Regular Pentagon Interior Angle Relationships

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Theorem 6.6. Let $ABCDE$ be a regular pentagon. Then angle ACD is congruent to angle CAD taken twice.

Proof. Let $ABCDE$ be a regular pentagon. Draw diagonals AC and AD so that angles ACD and CAD exist.

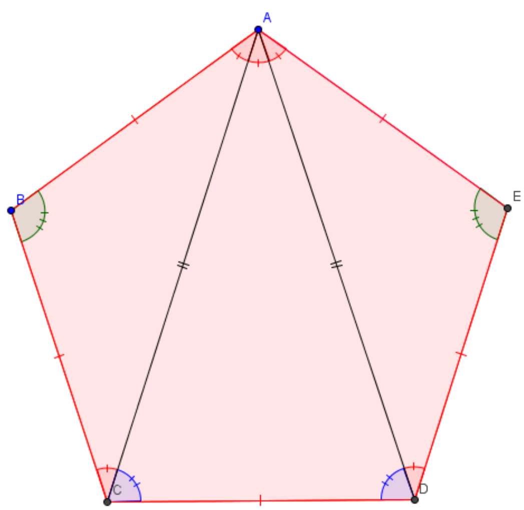

Figure 1: Pentagon $ABCDE$

Since pentagon $ABCDE$ is regular, all angles are congruent and all sides are congruent. By Theorem 5.2 (Hegewald), the sum of the interior angles of a pentagon is six right angles. Therefore, each interior angle is $\frac{6}{5}$ of a right angle. By Euclid I.32, the sum of the angles in a triangle is $\frac{10}{5}$ of a right angle. Since triangle ABC is isosceles, by Euclid I.5, angles BAC and ACB are congruent, and therefore must each be $\frac{2}{5}$ of a right angle.

By Theorem 6.5 (Stuhr), triangles ABC and AED are congruent. Therefore, angles BAC and EAD are congruent, and angle EAD must be $\frac{2}{5}$ of a right angle. Since angle BAE is $\frac{6}{5}$ of a right angle, and angles BAC and EAD are each $\frac{2}{5}$ of a right angle, the remaining angle CAD must be $\frac{2}{5}$ of a right angle.

Triangle ACD is isosceles, and therefore angles ACD and ADC are congruent. Since angle CAD is $\frac{2}{5}$ of a right angle, and the sum of the angles of a triangle is $\frac{10}{5}$ of a right angle, angles ACD and ADC must each be $\frac{4}{5}$ of a right angle.

Therefore, angle ACD is congruent to angle CAD taken twice.

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