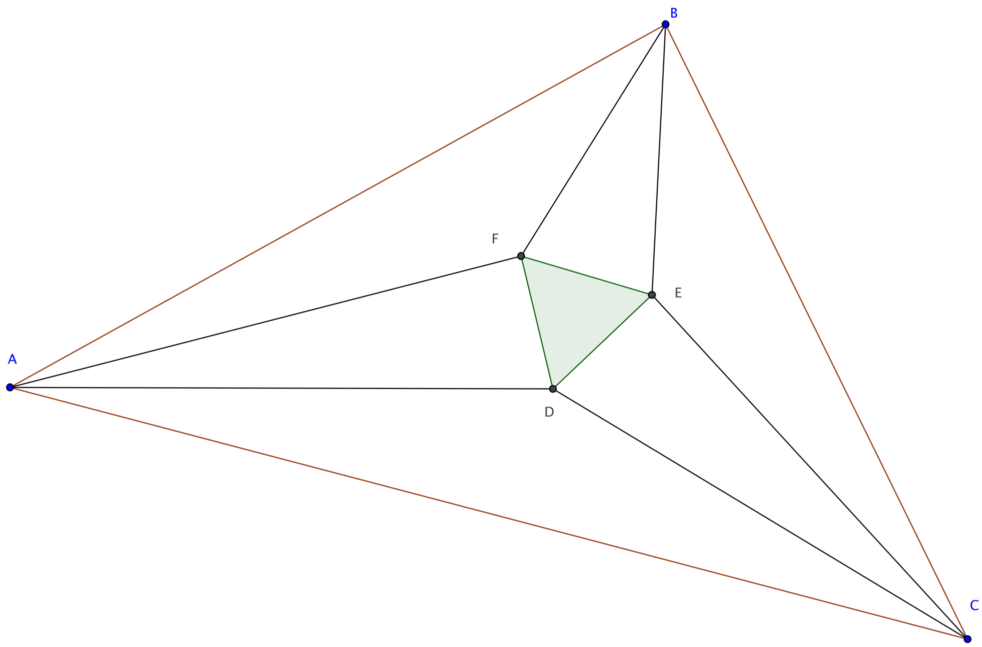
Transactions

in

Euclidean Geometry



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Rectangle Construction

Jaclyn Miller

October 28, 2018

Communicated by: Ms. DeVore.

Theorem (Miller). It is possible to construct a rectangle, ABCD, with a compass and straightedge.

Proof. Let circle X be a circle centered at point X, with radius XC and diameter AC.

1. Pick an arbitrary point on circle X and call it point B. Draw line segments AB and BC by Euclid Postulate I.1.

2. By Euclid Postulate I.1, draw a diameter through point B, called line L. Since L is a diameter, it also passes through the center of circle X, point X.

3. Line L will cut the circle in two places, once at point B. Name the other place line L cuts circle X point D.

4. Draw line segments AD and CD by Euclid Postulate I.1.

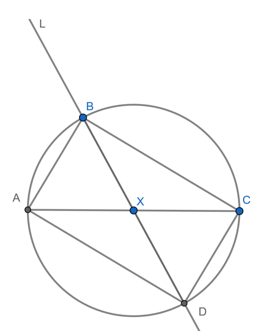


Figure 1: Construction of Rectangle ABCD.

Since point B is on circle X, by Ms. DeVore's Theorem 7.4, angle ABC is a right angle. Similarly, since D is on circle X, by Ms. DeVore's Theorem 7.4, angle CDA is a right angle.

Since XA, XB, XC, XD are all radii of circle X, line segments XA, XB, XC and XD are congruent. Since AC and BD cut each other, vertical angles AXB and DXC are congruent by Euclid Proposition I.15. Since line segments XB, XA, XC, and XD are congruent and angles AXB and DXC are congruent, triangles AXB and DXC are congruent by Euclid Proposition I.4.

Since XD and XC are congruent, triangle DXC is isosceles by Euclid Definition 20. Thus, base angles XDC and XCD are congruent by Euclid Proposition I.5. Since XB and XA are congruent, triangle AXB is isosceles by Euclid Definition 20. Thus, base angles XBA and XAB are congruent by Euclid Proposition I.5. Since triangles AXB and DXC are congruent, angles XCD, XDC, XBA, and XAB are congruent because corresponding parts of congruent triangles are congruent.

Since AC and BD cut each other, vertical angles AXD and BXC are congruent by Euclid Proposition I.15. Thus, since line segments XB, XC, XA, and XD are congruent and angles AXD and BXC are congruent, triangles AXD and BXC are congruent by Euclid Proposition I.4.

Similarly, since XA and XD are congruent, triangle AXD is isosceles by Euclid Definition I.20. Thus, base angles XAD and XDA are congruent by Euclid Proposition I.5. Since XB and XC are congruent, triangle BXC is isosceles by Euclid Definition 20. Thus, base angles XBC and XCB are congruent by Euclid Proposition I.5. Since triangles AXD and BXC are congruent, angles XBC, XCB, XAD, and XDA are congruent because corresponding parts of congruent triangles are congruent.

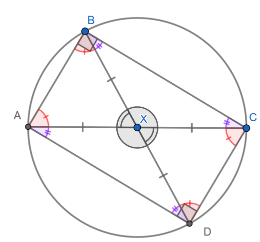


Figure 2: Construction of Rectangle ABCD.

Notice that angle XDC and angle XDA compose angle CDA. Recall that angle CDA is a right angle. Thus, angles XDC and XDA taken together form a right angle. Since angle XAB is congruent to angle XDC, and angle XAD is congruent to XDA, angles XAB and XAD taken together form a right angle, by Euclid Common Notion I.1. Since angles XAB and XAD compose angle BAD, angle BAD is a right angle. Since angle BCD is opposite to angle BAD in the quadrilateral, angles BCD and BAD, taken together, are equal to two right angles by Euclid Proposition III.22. Since angle BAD is a right angle, BCD must be a right angle as well. Thus, angles ABC, BCD, BAD, and CDA are all right angles. Therefore, ABCD is a rectangle by definition.

