

Constructing a Circumcircle

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April 29, 2015

Challenge 12.2. Construct a circle circumscribed about a given triangle ABC.

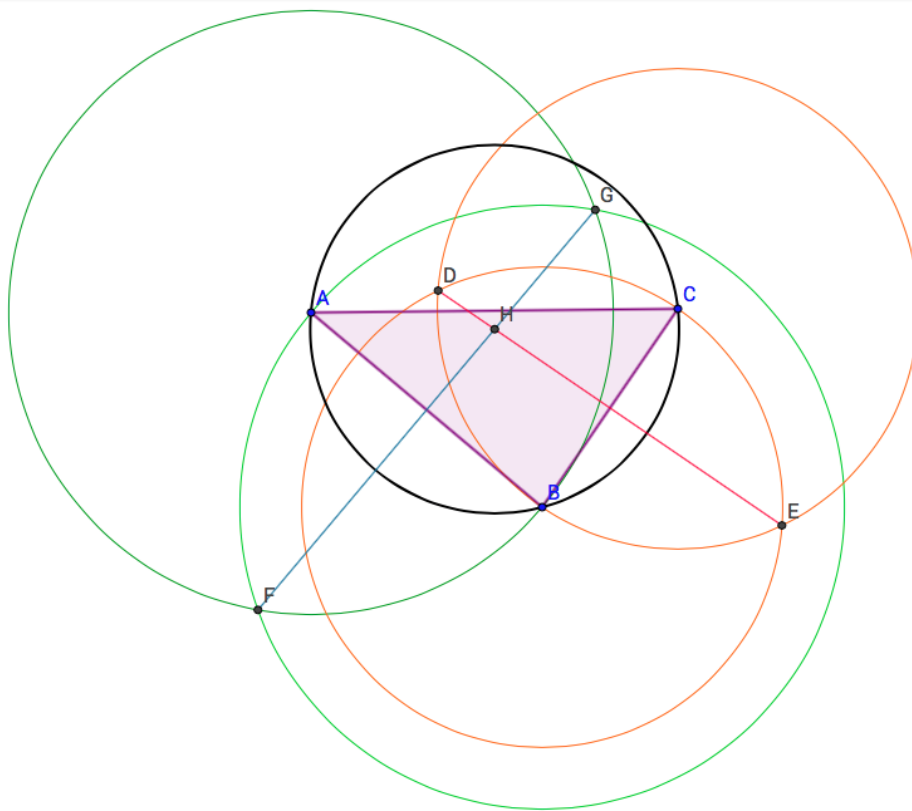


Figure 1: Construction of a circumcircle

1. Draw a circle around point B through point C.
2. Draw a circle around point C through point B. Label the points of intersection D and E.
3. Draw line segment DE. This is the perpendicular bisector of BC.
4. Draw a circle B through point A.

5. Draw a circle A through point B. Label the points of intersection F and G.
6. Draw line segment FG. This is the perpendicular bisector of AB. Label the point of intersection of segment DE and segment FG point H.
7. Draw a circle around H through point B.

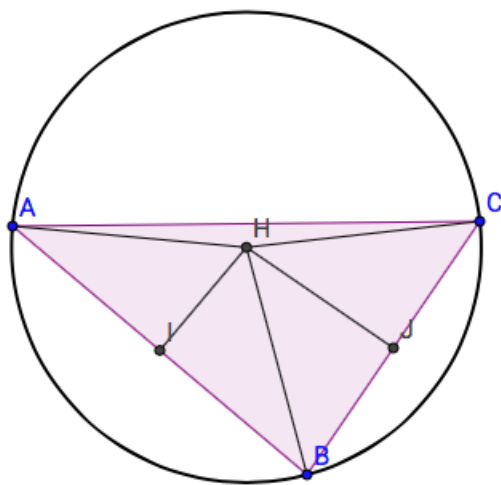


Figure 2: A circumscribed triangle

Proof. Since point B was used to construct the circle, we must show points A and C also lie on the circle.

Triangles BJH and CJH share side HJ. Since HJ is the perpendicular bisector of BC, JB and CJ are congruent, and angles HJB and HJC are right angles. Then by Side-angle-side, triangles BJH and CJH are congruent. Thus, HC is congruent to HB, and is a radius of the circle around H through B.

Similarly, triangles BIH and AIH share side HI. Since HI is the perpendicular bisector of AB, AI and BI are congruent, and angles AIH and BIH are right angles. Then by Side-angle-side, triangles BIH and AIH are congruent. Thus, HA is congruent to HB, and is a radius of the circle around H through B.

Since all three vertices of the triangle lie on circle H through B, the circle is the circumcircle of ABC. \square

Refereed by Toby Maggert.