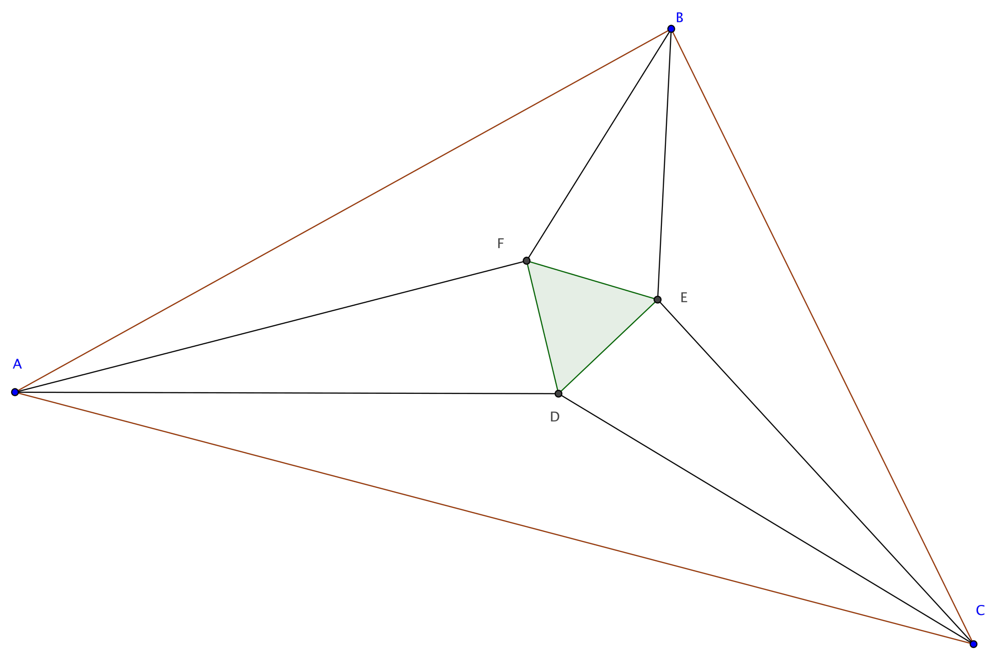
# Transactions
# in
# **Euclidean Geometry**




Volume 2018F

Issue # 3

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# Kite Diagonal Angle Measure

Alexa DeVore

October 1, 2018

*Communicated by: The Editor.*

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**Conjecture 2.5.** If the diagonals of a kite meet, they meet at a right angle.

*Proof.* Let ABCD be a kite. Let there exist segments AC and BD such that they are the diagonals. Where they cross will be point X. Now consider triangles ABD and CBD. Based off the definition of a kite, we know that there will be two pairs of adjacent congruent sides. Therefore, we know AB is congruent to CB, DA to DC. BD is congruent to itself from Common Notion 1. Therefore, triangle ABD is congruent to CBD by Euclid Proposition 1.8.

Now consider the triangles BXA and BXC. We know that angles ABX and CBX must be congruent since they are corresponding angles from triangles ABD and CBD. We also know that AB is congruent to CB and that XB must be congruent to itself. Therefore, we have congruent triangles by SAS or Euclid Proposition 1.4. Since triangles are congruent, we also know that angle AXB must be congruent to angle CXB. Since these two angles lie on a straight line, we know from Euclid Proposition 1.13 that the total sum of these two must be equal to two right angles. However since AXB is congruent to CXB, they must each be a right angle. Furthermore, due to the property of vertical angles, we know that AXD and CXD must also be right angles. Therefore, the diagonals of a kite meet at a right angle. □

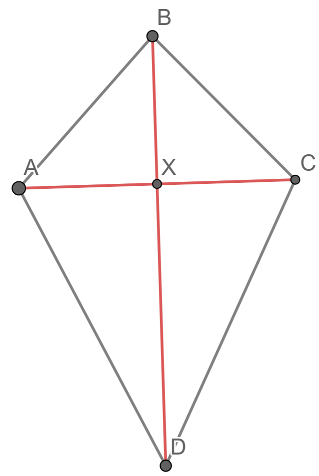


Figure 1: This is a picture of the kite

# A Rectangle is a Parallelagram

Lauren Falck

October 1, 2018

*Communicated by: Mr. Warner.*

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**Conjecture 3.1.** Let R be a rectangle. Then R is a parallelogram.

**Definition Rectangle.** A quadrilateral which has all four interior angles that are right angles.

*Proof.* By the definition of a rectangle, angles BAC, ABD, ACD, and BDC are right angles. By P.2, we can extend the lines of the rectangle. We can also add points to those extended lines to help with communication.

By Euclid I.13 there exist lines EH and FK that are straight lines, which create angles. Since lines EH and FK are straight lines that make angles, angle BAC and angle EAC must add up to two right angles or be two right angles. Since angle BAC is a right angle, angle EAC must also be a right angle. Then by Euclild I.15, Since angle BAC is a right angle, angle FAE must also be a a right angle, and since angle EAC is a right angle, angle FAB must also be a right angle.

Similarly, using Euclid I.13, there exist lines EH and GJ, which are straight lines that make angles. Since lines EH and GJ are straight lines that make angles angle ABD and angle HBD must add up to two right angles or be two right angles. Since angle ABD is a right angle, angle HBD must also be a right angle. Then by Euclild I.15, Since angle ABD is a right angle, angle GBH must also be a a right angle, and since angle HBD is a right angle, angle GBA must also be a right angle.

