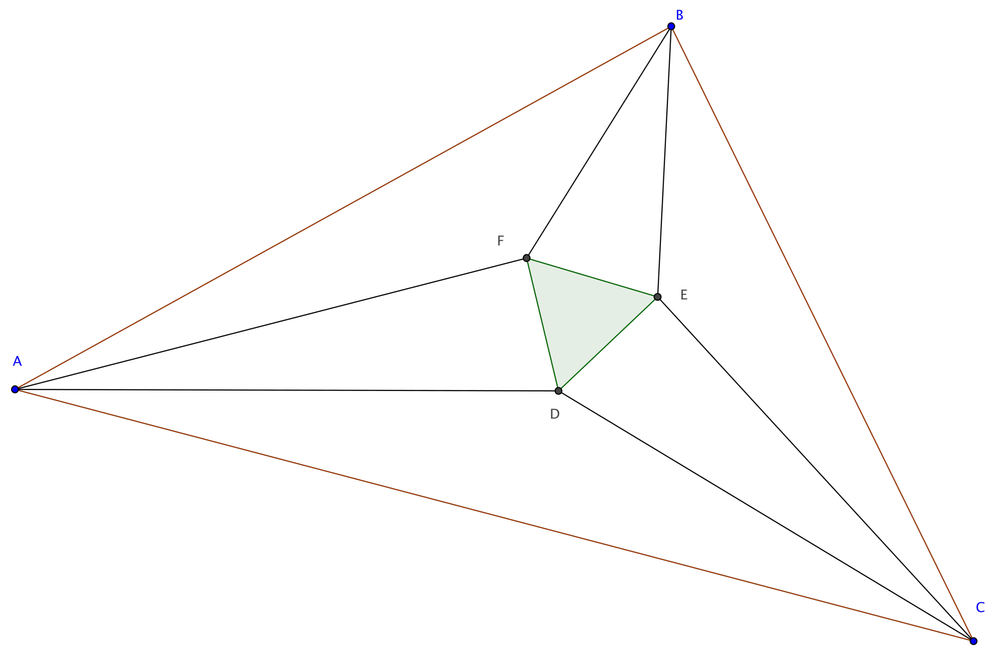
Transactions

in

Euclidean Geometry



Issue # 5

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Construction of a Kite

Kevin Conger

November 13, 2016

Theorem 2.3. It is possible to construct a kite with a compass and a straightedge.

1. Let circle A be a circle with center A.
2. Let circle C be a circle with center C. such that circle C intersects circle A.
3. Let the intersections of the circles be point B and point D (Circle-Circle Intersection Property).
4. Create line segments AB, AD, BC, and DC (Postulate 1).
5. The resulting quadrilateral ABCD is a kite.

(a) Circle A intersecting Circle C at point B and point D

(b) Creating segments AB, AD, CB, and CD

Figure 1: The construction of quadrilateral ABCD

Proof. By the Circle-Circle Intersection Property, circle A and circle C intersect at two points, namely B and D. Because AB and AD are radii of circle A, and by the definition of a circle, AB is congruent to AD. AB and AD meet at A, so they are a pair of adjacent line segments. Then AB and AD are a pair of congruent and adjacent sides of quadrilateral ABCD. Because CB and CD are radii of circle C, and by the definition of a circle, CB is congruent to CD. CB and CD meet at C, so they are also a pair of adjacent line segments. Then CB and CD are a pair of congruent and adjacent sides of quadrilateral ABCD. There now are two pairs of congruent and adjacent sides in the quadrilateral ABCD. Thus quadrilateral ABCD is a kite.

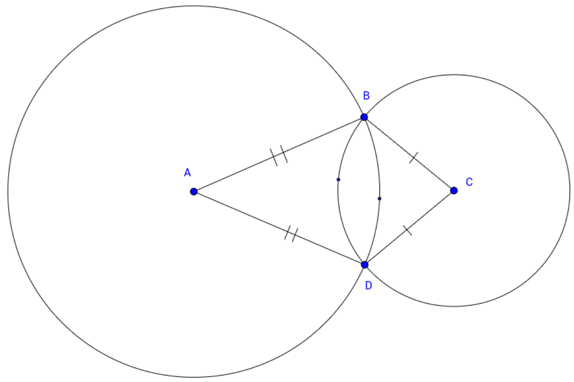


Figure 2: Quadrilateral ABCD is a kite.