□

There exist an Isosceles Triangle in a Regular Pentagon

Brad Warner

November 6, 2018

Communicated by: Jason Stine

Theorem 6.5. If ABCDE be a regular pentagon then the triangle ACD is isosceles.

Proof. Let ABCDE be a regular Pentagon. Since, ABCDE is a regular pentagon, we know all sides are congruent and all angles are congruent. Consider the triangles ABC and AED. Notice, sides AB is congruent to AE and BC is congruent to ED. Moreover, angle B is congruent to angle E since ABCDE is a regular pentagon. Since, we have two congruent sides and a congruent angle, then triangle ABC is congruent to triangle AED by Euclid's Proposition i.4. Therefore, AC is congruent to AD by corresponding parts of congruent triangles. Since, AC is congruent to AD, then triangle ACD is isosceles because we have two congruent sides.

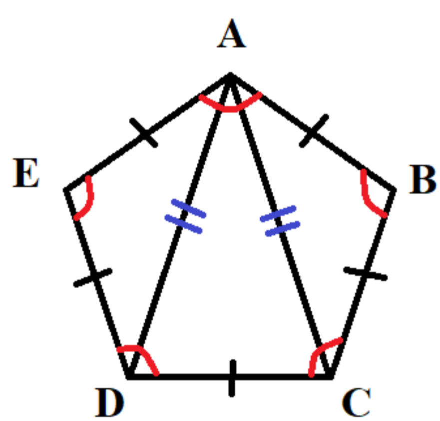


Figure 1: Regular Rhombus ABCD

□

Two Congruent Triangles

Lexis Wiegmann

October 24, 2018

Communicated by: Ms. Falck.

I am giving credit for the help of this proof to Mr. Stine and Mr. VandeLune.

Theorem 7.2. Let ABC (triangle 1) and DEF (triangle 2) be two right triangles, with angle A and angle D being the right angles. Suppose that BC is congruent to EF and AB is congruent to DE. Then the triangles are congruent.

Proof. Let ABC and DEF be triangles with the above conditions. By Euclid Proposition I.23, make angle alpha from E. Put that angle at B to make another triangle off of 1 (triangle 3). Extend line AC. Where the two lines meet, make point X. Because we extended the straight line, we know that there is another right angle at A. We know that angles D and A are right angles, the alphas are congruent, and sides AB and DE are congruent as well. From Euclid Proposition I.26, (ASA), triangle 2 is congruent to triangle 3. Therefore, all of the corresponding angles and sides are congruent.

Now, we will look at triangles 3 and 1. We know that sides XB and BC are congruent. That makes the whole triangle isosceles by Euclid Definition I.20. Now we now angles X and C are congruent as well as angle A is congruent to angle A. Because triangles 1 and 3 share side AB, by Euclid Proposition I.26 (AAS), triangles 1 and 3 are congruent. Therefore, all of the corresponding angles and sides are congruent.

From above, we know that triangles 2 and 3 are congruent. We also know triangles 3 and 1 are congruent. Using the transitive property, if triangle 2 is congruent to triangle 3 and triangle 3 is congruent to triangle 1, then triangle 2 is congruent to triangle 1. □

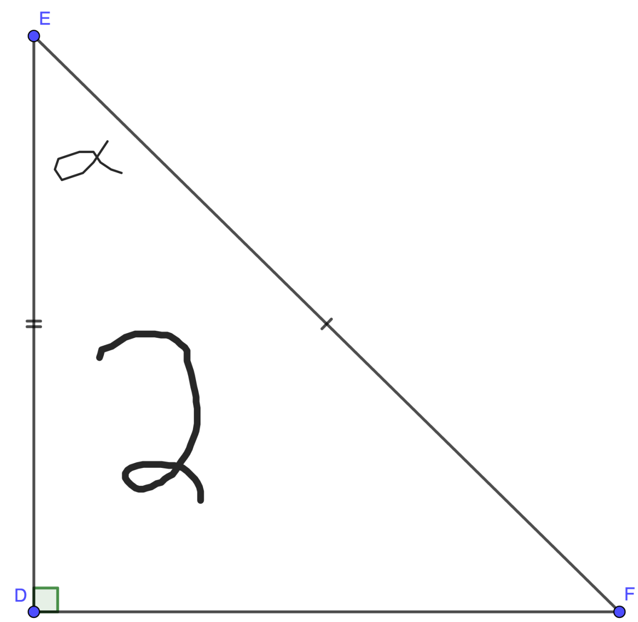


Figure 1: Triangle DEF (triangle 2) with angle alpha at angle E.

