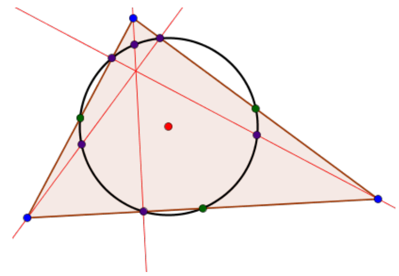
Euclidean Geometry:
An Introduction to Mathematical Work

Math 3600

Fall 2016



The Nine Point Circle

Definition. Let ABC be a triangle with orthocenter H. Let e_A, e_B and e_C be the midpoints of the segments AH, BH and CH, respectively. These three points are called the *Euler points* of triangle ABC.

18.1 Problem. Let ABC be a triangle with Euler points e_A, e_B and e_C. Show that the triangle $e_Ae_Be_C$ has the same orthocenter as triangle ABC.

18.2 Problem (The Nine-Point Circle)**.** Let ABC be a triangle. Let D, E, F be the midpoints of the sides, and H the orthocenter. Let e_A, e_B and e_C be the Euler points of ABC. Let A_F, B_F and C_F be the feet of the altitudes of ABC. Show that $D, E, F, e_A, e_B, e_C, A_F, B_F$ and C_F lie on a common circle.

Definition. The center of the circle just described is called the *nine-point center* of triangle ABC, and is commonly denoted N.

18.3 Conjecture. The nine-point center of ABC is the midpoint of the segment OH from circumcenter O to the orthocenter H.

18.4 Conjecture. Let ABC be a triangle. The nine-point circle of ABC and the inscribed circle of ABC are internally tangent.