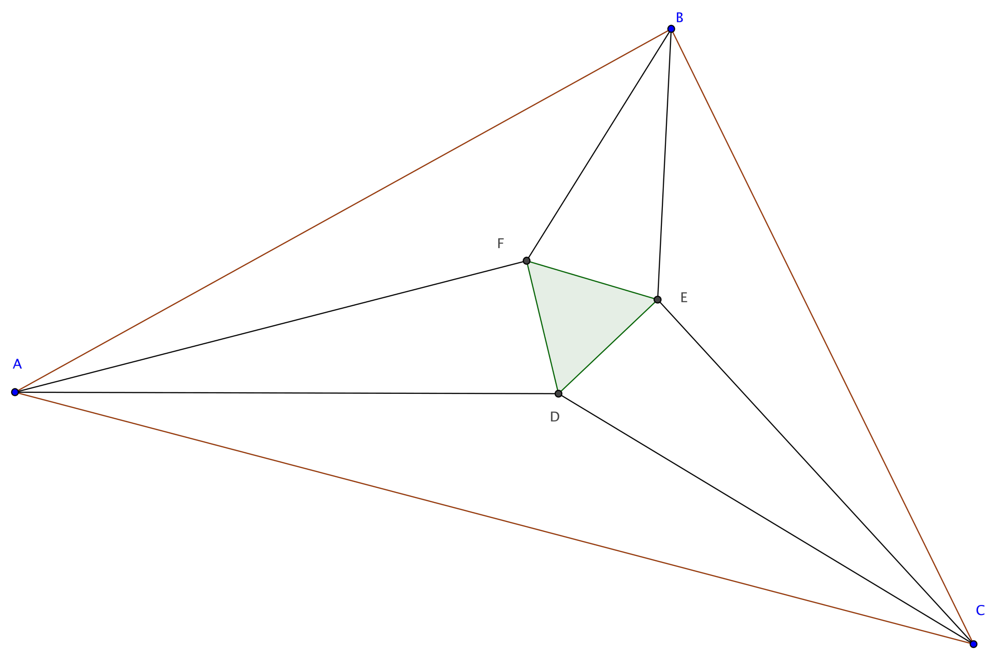
Transactions in **Euclidean Geometry**



Issue # 3

Angle Congruency and Squares

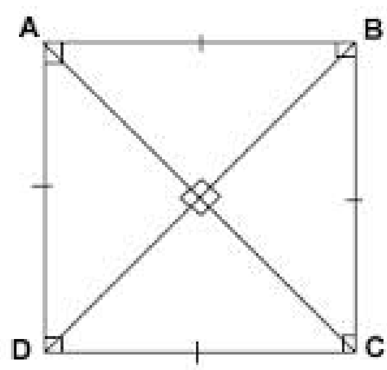
Kevin Walters, Thomas Bieber, Nathan Opheim, and Timothy Nieman

September 25, 2014

In class we discussed the perameters that were involved in order for certain angles in a rhombus ABCD to be congruent. The following article will discuss what happens to a rhombus when angle BAC is congruent with angle BDC.

Theorem A. When given a rhombus ABCD, if angle BAC is congruent to angle BDC, then ABCD is a square.

Proof. By the Benson, Herbst, Nieman, Westervelt theorem we know that if angle BAC is greater than angle BDC then segment BD is greater than segment AC and also if angle BDC is greater than angle BAC then segment ACis greater than segment BD.We also know by Fisher's theorem that if angle BAC is congruent to angle BDC then segment AC is congruent to segment BD. Using this information we can say that the only way for angle BAC to be congruent to angle BDC is if segment AC is congruent to segment BD. By definition of a rhombus we also know that AB, BC, CD, and DA are all mutually congruent.With this knowledge we can use propostion 8 to show that angle DAB is congruent to angle BCD and also that angle ABC is congruent to angle CDA. Because we know the diagonals are congruent we can use proposition 8 to show that angle ABC is congruent to angle BCD and angle CDA is congruent to angle DAB. Since angle ABC is congruent to angle CDA and angle DAB is congruent to angle BCD we can say that angle ABC is congruent to angle CDA which is congruent to angle DAB which is congruent to angle BCD. Since all four angles are now congruent we can use theorem E to show that the angles must be right. Because all four angles are right, by definition of a square, rhombus ABCD is a square. Therefore, whenever angle BAC is congruent to angle BDC, rhombus ABCD must be a square. □


