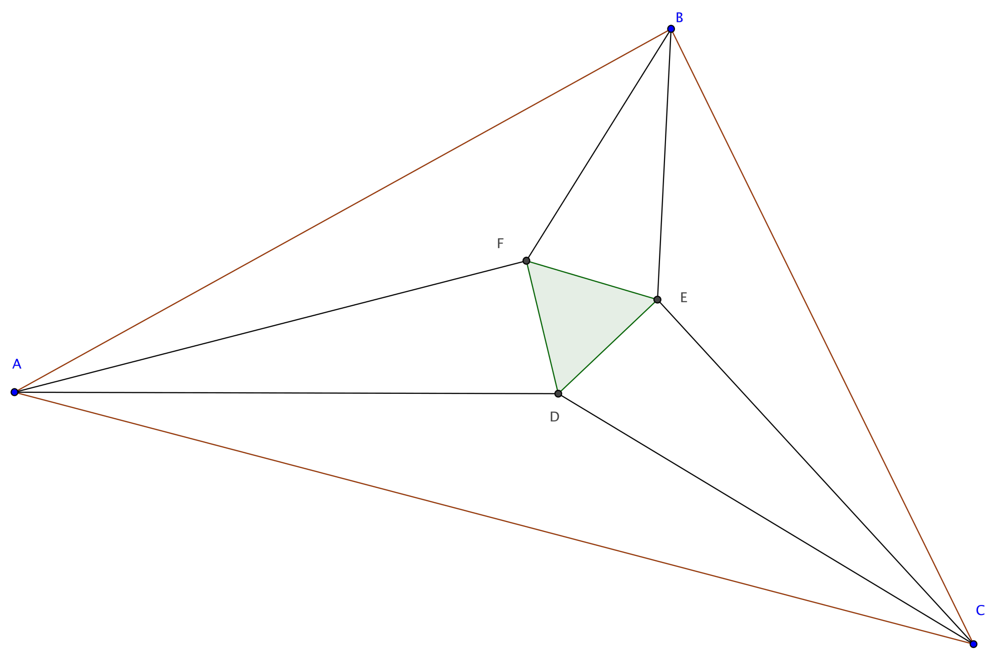
Transactions
in
Euclidean Geometry



Issue # 5

A QUADRILATERAL WITH PARALLEL SEGMENTS IS A RECTANGLE

JESSICA TESKE

Communicated by Emily Herbst

In order to prove Theorem 3.5, we will need Theorem E.

Theorem E. The interior angles of a parallelogram taken together make four right angles.

Theorem 3.5. Let ABCD be a quadrilateral such that angles ABC and ADC are right angles. If segments AB and CD are parallel, then ABCD is a rectangle.

Proof. Let ABCD be a quadrilateral. Within ABCD let angles ABC and ADC be right angles and let AB be parallel to CD. Assume that AB is congruent to CD. By Euclid Propostion I.33, since AB and CD are congruent and parallel the straight lines joining them at the extremities which are in the same directions respectively are also congruent and parallel themselves. Meaning AD and BC are also conguent and parallel.Then, by the definition of a parallelogram, this quadrilateral is now a parallelogram. By Euclid Postulate 1, let line segment BD be joined. By Euclid Proposition I.8 triangles BAD and BCD are congruent because BD is congruent to itself, AB is congruent to CD, and AD is congurent to BC. This makes angles BAD and BCD congruent. Since angles ABC and ADC are right angles, angles BAD and BCD also have to be right angles since they have to equal two right angles while also being congruent by Theorem E. Thus, by the definition of a rectangle, ABCD is a rectangle.

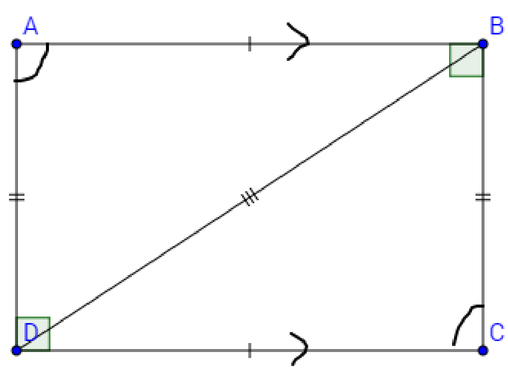


FIGURE 1. Quadrilateral ABCD

Above I stated that by Euclid Proposition I.33 since AD and BC join parallel and congruent segments AB and CD at extremities in the same directions respectively they are parallel and congruent themselves. What is meant by this is that AD and BC must join AB and CD at points A, B, C, and D, not at another point on the line segments. Also, same directions respectively means that AD and BC need to be either straight up and down, or side to side so that they are parallel. This means that AD and BC cannot vertices on opposite sides. Vertices A and D have to be on the same side of BC. Similarly vertices B and C have to be on the same side of AD. I have shown this below to make it more clear.

