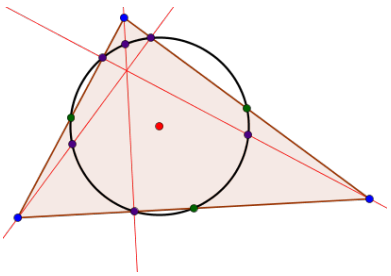
Euclidean Geometry: An Introduction to Mathematical Work

Math 3600, Fall 2014

11 October



Polygons

Now it is time to extend our venue to *polygons* with an arbitrary number of sides.

Definition. Let n be a natural number. An *n-gon* is a figure consisting of n points $A_1, A_2, \ldots, A_n$, prescribed in order and called *vertices*, and the n line segments, called *sides*, $A_1A_2, A_2A_3, \ldots, A_{n-1}A_n, A_nA_1$.

A *polygon* is an n-gon where n has not been specified.

Note: Commonly used terminology includes the following: 3-gon = triangle, 4-gon = quadrilateral, 5-gon = pentagon, 6-gon = hexagon.

5.1 Problem. Suppose that A, B, C are three consecutive vertices of a polygon. If at the vertex B we extend one of the two sides through B to a ray, then we create a new angle, called an *exterior angle* to the polygon at B.

This construction has a choice in it. In principle, this could be a problem. Describe the problem, then state and prove a theorem that resolves the issue.

5.2 Conjecture. The exterior angles of a pentagon, one choice made at each vertex, add up to four right angles.

5.3 Question. What is the sum of the exterior angles of a hexagon? What about a general n-gon? Can you find a way to build on our understanding from small values of n, to general values of n?