Temă - Exerciții teorie

1. a. $X = \begin{pmatrix} 2 & 3 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$, $Y = \begin{pmatrix} -3 & -2 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

$3X = \begin{pmatrix} 6 & 9 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$ $X^{-1} = \begin{pmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$

$\cos(\frac{\pi}{2} \cdot X) = \begin{pmatrix} -1 & 0 \\ \frac{1}{5} & \frac{4}{5} \end{pmatrix}$ $Y^2 = \begin{pmatrix} 9 & 4 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

$Y+3 = \begin{pmatrix} 0 & 1 \\ \frac{4}{5} & \frac{1}{5} \end{pmatrix}$

b. $X = \begin{pmatrix} 0 & 9 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ $Y = \begin{pmatrix} -3 & 1 \\ \frac{1}{7} & \frac{6}{7} \end{pmatrix}$

$X-1 = \begin{pmatrix} -1 & 8 \\ 1/2 & 1/2 \end{pmatrix}$ $X^{-2} = \begin{pmatrix} 0 & 9^{-2} \\ 1/2 & 1/2 \end{pmatrix}$

$\sin\left(\frac{\pi}{4} X\right) = \begin{pmatrix} 0 & \sqrt{2}/2 \\ 1/2 & 1/2 \end{pmatrix}$

$Y \cdot 5 = \begin{pmatrix} -15 & 5 \\ 1/7 & 6/7 \end{pmatrix}$

$e^Y = \begin{pmatrix} e^{-3} & e \\ 1/7 & 6/7 \end{pmatrix}$

Analog se rezolvă și punctele c și d, folosind:

$$A = \begin{pmatrix} x_1 & x_2 & \dots & x_k \\ p_1 & \dots & & p_k \end{pmatrix}$$

$$\Downarrow$$

$$f(A) = \begin{pmatrix} f(x_1) & \dots & f(x_k) \\ p_1 & \dots & p_k \end{pmatrix}.$$

2.

a. $X = \begin{pmatrix} 2 & 3 \\ 1/5 & 4/5 \end{pmatrix} \quad Y = \begin{pmatrix} -3 & -2 \\ 4/5 & 1/5 \end{pmatrix}$

$$\Rightarrow 2X + 3Y = \begin{pmatrix} 4-9 & 4-6 & 6-9 & 6-6 \\ 1/5 \cdot 4/5 & 1/5 \cdot 1/5 & 4/5 \cdot 4/5 & 4/5 \cdot 1/5 \end{pmatrix}$$

$$= \begin{pmatrix} -5 & -2 & -3 & 0 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix}$$

$$3X - Y = \begin{pmatrix} 6+3 & 6+2 & 9+3 & 9+2 \\ 1/5 \cdot 4/5 & 1/5 \cdot 1/5 & 4/5 \cdot 4/5 & 4/5 \cdot 1/5 \end{pmatrix}$$

$$= \begin{pmatrix} 9 & 8 & 12 & 11 \\ 4/25 & 1/25 & 16/25 & 4/25 \end{pmatrix}$$

$$X^2 \cdot Y^3 = \begin{pmatrix} 4 \cdot (-27) & 4(-8) & 9 \cdot (-27) & 9 \cdot (-8) \\ 1/5 \cdot 4/5 & 1/5 \cdot 1/5 & 4/5 \cdot 4/5 & 4/5 \cdot 1/5 \end{pmatrix}$$

$$= \begin{pmatrix} -108 & -32 & -243 & -72 \\ 1/25 & 4/25 & 16/25 & 4/25 \end{pmatrix}$$

b. $X = \begin{pmatrix} 0 & 3 \\ 1/2 & 1/2 \end{pmatrix} \quad \cancel{Y = \begin{pmatrix} 1 & -1 \\ 1/6 & 5/6 \end{pmatrix}} \quad Y = \begin{pmatrix} -3 & 1 \\ 1/7 & 6/7 \end{pmatrix}$

$$X - Y = \begin{pmatrix} 0+3 & 0-1 & 3+3 & 3-1 \\ 1/2 \cdot 1/7 & 1/2 \cdot 6/7 & 1/2 \cdot 1/7 & 1/2 \cdot 6/7 \end{pmatrix}$$

$$= \begin{pmatrix} 3 & -1 & 12 & 8 \\ 1/14 & 6/14 & 1/14 & 6/14 \end{pmatrix}$$

$$\cos(XY\pi) = \begin{pmatrix} \cos(-30) & \cos(0) & \cos(-27\pi) & \cos(9\pi) \\ 1/14 & 6/14 & 1/14 & 6/14 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & -1 & -1 \\ 1/14 & 6/14 & 1/14 & 6/14 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & -1 \\ 1/2 & 1/2 \end{pmatrix}$$

$$x^2+3y=\begin{pmatrix}0-9 & 0+3 & 81-9 & 81+3\\ 1/2\cdot 1/7 & 1/2\cdot 6/7 & 1/2\cdot 1/7 & 1/2\cdot 6/7\end{pmatrix}$$

$$=\begin{pmatrix}-9 & 3 & 72 & 84\\ 1/14 & 6/14 & 1/14 & 6/14\end{pmatrix}$$

Analog se ~~[illegible]~~ rezolvă și c.d. folosind formula:

~~$f(x,y)=$~~ $X=\begin{pmatrix}x_1 & --- & x_k\\ p_{x_1} & --- & p_{x_k}\end{pmatrix}$

