# Parallel Lines and Tangent Circles

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**Theorem 9.5.** Let two circles be tangent to a point $A$. If two lines are drawn through $A$ meeting one circle at further points $B$ and $C$ and meeting the other circle at points $D$ and $E$, then $BC$ is parallel to $DE$.

*Proof.* Let two circles be tangent at point $A$. Let two lines be drawn through point $A$ such that they meet on one circle at points $B$ and $C$, and meet the other circle through points $D$ and $E$. Let line $MN$ be tangent to both circles through $A$ by construction of Euclid III.17.

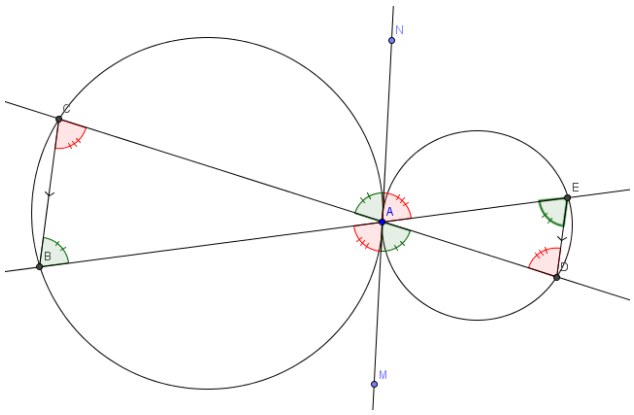


Figure 1: Two tangent circles with tangent line $MN$ through $A$, and lines $BE$ and $CD$ drawn through $A$

By Euclid III.32, angles $CAN$ and $CBA$ are congruent. By Euclid I.15 (vertical angles), angles $CAN$ and $DAM$ are congruent. By Euclid III.32, angles $DAM$ and $DEA$ are congruent. Therefore, angles $CBA$ and $DEA$ are congruent.

Since angles $CBA$ and $DEA$ are congruent, by Euclid I.29 (opposite interior angles congruent), $BC$ and $DE$ are parallel.

□