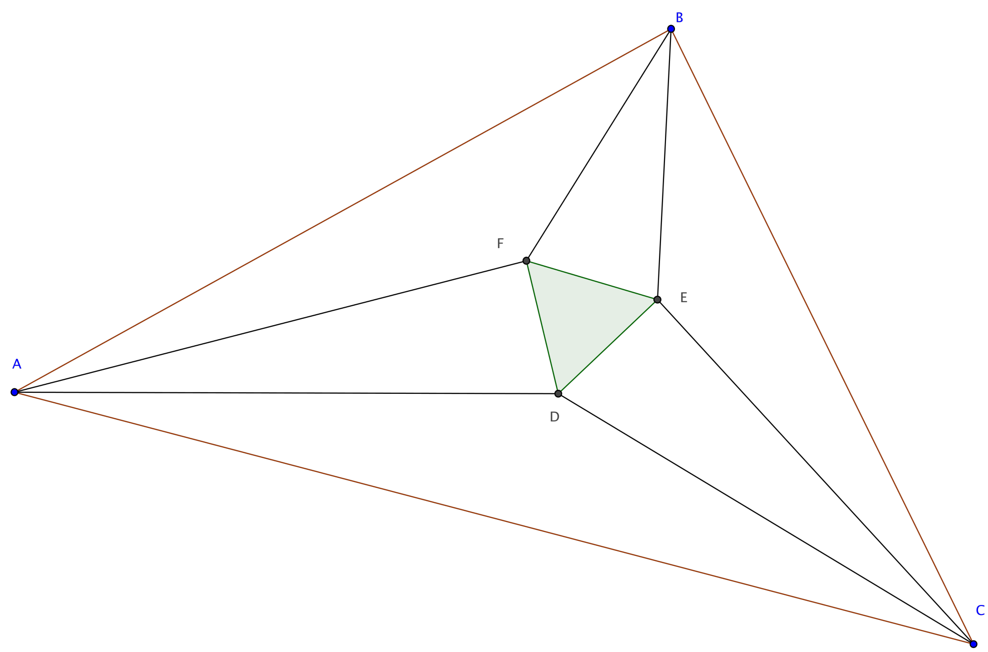
Transactions in **Euclidean Geometry**



Issue # 2

Proving Collinearity of Points

Eric Scheidecker

September 16, 2014

Theorem D. Let $\overline{AC}$ be a line, and X a point lying on that line. If there are points B and D such that angles $\angle BXA$ and $\angle DXA$ are right angles, then the points B, D, and X are collinear.

Proof. Let $\overline{AC}$, $\overline{BX}$, and $\overline{DX}$ be line segments. Let X be a point on $\overline{AC}$. Suppose $\angle BXA$ and $\angle DXC$ are right angles.

Since $\angle BXA$ and $\angle DXC$ are right angles and $\overline{BX}$ and $\overline{DX}$ share a point, by proposition 14, $\overline{BX}$ and $\overline{DX}$ will be in a straight line with one another. Thus B, X, and D are collinear. $\square$

