Incircle Construction

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Challenge 12.1. Construct a circle inscribed in a given triangle ABC.

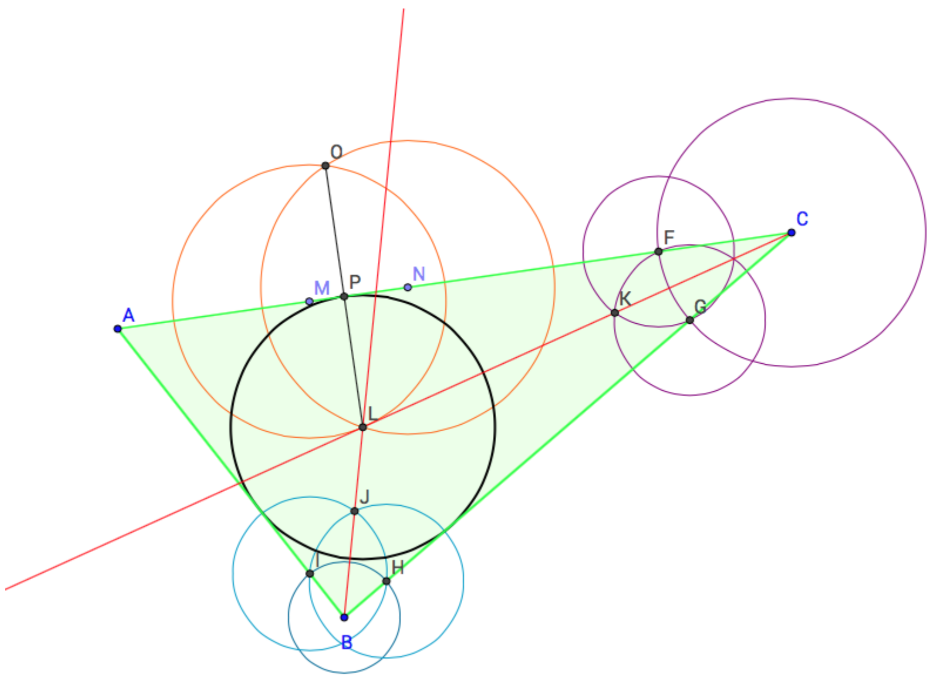

Figure 1: Construction of an incircle

1. Construct a circle around B so that the circle intersects with BC and BA. Label the points of intersection I and H.
2. Draw circle I through H.
3. Draw circle H through I.
4. Label one of the intersections of circles H and I as point J and draw ray BJ.
5. Draw a circle around C that intersects lines AC and BC. Label the points of intersection F and G.

6. Draw circle F through G.

7. Draw circle G through F.

8. Label one of the intersections of circles F and G as point K and draw ray CK.

9. Label the intersection of rays AH and BI as point L. Choose a point M on segment AC and draw a circle around M through L.

10. Pick a point N on AC and draw a circle around N through L.

11. Label the intersection of circles M and N as point O and draw line segment LO. Label the intersection of LO and AC as point P.

12. Draw a circle around L through P. This is the incircle.

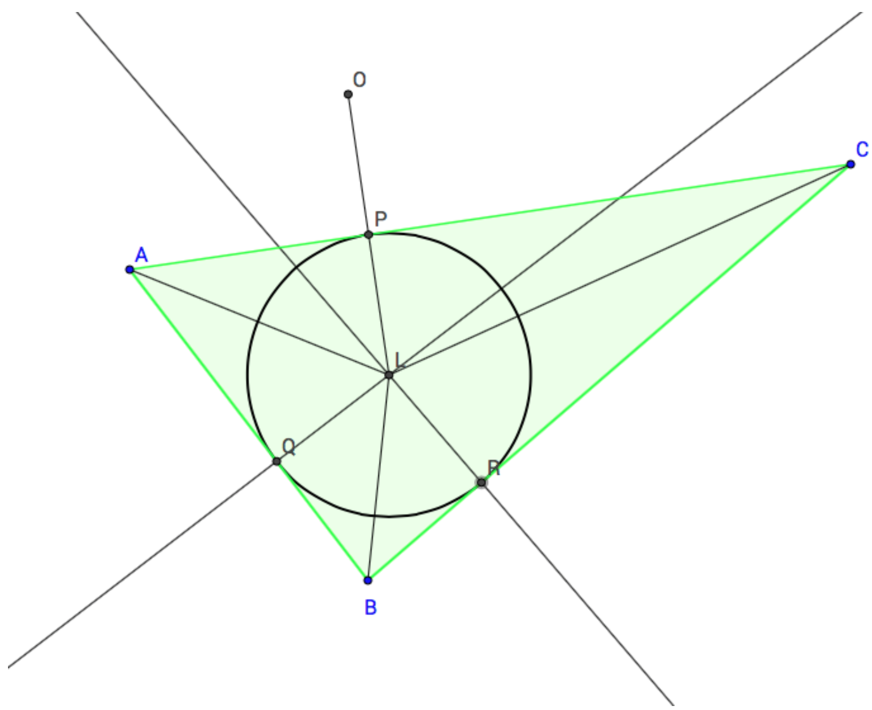

Figure 2: An incircle

Proof. Only one side of the triangle (AC) was used to construct the circle L through P. To truly be the incircle, this circle must also be tangent to AB and BC. We can construct a line through point L perpendicular to AB (meeting AB at point Q) and through point L perpendicular to BC (meeting BC at point R).
Consider the triangles LPC and LRC. LPC and LRC share side LC. Since PL is perpendicular to AC and LR is perpendicular to BC, angle CPL and angle CRL are congruent. Since CL is the angle bisector of ACB, angles PCL and RCL are congruent. Then by Angle Angle Side, triangle LPC is congruent to triangle LRC. Thus, LR is congruent to PL and is a radius of

the circle L through P.
Similarly, consider triangles PLA and QLA. Since LP and LQ are perpendicular to AC and AB respectively, AQL and APL are right angles. Since AL bisects angle CAB, PAL and QAL are congruent. AL is a shared side, so by AAS PLA and QLA are congruent. Thus, QL is congruent to PL and is also a radius of the circle.
Since all three segments perpendicular to the triangle's sides are radii of the circle around point L, this is a valid construction of the incircle. □

Refereed by Sladana Bulic