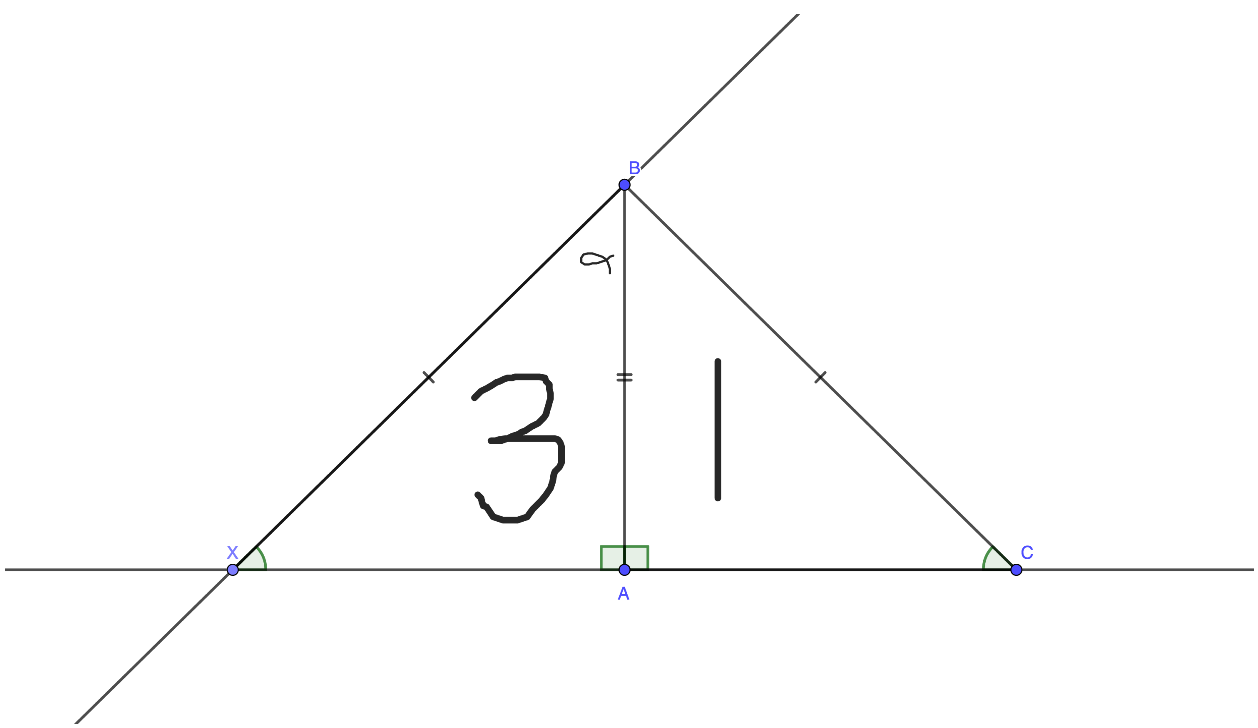


Figure 2: Triangles ABC (triangle 1) and ABX (triangle 3).

AB and AC are Congruent Tangent Lines

Brad Warner

November 3, 2018

Communicated by: Lexis Wiegmann

Theorem 9.1. Let AB and AC be two tangent lines from a point A outside of a circle. Then AB is congruent to AC.

Proof. Let B and C be on the circle, where lines AB and AC are tangent lines to the circle through A. By Euclid Postulate 1, draw lines BX and CX being radii of the circle. Draw a line between A and X by Euclid postuale 1. Consider the two triangles AXB and AXC. Since AX is a common line segment then AX is congruent to itself. Since BX and CX are radii of circle X then BX is congruent to CX. By Euclid's Proposition III.18 there exist right angles at ABX and ACX. By Ms. Wiegmann's 7.2 Theorem we say that triangles AXB and AXC are congruent by right angle, side, side. Thus, AB is congruent to AC.

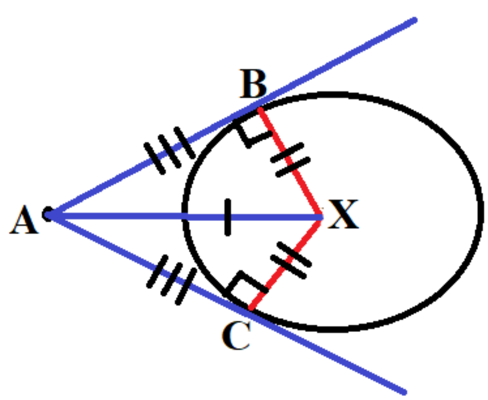


Figure 1: Tangent lines AB and AC touching circle X

□

Kite of Circle Radii

Alexa DeVore

November 6, 2018

Communicated by: Ms. Falck.

Conjecture 9.2. Let I and Ω be two circles with centers G and O, respectively. Suppose these circles meet at two points A and B. If GAO is a right angle, then GBO is a right angle.

Proof. Let I and Ω be circles intersecting at points A and B. By Euclid Postulate I.1, connect center of circle I, point G, to points A and B respectively and connect center of Ω, point O, to points A and B. Since points A and B both lie on circle I, lines GA and GB must both be radii of the circle and therefore congruent. Similarly, OA and OB are radii of circle Ω and therefore congruent. Therefore, GOAB has two pairs of adjacent, congruent and must be a kite by definition. As proven in theorem 2.2, the angles that are formed by the two different pairs of sides, are congruent. Therefore, if GAO is a right angle, GBO must also be a right angle.

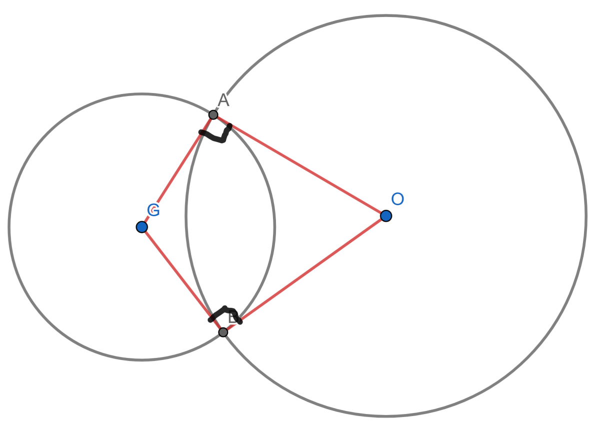


Figure 1: This is a picture of the circles and radii.

□

Rectangles are Cyclic

Jaclyn Miller

October 28, 2018

Communicated by: Mr. Warner.

Theorem 9.3. Rectangles are always cyclic.

Proof. Let ABCD be a rectangle. Draw diagonals AC and BD by Euclid Postulate I.1. By Mr. Vandelune's Theorem 3.3, line segment AC is congruent to line segment BD, and line segments AC and BD meet at a point E. Line segments AC and BD also bisect each other at E, by Mr. Vandelune's Theorem 3.3. Thus line segments EA, EB, EC, and ED are congruent.

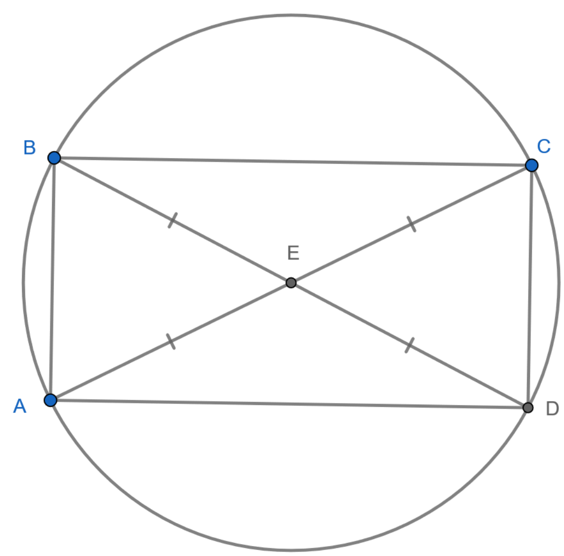


Figure 1: Rectangle ABCD inscribed in circle Z.