A
B
D
C

Some Kites are Not Parallelograms

Emily Herbst

October 22, 2014

Theorem 2.4. If ABCD is a kite but not a rhombus, then ABCD is not a parallelogram.

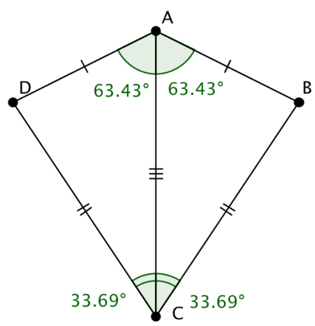


Figure 1: Kite ABCD

Proof. Let ABCD be a kite. Then line AD is congruent to line AB and line CD which is congruent to line CB. Assume that line AD and line AB are not congruent to line CD and line CB. Let there be the line AC. Since line AD is congruent to line AB, line CD is congruent to line CB, and line AC is congruent to line AC, then by Euclid Proposition I.8, triangle ADC is congruent to triangle ABC. Then angle ABC is congruent to angle ADC, angle CAB is congruent to angle CAD, and angle ACB is congruent to angle ACD.

Assume that line AB and line CB are the sides of a triangle that are not the base. Since line AB is not congruent to line CB then by Euclid Proppposition I.5, angle CAB is not congruent to angle ACB. Similarly for triangle ADC, since line AD is not congruent to line CD, then by Euclid Proposition I.5, angle CAD is not congruent to ange ACD. Since angle ACB is congruent to angle ACD and angle CAB is not congruent to angle ACB, then angle CAB is not congruent to angle ACD.

Since angle ACB is congruent to angle ACD and angle CAD is not congruent to angle ACD, then angle CAD is not congruent to angle ACB. Then by Euclid Proposition I.29, since line AC cuts through line AB and line CD and angle CAB is not congruent to angle ACD and angle CAD is not congruent to angle ACB, line AB is not parrallel to line CD. Also by Euclid Proposition I.29, since line AC cuts through line AD and line CB and angle CAB is not congruent to angle ACD and angle CAD is not congruent to angle ACB, line AD is not parrallel to line CB. Since line AB is not parallel to line CD and line AD is not parallel to line CB, then the kite ABCD is not a parallelogram. □

Theorem 3.7

Diann Herington

October 20, 2014

Theorem 3.7. Let $ABCD$ be a quadrilateral. The midpoints of the four sides are the vertices of a parallelogram.

Proof. Since the diagonal AC forms triangle ABC, line segment EF is parallel to line segment AC by Mr. Bieber's theorem 3.6. Similarly, with triangle ADC, segment HG is parallel to segment AC. Therefore, EF is parallel to HG by Euclid I.30.
Since the diagonal BD forms triangle DAB, segment HE is parallel to segment DB by Theorem 3.6. Similarly, with triangle DCB, segment GF is parallel to segment DB. Therefore, segment HE is parallel to segment GF by Euclid I.30.
So, since segment EF is parallel to segment HG and segment HE is parallel to segment GF, by definition, $EFGH$ is a parallelogram. □

