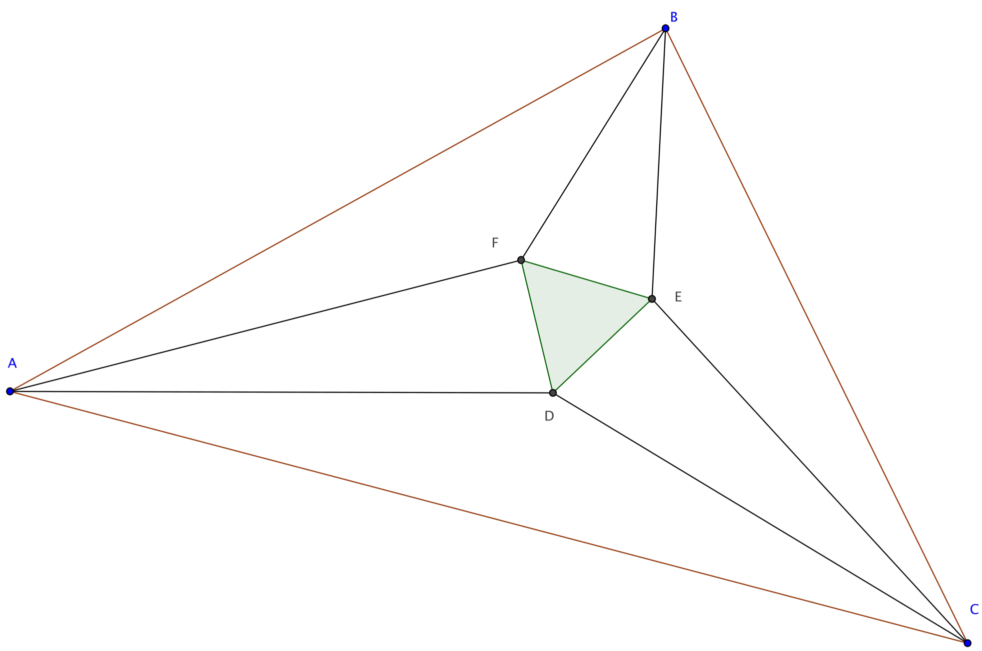
Transactions
in
Euclidean Geometry



Issue # 1

Table of Contents

Angles in Rhombi

Mackenzie Mitchell

September 14, 2016

Theorem. Opposite angles in a rhombus are congruent.

Proof. Let ABCD be a rhombus.

First we must prove that angle ABC and ADC are congruent to each other.

By Postulate 1, we may draw the segment AC. This forms two triangles ABC and ADC, by definition of a triangle (refer to figure 1). By the definition of a rhombus, all sides of a rhombus are congruent; meaning line segments AB, BC, CD, and DA are all congruent to each other. Comparing the two triangles, we have AD congruent to BC and DC congruent to AB. By Common Notion 4, AC is congruent to itself. Since triangles ABC and ADC have two sides congruent to the corresponding two sides and also have the base congruent to the corresponding base, the angles will also be congruent by Euclid Proposition I.8. Thus we have angle ABC congruent to angle ADC.

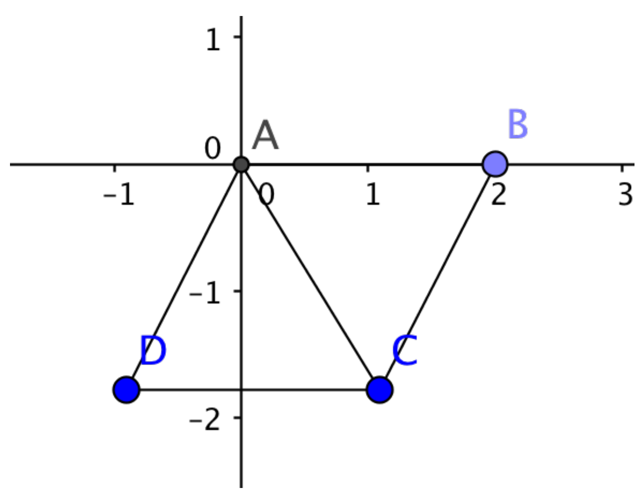


Figure 1: Rhombus 1 with triangles ACD and ABC

What remains is to show that angle BAD and angle BCD are congruent to each other.

