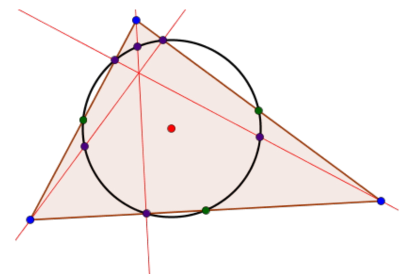
Euclidean Geometry: An Introduction to Mathematical Work

Math 3600

Fall 2016



More Triangle Centers

Now it is time to play. We have studied lots of interesting topics, and we can use our understanding to prove beautiful theorems.

17.1 Conjecture. Let ABC be a triangle with D the midpoint of AB and E the midpoint of AC. Then BC is twice DE.

Definition. Let ABC be a triangle, and let D be the midpoint of side BC. The segment AD is called the *median* of ABC at A.

17.2 Conjecture. Suppose that m and ℓ are two medians of a triangle ABC. The point where m and ℓ intersect lies on each median $2/3$ of the way from the vertex to the opposite side.

17.3 Conjecture. The medians of a triangle are concurrent.

Definition. The point just found is called the *centroid* of the triangle.

A many hundreds of notions of what could be the "center" of a triangle have been investigated. A detailed list is compiled at the web page `http://faculty.evansville.edu/ck6/tcenters/`.

Definition. Let ABC be a triangle. A line from a vertex which is perpendicular to the opposite side is called an *altitude*.

Definition. Let ABC be a triangle. The triangle formed by joining the midpoints of the sides of ABC by segments is called the *medial triangle* of ABC.

17.4 Conjecture. Let ABC be a triangle with DEF its medial triangle. An altitude of DEF is a perpendicular bisector of one of the sides of ABC.

17.5 Conjecture. The three altitudes of a triangle are concurrent.

Definition. The point of concurrence of the altitudes of a triangle is called the *orthocenter* of the triangle. The traditional notation is to label this point H.

17.6 Problem. Let ABC be a triangle with circumcenter O, centroid G and orthocenter H. Show that O, H and G are collinear, and GH is twice OG.

Definition. The line found in the last problem is called the *Euler line* of triangle ABC.