Similarly, using Euclid I.13, there exists lines LI and FK, which are straight lines that make angles. Since lines LI and FK are straigh lines with angles, angle DCA and angle LCA must add up to two right angles or be two right angles. Since angle DCA is a right angle, angle LCA must also be a right angle. Then by Euclild I.15, Since angle DCA is a right angle, angle LCK must also be a a right angle, and since angle LCA is a right angle, angle KCD must also be a right angle.

Similarly, using Euclid I.13, there exists lines LI and GJ, which are straight lines that make angles. Since lines LI are straight lines that make angles, angle BDC and angle BDI must add up to two right angles or be two right angles. Since angle BDC is a right angle, angle BDI must also be a right angle. Then by Euclild I.15, Since angle BDC is a right angle, angle JDI must also be a a right angle, and since angle BDI is a right angle, angle CDJ must also be a right angle.

By Euclid I.27, since the alternate angles are congruent, lines AB and CD are parallel to each other and lines AC and BD are parallel to each other. Since the opposite sides of the quadrilateral are parallel, the rectangle is a parallelogram.

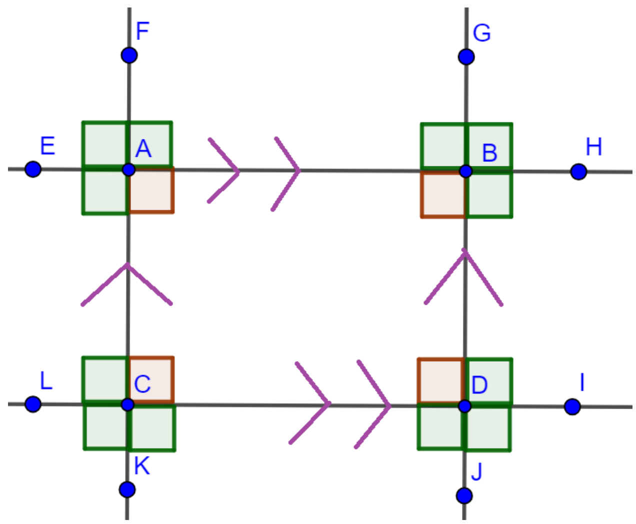
Parallel Proof.png



Figure 1: This is a picture of rectangle R.

□

# Rectangles and Parallel Lines

Alexa DeVore

October 1, 2018

*Communicated by: The Editor.*

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For the sake of this proof, we are going to assume that Conjecture $I$ stating "Let line AB be parallel to CD. If line BC is perpendicular to AB and line AD is perpendicular to CD, then line BC is perpendicular to AD," is true.

**Conjecture 3.5.** Let ABCD be a quadrilateral such that angles ABC and ADC are right angles. If AB and CD are parallel then ABCD is a rectangle, assuming Conjecture $I$ is true.

*Proof.* Let ABCD be a quadrilateral such that the previous conditions are met. Based off of Conjecture $I$, we can conclude that BC is parallel to AD. Based off postulate 2, we can extend the lines of the rectangle to be able to utilize the exterior angles. I will extend line CD and add a point E above C. Since BC and AD are parallel, we know that the corresponding angles are congruent from Euclid 1.29. Therefore, angle BCE must also be a right angle. Since angle BCE is on line DC, we know that the two angles that fall on this line must be equal to two right angles from Euclid 1.13. Therefore angle BCD must also be a right angle. Similarly, I will extend line AB down and name a point F. Therefore for the same reasons, angles FAD and BAD must be right angles. Therefore this quadrilateral has four right angles and must be a rectangle. □

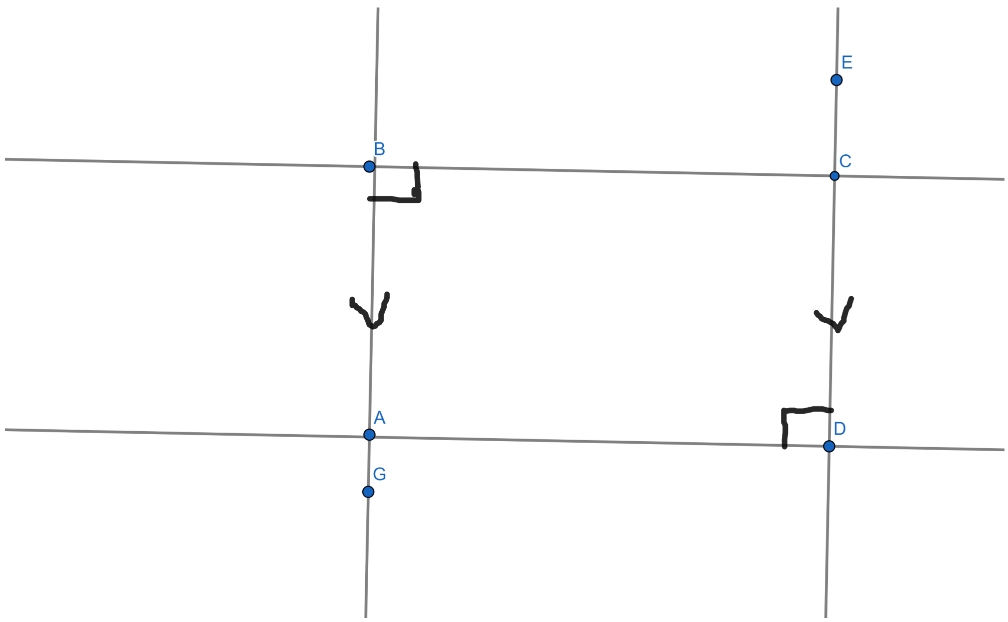


Figure 1: This is a picture of the quadrilateral.

