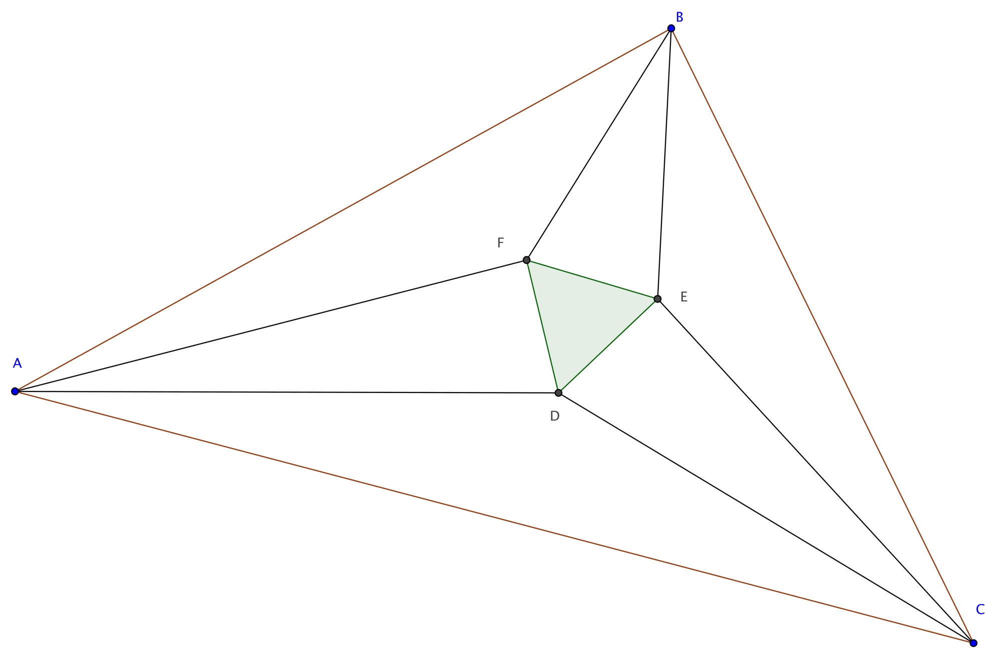
Transactions

in

Euclidean Geometry



Issue # 2

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The Diagonals of a Rhombus

Rebecca Shere

September 27, 2016

Theorem 1.2. The diagonals of a rhombus must cross inside the rhombus.

Proof. Let ABCD be a rhombus.

Let inside the rhombus be thought of as in the area contained by the line segments AB, BC, CD, and AD.

By Euclid's 1st postulate, we will create line segment AC. By definition of a rhombus, we know line segments AB, BC, CD, and AD are congruent. We also know line segment AC is congruent to itself. By Euclid.I.8, triangles ABC and ADC are congruent. Therefore angle CBA is congruent to angle CDA, angle BAC congruent to angle DAC, and angle ACB congruent to angle ACD. By Euclid.I.5, we know base angles in isosceles triangles are congruent. Looking at triangle ACD and triangle ACB, which are both isosceles, base angles CAD and DCA are congruent and base angles ACB and BAC are congruent. Thus we know angles BAC, CAD, ACB, and DCA are all congruent to each other. Furthermore, angles BAC and CAD taken together are congruent to angle BAD, so we know AC bisects angle BAD. Also angles ACB and ACD taken together are congruent to angle BCD. Thus, line segment AC bisects angle BCD.

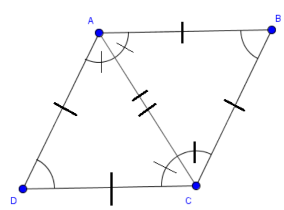

Figure 1: Line segment AC bisects the angles BAD and BCD

Similarly, by Euclid's postulate 1 we can create line segment BD. Again, by Euclid.I.8, the triangles created by line segment BD namely triangle BAD and triangle BCD are congruent, so angle BAD is congruent to angle BCD, angle ADB is congruent to angle CDB and angle ABD is congruent to angle CBD. By Euclid.I.5, using the 2 isosceles triangles BAD and BCD we know angle ABD is congruent to angle ADB and angle CBD is congruent to angle CDB. Thus, angles ADB, ABD, CDB, and CBD are all congruent. Since angle ABD and angle DBC taken together are congruent to angle ABC, line segment BD bisects angle ABC. Also

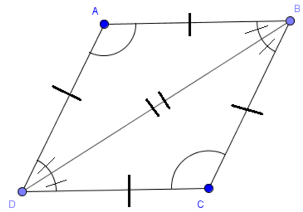

Figure 2: Line segment DB bisects angles ABC and ADC

angle BDC and angle BDA taken together are congruent to angle ADC, thus line segment BD bisects angle ADC.

