Kite Angle Congruence

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Theorem 2.1. A kite only has one pair of opposite angles that are congruent.

Proof. Let ABCD be a kite. We know segments AB and AD are congruent to each other and segments BC and DC congruent to each other by definition of a kite. If we draw diagonal AC, then we get triangle ABC and triangle ADC, both sharing side AC. From Euclid's Proposition I.8, we know these triangles are congruent to each other. This means that angle ABC and ADC are congruent to one another because of corresponding angles of congruent triangles.

Looking at the other opposite pair of angles in the kite, they will not be congruent to each other. Because of the congruent triangles ABC and ADC, we know angles BCA and DCA are congruent to one another. We also know angle BCD is the sum of the angles BCA and DCA, or twice the angle BCA. Similarly, we know angle BAD is the sum of the angles BAC and DAC, or twice the angle BAC. Euclid's Proposition I.6 states that if two angles in a triangle are congruent, then the sides that subtend to the angles will also be congruent. Using Proposition I.6, we have two cases for our kite. In case one, we will use the definition of a kite, so BA must be congruent to BC for angles BCA and BAC to be congruent. Since the definition of a kite says BA and BC don't have to be congruent to each other, we know angles BAD and BCD aren't necessarily congruent to each other. And since angle BAD is twice BAC and angle BCD is twice angle BCA and we know BAC and BCA can be different, then angles ABC and ADC aren't always congruent. In case two, sides BA and BC can be congruent, so all four side of the kite would be congruent, resulting in a equilateral kite which is also known as a rhombus. Using Euclid I.6, this would mean angles BCA and BAC are the same, resulting in opposite congruent angles.

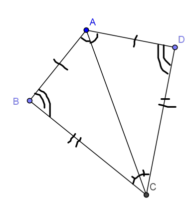

Figure 1: Kite ABCD

Figure 1 shows all the congruent sides and angles described in the proof.

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