

Rectangles are Parallelograms

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Theorem 3.1. Let R be a rectangle. Then R is a parallelogram.

Proof. Start with rectangle $ABCD$. The definition of a rectangle is a quadrilateral which has all four interior angles that are right angles. Then extend all four sides. Then using Book 1 Proposition 15, we are able to make the opposite angles right angles. The opposite angles are the angles on the opposite side of the line extended. Proposition 15 is the vertical angle proposition which states that "If two straight lines cuts one another, they make the vertical angle equal to one another."

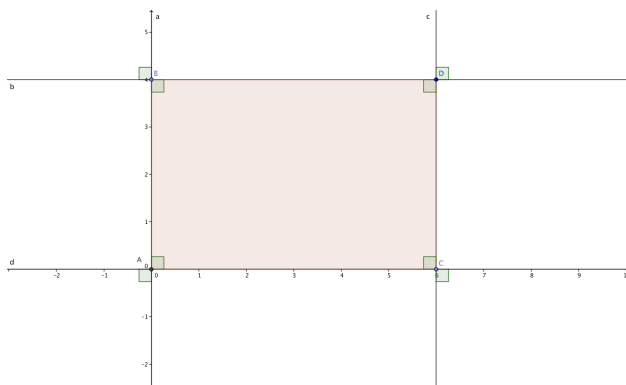


Figure 1: The rectangle proving that the rectangle is a parallelogram

Then since we have right angles, we can use Book 1 Proposition 28 which states that if a straight line falling on two straight lines make the exterior angle equal to the interior and opposite angle on the same side, or the interior angles on the same side equal to two right angles. Using this, the rectangle is able to have right angles for all of the interior angles. Since the opposite sides have the same angle, those lines are parallel. Since there are two pairs of opposite sides, we have two sets of parallel lines, thus making a rectangle a parallelogram.

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