`Note: When the vertices B, C, and D are collinear, the resulting figure is not a kite. Since B, C, and D are collinear, BC and CD are collinear. Then BC taken with CD results in one side, namely BD, of the polygon. Then ABD is a triangle.`

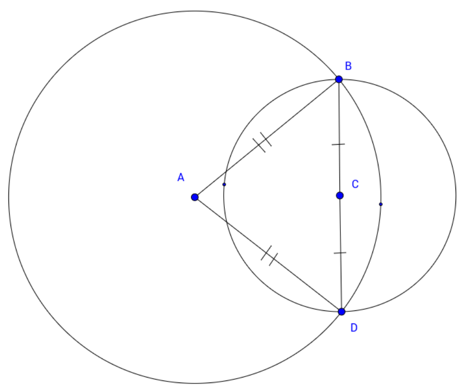


Figure 3: In this figure, circle A and circle C have been chosen such that points B, C, and D are collinear. Therefore, this figure is not a kite.

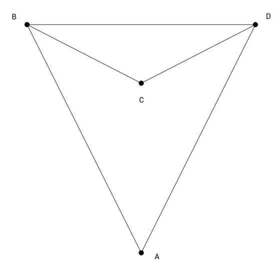
□

Defining a Non-Convex Kite

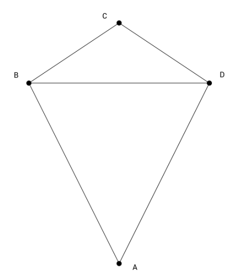
Kevin Conger

November 7, 2016

Definition . Let ABCD be a kite such that AB is congruent to AD and CB is congruent to CD. ABCD is a non-convex kite if the vertices A and C are on the same side of the diagonal created by B and D. The kite is said to be a convex kite if the vertices A and C are on opposite sides of the diagonal created by B and D.
(This definition utilizes Miss Bavido's definition of "opposite sides" which includes what it means for two points to be on the same side of a given line.)



(a) Non-Convex Kite



(b) Convex Kite

Figure 1: (a) The vertices A and C of this kite ABCD are on the same side of diagonal BD, so this kite is a non-convex kite. (b) The vertices A and C of this kite ABCD are NOT on the same side of diagonal BD, so this kite does not fit the definition of a non-convex kite. It is a convex kite.

Cohen and Conger Kites

Tessa Cohen

December 2, 2016

Communicated by Ms. Maus.

At the conclusion of our discussion about Convex and Non-Convex kites, we ended with two different definitions of Non-Convex kites. Miss Cohen's definition deals with betweenness of points, whereas Mr. Conger's definition deals with points on the same or opposite sides of a line segment. How are these definitions related? Can they be used interchangeably?

Theorem N. The notions of Cohen convexity and Conger convexity are the same.

Note: I will show my argument using convex kites. The same argument would also hold for non-convex kites.

Proof. **Case 1.** If a convex kite fits the Conger definition of convexity, then it also fits the Cohen definition.
Let ABCD be a convex kite that fits Mr. Conger's definition of convexity. Then ABCD will be a kite such that AB is congruent to AD and CB is congruent to CD. By Euclid Postulate 1, we can create segment BD and line AC. By Mr. Conger's definition, A and C lie on opposite sides of segment BD.

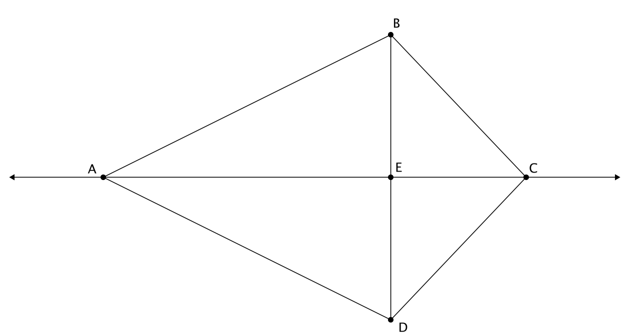


Figure 1: Conger convex kite. A and C lie on opposite sides of segment BD.

In order to analyze how a Conger kite is related to a Cohen kite, we look at Miss Bavido's definition of "on opposite sides of." By Miss Bavido's definition, we know there exists a point, E, that intersects the line segment BD and the line AC. We also know by Miss Bavido's definition that point E must be in between points A and C. Miss Cohen's definition says that a kite is convex when point E, created at the intersection of segment BD and line AC, is in between points A and C. In the Conger convex kite we found that point E is at the intersection of segment BD and line AC and that E is in between points A and C. Therefore, a Conger convex kite will also be a Cohen convex kite.

Case 2. If a kite fits the Cohen definition of convexity then it also fits the Conger definition.
Let ABCD be a convex kite that fits Miss Cohen's definition of convexity. Then ABCD will be a kite such that AB is congruent to AD and CB is congruent to CD. By Euclid Postulate 1, we can create segment BD and line AC. Let E be the point at the intersection of segment BD and line AC. By Miss Cohen's definition of a convex kite, point E will be in between points A and C.

