Convex Polygon Angles

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Theorem 5.3. The sum of the exterior angles of a convex hexagon add to four right angles. The sum of the exterior angles of a general convex n-gon also add to four right angles.

Proof. We have hexagon ABCDEF. Extend each of the vertices (one choice made at each) to construct the exterior angles. Euclid's Proposition I.13 says if we add the interior and exterior angles at a given corner, we get two right angles. Since we have six corners on the hexagon, we get twelve right angles. These twelve right angles make up both the interior and exterior angles. At vertex A, extend the diagonals to AE, AD, and AC. Since the hexagon is convex, the diagonals lie inside the polygon. This gives us four triangles inside the polygon: AFE, AED, ADC, and ACB. Euclid's Proposition I.32 says that the three angles of a triangle add up to two right angles. Since we have four triangles, then we get a total of eight interior right angles. When we subtract the interior total from the whole total (twelve minus eight) we get a total of four exterior right angles.

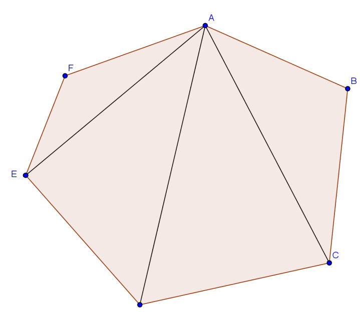

Figure 1: Hexagon ABCDEF with the diagonals drawn in

By using the same method for other polygons, we can use the equation 2n - 2(n-2), where n is the number of vertices, to get the total number of exterior angles. 2n stands for the total number of exterior and interior right angles and 2(n-2) stands for the number of triangle in the polygon times the number of right angles in the polygon. When we simplify the equation, you just get 4 exterior right angles for any n-gon.

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Refereed by Hailey Manternach