Convex Pentagon Angles

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Theorem 5.2. The exterior angles of a convex pentagon (one choice made at each vertex) add to four right angles.

Proof. Let ABCDE be a convex pentagon. Using Theorem 5.1, extend one of the two sides at each vertex to a ray to construct the exterior angles (one choice made at each vertex). Proposition I.13 says if we add the interior and exterior angles at a given vertex we get two right angles. Since we have five vertices on the pentagon, we get ten right angles. These ten right angles make up both the interior and exterior angles. At vertex A, extend the diagonals AD and AC. Since the pentagon is convex, the diagonals will lie inside the pentagon. This gives us three triangles, which lie inside the pentagon: ABC, CAD, and DEA. Proposition I.32 says that the three angles of a triangle add up to two right angles. Since we have three triangles, then we get a total of six interior right angles. Subtract the interior total from the whole total (ten minus six) to get a total of four exterior right angles.

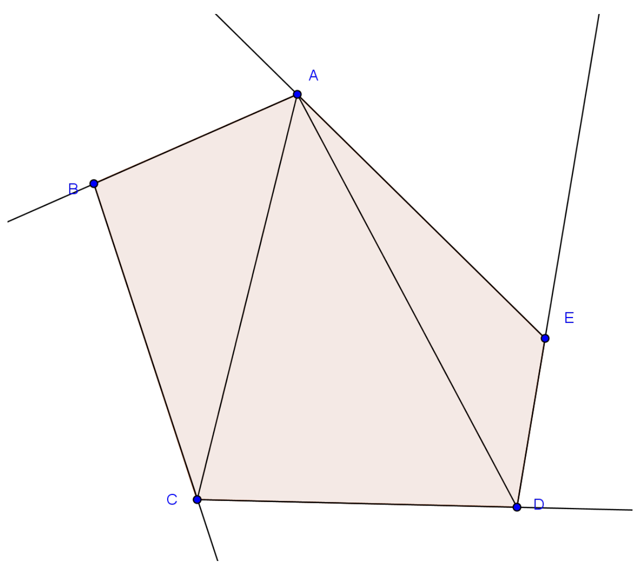

Figure 1: Pentagon ABCDE with each side extended once at each vertex and split into three triangles.

Refereed by Hailey Manternach □