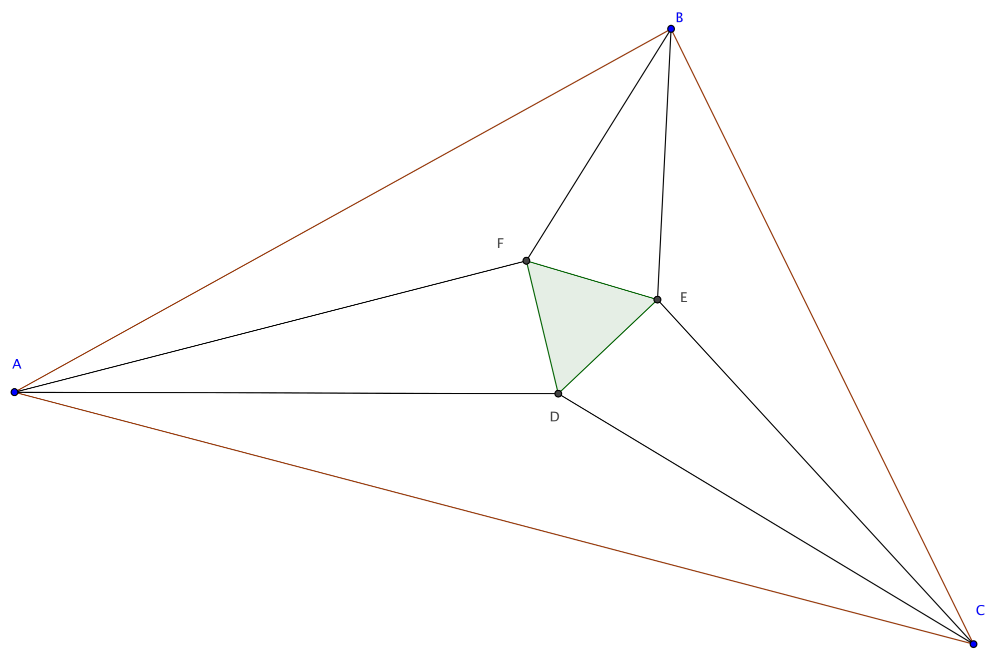
Transactions
in
Euclidean Geometry



Issue # 4

DEFINING SIMPLE AND COMPLEX POLYGONS

JOSHUA HAWKINS

Communicated by Emily Herbst

The following definitions were developed by the whole class, during one of our meetings.

Definition Simple Polygon. A simple polygon is one in which for any two sides of the polygon if the sides meet, they meet at exactly one vertex.

Definition Complex Polygon. A complex polygon is one for which there exists two sides of the polygon which intersect at a point which is not a vertex.

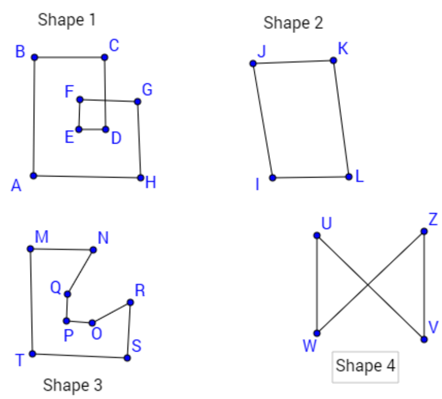


FIGURE 1. Shape 1 and Shape 3 are complex polygons. Shape 2 and Shape 4 are simple polygons.

A SIMPLE QUADRILATERAL WITH OPPOSITE SIDES CONGRUENT AND OPPOSITE ANGLES RIGHT IS A RECTANGLE

JOSHUA HAWKINS

Communicated by Eric Scheidecker

Theorem 3.4.a. Let ABCD be a quadrilateral such that angles ABC and ADC are right angles. If ABCD is simple and segments AB and CD are congruent, then ABCD is a rectangle

Proof. Let ABCD be a simple quadrilateral with segment AB congruent to segment CD, and angles ABC and ADC are right. Let circles CD and AD be drawn. Notice point D is in circle CD and circle AD, so by the Circle-Circle Intersection Property circles CD and AD intersect at two points. Label the point that is not point D, E. Draw line AE, line AC, and line EC. Since line CD and line EC are radii of circle CD, they are congruent. Also since line AD and line AE are radii of circle AD, they are congruent. Since lines DC and EC are congruent, lines AD and AE are congruent, and line AC is congruent to itself, then triangles ADC and AEC are congruent by proposition 8.

