# Rezolvare probleme

1.3 i. $A=(3,3)$
$B=(2,4)$
$C=(5,1)$

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 2 & 5 \\ 3 & 4 & 1 \end{vmatrix} = (2-20)-(3-15)+(12-6) =$$
$$= -18 + 12 + 6 = 0$$

$\Rightarrow A, B, C$ coliniare

$\Rightarrow r(A,B,C)$ : $\overrightarrow{AB} = B-A = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\overrightarrow{BC} = C-B = \begin{pmatrix} 3 \\ -3 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} -1 \\ 1 \end{pmatrix} = r\begin{pmatrix} 3 \\ -3 \end{pmatrix} \Leftrightarrow r = -3$$

ii. $A=\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix}$, $B=\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$, $C=\begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$ $\Rightarrow \Delta = \begin{vmatrix} 1 & 2 & 4 \\ 4 & 3 & 1 \\ -2 & -1 & 1 \end{vmatrix} = (3+1) - 2(4+2) + 4(-4+6)$
$= 4 - 12 + 8 = 0$

$B-A = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $C-B = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = r \cdot \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix} \Rightarrow r = 2$$

1.4. $A, P, B$ coliniere $\Rightarrow$

$$\begin{vmatrix} 1 & 1 & 1 \\ 6 & \alpha & 2 \\ 2 & \beta & -2 \end{vmatrix} = 0 \iff -2\alpha - 2\beta - (-12-4) + 6\beta - 2\alpha = 0$$

$$\iff -4\alpha + 4\beta + 16 = 0$$

$$\iff \alpha = \beta + 4$$

$$r(APB) = 2 \iff P - A = 2 \cdot (B - P) \iff P = \frac{2B + A}{3}$$

$$= \begin{pmatrix} 10/3 \\ -2/3 \end{pmatrix}$$

1.5 $P = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$Q = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$

$R_1 = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$

$R_2 = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

$R_3 = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

$$\Delta_1 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 2 & 4 & 8 \end{vmatrix} = 0 \Rightarrow \text{pe dreaptă}$$

$$\Delta_2 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 6 \\ 2 & 4 & 0 \end{vmatrix} = (-24) - (-12) + (8-8) = 12 - 24 = -12$$

$\hookrightarrow$ în dreapta

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -2 \\ 2 & 4 & -1 \end{vmatrix} = (-4+8) - (-2+4) + (8-8) = 4 - 2 = 2$$

$\hookrightarrow$ în stânga.

1.6 Luăm 3 puncte la mișto: $P_1, P_2, P_3$.

$P_3$ o să afle fie în stânga, fie în dreapta lui $P_1P_2$

Alegem $R_4$ ca fiind simetricul față de mij. lui $P_1P_2$

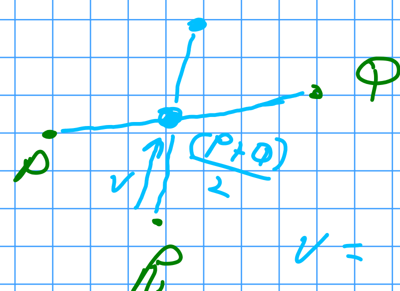
$$P = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

$$Q = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

$$R_1 = \begin{pmatrix} 4 \\ 6 \\ 2 \end{pmatrix}$$

Verificăm că nu sunt coliniare:

$$\begin{vmatrix} 1 & 2 & 4 \\ 2 & 3 & 6 \\ 3 & 4 & 2 \end{vmatrix} = (6-24) - 2(4-18) + 4(8-9)$$
$$= -18 + 28 - 4 = 6 \neq 0$$



