Construction of a Rhombus

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After the conclusion that Mr. Baker's construction of a rhombus leads to only one rhombus, up to congruence, I proposed a different, more flexible construction.

Lemma 1.4. Given a line segment AB, construct rhombus ABCD.

Given segment AB,

1. Construct a circle around A, through B.
2. Construct a circle around B, through A.
3. Choose any point C on the circle around B, so that C is not colinear with AB. Construct circle C through B.
4. Label the point D at the intersection of Circles A and C. Connect the dots!

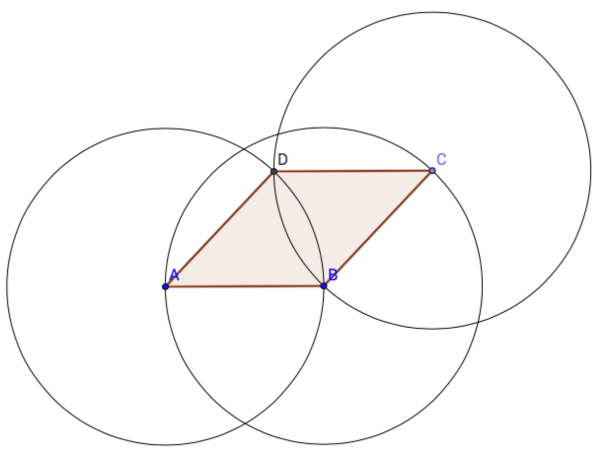

Figure 1: This is a rhombus.

Proof. To show this is a rhombus, all sides must be congruent. Since AB and BC are radii of the same circle, they are congruent. Similarly, AB and DA are radii of the circle, and the same goes for BC and CD. Thus, all sides are congruent, so ABCD is a rhombus. □

Question 1.5. How flexible is this construction?

Proof. Since point C can be chosen as any point on the circle B through A, an infinite number of points can be chosen. Therefore, infinitely many rhombi can be constructed.

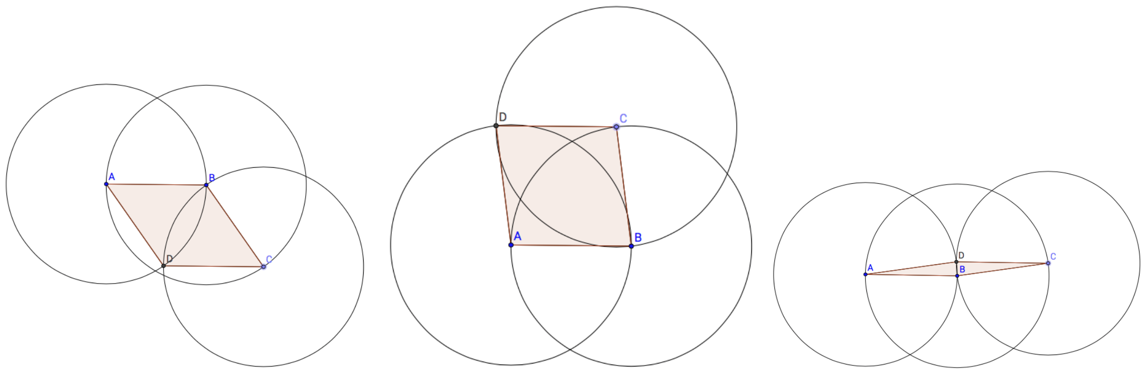

Figure 2: Several different rhombi which can be constructed.

□

Question G. Do any pairs of choices of point C lead to congruent rhombi?

Conjecture K. If BC meets AB at a right angle, there is one other point C which leads to a congruent rhombus. If BC does not meet AB at a right angle, there are three other choices of point C which construct congruent rhombi. If segment AB is extended to be the diameter of circle B, and a line is drawn from the chosen point C to meet line AB at a right angle, then the point where the line again meets circle B is one alternate choice of point C. In this figure, let's call it E. The second point exists at the point where, if segment BC is extended to be the diameter of circle B, BC again meets circle B. Let's call it G. Similarly, the final point lies where, if a line is drawn to be the diameter of circle B, through point E, the diameter again meets the circle. In this diagram, it is point F.

Proof. Ms. Rehnstrom has previously proved that the rhombus constructed using point G is congruent to the rhombus constructed using point C.

To show point E leads to a congruent rhombus, angle ABC must be congruent to angle ABE. To show this, we will show triangle BHC is congruent to triangle BHE. Because BC and BE are radii of the same circle, they are congruent. Since line GE meets AB at a right angle, angles BHE and BHC are both right angles and thus are congruent. Triangles BHC and BHE share side BH, which is, of course, congruent to itself. Using Euclid I.12, segments HC and HE are congruent. Then by Side-Side-Side, triangles BHC and BHE are congruent, so angles HBC and HBE are congruent. Then angles ABC and ABE are also congruent.

