Quadrilateral with Opposite Right Angles and Opposite Congruent Sides form a Rectangle

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Theorem 3.4. Let ABCD be a quadrilateral such that angles ABC and ADC are right angles. If segments AB and CD are congruent, then ABCD is a rectangle.

Proof. Construct circle with center B through A, and let point E be the intersection of circle B and line AB. Construct circle with center D through C, and let point F be the intersection of circle D and line DC. By definition of a circle, segments AC, CE, and AF are congruent. Consider isosceles triangles CAF and ACE. By conjecture H, point B forms congruent segments BE and AB, and point D forms congruent segments FD and DC. Because segments AB, BE, FD, and DC are congruent to one another, segments AE and FC are congruent. Triangles ACE and FAC are congruent by Proposition 8.

Corresponding angles CAE and ACF are congruent. By Proposition 4, triangles ABC and CDA are congruent, and have congruent angles BCA and DAC. By Proposition 32, angles BAC and DAC form a right angle, and angles ACB and DCA form a right angle. Quadrilateral ABCD has four right angles, conforming to the definition of a rectangle. □

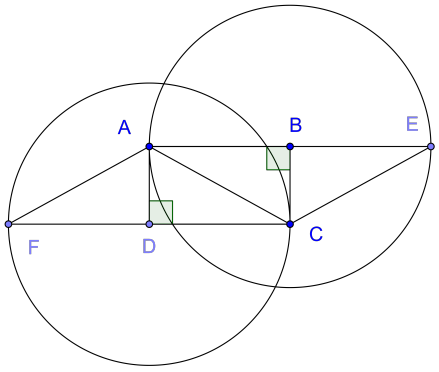

Figure 1: Construction of Triangles