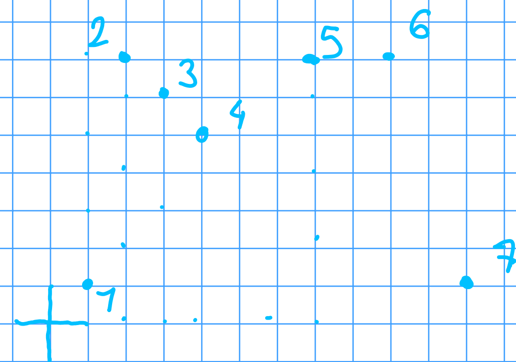

$$v = \frac{P+Q}{2} - R_1$$

$$R_2 = R_1 + 2v = P + Q - R_1$$

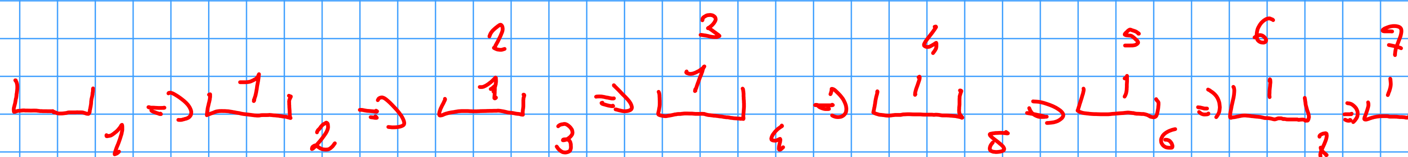
$$R_2 = \begin{pmatrix} -1 \\ -1 \\ 5 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & -1 \\ 2 & 3 & -1 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -1 & 1 \\ 3 & -2 & 8 \end{vmatrix} = -8 + 2 = -6$$

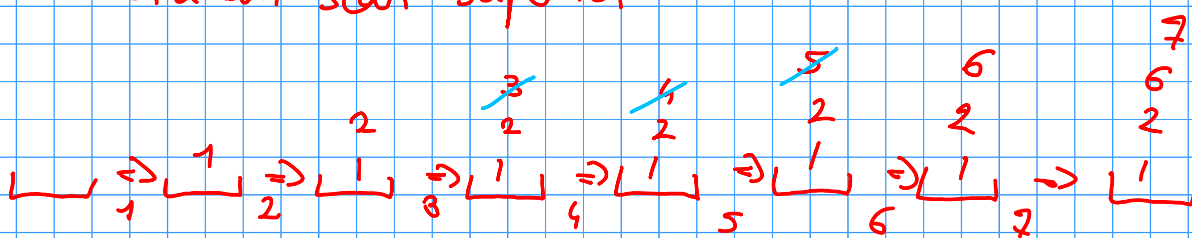
2.4



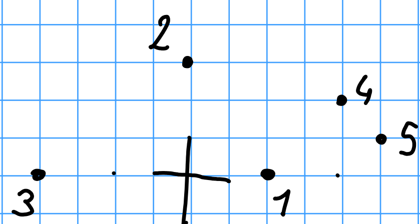

Graham Scan Inferior:




Graham Scan Superior




2.6




Incepem Jarvis march cu cel mai mic punct, care este 3. Ca pivot, il alegem ca pivot primul punct ($P_1$).

Incercăm să il înlocuim cu $P_2$, dar $P_1$ este mai în dreapta.

Incercăm să il înlocuim cu 4, 5, —— " ——

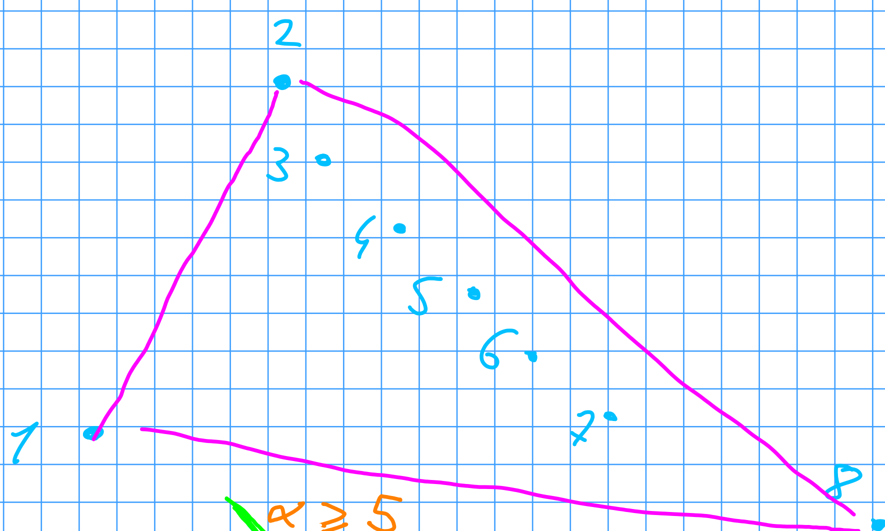
Așadar, continuăm cu $P_1$. Il alegem ca pivot ~~pe~~ 2.

Incercăm să il înlocuim cu 3, dar 3 e în stânga.

