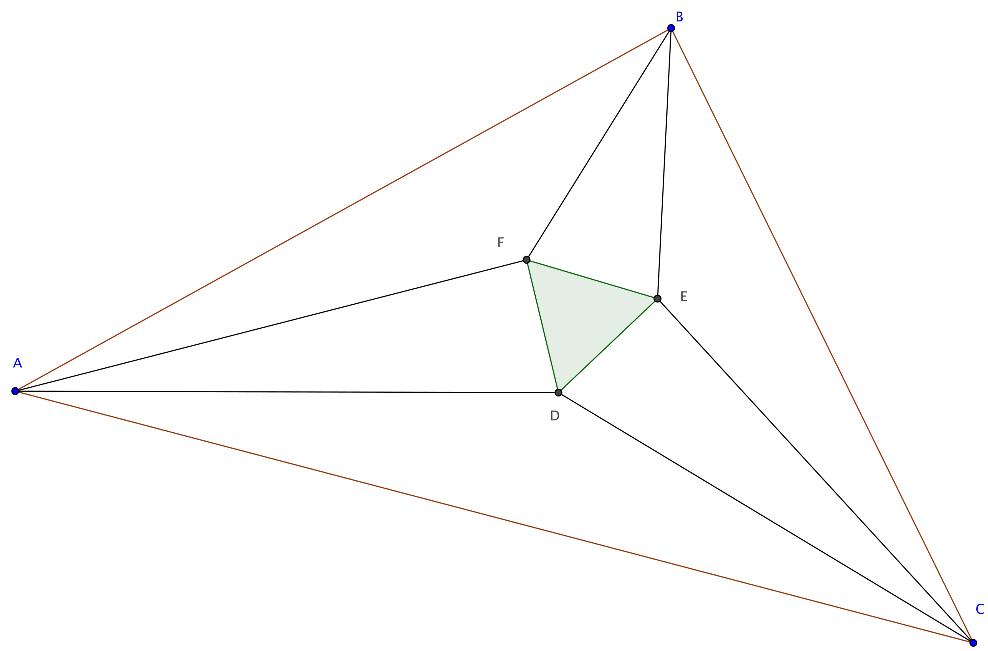
Transactions in **Euclidean Geometry**



Issue # 3

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Construction of a Rhombus

Maria Ahrens

October 7, 2016

The construction of a rhombus is as such:
1) Create a line segment AB
2) Create a circle AB and another circle BA
3) Place a point, C, anywhere along Circle BA
4) Create a circle BC
5) Place a point D on the intersection of Circle AB and Circle BC
6) Create line segments BC, CD, and DA to form a rhombus.

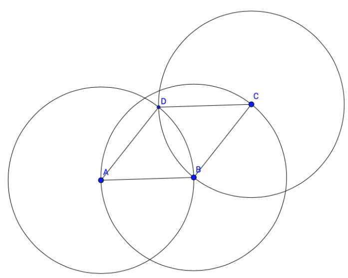

Figure 1: Rhombus ABCD

Theorem 4.1. The construction of the figure ABCD is a rhombus.

Proof. The goal of this proof is to show that all of the line segments are congruent to one another, thus, creating a rhombus. The recipe for our construction begins with a line segment AB, which becomes the radius of two distinct circles, Circle AB and Circle BA. By the Circle-Circle-Intersection Postulate, We can conclude that Circle AB and Circle BA intersect.

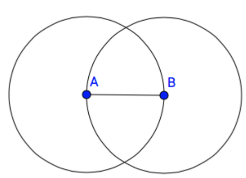

Figure 2: The intersection of Circles AB and BC by the Circle-Circle-Intersection Postulate.

The third step in the construction of a rhombus from the recipe above says that a point, C, can be placed anywhere on the circumference of Circle BA. By doing this, we know that when a line segment is drawn from Point B to Point C, BC will be congruent to line segment AB by the definition of a circle since both segments are radii of Circle BA.

