Constructing a Congruent Line Segment

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Challenge 12.3. Given a line ℓ, a line segment d and a point C, construct a circle with center C that cuts off a segment from line ℓ which is congruent to d.

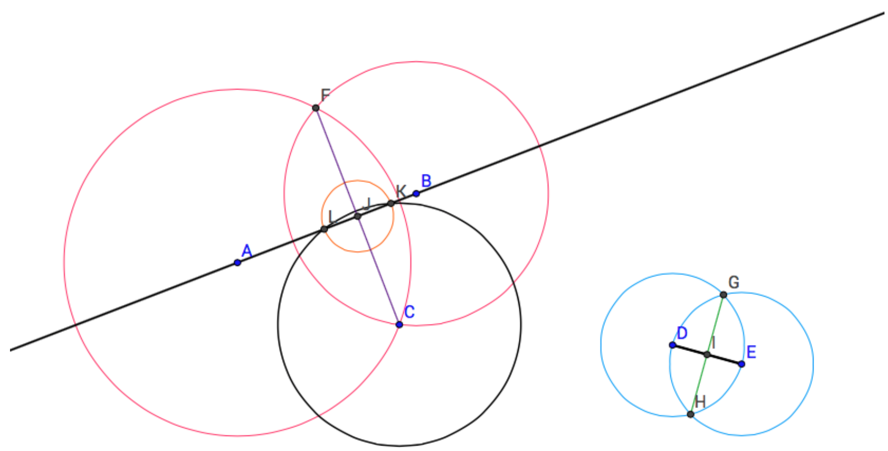

Figure 1: construction of a congruent line segment

1. Choose point A on ℓ and draw circle A through C.
2. Choose point B on ℓ and draw circle B through C.
3. Label the point of intersection of the circles F and draw line segment CF. This is the segment perpendicular to ℓ through C. Label its intersection with ℓ as point J.
4. Label the endpoints of the line segment d as D and E. Draw circle D through E.
5. Draw circle E through D. Label the points of intersection of the circles G and H.
6. Draw line segment GH. This is the perpendicular bisector of DE. Label the intersection of DE and GH as point I.

7. Draw a circle with radius IE at point J. Label the points where the circle intersects with ℓ L and K.

8. Draw circle C through L.

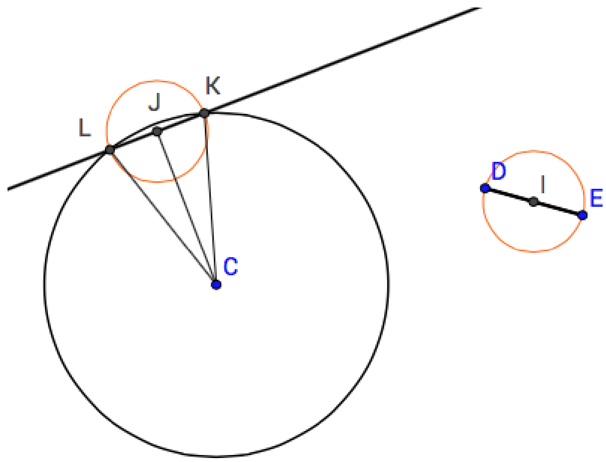

Figure 2: Line segment LK is congruent to DE

Proof. Since a circle radius IE was used to find points L and K, LK is congruent to DE. However, the challenge was to use a circle around point C to make this line segment. Point L was used to draw the circle around C, so we must show that point K also lies on the circle. Consider the triangles LJC and KJC. JC is a shared side between the two triangles. Since JC is perpendicular to ℓ, angles LJC and KJC are right angles and thus are congruent. Since LJ and KJ are radii of the circle at J, they are congruent. Then using SAS, triangles LJC and KJC are congruent. Therefore KC is congruent to LC and is also a radius of the circle C through L.

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Refereed by Ellen Barbaresso.