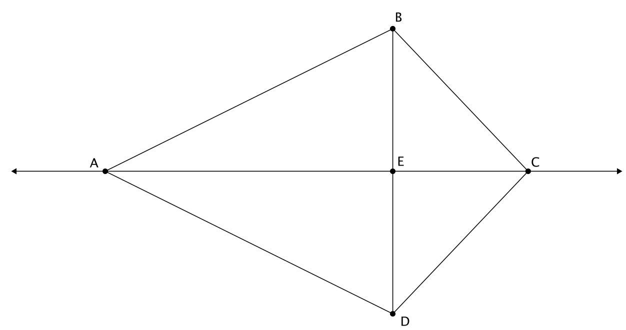


Figure 2: Cohen convex kite. Point E lies in between points A and C.

Again, Mr. Conger's definition says that in a convex kite, A and C lie on opposite sides of segment BD. We know that point E is on the intersection of segment BD and line AC. We also know that E lies in between A and C. Therefore, by Miss Bavido's definition of "on the opposite side of," A and C are on opposite sides of segment BD. Thus, a Cohen convex kite will also be a Conger convex kite.

Since both cases have the same results, we can conclude that the statements are equivalent, therefore, the notions of the definitions are the same. Knowing this allows us to use these two definitions interchangeably.

□

Convex or Non-Convex Polygons

Staci Schmeling and Mackenzie Mitchell

December 2, 2016

Communicated by Ms. Bavido.

During some presentations, we came along some challenges about convex and non-convex polygons. This report will give us a definition of non-convex and convex polygons.

Definition P. Let P be a polygon, we say P is non-convex when there exists a diagonal which lies outside of P.

By using postulate 1 we can draw a straight line from any point to any point. When connecting the points and creating a diagonal in each polygon pictured below, we can see that there are some diagonals outside the figures. Thus, the figures with at least one diagonal outside of the polygon are considered non-convex. Also, the polygons with all possible diagonals contained inside of the polygon are considered convex.

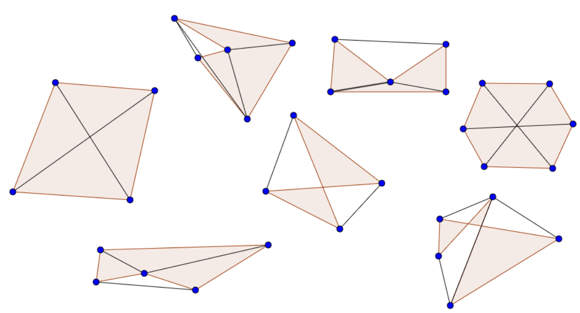


Figure 1: Non-Convex and Convex Polygons

In Figure 1, we can see that there are two polygons where all the possible diagonals are inside or contained within the figure which would be defined as convex. The other five polygons have diagonals lying outside of the figure so they are considered non-convex polygons.

Fixing Euclid Proposition 7

Tessa Cohen and Erica Schultz

December 2, 2016

Communicated by Ms. Shere.

In Proposition 7, Euclid makes the mistake of using a specific figure to come to a conclusion and fails to consider different constructions of the figure. Here we outline the other possible constructions and complete the proof of Proposition 7.

Euclid Proposition 7. Given two straight lines constructed on a straight line (from its extremities) and meeting in a point, there cannot be constructed on the same straight line (from its extremities), and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each to that which has the same extremity with it.

Proof. **Case 1** Segments AD and BC cross or segments AC and DB cross.

Subcase 1: Segments AD and BC cross. This is the subcase that Euclid proves in Euclid's Elements.

Figure 1: The constructions for Case 1. Left is Subcase 1, right is Subcase 2.

Subcase 2: Segments AC and DB cross. When applying the argument to Subcase 2, it is similar to Subcase 1.

Let segments AC and CB be constructed on the straight line AB, from its extremities, meeting at point C. Let segments AD and DB be constructed on the same line AB, again from its extremities, meeting at point D. Let AD and DB be congruent to AC and CB, respectively, such that segment AC crosses segment DB. By Euclid Proposition I.5, since segment BD is congruent to segment BC, triangle BCD is isosceles, thus angle BCD is congruent to angle BDC. Since angle DCA plus angle BCA is congruent to angle BCD and angle BCD is congruent to angle BDC, then angle BDC is greater than angle DCA by Euclid Common Notion 5. Since angle BDC is greater than angle DCA and angle BDC plus angle ADB is congruent to angle ADC, then angle ADC is greater than the angle ACD. However, by Euclid Proposition I.5, since AD is congruent to AC, triangle ADC is isosceles, thus angle ADC is congruent to angle ACD. Therefore, segments AC and CB cannot be congruent to segments AD and DB, respectively.

Case 2: AD does not cross BC and AC does not cross BD.

There are two subcases that result from this case; either point D lies inside of triangle ABC or D lies outside of triangle ABC.

