Concurrency of a Triangle

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Theorem(8.4). The three perpendicular bisectors of any triangle are concurrent.

Proof. Let T be a triangle. Draw a perpendicular bisector of sides AB and BC. Call the point, where these two lines intersect, D. Then, connect B to D and C to D, also connect A to D, so the triangles BDC and BDA are formed. Also M and M' are the midpoints of the sides AB and BC. In a triangle BDC, we know that BM' is congruent to CM' since M' is the midpoint. Then, triangle BM'D is congruent to the triangle CM'D since they form right right angles and share a side. BM'D is congruent to CM'D by Side-Angle-Side. Thus, BD is congruent to CD. Similar argument goes for triangle BDA.

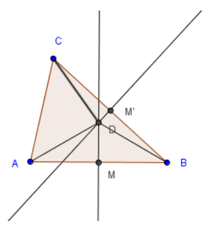

Figure 1: FIGURE 1

In a case of the triangle ADC, we draw an altitude from point D, called E. E is the midpoint of the side AC. Then, the triangles AED and CED have the right angle and share the side. Then, we can say that AD, BD, and CD are congruent by Side-Angle-Side. Thus, these three perpendicular bisectors of a triangle T are concurrent. Also, we have circumscribed circle.

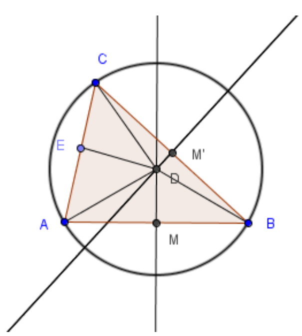

Figure 2: FIGURE 2

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