Equal Content Construction

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Theorem 13.8. Given a rectangle, construct a square of equal content.

Proof. Construct a rectangle ABCD. Extend line DC. Construct circle C with radius CB and label the point of intersection with circle C and DC as point E. Using Theorem 11.2, find the midpoint of DE and label it M. Construct circle M with radius MD. Extend CB to circle M and label the point N. Create circle C with radius CN and label the point of intersection of circle C and DC as point O. Create circle O with radius CO. Draw circle N with radius NC and label the point of intersection between circle N and circle O as point P. Draw line segment NP and PO. Extend NC until it meets circle M and label that point Q. By Euclid I. 14, angle NCO is a right angle. We know CN is congruent to CO because they are both the radius of circle C. Then we drew circle O with radius CO, so OP is also congruent to CN and CO. Then we drew circle N with radius NC, so NP is also congruent to CN, CO, and OP. So, we know all four sides are congruent because they are all the radius of congruent circles. So, CNPO is a rhombus. The opposite angles in a rhombus must be congruent to each other. Since angle NCO is a right angle, then angle NPO is also a right angle. We know four right angles make up a rhombus, so angle CNP and COP must be right angles too. Therefore, CNPO is a square. By Euclid III. 3, since DE passes through the center of the circle and cuts NQ at right angles, it bisects NQ, so NC is congruent to CQ. By Euclid III. 35, the rectangle formed by DC and CE have the same content as the rectangle formed by NC and CQ. Therefore, this is a construction of a square of equal content as a given rectangle. ∎

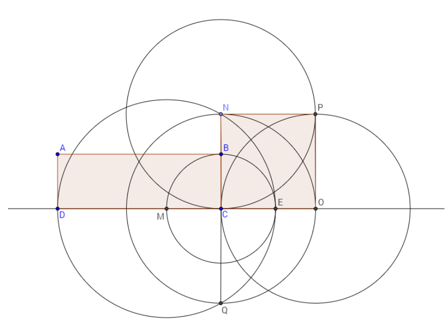

Figure 1: Construction of a square of equal content as a given rectangle.

Refereed by Charlotte Brandenburg

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