Math 3600: Euclidean Geometry

Mondays, Wednesdays, & Fridays 2:00-2:50pm, Wright Hall 08

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Office Hours:

M, T, Th, & F 3:00-5:00 pm.
Please use this to schedule:
https://theronhitchman.youcanbook.me/
you can email me for other appointments

"A good stack of examples, as large as possible, is indispensable for a thorough understanding of any concept, and when I want to learn something new, I make it my first job to build one."
--- Paul Halmos

A bit about the course

You have certainly studied some geometry before, so the basic objects of this course will be familiar to you. We will run across triangles, rectangles, and other polygons, and circles, too. We will study congruence of figures and an early concept of area. We will construct diagrams using the classical tools of compass and straightedge. (It's really *a pair of compasses*, like a pair of pants, but nevermind that for now.)

The subject is ancient (some of it dates back to Thales of Miletus in *circa* 600 B.C.E.), so it may surprise you that the approach is very modern: Euclid put the subject together in a way that we now call *the axiomatic method*. This is our deepest mathematical heritage, and is essentially unchanged in the last 2500 years. Mathematicians still use the axiomatic method, and it is what distinguishes us from other fields of academic work. This course is partly another chance for you to learn how to work this way, by writing arguments we call "proofs."

But the real work of our class is *learning how to do mathematics independently*. Yes, we will find and make proofs, but we will also have to learn what a mathematician means by the words *definition* and *conjecture*, and figure out how to make those things, too.

By the way, the official catalog description for this course is a lie, and it has been for about ten years. (I put it below.) This course will be a *first course* in planar geometry, but from an *advanced perspective*. We will start from the bottom, and build our stairs as we climb up.

MATH 3600/5600 (800:166g). Euclidean Geometry - 3 hrs.

Topics of plane geometry beyond a first course; compass and straightedge constructions, the nine-point circle, Ceva's and Menelaus' theorems, triangle centers, conics, and tessellations. Prerequisite(s): <u>MATH 1420 (800:060)</u> or equivalent; junior standing. (Fall and Spring)

What you will learn

This course is about *process* as much as it is *content*. That is, it is more important that you learn how to think and work like a mathematician does than that you learn lots of geometry theorems. But you will learn some geometry theorems! Some will be familiar, and you will get a new chance to really understand them, and some of them will be new to you. And you will see some material you haven't heard about before.

Since our subject (classical planar geometry) is so old, there is a lot of it. We have millennia of progress we could survey! The best we can do is to pick some interesting and accessible things which help you learn the structure of mathematical work. I hope you learn some pretty things which are new to you. Geometry is beautiful and I enjoy it very much.

General skills you will be working on

Mathematical Exploration: You will have many opportunities to settle conjectures and ask questions. This is the heart of mathematical work. You will learn about the mathematician's mindset for work: accepting "being stuck," but finding a balance of patience, persistence, and determination. You will learn general methods for answering the question, "What do I do now?"

Developing Mathematical Arguments: We will use the axiomatic method to make formal arguments, called *proofs*. We will also learn to make *counterexamples*, *conjectures*, and *definitions*.

Communication: You will get many chances to practice speaking about mathematics with clarity and precision. This is rather a tough thing to learn, and so we will practice every day. This will happen when you give presentations to the class about your work, and when you participate in discussion of presentations your classmates give. You will practice writing clearly and concisely when you submit your work to the class journal, *Transactions in Euclidean Geometry*.

Use of technology: You will learn how to use GeoGebra to make diagrams and explore.

"On a bien souvent répété que la Géométrie est l'art de bien raisonner sur des figures mal faites..." --- Henri Poincaré

Specific Euclidean Geometry content you will learn

We'll learn about triangles and circles and their basic properties. We will explore quadrilaterals of special types, like rhombuses and kites and rectangles. We will figure out how to think about general polygons. We will learn to do constructions of diagrams using the classical tools. We will learn about the way the ancients thought about area, and leverage it to construct the regular pentagon. If time permits, we will learn other things, too.

How you will know you are learning

The best way to learn mathematics is (1) to DO mathematics to the best of your ability, and (2) to DISCUSS mathematics with others as often as you can. Your successful growth as a mathematical thinker depends upon your engagement with the material. The most important feature of this course is the regular assignment, which is one thing with four facets:

Daily Work: At home, you will settle conjectures, by either making arguments for their validity (a proof), or by constructing counterexamples to disprove them. You will work on this individually, and occasionally in small groups. Your work will rely on Euclid's *Elements*, so you will read its first four books.

