Broken Chord Theorem

Ange Rehnstrom

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Theorem 10.7. Let AB and BC be two chords of a circle C, where BC is greater than AB. (Such a con-figuration is sometimes called a broken chord.) Let M be the midpoint of arc ABC and F the foot of the perpendicular from M to chord BC. Then F is the midpoint of the broken chord, that is, AB and BF taken together are congruent to FC.

Proof. In order to show that F is the midpoint of the broken chord, we will show that AB and BF taken together are congruent to FC.

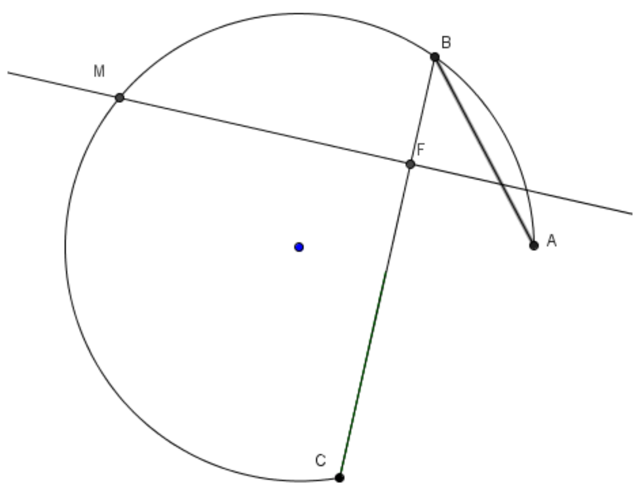

Figure 1: Construction of the Theorem

Draw a circle with radius AB around point C. Label the where the circle intersects segment BC point E, and AB is congruent to CE. Draw triangles ABM and CEM. Since M is the midpoint of arc ABC we know arc AM is congruent to arc MC, then by Euclid III.29 we know that segment AM is congruent to segment CM. By Euclid III.21 we have that angle BAM is congruent to BCM because they are both in segment BM. Since we have CE is congruent to AB, AM is congruent to CM, and angle BAM is congruent to BCM by Euclid I.4 triangle ABM is congruent to triangle CEM. Since these are congruent triangles we now have segment ME is congruent to MB (see Figure 2).

Now draw triangle MEF and MBF. We know that segment ME is congruent to MB. We also know that triangles MEF and MBF are right triangles because F is the foot of the

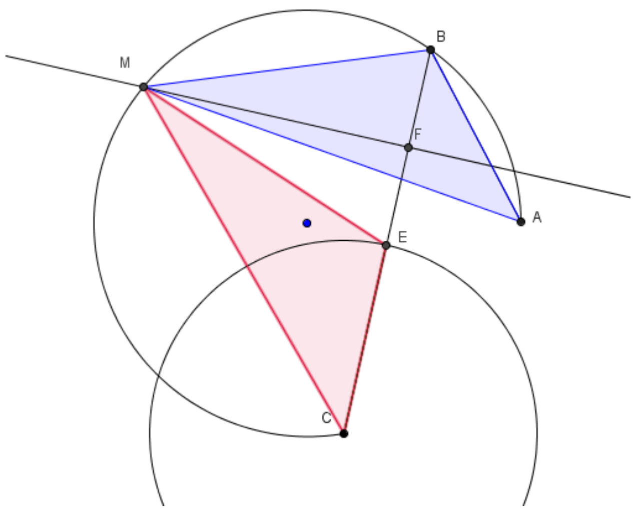

Figure 2: Triangles ABM and CEM

perpendicular from M to segment BC by construction, so segment MF is perpendicular to BC. We have that triangles MEF and MBF are right triangles, ME is congruent to MB and the side MF is shared. Therefore by Theorem 7.2 by Ms. Freking triangles MEF and MBF are congruent. Since these are congruent triangles we have that segment BF is congruent to segment FE (See Figure 3).

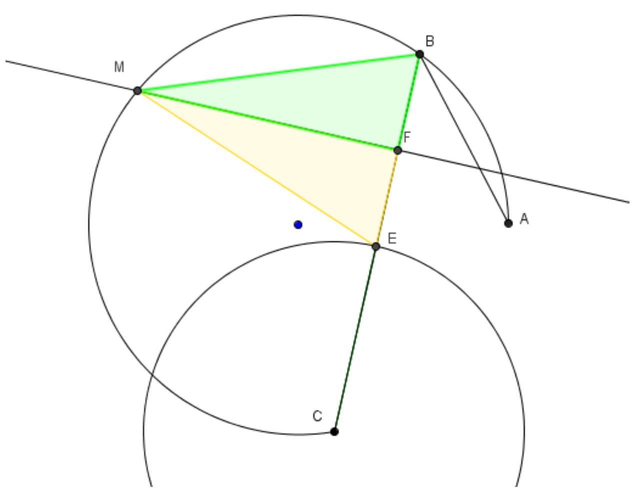

Figure 3: Triangles MEF and MBF

Now since we have BF is congruent to FE, and we have AB is congruent to CE we can say that AB and BF taken together is congruent to CE and FE taken together. CE and FE taken together make segment FC. Therefore, we have AB and BF taken together are congruent to FC.

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Refereed by: Grace Freking and Austin Henry