Euclid's Unsupported Proposition I.4

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In Euclid's proposition I.4, we discuss the congruence of two triangles, related by two corresponding congruent sides and a congruent angle contained between the two sides.

Conjecture 4.3. Euclid constructs two triangles, ABC and DEF. He supposes that segment AB is congruent to segment DE, and segment AC is congruent to DF. He also supposes that angle BAC is congruent to angle EDF. He then claims that the triangles are congruent.He proves this by superposition, or by placing one triangle over the other. He says:

> For, if the triangle ABC be applied to triangle DEF, and point A be place on the point D and the straight line AB on DE, then the point B will also coincide with E, because AB is equal to DE.

The problem stems from a lack of support for superposition. His definitions, postulates, common assumptions, and three previous postulates make no mention of "applying" or "placing" one object to another to determine congruence. Because of this, Euclid's leap to congruence is axiomatically unfounded, and he makes no attempt to reconcile the gap in logic.