# Rectangle Midpoints Form a Parallelogram

Lexis Wiegmann

October 3, 2018

*Communicated by: Ms. Miller.*

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This proof will only work for a certain case.

**Theorem 3.7.** Let ABCD be a rectangle. The midpoints of the four sides are the vertices of a parallelogram.

*Proof.* Let ABCD be a rectangle. Because it's a rectangle, we know from Mr. Warner's conjecture 3.2 that the pair of opposite sides of a rectangle are congruent. Line segments AE, EB, DG, and GC are all congruent by the definition of midpoint. Similarly, line segments AH, HD, BF, and FC are all congruent by the definition of midpoint.

By Euclid Postulate I.1, draw line segments EF, FG, GH, and HE. We will first look at triangle HAE and triangle EBF. Since line segments AE and EB are congruent, line segments AH and BF are congruent, and angles HAE and EBF are congruent, then triangles HAE and FBE are congruent by Euclid Proposition I.4, (SAS). Because the triangles are congruent, we know the corresponding sides and angles are congruent as well. Therefore, line segments EF and HE are congruent. In the same way, it can be shown that each of the triangles AEH, EBF, CFG, and HGD are congruent to the other.

We will now look at quadrilateral EFGH. We know that all of the sides of this quadrilateral are congruent because of the paragraph above. By Euclid Postulate I.1, draw line segment EG. The two triangles HEG and FEG share the line EG. Because line segments EF, FG, GH, and HE are congruent, and line segment EG is common, triangles HEG and FEG are congruent by Euclid Proposition I.8 (SSS). Also, because the four sides of this quadrilateral are all congruent, by definition, this figure is a rhombus. The four midpoints of a rectangle are the vertices of a rhombus. By Ms. Miller, Mr. Stine, and Mr. Warner's Theorem 1.6, (A Rhombus is a Parallelogram) we know that a rhombus is a parallelogram. Therefore, the midpoints of a rectangle are the vertices of a parallelogram.

□

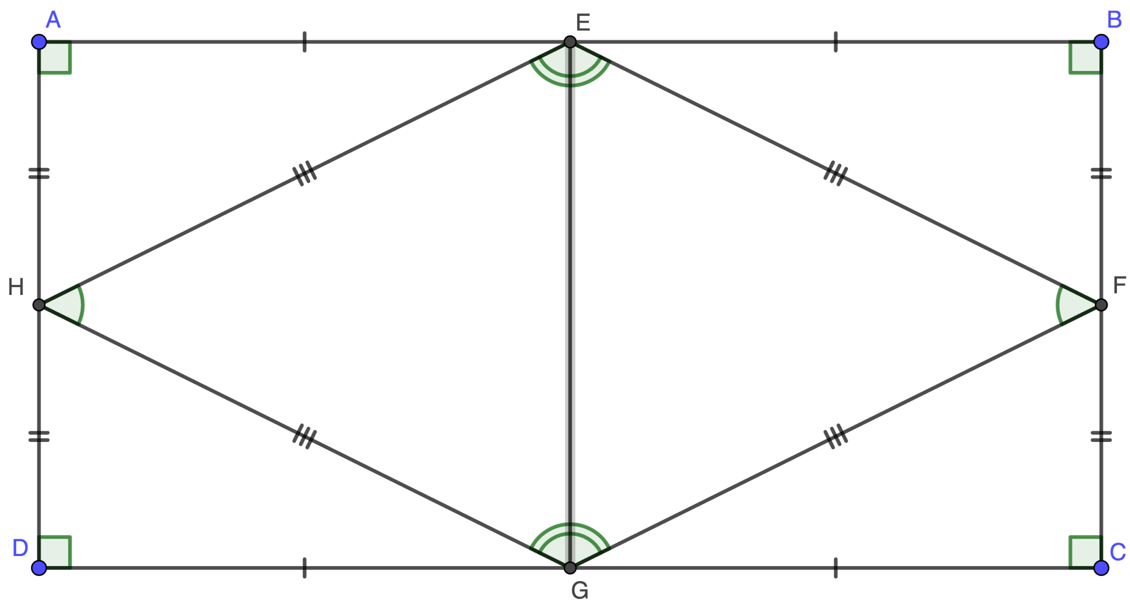


Figure 1: ABCD is a rectangle with EFGH being a rhombus.

# This Figure Is a Parallelogram

Lexis Wiegmann

October 3, 2018

*Communicated by: Mr. Stine.*

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**Conjecture 3.8.** Let ABCD be a quadrilateral. If AB is congruent to CD and BC is congruent to AD, then ABCD is a parallelogram.

*Proof.* Let ABCD be a quadrilateral. By Euclid Postulate I.1, draw the line AC. Line BC is congruent to line AD, line AB is congruent to line DC, and line AC is shared by the two triangles. Therefore, by Proposition I.8, (SSS) triangle ABC is congruent to triangle ADC. Since the triangles are congruent, that means the corresponding angles are congruent as well. Angle ABC is congruent to angle ADC, angle BAC is congruent to angle DCA, and angle DAC is congruent to angle BCA. From Euclid Proposition I.27, because angle DCA is congruent to angle BAC, line AB and line CD are parallel. Similarly, line BC and line AD are parallel. Therefore, this quadrilateral is a parallelogram. □

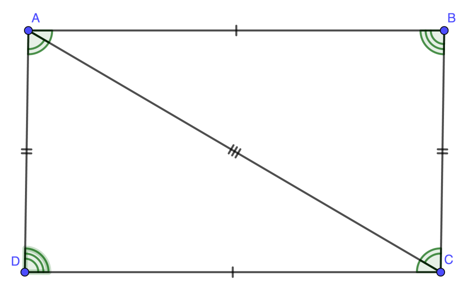


Figure 1: ABCD is a quadrilateral with line BC congruent to line AD and line AB congruent to line DC.