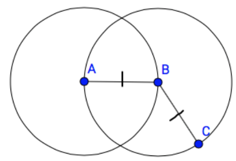

Figure 3: Line Segments AB and BC are both radii of Circle AB.

When Circle CB is constructed, it will intersect Circle AB by the Circle-Circle-Intersection Postulate. Point B should already be located on one of the intersections of these two circles, and Point D should be placed on the second intersection of Circle AB and Circle BC. Notice how Point D lies on the circumference of Circle CB, making it a radius of that circle. This is important since we also know that the line segment, BC, is a radius of the same circle. Thus, Line Segments BC and CD are congruent by the definition of a circle. And since BC is congruent to AB from what was proven previously, we can conclude that CD is congruent to both BC and AB by Euclid's Common Notion 1.

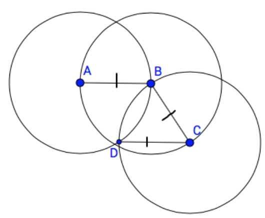

Figure 4: Line Segments AB, BC, and CD are congruent.

The last line segment, DA, still needs to be shown that it is congruent to the other three line segments. Note that when a line segment, DA, is created, it forms a radius of Circle AB. AD is then congruent to AB by the definition of a circle. Since AB is congruent to BC and CD, we can conclude that DA is congruent to AB, BC, and CD by Common Notion 1, resulting in all congruent line segments. Therefore, ABCD is a rhombus.

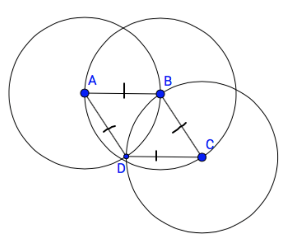

Figure 5: Line Segments AB, BC, CD, and DA are congruent forming Rhombus ABCD.

□

The Geometry of Kites

Samantha Ancona

October 3, 2016

Theorem 2.1. Let ABCD be a kite with AB congruent to AD and BC congruent to DC. Then angle ABC is congruent to angle ADC.

Proof. Let ABCD be a kite. By definition of a kite, AB is congruent to AD and BC is congruent to DC. With Postulate 1 let there be a straight line so that the line segment AC is formed. Suppose that the kite is split into two triangles ABC and ADC. We will show angle ABC is congruent to angle ADC. We have line segment AB congruent to AD, and BC congruent to DC by the definition of a kite. Also, AC is congruent to itself, see Figure 1. Therefore, we have two triangles that satisfy Euclid I.8 (SSS). Thus, angle ABC is congruent to angle ADC.

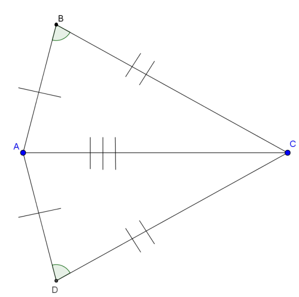

Figure 1: Angle ABC is Congruent to Angle ADC

Conjecture F. Let ABCD be a kite with AB congruent to AD and BC congruent to CD. Generally, angle DAB is not congruent to angle DCB.

Question G. With the same hypothesis, is it true that when angle DAB is congruent to angle DCB we must necessarily have that ABCD is a rhombus?

□

The Truth Behind Angles in a Kite

Danielle Maus

October 5, 2016

Ms. Ancona partially proved Theorem 2.1 where she proved only one pair of opposite angles in a kite are congruent. Conjecture 2.1 states that both pairs of opposite angles in a kite are congruent and I argue this conjecture to be false and here is why.

Theorem F. Let ABCD be a kite where side AB is congruent to side BC, side AD is congruent to side CD. Then angle ABC is not congruent to angle ADC. For specification purposes, this kite is not a rhombus therefore side AB is not congruent to side AD or side CD and side BC is not congruent to side AD or side CD.

Proof. Suppose ABCD is a kite where side AB is congruent to side BC and side AD is congruent to side CD. By Ms. Ancona's Theorem angle DAB is congruent to angle DCB.