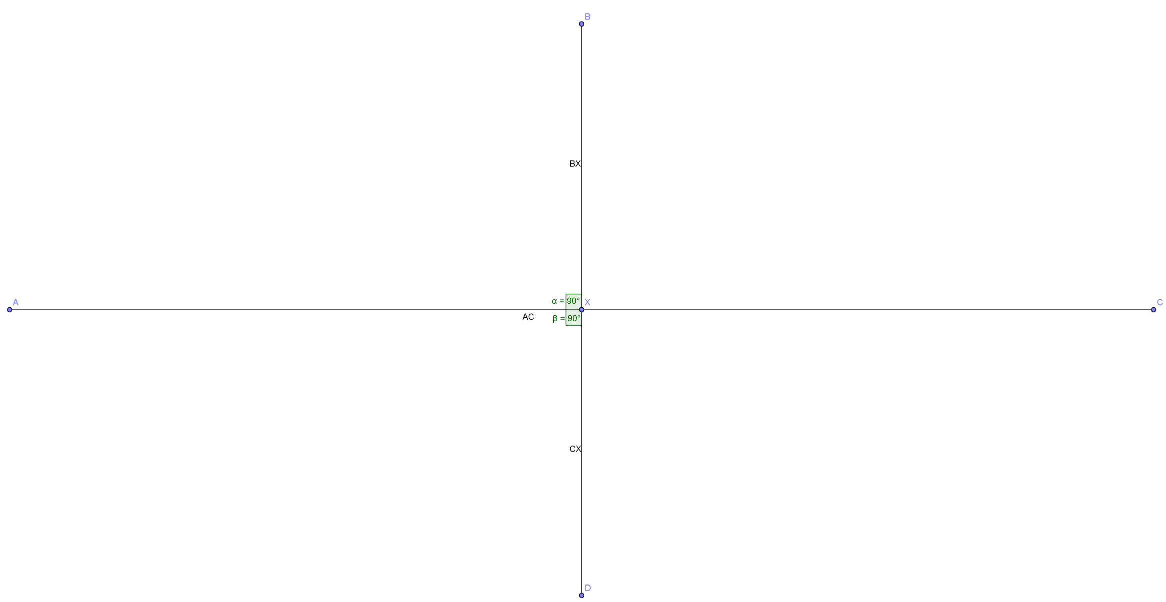


Figure 1: $\overline{BX}$ and $\overline{DX}$ meeting $\overline{AC}$ at right angles.

Two Lines Meeting at Right Angles to a Common Line at the Same Point are Themselves a Straight Line

Joshua Hawkins

September 19, 2014

Theorem D. Let lines BX and DX meet line AC at right angles at point x. Then points D, X and B are collinear.

Proof. Let lines BX and DX meet line AC at right angles at point x. Then either points D, X, and B are collinear or not collinear. Let points D, X, and B be not colliear. Next extend line BX through point X to Y. Then points B, X, and Y are collinear by definition 4. By Proposition 13, since line BY is cut by line AC, then angle BXC and BXD are congruent to two right angles. Next, Angle BXC is a right angle, so angle CXY is also a right angle by common notice 3. Also, angle CXD is a right angle and is comprised of angles DXY and CXY. Since angle CXY is a part of angle DXC, then angle DXC is greater than angle CXY by common notice 5. But, DXC and CXY are both right angles. So, a right angle is greater than a right angle. This is a contradiction Therefore D, X, and B must be colliear.

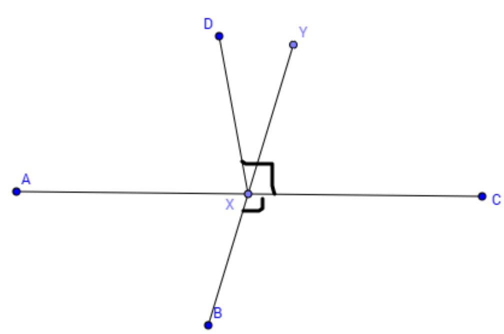


Figure 1: Two lines at right angles to the same line meeting at the same point.

□

Conjecture E: Interior Angles of a Rhombus

Diann Herington

October 6, 2014

Theorem . Let $ABCD$ be a rhombus. Then angles ABC, BCD, CDA, DAC, when taken together, make four right angles.

Proof. Segments AB and DC can be extended by Postulate 2. By Proposition 13, segment AD set up on line AB will either make two right angles, or angles equal to two right angles. So, angle r and angle s taken together must equal two right angles. Similarly, angle t and angle u taken together must equal two right angles, angle v and angle w taken together must equal two right angles, and angle z and angle q must equal two right angles when taken together.
By Theorem 1.6 in the first issue of Transactions in Euclidean Geometry, since $ABCD$ is a rhombus, $ABCD$ is also a parallelogram. Therefore, segment AB is parallel to segment DC, and segment AD is parallel to segment BC.
By Proposition 29, angle r is congruent to angle z and angle s is congruent to angle q. Similarly, angle t is congruent to angle v and angle u is congruent to angle w.
Since angle r and angle s taken together must equal two right angles, and angle r is congruent to angle z, then angle s and angle z taken together must equal two right angles. Similary, angle t and angle w must equal two right angles when taken together.
Therefore, angle s, angle z, angle t and angle w, when taken together, must equal four right angles. $\square$

Corollary . The interior angles of a parallelogram taken together make four right angles.