By Postulate 1, we may draw the segment BD. This forms two triangles BAD and BCD, by definition of a triangle (refer to figure 2). By the definition of a rhombus, all sides of a rhombus are congruent; meaning line segments AB, BC, CD, and DA are all congruent to each other. Comparing the two triangles, we have DA congruent to BC and AB is congruent to CD. By Common Notion 4, BD is congruent to itself. Since triangles BAD and BCD have two sides congruent to the corresponding two sides and also have the base congruent to the corresponding base, the angles will also be congruent by Euclid Proposition I.8. Thus we have angle BAD congruent to angle BCD.

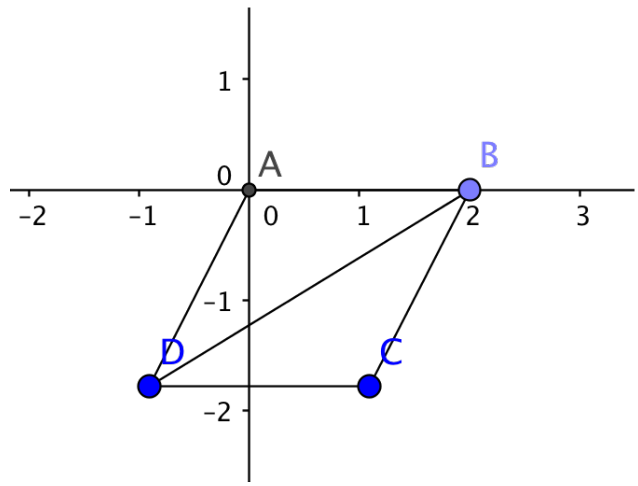


Figure 2: Rhombus 2 with triangles BAD and BCD

Therefore, in rhombus ABCD we have opposite angles congruent to each other. Thus, in any given rhombus, the opposite angles will be congruent to each other. □

Rhombi in Relation to Squares

Mackenzie Mitchell

September 24, 2016

The first day of class, we were given a handful of conjectures, Conjecture 1.1 being the first. Conjecture 1.1 states: Let ABCD be a rhombus. Then angle ABC is congruent to angle ADC. Similarly, angle BAC is congruent to angle BDC. With further exploration, we were able to see that the first half of this conjecture is true and the second is false. Thus Miss Goedken and Miss Ahrens proposed Conjecture A. Conjecture A states: The second statement is true exactly when ABCD is a square. Conjecture A contradicts Conjecture 1.1 because any given rhombus is not necessarily a square and if angles BAC and BDC are only congruent when the quadrilateral is a square, then conjecture 1.1 will not hold. I resolve this issue by showing that if angle BAC is congruent to angle BDC, then all the angles in the rhombus are congruent to each other. Then continue to strengthen this argument by proving they are all right angles. This will resolve this contradiction among the two conjectures.

Theorem. Let ABCD be a rhombus. If angle BAC is congruent to angle BDC, then ABCD is a square.

Proof. Let ABCD be a rhombus. By Postulate 1, we can create segment BD. We want to show that BD bisects the angles ABC and ADC.

By creating BD, by definition of a triangle, we have created two triangles: DAB and DCB (refer to figure 1). Since triangle DAB and triangle DCB have two sides congruent to each other in each triangle (DA congruent to DB and DC congruent to CB), they can be defined as isosceles triangles. By Euclid Proposition I.5, the angles at the base of each triangle are congruent; meaning angle ABD is congruent to angle ADB and angle DBC is congruent to angle BDC. Since triangle DAB and triangle DCB have three corresponding congruent sides, (DA, AB, DC, and BC congruent to each other and DB congruent to itself by common notion 4) by Euclid Proposition I.8, triangle DAB is congruent to triangle DCB. Thus, the angles in the bases are congruent; meaning angles ADB, ABD, DBC, and BDC are all congruent to each other. Also, by Mitchell's Theorem 1, angles DAB and DCB are also congruent to each other. Since angles ADB and BDC are congruent and angles ABD and DBC are congruent, BD bisects angles ABC and ADC.