I am going to prove this conjecture by two cases. The first case addresses when diagonal BD is not congruent to any sides of our kite and the second case addresses when diagonal BD is congruent to one side of our kite. I am not addressing the third case of when BD is congruent to two different sides of our kite (sides that are not already congruent to one another) because this case would create a rhombus which goes against what I am trying to prove. These cases are exhaustive because they cover all kite constructions and side congruencies in relation to diagonal BD.

Because this paper relies heavily on Euclid's I.18, I am going to include the proposition here, denoting it as Theorem 1, for the reader to refer back to.

Theorem 1 [Euclid I.18] In any triangle the greater side subtends the greater angle.

By Euclid's Postulate 1 we can create line segment BD

Case 1: Let diagonal BD not be congruent to any sides of kite ABCD. (see figure 1)

Since side AB is congruent to side BC, side AD is congruent to side CD, and BD is congruent to itself, by Euclid's I.8 (SSS) triangle BAD is congruent to triangle BCD. Since triangle BAD is congruent to triangle BCD, we can say angle ABD is congruent to angle CBD and angle ADB is congruent to angle CDB. Since BD is not congruent to any two sides of the triangles we can conclude the triangles are scalene. Therefore in triangle BAD and triangle BCD, by Euclid's I.18 (refer to Theorem 1), we can conclude that the angles in triangle BAD and triangle BCD are all distinct. Therefore, no two angles in the same triangle will be congruent to one another.

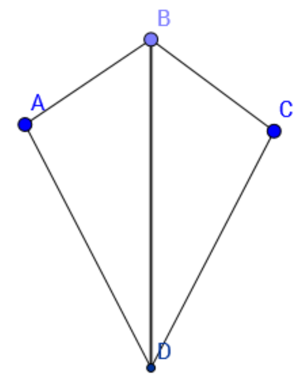

Figure 1: Kite ABCD

Since BD is an angle bisector of angle ABC and angle ADC we can say, two of angle ABD is not congruent to two of angle ADB. Since angle ABD is congruent to angle CBD and angle ADB is congruent to angle CDB, we can say angle ABD taken together with angle CBD is not congruent to angle ADB taken together with angle CDB. Therefore we can conclude angle ABC is not congruent to angle ADC.

Case 2: Let diagonal BD be congruent to one side of kite ABCD.(see figure 2)

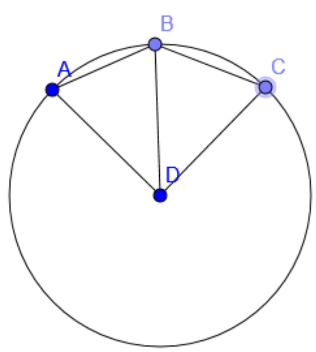

Figure 2: Kite ABCD with BD congruent to side AD and CD

Suppose BD is congruent to side AD and CD. Since BD is congruent to AD and CD, triangle ABD and triangle CBD are isosceles. By Euclid's I.5, angle DAB is congruent to angle DBA and angle DCB is congruent to angle DBC. Since base angles are congruent in either triangle and side AB is congruent to side CB, by Euclid's I.26 triangle ABD is congruent to triangle CBD. This makes angle ADB congruent to angle CDB. Therefore in triangle ABD and triangle CBD, by Euclid's I.18 (refer to Theorem 1), we can say that the base angles in triangle ABD and triangle CBD will be distinct from angle ADB and angle CDB.

Since BD is an angle bisector of angle ABC and angle ADC we can say two of angle ABD is not congruent to two of angle ADB. We then can say, angle ABD taken together with angle CBD is not congruent to angle ADB taken together with angle CDB. Therefore, angle ABC is not congruent to angle ADC.

This case 2 argument can be used to prove BD congruent to side AB and side CB as well. The result will be the same.

Since we concluded in Case 1 and Case 2 that angle ABC is not congruent to angle ADC, we can conclude in kite ABCD angle ABC is not congruent to angle ADC.