Let E be the center of a circle with radius EA, called circle Z. Then, since EA is a radius of the circle, and line segment EA is congruent to line segments EB, EC, and ED, line segments EB, EC, and ED are also radii of circle Z. Thus, points A, B, C, D all lie on circle Z, because they are each endpoints of a radius of circle Z. Therefore, rectangles are always cyclic.

□

Parallel Lines and Circles

Jaclyn Miller

November 6, 2018

Communicated by: Mr. Stine.

Theorem 9.5. Let two circles be tangent at a point A. If two lines are drawn through A meeting one circle at further points B and C and meeting the other circle at points D and E, then BC is parallel to DE.

Proof. Let two circles be tangent at point A.

Case One: The two lines fall on opposite sides of the extended diameter.

Draw two lines through A such that the lines fall on opposite sides of the extended diameter. Let points B, C, D, and E be the points that the lines meet the circles, such that B and C are on the same circle and D and E are on the same circle. By Euclid Proposition III.1, find the center, point X, of the circle points D and E fall on. Similarly, find the center, point Y, of the circle points B and C fall on. Draw line segment XY by Euclid Postulate I.1. By Euclid Proposition III.12, line segment XY will pass through the point that circle X and circle Y intersect, namely, point A. Extend line segment XY into a line by Euclid Postulate I.2.

Draw line segments DX, EX, BY, and CY by Euclid Postulate I.1. Since DX, AX, and EX are radii of circle X, DX AX and EX are congruent. Similarly, since BY, AY, and CY are radii of circle Y, BY, CY, and AY are congruent. Since line segments EX and AX are congruent, triangle EXA is isosceles by Euclid Definition I.20. Since triangle EXA is isosceles, the base angles XEA and XAE are congruent by Euclid Proposition I.5. Similarly, since lines segments AY and BY are congruent, triangle ABY is isosceles by Euclid Definition I.20. Since triangle ABY is isosceles, the base angles YBA and YAB are congruent by Euclid Proposition I.5. Angles XAE and YAB are vertical angles and thus congruent by Euclid Proposition I.15. Since angles XAE and XEA are congruent, angles YBA and YAB are congruent, and angles XAE and YAB are congruent, angles XAE, XEA, YAB, YBA are congruent by Euclid Common Notion I.1.

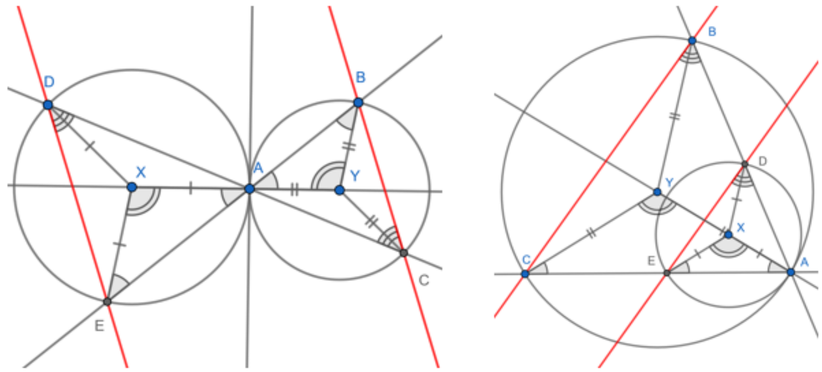


Figure 1: Case One.

By Euclid Proposition I.32, angles XAE, XEA, and AXE taken together are congruent to two right angles. Similarly, angles YBA, YAB, and AYB taken together are congruent to two right angles by Euclid Proposition I.32. Since angles XAE, XEA, and AXE taken together are congruent to two right angles, and angles YBA, YAB, and AYB taken together are congruent to two right angles, then angles XAE, XEA, and AXE taken together are congruent to angles YBA, YAB, and AYB by Euclid Common Notion I.1. Thus, since angles XAE, XEA, YAB, YBA are congruent, then angle AXE is congruent to angle AYB.

In circle X, since angle AXE and angle EDA share the same base, line segment EA, and D lies on the circle, and X lies within the circle, angle EXA is double angle EDA by Euclid Proposition III.20. Similarly, in circle Y, since angle AYB and angle ACB share the same base, line segment AB, and C lies on the circle, and Y lies within the circle, angle AYB is double angle ACB by Euclid Proposition III.20. Since angles EXA and AYB are congruent, and angles EDA and ACB are half of angles EXA and AYB respectively, angles EXA and AYB are congruent.

Then, since line segment DC falls on line segments DE and BC, and makes alternate interior angles EXA and AYB are congruent, line segments DE and BC are parallel by Euclid Proposition I.27.

Case Two: One of the two lines is the extended diameter of the circles.

Find the center of one of the circles, and call it point X by Euclid Proposition III.1. Similarly, by Euclid Proposition III.1, find the center of the other circle, and call it point Y. Draw line segment XY by Euclid Postulate I.1. By Euclid Proposition III.12, line segment XY will pass through the point that circle X and circle Y intersect, namely, point A. Extend line segment XY into a line by Euclid Postulate I.2. Let the point that XY intersects circle X be point D, and let the point that XY intersects circle Y be point C. Draw another line through A, called line L. Let the point where L intersects circle X be point E, and let the point where L intersects circle Y be point B.