# Exterior Angle Congruence

Jason Stine

November 30, 2018

*Communicated by: Ms. Falck*

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**Theorem 5.1.** Exterior angles on a polygon that are centered at the same vertex will be congruent.

*Proof.* Let A, B, and C be consecutive vertices on a N sided polygon, where N is a natural number greater than 2. Construct two rays, ray AB with endpoint A and ray CB with endpoint C. By Euclid Proposition 1.15, angles CBY and ABX are congruent. Thus the exterior angles centered at the same vertex will be congruent. □

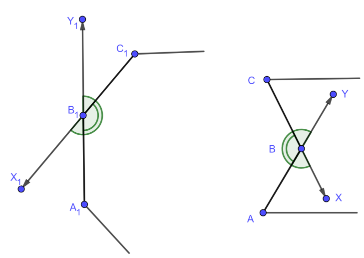


Figure 1: N Sided Polygons with Consecutive Vertices A,B,C.

# Pentagon Exterior Angles

Alexa DeVore

October 7, 2018

*Communicated by: Mr. Stine.*

**Conjecture 5.2.** The exterior angles of a pentagon, one choice made at each vertex, add up to four right angles.

*Proof.* Let ABCDE be a pentagon. For the sake of this proof, we will extend each of the lines into a ray. Now consider the two angles made at the point on each ray. Euclid 1.13 says that two angles that lie on a line will sum up to that of two right angles. Since this pentagon has five lines, that means these lines create a shape that is equal to 10 right angles. Now create the diagonals of this pentagon by connecting points A and D and connecting points A and C. Now this pentagon can be viewed it comprised of three triangles: ADE, ACD, and ABC. Euclid 1.32 states that the interior angles of a triangle are always equal to that of two right angles. Since our pentagon is comprised of three triangles, that means interior angles of our pentagon is equal to six right angles. Since our five points contain a total of 10 right angles, and the interior contains six right angles, that means that four right angles are not accounted for in the interior of the pentagon. Therefore, the exterior angles must be equal to four right angles.

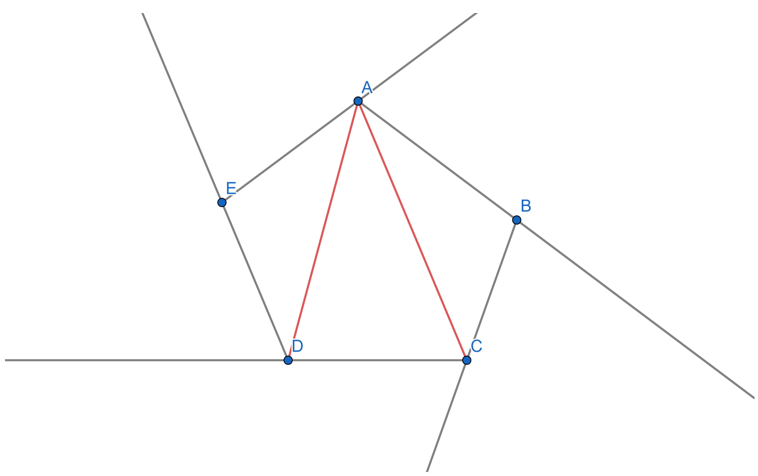


Figure 1: This is a pentagon with lines extended and diagonals.

□

# Exterior Angles of a Hexagon and N-gon

Jaclyn Miller

October 3, 2018

*Communicated by: Mr. Vandelune.*

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**Theorem 5.3.** The sum of the exterior angles of a convex hexagon is congruent to four right angles. This is true for all convex n-gons.

*Proof.* Let ABCDEF be a hexagon. By Euclid Postulate I.2, extend all line segments out to one side. At each vertex, two lines meet. By Euclid Proposition I.13, each vertex–including exterior and interior angle measure–is congruent to the sum of two right angles. Since there are six verticies in a hexagon, the measure of all interior and exterior angles taken together is congruent to the sum of 12 right angles.

Draw line segments AC, AD, and AE. Notice that the diagonals divide the convex hexagon into 4 triangles. By Euclid Proposition I.32, triangles ABC, ACD, ADE, and AEF have interior angle measures congruent to two right angles. Since there are 4 triangles, and each triangle has angles that, when taken together, are congruent to 2 right angles, the interior angles of the hexagon taken together are congruent to 8 right angles. Since the total number of right angles in the hexagon–including interior and exterior angles–was congruent to 12 right angles, and the interior angles taken together are congruent to 8 right angles, the sum of the exterior angles of a convex hexagon are congruent to 4 right angles.

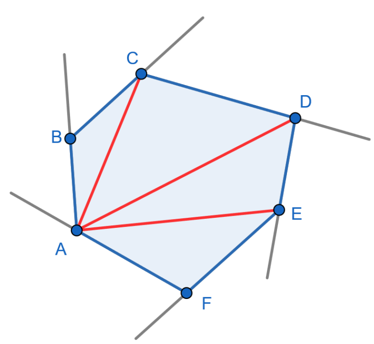


Figure 1: Hexagon ABCDEF.

