Rhombus

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Theorem 1.2. The diagonals of a rhombus must cross.

Proof. Let $ABCD$ be a rhombus; therefore, it has four congruent sides. Draw diagonal BD. Then, two triangles are formed, triangle ABD and triangle BCD. These two triangles share a side BD. By Proposition I.5, triangles ABD and BCD are isosceles triangles since they have the same base angles; which tells us that angles are congruent. Thus, these two triangles are congruent. By Ms. Van Nevele's Theorem H, we know that there exists the point which is the midpoint of the foot. Now, we draw an altitude from vertice A down to the side BD, which is also perpendicular to the side BD. We name that point E. Then, we get two right triangles, ABE and ADE. Angle E is a right angle in both triangles. By Proposition I.4, these two triangles are symmetric which means that they are mirror images of each other. Similarly, we draw an altitude from vertice C to the side BD. We call that point F. By Theorem H, E and F form right angles. Then, E and F are the same point because they represent the median of side BD. By Proposition I.14, A, E, and C are collinear points. Then, BD intersects AC at right angles. Thus, diagonals of a rhombus meet.

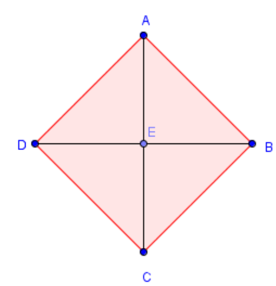

Figure 1: Rhombus with altitude E

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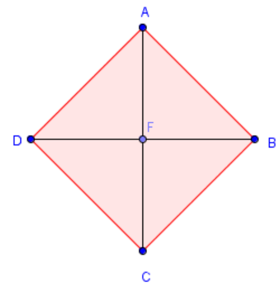

Figure 2: Rhombus with altitude F

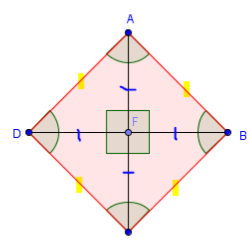

Figure 3: Diagonals of a rhombus meet