Next, draw line EX parallel to AE and draw line BY parallel to CB by proposition 31. Draw line EB. Notice since triangles ADC and AEC are congruent, then angle AEC is congruent to angle ADC and both AEC and ABC are right angles. Since angles AEC and CBA are right, then angles XEC and YBA are right by proposition 13. Notice angle FEB is a part of angle FAC, so angle FEB is less then a right angle. Similarly, angle FBE is less than a right angle. So, by postulate 5 lines EX and and BY must intersect at a point. Label the intersection of lines EX and BY F.

Since triangles ADC and AEC are congruent, then lines EC and AB are congruent. Since angle AEC is congruent to angle CBA, line EC is congruent to line BA, and angle EFB is congruent to itself, then triangles ABF and CEF are congruent by proposition 26. Since triangles ABF and CEF are congruent, then line AF is congruent to line CF. Since lines AF and CF are congruent, then triangle AFC is an isosceles triangle. Since AFC is an isosceles triangle, then by proposition 5 angles FAC and FCA are congruent. Since line AB is congruent to line EC, line AC is congruent to itself, and angle EAC is congruent to angle BCA, then triangle EAC is congruent to triangle BCA by proposition 4. Since triangle BCA is congruent to triangle EAC and triangle EAC is congruent to DAC, then triangel DAC is congruent to triangle BCA. Since, triangle BCA is congruent to triangle DAC, then angle BAC is congruent to angle DCA. Since angle BAC is congruent to DCA, then lines AB an CD are parallel to each other by proposition 29. Therefore by theorem 3.5, ABCD is a rectangle.

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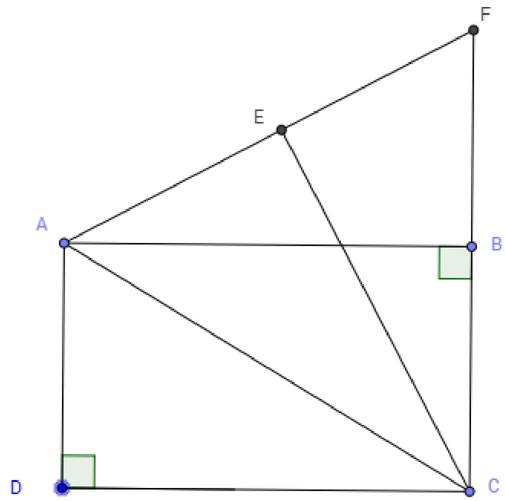

FIGURE 1. Image of constructions used in the proof.

ANALYZING EUCLID PROPOSITION I.4

MAGGIE DVORAK AND EMILY HERBST

Communicated by Joshua Hawkins

Figure out what the problem is with Euclid Proposition I.4.

Theorem I.4. If two triangles have two sides equal to two sides respectively and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively namely those which the equal sides subtend.

There were two issues with this proof that were discussed extensively:

Issue One: Euclid suggests if the triangle ABC be applied to the triangle DEF, and if the point A be placed on the point D and the straight line AB on DE, then the point B will also coincide with E, because AB is equal to DE . . . angle BAC is equal to angle EDF . . . thus the whole triangle ABC will coincide with the whole triangle DEF, and will be equal to it. Notice that the proposition does not address other possibilities. Euclid is assuming that aligning line AB to line DE will result in line AC aligning to line DF and angle BAC aligning to angle EDF. The proof should address that there is no other triangle that can be constructed, and thus triangle ABC is congruent to triangle DEF. To go into detail, if line AB is aligned to line DE, then line AC must align to line DF and not some line DG and angle BAC must align to angle EDF and not some angle EDG.