Figure 2: The constructions for Case 2. Left is Subcase 1, right is Subcase 2.

Subcase 1: Point D lies inside of triangle ABC.

Let segments AC and CB be constructed on the straight line AB, from its extremities, meeting at point C. Let segments AD and DB be constructed on the same line AB, from its extremities, meeting at point D. Let AD and DB be congruent to AC and CB respectively, such that AC does not cross BD and AD does not cross BC, and point D lies inside of the triangle ABC. By Euclid Postulate 2, extend segments BD and BC into lines. Place point E on the intersection of line BD and segment AC. Place point F on line BC such that segment CF is congruent to segment DE. By Euclid Postulate 1, create line segment DF.

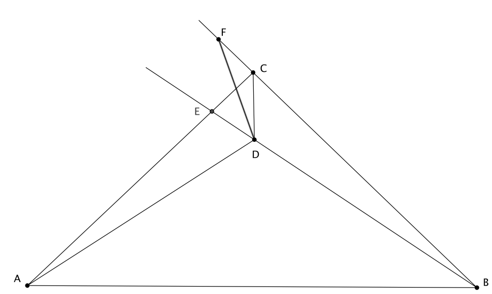


Figure 3: Extend lines BD and BC, create points E and F, and create segment DF.

In the proof of Euclid Proposition I.5, it is found that triangle CDF will be congruent to triangle CDE, by shared side CD, congruent sides CF and DE and congruent sides CE and DF. Therefore, angle DCE is congruent to angle CDF. Since AD is congruent to AC, triangle ADC is isosceles, thus angle CDA is congruent to angle DCE, by Euclid Proposition I.5. Since angle CDF plus angle FDA is congruent to angle CDA, then angle CDF is less than angle CDA, by Euclid Common Notion 5. Since angle CDA is congruent to angle DCE, and angle CDF is less than angle CDA, angle CDF must be less than angle DCE. However, it was previously proven that angle CDF is congruent to angle DCE. Therefore, segments AC and CB cannot be congruent to segments AD and DB, respectively.

Subcase 2: Point D lies outside of the triangle ABC.

Let segments AC and CB be constructed on the straight line AB, from its extremities, meeting at point C. Let segments AD and DB be constructed on the same line AB, from its extremities, meeting at point D. Let AD and DB be congruent to AC and CB, respectively, such that AC does not cross BD and AD does not cross BC, and point D lies outside of the triangle ABC. Extend segments BC and BD into lines. Place point E on the intersection of line BC and segment AD. Place point F on line BD such that segment CE is congruent to segment DF. By Euclid Postulate 1, create line segment CF.

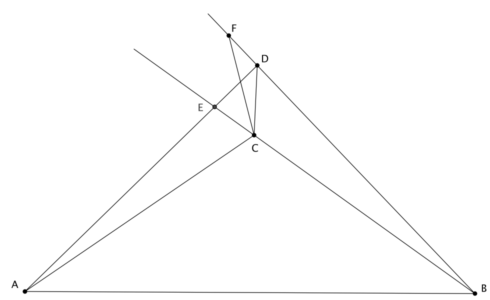


Figure 4: Extend lines BC and BD, create points E and F, and create segment CF.

In the proof of Euclid Proposition I.5, it is found that triangle CDF will be congruent to triangle CDE, by shared side CD, congruent sides DE and CF and congruent sides CE and DF. Therefore, angle CDE is congruent to angle DCF. Since AC is congruent to AD, triangle ADC is isosceles, thus angle CDA is congruent to angle ACD, by Euclid Proposition I.5. By Euclid Common Notion 5, since angle DCF plus angle FCA is congruent to angle DCA, angle DCF is less than angle DCA. Since angle DCA is congruent to angle CDE and angle DCF is less than angle DCA, then angle DCF must also be less than angle CDE. However, it was previously proven that angle DCF is congruent to angle CDE. Therefore, segments AC and BC cannot be congruent to segments AD and DB, respectively.

□

A Regular Rhombus is a Square

Staci Schmeling

December 2, 2016

Communicated by Ms. Shere.

Theorem 6.2. Let ABCD be a rhombus. If angle A is congruent to B, then ABCD is regular.

Proof. We start with rhombus ABCD and angle A is congruent to angle B.

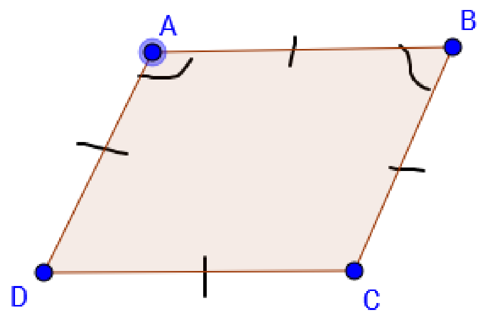


Figure 1: Given information from theorem

By using Miss Mitchell's Theorem 1.1, we know that the opposite angles in a rhombus are congruent. So, angle A is congruent to angle C, and angle B is congruent to angle D.