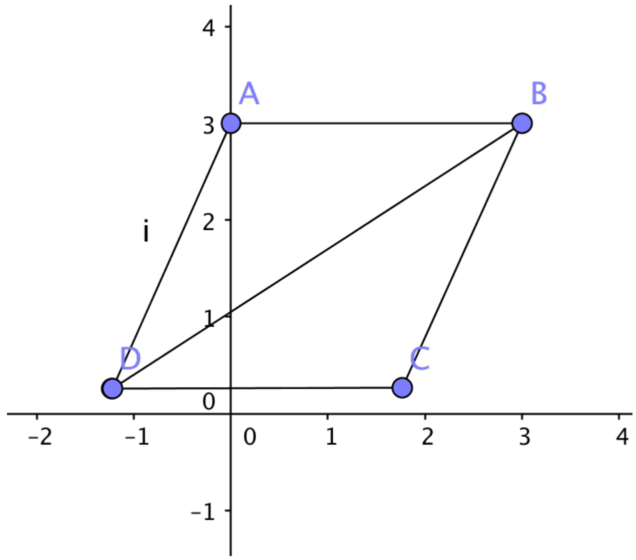


Figure 1: Rhombus ABCD with triangles DAB and DCB

By postulate 1, we can create segment AC. We want to show that AC bisects angles DAB and DCB. By creating AC, by definition of a triangle, we have created two triangles: ABC and ADC (refer to figure 2). Similarly, we can see by arguments like above, we also get angles BAC, BCA, DAC, and DCA are all congruent to each other and angles ADC and ABC are congruent to each other by Mitchell's Theorem 1. Since angles DAC and BAC are congruent and angles DCA and BCA are congruent, AC bisects angles DAB and DCB.

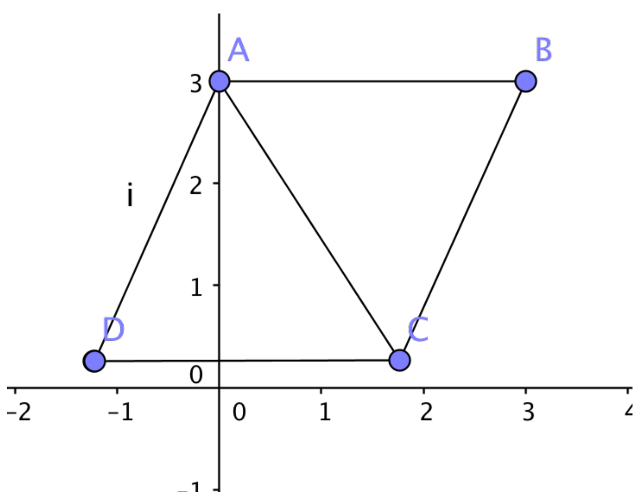


Figure 2: Rhombus ABCD with triangles ABC and ADC

Now that we have BD bisecting angles ABC and ADC and AC bisecting angles DAB and DCB, we want to show that all four angles in the rhombus (Angles ABC, ADC, DAB, and DCB) are all congruent to each other.

By definition of angle bisector, angle BAC is congruent to half of angle DAB and angle BDC is congruent to half of angle ADC. Since our goal is having angle BAC congruent to angle BDC, we must show that half of angle DAB is congruent to half of angle ADC. We can use common notion 1 to say that those two angles are congruent since common notion 1 states things which are equal (congruent) to the same thing are also equal (congruent) to each other. Since we have half of angle DAB congruent to half of angle ADC, we can multiply by 2 on each side of the congruence statement to make half of the angles (DAB and ADC) the full angles. Thus, we have angle DAB congruent to angle ADC. By Mitchell's Theorem 1, angle ADC is congruent to angle ABC and angle DAB is congruent to angle DCB. Thus, by common notion 1, we have all angles congruent to each other; meaning angles ADC, ABC, DAB, and DCB are all congruent to each other. Thus we have all four angles in the rhombus congruent to each other.

Now that we have all angles congruent to each other, we must show that each angle is a right angle to get to our definition of a square.

Consider triangle ABC. By Euclid Proposition I.32, any triangle has the angles summing to 2 right angles. Thus triangle ABC's angles will sum up to 2 right angles. Since triangle ADC is also in rhombus ABCD, we have triangle ADC's angles summing up to 2 right angles. With two triangles in rhombus ABCD, we have the total number of angles summing up to 4 right angles. Since there are 4 angles congruent to each other in rhombus ABCD (angles ABC, ADC, DAB, and DCB), each angle must be a right angle. Since rhombus ABCD has congruent sides and all the angles are right-angled, ABCD is a square (refer to figure 3). Thus, when angle BAC is congruent to angle BDC in a rhombus ABCD, ABCD must be a square.