Proof. By Theorem 1.6 in the first issue of Transactions in Euclidean Geometry, if $ABCD$ is a rhombus, then $ABCD$ is a parallelogram. If the interior angles of a rhombus must make four right angles by the above theorem, and since a rhombus is a parallelogram, the interior angles of a parallelogram when taken together must also make four right angles. $\square$

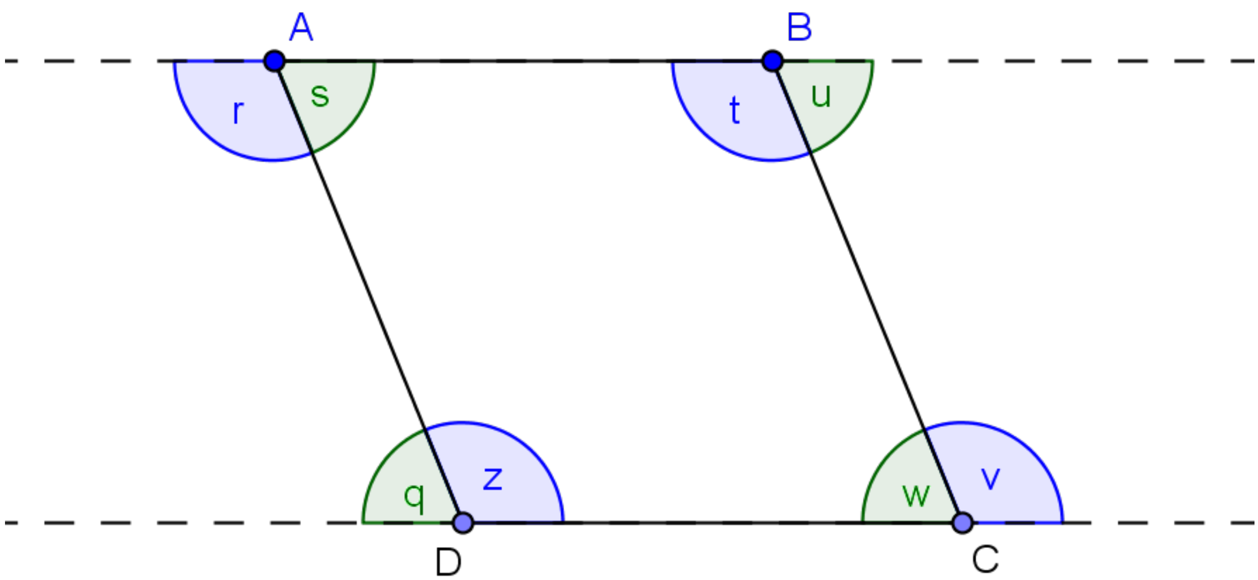


Figure 1: Rhombus ABCD

Opposite Angles of a Kite

Joellen Hatchett

October 6, 2014

Theorem 2.1. Let ABCD be a kite, with AB be congruent to AD and BC be congruent to DC. Then angle ABC is congruent to ADC.

Proof. By definition of a kite line segnement AB is congruent to line segment AD and line segment BC is congruent to DC. Line segment AC is congtuent to line segment AC, since it is the same line segment. By Euclid I.8 triangle ABC is congruent to triangle ADC, so angle ABC is congruent to angle ADC. □

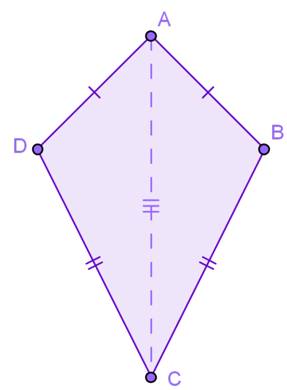


Figure 1: Kite ABDC with AB congruent to AD and BC congrent to DC.

Side Lengths and Their Corresponding Angle Measure in a Kite

John Fisher

September 13, 2014

Theorem 2.1b. Suppose that ABCD is a kite with AB congruent to AD and DC congruent to BC. If DC is greater than AD, then angle DAB is greater than angle DCB.

Proof. Let ABCD be a kite such that DC is greater than AD, and AC is congruent to itself. Then, by definition of a kite, BC must also be greater than AB.

By proposition 8 we know that triangles DCA and BCA are congruent. This means that angle DAC is congruent to angle BAC, and angle DCA is congruent to angle BCA. Since we know that DC is greater than AD, we know that the angle opposite DC must be greater than the angle oppostite AD by proposition 19. This means that angle DAC must be greater than DCA, and similarly angle BAC is greater than angle BCA. Since angle DAB is composed of angles DAC and BAC, and angle DCB is composed of angles DCA and BCA, angle DAB must be greater than DCB. Thus, whenever DC is greater than AD we have angle DAB is greater than angle DCB. □

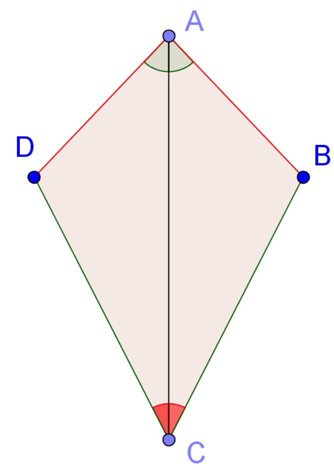


Figure 1: Kite ABCD. Notice that the color of the angles in question correspond with the color of the side in question.

