Congruent Right Triangles

Grace Freking

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Theorem 7.2. Let ABC and DEF be two right triangles with the angles at A and D right angles. Suppose that BC is congruent to EF and AB is congruent to DE. Then the triangles are congruent.

Proof. Construct triangle ABC. Extend line AC. By Euclid I.14, both angles at A are right angles. Make circle B with radius BC. Connect B to the point of intersection of line AC and circle B. Label that point E. So, BE is congruent to BC because they are both the radius of the circle B. By Euclid I.5, angle BEA is congruent to angle BCA because triangle EBC is an isosceles triangle. By Theorem 11.6, make a line through B that is parallel to EC. Construct circle E with radius BE. Then connect E to the point of intersection of line B and circle E. Label this point F. So, EF is congruent to EB because they are both the radius of circle E. By Euclid I.5, angle EFB is congruent to angle EBF because triangle BEF is isosceles. By Euclid I.29., angle FBE is congruent to angle BEC because a straight line falling on parallel straight lines makes the alternate angles congruent. Draw line E that hits line BF that is also parallel to AB. Label this point D. ED is congruent to AB because BAED is a parallelogram and the pairs of opposite sides of a parallelogram are congruent. By Euclid I.29, angle BDE and angle FDE are right angles. So, angle BAC is congruent to angle EDF, angle ACB is congruent to angle DFE, and BC is congruent to EF. This is AAS. Therefore, by Euclid I.26, triangle ABC is congruent to triangle DEF.

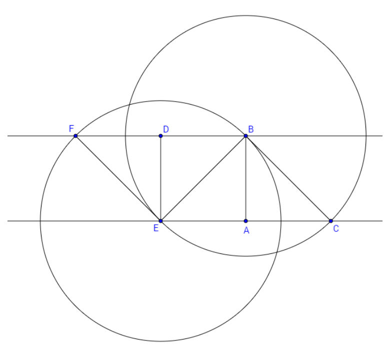

Figure 1: Triangles ABC and DEF

Refereed by Emily Jacobs

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