

*Euclidean Geometry:  
An Introduction to Mathematical Work*

*Math 3600*

*Spring 2015*

*More Content*

The following are some challenging tasks having to do with the theory of equal content. The first two are important to us. The rest are opportunities to show off.

**14.1 Problem.** Given two rectangles,  $P$  and  $R$ , construct a square  $S$  so that, when taken together the content of  $R$  and  $S$  is equal to the content of  $P$ .

**14.2 Problem.** Use the theory of content to give a new proof of the midline theorem, Theorem 3.6.

**14.3 Challenge.** Given a line  $\ell$  and given two points  $A$  and  $B$  not lying on  $\ell$ , construct a circle passing through  $A$  and  $B$  and tangent to  $\ell$ .

**14.4 Challenge.** Given two lines  $\ell$  and  $m$  and a point  $P$  not on either line, construct a circle through  $P$  and tangent to both  $\ell$  and  $m$ .

**14.5 Conjecture.** Let  $ABC$  be a triangle,  $\overleftrightarrow{DE}$  a line parallel to the base  $BC$ , and  $F$  the midpoint of segment  $DE$ , where  $D$  lies on ray  $AB$  and  $E$  lies on ray  $AC$ . Let  $AF$  meet  $BC$  at  $G$ . Then  $G$  is the midpoint of  $BC$ .

**14.6 Conjecture.** Let  $\Gamma$  be a circle with center  $O$ . Let  $A$  be a point outside the circle, and  $AB$  and  $AC$  tangents to  $\Gamma$  from  $A$ , with  $B, C$  lying on  $\Gamma$ . Let  $BC$  meet  $OA$  at  $D$ . Then the rectangle on  $OA$  and  $OD$  has equal content with the square on  $OB$ .

**14.7 Conjecture.** Let  $ABC$  be a right triangle with right angle at  $A$ . Let  $AD$  be the altitude from  $A$  to side  $BC$ . The square on side  $AD$  has equal content with the rectangle on  $BD$  and  $DC$ .

**14.8 Conjecture.** Let  $ABC$  be a triangle and  $D$  a point on side  $BC$ . Let  $E$  be the midpoint of  $BC$  and draw the parallel to  $AD$  through  $E$ . Let this new line meet the union of  $AB$  and  $AC$  at a point  $F$ . Then the segment  $DF$  cuts the triangle into two polygons of equal content.