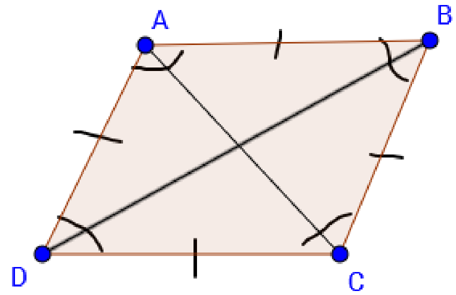


Figure 2: All angles and all sides are congruent

By Euclid's Common Notion 1, things that are equal to the same thing are also equal to one another, we know angle D is congruent to angle B, and angle B is congruent to angle C. Thus, angles A, B, C and D are congruent to each other. Since we know that for a rhombus to be regular all the sides must be congruent and all the angles must be congruent to each other, we can say that rhombus ABCD is a regular rhombus. Since we know that both a square and a regular rhombus has to have four congruent sides and four congruent angles, we can conclude that whenever we have a regular rhombus we also have a square.

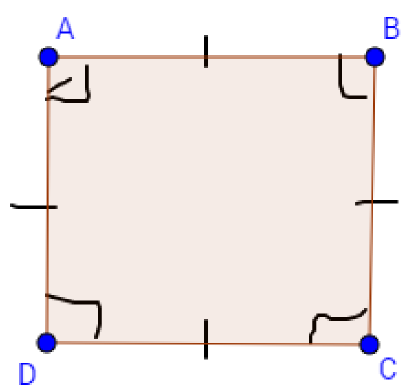


Figure 3: This regular rhombus is a square

□

Pentagon Regularity Given Certain Angles

Danielle Maus

December 2, 2016

Communicated by Ms. Shere.

Conjecture 6.4 states, "Let ABCDE be an equilateral pentagon. If angle A is congruent to angle B, then ABCDE is regular." Mr. Conger gave a counterexample for conjecture 6.4 by proving that if angle A and angle B are right angles, ABCDE will be not regular. By modifying the hypothesis of conjecture 6.4, we are able to prove that ABCDE is a regular pentagon.

Theorem 6.4-1. Let ABCDE be an equilateral pentagon. If angle B is congruent to angle D and angle E, then ABCDE is regular.

Proof. Let ABCDE be an equilateral pentagon (all sides are congruent to one another). Let angle B be congruent to angle D and angle E. By Euclid's Postulate 1, we can create line segment AC and line segment EC. (see figure 1) Because triangle ABC and triangle CDE have two sides congruent to two sides respectively and an angle congruent to an angle, by Euclid's I.4, triangle ABC is congruent to triangle CDE. This means line segment AC is congruent to line segment EC. Triangle ABC and triangle CDE are also isosceles triangles, by Euclid's I.5, the base angles are congruent in each triangle. Since triangle ABC and triangle CDE are congruent to one another and have base angles congruent in each, angle BAC, angle BCA, angle DCE, and angle DEC are all congruent to one another.

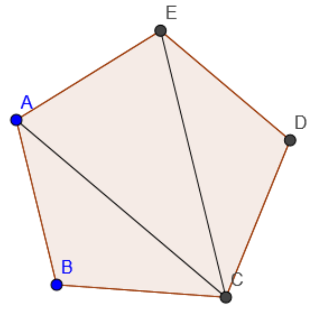


Figure 1: Pentagon ABCDE with diagonals AC and EC

Triangle ACE is isosceles, because line segment AC is congruent to line segment EC. By Euclid's I.5, the base angles in triangle ACE are congruent to one another. Angle E is made up of angle DEC taken together with angle AEC. Angle A is made up of angle BAC taken together with angle EAC. Angle DEC is congruent to angle BAC and angle AEC is congruent to angle EAC. By Euclid's Common Notion 2, which states "when equals be added to equals the wholes are equals", we can say angle A is congruent to angle E.

Because angle A is congruent to angle E and angle E is congruent to angle B and angle D, then by Euclid's Common Notion 1, we can say angle A is congruent to angle B, angle D, and angle E.

Now by Euclid's Postulate 1, we can create line segments BE and CE. Triangle AEB is congruent to triangle DEC, by Euclid's I.4, because the triangles have a side, an angle, and a side congruent to one another respectively. Because triangle AEB is congruent to triangle DEC, line segment BE is congruent to line segment CE. Triangle AEB and triangle DEC are also isosceles triangles, so their base angles are congruent to one another by Euclid's I.5. Because both triangles are congruent to one another and have base angles congruent, this means angle ABE, angle AEB, angle DEC, and angle DCE are all congruent to one another.

