

*Euclidean Geometry:  
An Introduction to Mathematical Work*

*Math 3600*

*Fall 2016*

*Regular Figures, A Warm-up*

A great part of the allure of geometry is figures with symmetry. Inspired by this, let us study some polygons that have a lot of symmetry.

**Definition.** A polygon is said to be *equilateral* if all of its sides are congruent, *equiangular* if all of its angles are congruent, and *regular* if it is both equilateral and equiangular.

**6.1 Conjecture.** An equilateral triangle is equiangular, hence regular.

**6.2 Conjecture.** Let  $ABCD$  be a rhombus. If angle  $A$  is congruent to  $B$ , then  $ABCD$  is regular.

**Definition** (reminder). A regular quadrilateral is called a *square*.

**6.3 Problem.** Does Conjecture 6.2 hold if we replace “angle  $B$ ” by “angle  $C$ ”? State a result and prove it.

**6.4 Conjecture.** Let  $ABCDE$  be an equilateral pentagon. If angle  $A$  is congruent to angle  $B$ , then  $ABCDE$  is regular.

**6.5 Conjecture.** Let  $ABCDE$  be a regular pentagon. The triangle  $ACD$  is isosceles.

**6.6 Problem.** Let  $ABCDE$  be a regular pentagon. State the relationship between the angles  $CAD$  and  $ACD$  that shows how special the triangle is. Prove your observation.

**6.7 Problem.** Find experimental evidence for the number of regular pentagons with a given side. (Try using five toothpicks!)