Construction and Proof of a Kite Using a Compass and Straightedge

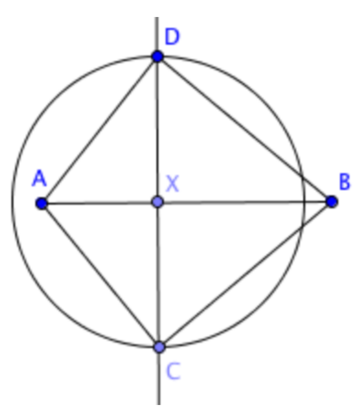
Ashley Stuffelbeam

October 6, 2014

Theorem 2.3. Given segment AB, a kite can be constructed using a compass and a straightedge.

Construction

1. Draw line segment AB.
   Select a point X on segment AB, not A or B.
2. Construct a perpendicular line to AB from X (Proposition 11).
   Select any point on the new line and call it C.
3. Continue line CX through X (Postulate 2).
4. Construct a circle with center X and through the point C.
   Make a point where circle XC and ray CX intersect, call it D.
5. Draw segment AD.
6. Draw segment DB.
7. Draw segement BC.
8. Draw segment CA.



Proof. Let AB be a line segment, and select a point X on that segment, but not A or B. Using Proposition 11, construct a perpedicular line to AB from point X. Select any point on this new line, not X, and call it C. Using Postulate 2, extend line CX through X. Then by creating a circle with center X through point C and naming the point of intersection between the circle and line CX D, we can say segment CX is congruent to segment DX because both are radii of the circle. Then we can say segment XB is congruent to itself, and finally by using Proposition 11 to construct a perpendicular line, we can say both angles DXB and CXB are right, and thus congruent. Therefore, triangle DXB is congruent to triangle CXB by Proposition 4. Since these triangles are congruent, we can conclude segment DB is congruent to segment CB.

A similar argument also using Proposition 4 holds to prove triangle DXA is congruent to triangle CXA, and thus segment DA is congruent to segment CA.

Since the figure ACBD has two pairs of adjacent sides that are congruent, the figure is a kite. □

Kite Construction

Joellen Hatchett, Megan Westervelt

October 6, 2014

Theorem 2.3. A kite is constructable with a standard straight edge and a compass.

Proof. Begin by drawing the line segment AB. Next, draw a circle centered at point B, with radius BA. Pick a point C on circle BA such that C is not collinear with line segment AB. Construct line segment BC and line segment AC. By Euclid Propostion I.10, define the point X to bisect line segment AC. Given line segment AC and the point B, construct the perpendicular line segment XB by using Euclid Proposition I.12. By Euclid Postulate 2, it is understood that line segment XB can be extended on either side. Let point D be defined as the intersection of line XB with circle BA. Finally draw line segements AD and CD to construct quadrilateral ABCD.

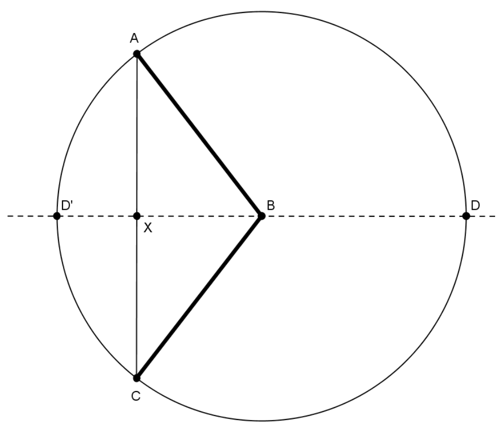
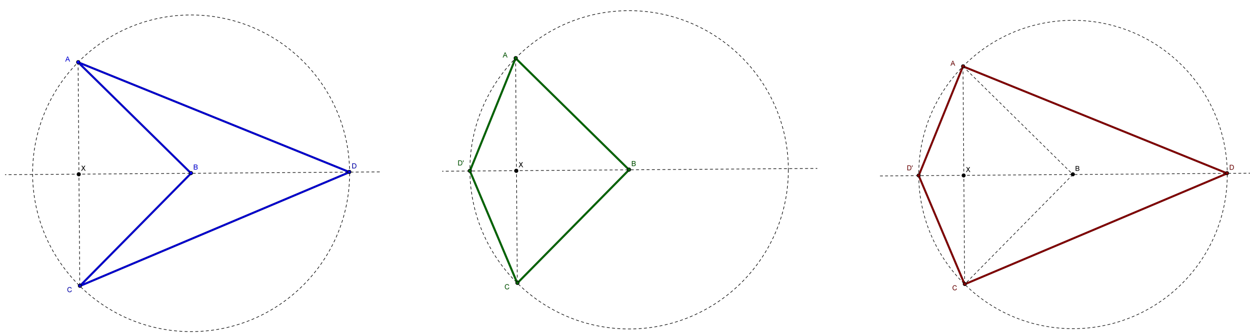