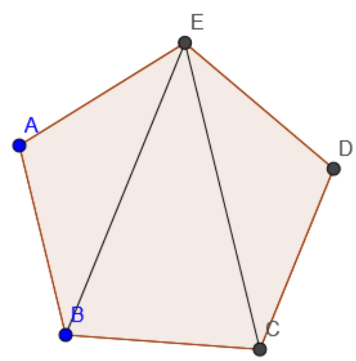


Figure 2: Pentagon ABCDE with diagonals BE and CE

Triangle BEC is isosceles because line segment BE is congruent to line segment CE. By Euclid's I.5, the base angles in triangle BEC are congruent to one another. Angle B is made up of angle ABE taken together with angle CBE. Angle C is made up of angle DCE taken together with angle BCE. Angle ABE is congruent to angle DCE and angle CBE is congruent to angle BCE. By Euclid's Common Notion 2, we can say angle B is congruent to angle C.

Since angle B is congruent to angle C and angle B is congruent to angle A, angle E, and angle D, then by Euclid's Common Notion 1, we can say angle C is congruent to angle A, angle B, angle D, and angle E. Therefore we can conclude pentagon ABCDE is equiangular. Since ABCDE is equiangular and equilateral, ABCDE is a regular pentagon.

□

Angle Bisectors of a Triangle are Concurrent

Taryn Van Ryswyk

December 2, 2016

Communicated by Ms. Maus.

Lemma 8.2. Angle bisectors of any two angles in a given triangle will intersect inside the given triangle.

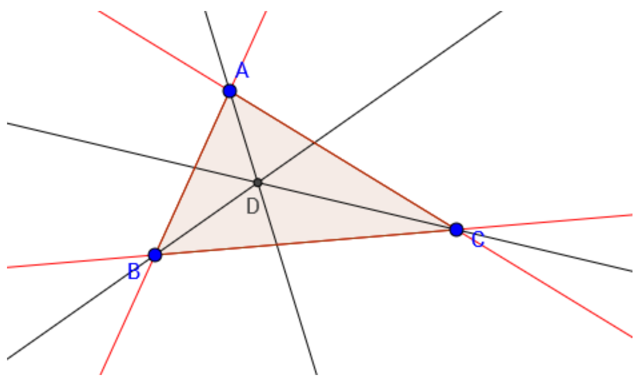


Figure 1: Triangle ABC with angle bisectors at angle A, angle B, and angle C.

Proof. By Euclid's Proposition I.9, we can bisect angle A, angle B and angle C. Since the triangle itself is the sum of two right angles by Euclid's Proposition I.32, then the sum of the angles at BAC and ABC are less than the sum of two rights. Since the sum of the angles are less than the sum of two right angles, when one of the angles is bisected the angle must be less than a right angle. Thus making the bisected angle at BAC and ABC an acute angle. By Euclid's Postulate 2 we can extend segment AB and the angle bisectors of angle BAC and angle ABC. Then by Euclid's Postulate 5, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles the two straight lines will meet on the side which the angles less than the two right angles are located. Thus, since we know that the bisected angles are acute we can conclude that the angle bisector of BAC will intersect the angle bisector of ABC at some point, which we will denote as D. Similarly for angle bisector ABC and BCA, and angle bisector BCA and BAC. Since all three interior angles of triangle ABC are acute when bisected, then the angle bisectors will intersect within the triangle. (Note this Lemma does not state that all three angle bisectors are concurrent at point D. This will be shown in Theorem 8.2.)

□

Theorem 8.2. The three angle bisectors of a triangle are concurrent.

Proof. From Lemma 8.2 the angle bisector at BAC and the angle bisector at ABC will meet at some point D, located inside triangle ABC.

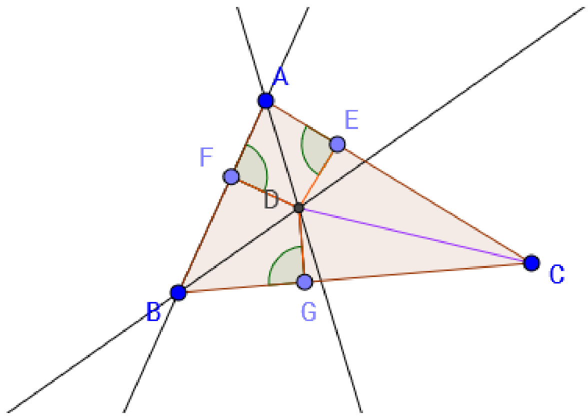


Figure 2: Triangle ABC

By Euclid's Proposition I.12, we can construct a line perpendicular to AB from point D, a line perpendicular to BC from point D, and a line perpendicular to AC from point D. The point where the perpendicular line intersects AB is labeled F. Similarly, the point where the perpendicular line intersects BC is labeled G and the point where the perpendicular line intersects AC is labeled E.

