Euclidean Geometry:
An Introduction to Mathematical Work

Math 3600

Spring 2015

Regular Figures, A Warm-up

A great part of the allure of geometry is figures with symmetry. Inspired by this, let us study some polygons that have a lot of symmetry.

Definition. A polygon is said to be *equilateral* if all of its sides are congruent, *equiangular* if all of its angles are congruent, and *regular* if it is both equilateral and equiangular.

6.1 Conjecture. An equilateral triangle is equiangular, hence regular.

6.2 Conjecture. Let $ABCD$ be a rhombus. If angle A is congruent to B, then $ABCD$ is regular.

Definition (reminder)**.** A regular quadrilateral is called a *square*.

6.3 Problem. Does Conjecture 6.2 hold if we replace "angle B" by "angle C" ? State a result and prove it.

6.4 Conjecture. Let $ABCDE$ be an equilateral pentagon. If angle A is congruent to angle B, then $ABCDE$ is regular.

6.5 Conjecture. Let $ABCDE$ be a regular pentagon. The triangle ACD is isosceles.

6.6 Problem. Let $ABCDE$ be a regular pentagon. State the relationship between the angles CAD and ACD that shows how special the triangle is. Prove your observation.

6.7 Problem. Find experimental evidence for the number of regular pentagons with a given side. (Try using five toothpicks!)