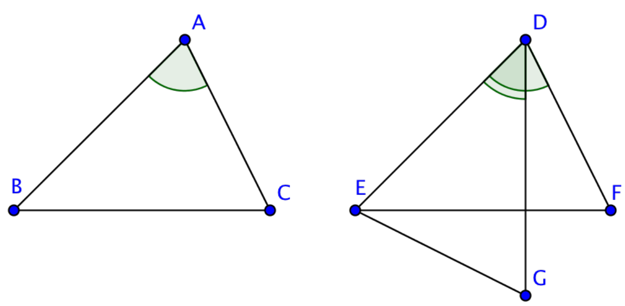

FIGURE 1. Exploring other possibilities

Issue Two: Euclid suggests "if the triangle ABC be applied to the triangle DEF, and if the point A be placed on the point D and the straight line AB on DE, then the point B will also coincide with E, because AB is equal to DE." To suggest one triangle can be "applied" to another triangle suggests tools or methods beyond the use of compass and straight edge. While today, within certain mediums, we can use operations like reflect, transpose, etc., this Proposition suggests that one would have the capability of superimposing one triangle over another. With only a compass and straightedge, one simply could not apply one triangle to another.

PROVING EXTERIOR ANGLES AT A VERTEX ARE EQUIVALENT USING VERTICLE ANGLES

JOHN FISHER

Communicated by Emily Herbst

This theorem shows that no matter which side of the polygon (A or C) we chose to extend through vertex B, the exterior angle will be the same.

Theorem 5.1. Let A,B, and C be consecutive vertices of a polygon. The exterior angle at B created by extending one of the two sides (A or C) through B is equivalent to the other.

Proof. Let A,B, C be consecutive vertices of a polygon. Extend side C through B creating a ray, g. This creates an exterior angle, X, at vertex B. Now extend side A through vertex B creating a ray, f. This creates an exterior angle, Y, at vertex B. By Euclid Proposition I.15, rays g and f make vertical angles equal to one another. For X and Y together with the verticle angle B2 make two right angles. Since X and angle B2 make two right angles and Y and the angle B2 make two right angles, X must equal Y. ☐