□

Congruency of Angles in a Kite

Danielle Maus

October 6, 2016

Ms. Mitchell's Theorem 1.1 states that opposite pairs of angles in a rhombus are congruent. The partial result for kites in Ms. Ancona's Theorem 2.1 states that only one pair of opposite angles are congruent. Conversely, we were able to conclude that if opposite pairs of angles in a kite are congruent, the kite is a rhombus.

Theorem G. Let ABCD be a kite where line segment AB is congruent to line segment BC and line segment AD is congruent to line segment CD. If angle ABC is congruent to angle ADC, then ABCD is a rhombus.

Proof. Let ABCD be a kite with the above hypothesis. Let angle ABC be congruent to angle ADC (see figure 1).

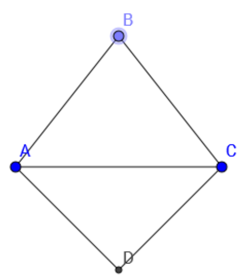

Figure 1: Kite ABCD

By Euclid postulate 1 we can create line segment AC. Now, the kite is split into two triangles, triangle ABC and triangle ADC. Since we assumed in our theorem that line segment AB is congruent to line segment BC and line segment AD is congruent to line segment DC, we can conclude that both triangle ABC and ADC are isosceles. Since both triangles ABC and ADC are isosceles, by Euclid I.5 we can conclude that the base angles in each triangle are congruent to one another. This means angle BAC is congruent to angle BCA and angle DAC is congruent to angle DCA.

Since we know angle ABC is congruent to angle ADC, and the base angles in triangle ABC and triangle ADC are congruent to one another, and by Euclid I.32 all three interior angles in a triangle need to add up to two right angles, we can conclude the following:

Angle ABC taken together with two of angle BAC is congruent to Angle ADC taken together with two of angle DAC. Since angle ABC is congruent to angle ADC, we know two of angle BAC is congruent to two of angle DAC. Since two of angle BAC is congruent to two of angle DAC, this means angle BAC is congruent to angle DAC.

By Euclid common notion 1 we can then conclude, since angle BAC is congruent to angle DAC, angle BAC is congruent to angle BCA, angle DAC is congruent to angle DCA, then angle BCA is congruent to angle DCA.

Finally, since triangle ABC and triangle ADC have two angles and one side congruent to each other respectively, by Euclid I.26 we can conclude that all sides of the kite are congruent to one another.

Since the definition of a rhombus is a quadrilateral with all sides congruent to one another, we can conclude that our kite ABCD is a rhombus. □

The Extended Diagonals of a Kite Cross

Abigail Goedken

October 9, 2016

Theorem 2.2, The extended diagonals of a kite cross.

Proof. We want to prove that the extended diagonals of a kite cross. First, let ABCD be a kite such that line segment AB is congruent to line segment AD and line segment CB is congruent to line segment CD. By Euclid postulate 1, we can draw a straight line from point B to point D creating line segment BD. Let point E be the midpoint of of line segment BD, then segment BE is congruent to segment DE. By Euclid postulate 1, we can create line segment AE. Since line segment AE is congruent to itself, line segment BE is congruent to line segment DE, and line segment AB is congruent to line segment AD, then triangle AEB is congruent to triangle AED by Euclid 1:8. Since triangle AED is congruent to triangle to AEB, angle AED is congruent to angle AEB. Euclid's 1:13, a straight line falling on a straight line either creates two right angles or angles equal to two right angles, since line AE falls on line BED and angles AED and AEB are congruent they both must be right angles.