Looking at the triangle BFD and triangle BGD, we know that by the definition of an angle bisector, angle FBD is congruent to angle GBD. We know that angle BFD is congruent to angle BGD, by Euclid's Postulate 4, a right angle is equal to a right angle. Segment BD is congruent to itself. After finding two angles and a side congruent in each triangle, by Euclid's Proposition I.26 we can conclude that triangle BFD is congruent to triangle BGD. Thus, segment GD is congruent to FD. The same argument holds for triangle DFA and triangle DEA. By Euclid's Proposition I.26, we can conclude triangle DFA is congruent to triangle DEA. Thus, segment FD is congruent to segment ED. By Euclid's Common Notion 1, since segment FD is congruent to segment GD and segment FD is congruent to segment ED, then segment GD is congruent to segment ED.

By Euclid's Postulate 1, create segment CD. Next, we will look at triangle CGD and triangle CED. We know that segment GD is congruent to segment ED. By Euclid's Postulate 4 angle CGD is congruent to angle CED because a right angle is equal to a right angle. Segment CD is congruent to itself. After proving two sides congruent and an angle congruent respectively in triangle CGD and triangle CED, by Theorem 7.2 (Hypotenuse-Leg Theorem) triangle CGD is congruent to triangle CED. Thus we know that angle DCE is congruent to angle DCG. Therefore, CD is the angle bisector of BCA. Thus all angle bisectors of triangle ABC intersect at point D, making angle bisectors of a triangle concurrent. □

Tangent Line Congruency

Christopher Merck

December 2, 2016

Communicated by Ms. Maus.

Theorem 9.1. Let AB and AC be two tangent lines from a point A outside a circle. Then AB is congruent to AC.

Proof. Let a circle exist with a center at O. Let point A exist outside of Circle O. By Euclid Proposition III.17, construct lines tangent to Circle O from point A. Let the points where the tangent lines touch the circle be called point B and point C. By Postulate I draw a line segment from B to O, and from C to O. By Euclid Definition I.15, lines that fall on the circle from a point lying within the figure are equivalent to each other. Since OB and OC are both contained within the circle, and both originate from the same point, they are congruent. By Euclid Proposition III.18, which states "when a straight line from the center is joined to the point where the tangent line touches the circle, then the lines are perpendicular." Hence, segment AB is perpendicular to segment OB, and segment AC is perpendicular to segment OC. Hence, right angles will be formed at angles OCA and OBA, and these angles will be congruent to each other by Euclid Postulate 4. Next, by Euclid Postulate I, draw a line segment from O to A, to form line segment OA. By Euclid Common Notion 4, segment OA is congruent to itself. Since OA is the hypotenuse of both triangles OBA and OCA and segments OB and OC are congruent radii, and both triangles OBA and OCA are right triangles, then by the hypotenuse-leg theorem, triangles OBA and OCA are congruent. Since triangles OBA and OCA are congruent to one another, tangent lines AB and AC are congruent to each other. ☐

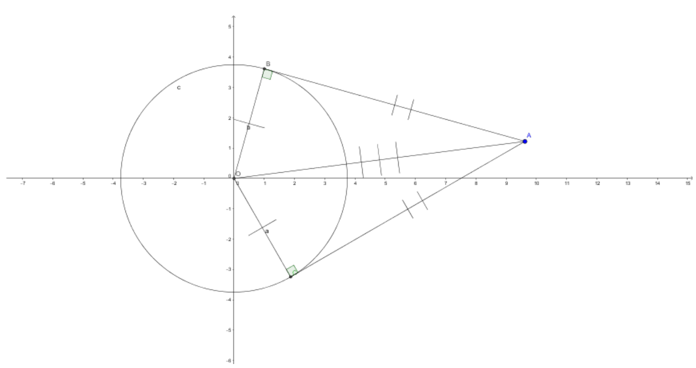


Figure 1: By the Hypotenuse Leg Theorem, tangents AB and AC are congruent.

SSS Theorem and Circle Intersections

Christopher Merck

December 2, 2016

Communicated by Ms. Van Ryswyk.

Theorem 9.2. Let Γ and Ω be two circles with centers G and O, respectively. Suppose that these circles meet at two points A and B. If GAO is a right angle, then GBO is a right angle.