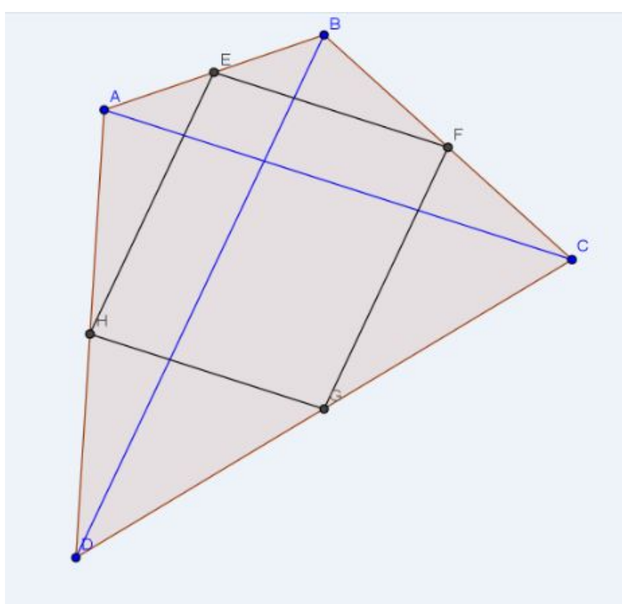


Figure 1: Quadrilateral ABCD with midpoints E, F, G, and H

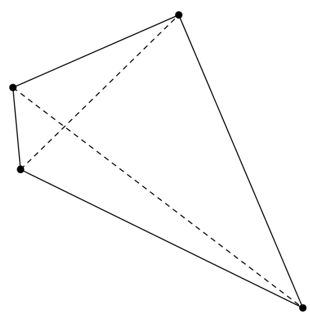
Convex and Not Convex Quadrilaterals

Megan Westervelt

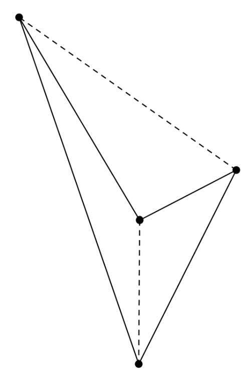
October 6, 2014

Definition Convex Quadrilateral. A *convex quadrilateral* is one in which the diagonals of the quadrilateral cross. A *not convex quadrilateral* is one in which the diagonals do not cross.

Below are some examples of simple convex and simple not convex quadrilaterals.



(a) Convex



(b) Not Convex

Figure 1: Simple Quadrilaterals

Also notice that this defintion applies to complex quadrilaterals as well.

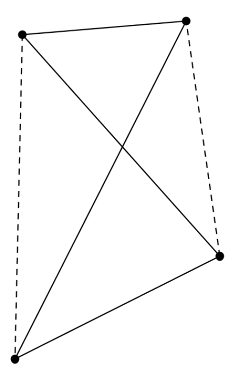


Figure 2: Complex Not Convex Quadrilateral

DEFINITION OF THE "INTERIOR" OF A POLYGON

JOHN FISHER

Communicated by Eric Scheidecker

Definition . Let P be a polygon and let X be a point not lying on one of the sides of P. Choose a ray r, which does not meet a vertex of P, with initial point X. Let n be the number of sides of P which meet r. If n is odd, we say that X is inside of P. If n is even, we say X is outside of P.

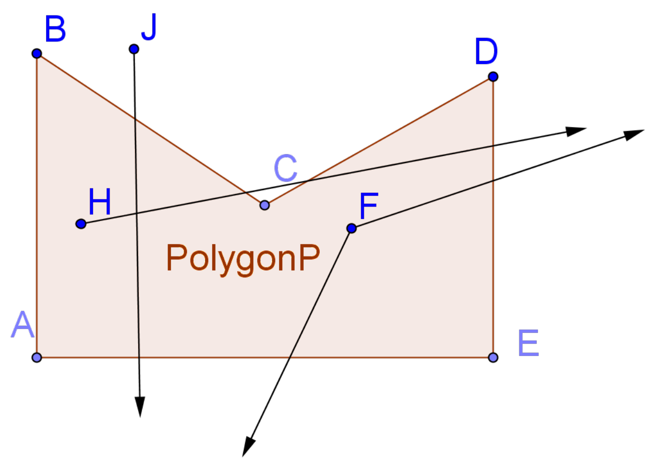


FIGURE 1. Polygon P, with vertices ABCDE. Notice that none of the rays extending from point F,H, and J meet vertices. J intersects an even number of sides, thus J is outside of P. F and H intersect an odd number of sides, thus F and H are on the interior of P.