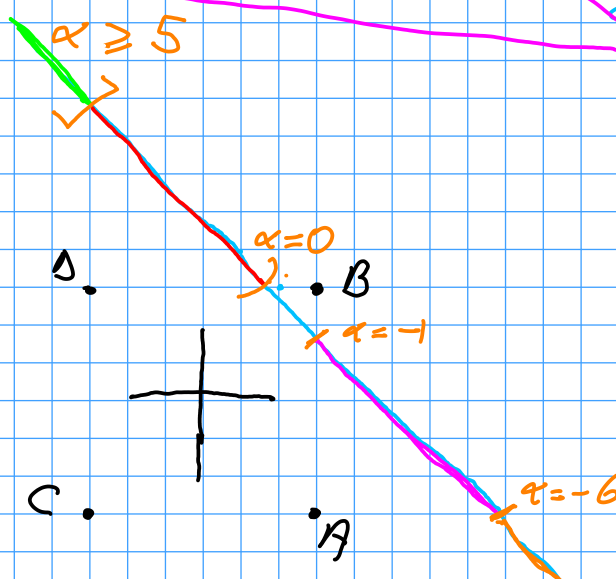
Il înlocuim cu 4, că e mai în dreapta

Il înlocuim pe P4 cu P5 că e mai în dreapta

## 2.7




## 2.8




I $\alpha \geq 5 \Rightarrow F = \{M, A, B, C\}$

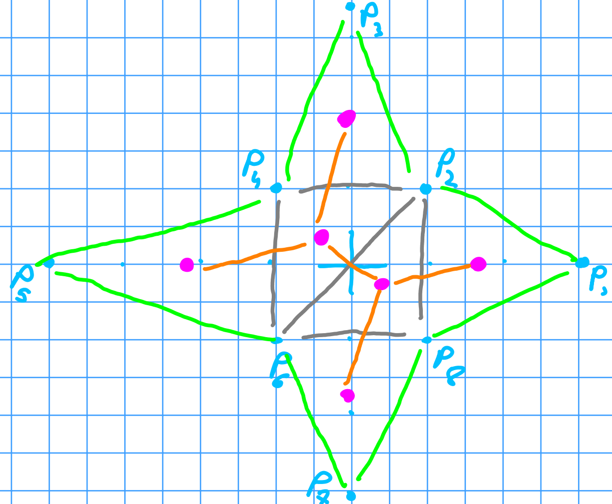
II $0 < \alpha < 5 \Rightarrow F = \{M, A, B, C, \Delta\}$

III $-1 \leq \alpha \leq 0 \Rightarrow F = \{A, B, C, \Delta\}$

IV $-6 < \alpha < -1 \Rightarrow F = \{M, A, B, C, \Delta\}$

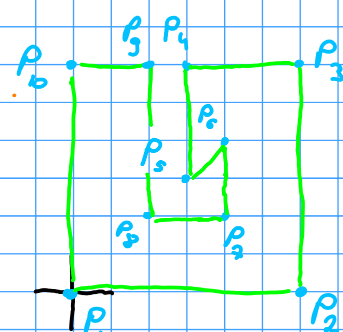
V $\alpha \leq -6 \Rightarrow F = \{M, B, C, \Delta\}$

3.4

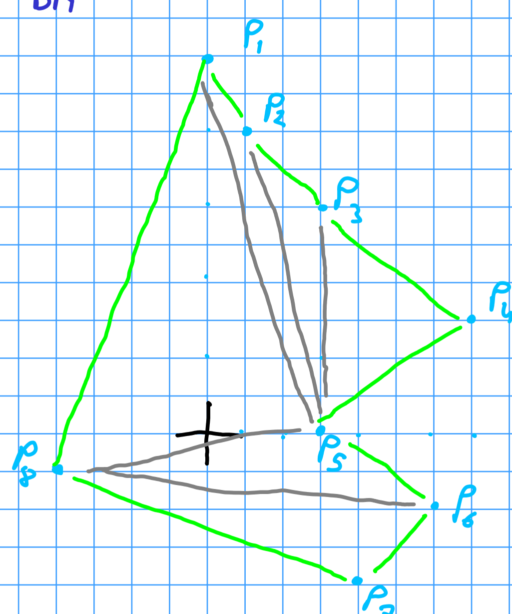



3.6

| | Principal | Convex |
|---|---|---|
| $P_1$ | NU | DA |
| $P_2$ | NU | DA |
| $P_3$ | NU | DA |
| $P_4$ | NU | DA |
| $P_5$ | DA | DA |
| $P_6$ | DA | NU |
| $P_7$ | NU | NU |
| $P_8$ | NU | NU |
| $P_9$ | DA | DA |
| $P_{10}$ | DA | DA |



