A Quadrilateral with a set of Opposite Angles that are both Right Angles and a set of Opposite Sides that are Parallel is a Rectangle.

Emily Jacobs

April 29, 2015

Theorem 3.5. Let ABCD be a quadrilateral such that angles ABC and ADC are right angles. If segments AB and CD are parallel, then ABCD is a rectangle.

Proof. Let angles ABC and ADC are right angles and line segment AB be parallel to line segment CD. If we split the quadrilateral into triangle DAC and triangle BCA, then that angle DAC and angle DCA add up to a right angle due to Euclid I.32.

Similarly, angle BAC and angle BCA have to add up to a right angle by Euclid I.28, we know that alternate interior angles are congruent. Therefore, angle DCA is congruent to angle BAC and angle DAC is congruent to angle BCA. Since angle DCA is congruent to angle BAC and and we know that angle DAC and angle DCA add to a right angle, then angle DAC and angle BAC also add to a right angle.

Similarly, angle DCA and angle BCA also add to a right angle. Therefore, we know that angle DAB and angle DCB are both right angles. Since angle DAB, angle DCB, angle ABD, and angle ADC are all right angles then we know that this quadrilateral is a rectangle by the definition of a rectangle.

Refereed by: Connor Schulte

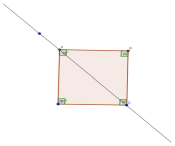

Figure 1: Quadrilateral ABCD

□