By Euclid.I.17, any 2 angles of the same triangle added together must always be less than 2 right angles; therefore, the congruent angles BAC, CAD, ACB, and DCA must all be acute. Similarly, by Euclid.I.17, angles ABD, DBC, BDC, and ADB are also acute as they were all proven to be congruent. Thus, angles BAC, CAD, ACB, DCA, ABD, DBC, BDC, and ADB are all acute.

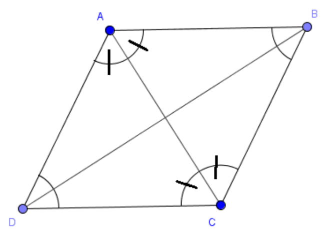

Figure 3: All bisected angles are acute

Now using Euclid's 5th postulate, line segment AD falling on line segments AC and DB make interior angles CAD and ADB which are both acute, so the line segments AC and DB must intersect on the side of line segment AD containing these acute interior angles. Next, line segment BC falling on line segments AC and DB make interior angles DBC and ACB both proven to be acute. Thus, line segment AC and DB must cross on the side of BC which contains these acute interior angles. Similarly, line segment AB falling on line segments AC and DB make interior angles BAC and ABD which are both acute so line segments AC and BD must intersect on the side of line segment AB which contains these acute interior angles. Lastly, line segment DC falling on line segments AC and DB make acute interior angles BDC and DCA, thus line segments AC and DB must intersect on the side of line segment DC which contains these acute interior angles. Following from above, the interior angles CAD, ADB, DBC, ACB, BAC, ABD, BDC, and DCA are all in the area contained by line segments AB, BC, CD, and AD. Therefore, the diagonals must cross inside the rhombus.

□

Diagonal Intersection of a Rhombus

Taryn Van Ryswyk

September 27, 2016

Theorem 1.2. The diagonals of a rhombus must cross.

Proof. Let ABCD be a rhombus. By Euclid's definition of a rhombus all sides are congruent.By Euclid Postulate 1, construct segment AC. By Ms. Mitchell's Theorem,1.1, we can conclude angle ABC is congruent to angle ADC. Euclid Proposition I.9 allows angle ABC to be bisected from point B to segment AC, creating point F. Similarly for angle ADC to be bisected from point D to segment AC, creating point E.Thus making angle ADE, angle CDE, angle ABF, and angle CDF all congruent to one another.

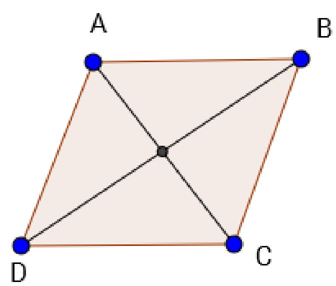

Figure 1: Rhombus ABCD with point in the middle representing point E and point F.

We will now show that point E equals point F. By Euclid Proposition I.8, since all sides of triangle ABC are congruent to all sides of triangle ADC then triangle ABC is congruent to triangle ADC. We can also concluded that by Euclid Proposition I.4, since triangle ABF and triangle CBF have two sides and the angle created by the two sides congruent, then triangle ABF is congruent to triangle CBF. Similarly, triangle ADE is congruent to triangle CDE.

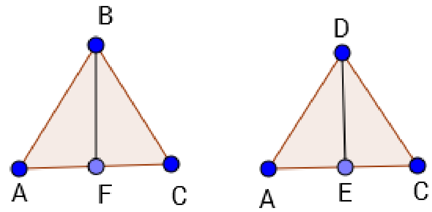

Figure 2: Congruent Triangles ABC and ADC.

Then segment AF is congruent to segment FC, thus point F is the midpoint of segment AC. Similarly segment AE is congruent to segment EC, thus point E is the midpoint of segment AC. Since triangle ABC and triangle ADC are congruent, the midpoints of the base AC are equal. Thus point E is equal to point F. I will now state the midpoint of AC to be labeled G.