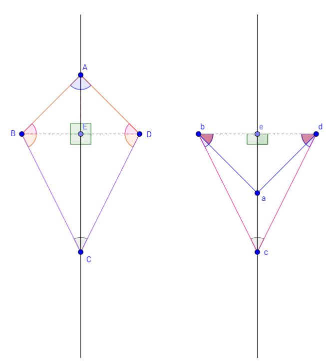

Figure 1: Diagonals of a kite crossing

Second, by Euclid postulate 1, we can draw line segment CE from point C to point E. Since CE is congruent to itself, CB is congruent to CD, and BE is congruent to DE then triangle CEB is congruent to triangle CED by Euclid 1:8. Since triangle CEB is congruent to triangle CED, angle CEB is congruent to angle CED. By Euclid's 1:13, since line CE falls on line BED and angles CEB and CED are congruent they both must be right angles. By Euclid postulate 2 we can extend the finite line segments AE and CE to create infinite lines. By Euclid 1:13 a straight line falling on a straight line must create two right angles or two angles equal to two right angles. Since AE is extended, BD is now a straight line falling on a straight line. Since angle BEA is a right angle then the new angle created by BE and the extended line AE is also a right angle. Since angle DEA is a right angle then the new angle created by DE and the extended line AE is also a right angle. Therefore where AE crosses BD 4 right angles are created. Similarly, since CE is extended, BD is a straight line falling on a straight line. Since angle BEC is a right angle the new angle created by BE and the extended line CE is also a right angle. Since angle DEC is a right angle the new angle created by DE and the extended line CE is also a right angle. Therefore, where CE crosses BD 4 right angles are created. Since CE falling on BD and AE falling on BD both create right angles, by Euclid common notion 4 AE and CE must be the same line. Therefore the points A, E, and C are co- linear and the extended diagonal AC crosses diagonal BD at point E.

□

Non-Convex Kite

Tessa Cohen

October 7, 2016

Definition . Let ABCD be a kite with AB congruent to AD and CB congruent to CD. Let E be the point where the segment BD meets the line AC. We say that ABCD is a non-convex kite when E does not lie between A and C. ABCD is a convex kite when E lies in between A and C.

Figure 1: The kite on the left is non-convex, the kite on the right is convex.

Note: Miss Bavido's definition of betweenness requires three points that are collinear. By Miss Goedken's Theorem 2.2, we know that the diagonals of a kite will always cross, therefore, points E, A and C will always be collinear.

The Fallacy in W. W. Rouse Ball's Proof

Danielle Maus

October 5, 2016

W.W. Rouse Ball's Theorem states that all triangles are isosceles. Professor Ball argues for this theorem, however we came across multiple errors in his proof.

Conjecture 4.1. Read Professor Ball's argument and figure out what went wrong.

Proof. Professor Ball's proof starts as follows. Let ABC be a triangle. Let D be the midpoint of segment BC. Let the perpendicular to BC at D meet the angle bisector of A at the point E.

He splits up his argument in three separate cases.

His first case argues that if point E is inside triangle ABC, triangle ABC is isosceles. He does this by manipulating Euclid's I.26, I.4, Theorem on Hypotenuse-Leg for Right Triangles, and Euclid's Common Notion 2.

His second case argues that if point E is outside triangle ABC, then by utilizing the same argument contained in case 1 and substituting Common Notion 3 in for Common Notion 2, triangle ABC is isosceles.

His third case argues that if point E coincides with point D "the proof is even easier" and he leaves the rest of case 3 for the reader to figure out.

Now, I am going to look at each individual case and point out errors in Professor Ball's argument.

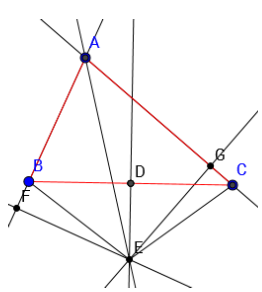

Figure 1: Point E falls outside of triangle ABC

The error in case 1 is when he supposes that point E is inside the triangle. By constructing different arbitrary triangles letting point D be the midpoint on segment BC and letting the angle bisector of angle A meet the intersection of the perpendicular on BC at point E, point E will never fall inside triangle ABC. (see figure 1) I can make this conclusion because of various constructions of triangles made in Geogebra. Point E never falls inside triangle ABC and for this reason Professor Ball's case 1 is invalid.

