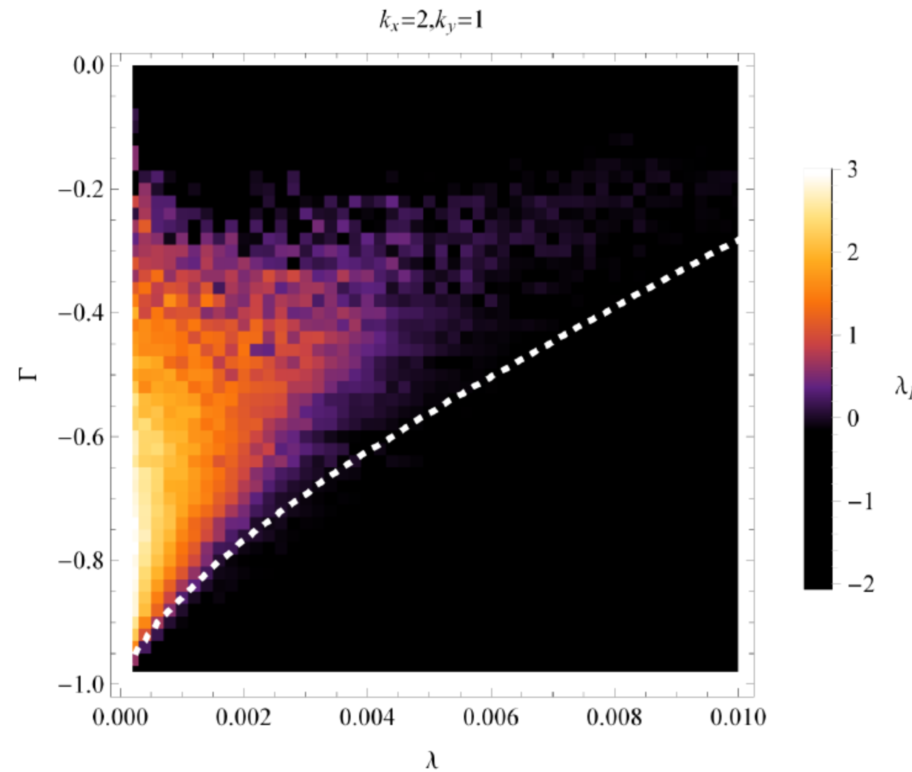
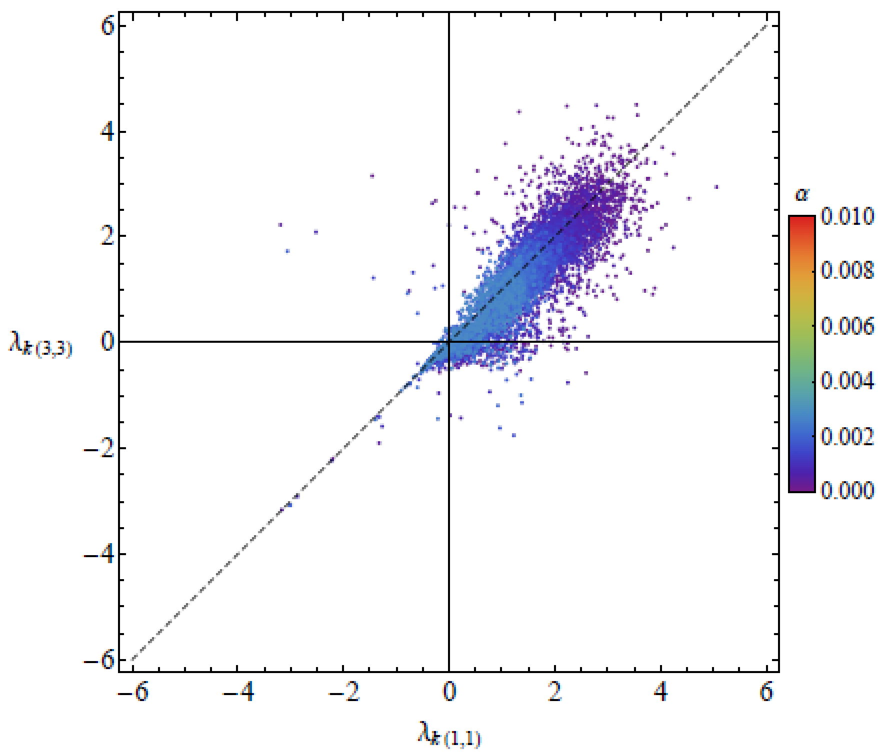
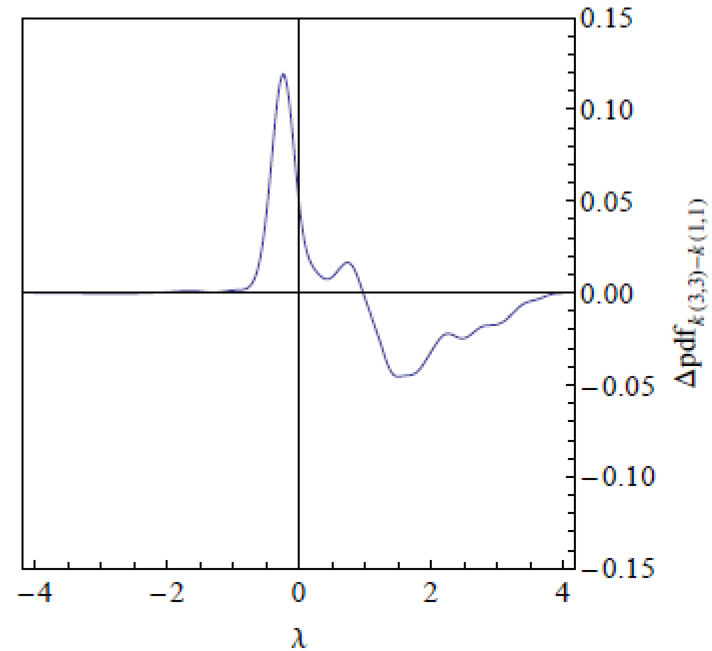




k(2,2), k(2,1) very similar to k(1,1)
No significant net shift in predictability

Comparative behaviour:
LARGE RANDOM GAMES





k(3,3) vs. k(1,1)

Net shift toward more predictable behaviour in 'hot' region.

Similar effect observed in k(5,5)

Summary

- Extended the EWA learning model *(Camerer & Ho 1999)*
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  - Recursive anticipation of opponent's reasoning to an arbitrary depth; k(*n*).

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- Extended the EWA learning model *(Camerer & Ho 1999)*
  - p-*Beauty Contest* logic: players may pre-empt their opponent's adaptation.
  - Recursive anticipation of opponent's reasoning to an arbitrary depth; k(*n*).

- Compared predictability of behaviour in higher k-level dynamics with k(1,1) EWA.
  - Changes in predictability for games, both simple and complicated.
  - General schema unchanged when averaged over large ensembles of games.

Conclusions

- Behaviour of players in a deterministic model of human decision making displays inherent unpredictability.
  - Our model of k-level reasoning has limited stabilising effect.

- Experimental studies suggest human decision-making lies predominantly in the k($n \leq 3$) regime. (Nagel 1999, Camerer & Ho 2004)
  - Higher orders are unlikely to contribute.