Figure 1: Kite Setup

Since line segment XB is perpendicular to line segment AC it is understood by Euclid Defintion 10 that angle AXB is a right angle. Similarly, angle CXB is also a right angle. Note, line segment AX is congruent to line segment CX because X bisects line segment AC. Line segment XD is congruent to itself. Using Euclid Proposition I.4, we can conclude that triangle AXD is congruent to triangle CXD. From this we can determine that corresponding parts AD and CD are also congruent. Also note that line segment AB is congruent to line segment BC by Euclid Definition 15. Therefore we know that quadrilateral ABCD is a kite because it has two pairs of adjacent and congruent sides.

□

It is important to note that this construction can create three different variations of a kite. There are two places where line XB intersects circle BA. Figure 1 shows the two locations where D could be located. These are marked as D and D'. When the point marked D is choosen, we are able to create the group of kites such as the one in Figure 2a. Similarly when point D' is choosen, we can create the group of kites such as the one in Figure 2b. If D' is allowed to replace D in the previous proof, it can be shown that quadrilateral ABCD' will also create a kite. The final case involves the construction of quadrilateral ADCD'. We can determine that line segment AD' is congruent to line segment CD' using triangles AXD' and CXD'. Similarly, line segment AD is congruent to line segment CD using triangles AXD and CXD. Therefore, we can also determine that quadrilateral ADCD' is a kite.



(a) Kite ABCD (b) Kite ABCD' (c) Kite ADCD'

Figure 2: Various Kite Constructions

Diagonals That Meet in a Kite are Right

Nathan Opheim

September 29, 2014

Theorem 2.5. If the diagonals of a kite meet, then they meet at a right angle.

Proof. Let ABCD be a kite with AB congruent to AD and BC congruent to DC.

Assume Conjecture D (Herbst) *[Let AC be a segment and X a point on this segment. Suppose that segment BX meets AC at right angles, and segment DX meets AC at right angles. Then points B,X, and D are collinear.]* is true.

Assume X is the midpoint of BD. Then by Proposition 15, angle DXA is congruent to angle BXC and angle DXC is congruent to angle BXA. Then triangle DXA is congruent to triangle BXA

So angle AXD is congruent to AXB, which are both right by Conjecture D, as BX and AC meet at right angles.

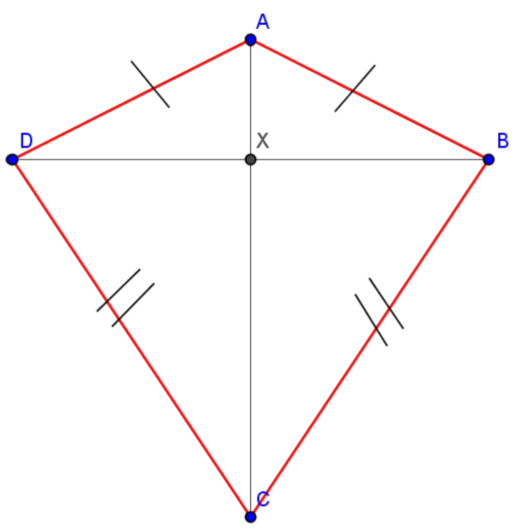


Figure 1: Kite ABCD

□

Proving Rectangles are Parallelograms

Emily Herbst

September 21, 2014

Theorem 3.1. Let R be a rectangle. Then R is a parallelogram.

Proof. Let R be a rectangle with points A, B, C, and D. Since R is a rectangle, angle DAB, angle ABC, angle BCD, and angle CDA are all right angles. Because the straight line AD lays on the straight lines AB and DC with the interior angles DAB and ADC equal to two right angles, then by Euclid Proposition I.28, line AB and line DC are parallel to one another. Because the straight line AB lays on the straight lines BC and AD with the interior angles DAB and ABC equaling two right angles, then by Euclid Proposition I.28, line BC and line AD are parallel to one another. Since line AB is parallel to line DC and line BC is parallel to line AD, then rectangle R is a parallelogram.