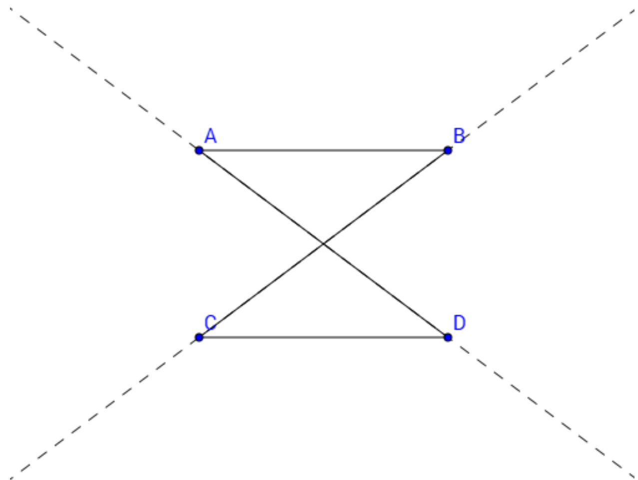


FIGURE 2. AD and CB cross, so they are not in the same directions and are not parallel.

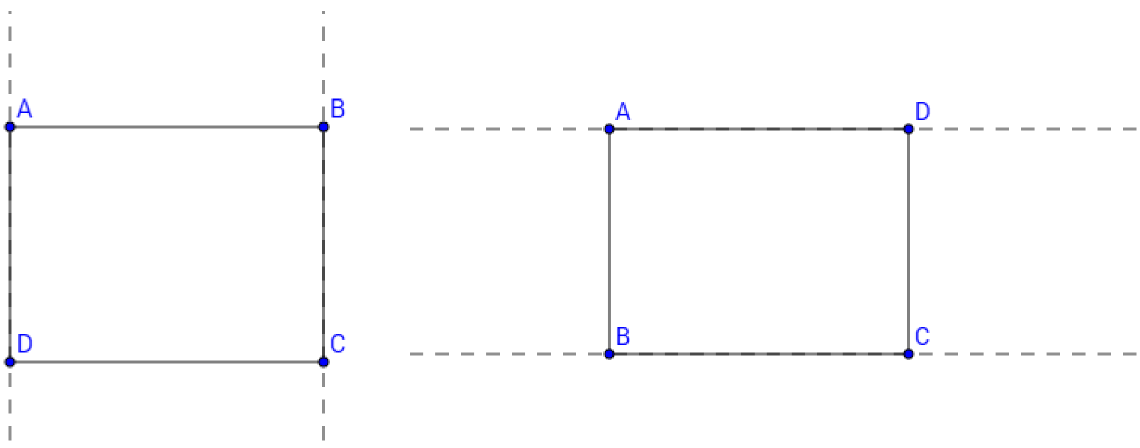


FIGURE 3. AD and CB do not cross and as you can see they are in the same directions and they are parallel.

□

ARE ALL TRIANGLES ISOSCELES?

KATY GOODMUNDSON

Communicated by Kaylee Benson

Figure out what went wrong with Professor Ball's Argument.

Theorem 4.1. There exists a triangle which is not isosceles.

Proof. Let ABC be a triangle. Let M be the midpoint of segment BC. From Euclid I.11, draw a perpendicular line from M outside the figure. From Euclid I.9, draw an angle bisector at angle A. Let the perpendicular line from M meet the angle bisector of A at the point X.

Suppose X falls outside the triangle. Draw CX and BX. CX is congruent to BX because M is the midpoint of BC. Extend AB and AC. Draw the shortest distance from X to AB and AC. Label these points B2 and C2. XB2 is congruent to XC2 because X lies on the angle bisector, therefore, they are equal distance from the point X. By Hypotenuse-Leg-Theorem, triangle XBB2 is congruent to triangle XCC2. By corresponding parts of congruent triangles, BB2 will be congruent to CC2. Similarly, by Hypotenuse-Leg-Theorem, triangle AXB2 is congruent to triangle AXC2. By corresponding parts of congruent triangles, AB2 is congruent to AC2. Now subtracting off the extended sides of the triangle, this follows:

AB2-BB2 = AB2

AC2-CC2 = AC

From the proof above, it is concluded that AB2 is congruent to AC2 and BB2 is congruent to CC2. Thus, AB2 is congruent to AC. Hence, the segment AB is not congruent to AC, which proves that triangle ABC is not isoseles.

