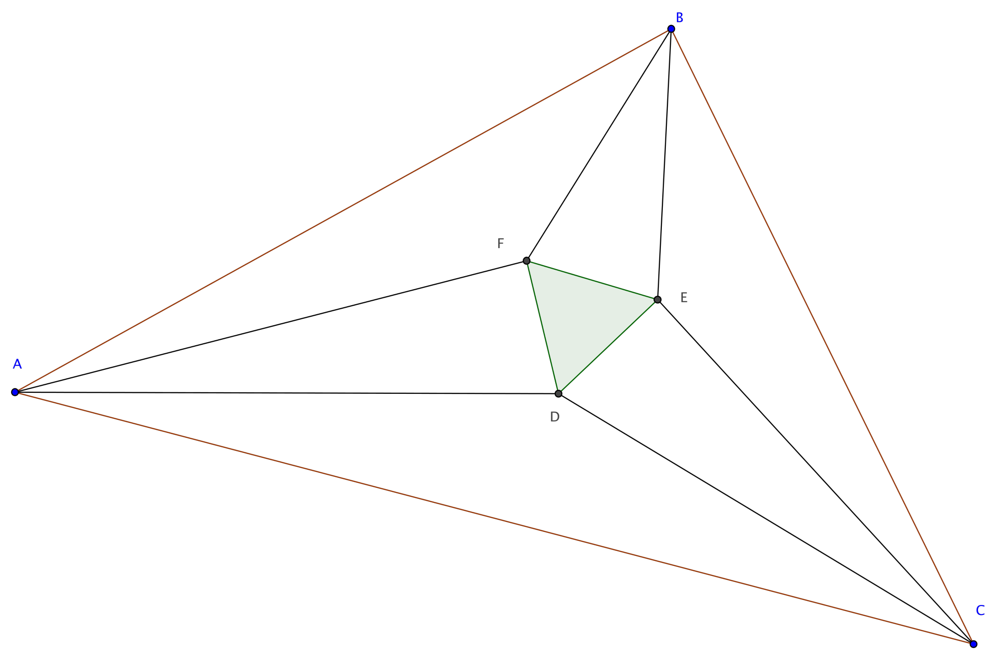
Transactions
in
Euclidean Geometry



Volume 2018F

Issue # 7

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Pairs of Opposite Angles in a Kite are Congruent

Brad Warner

December 7, 2018

Communicated by: The Editor

Theorem 2.1A. Let ADCB be a Kite, such that AB is congruent to BC and AD is congruent to DC. Then angle A is congruent to angle C

Proof. Since ADCB is a kite, the kite has adjacent congruent sides. Therefore, side AB is congruent to side BC, side AD is congruent to side DC, and by Postulate 1 side BD is common. Consider triangle ABD and triangle CBD. Since side AB is congruent to side BC, side AD is congruent to side DC, and side BD is common. Therefore, by deVore Theorem 1.1 triangle ABD is congruent to triangle CBD. Thus, angle A is congruent to angle C. Since angle A is congruent to angle C then there exist a pair of opposite angles in the kite ADCB. Thus, there exist an opposite angles of ADCB are that congruent to one another.

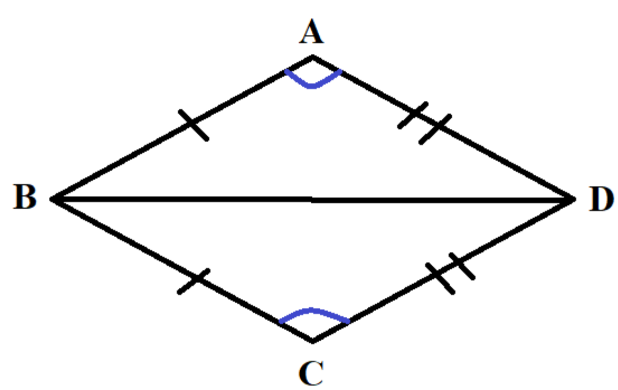

Figure 1: Rectangle ADCB

□

Finding the Parallel Line

Jimmy Pham

December 8, 2018

Communicated by:Jason Stine

Conjecture 11.6. Construct a tangent line to a given circle through a point A on that circle B.

Proof. Here are the steps to construct a tangent line.

