Angle Relationships In Circles

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Theorem 10.1. Let Γ be a circle with center O. Let X be a point in the interior of the circle, and suppose that two lines ℓ and m intersect at X so that ℓ meets Γ at points A and A' and m meets Γ at B and B'. The twice angle AXB is congruent to angle AOB and $A'OB'$.

Proof. Let Γ be a circle with center O. Let X be the intersection of lines ℓ and m. Let ℓ meet Γ at A and A', and let m meet Γ at B and B'.
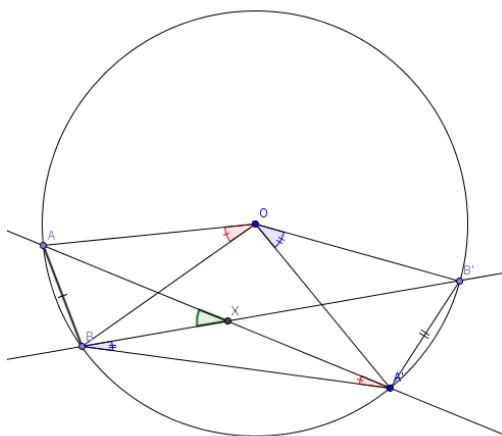

Figure 1: Circle Γ with two lines intersecting at X

By Euclid III.20, angle AOB is double of angle $AA'B$. Similarly, angle $A'OB'$ is double of angle $B'BA'$.

By Euclid I.32, the sum of the angles of triangle ABA' is two right angles. Therefore, angle $A'AB$, angle ABX, angle XBA', and angle $XA'B$ taken together is two right angles. Similarly, in triangle AXB, angle XAB, angle ABX, and angle AXB taken together is two right angles.

Therefore, double of angle AXB is congruent to angle AOB and angle $A'OB'$.

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