If the angle bisector line AX falls on the perpendicular line MX, then one can use the same proof to find that AB will be congruent to AC. Hence, the triangle will be isosceles. When AX and MX are not the same line and intersect at some point outside the figure, the triangle will not be isosceles.

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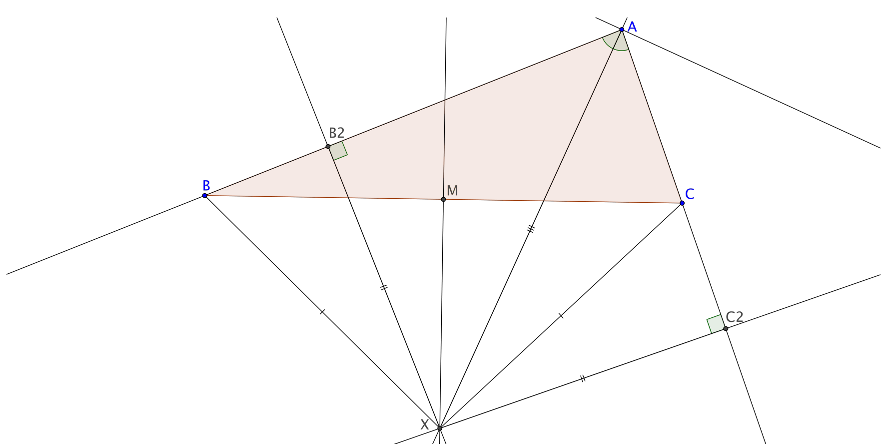

FIGURE 1. Triangle ABC: A non-isosceles triangle

Date: November 21, 2014.

CONSTRUCTING AN ANGLE THAT IS NOT RIGHT

EMILY HERBST

Communicated by Eric Scheidecker

Theorem G. Construct a point C not lying on a segment AB, so that angle ACB is not a right angle.

Proof. 1. Draw line GD. Choose a point E not on line GD.

2. Draw a circle with center E that intersects line GD at points F and B.

3. By Euclid Proposition I.12, from point E, draw a stright line perpendicular to line GD. Call the intersection of this line to line GD, point A.

4. Draw a circle with center A throught point B. Chose point H where line EA intersects this circle.

5. Extend line HA to be the diameter of the circle AB. Call point I the intersection of line HA and circle AB. Choose a point C on the circle AB, not on point F, such that C and B lie on the opposite sides of line HI.

6. Connect line AC and line CB.

Since line AC and line CB are both radii of the circle AB, then AC is congruent to AB. Since line AC is congruent to line AB, then triangle CAB is isosceles. By Euclid Proposition I.5, since triangle CAB is an isosceles triangle, angle ACB is congruent to angle ABC. By Euclid Proposition I.32, angle ACB, angle ABC, and angle CAB (the three interior angles of triangle CAB) sum to two right angles. Notice that point C and point B lie on opposite sides of line HAI. Since HAB is equal to one right angle and point C and point B are on the opposite sides of line HAI, then angle CAB is greater than HAB. Because angle HAB is equivalent to one right angle, this means CAB is greater than one right angle. If angle CAB is greater than one right angle, angle ACB, angle ABC, and angle CAB sum to two right angles, and angle ACB is congruent to angle ABC, then angle ACB and ABC are both less than one right angle. Thus angle ACB is not a right angle.

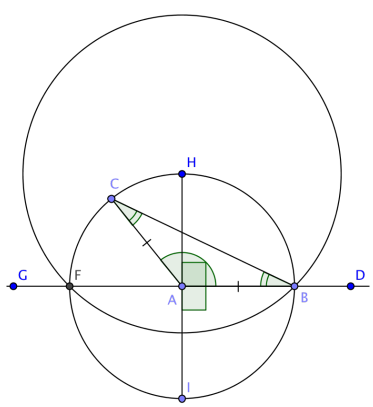

FIGURE 1

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EXTERIOR ANGLES OF A PENTAGON ADD UP TO FOUR RIGHT ANGLES

LOGAN BIEGHLER

Communicated by Katy Goodmundson

Theorem 5.2. The exterior angles of a pentagon, one choice made at each vertex, add up to four right angles.