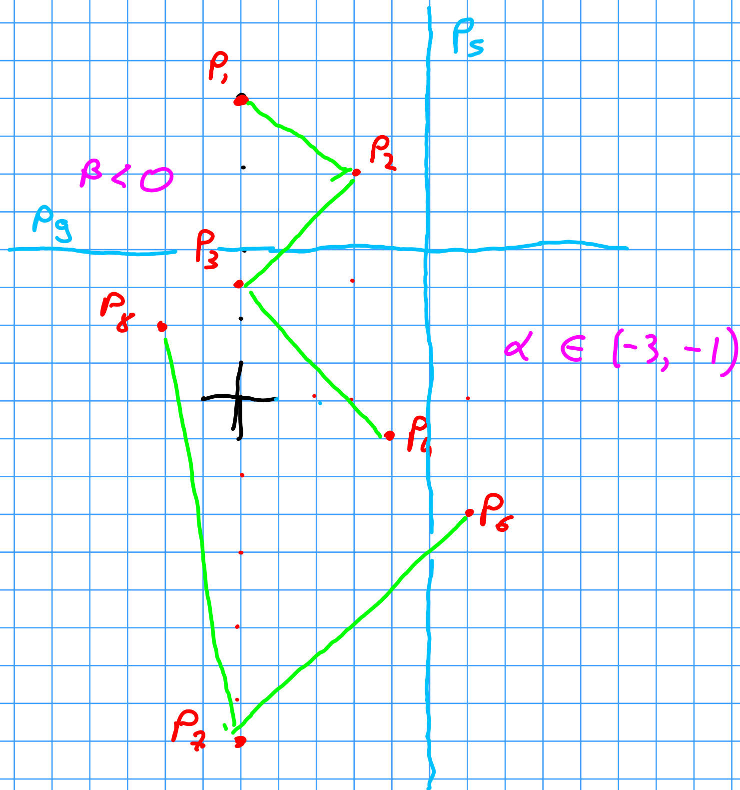

3.7




| $P_1$
| $P_1$ $P_2$
| $P_1$ $P_2$ $P_3$
| $P_1$ $P_2$ $P_3$ $P_4$
| $P_1$ $P_5$
| $P_5$ $P_8$
| $P_8$ $P_6$

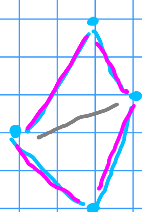
Poligonul este y-monoton

3.8

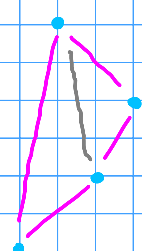



3.9

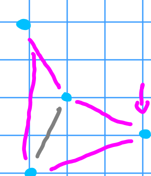
Ca7 I



Ca8 II A



Ca7 II B



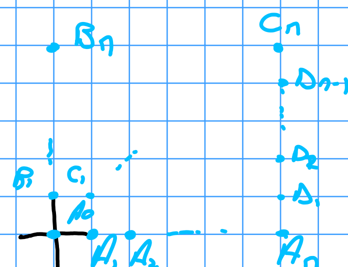
4. 4

$N = 4n - 4$

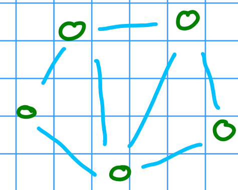
$k = 3n - 2$

$\Rightarrow \Delta = 2N - k - 2 =$
$8N - 8 - 3n + 2 - 2$
$= 5N - 8$

$m = 3N - k - 3 = 12n - 12 - 3n + 2 - 3$
$= 9n - 13$



4.5 $\begin{cases} 2N - k - 2 = 3 \\ 3N - k - 3 = 7 \end{cases}$

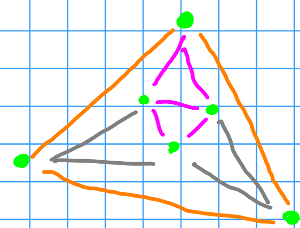


$\Leftrightarrow \begin{cases} N = 5 \\ k = 5 \end{cases}$

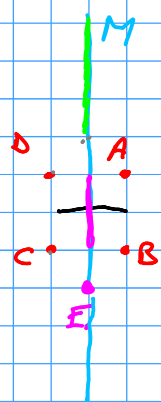
4.6

$N = 6$

$3N - k - 3 = 12$
$18 - k = 15$
$k = 3$



4.7




I $\gamma > 1 \Rightarrow N = 6$, $k = 6$

II $\gamma \in (-2, 1) \Rightarrow N = 6$, $k = 5$

III $\gamma = -2 \Rightarrow N = 5$, $k = 5$

IV $\gamma < -2 \Rightarrow N = 6$, $k = 5$