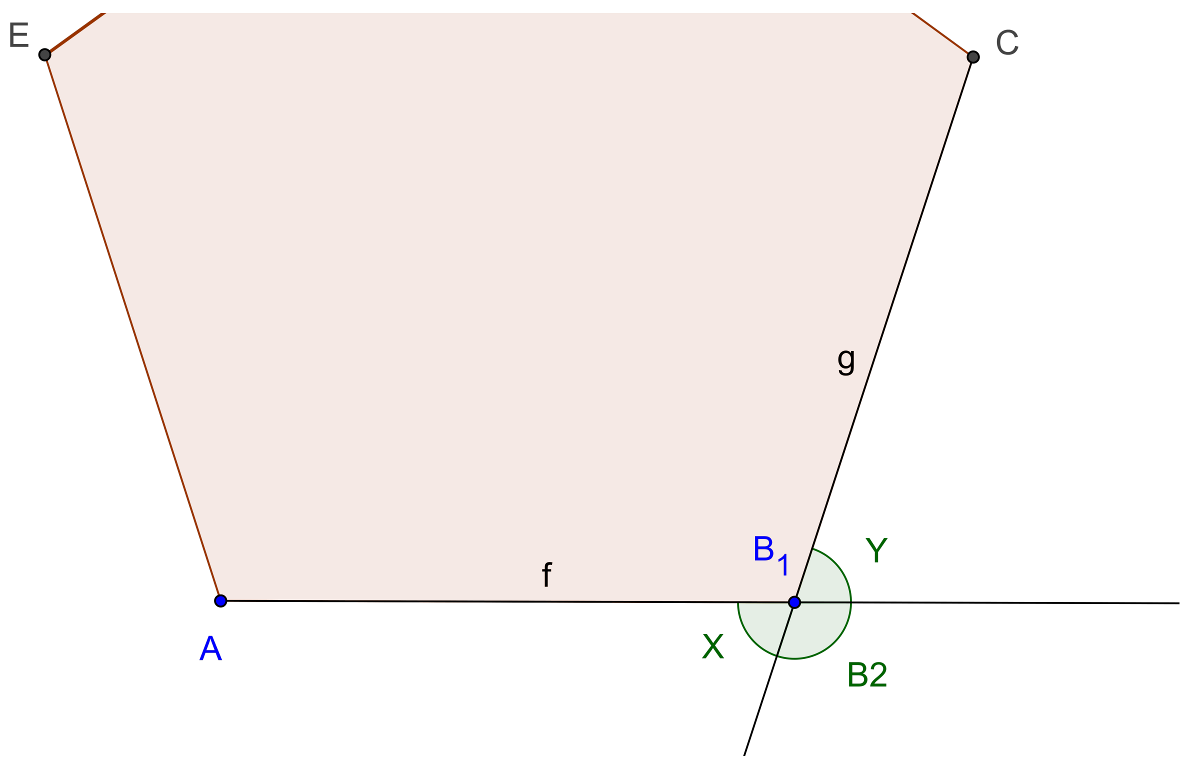

FIGURE 1. Notice that extending rays g and f through vertex B make vertical angles X and Y, respectively.

EQUILATERAL TRIANGLES ARE REGULAR

DIANN HERINGTON

Emily Herbst

Theorem 6.1. An equilateral traingle is equiangular, hence regular.

Proof. By way of contradiction, assume angle a is greater than angle b and greater than angle c. Since ABC is equilateral, all sides must be equal. By Euclid Proposition I.19, since angle a is the greater angle, it must subtend the greater side. However, all sides are equal so there cannot be a greater side. This is a contradiction so angle a is not the greater angle. Similarly, suppose angle b is the greater angle. By Euclid I.19, angle b must subtend the greater side. Since all sides are equal, this is a contradiction and angle b cannot be the greater angle.

Similarly, suppose angle c is the greater angle. By Euclid I.19, angle c must subtend the greater side. Since all sides are equal, this is a contradiction and angle c cannot be the greater angle.

Since no angle is greater, there cannot be a lesser angle. Since there is no greater or lesser angle, all angles must be equal. Therefore, an equilateral triangle is equiangular, hence regular. □

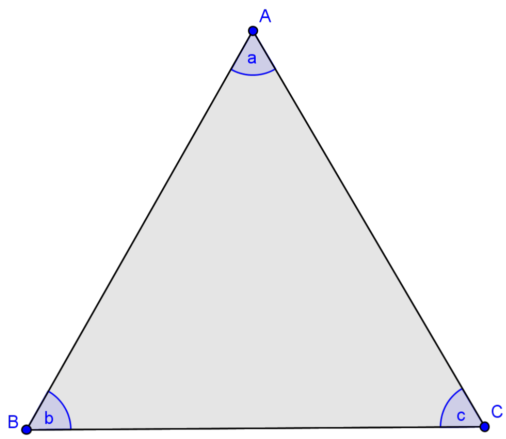

FIGURE 1. Equilateral triangle ABC

THREE LINE SEGMENTS WHICH FAIL TO CREATE A TRIANGLE

MEGAN WESTERVELT

Communicated by Emily Herbst

Theorem 7.1. It is possible to construct three line segements with a compass and straight edge which will not be congruent to the sides of any triangle.

Proof. Begin by drawing a circle with center X and diameter AB. Then construct a circle with center A and radius AX. Let C be any point on this circle that is not equal to X. Next construct a circle with center B and radius BX. Let D be any point on this circle not equal to X. *(Refer Figure 1 Below)*

Notice that line segment AX will be congruent to line segment BX because both are radii of the cirlcle centered at X. Line segment BX will also be congruent to the line segment BD, again because both are radii of the circle centered at B. Finally line segment AX will be congruent to line segment AC because they are both radii of the circle centered at A. Therefore, we can conclude that line segments AC, AX, BX, and BD are all congruent to one another. Also note that the line segment AB has twice the length of line segments AC, AX, BX, and BD.