Proof. Begin by constructing a pentagon ABCDE. Next, draw the line segments AC and AD. Since we have drawn the line segments of AC and AD, we now have created three triangles inside of the pentagon. The three triangles created are ABC, ACD, and ADE. By Euclid Proposition I.32, we know that each of the three triangles that were created is made up of two right angles. By that statement we know that there are a total of six right angles inside of the pentagon ABCDE. Since we know there are five sides to a pentagon, we want to find the total number of right angles that a pentagon is made of. Extend the sides of the pentagon AB, BC, CD, DE, and EA. By extending the sides of the pentagon, at each vertex we have created an exterior angle. By extending side of the pentagon,Eulcid Proposition I.13, we know that a straight line is equal to two right angles.After you extend AB, BC, CD, DE, and EA, now chose points G, H, I, J, and F on each extended line. In the figure you can see that each segment AG, BH, CI, DJ, and EF add up to two right angles. Since there are five sides to a pentagon then the total sum of right angles of a pentagon is ten right angles. Since we know that a pentagon is made up of ten right angles and the inside of the pentagon contains six right angles, we can say the exterior angles of a pentagon, one choice made at each vertex, do add up to four right angles. □

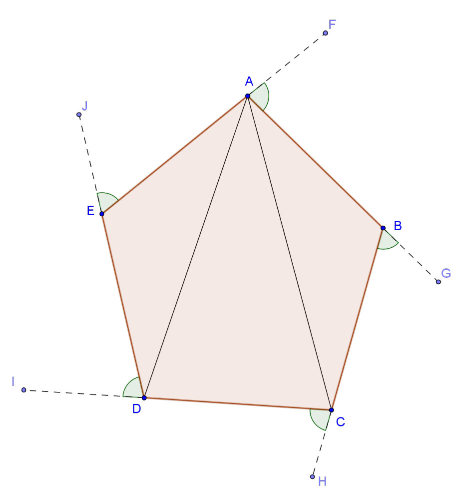

FIGURE 1. Pentagon ABCDE

Date: November 13, 2014.

IF ANGLE ACB IS A RIGHT ANGLE, C LIES ON THE CIRCLE WITH DIAMETER AB

JOHN FISHER

Communicated by Thomas Bieber

Theorem 7.5. If angle ACB is a right angle, then C lies on the circle with diameter AB.

Proof. Let ACB be a triangle, with angle ACB being right. Draw circle, X, with diameter AB. Let Y be the midpoint of AB. Extend a ray starting at Y through C. Let Z be the interesection of the ray from Y through C and circle X.

By 7.4 (Bieber, Wendt) angle AZB must be right, because Z is on the circle with diameter AB. Since we already have angle ACB being right, Z and C must be the same point. For if Z were a point other than C on the outside of triangle ACB (see Z2) along the ray from Y through C, the angle AZB would be less than right by Proposition 21. Similarly (also by Proposition 21), if Z were inside triangle ACB (see Z3) along the ray from Y through C, angle AZB would be greater than one right angle. Since we have AZB being right and Z cannot possibly be in any other location without breaking this conclusion, Z must equal C. Thus if we have angle ACB being right, then C lies on the circle with diameter AB.

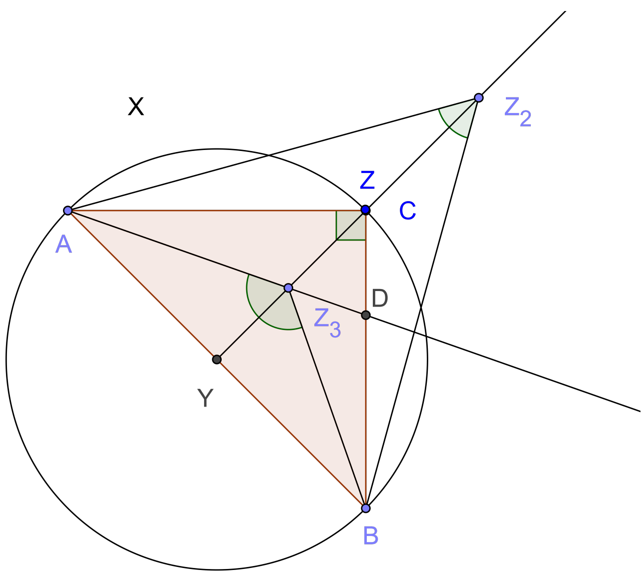

Figure 1. Notice circle with AB, and angle ACB is right. If Z is any point other than C, then by Proposition 21 angle AZB would be greater than or less than right.

