Constructing a Tangent Line Theorem

Mr. Austin Henry

May 1, 2015

This construction was aided by Mr. Toby Maggert. Refereed by Ms. King.

Theorem 11.8. Given a circle with center O, and given a point A outside the circle, it is possible to construct a line l through A which is tangent to the circle.

1. Construct line segment OA.
2. Construct circle centered at A through O.
3. Construct circle centered at O through A.
4. Where circle AO meets circle OA two points are created, named Q and Z. Connecting point Q to point Z another point, X, is made at the intersection of line segment OA with line segment QZ. This point X is the mid-point of line segment OA by Mr. Stuhr's Theorem 11.2.
5. Create circle centered at X through points O and A.
6. Where circle XOA meets circle O, a point Y is noticed. Create line l through AY, this line is tangent to circle O and goes through point A.

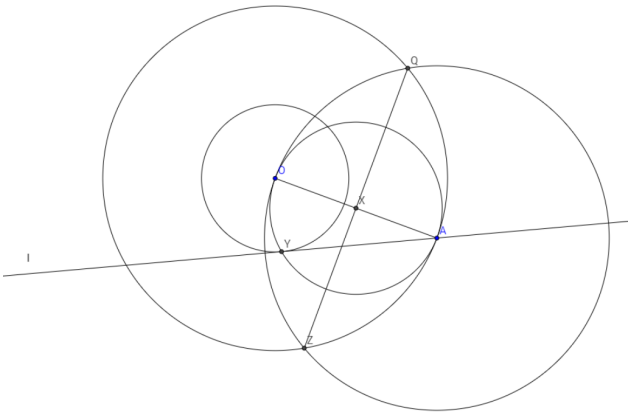

Figure 1: Line l is tangent to circle O

Proof. We say that line l meets line segment OY at a right angle and therefore is tangent to circle O, by Euclid III.16 "The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle..." Since OA is a diameter of circle XOA and Y lies on circle XOA, it is true by Mr. Baker's Theorem 7.4 that angle OYA is right. Therefore line l, drawn on line segment AY, meets OY at a right angle and by Euclid III.16 is then tangent to circle O.

□