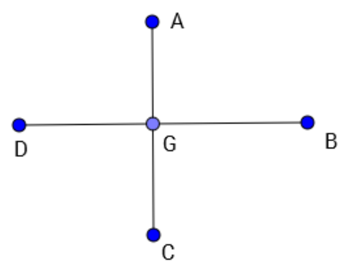

Figure 3: Straight line BD crosses Segment AC.

Euclid Proposition I.13 allows us to state that angle BGC and angle BGA, similarly angle DGC and angle DGA are all right angles because a straight line set up on a straight line will create two right angles or angles equal to two right angles. Thus segment BG is perpendicular to segment AC, and similarly segment DG is perpendicular to segment AC. By Euclid Proposition I.14, if two straight lines not lying on the same side make adjacent angles equal to two right angles then the two straight lines will be a straight line with one another. Thus segment DG and BG lie on opposite sides of segment AC and since angle AGB and angle AGD are both right angles and they are adjacent angles created by segment AC, points B,G,D are collinear. Then segment AC and BD share point G, thus lines of a rhombus must cross.

□

Rectangles are Parallelograms

Staci Schmeling

September 28, 2016

Theorem 3.1. Let ABCD be a rectangle. Then ABCD is a parallelogram.

Proof. Start with rectangle ABCD. By the definition of a rectangle, it is a quadrilateral which has all four interior angles that are right angles.

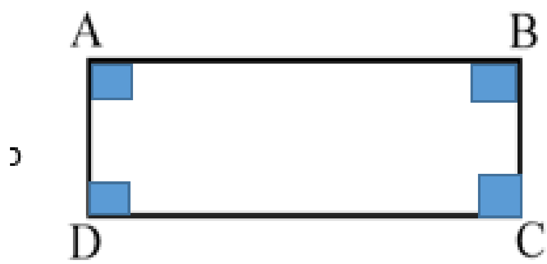

Figure 1: Rectangle ABCD with all interior angles as right angles.

Next, by postulate 2 we can extend all four sides of rectangle ABCD.Then by Euclid I.13, we know that a straight line set up on another straight line must by congruent to two right angles or be two right angles. Since a straight line has to be congruent to two right angles, and we already know that all interior angles are right angles then the exterior angles are right angles.

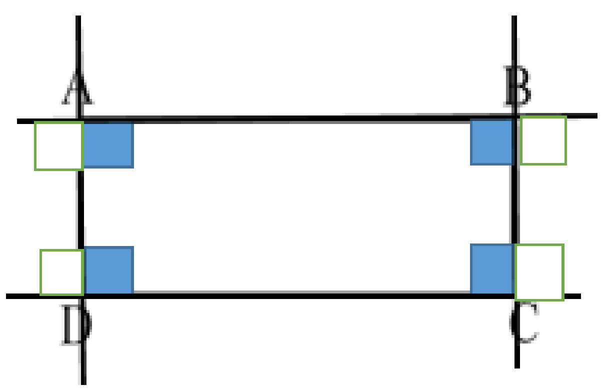

Figure 2: Extension of each line segments and Euclid I.13.

Euclid I. 27 states that if a straight line falling on two straight lines make the alternate interior angles congruent to one another, the straight lines will be parallel to one another. Using Euclid I.27, we can prove that line segments AB and DC are parallel, and the line segments AD and BC are parallel.

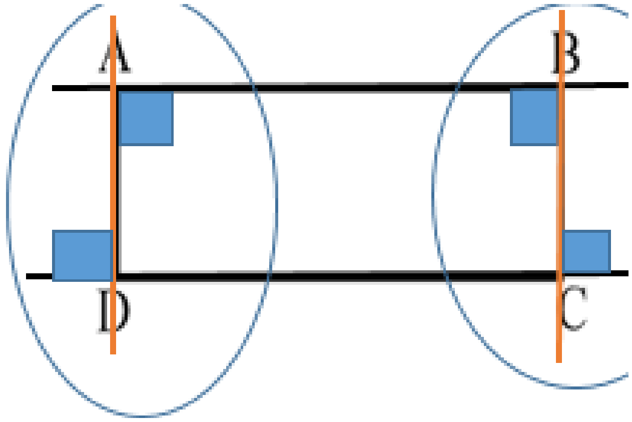

Figure 3: Proving line segments AB and DC are parallel.

