Equilateral Pentagons

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April 29, 2015

Theorem 6.4. Let ABCDE be an equilateral pentagon. If angle A is congruent to angle B, then ABCDE is not always regular.

Proof. 1. Draw a line and label it I. Pick a point on I and label it A.

2. Draw line M perpendicular through I at A.

3. Draw circle with center A and any radius. Choose one of the intersections of the circle through I and label it B. The intersection of circle AB and line M is point E.

4. Draw line N perpendicular to I through B.

5. Draw circle B with radius A. The intersection of circle BA and N is point C.

6. Draw circle with center C radius B.

7. Draw circle E with radius A. The intersection of circles CB and EA is point D. Connect segments CD and ED.

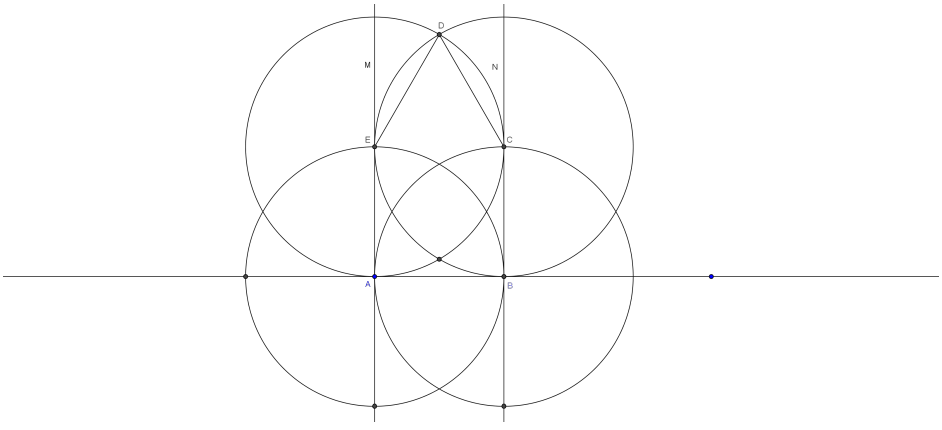

Figure 1: Equilateral pentagon ABCDE.

All the sides are congruent because all the circles used to construct the pentagon have the same radius. Angles EAB and CBA are right angles because of construction. Draw segments CE and AC. By Euclid's Proposition I.4, triangles ABC and CEA are congruent.

That means angles EAC and BCA are congruent by Euclid's Proposition I.29. Then angles ABC and CEA are both right angles. But the angle AED is composed of AEC and CED. So if part of angle AED is a right angle added to CED will be greater than a right angle. Then it's not congruent to ABC, therefore not equiangular or regular.

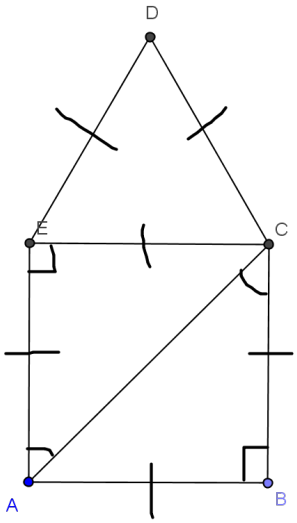

Figure 2: This shows the pentagon created when all the circles and lines are erased. It shows all the congruent sides and angles.

Refereed by Connor Schulte

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