In general, any convex n-gon will have the exterior angles congruent to 4 right angles by a similar process. Look back at the convex hexagon example. The total sum of the interior and exterior angles was the same as 2n, where n is the number of verticies in the convex n-gon. For example, there are six verticies in a hexagon, and the total sum of the interior and exterior angles of the hexagon is congruent to 12 right angles, which is 2(6). Then, the sum of the interior angles is equal to 2(n-2). For example, in a convex hexagon, we found that the sum of the interior angles is congruent to 8 right angles, which is 2(6-2). To find the sum of the exterior angles, take the total number of right angles that the figure's angles are congruent to, and subtract the total number of right angles that is congruent to the measure of the interior angles. Then, the sum of the exterior angles of an n-gon is 2n - 2(n-2). Using this equation, four right angles will always be the sum of the exterior angles.

$\square$

# Inside of a Convex Polygon

Jaclyn Miller

October 23, 2018

*Communicated by: Ms. Wiegmann.*

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**Definition C.** Given a convex polygon, we say a point is inside the polygon if the point lies on a line segment with two endpoints on different sides of the polygon.

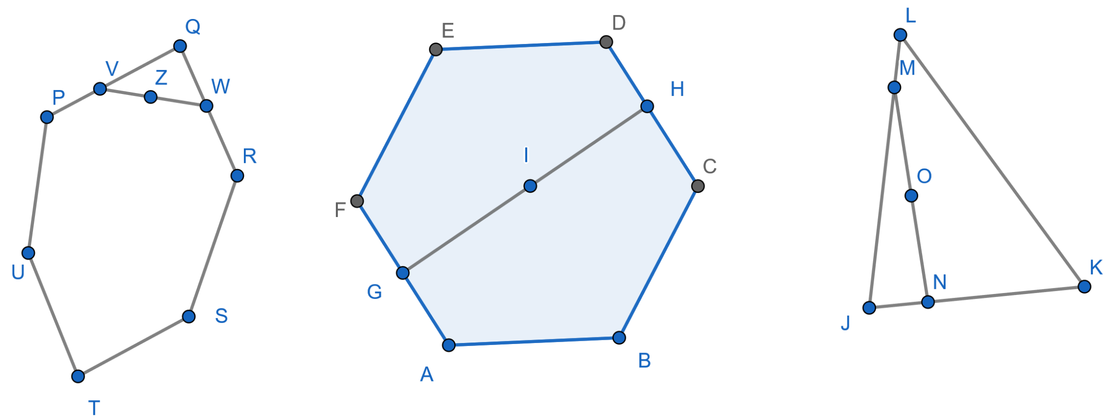


Figure 1: Three examples of points inside polygons.

**Examples C.** (Left) Polygon TUPQRS has point Z. Point Z is inside TUPQRS because it lies on line segment VW, where V and W lie on different sides of TUPQRS.
(Center) Hexagon AFEDCB has point I. Point I is inside AFEDCB because it lies on line segment GH, where G and H lie on different sides of AFEDCB.
(Right) Triangle JLK has point O. Point O is inside JLK because it lies on line segment MN, where M and N lie on different sides of JLK.

# Given Angle A and B a Rhombus is Regular

Brad Warner

October 2, 2018

*Communicated by: Lauren Falck*

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**Definition 1.** Transitivity is when A is congruent to B and B is congruent to C, then A is congruent to C.

**Theorem 6.2.** Let ABCD be a rhombus. If angle A is congruent to B, then ABCD is regular.

*Proof.* Let ABCD be a rhombus. By Theorem 1.6, we know a Rhombus is a parallelogram. Therefore, a rhombus is equilateral, since all sides are congruent. By Theorem 1.1, we know a rhombus has opposite angles are congruent. Given angle A is congruent to angle B, angle A is congruent to angle C, and angle B is congruent to angle D then by transitivity all angles are congruent. Thus, ABCD is equiangular. Since ABCD is equilateral and equiangular then ABCD is regular.

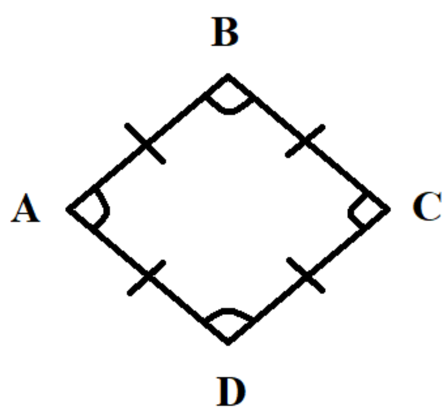


Figure 1: Regular Rhombus ABCD

□

# Equilateral Pentagons are not always Regular

Jaclyn Miller

October 3, 2018

*Communicated by: Mr. Vandelune.*

The counter-example to this conjecture will only work if it is possible to construct a regular pentagon. If it is not possible to construct a regular pentagon at all, then it is not possible to construct a regular pentagon with the conditions given in the conjecture.

**Conjecture 6.4.** Let AEDCB be an equilateral pentagon. If angle A is congruent to angle B, then ABCDE is regular.

**Counter Example 6.4.** Let AEDCB be a regular pentagon. Since AEDCB is a regular pentagon, line segments AE, ED, DC, CB, and AB are congruent by definition. By Euclid Postulate I.2 draw a circle with center point E and radii AE and ED, called circle E. Similarly, construct a circle with center at point C and radii CD and CB, called circle C. Notice that point D is a point of intersection for circle E and circle C. Thus, there exists a point M that is inside both circle C and circle E; and there exists a point N that is inside circle C but outside circle E. Then, by the Circle-Circle Intersection Property, circle C and circle E intersect twice.

