Right Triangle Lies on a Circle

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May 1, 2015

This is one half of Thales' Theorem

Theorem 7.5. If ACB is a right angle, then C lies on the circle with diameter AB.

Proof. Using proposition 31, construct a straight line parallel to BC at point A, and a straight line parallel to AC at point B. Because the quadrilateral ACBD has two pairs of parallel sides, it is a parallelogram, and angle ADB is a right angle by the definition of a parallelogram. Because ACBD has two pairs of parallel sides and opposite right angles, it is a rectangle by Theorem 3.5. Draw segment CD, and let point E be the intersection of CD and AB proven by Theorem 3.3. By Theorem 3.3, segments AE,BE,CE,DE are congruent to one another. Construct circle E through A. By definition of a circle, points A, B, C lie on circle E. Because points A and B lie on circle E, and the center of the circle, E, lies on segment AB, segment AB is a diameter of circle E. □

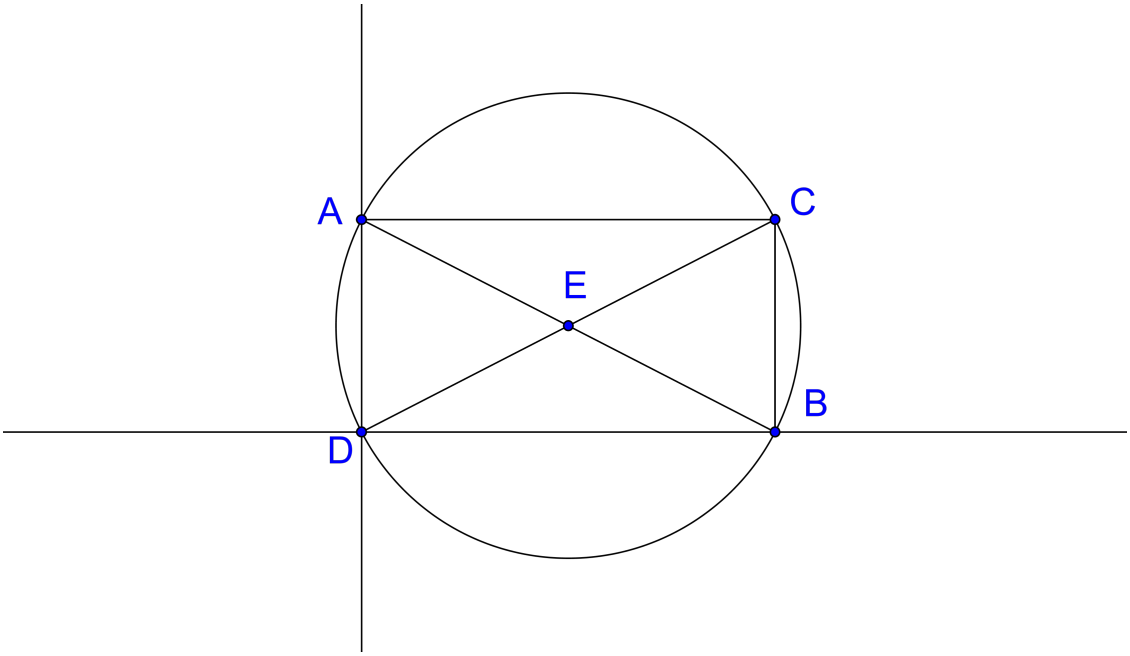

Figure 1: Quadrilateral ACBD