Proof. Let circles Γ and Ω exist, with circle Gamma centered at point G and circle Omega centered at point O. Let the circles intersect each other at points A and B. Let angle GAO be a right angle. By Euclid Postulate I, create line segments GA, GB, OA, and OB. Since line segments GA and GB fall on the circle Γ and originate from the center G, they are congruent by Euclid's Definition I.15. Similarly, by Euclid's Definition I.15, line segments OA and OB fall on the circle Ω and originate from the center O, thus line segments OA and OB are congruent. By Euclid Postulate I, construct a line segment from point G to point O, forming line segment GO, which is congruent to itself. By Euclid Proposition I.8, since both triangle GAO and GBO have three congruent sides, they are congruent to each other. Since both triangles are congruent to each other, corresponding parts of the triangles are congruent to each other. Hence angle GBO is congruent to angle GAO, making angle GBO a right angle.

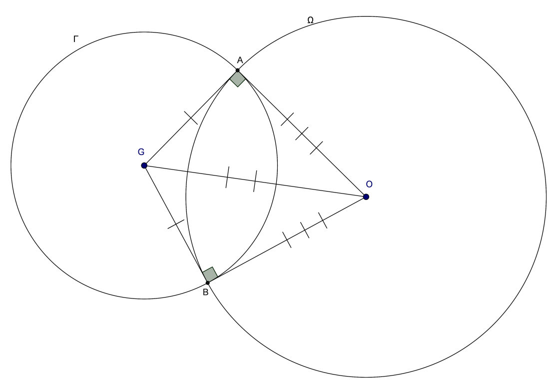


Figure 1: Circle Γ and Circle Ω are shown where angle GBO is congruent to angle GAO.

$\square$

Cyclic Quadrilaterals

Danielle Maus

December 2, 2016

Communicated by Ms. Worsfold.

Theorem 9.3. A rectangle is always a cyclic quadrilateral.

Proof. Let ABCD be a rectangle. By Euclid's Postulate 1 create diagonal AC. By Euclid's I.10 bisect line segment AC at a midpoint which we will denote as O. Because point O is the midpoint of segment AC, line segment AO is congruent to line segment CO. By Euclid's Postulate 3 we can construct a circle centered at point O with radius AO. We now know points A and C are on our circle O because both segments AO and CO are radii of our circle, but what about points B and D?

By definition of a rectangle all interior angles are right angles. Therefore, angle CDA and angle ABC are right angles. By Ms. Ahren's Theorem 7.5, because AC is our diameter of circle O and angle CDA and angle ABC are right angles, points B and D will lie on circle O. (see figure 1)

Since points A, B, C, and D of our rectangle all lie on circle O, rectangle ABCD is a cyclic quadrilateral.

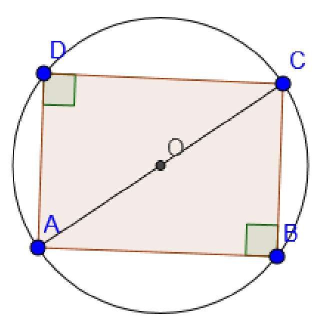


Figure 1: Rectangle ABCD

□

A Circle Contains the Sum of Four Right Angles

Taryn Van Ryswyk

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Communicated by Mr. Phaly.

Theorem AE. Let points A, B, C, and D be four points on the circumference of a circle centered at O. The four angles formed by the rays OA, OB, OC, and OD taken together make the sum of four right angles.

Proof. Extend segment OD by Euclid's Postulate 2. Let point E be the intersection point of line OD and the circle centered at O. By Euclid's Definition I.17, segment DE is the diameter of the circle centered at O. Euclid's Proposition I.13 states that a straight line set up on a straight line make angles equal to the sum of two right angles. Thus, angles DOC, COB, and BOE equal the sum of two right angles. Similarly, angle DOA and angle AOE equal the sum of two right angles. Therefore, when you sum angles DOC, COB, BOE, DOA, and AOE you will get a sum equal to four right angles. Since the circle centered at O contains all of these angles, it will also equal the sum of four right angles.

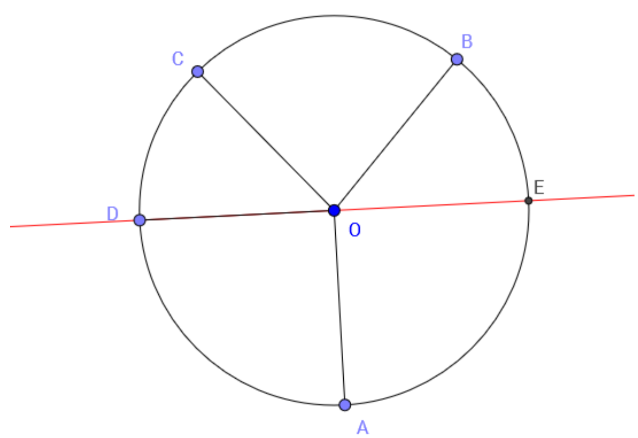

Figure 1: Points A, B, C, and D on the circumference of circle centered at O.

$\square$