1. Let point B be the center of circle B.

2. Create a circle using point A as the center and use the radius AB to create point C using the intersections.

3. Create a circle using point C that uses the radius CB and CA.

4. Use the radius of BC the make the diameter of circle C the create point D

5. Use point D the create DA and extend the line to make it a tangent line.

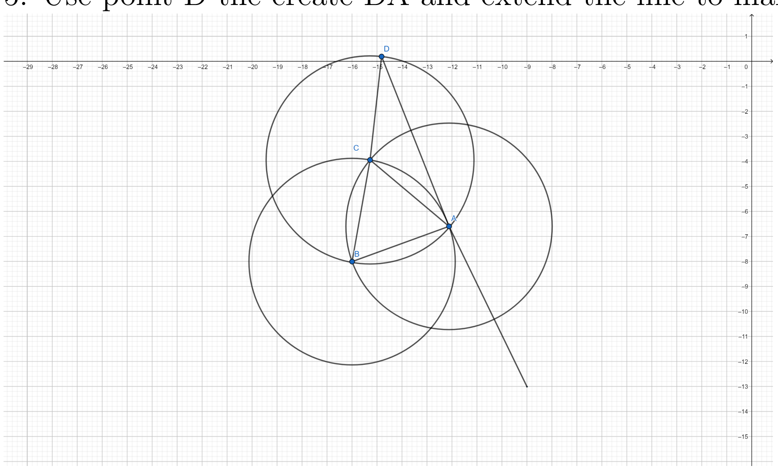

Figure 1: This is a picture of construction.

By Euclid Postulate 3, make a circle with the radius AB and use book 1 proposition 1 to find point C. Using Euclid Postulate 3 again to create a circle with point using the radius CB and CA. Now using Euclid Postulate 2, using radius CB to make the diameter in circle C to get point D. Also from Euclid 1, draw a line DA and using Euclid Postulate 2 to make it a tangent line. In book 3 proposition 36 uses point D to make a tangent line with point A.

Now using Theorem 7.4, the angle A would be a right angle which makes BA and line DA perpendicular and using book 3 proposition 18 to show that line DA is a tangent line. $\square$

Finding the Parallel Line

Jimmy Pham

December 8, 2018

Communicated by:Jason Stine

Theorem 11.6. Given a line l and a point A noting laying on l, construct a line parallel to l which passes through A.

Proof. Here are the steps to construct a parallel line.

1-4. Construct a perpendicular line to l through point A to create a new point called B.

5. Make a semi-circle using point B with the radius AB.

6. Draw a line through point A that is perpendicular to AB.

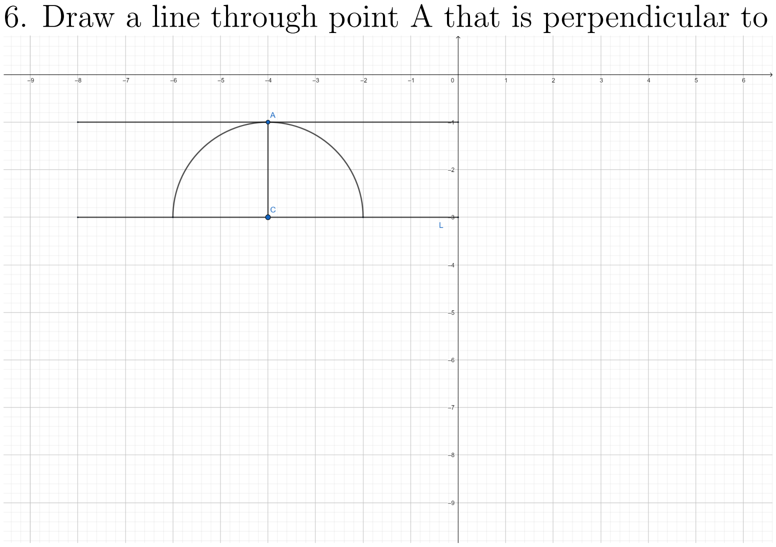

Figure 1: This is a picture of construction.

From Theorem 11.4 that Ms. Wiegmann did, make a perpendicular line that connects point A and line l to create a new point B. By Euclid Postulate 3, make a semi-circle with the radius AB. Also from Euclid postulate 2, draw a line that is perpendicular to AB through the point A. Using book 3 proposition 18 to identify that the line going through point A is a tangent line. Using book 3 proposition 19, if a straight line touch a circle, and from the point of contact a straight line be drawn at right angle to the tangent, the centre of the circle will be on the straight line so drawn and proposition 28 in book 1 make this construction true.

Also Ms. DeVore's conjecture I also proves that it is parallel. If we can prove conjecture R on creating a tangent line through a point on a circle than this proof is true. $\square$

Squares Inscribed in a Triangle

Payson VandeLune and Lexis Wiegmann

December 9, 2018

Communicated by: Mr. Stine.

Lemma 1. Let ABC be an isosceles triangle with line segments AB congruent to BC. The angle bisector of angle ABC bisects line segment AC.

Proof. Let ABC be an isosceles right triangle with line segments AB and BC congruent. Draw in the angle bisector BX, the point where the bisector meets line segment AC, let that be X. By Euclid Proposition I.4 (SAS), triangles ABX and XBC are congruent. Because of this, line segments AX and XC are congruent, making the angle bisector the bisector of line segment AC. □

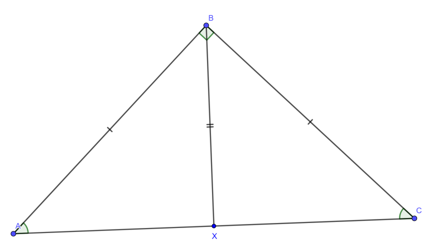

Figure 1: Line segment BX bisects line segment AC.