To show point F creates a congruent rhombus, we apply Ms. Rehnstrom's theorem again, using point E as the original chosen point.

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Refereed by Harmony Van Nevele.

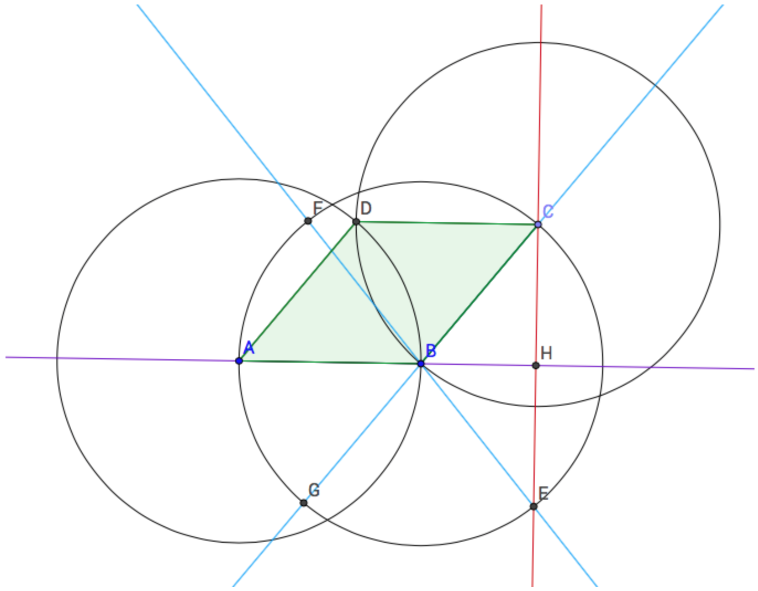

Figure 3: Three other choices of points.

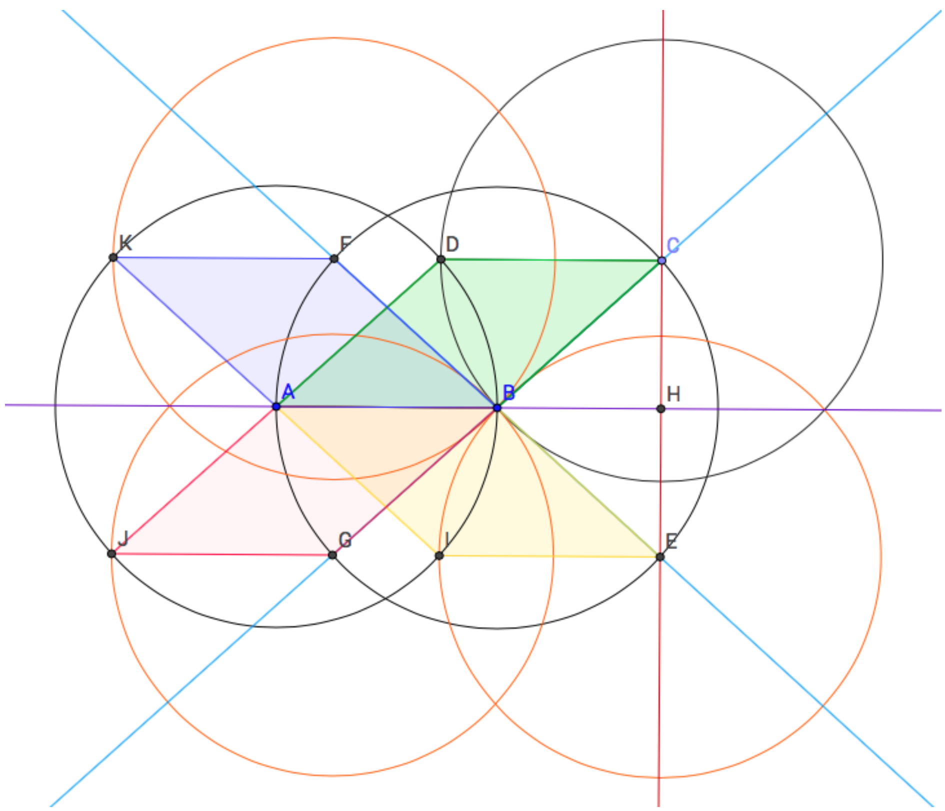

Figure 4: Here are the four congruent rhombi. Very pretty.