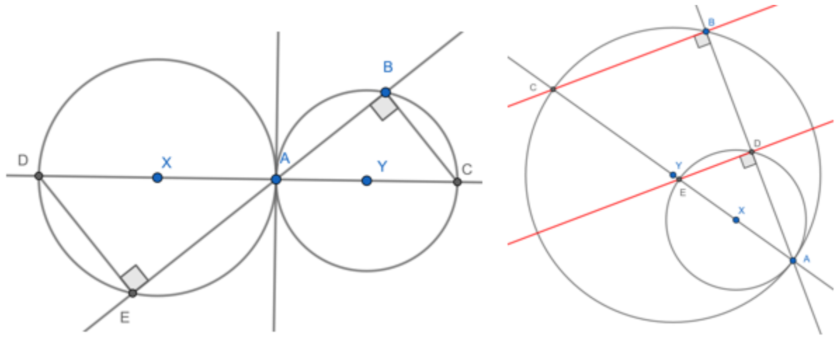


Figure 2: Case Two.

Draw line segments DE and BC by Euclid Postulate I.1. Since DA is the diameter of circle X, and E is a point on circle X, angle DEA is a right angle by Ms. DeVore's Theorem 7.4. Similarly, since CA is the diameter of circle Y, and B is a point on circle Y, angle CBA is a right angle by Ms. DeVore's Theorem 7.4. Thus angles CBA and DEA are both right angles and thus congruent. Since EB falls on line segments DE and BC, and makes alternate interior angles DEA and CBA congruent, line segments DE and BC are parallel by Euclid Proposition I.27.

Case Three: The two lines fall on the same side of the extended diameter.

Draw two lines through A such that the lines fall on the same side of the extended diameter. Let points B, C, D, and E be the points that the lines meet the circles, such that B and C are on the same circle and D and E are on the same circle. By Euclid Proposition III.1, find the center, point X, of the circle points D and E fall on. Similarly, find the center, point Y, of the circle points B and C fall on. Draw line segment XY by Euclid Postulate I.1. By Euclid Proposition III.12, line segment XY will pass through the point that circle X and circle Y intersect, namely, point A. Extend line segment XY into a line by Euclid Postulate I.2. Let the second point where line XY intersects circle X be point J, and let the second point where line XY intersects circle Y be point K.

Draw line segments BC, CK, JD, and DE by Euclid Postulate I.1. Since JA is the diameter of circle X and point D is on circle X, angle JDA is a right angle, by Ms. DeVore's Theorem 7.4. Similarly, since KA is the diameter of circle Y and point C is on circle Y, angle KCA is a right angle, by Ms. DeVore's Theorem 7.4. By Euclid Proposition I.15, angle JAD is congruent to angle CAK because JAD and CAK are vertical angles. Similarly, vertical angles BAC and DAE are congruent by Euclid Proposition I.15.

The polygon with sides JA, AE, DE, and JD is a cyclic quadrilateral, thus opposite angles JDE and JAE taken together are congruent to two right angles by Euclid Proposition III.22. Angle JDE is composed of angles JDA and ADE, and angle JAE is composed of angles JAD and DAE. Thus, angles JDA, ADE, JAD, and DAE taken together are congruent to two right angles. Similarly, the polygon with sides AB, BC, CK, and AK is a cyclic quadrilateral, thus

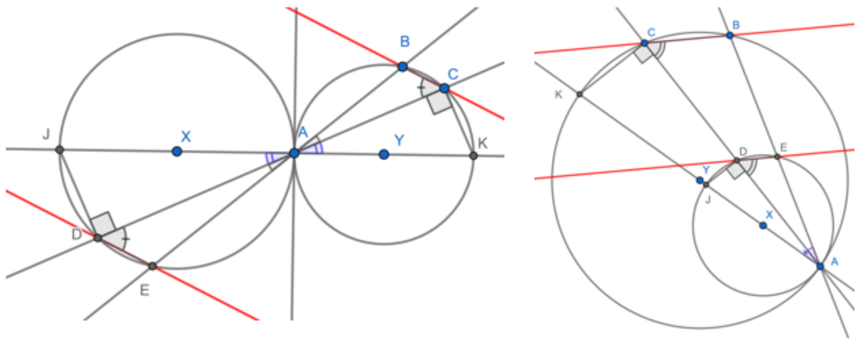


Figure 3: Case Three.

opposite angles BCK and BAK taken together are congruent to two right angles by Euclid Proposition III.22. Angle BCK is composed of angles BCA and ACK, and angle BAK is composed of angles BAC and CAK. Thus, angles BCA, ACK, BAC, and CAK taken together are congruent to two right angles.

Since angles JDA, ADE, JAD, and DAE taken together are congruent to two right angles, and angles BCA, ACK, BAC, and CAK taken together are congruent to two right angles, then angles BCA, ACK, BAC, and CAK taken together are congruent to angles JDA, ADE, JAD, and DAE taken together by Euclid Common Notion I.1.

JDA and ACK are both right angles, thus congruent. Recall that angle JAD is congruent to angle CAK, and angles BAC and DAE are congruent. Then, angles ADE and ACB are congruent. Since line segment DC falls on line segments DE and BC, and makes alternate interior angles ADE and ACB congruent, DE and BC are parallel by Euclid Proposition I.27. □

Chords and Acute Angles

Jaclyn Miller

November 5, 2018

Communicated by: Mr. Vandelune.

Theorem 10.3. If two chords of a circle subtend different acute angles at points of a circle, then the smaller angle belongs to the shorter chord.

Proof. Let X be a circle centered at point X. Let A, B, C, D, be points on circle X such that line segment AB is shorter than line segment CD. Let points E and F be on circle X such that angles AEB and DFC are acute angles.

By Euclid Postulate I.1, draw line segments AX, BX, CX, and DX. Then, since angle AXB is an angle at the center and angle AEB is an angle at the circumference, angle AXB is twice as large as angle AEB by Euclid Proposition III.20. Similarly, since angle DXC is an angle at the center and angle DFC is an angle at the circumference, angle DXC is twice as large as angle DFC by Euclid Proposition III.20.

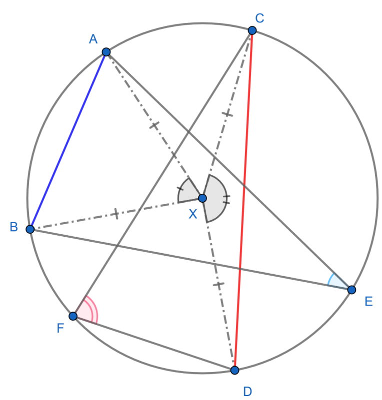


Figure 1: Circle X with Chords AB and CD.

