Pythagorean Theorem

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The following will prove the Pythagorean Theorem based off of Euclid I.47.

Theorem 13.1. In a right triangle, the square opposite the right angle has equal content with the other two squares.

Proof. Let ABC be a triangle with angle A as a right angle. Let the squares of each side of a triangle. Construct segment AL which is parallel to segment BD. Also construct segment AD and FC. Because angle BAG and angle BAC are right angles we know the segment GAC is a straight line by Euclid I.14. Likewise, angle BAC and CAH are each right angles forming straight line HAB by Euclid I.13.

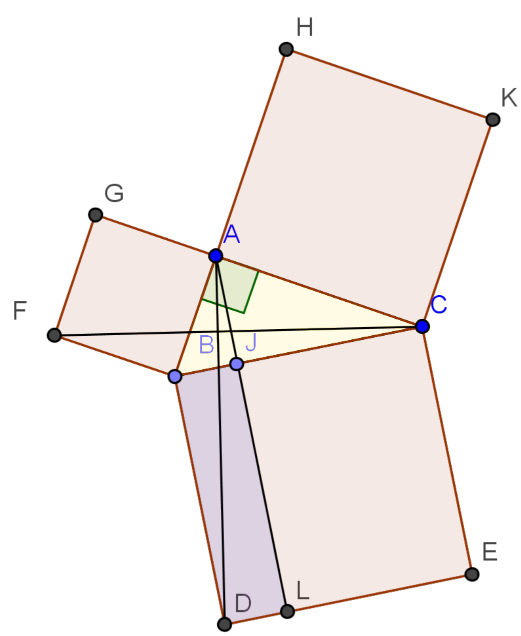

Figure 1: Proving that two of triangle ABD has the equal content with quadrilateral BL. Also proving two of triangle FBC has equal content with square BG. Proving that quadrilateral BL and square BG have equal content.

By Euclid I.46, angle FBA and angle DBC are congruent. When you combine angle ABC with angle FBA and angle DBC possible by Euclid Common Notion 2, then angle DBA is congruent to angle FBC. By Euclid I.46, segment DB is congruent to segment BC. Also segment FB is congruent to segment BA. Using this we know triangle ABD is congruent to FBC by Euclid I.4

By Euclid I.41 we know that two of triangle ABD will have equal content with quadrilateral BL. Likewise, by Euclid I.41, two of triangle FBC will have equal content with BG. Because doubles of equals are still equal to each other, square BG has equal content with quadrilateral BL.

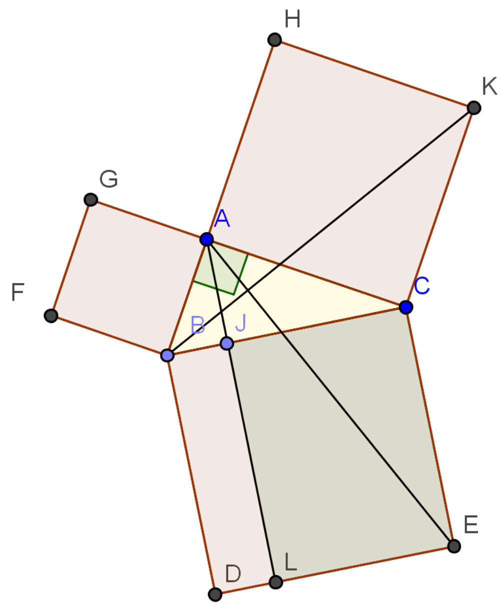

Figure 2: This shows segments AE and BK to complete the same argument.

Let AE and BK be segments. Repeating this same process with triangles ACE and BCK we know that quadrilateral CL has equal content with square CH.

We know that quadrilateral BL has equal content with square BG and quadrilateral CL has equal content with square CH. We also know that quadrilaterals CL and BL make up square CE. Therefore we can conclude that the square on the side BC is equal to the squares on the sides BA and AC.Refereed by Nicole Hegewald. □