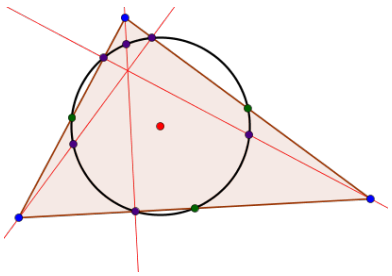
*Euclidean Geometry:*
*An Introduction to Mathematical Work*

*Math 3600*

*Spring 2015*



## *Circles*

We have learned quite a bit about basic polygonal shapes, especially triangles, and various species of quadrilaterals. Now we turn our attention to circles. This is the subject of Book III in Euclid's *Elements*. We already have one beautiful theorem about circles, that of Thales, but we'd like to have more.

Read the *Elements* Book III Propositions 1-34. For the following propositions you should work in the axiomatic style of Euclid using I.1-34, III.1-34 and any previously proved results.

Pay special attention to III.16, III.18, III.20, III.21, III.31 and III.32.

**9.1 Conjecture.** Let $AB$ and $AC$ be two tangent lines from a point $A$ outside a circle. Then $AB$ is congruent to $AC$.

**Definition.** We say that two circles *meet at right angles* if the radii of the two circles to a point of intersection make a right angle.

**9.2 Conjecture.** Let $\Gamma$ and $\Omega$ be two circles with centers $G$ and $O$, respectively. Suppose that these circles meet at two points $A$ and $B$. If $GAO$ is a right angle, then $GBO$ is a right angle.

**Definition.** A quadrilateral $ABCD$ is said to be a *cyclic quadrilateral* if there is a circle $\Gamma$ such that the four vertices $A, B, C$ and $D$ lie on $\Gamma$.

**9.3 Conjecture.** A rectangle is always a cyclic quadrilateral.

**9.4 Conjecture** (Cyclic Quadrilateral Theorem)**.** Let $A, B, C$ and $D$ be four points. The quadrilateral $ABCD$ is cyclic if and only if angle $DAC$ is congruent to $DBC$.

**9.5 Conjecture.** Let two circles be tangent at a point $A$. If two lines are drawn through $A$ meeting one circle at further points $B$ and $C$ and meeting the other circle at points $D$ and $E$, then $BC$ is parallel to $DE$.