Error's in W.W Rouse Ball's Isosceles Triangle Argument

Ange Rehnstrom

April 30, 2015

Professor Ball's conjecture was that all triangles are isosceles. I have pin pointed his errors and will present them to you in this paper. You may want to begin by reading through his argument.

First create triangle ABC. Follow Professor Ball's construction to create points D and E. The author then supposes that E lies inside the triangle. Through observation by this construction E does not, and will not lie inside the triangle.

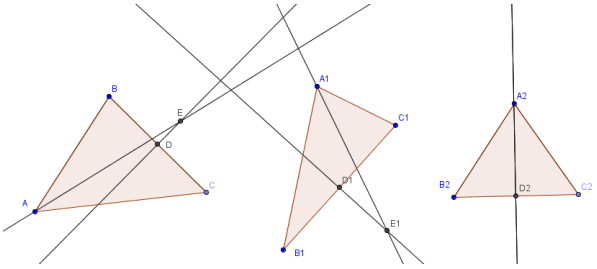

Figure 1: E will only land outside of the triangle, or coincide with point D. We will look at both of these cases

Now we will look at the two cases that are possible.

Case 1: Point E lies outside of the triangle. The author tells us to follow the same proof as was used for point E lying inside of the triangle and conclude that AB and AC are the differences of equals, therefore equal. We will start by following his construction and creating triangle ABC with points D and E. We then drop perpendiculars EF and EG onto the sides of the triangle. One of the perpendiculars will not touch the side of the triangle, so you will extend that side. In our figure side AC will be extended. You will also draw segments BE and CE. Please refer to Figure 2.

Professor Ball then states 'AEF and AEG have the side AE common and two angles congruent, so they are congruent by Euclid I.26 (AAS). Hence AF is congruent to AG and EF is congruent to EG. The triangles BDE and CDE have DE common, two other sides congruent, and the included right angles equal. Hence they are congruent by Euclid I.4 (SAS). In particular, BE is congruent to CE.

Now, the triangles BEF and CEG are right triangles with hypotenuses and a pair of legs congruent, so by Theorem on Hypotenuse-Leg for Right Triangles, they are congruent. Hence BF is congruent to CG.' All of which is correct.

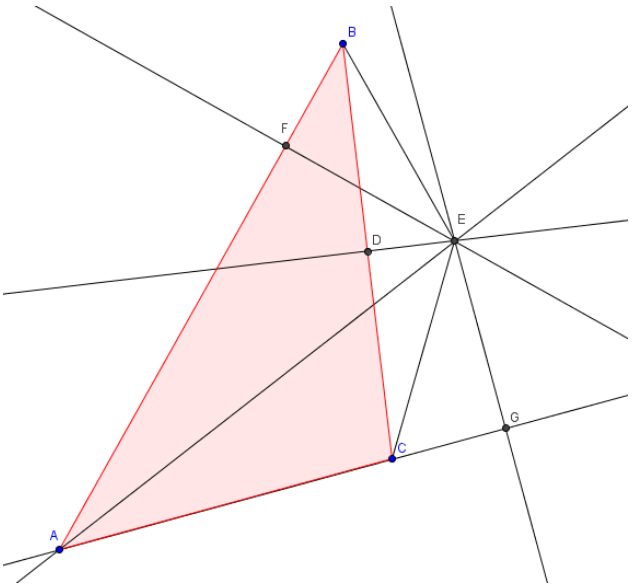

Figure 2: Professor Ball's Construction

From this he believes we should be able to conclude that AB and AC are the difference of equals. However, by looking at our figure we can see that the line segments you would be able to remove are BE and CE from the segments AB and AG. Leaving you with line segments AF and AC as a difference of equals. Therefore AB is not congruent to AC and the author's argument fails.

Case 2: Point E coincides with point D. The author states that this is an easy proof and leaves it to the reader to complete. Professor Ball's construction where point E coincides with point D is shown in figure 3.

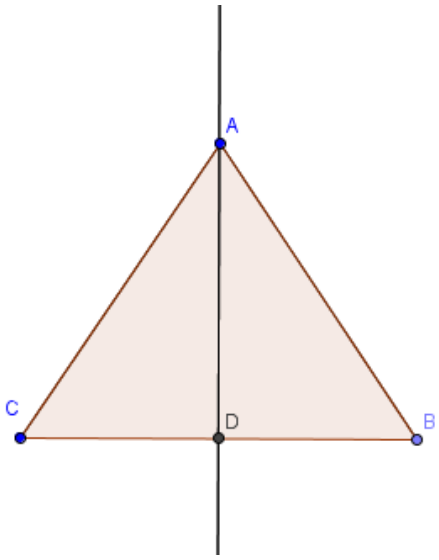

Figure 3: Point E coincides with point D

The triangles ADC and ADB share side AD. CD and BD are congruent because point D

is a midpoint of segment CB by construction. Angles ADC and ADB are both right angles because the line segment through point D is perpendicular to segment CB by construction. Therefore they are congruent by Euclid I.4 (SAS). I agree with the author that in this case the triangle is isosceles.

Refereed by: Toby Maggert