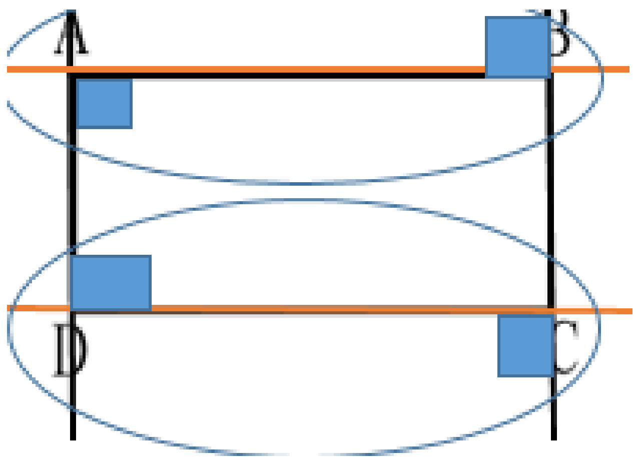

Figure 4: Proving line segments AD and BC are parallel.

Since those line segments are parallel, rectangle ABCD is a parallelogram.

□

Diagonals of a Rectangle

Megan King

September 28, 2016

Theorem 3.3. The two diagonals of a rectangle are congruent and bisect each other.

Proof. Let ABCD be a rectangle. By the definition, a rectangle has the four interior angles being right angles. By Postulate 1, we can create diagonals AC and BD. By Miss Schultz Theorem 3.2, it is known that ABCD is a parallelogram and side AB is congruent to side CD. Similarly, side AD is congruent to side BC.

By Euclid's Proposition I.4 in *Euclid's Elements*, triangle ABC is congruent to triangle DCB. Knowing this, angle BCA is congruent to angle CBD. Using Postulate 5,line segment BC falls on diagonals AC and BD making the interior angles on the same side less than two right angles. Then the two straight lines meet on the side on which the angles are less than two right angles. Therefore, diagonal AC crosses diagonal BD and there exists a point X, at the intersection.

Triangle ADC is congruent to triangle DAB. Therefore, angle CAD is congruent to angle ACB by alternate interior angles in Proposition 29, which is true because ABCD is a parallelogram. Similarly, angle BDA is congruent to angle DBC. By Euclid's Proposition I.26, triangle AXD is congruent to triangle BXC. Using Euclid's Proposition I.15, angle AXD is congruent to angle BXC.

Hence, using Euclid's Proposition I.6, segment AX is congruent to segment XD and segment BX is congruent to CX, because of congruent triangles. Thus by those congruent triangles, segments AX, CX, BX, and DX are congruent to each other. Therefore, diagonal AC is congruent to diagonal BD. Similarly, because of congruent triangles and congruent diagonals, the diagonals share a midpoint and bisect each other. □

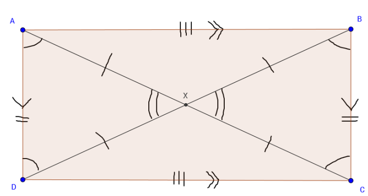

Figure 1: Congruent diagonals that bisect each other

Making a Quadrilateral a Rectangle

Rebecca A Shere

September 29, 2016

Theorem 3.4. Let ABCD be a quadrilateral such that angle ABC and angle ADC are right angles. If line segment AB is congruent to line segment CD then ABCD is a rectangle.

Proof. By Euclid's 1st postulate create line AC. By Euclid's 2nd postulate extend line segment AD. By Euclid.I.2, we can create a line segment that has the length congruent to line segment BC and have it begin at point D. Let's call this line segment DF. Since we don't know that line segment DF lies on line AD, we create a circle with center D and radius DF. Since D is the center of the circle, line AD will intersect the circle twice. We shall denote E as the intersection of the circle, with radius DF centered at D, and the line AD such that line segment AD and line segment DE do not share any points other than D. This line segment DE is also a radius of our circle, so line segments DE and DF are congruent. Thus line segment DE is congruent to line segment BC.

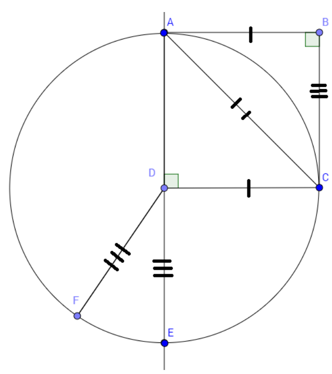

Figure 1: DE is congruent to BC

By Euclid's 1st postulate, we are able to create line segment CE. This line segment creates the triangle CDE. Since line AE is a straight line, by Euclid.I.13, angle CDE and angle ADC taken together must be congruent to 2 right angles, since angle ADC is right then angle CDE must also be right. By Euclid.I.4, triangle CDE is congruent to triangle ABC, as they have a side, an angle and a side congruent respectively. Thus angle BAC is congruent to angle DCE, angle BCA is congruent to angle DEC, and line segment AC is congruent to line segment CE. Looking at triangle ACE, we have 2 sides, namely line segment AC and line segment CE, that are congruent; therefore, triangle ACE is isosceles by definition. By Euclid.I.5, we know base angles of isosceles triangles are congruent, so angles CED and CAD are congruent.

