# Steiner-Lehmus Theorem

Trent Baker

May 1, 2015

**Theorem 10.5.** If two angle bisectors of a triangle are congruent, the triangle is isosceles.

*Proof.* By way of contradiction, assume that triangle ABC, with congruent angle bisectors BE and CD, is not isosceles. Then segment AC is not congruent to AB. Without loss of generality, let AC be greater than AB. By Proposition 18, angle ABC is greater than ACB, and angle EBC is greater than angle DCB. Consider triangles BEC and CDB that share side BC and have congruent sides BE and CD. By Proposition 24, because angle EBC is greater than angle DCB, segment EC is greater than BD.

By proposition 31, construct a straight line at point E parallel to BD, and a straight line parallel to BE at point D. Consider quadrilateral BDFE, which is a parallelogram by construction. By definition of a parallelogram, angle EFD is congruent to angle DBE. Then angle EFD is congruent to angle EBC and greater than either angle DCB and angle DCE. By definition of a parallelogram, segment DF is congruent to BE, and congruent to segment CD.

Consider isosceles triangle FDC. By Proposition 5, angles EFD and EFC together are congruent to angles DCE and ECF together. Because angle EFD is greater than angle DCE, the angle of EFC must be less than the angle of ECF. Consider triangle EFC. By proposition 19, segment EF is greater than EC.

However, segment EF is congruent to segment BD, which is less than EC. From this impossibility, triangle ABC must be isosceles.

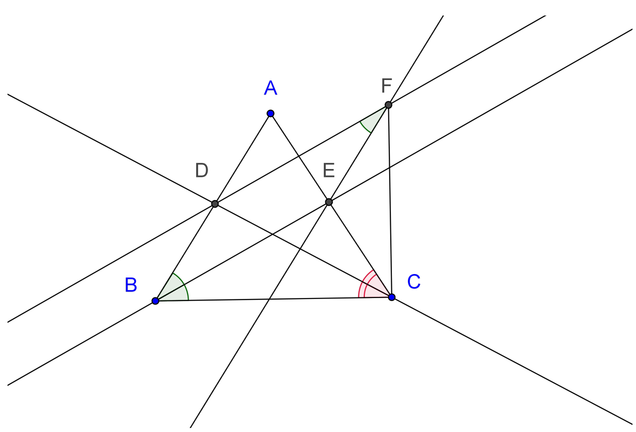


Figure 1: Construction of Triangle FDC and FEC

□