Euclidean Geometry: An Introduction to Mathematical Work Math 3600, Fall 2014 23 October

Regular Figures, A Warm-up

A great part of the allure of geometry is figures with symmetry. Inspired by this, let us study some polygons that have a lot of symmetry.

Definition. A polygon is said to be *equilateral* if all of its sides are congruent, *equiangular* if all of its angles are congruent, and *regular* if it is both equilateral and equiangular.

- **6.1 Conjecture.** An equilateral triangle is equiangular, hence regular.
- **6.2 Conjecture.** Let ABCD be a rhombus. If angle A is congruent to B, then ABCD is regular.

Definition (reminder). A regular quadrilateral is called a *square*.

- **6.3 Problem.** Does Conjecture 6.2 hold if we replace "angle B" by "angle C"? State a result and prove it.
- **6.4 Conjecture.** Let ABCDE be an equilateral pentagon. If angle A is congruent to angle B, then ABCDE is regular.
- **6.5 Conjecture.** Let *ABCDE* be a regular pentagon. The triangle *ACD* is isosceles.
- **6.6 Problem.** Let *ABCDE* be a regular pentagon. State the relationship between the angles *CAD* and *ACD* that shows how special the triangle is. Prove your observation.
- **6.7 Problem.** Find experimental evidence for the number of regular pentagons with a given side. (Try using five toothpicks!)