Lemma 2. Let ABC be a triangle with line segments AB congruent to BC and angle B right. Let points X, Y, and Z be the midpoints of line segments AB, BC, and CA respectively. Then XBYZ is a square.

Proof. Let ABC be an isosceles right triangle with line segments AB and BC be congruent and angle ABC is right. Let points X, Y, and Z be the midpoints of line segments AB, BC and CA respectfully. By Euclid Postulate 1, draw line segment BZ, which is the angle bisector of ABC. By Euclid Proposition I.4 (SAS), triangles XBZ and YBZ are congruent. By Theorem 3.6 (Midline Theorem) line segment XZ is parallel to line segment BC. By Euclid Proposition I.29, angle ZBY is congruent to angle BZX. But, since line segment BZ is the angle bisector and triangles XBZ and YBZ are congruent, then these triangles are isosceles and angle XZY is a right angle. Since line segment XZ is parallel to BC, angle BXZ is also right by Euclid Proposition I.29. Similarly, angle BYZ is right. So, XBYZ has four congruent sides and four right angles. Therefore, XBYZ is a square. □

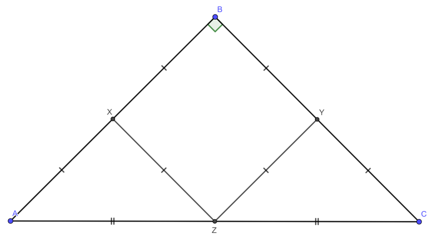

Figure 2: Square XBYZ.

Theorem 13.5. There are two ways to inscribe a square in an isosceles right triangle. Which one has the greater content?

Proof. Let ABC be an isosceles right triangle with line segments AB and BC congruent and angle B be right. From lemma 2, we know that XBYZ is a square. Because it is a square, we know that line segments XB, BY, YZ, and ZX are congruent. We also know that angles XBY, BYZ, YZX, and ZXB are congruent. Line segments AZ and ZC are congruent because of the midpoint. Line segments AX and XB are congruent as well as BY and YC. By Euclid Proposition I.8 (SSS), triangles AXZ and ZYC are congruent. Making the corresponding angles congruent as well. By Euclid Postulate 1, create line segment XY. By Euclid Proposition I.8 (SSS), triangles XBY is congruent to XZY. Because AB and BC are congruent the line segments AX and XZ are congruent as well. They are all the sides of that square. By Euclid Proposition I.4 (SAS), triangles AXZ and XZY are congruent. Without loss of generality, triangles AXZ, XZY, XBY, and ZYC are all congruent because of Euclid Proposition I.4 (SAS). Both triangles AXZ and ZYC fit in the square XBYZ perfectly, with no space left over.

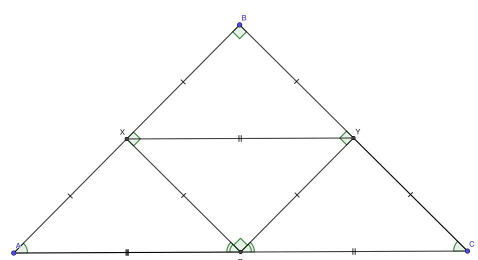

Figure 3: Triangle ABC where the square is half of the triangle.

Let ABC be an isosceles right triangle with line segments AB and BC congruent and angle B be right. By the definition of isosceles, we know that angles BAC and BCA are congruent. By lemma 1, we know that line segment AC is split up into thirds. Therefore, line segments AW, WZ, and ZC are congruent. By the definition of a square, WZ, ZY, WY, and XW are congruent as well. By Euclid Proposition I.13, we know that angles AWX and YZC are right angles. By Euclid Proposition I.4 (SAS), triangle AWX is congruent to triangle CZY. The corresponding side and angles are congruent as well. Also, by Euclid Proposition I.4 (SAS), we know triangles WXZ and XYZ are congruent as well as AWX is congruent to XWZ. This makes triangles AXW, WXZ, XZY, and ZCY all congruent triangles. Other than the top triangle, we are going to cut each of the triangles in half. This means that each leg of the bottom eight triangles will be a third of the length of line segment AC. Now we have nine congruent triangles within the original isosceles right triangle. The two triangles in triangle AWX and the two triangles within triangle ZYC will all fit in the square WXYZ perfectly because they are all congruent triangles. We are left with one triangle left over, namely, triangle XBY. So, out of the 9 triangles we have within the first isosceles right triangle, we can make up two of the same size squares.

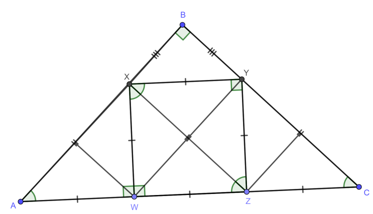

Figure 4: Triangle ABC where the square is four ninths of the triangle.

In the first construction, we didn't have any triangles left over. The square was half of the original triangle. In the second construction, we had triangle XBY left over. So, the square was four ninths of the original triangle. Because of this reason the first construction has greater content than the second construction. □