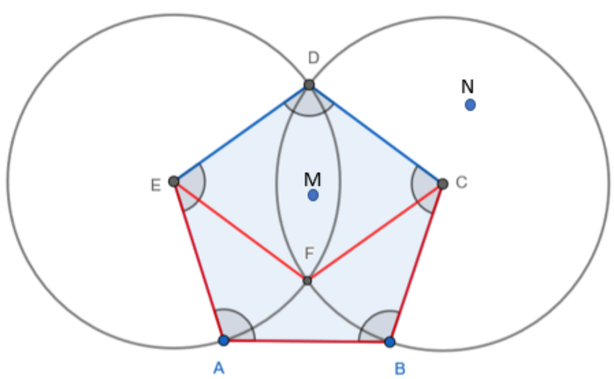


Figure 1: Regular Pentagon AEDCB and Equilateral Pentagon AEFCB.

Let F be the other point of intersection. By Euclid Postulate I.1, draw line segments EF and FC. Since line segment EF is a radius of circle E, line segment EF is congruent to line segment EA. Since line segment FC is a radius of circle C, line segment FC is congruent to line segment CB. Since line segments EA, AB, and CB are sides of regular pentagon AEDCB, they are congruent by definition. Thus, line segments EA, EF, FC, CB, and AB are congruent. Therefore, pentagon AEFCB is equilateral.

Since AEDCB is a regular pentagon, it is equiangular by definition. Thus, angles A, B, C, D, and E are congruent. In the new pentagon AEFCB, angle A and angle B are still congruent. However, line segment EF cuts angle AED. By Euclid Common Notion I.5, angle AEF is less than angle DEA. Similarly, by Euclid Common Notion I.5, angle FCB is less than angle DCB. Thus, AEFBC is a equilateral pentagon with angles A and B congruent that is not equiangular and thus, not regular.

# Angles of a Regular Pentagon

Jaclyn Miller

September 28, 2018

*Communicated by: Ms. DeVore.*

**Theorem 6.6.** Let ABCDE be a regular pentagon. Then the measure of angle ACD and the measure of angle CDA is exactly twice as large as the measure of angle CAD.

*Proof.* Let ABCDE be a regular pentagon. Since ABCDE is regular, it is equiangular and equilateral. By Ms. DeVore's Theorem 5.2, the measure of all the interior angles of ABCDE is congruent to six right angles. Since there are five congruent interior angles, each angle must be $\frac{6}{5}$ of a right angle.

By Euclid Postulate I.1, draw line segments AC and AD. Since line segment AB is congruent to line segment BC, triangle ABC is isosceles by definition. Since triangle ABC is isosceles, base angles BCA and BAC are congruent by Euclid Proposition I.5. Similarly, since line segment AE is congruent to line segment ED, triangle AED is isosceles by definition. Since triangle AED is isosceles, base angles ADE and DAE are congruent by Euclid Proposition I.5.

The measure of the interior angles of triangle ABC is congruent to two right angles by Euclid Proposition I.32. Since angle ABC is congruent to $\frac{6}{5}$ of a right angle, and angles BCA and BAC are congruent, angle BCA is congruent to $\frac{2}{5}$ of a right angle and angle BAC is congruent to $\frac{2}{5}$ of a right angle. Similarly, the measure of the interior angles of triangle AED is congruent to two right angles by Euclid Proposition I.32. Since angle AED is congruent to $\frac{6}{5}$ of a right angle and angles DAE and ADE are congruent, angle DAE is congruent to $\frac{2}{5}$ of a right angle and angle ADE is congruent to $\frac{2}{5}$ of a right angle.

By Mr. Warner's Theorem 6.5, triangles ABC and AED are congruent, so triangle CAD is isosceles. Thus, base angles ACD and CDA are congruent by Euclid Proposition I.5. Since angle BCA is congruent to $\frac{2}{5}$ of a right angle, and angle BCD is congruent to $\frac{6}{5}$ of a right angle, angle ACD is congruent to $\frac{4}{5}$ of a right angle by subtraction. Similarly, since angle ADE is congruent to $\frac{2}{5}$ of a right angle and angle CDE is congruent to $\frac{6}{5}$ of a right angle, angle CDA is congruent to $\frac{4}{5}$ of a right angle by subtraction.

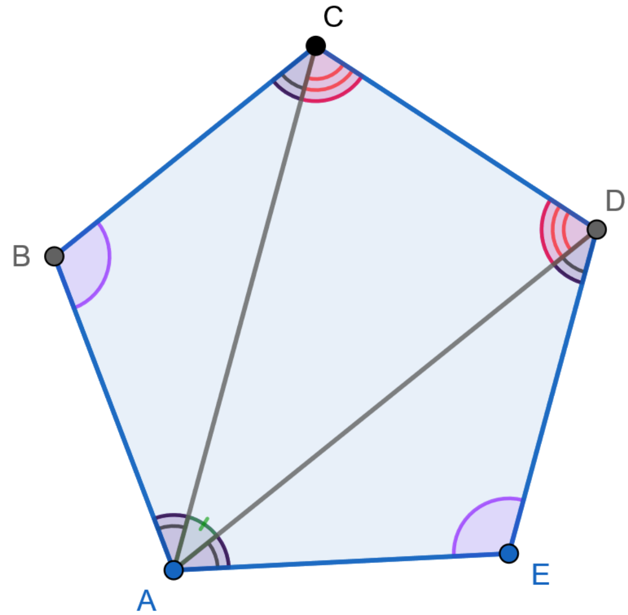


Figure 1: Regular Pentagon ABCDE.