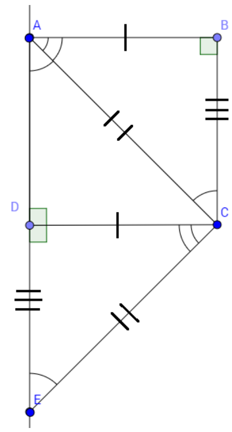

Figure 2: Base angles equal

By Euclid.I.26, triangle ADC is congruent to triangle EDC. This happens as we have 2 congruent angles and a congruent side that subtends one of those angles. It then follows that triangle ADC is congruent to triangle ABC.

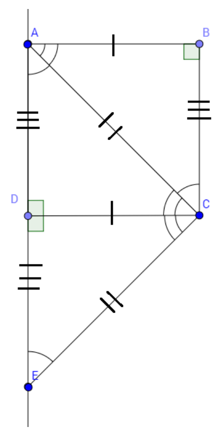

Figure 3: Triangles ABC, CDA, and CDE are all congruent

By Euclid.I.32, when taken together the interior angles of a triangle must be congruent to 2 right angles. We have 1 right angle in triangle ABC and 1 in triangle CDA, so in each of these triangles the remaining 2 angles when taken together must be congruent to 1 right angle. Therefore angle BAC and angle BCA taken together make a right angle, and angle DCA and angle CAD taken together make a right angle. Since BAC is congruent to DCA, we know taken together angles BCA and DCA make a right angle. Since DAC is congruent to BCA, we know BAC taken together with DAC also makes a right angle. Thus, all the angles in the quadrilateral ABCD are right angles, and ABCD is by definition a rectangle.

□

Parallel Lines and Right Angles in a Rectangle

Christopher Merck

September 29, 2016

Theorem 3.5. If ABCD is a quadrilateral such that angles ABC and ADC are right angles and segments AB and CD are parallel, then ABCD is a rectangle.

Proof. Let ABCD be a quadrilateral. Let angle ABC and angle ADC be right angles. Let line segment AB and line segment CD be parallel to each other.

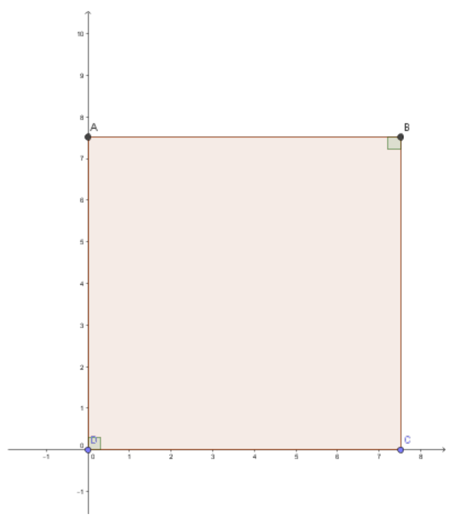

Figure 1: Quadrilateral ABCD with right angles ABC and ADC

By Euclid Postulate 2, extend line segments AB, BC, CD, and DA to form lines AB, BC, CD, and DA.

By Euclid I.29, when a straight line falls upon parallel lines, then the exterior angle is congruent to the interior and opposite angle. Here, line AC falls along parallel lines AB and CD. As the interior opposite angle ADC is a right angle, then the exterior angle must also be a right angle.

By Euclid I.13, since a straight line stands on a straight line, then it makes either two right angles or angles whose sum equals two right angles. Here, angle DAB must also be a right angle, as there is already one right angle present.

The other missing angle can be found in the same manner. Angle ABC is a right angle, and line BC falls on parallel lines AB and CD. Then the Angle opposite of ABC is also a right angle.

