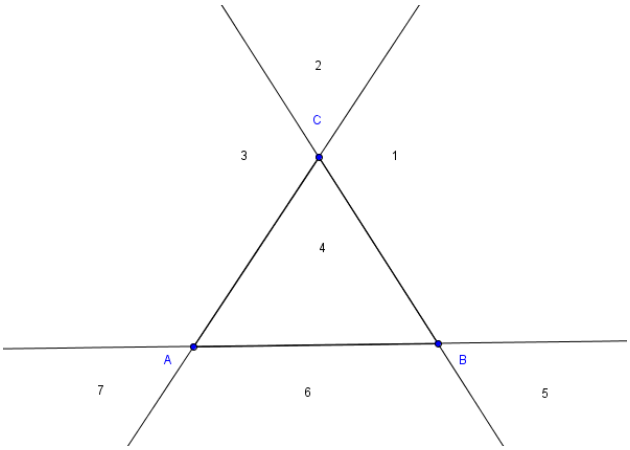
Completion of Euclid I.7

Mr. Toby Maggert

April 8, 2015

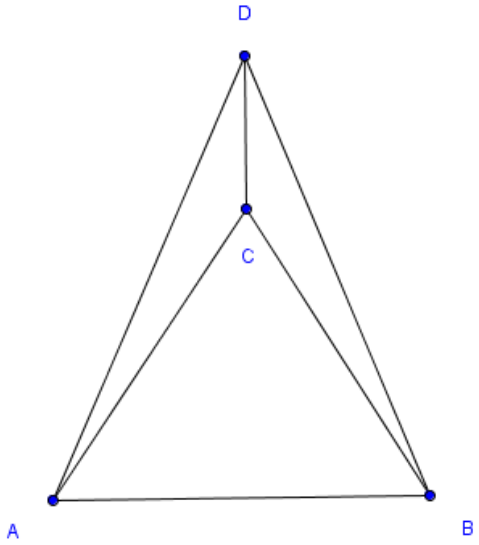
Theorem Euclid I.7. Given two straight lines constructed from the ends of a straight line and meeting in a point, there cannot be constructed from the ends of the same straight line, and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each equal to that from the same end.

Proof. Note that Euclid's proof is only applicable to certain choices for point D. We will continue his proof to be applicable to any selection of point D. We have seven regions to select for our point D.

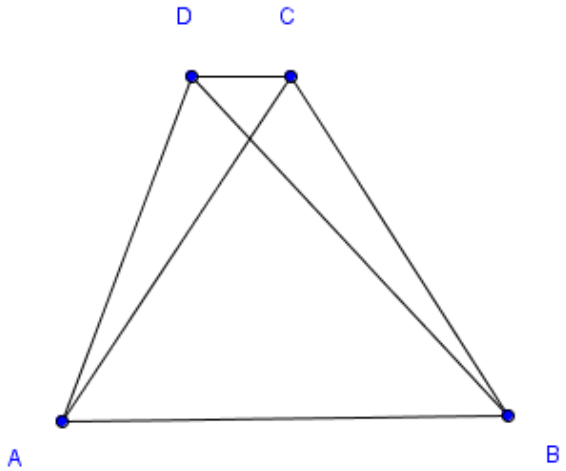


Region 1: Euclid's proof is applicable to all the points in region 1.

Region 2: By Euclid 1.32, we know that angles CBD, BCD, and BDC add up to two right angles. Therefore, angle CBD is congruent to two right angles minus angles BCD and BDC.Therefore we know that angle CBD is congruent to angles ABD, ADB, and DAB minus angles BCD and BDC. Since angle ADB is congruent to angles ADC and BDC, we have angle CBD is congruent to angles ABD, ADC, and DAB minus angle BCD. This implies that angle BCD is congruent to angles ABC, ADC, and DAB. Since angle BAD is congruent to angles BAC and CAD, we have angle BCD is congruent to angles ABC, ACD, CAD, and BAC. Since angles ABC, BAC, and ACD add up to two right angles, and angle BCD is congruent to two right angles minus angles CBD and CDB, we have 2 right angles minus CBD minus CDB is congruent to two right angles plus angle CAD. Therefore we have zero is congruent to angles CAD, CBD, and CDB, which is impossible.



Region 3: By Euclid I.5 we know that angle BCD is congruent to angle BDC. Since angle BDC is strictly less than angle ADC, we have angle BCD is less than angle ADC. By Euclid I.5 we know angle ADC is congruent to angle ACD. Therefore angle BCD is less than angle ACD, but angle ACD is less than angle BCD, so we have a contradiction.



Region 4: We follow the proof for region 2, but replace point C with point D, and visa-versa. Regions 5,6,7: Regions 5, 6 and 7 are not applicable choices for point D due to being on different sides of line AB as point C. □