By way of contradiction, assume that line segments AC, AB, and BD form a triangle. According to Euclid Proposition I.20, the sum of any two sides are greater than the remaining side. The sum of line segments AC and AB will be greater than the length of BD. Similarly, the sum of line segments BD and AB will be greater than the length of AC. The problem arises when line segment AC is added to BD. The sum of segments AC and BD will be exactly equal to the length of line segment AB. Therefore, the sum of AC and BD is not greater than the remain side AB. Thus the line segments AC, AB, and BD will not form a triangle.

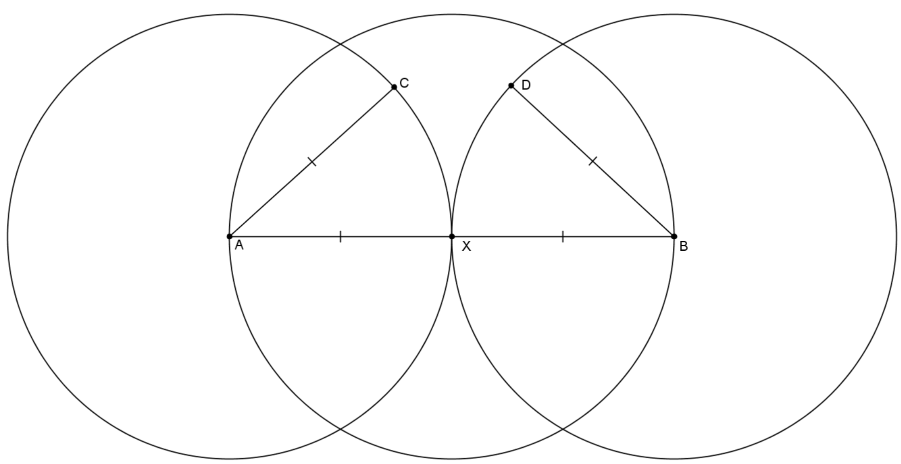

FIGURE 1. Construction of the Line Segments

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ALL THREE ANGLE BISECTORS OF A TRIANGLE ARE CONCURRENT

KAYLEE BENSON

Communicated by Katy Goodmundson

Theorem 8.2. The three angle bisectors of a triangle are concurrent.

Proof. Let ABC be a triangle. Construct angle bisectors of angles BAC and ABC. Call the intersection point I. Draw perpendicular lines l, m, and n from I through AC, AB, and BC respectively, this is done by Euclid's I.12. Label the intersection of AB and line m point F. Label the intersection of AC and l point E. Finally, label the intersection of BC and n point D. Let CI be joined. Now, using Ms. Stuffelbeam's Theorem 8.1, it is known that triangles AFI and AEI are congruent. Also, triangles BFI and BDI are congruent. Lastly, triangle DIC is congruent to triangle EIC. Triangles DIC and EIC are congruent therefore by corresponding parts of congruent triangles, angle DCI is congruent to angle EIC, angle AFI is congruent to angle AEI, and angle BFI is congruent to angle BDI. Therefore lines AI, CI, and BI are all angle bisectors and meet at point I. Hence, angle bisectors of a triangle are concurrent.

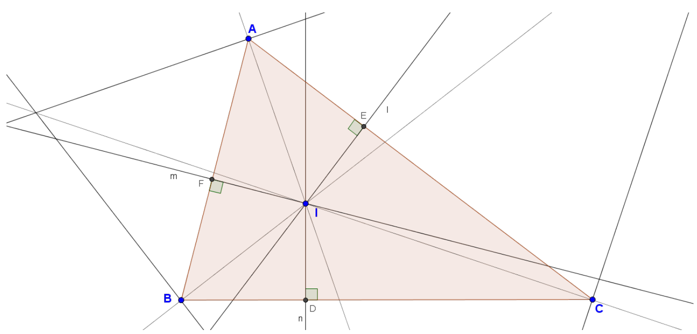

Figure 1. Triangle ABC

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