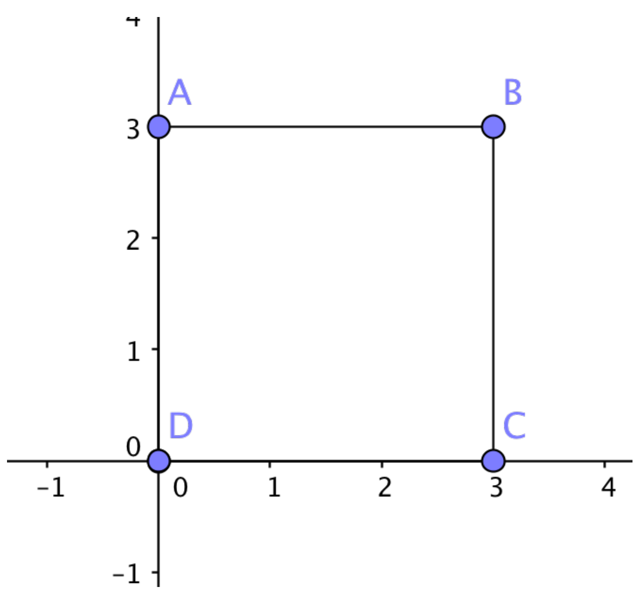


Figure 3: Square ABCD

□

All Rhombi are Parallelograms

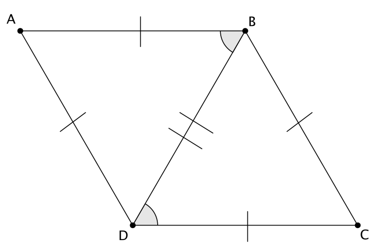
Tessa Cohen

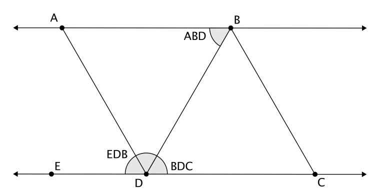
September 25, 2016

Theorem 1.6. Let ABCD be a rhombus. Then ABCD is a parallelogram.

Proof. Let rhombus ABCD exist. Let segment DB exist by Euclid I Postulate 1.

By Euclid I defintion 22, since ABCD is a rhombus, all sides are congruent to each other. Segment DB is congruent to itself by Euclid I Common Notion 4. Therefore, triangle DAB and triangle BCD are congruent by Euclid I.8. Therefore, we can say that angles ABD and BDC are congruent.





Let segment AB extend into infinite line AB and segment DC extend into infinite line DC by Euclid Postulate 2. Let there be a point E on line DC such that D is between E and C. By Euclid I.13, since segment DB is a straight line set up on another straight line, DC, the angles created, angles EDB and BDC, are equivalent to two right angles.

By Euclid Common Notion 2, since angle ABD is congruent to angle BDC, and angle BDE and angle BDC create two right angles, then angle BDE and angle ABD also create two right angles.

Since angle ABD and angle EDB, or the interior angles on one side, create two right angles, line AB is parallel to line CD by Euclid I.28. Since side AD is congruent to side BC and both are extremities off of line AB and line DC in the same direction, and line AB and line DC are parallel, side AD is also parallel to side BC by Euclid I.33. Remembering that the definition of a parallelogram is a quadrilateral in which two sets of opposite sides are parallel, we know side AB is parallel to side DC and side AD is parallel to side BC. Therefore, ABCD is a parallelogram.

□

Rhombus Diagonals Meeting at a Point

Heather Bavido

September 23, 2016

Theorem 1.7. Let ABCD be a rhombus. Suppose that the diagonals AC and BD meet at a point X. The angle AXB is a right angle.

Proof. To begin, all sides of a rhombus are congruent. Therefore, since line segment AB is congruent to BC, we conclude that triangle ABC is an isosceles triangle. By Euclid I.5, in an isosceles triangle, the angles at the base are equal to one another. Thus, we have angle BAC is congruent to angle BCA.

By Euclid I.8, if two triangles have two sides equal to two sides respectively, and have also the base equal to the base, they will also have the angles equal. As follows, by the congruent sides of the rhombus and the shared diagonal BD, we have triangle DAB congruent to triangle BCD. Since the angles of triangles DAB and BCD are congruent, we have angle DBC congruent to angle ABD, and angle BDA is congruent to angle BDC.

