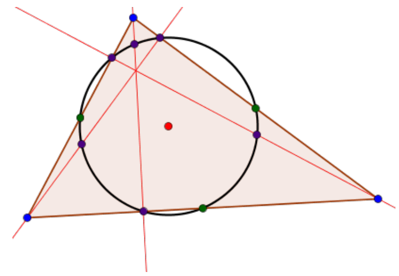
# *Euclidean Geometry:*
# *An Introduction to Mathematical Work*

*Math 3600*

*Spring 2015*



## *The Nine Point Circle*

**Definition.** Let $ABC$ be a triangle with orthocenter $H$. Let $e_A$, $e_B$ and $e_C$ be the midpoints of the segments $AH$, $BH$ and $CH$, respectively. These three points are called the *Euler points* of triangle $ABC$.

**18.1 Problem.** Let $ABC$ be a triangle with Euler points $e_A$, $e_B$ and $e_C$. Show that the triangle $e_Ae_Be_C$ has the same orthocenter as triangle $ABC$.

**18.2 Problem** (The Nine-Point Circle)**.** Let $ABC$ be a triangle. Let $D, E, F$ be the midpoints of the sides, and $H$ the orthocenter. Let $e_A, e_B$ and $e_C$ be the Euler points of $ABC$. Let $A_F$, $B_F$ and $C_F$ be the feet of the altitudes of $ABC$. Show that $D, E, F, e_A, e_B, e_C, A_F, B_F$ and $C_F$ lie on a common circle.

**Definition.** The center of the circle just described is called the *nine-point center* of triangle $ABC$, and is commonly denoted $N$.

**18.3 Conjecture.** The nine-point center of $ABC$ is the midpoint of the segment $OH$ from circumcenter $O$ to the orthocenter $H$.

**18.4 Conjecture.** Let $ABC$ be a triangle. The nine-point circle of $ABC$ and the inscribed circle of $ABC$ are internally tangent.