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THE CONGRUENCY OF TRIANGLES WITHIN A TRIANGLE

ASHLEY STUFFELBEAM

Communicated by Diann Herington

Theorem 8.1. Let ABC be a triangle, with rays r and s the angle bisectors at A and B, respectively. Suppose that r and s meet at the point I which lies inside the triangle. Draw lines l and m through I that are perpendicular to AC and BC respectively. If l meets AC at point X and m meets BC at Y, then triangle IXC is congruent to triangle IYC.

Proof. To begin, construct a segment from point I that is perpendicular to segment AB using Euclid Proposition I.12, and call the intersection of this segment with AB point Z. Ray r bisects angle BAC, therefore angle ZAI is congruent to angle XAI. We can also conclude that angles AZI and AXI are congruent because both are right angles, and segment AI is congruent to itself. Therefore, triangle AZI is congruent to triangle AXI by Euclid Proposition I.26.

Similarly, ray s bisects angle ABC, therefore angle ZBI is congruent to angle YBI. We also know that angles BZI and BYI are congruent because both are right angles, and segment BI is congruent to itself. Therefore, triangle BZI is congruent to triangle BYI by Euclid Proposition I.26.

Since triangles BZI and BYI are congruent, segments ZI and YI are congruent by corresponding parts of congruent triangles. Similarly, since triangles AZI and AXI are congruent, segments ZI and XI are congruent. Therefore, segments ZI, YI, and XI are congruent.

Construct segment IC. As previously stated, segments YI and XI are congruent. Segment IC is congruent to itself, and angles YIC and XIC are congruent because both are right angles. Therefore, triangle YIC is congruent to triangle XIC by Mr. Scheidecker's Theorem 7.2.

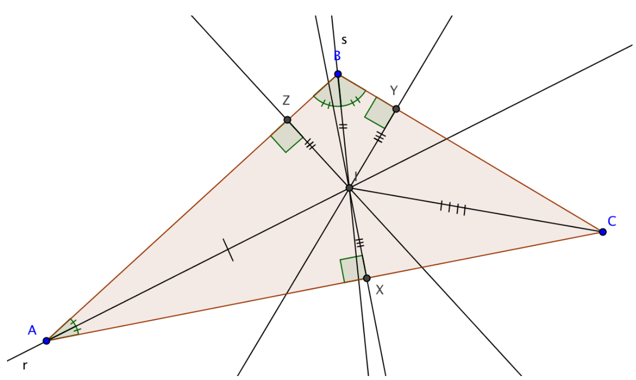

FIGURE 1. Triangle ABC

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THEOREM 9.2: CIRCLES

EMILY BACHMEIER

Communicated by Diann Herington

Theorem 9.2. Let Γ and Ω be two circles with centers G and O, respectively. Suppose that these circles meet at two points A and B. If GAO is a right angle, then GBO is a right angle.

Proof. Given two circles with centers G and O with intersection points A and B:

(1) Construct line segments GA, GB, OA, and OB from the centers of the circles to the two points of intersection.
(2) Construct line segment GO to connect the centers of the circles.

By Euclid I.1, line segments OA and OB are congruent because they are radii of the circle. Similarly, line segments GA and GB are also congruent. The constructed line GO is congruent to itself. Therefore, by Euclid I.8, triangle GAO is congruent to triangle GBO. Then, by Congruent Parts of Congruent Triangles, we can say that since angle GAO is right, then angle GBO must also be right.

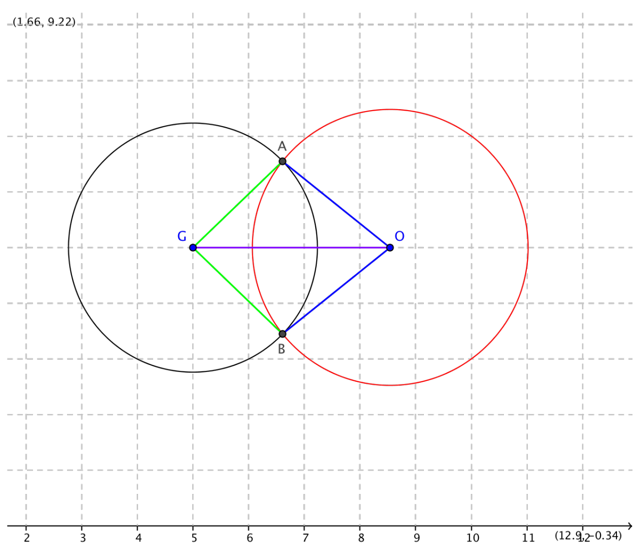

FIGURE 1. The construction for Theorem 9.2

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