Presenting Work to Class: When you are convinced that you have settled a conjecture and that your argument is good enough to constitute a proof, you will present your argument to the class. How this works is flexible. Most of the time it is convenient to use the chalkboard and write and sketch as you talk. Sometimes you might want to use a document camera.

Participating in Class Discussion: After a presentation, we will take time as a class to think through the argument and discuss it. Here you will ask questions when you don't understand, and you will be stubborn in those cases until things become clear to you. During class, you may make a conjecture, or formulate an official Question, each of which would be added to the *Class Conjectures List*.

Writing for the Class Journal: When you have successfully presented an argument for a new theorem to the class, you will then write a short paper detailing the arguments. This paper will get reviewed ("refereed") and revised, and eventually published in the class journal.

Refereeing Work for the Class Journal: You will be asked to review the work of your peers. You will comment on their written work in a good-faith effort to understand it and help improve it.

Other Things:

There are three other activities for the course.

- ☐ Three written reflection assignments, each paired with a short one-on-one discussion with me in my office. These will happen in weeks 2-3, 9, and 13.
- An in-class, midterm exam. This is really just a formative assessment, and won't officially affect your grade. (We'll discuss that when it gets close.)
- ☐ A week-long, take-home Final Exam paired with a summative CV assignment. This will be due at the last possible moment allowed by UNI's registrar.

How I will determine your grade

The structure and aims of Math 3600:01 are so radically different from a traditional "computation based" mathematics course that I find the general structure of assigning grades from an accumulation of points on homework, quizzes, and exams to be completely unworkable. If I could get away with it, I would not assign grades at all, and focus all of my energy on helping you develop. However, the registrar demands that I assign grades.

I can usually sort out a reasonable "grade" for each student by the end of the semester by the mysterious process of watching and listening in class, and reading student work. There are several difficulties with this, which are probably apparent to you:

- 1. It requires that you trust me and my professionalism.
- 2. It mixes up formative assessment and summative assessment pretty badly.
- 3. It has no formal feedback mechanism.
- 4. Students do not necessarily identify the same things as important that I do.

I can not see any way around problem (1). Even the traditional "points accumulation" process requires this. I choose not to deal with item (2). In fact, I am deliberately leaving it mixed up so that I can help you all as much as possible. But items (3) and (4) are about clear communication of expectations and of perceived progress. That I can do something about.

Something about communication:

Our assessment interviews are chances for us to discuss your progress. If you want to talk about this at other times, please just ask. I would be happy to discuss your continued growth as a mathematician.

Something about my expectations:

The fundamental unit of credit in this course is the completed, published paper. I will set grades by considering the totality of your work throughout the semester -- both the quantity and quality of your papers count -- including your final exam, and your work as a referee. This is part of the purpose of the CV assignment: you will make a short document which will help remind me of the work you have done throughout the term. Your CV will list the papers you have published and the papers you have refereed. You will staple it to the top of your final exam and turn the two in together.

I am looking for students to display competence with the list of General Skills above, and hoping to find evidence of that competence in their published papers. I don't have set score cut-offs, but I can share the "typical outcomes" below. Again, these aren't *rules*. Instead, they are descriptions of the kinds of students I have seen achieve different grades over the last decade.

Some Rough Guidelines to grades	
A	This student has published many papers (in the neighborhood of 10), and at some point has done something special: found a creative argument, displayed unusual persistence, or mastered something technically challenging. This student has refereed many papers for peers. This student has demonstrated intellectual leadership during class discussions. This student has settled three tasks on the final exam, and made significant progress on the fourth.
В	The student has several published papers (say, 5 or 6), and has demonstrated they can handle finding and writing routine arguments. This student has refereed a few papers for others, and helped them improve those papers. This student is active in class discussions. This student solved 3 of the problems on the final exam.
С	The student has published a few papers (say, 3), most of them late in the term. This student has demonstrated that they can sometimes handle routine arguments, but can only reliably do so with outside help. This student has refereed the work of others rarely, and did not improve the papers much. This student participates in class infrequently. This student has completed one or two tasks from the final exam, and maybe had something to say about another task.
D	This student has rushed to finish their first or second paper at the end of the term. This student has not demonstrated a regular ability to handle the routine type of task we face in class, and has not refereed the work of any peers. This student does not participate in class. This student may have settled one task on the final exam.
F	At some point, this student gave up on the process, and has not truly participated in a meaningful way. This student has chosen not to be assessed by presenting in class, and does not participate in class discussion. This student mangles an attempt at one of the four tasks on the final exam, and does not attempt the others.