Again by Euclid I.5, the congruent triangles DAB and BCD also have each of the base angles congruent to each other. Thus, angles DBC, BDC, ABD and ADB are all congruent. By Euclid I.4, since line segment AB is congruent to segment BC, angle ABD is congruent to angle CBD, and line segment XB is congruent to itself, triangles ABX and CBX are congruent. Because triangle ABX and triangle CBX are congruent, they will also have the remaining angles congruent. Hence, angle AXB is congruent to angle BXC.

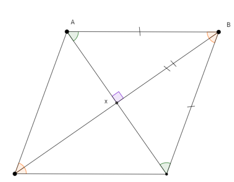


Figure 1: Rhombus ABCD

Lastly, by Euclid I.13, if a straight line (XB) set up on a straight line (AC) make angles (AXB, BXC), it will make either two right angles or angles equal to two right angles. Since AXB and BXC are congruent they must be exactly two right angles. Thus angle AXB is a right angle. □

Diagonals of a Kite

Amanda Worsfold

September 20, 2016

Theorem 2.5. If the diagonals of a kite meet, then they meet at a right angle.

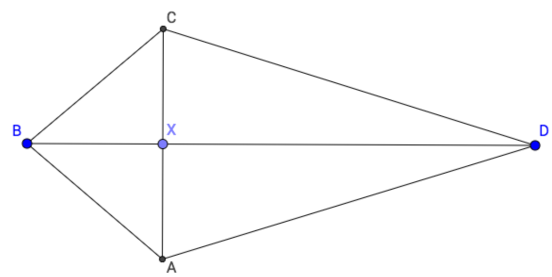


Figure 1: Kite ABCD

Proof. Let ABCD be a kite where the diagonals meet at point X (See Figure 1). By definition of a kite, AB is congruent to BC and AD is congruent to CD. The triangles DAB and DCB have the side BD in common, as well as DA congruent to DC and AB congruent to CB. The triangles are congruent by Euclid I.8, as well as their corresponding angles. Hence, angle ABD is congruent to angle CBD.

Now, triangles ABX and CBX share a common side BX, have congruent angles ABX and CBX, and side AB congruent to side BC. Thus, the triangles are congruent by Euclid I.4. This also means the bases AX and XC are congruent. Using Euclid I.13, angle AXB and angle CXB make angles that add up to two right angles. Because triangles ABX and CBX are congruent, angles AXB and CXB are congruent and therefore they must be right angles. Thus, the diagonals of a kite meet at a right angle. □

Congruency of Opposite Sides in Rectangles

Erica Schultz

September 16, 2016

Theorem 3.2. Each pair of opposite sides in a rectangle is a pair of congruent segments.

Proof. Let $ABCD$ be a rectangle.

By definition, we have a quadrilateral which has all four interior angles that are right angles. The rectangle $ABCD$ has a straight line AD falling on the two straight lines AB and DC. The interior angles BAD and ADC are congruent to two right angles, so the lines AB and DC are parallel to one another by Euclid 1.28. Similarly, the straight line AB falls on the straight lines AD and BC and the interior angles BAD and ABC are congruent to two right angles, so the lines AD and BC are parallel to one another by Euclid 1.28 also.

By Postulate 1, we will draw the diagonal AC. The lines AD and BC are parallel to one another, so the alternate angles CAD and ACB are congruent by Euclid 1.29. Similarly, the lines AB and DC are parallel to one another, so the alternate angles BAC and DCA are congruent by Euclid 1.29 also. We know that line AC is congruent to itself, and the angles CAD and ACB are congruent to one another, as well angles BAC and DCA are congruent to one another, so the triangles ACD and CAB are congruent to one another by Euclid 1.26. Figure 1 helps to make clear the angles and triangles that we are talking about. Because triangles ACD and CAB are congruent to one another, we know that the sides AB and DC are congruent to one another, and the sides AD and BC are congruent to one another. Thus, the opposite sides of a rectangle are congruent segments.

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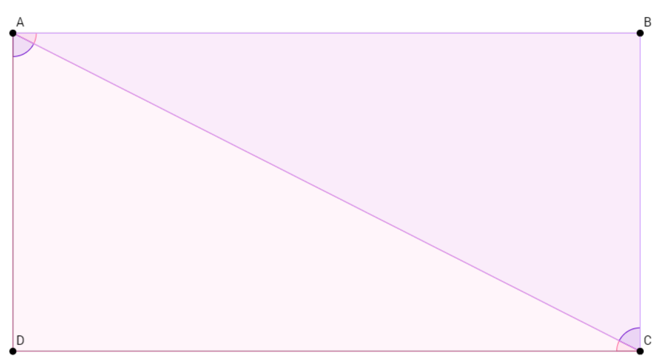

Figure 1: Rectangle $ABCD$

Midpoints in a Quadrilateral

Danielle Maus and Mackenzie Mitchell

September 22, 2016

Theorem 3.7. Let ABCD be a quadrilateral. The midpoints of the four sides are the vertices of a parallelogram.