EXTERIOR ANGLES OF ANY POLYGON ARE EQUAL TO FOUR RIGHT ANGLES

KATY GOODMUNDSON

Communicated by Joellen Hatchett

Question 5.3. What is the sum of the exterior angles a Hexagon? What is the sum of the exterior angles of a general n-gon?

Proof. First, we will begin by proving that the exterior angles of any polygon will sum to four right angles. Suppose we have a polygon with n-vertices. Then choose one of those vertices on the polygon. Let T = the number of triangles that can be formed inside the polygon and let n = the number of sides of the polygon. From Conjecture T, proved by Mr. Hawkins, we can divide any n-gon into (n-2) non-overlapping triangles by inserting diagonals. From there, we can extend all the sides of the polygon. There are now two exterior angles formed on each of these vertices. Of those exterior angles, we will choose the smaller angle of each and call each of them Theta. Since we know T = number of triangles inside the polygon and n = the number of sides of the polygon, we can say that the number of right angles that make up the sum of the exterior angles of any given polygon is E = (2*(n-T)), or four right angles.

Now we will show how this works for a Hexagon.

Suppose we have a Hexagon with vertices A, B, C, D, E, and F. Choose point A to be the vertex that will be used. Draw diagonals AC, AD, and AE. With the diagonals, there are four triangles inscribed in this hexagon. By Euclid Proposition I.32, there are a total of two right angles in each triangle. Therefore, the interior angles of this Hexagon sum to eight right angles. Now, extend AB, BC, CD, DE, EF, and FA. There are now two exterior angles formed on each of these vertices. Of those exterior angles that are formed, we will choose the smaller angle of each and call each of them Theta. The sum of the exterior angles of this Hexagon is E = (2*(n-T)). In our case, T = 4 and n = 6. So the total sum of the exterior angles will be: E = 2(*(6-4)) = (2*2) = 4 Right Angles.

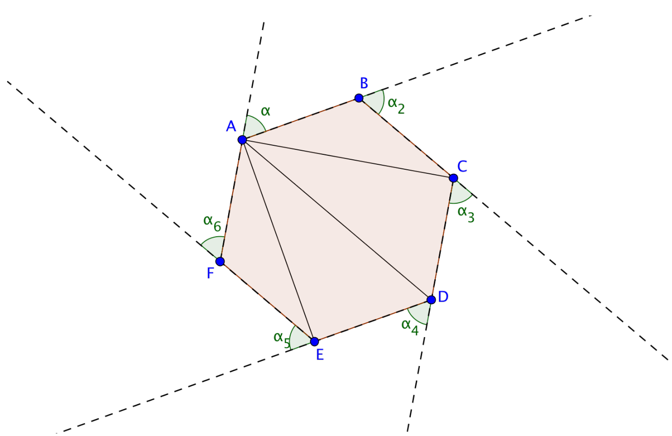


FIGURE 1. Hexagon ABCDEF

Date: October 30, 2014.

Theorem 5.3. Let P be a polygon with n sides which can be divided up into T triangles by using diagonals. Then the exterior angles of P taken together make (2*(n-T)) or four right angles.

□

A RHOMBUS IS REGULAR IF ANGLE A AND ANGLE B ARE CONGRUENT.

KAYLEE BENSON

Communicated by Emily Herbst

Theorem 6.2. Let ABCD be a rhombus. If angle DAB is congruent to angle ABC, then ABCD is regular.