Recall that points A, B, C, and D all lie on circle X. Thus line segments AX, BX, CX, and DX are radii of circle X, and therefore congruent. Thus, triangles DXC and AXB have two pairs of congruent sides, but triangle DXC has a longer base. Therefore, by Euclid Proposition I.25, angle DXC is greater than angle AXB.

Since angle AEB is half of angle AXB and angle DFC is half of angle DXC, and angle DXC is greater than angle AXB, angle AFC is greater than angle AEB. Thus, the smaller angle, AEB, belongs to the shorter chord, AB.

$\square$

Archimedes' Theorem of the Broken Chord

Jaclyn Miller and Payson VandeLune

November 9, 2018

Communicated by: Ms. Wiegmann.

Theorem 10.7. Let AB and BC be two chords of a circle O, where BC is greater than AB. Let M be the midpoint of arc ABC and F the foot of the perpendicular from M to chord BC. Then F is the midpoint of the broken chord, that is, AB and BF taken together are congruent to FC.

Proof. Let there be a circle such that points A, B, and C lie on the circle and chord BC is greater than chord AB. Let M be the midpoint of the arc ABC and F the foot of the perpendicular from M to chord BC. Since M is the midpoint, arc AM is congruent to arc MC.

By Euclid Postulate I.1, draw chords AM and MC. Since chord AM subtends arc AM, and chord MC subtends arc MC, chords AM and MC are congruent by Euclid Proposition III.29. Then, since angle BAM stands upon arc BM and angle BCM stands upon arc BM, angles BAM and BCM are congruent by Euclid Proposition III.27.

Construct a circle centered at C, with radius congruent to chord AB by Euclid Postulate I.3, and call the place that the circle cuts chord BC point P. We know such a point exists because the radius, AB, is shorter than chord BC. Thus, line segment PC is congruent to chord AB by construction. By Euclid Postulate I.1, draw line segments MP and MB.

Since line segments PC and AB are congruent, line segments AM and MC are congruent, and angles BAM and BCM are congruent: triangles MAB and MCP are congruent by Euclid Proposition I.4. Corresponding parts of congruent triangles are congruent, so line segment MP is congruent to line segment MB.

Since line segments MP and MB are congruent, triangle MBP is isosceles by Euclid Definition I.20. Since triangle MBP is isosceles, the base angles MBP and MPB are congruent by Euclid Proposition I.5. Recall that F is the foot of the perpendicular from M to chord BC. Thus angle MFB is congruent to angle MFP, because each is a right angle. Therefore, since line segments MB and MP are congruent, angles MBP and MPB are congruent, and angles MFB and MFP are congruent: triangles MFB and MFP are congruent by Euclid

Proposition I.26. Corresponding parts of congruent triangles are congruent, so line segments BF and FP are congruent.

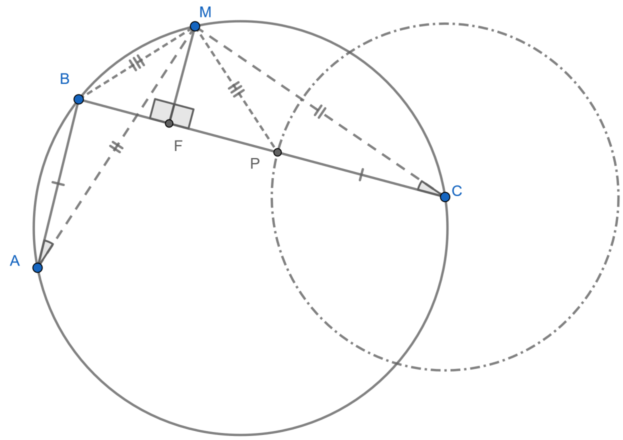


Figure 1: The Broken Chord.

Hence, line segments AB and PC are congruent, and line segments BF and FP are congruent. Notice that line segment FC is composed of line segments FP and PC. Then line segments AB and BF taken together are the same as line segments PC and FP taken together, which is the same as line segment FC. Thus, AB and BF taken together are congruent to FC, and F is the midpoint of the broken chord.

□

Construction of an Angle Bisector

Lauren Falck

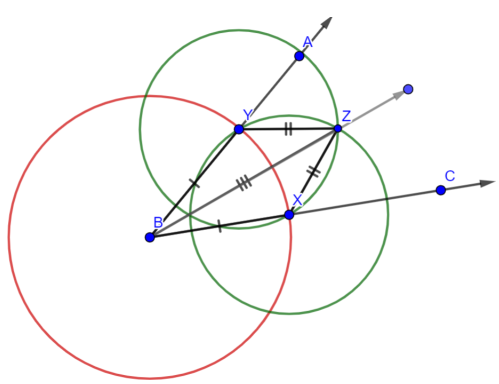
October 31, 2018

Communicated by: Ms. Wiegmann.

Conjecture Conjecture 11.1. Given an angle, construct the angle bisector.

Proof. By Euclid Proposition I.9, given angle ABC, place point X on ray BC. The first step is to construct a circle centered at B with radius BX. Point Y is created from the intersection of circle BX and ray BA. The second step is to create a circle centered at X with radius XY. The third step is create a circle centered at Y with radius YX. Point Z is created from the intersection of circle XY and circle YX. The final step of the construction is to create ray BZ.

Segment BX is congruent to segment BY because they both have the same radius of circle BX. Segment XZ is congruent to segment YZ because the have the same radius of circle XY. Segment BZ is a shared segment between triangle BYZ and triangle BXZ. By I.8, since segment BX is congruent to BY, segment XZ is congruent to YZ, and BZ is congruent to BZ, triangle BXZ is congruent to triangle BYZ. Since triangle BXZ is congruent to triangle BYZ, all of their corresponding angles are congruent. Since all of their corresponding angles are congruent, angle CBZ is congruent to angle ABZ. Since angle CBZ is congruent to angle ABZ, ray BZ is the angle bisector of angle ABC. □



bisector.png

Figure 1: This is a picture of Angle ABC and how it would look like to construct the angle bisector BZ.

