Triangle Midline Theorem

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Theorem 3.6. Let ABC be a triangle, D the midpoint of AB and E the midpoint of AC. Then the line through E and D, called a *midline*, is parallel to the line through B and C.

Proof. Let ABC be a triangle. Let D be the midpoint of AB. Let E be the midpoint of AC. Let AM exist parallel to BC by construction of Euclid I.31. Let MN exist parallel to AB by construction of Euclid I.31.

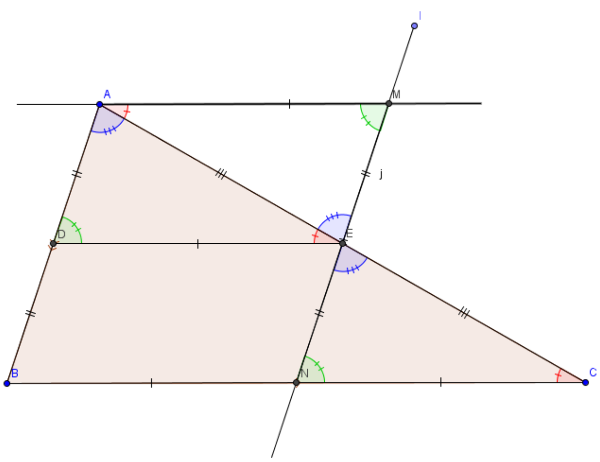

Figure 1: Triangle ABC

Angle MEA is congruent to angle CEN by Euclid I.15 (vertical angles). Since lines AM and BC are parallel, angle AME is congruent to angle CNE by Euclid I.27. Since E is the midpoint of AC, then AE is congruent to CE. Then by Euclid I.26 (ASA), triangle AME is congruent to triangle CNE.

Since AB is parallel to MN, angle DAE is congruent to angle MEA by Euclid I.27. Similarly, angles EAM and AED are congruent by Euclid I.27. Since AE is a shared side, then by Euclid I.26 (ASA) triangle AME is congruent to triangle EDA.

Since opposite pairs of angles are congruent, quadrilateral $AMED$ is a parallelogram by Euclid I.34. Therefore, midline DE is parallel to BC.

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