Perpendicular Bisector Intersection Theorem

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Theorem . Let T be a triangle. For any pair of sides of T, the perpendicular bisectors of those sides meet.

Proof. We will show that any two given perpendicular bisectors of a triangle will intersect. We have triangle ABC. Let line ℓ be a perpendicular bisector of side extension BC and line m be a perpendicular bisector of side extension AB. By way of contradiction, suppose that ℓ and m are parallel.

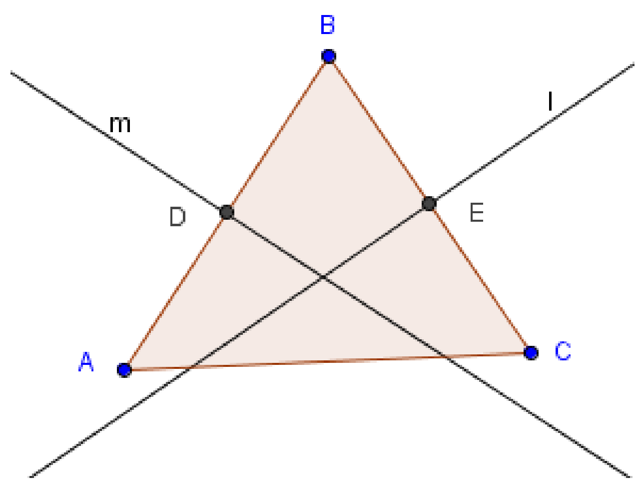

Figure 1:

We know that m meets BC because m is parallel to BC and m is parallel to ℓ. If m and BC were parallel, ℓ would be parallel and perpendicular to BC, which is impossible. similarly, we know that ℓ meets AB. Since ℓ is parallel to m, and ℓ is perpendicular to side BC, we know that m is perpendicular to BC by Euclid I.29.

We know that AB and BC meet at point B. Since triangle ABC is a triangle, we know that AB and BC are not collinear or parallel. Since AB meets line m, it also meets line ℓ by Euclid I.29. We call the intersections of ℓ and BC point W, ℓ and AB point X,m and BA point Y, and m and BC point Z.

We note that BWX makes a triangle. By Euclid I.29 points X and Y are congruent. Since we know the interior angles of a triangle added together are congruent to two right

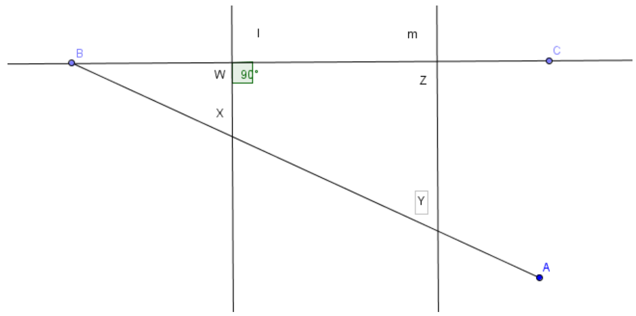

Figure 2:

angles, and BWX is a right angle, we know that WBX and BXW add up to a right angle. Neither of them are a right angle by definition. Therefore by Euclid I.29 since angle BXW is not a right angle and angle BXW is congruent to XYZ, angle XYZ is not a right angle, but m is perpendicular to BA. This is a contradiction. We conclude that any two given perpendicular bisectors must intersect.

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Refereed by Ms. Megan King