Finding the Midpoint of a Segment

Lexis Wiegmann

November 1, 2018

Communicated by: Mr. Warner.

Question 11.2. Given a segment, find the midpoint. (Par 3)

Proof. 1. Make a circle with center A and radius AB.

2. Make a circle with center B and radius AB. Make the two intersection points of the two circles C and D.

3. By Euclid Postulate 1, connect CD. Where line CD intersects line AB is the midpoint of segment AB. By Euclid Postulate 1, connect lines AD, DB, BC, and CA. All of these lines are the radii of the same circle, so they are congruent to each other. Because all of the sides of this quadrilateral are congruent, this figure is a rhombus. From Ms. Miller's Lemma (Diagonals of a Rhombus), we know that the diagonals of this rhombus will bisect each other. Therefore, the midpoint is the intersection of the bisectors AB and CD.

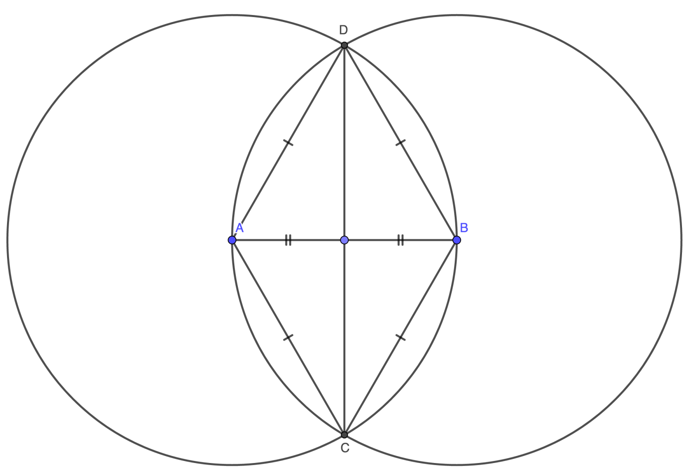


Figure 1: Segment CD intersects segment AB which is the midpoint of AB.

□

Construction of a Perpendicular Line Through a Point

Lauren Falck

November 12, 2018

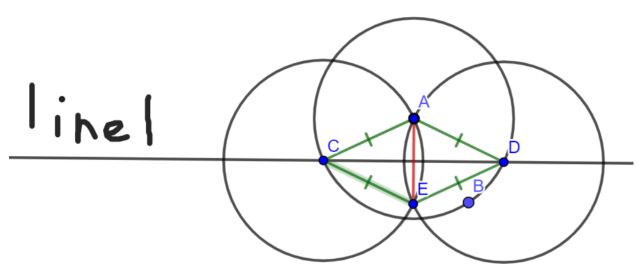
Communicated by: Mr. Stine.

Conjecture 11.3. Given a line l and a point A not lying on l, construct a line perpendicular to line l through A.

Proof. By Euclid I.12, Pick a point B where it lies on the other side of line l from point A.

1. Then construct a circle centered at A with radius AB. The intersection on line l and the circle creates points C and D.
2. Then construct a circle centered at C with radius CA.
3. Construct another circle centered at D with center DA. The intersection of the two circles creates point E.
4. Connect points A and E to form line segment AE, which is perpendicular to line l.

Since line segment CA is the radius for both circle A and circle C and line segment AD the radius for both circle A and D, circles A, C, and D are congruent. Therefore, the radi AC, AD, CE, and ED are congruent. A quadrilateral with congruent sides is called a rhombus. By Weigmann and Stine's Theorem 1.7 , we have proven that the diagonals of a rhombus meet at right angles. Therefore, line AE is perpendicular to line CD. □



Construction.png

Figure 1: Construction of a perpendicular line through a point off of the line.

Finding a Perpendicular Line

Lexis Wiegmann

November 9, 2018

Communicated by: Mr. Stine.

Question 11.4. Given a line l and a point A lying on l, construct a line perpendicular to l through A. (Par 4, possibly 3)

Proof. Here are the steps to construct a perpendicular line.

1. Make an arbitrary circle around point A. The two intersection points to line l, make them points B and C. The only restrictions this circle has to have is that it crosses the line l at two different points.
2. Make a circle centered at B with radius BC.
3. Make a circle centered at C with radius BC. Where the two circles with radius BC intersect, we get points D and F.
4. By Euclid Postulate 1, draw line DE.

By Euclid Postulate 1, make lines BD, DC, CE, and EB. Those line segments are all radii of the circles B and C. Because those line segments are congruent, this figure is a rhombus. From Ms. Miller's 1.2 conjecture (Diagonals of a Rhombus) lines DA, AE, BA, and AC are the diagonals of this rhombus. Also from Mr. Stine and Ms. Wiegmann's 1.7 theorem (Diagonals Form Right Angles) we know that this rhombus will form a right angle where the diagonals meet. Therefore, line DE through point A is perpendicular to line l. □

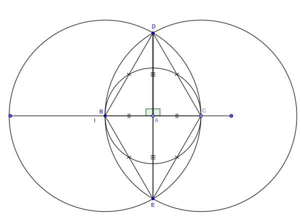


Figure 1: Line DE is perpendicular to line l through point A.

Center of a Circle Construction

Lauren Falck and Jaclyn Miller

November 13, 2018

Communicated by: Ms. DeVore.

Theorem 11.7. Given the circumference of a circle, find the center of the circle.

Proof. Let there be a circle, called circle M. To construct the center of the circle:

1. Pick two points A and B that lie on the circle and connect them.

2. Construct circle a circle centered at A with radius AB by Euclid Postulate I.2.

3. Construct circle a circle centered at B with radius AB by Euclid Postulate I.2.

4. The intersections of circle AB and circle BA creates points W and X. Construct a line through points W and X by Euclid Postulate I.1. This line, WX, is the perpendicular bisector of line segment AB by Euclid Proposition I.12 and Euclid Proposition I.10.

5. The intersection of circle M and line WX creates points C and D. Construct a circle centered at C with radius CD by Euclid Postulate I.2.

