Diagonals of a Kite Meet at a Right Angle

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I thank Ms. Dvorak for a useful conversation about this work.

Theorem 2.5. If the diagonals of a kite meet, then they meet at a right angle.

Proof. Let $ABCD$ be a kite. Let line NO be parallel to diagonal AC through point B by construction of Euclid I.31. Extend line BA past point A to point E. Extend line DA past point A to point F. Extend line CA past point A to point G. Extend line DC past point C to point H. Extend line AC past point C to point I. Extend line BC past point C to point J. Extend line DB past point B to point K. Extend line AB past point B to point L. Extend line CB past point B to point M. Let Q be the intersection of the diagonals of kite $ABCD$.

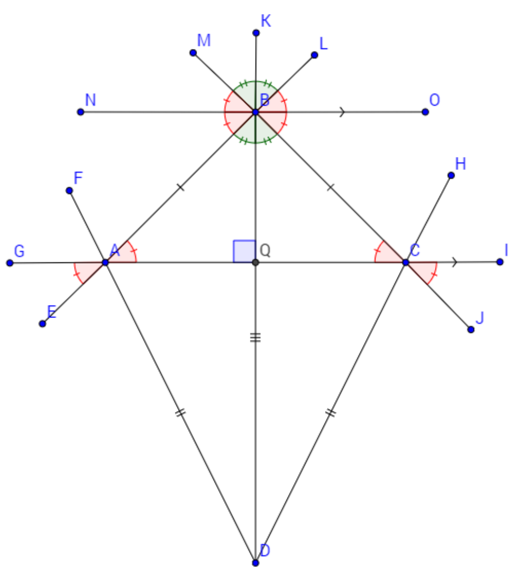

Figure 1: Kite $ABCD$

Since $ABCD$ is a kite, then triangle ABC is an isosceles triangle, By Euclid I.5, triangle ABC has congruent base angles BAC and BCA. By Euclid I.15 (vertical angles), angle GAE is congruent to angle BAC. Similarly, by vertical angles, angle ICJ is congruent to angle BCA. By Euclid I.28, and since line NO is parallel to diagonal AC, then angle BCA is congruent to angle MBN.

Since diagonal BD is shared by triangles ABD and CBD and since side AB is congruent to CB and side AD is congruent to CD, two congruent triangles are formed by Euclid I.8. Hence, angle ABD is congruent to angle CBD. By vertical angles, angle ABD is congruent to angle KBL, and similarly, angle CBD is congruent to angle MBN.

By Euclid I.13, the angles KBN and KBO will make two right angles when taken together. Since angle KBN is congruent to angle KBO, each angle should make one right angle. By Euclid I.28, angle KBN is congruent to angle BQA. Therefore, the diagonals of a kite meet at a right angle.

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