Proof. Let BD be joined. By the definition of a rhombus, line AB is congruent to line BC which is congruent to line AD and also congruent to line CD. By Euclid proposition I.8, we know that triangle ABD is congruent to triangle CDB. So by corresponding parts of congruent triangles, angle DAB is congruent to angle BCA, angle CBD is congruent to angle ADB and angle ABD is congruent to angle CDB. So therefore angle ADC is congruent to angle CBA. Since angles DAB and BCD are congruent, angles ABC and BCD are congruent and we were given that angles DAB and ABC were congruent then the rhombus is equiangular. Therefore ABCD is both equiangular and equilateral hence regular.

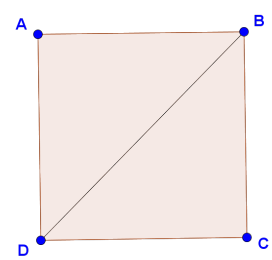


Figure 1. Rhombus ABCD

$\square$

EXPLORING ANGLES OF A RHOMBUS

EMILY HERBST

Communicated by John Fisher

Theorem 6.3. Let ABCD be a rhombus. If angle A is congruent to angle C and angle A and C are right angles, then ABCD is regular. If angle A is congruent to angle C and angle A and C are not right angles, then ABCD is not regular.

Proof. Let ABCD be a rhombus. By definition of a rhombus, sides AB, BC, CD, and DA are congruent. Let there be the line BD. Since line AB is congruent to line CD, line AD is congruent to line CB, and line BD is congruent to line BD, then triangle DAB is congruent to triangle BCD by Euclid Proposition I.8. Since triangle DAB is congruent to triangle BCD, then angle DAB is congruent to angle BCD, angle ABD is congruent to angle CDB, and angle ADB is congruent to angle CBD. Since line AB is congruent to line AD and line CB is congruent to line CD, then both triangle DAB and triangle BCD are isosceles triangles. By Euclid Proposition I.5, since triangle DAB is an isosceles triangle, angle ABD is congruent to angle ADB. By Euclid Proposition I.5, since triangle DCB is an isosceles triangle, angle CBD is congruent to angle CDB. Since angle ABD is congruent to angle CDB, angle ADB is congruent to angle CBD, angle ABD is congruent to angle ADB, and angle CBD is congruent to angle CDB, then angle ABD is congruent to angle CBD which is congruent to angle CDB which is congruent to angle ADB.

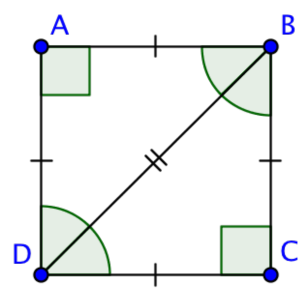


Figure 1. Rhombus ABCD with angle A and angle C as right angles.

Case 1: Suppose angle A is congruent to angle C such that they are right angles. By Euclid Proposition I.32, triangle DAB has two right angles and triangle BCD also has two right angles. Then for triangle DAB, angle ABD and angle ADB would sum to equal one right angle. Then for triangle DCB, angle CDB and angle CBD would sum to equal one right angle. Since angle ABD is congruent to angle CBD which is congruent to angle CDB which is congruent to angle ADB, angle ABD and angle ADB sum to one right angle, and angle CDB and angle CBD sum to one right angle, then angle ABD and angle CBD would sum to equal one right angle called angle B and angle ADB and angle CDB would sum to equal one right angle called angle D. Since angle A and angle C are right angles and angle B and angle D are right angles, then ABCD is equiangular and thus regular.

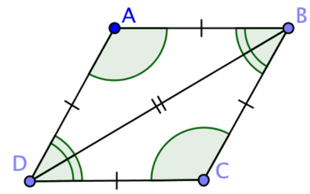


Figure 2. Rhombus ABCD with angle A and angle C as obtuse angles.

