

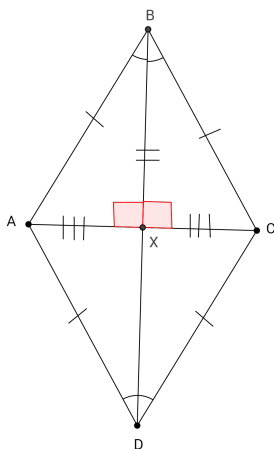
# The Angle Formed by the Diagonals of a Rhombus

Megan King and Harmony Van Nevele

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**Theorem 1.7.** If the diagonals of a rhombus  $ABCD$  cross at point  $X$ , the angle  $AXB$  will be a right angle.

*Proof.* Let  $ABCD$  be a rhombus. Suppose the diagonals  $AC$  and  $BD$  meet at point  $X$ . Euclid's book I, proposition 8 states that two triangles with three congruent sides are also congruent triangles. Since  $ABCD$  is a rhombus, we know that sides  $AB$ ,  $BC$ ,  $CD$ , and  $DA$  are all congruent, and since triangle  $ABD$  and triangle  $CBD$  share side  $BD$ , then they have three congruent sides. By Euclid I.8, triangles  $ABD$  and  $CBD$  are congruent. Therefore, angles  $ABX$  and  $CBX$  are congruent. Since sides  $AB$  and  $BC$  are congruent and the angles  $ABX$  and  $CBX$  are congruent, the triangles  $ABX$  and  $CBX$  are congruent by Euclid's book I, proposition 4. This means that angles  $AXB$  and  $CXB$  must be congruent. Since they are two angles set up on a straight line,  $AC$ , they must make two right angles by Euclid's book I, proposition 13. They must both be right angles since they are congruent, therefore if the diagonals of a rhombus  $ABCD$  cross at point  $X$ , the angle  $AXB$  will be a right angle.



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