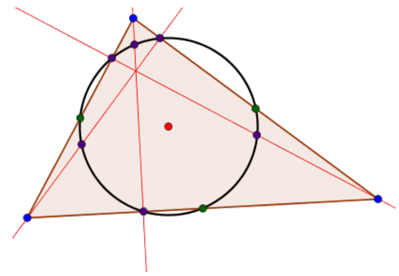
Euclidean Geometry: An Introduction to Mathematical Work

Math 3600

Fall 2016



Here are some experimental problems I have never reached with this course.

The Simpson Line

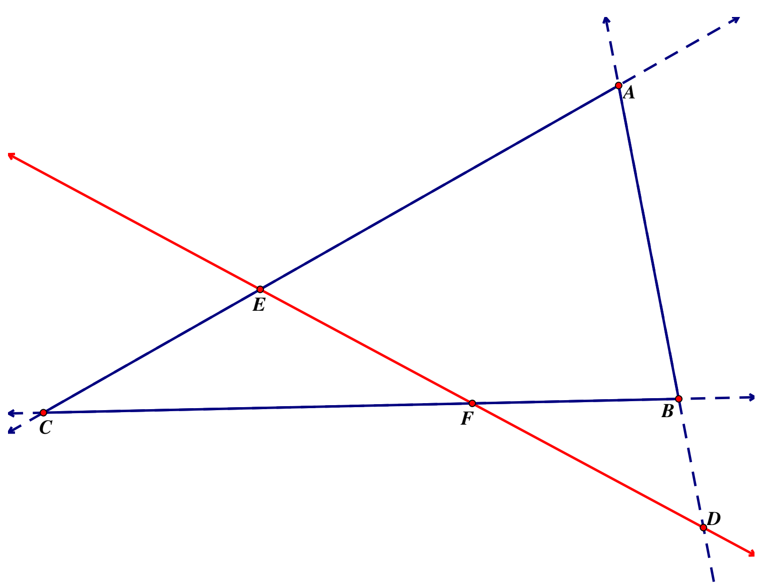
19.1 Problem. Let ABC be a triangle. Let P be a point on the circumscribed circle of ABC. Let D, E, F be the feet of the perpendiculars from P to the sides of the triangle (possibly extended). Show D, E and F are collinear.

Definition. The line just found is called the *Simson line* of P with respect to ABC.

Some Basic Projective Geometry

19.2 Problem (Menelaus' Theorem)**.** Let ABC be a triangle. Let a line ℓ cut the (extended) sides of ABC at D, E, F. Then

$$AD \cdot BF \cdot CE = BD \cdot CF \cdot AE.$$



19.3 Problem. Show the converse is also true!

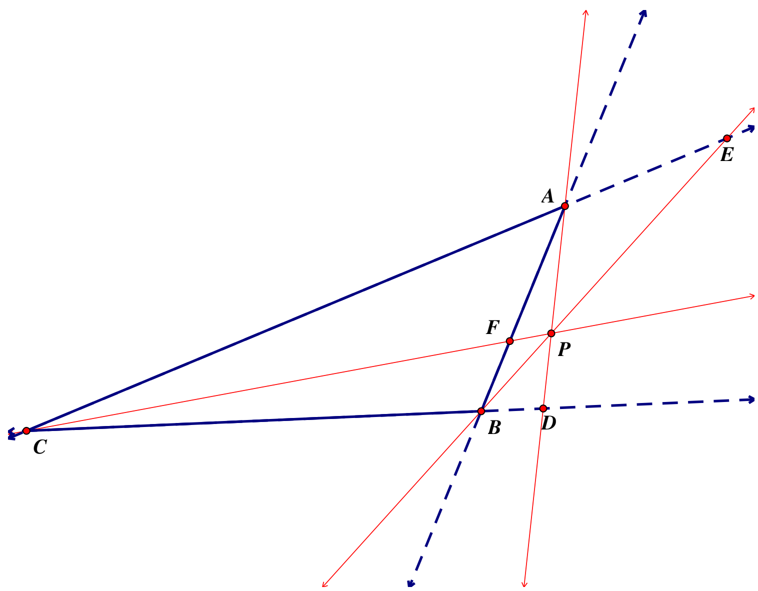
Note: This result is usually stated

$$\frac{AD}{DB}\frac{BF}{FC}\frac{CE}{EA} = -1$$

and the segments are interpreted as *signed* segments, where the direction of travel matters!

19.4 Problem (Ceva's Theorem)**.** Let ABC be a triangle, and let P be any point inside the triangle. Draw lines from the vertices through P meeting the opposite sides at D, E, F. Show that

$$AD \cdot BF \cdot CE = BD \cdot CF \cdot AE$$



19.5 Problem. Show the converse!

Note, this also has a more standard restatement. What should it be?

19.6 Problem (Desargues' Theorem)**.** Let ABC and $A'B'C'$ be two triangles. Suppose that the lines AA', BB' and CC' are concurrent at a point O. Suppose that AB is parallel to $A'B'$ and BC is parallel to $B'C'$. Show that AC is parallel to $A'C'$.