The Intersection of Angle Bisectors in a Triangle

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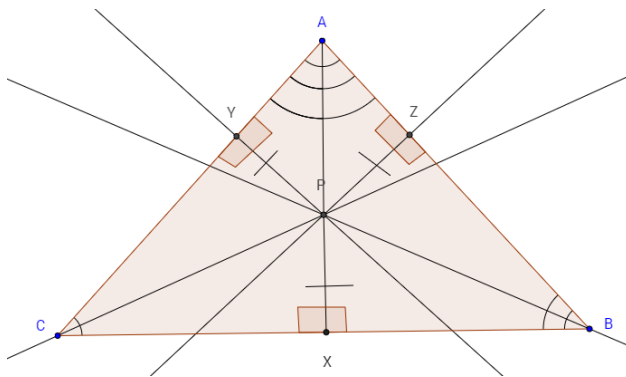
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Theorem 8.2. The three angle bisectors of a triangle are concurrent.

Proof. Let ABC be a triangle. Draw in the angle bisectors of angle BCA and angle ABC. Assuming they intersect, label that point P. From point P to segment AB, draw a perpendicular and label the point, where it crosses segment AB, Z. Draw another perpendicular from point P to side BC. Label the point, where the perpendicular crosses AB, X. Do this again with side AC and label the point Y.

Since angle ABC is bisected, then we know that angles ABP and CBP are congruent. By Euclid I.26, we know that triangles PXB and PZB must be congruent since they share side BP. The same holds true for triangles PYC and PXC. Since the triangles are congruent, we know that the segments PZ, PX, and PY must also be congruent.

Next, draw a line segment from point P to point A. Since triangles AYP and AZP share side AP and sides PY and PZ are congruent, then the triangles are congruent by MS. Freking's Theorem 7.2. Since triangles AYP and AZP are congruent, then angles YAP and ZAP must also be congruent, making the line segment AP also an angle bisector. Therefore, all three angle bisectors are concurrent.

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