EUCLIDEAN GEOMETRY
MIDTERM EXAM

OCTOBER 14, 2015

Instructions: Please use complete sentences and explain yourself as clearly as you can. You may use your notes and your copy of the *Elements*, but I think you are better off just using your brain. It is not necessary to quote results by their proper numbers, but results you use should be stated and used properly. As usual, any item from the *Elements* I.1-34 and any item proved in class may be used in your work.

This is a one hour exam.

Task 1. Prove that the diagonals of a parallelogram bisect each other.

Task 2. Recall that we have already proved the following theorem:

> **Theorem 3.6:** (Wulfekuhle) Let ABC be a triangle with D the midpoint of AB and E the midpoint of AC. The line DE is parallel to the line BC.

I am interested in a new proof of this theorem that uses an argument by contradiction. Write out the beginning of such a proof. You need not complete the proof, but you should begin it well.

Task 3. We have considered several ways to express ideas by making new definitions. Often, there is more than one way to make a technical condition encompassing an idea. To reduce confusion, it becomes important to show that these conditions are equivalent.

In class, we made this definition:

> **Definition A:** We say that a quadrilateral is convex when for each diagonal of the quadrilateral, that diagonal lies inside the quadrilateral.

But sometimes, mathematicians use this definition:

> **Definition B:** We say that a quadrilateral is convex when for each pair of points P and Q which lie inside the quadrilateral, the segment PQ also lies inside the quadrilateral.

Of course, we would really like this to be true.

> **Theorem:** The two definitions above are equivalent.

Without actually giving a proof, explain how you would structure a proof of this theorem. That is, do not write a complete proof of this result, but give a high-level outline of what would be necessary in an argument for this result.

Task 4. Let ABC be an equilateral triangle. We wish to choose a point D in the plane so as to construct a quadrilateral $ABCD$. Where can we put this new point?

Draw a diagram and label the places where D cannot be placed to make a quadrilateral. For each part of your figure where D cannot be placed, label it with the reason that D cannot be placed there. Also, for those parts of your diagram where one may place D, label it with any interesting description of the shape of the resulting quadrilateral that might be relevant.