$Y=\begin{pmatrix}y_1 & --- & y_l\\ p_{y_1} & --- & p_{y_l}\end{pmatrix}$

$\Rightarrow f(X,Y)=\begin{pmatrix}f(x_i,y_j)\\ p_{x_i}\cdot p_{y_j}\end{pmatrix}$

Ex 3: a. $X=\begin{pmatrix}1 & 2\\ p & q\end{pmatrix}$, $Y=\begin{pmatrix}3 & 9\\ 0{,}1 & \frac{p^2+0{,}02}{2}\end{pmatrix}$

X, Y bine definite $\Rightarrow p+q=0{,}1+\frac{p^2+0{,}02}{2}=1$

$p, q, 0{,}1, \frac{p^2+0{,}02}{2}>0$

$\Rightarrow q=1-p$

$p^2+0{,}2+0{,}02=1$

$\Leftrightarrow p^2=0{,}78 \Leftrightarrow p=\pm\sqrt{0{,}78}=\pm 0{,}881\ldots$

dar $p>0 \Rightarrow p=0{,}881\ldots=\sqrt{0{,}78}$

$q=0{,}119\ldots=1-\sqrt{0{,}78}$

b. $X=\begin{pmatrix}1 & 2 & 3\\ 1/3 & p & q\end{pmatrix}$ $X^2=\begin{pmatrix}1 & 4 & 9\\ p & p & p^2\end{pmatrix}$

dar $X^2=\begin{pmatrix}1 & 4 & 9\\ 1/3 & p & q^2\end{pmatrix}$

$\Rightarrow p=1/3,\ p^2=q^2=1/9 \Rightarrow X=\begin{pmatrix}1 & 2 & 3\\ 1/3 & 1/3 & 1/9\end{pmatrix}$ ~~OK~~

Suma probabilităților nu este 1.

c. $X = \begin{pmatrix} -1 & 0 & 1 \\ p & p^2 & q \end{pmatrix}$ $\quad X^2 = \begin{pmatrix} 0 & 1 \\ 9/25 & 16/25 \end{pmatrix}$

dar $X^2 = \begin{pmatrix} 0 & 1 \\ p^2 & q+p \end{pmatrix}$

$\Rightarrow p+q = \frac{16}{25} \quad p^2 = \frac{9}{25}$

~~$\Leftrightarrow 25p^2 + p = \frac{16}{25}$~~

Dar $p > 0 \Rightarrow p = \frac{3}{5} \Rightarrow q = \frac{16}{25} - \frac{15}{25} = \frac{1}{25}$.

observăm că $X = \begin{pmatrix} -1 & 0 & 1 \\ 3/5 & 9/25 & 1/25 \end{pmatrix}$ este o variabilă aleatoare validă

$\Rightarrow p = \frac{3}{5} \quad q = \frac{1}{25}$

d. $X = \begin{pmatrix} -1 & 1 \\ 2p & q \end{pmatrix}$ $\quad Y = \begin{pmatrix} 0 & 1 \\ q & 7q \end{pmatrix}$

$\Rightarrow q + 7q = 1 \Leftrightarrow q = \frac{1}{8}$

$2p + q = 1 \Leftrightarrow p = \frac{1-q}{2} = \frac{7}{16}$

observăm că $\begin{pmatrix} -1 & 1 \\ 7/8 & 1/8 \end{pmatrix}$ și $\begin{pmatrix} 0 & 1 \\ 1/8 & 7/8 \end{pmatrix}$ sunt variabile aleatoare valide

$\Rightarrow p = \frac{7}{16} \quad q = \frac{1}{8}$

Ex 4:

a. $2X + 3Y = \begin{pmatrix} 4-9 & 4-6 & 6-9 & 6-6 \\ \frac{4}{25} & \frac{1}{25} & 16/25 & 4/25 \end{pmatrix} = \begin{pmatrix} -5 & -3 & -2 & 0 \\ 4/25 & 16/25 & 1/25 & 4/25 \end{pmatrix}$

$P(2X+3Y) > 1 = 0$

$P(2x+3y>0 \mid x>0) = \frac{P(2x+3y>0 \cap x>0)}{P(x>0)} = \frac{0}{1} = 0$

$P(2x+3y<3 \mid y<-2) = \frac{P(2x+3y<3 \cap y<-2)}{P(y<-2)} = \frac{4/5}{4/5} = 1$

$P(X^2 \cdot Y^3 > 3) = 0$

$P(X^2 \cdot Y^3 \leq 3) = 1$

$$2X+3Y < 3X-Y = \begin{pmatrix} 4-9<6+3 & 4-6<6+2 & 6-5<9+3 & 6-6<9+2 \\ 1/5 \cdot 4/5 & 1/5 \cdot 1/5 & 16/25 & 4/25 \end{pmatrix}$$

$$= \begin{pmatrix} \text{Adevărat} \\ 1 \end{pmatrix}$$

$\Rightarrow P(2X+3Y < 3X-Y) = 1$

b. $P(X-Y < X^2+3Y) = P(X-Y-X^2-3Y<0)$

$= P(X-X^2-4Y<0)$

$$X-X^2-4Y = \begin{pmatrix} 12 & -4 & -60 & -76 \\ 1/14 & 6/14 & 1/14 & 6/14 \end{pmatrix}$$

$\Rightarrow P(X-X^2-4Y<0) = 6/14 + 1/14 + 6/14 = \frac{13}{14}$

Celelalte exerciții se rezolvă analog