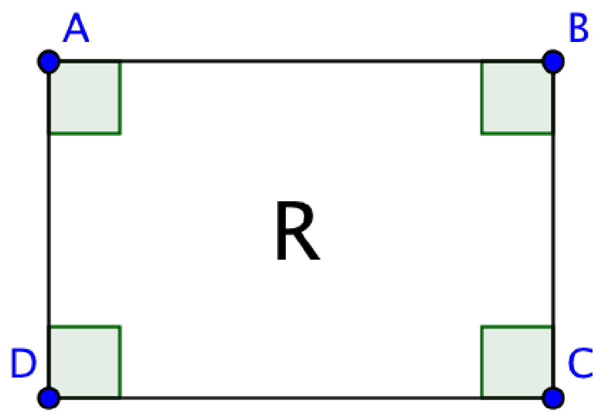


Figure 1: Rectangle R

□

Congruent Segments of a Rectangle

Kaylee Benson

September 24, 2014

Theorem 3.2. Let R be a rectangle. Then each pair of opposite sides of R is a pair of congruent segments.

Proof. Let SU be joined. By the definition of a rectangle angle STU is congruent to angle URS. By Ms. Herbst Theorem 3.1, rectangle R is a parallelogram. Then by proposition 29 angle RSU is congruent to angle TUS. SU is congruent to SU. By proposition 26, triangle TSU is congruent to triangle RUS. Thus by corresponding parts of a triangle, ST is congruent to UR and RS is congruent to TU. □

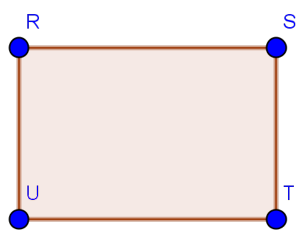


Figure 1: Rectangle R

Parallel segments and two right angles of a quadrilateral make ABCD a rectangle.

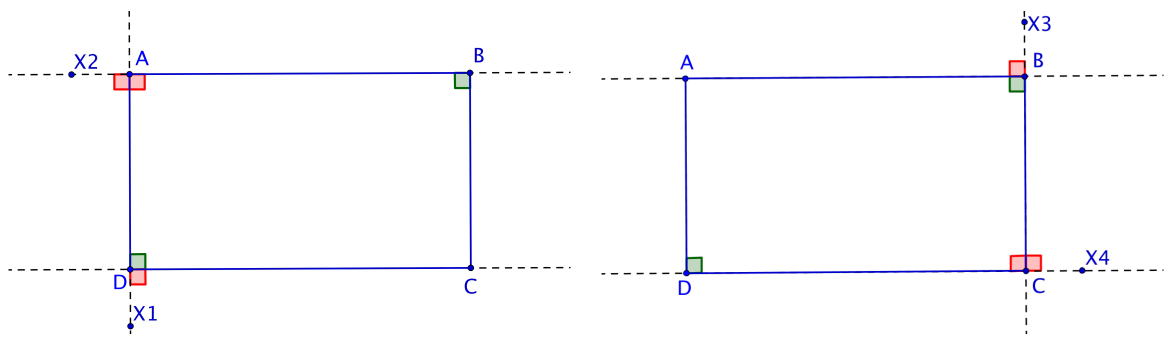
Katy Goodmundson

October 7, 2014

Theorem 3.5. Let ABCD be a quadrilateral such that angles ABC and ADC are right angles. If segments AB and CD are parallel then ABCD is a rectangle.

Proof. Let ABCD be a quadrilateral. Angles ABC and ADC are given as right angles. The segments AB and CD are parallel.

First, extend line AB and line AD. Choose point X1 on AD and point X2 on AB. By Definition 10, angle X1DC is a right angle. Since X1DC is a right angle, then by Proposition 29, angle DAX2 is a right angle. Thus, by Definition 10, angle DAB will be a right angle.



(a) Angle DAB is right (b) Angle BCD is right

Second, extend line BC and DC. Choose point X3 on BC and point X4 on DC. By definition 10, angle X3BA is a right angle. Since X3Ba is a right angle, then by Proposition 29, BCX4 is a right angle. Thus, by Definition 10, angle BCD is a right angle.

Hence, ABC, BCD, ADC, and DAB are all right angles. Therefore, by the definition of a rectangle, ABCD is a rectangle. □