Some course policies

Ш	I pride myself on having a good environment for working and learning. It is very
	important to me that we all treat each other with care and respect, in equal measure. I
	know that I ask students to take risks in class almost every day, and this can be
	challenging for many. I ask that you help me keep our classroom a supportive place for
	each of the people in it.
	There are very few deadlines to be met in this course, and only one in-class exam. So,
	these are really fixed things.
	All papers submitted for publication have to be in by our meeting on the last day
	of classes. Papers submitted later than that won't be considered for publication.
	If you need to rearrange the midterm, please give me at least a day's notice.
	☐ The deadline for the final exam and CV assignment is basically un-extendable.

Your resources

"Euclid alone has looked on Beauty bare." --- Edna St. Vincent Millay

One of the beauties of Euclidean Geometry is that it does not require much by way of materials. Euclid's *Elements* is one of the oldest surviving mathematics textbooks, and still a good resource. We will read Euclid, translated to modern-ish English by Sir Thomas Heath.

- ☐ Euclid's Elements, Dana Densmore, ed., Green Lion Press, 2002. ISBN 1888009-19-5
- You will use Geogebra to make diagrams and do some investigations. There is a version that can be downloaded to a machine you own, and also a way to use it online and link to a Google Drive account (like the one provided with your UNI gmail address). You can find both here: https://www.geogebra.org/
- ☐ (Optional) If you like the feel of traditional tools, you may wish to purchase a compass and straightedge. (That is, get a *pair of compasses* -- which is only one object -- and a ruler to be used as a straightedge.)

Dates to observe

Monday 8/21: First Day of classes & Solar Eclipse!

Week of 8/28: First Assessment Interviews start (1st reflection assignment)

Monday 9/4: Labor Day -- no class

Week of 9/4: First Assessment Interviews continue

Wednesday 10/11: Midterm Formative Assessment (in class)

Week of 10/16: Second Assessment Interviews (2nd reflection assignment)

Week of 11/13: Third Assessment Interviews (3rd reflection assignment)

Week of 11/20: Thanksgiving Break -- no classes

Friday, 12/7: Last day of class. Journal submissions deadline & final exam distributed

Thursday, 12/12, 5pm: Final Exam and CV assignment due.

A few things to help you along the way

There are a great many useful resources for students who study mathematics at UNI. Since most college work is done outside of class, though, you will have to be responsible for seeking help when you need it.

Office Hours

First, please remember that I am a resource for you. My job is to both challenge and support you, and I enjoy my job. I can help you make this course a successful experience. I will hold regular office hours on Mondays, Tuesdays, Thursdays, and Fridays from 3 to 5 pm. To make some sort of sense of those hours, I use an online booking service. You are free to drop by and I would be happy to talk if I am available. But sometimes I have meetings, and sometimes I will be booked. You can reserve a spot in my schedule by filling out the online form at:

https://theronhitchman.youcanbook.me/

This will make an appointment and put it directly on my calendar. To let me understand my days as I start them, I have set it up so that bookings can only be made more than 15 hours in advance. Again, you are free to stop by without a booking, but bookings take precedence.

Academic Learning Center

The Academic Learning Center (Innovative Teaching and Technology Center room 007) provides various kinds of support, including tutoring, for mathematics students. You can also get help with writing, and some advising. If you are interested, you can find more information here:

https://unialc.uni.edu/math-science-services

Student Disability Services

The office of Student Disability Services provides support to students with documented need. You can find more information here:

https://sds.uni.edu/

Their standard suggested syllabus blurb is as follows:

Please address any special needs or special accommodations with me at the beginning of the semester or as soon as you become aware of your needs. Those seeking accommodations based on disabilities should obtain a Student Academic Accommodation Request (SAAR) form from Student Disability Services (SDS) (phone 319-273-2677, for deaf or hard of hearing, use Relay 711). SDS is located on the top floor of the Student Health Center, Room 103.

Mathematics Tutoring Center and Student Lounge

The mathematics department has a tutoring center on the third floor of Wright Hall in room 338. (This is the north end of the building next to the stairs.) It should also be a decent place to sit and do mathematics, with the benefits that (1) it is down the hall from all the faculty offices, and (2) you can go there from class without going outside during the winter.