Angle Relationships In Circles II

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Theorem 10.1. Let Γ be a circle with center O. Let X be a point in the interior of the circle, and suppose that two lines ℓ and m intersect at X so that ℓ meets Γ at points A and A' and m meets Γ at B and B'. Then twice angle AXB is congruent to angle AOB and $A'OB'$.

Theorem 10.2. Let Γ be a circle with center O. Let X be a point in the exterior of the circle, and suppose that two lines ℓ and m intersect at X so that ℓ meets Γ at points A and A' and m meets Γ at B and B'. Then angle AOB and twice angle AXB taken together is congruent to angle $A'OB'$.

Proof. Let Γ be a circle with center O. Let X be the intersection of lines ℓ and m. Let ℓ meet Γ at A and A', and let m meet Γ at B and B'.

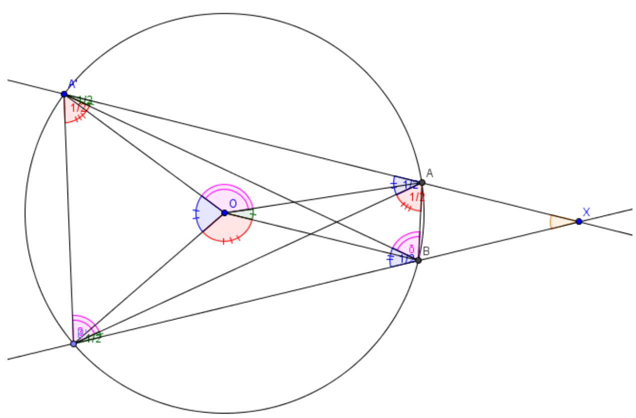

Figure 1: Circle Γ with two lines intersecting at X

By Euclid III.20, angle AOB is double of angle $AA'B$ and, and double of angle $AB'B$. Similarly, angle $A'OB'$ is double of angle $B'BA'$ and, and double of angle $B'AA'$. Similarly, angle $B'OB$ is double of angle $B'A'B$, and double of angle $B'AB$. Similarly, angle $A'OA'$ is double of angle $A'BA$, and double of angle $A'B'A$.

By Euclid I.32, the sum of the angles of triangle $A'XB'$ is two right angles. Therefore, angle $B'A'B$, angle $BA'A$, angle AXB, angle $AB'B$, and angle $A'B'A$ taken together is two right angles. Similarly, in triangle $A'BB'$, angle $A'BB'$, angle $AB'B$, angle $AB'A'$, and angle $B'A'B$ taken together is two right angles.

Therefore, angle AOB and twice angle AXB taken together is congruent to angle $A'OB'$.

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