6. Construct a circle centered at D with radius CD by Euclid Postulate I.2.

7. The intersections of circle CD and circle DC creates points Y and X. Construct a line through points Y and Z by Euclid Postulate I.1. This line, YZ, is the perpendicular bisector of line segment AB by Euclid Proposition I.12 and Euclid Proposition I.10; and the intersection of circle M and line YZ creates points E and F. The intersection of line WX and line YZ creates point O, which is the center of circle M.

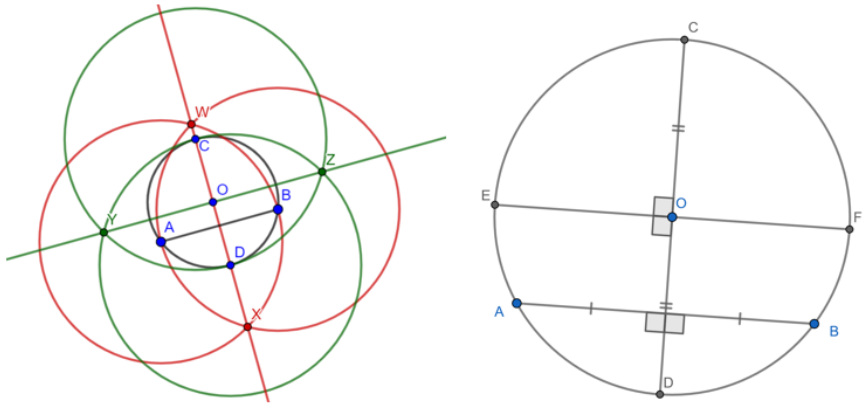


Figure 1: Left: Construction of Center of Circle M. Right: Proof that Point O is the Center.

Draw line segment CD by Euclid Postulate I.1, and notice that it is part of line WX. Since line CD is a perpendicular bisector of line segment AB, it cuts line segment AB into two congruent segments at right angles. Thus, by Euclid Proposition III.1, the center of circle M must be on line segment CD. Similarly, draw line segment EF by Euclid Postulate I.1, and notice that it is part of line YZ. Since line segment EF is a perpendicular bisector of line segment CD, it cuts line segment CD into two congruent segments at right angles. Thus, the center of circle M must be on line segment EF by Euclid Proposition III.1. Recall that the point of intersection of line CD and EF is point O. Then, since the center of the circle must be on line segments CD and EF, the center of the circle must be point O, because it is the intersection of line segments CD and EF, and thus on both segments. Thus, point O is the center of circle M. □

Tangent Line of a Circle

Lexis Wiegmann

November 15, 2018

Communicated by: Ms. Miller.

Question 11.8. Given a circle with center O, and given a point A outside the circle, construct a line l through A which is tangent to the circle. (Par 6)

Proof. Let there be a circle centered at O and let there be a point A outside of circle O.

1. By Euclid Postulate I.1, draw line segment OA.
2. By Euclid Postulate I.3 create a circle centered at point A with radius AO.
3. By Euclid Postulate I.3 create a circle centered at point O with radius AO. By the Circle Circle Intersection Theorem, we know two circles cut each other at two points, call those points B and C.
4. By Euclid Postulate I.1, draw line segment BC. From the intersection point of lines OA and BC, make point X. We know that this is a midpoint of OA from Ms. Wiegmann's construction 11.2 (Finding the Midpoint of a Segment).
5. By Euclid Postulate I.3 create a circle centered at X with radius XA. By Circle Circle Intersection Theorem, those two circles cut each other at two points, make one of those points D.
6. By Euclid Postulate I.1, draw line DA.

By Euclid Postulate I.1, draw line segment OD. Line segment OA is the diameter of the circle centered at X. Point D is an intersection of circle O and circle X, therefore point D is on the circle X. From Ms. DeVore's Theorem 7.4 we know that angle ODA is a right angle. Thus, DA is perpendicular to the radius of circle O, OD. Therefore, line AD is tangent to circle O. □

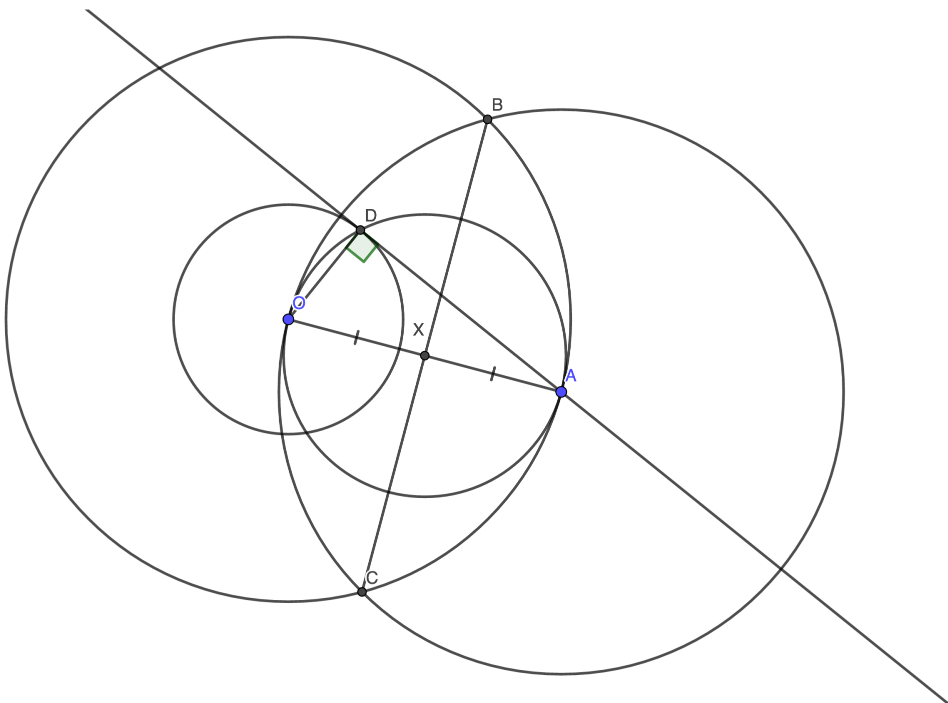


Figure 1: Line DA is tangent to circle with center O.