Finally, since angle BAC is congruent to $\frac{2}{5}$ of a right angle, angle DAE is congruent to $\frac{2}{5}$ of a right angle, and angle BAE is congruent to $\frac{6}{5}$ of a right angle, then angle CAD is congruent to $\frac{2}{5}$ of a right angle by subtraction.

Therefore angle ACD is congruent to $\frac{4}{5}$ of a right angle, angle CDA is congruent to $\frac{4}{5}$ of a right angle, and angle CAD is congruent to $\frac{2}{5}$ of a right angle. Thus, the measure of angles ACD and the measure of angle CDA is exactly twice as large as the measure of angle CAD. $\square$

# Circle Diameter Angle

Alexa DeVore

October 17, 2018

*Communicated by: Ms. Miller.*

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**Conjecture 7.4.** If AB is the diameter of a circle, and C lies on the circle then angle ACB is a right angle.

*Proof.* Let a line segment AB be the diameter of a circle centered at the midpoint X. Let a point C exist on the circle and create segments AC, CB, and XC, by Euclid Proposition 1.1. Note that since point X bisects diameter AB, AX and XB are radii and therefore congruent. Since segment CX connects the center of the circle to a point on the circle C, CX must be a radius of the circle and therefore congruent to AX and XB.

Now consider the triangles AXC and CXB. Since segment AX is congruent to CX, we can call triangle AXC isosceles by Euclid Definition 20 and therefore, base angles XAC and XCA must be congruent by Euclid Proposition 1.5. Similarly, triangle CXB has two congruent sides so angles XCB and XBC also are congruent by 1.5. Now consider triangle ABC. Since triangle ABC is composed of angles XAC, XCA, XCB, and XBC, we know that these four angles must congruent to two right angles.

Now since we know that we have two congruent angles from triangle AXC and two congruent angles from triangle CXB, and that these four angles sum up to two right angles, we know that one angle from AXC and one from CXB must sum to one right angle. Since angle ACB is comprised of one angle from AXC and CXB, it must be a right angle. Therefore when point C lies on a circle with diameter AB, ACB will be a right angle.

□

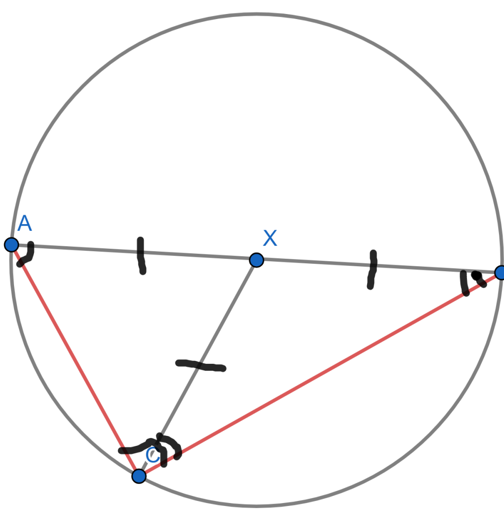


Figure 1: This is a picture of the circle and triangle.

# Right Triangles and Circles

Jaclyn Miller

October 9, 2018

*Communicated by: Ms. Wiegmann.*

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This is a proof of the contra-positive: If point C is not on the edge of the circle, then ACB is not a right angle. This proof will examine the only other options, point C inside the circle and point C outside the circle, through two cases. Case 1 is if point C is outside the circle. Case 2 is if point C is inside the circle.

**Theorem 7.5.** If angle ACB is a right angle, then C lies on the circle with diameter AB.

*Proof.* Let line segment AB be the diameter of a circle, called circle M.

**Case One**

1. Let point C be an arbitrary point outside of circle M.
2. By Euclid Postulate I.1, draw line segments AC and BC to create triangle ACB.
3. Let the second point where line segment AC crosses circle M (the first is at point A) be point E.
4. Let the second point where line segment BC crosses circle M (the first is at point B) be point F.
5. Choose any point on the arc EF, and call it D.
6. Draw line segments AD and BD, by Euclid Postulate I.1, to create triangle ADB.

Then, since D lies on the edge of the circle and AB is the diameter of the circle, angle ADB is a right angle by Ms. DeVore's Theorem 7.4. Since triangle ADB lies within triangle ACB, and the triangles share side AB, angle ACB is less than angle ADB by Euclid Proposition I.21. Since angle ADB is a right angle, and angle ACB is less than angle ADB, angle ACB is not a right angle. Therefore, point C cannot be outside the circle and create a right angle.

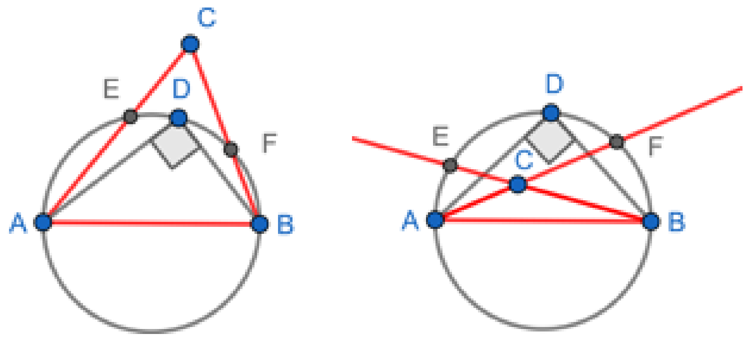


Figure 1: Construction of Circle M in Case One (Left) and Case Two (Right).