Case 2: Suppose angle A is congruent to angle C such that they are obtuse angles. By Euclid Proposition I.32, triangle DAB has two right angles and triangle BCD also has two right angles. Then for triangle DAB, angle ABD and angle ADB would sum to less than one right angle. Then for triangle DCB, angle CDB and angle CBD would sum to less than one right angle. Since angle ABD is congruent to angle CBD which is congruent to angle CDB which is congruent to angle ADB, angle ABD and angle ADB sum to less than one right angle, and angle CDB and angle CBD sum to less than one right angle, then angle ABD and angle CBD would sum to less than one right angle called angle B and angle ADB and angle CDB would sum to less than one right angle called angle D. Since angle A and angle C are right angles and angle B and angle D are less than one right angle (or acute), then ABCD is not equiangular and thus not regular.

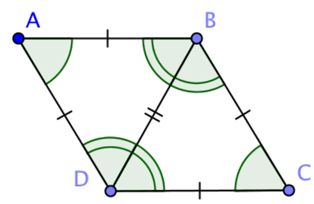


Figure 3. Rhombus ABCD with angle A and angle C as acute angles.

Case 3: Suppose angle A is congruent to angle C such that they are acute angles. By Euclid Proposition I.32, triangle DAB has two right angles and triangle BCD also has two right angles. Then for triangle DAB, angle ABD and angle ADB would sum to more than one right angle. Then for triangle DCB, angle CDB and angle CBD would sum to more than one right angle. Since angle ABD is congruent to angle CBD which is congruent to angle CDB which is congruent to angle ADB, angle ABD and angle ADB sum to more than one right angle, and angle CDB and angle CBD sum to more than one right angle, then angle ABD and angle CBD would sum to more than one right angle called angle B and angle ADB and angle CDB would sum to more than one right angle called angle D. Since angle A and angle C are right angles and angle B and angle D are more than one right angle (or obtuse), then ABCD is not equiangular and thus not regular. □

ISOSCELES TRIANGLE IN A REGULAR PENTAGON

ASHLEY STUFFELBEAM

Communicated by Megan Westervelt

Theorem 6.5. Let ABCDE be a regular pentagon. The triangle ACD is isosceles.

Proof. Begin by constructing the regular pentagon ABCDE. Next, draw the line segments AC and AD. Since the pentagon is regular, by definition we know that all sides are congruent and all angles are congruent. Because of this, we can say segment AE is congruent to segment AB, segment DE is congruent to segment CB, and angle ABC is congruent to angle AED. Therefore, by Euclid Proposition I.4 we can conclude triangle ACB is congruent to triangle ADE. Since these triangles are congruent, we can also say segment AC is congruent to segment AD. Since these two segments are congruent, we can then conclude that triangle ACD is isosceles using Euclid Definition 20.
Note: Triangle ACD can be equilateral, because an equilateral triangle is also isosceles since it still has two sides congruent, and triangle ACD satisfies this condition as proven above. However, it does not have to be equilateral, as we only need to show that it has two congruent sides.

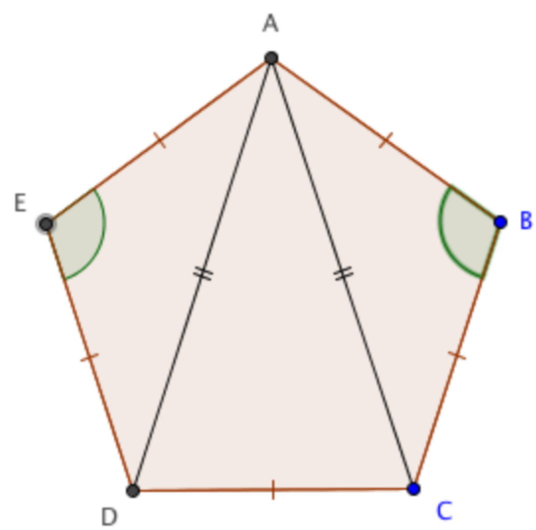


Figure 1. Regular Pentagon ABCDE

□