The error in case 2 is when Professor Ball states "by using the same argument as case 1 side AB and side AC are the differences of equals, hence equal." The problem with this argument is that the differences of equals will not make side AB congruent to side AC. The reason for this is because when Professor Ball states, "Drop perpendiculars EF and EG from E to the sides of the triangle", these perpendiculars will not both fall on the line segment AB and line segment AC at the same time. Perpendicular EF falling on line segment AB might be inside the triangle when perpendicular EG falling on line AC will not be inside the triangle. The reverse holds true. (see figure 2 and 3)

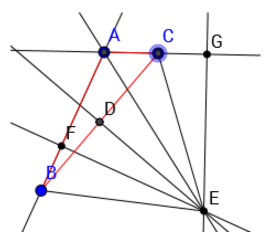

Figure 2: Perpendicular EF falls inside triangle ABC

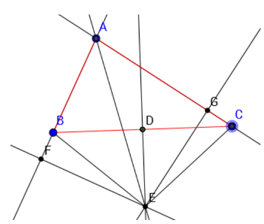

Figure 3: Perpendicular EG falls inside triangle ABC

Looking at figure 2 and figure 3 we can see the error regarding differences in equals (Euclid's Common Notion 3). First, looking at figure 2 side AF minus side BF will get us less than the desired side AB. Side AG minus side CG will get us the desired side AC. Therefore the differences in equals will not make side AB congruent to side AC. Looking at figure 3 side AF minus side BF will get us the desired side length AB. Side AG minus side CG will get us less than the desired side length AC, therefore AB is not congruent to side AC.

The error in case 3 is when Professor Ball states that "if E coincides with point D the triangle will be an isosceles triangle." With further constructions of arbitrary triangles on Geogebra, point E will only coincide with point D when triangle ABC is isosceles. However, when triangle ABC is isosceles the angle bisector at A and the perpendicular to BC at D will be the same line, the angle bisector and perpendicular line will coincide with one another. (see figure 4) Therefore, since the lines are the same there will be no one intersection point where point D coincides with point E. In other words we do not know where E is in this case, therefore this case is invalid.

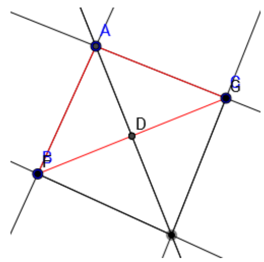

Figure 4: Angle Bisector and Perpendicular coincide-no point E

From my above findings and errors found in Professor Ball's argument I have made the following conjecture. I have concluded that the intersection of the angle bisector at A and the perpendicular to BC at D at the point E will always be outside of triangle ABC. The perpendiculars EF and EG will never both fall inside arbitrary triangle ABC at the same time. And finally, if triangle ABC is isosceles point E will not exist. Because of these findings I can conclude Professor Ball's theorem is invalid in that not all triangles are isosceles.

□

On Defining Polygons

Amanda Worsfold

October 6, 2016

Question P. Can we find a definition for convex/non-convex for general polygons?

Definition P. A polygon, P, is non-convex when there exists a line, L, such that:
1) L doesn't contain a side of P
2) L meets P at more than two points. This is shown in Figure 1.

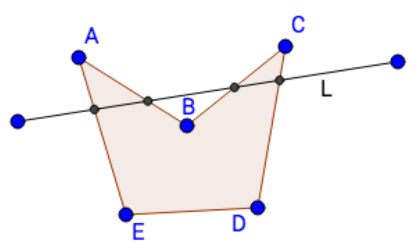

Figure 1: Non-Convex Pentagon with Line L

If these conditions cannot be met, then P is convex. This is shown in Figure 2.

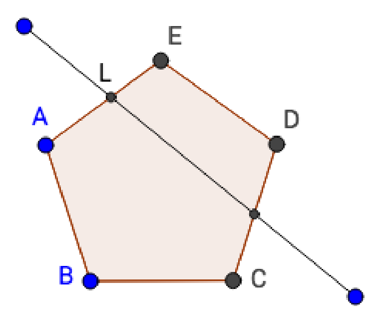

Figure 2: Convex Pentagon with Line L