Proof. Let ABCD be an arbitrary quadrilateral. Let vertices E, F, G, and H be the midpoints of our quadrilateral where vertex E lies on side AB, vertex F lies on side BC, vertex G lies on side CD, and vertex H lies on side DA. We want to show by connecting these vertices a parallelogram will form.

First we will show that EF and HG are parallel. By Postulate 1, we can create diagonal AC. This creates triangles ABC and ADC by the definition of a triangle. By Postulate 1, we can create EF and HG (refer to figure 1). By the Midline Theorem, EF is parallel to AC as the Midline Theorem states: In any triangle ABC, where D and E are the midpoints of AB and AC respectively, DE will be parallel to BC. Thus, we also can conclude HG is parallel to AC by the Midline Theorem. By Euclid Proposition I.30, since EF is parallel to AC and HG is parallel to AC, EF is parallel to HG. Thus, we have one set of parallel lines.

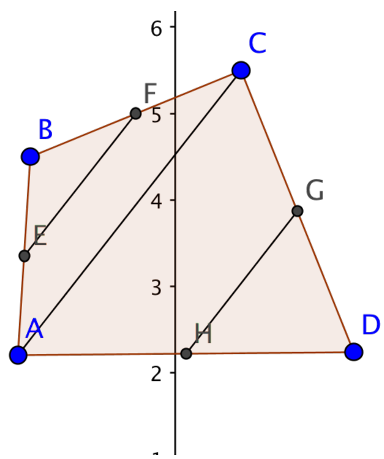

Figure 1: Quadrilateral ABCD with Triangles ABC and ADC

Now, we will show EH and FG are parallel. By Postulate 1, we can create diagonal BD. This creates triangles BCD and BAD by the definition of a triangle. By Postulate 1, we can create EH and FG (refer to figure 2). By using the same technique we used above, we can use the Midline Theorem to show EH is parallel to BD and FG is parallel to BD. We can also use Euclid Proposition I.30 again to show that EH and FG are parallel to each other as we did earlier. Thus, we have our second set of parallel lines.

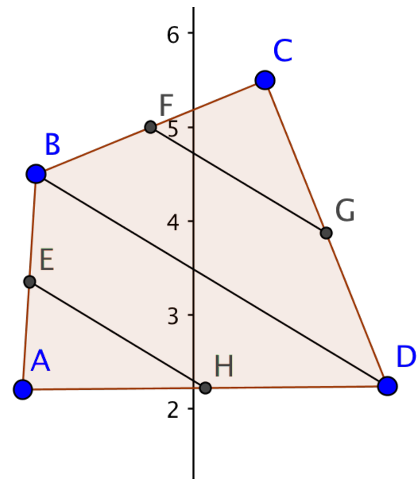

Figure 2: Quadrilateral ABCD with Triangles BCD and BAD

Thus, we have EF parallel to HG and EH parallel to FG. Since we have two sets of parallel lines, we have EFGH as a parallelogram by the definition of a parallelogram. Therefore, if ABCD is a quadrilateral, the midpoints of the four sides are the vertices of a parallelogram.

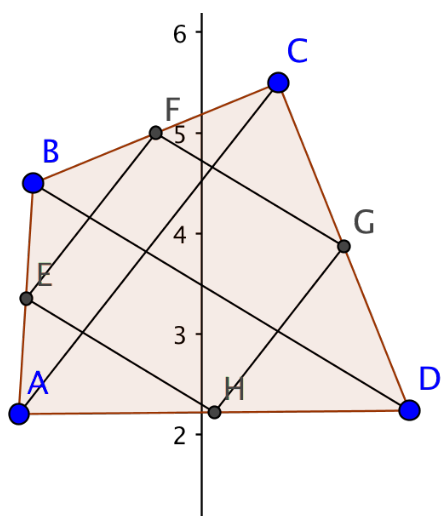

Figure 3: Quadrilateral ABCD with midpoints EFGH forming a parallelogram

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