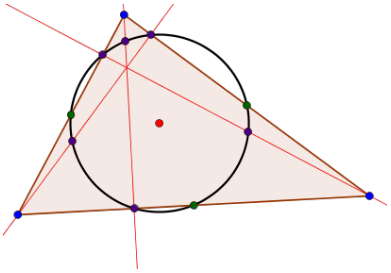
Euclidean Geometry: An Introduction to Mathematical Work

Math 3600, Fall 2015

11 December

Final Examination Problems



Instructions: This examination is due by 5pm on Thursday, December 17th. Be sure to adhere to the standards we have used all semester.

You may use your course notes, the *Elements* **through Book IV** and the class journal as references. You may not speak to anyone about the content of this exam except me until the deadline has passed. Explain yourself clearly and give complete arguments to receive full credit. You may turn in the exam by dropping it off at my office (327 Wright Hall) or at the Mathematics Department Office (220 Wright Hall).

You may write out your arguments and diagrams by hand if you choose. If you do so, please be sure that your work is legible. You may also type up your work using the journal template, and make figures using GeoGebra.

Good Luck!

F.1 Problem. Given a circle and its center, construct a regular octagon inscribed in the circle. Prove that your construction works.

F.2 Problem. Let ABC be a triangle. Extend sides AB and AC to rays AB and AC, forming exterior angles. Let r_A be the angle bisector of angle BAC, let r_B be the angle bisector of the exterior angle at B, and let r_C be the angle bisector of the exterior angle at C.

- Prove that these three rays are concurrent. The point just constructed is called an *excenter* of ABC. Let us call it E_A.
- Prove that E_A is the center of a circle which is tangent to the extended sides of the triangle.

F.3 Problem. This is a conditional task.

If 14.1 is not proved on December 11, prove Conjecture 14.1.

If 14.1 is proved on December 11 but 14.2 is not, do Task 14.2.

If 14.1 and 14.2 are both proved on December 11, prove Conjecture 14.7.

Definition. A quadrilateral is called *circumscriptable* if there exists a circle lying inside the quadrilateral which is tangent to each of the four sides.

F.4 Problem. Settle the following conjecture as completely as you can. I want to see as much as you can do with this.

> **Conjecture:** Let ABCD be a quadrilateral. Then $ABCD$ is circumscriptable if, and only if, AB and CD taken together are congruent to AD and BC taken together.