**Case Two**

1. Let point C be an arbitrary point inside of circle M.
2. By Euclid Postulate I.1, draw line segments AC and BC to create triangle ACB.
3. By Euclid Postulate I.2, extend line segments AC and BC.
4. Let the second point where line segment AC crosses circle M (the first is at point A) be point F.
5. Let the second point where line segment BC crosses circle M (the first is at point B) be point E.
6. Choose any point on the arc EF, and call it D.
7. Draw line segments AD and BD, by Euclid Postulate I.1, to create triangle ADB.

Then, since D lies on the edge of the circle and AB is the diameter of the circle, angle ADB is a right angle by Ms. DeVore's Theorem 7.4. Since triangle ACB lies within triangle ADB, and the triangles share side AB, angle ACB is greater than angle ADB by Euclid Proposition I.21. Since ADB is a right angle, and angle ACB is greater than angle ADB, angle ACB is not a right angle. Therefore, point C cannot be inside the circle and create a right angle.

Thus, C cannot be outside or inside the circle and create a right angle. Therefore, C must be on the circle. Hence, if angle ACB is a right angle, C must lie on the circle with diameter AB. □

# Perpendicular Bisectors are Concurrent

Jaclyn Miller

October 21, 2018

*Communicated by: Ms. Falck.*

**Theorem 8.4.** The three perpendicular bisectors of any triangle are concurrent.

*Proof.* Let ABC be a triangle. By Euclid Proposition I.10: let D be the midpoint of line segment AB; let E be the midpoint of line segment BC, and let F be the midpoint of line segment AC. By Euclid Proposition I.11, draw a line through point D and perpendicular to AB, called line L. By Euclid Proposition I.11, draw a line through point E and perpendicular to line BC, called line M. Then L and M are perpendicular bisectors of triangle ABC. By Mr. Vandelune's Theorem 8.3, L and M meet at a point, which we call point X. Draw line segments AX, BX, and CX by Euclid Postulate I.1.

Since point D is the midpoint of line segment AB, line segment AD is congruent to line segment BD. Since L is the perpendicular bisector of line segment AB, angles ADX and XDB are congruent, and in fact, right angles. Thus, since line segments AD and BD are congruent, angles ADX and XDB are congruent, and line segment DX is shared: triangles ADX and XDB are congruent by Euclid Proposition I.4. Since corresponding parts of congruent triangles are congruent, line segment BX is congruent to line segment AX.

Similarly, since point E is the midpoint of line segment BC, line segments BE and EC are congruent. Since M is the perpendicular bisector of line segment BC, angles BEX and CEX are congruent, and in fact, right angles. Thus, since line segments BE and EC are congruent, angles BEX and CEX are congruent, and line segment EX is shared: triangles BEX and CEX are congruent by Euclid Proposition I.4. Since corresponding parts of congruent triangles are congruent, line segment CX is congruent to line segment BX. Thus by Euclid Common Notion I.1, since line segments CX and AX are congruent to BX, then line segments CX and AX are congruent.

By Euclid Postulate I.1, draw line segment FX. Since point F is the midpoint of AC, line segment AF is congruent to line segment CF. Since line segments AX and CX are congruent, line segments AF and CF are congruent, and line segment XF is shared, triangles XAF and XCF are congruent by Euclid Proposition I.8. Since corresponding parts of congruent triangles are congruent, angle AXF is congruent to angle CXF. Since angles AXF and CXF

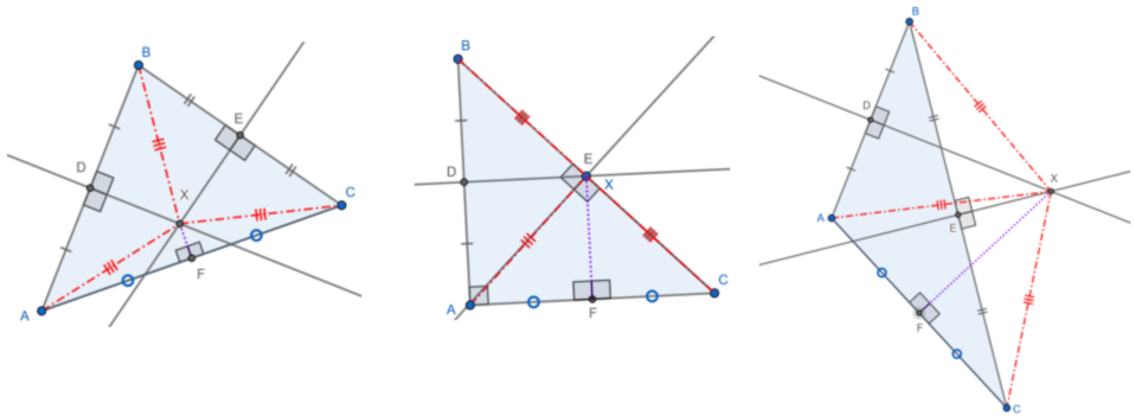


Figure 1: Depiction in Acute Triangle (left), Right Triangle (center), Obtuse Triangle (right).

are on a straight line, they are congruent to two right angles when taken together, by Euclid Proposition I.13. Since AXF and CXF are congruent and taken together are congruent to two right angles, angle AXF is a right angle and angle CXF is a right angle.

Since angles AXF and CXF are adjacent right angles, line segment FX is perpendicular to line segment AC, by Euclid Definition I.10. Thus, FX is the perpendicular bisector of line segment AC. Since FX, L, and M are the perpendicular bisectors of the three sides of the triangle, and pass through point X, point X is the intersection of the three perpendicular bisectors. Thus, the three perpendicular bisectors of any triangle are concurrent.

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