Since there is a straight line, CD, falling on a another straight line, BC, then the sum of the two angles must be the sum of two right angles. As there is already one right angle, then angle BCD is also a right angle. There are now four right angles. Hence, a quadrilateral ABCD with line segments AB and CD being parallel, and angles ABC and ADC being right angles, is a rectangle.

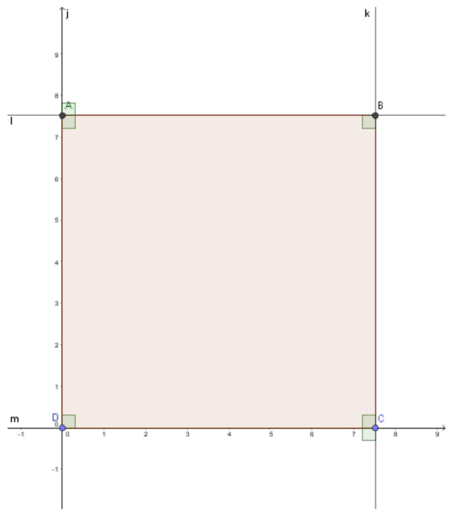

Figure 2: Quadrilateral ABCD is also a rectangle

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In Between and Opposite

Heather Bavdio

September 28, 2016

Question K. Let ABC be collinear points. What do we mean by ”B lies between A and C”?

Definition K. If point B lies between points A and C they form a relationship in which line segment AB taken with line segment BC will be congruent to line segment AC. This is shown in Figure 1.

Figure 1: Collinear points ABC where B lies between A and B

If line segment AB taken with line segment BC is greater than line segment AC, point B will not be in between points A and C. This is shown in Figure 2.

Figure 2: Collinear points ABC where B does not lie in between A and B

Question L. Let L be a line and A and B points. What do we mean by the phrase "A and B lie on opposite sides of the line"?

Definition L. The points A and B lie on opposite sides of line L if and only if there exists a point X that intersects the line segment AB and line L. This point X must lie between the points A and B, points A and B can not lie on line L. This is shown in Figure 3.

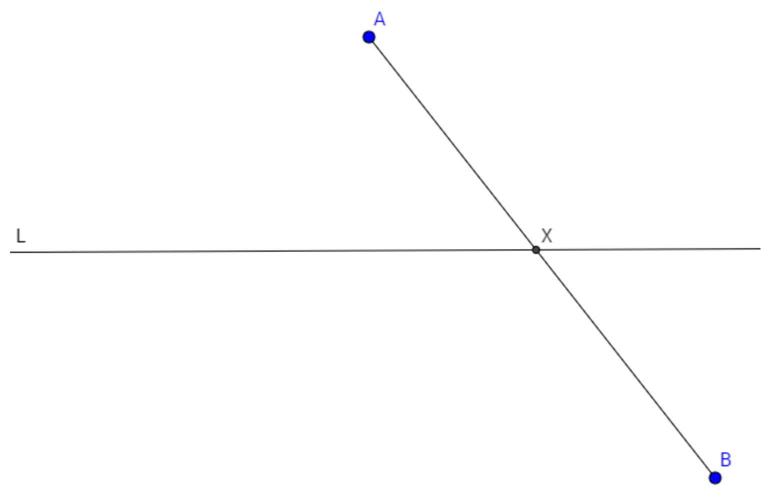

Figure 3: Point A and B on opposite sides of L

The points A and B lie on the same side of line L if the point of intersection does not occur between A and B. This is shown in Figure 4.

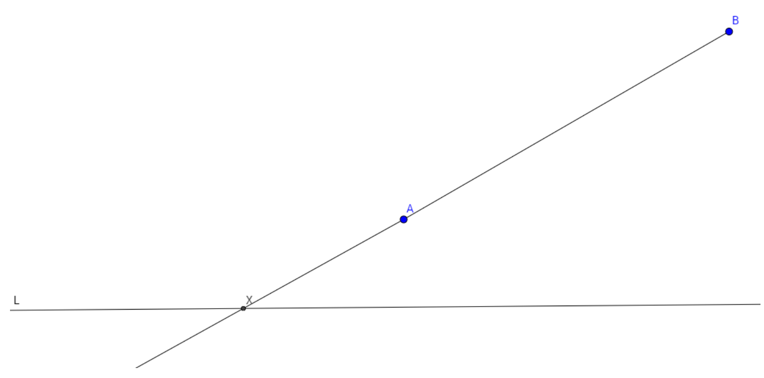

Figure 4: Point A and B on the same side of L