

Construction For a Perpendicular Line

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Theorem G. Given a line ℓ and a point C not lying on ℓ , we can construct a line perpendicular to ℓ through C in three strokes.

Step 1: Select any two different points A and B on line ℓ . Construct circle A through C.

Step 2: Construct circle B through C. Note that circles AC and BC will intersect twice by Euclid I.7 at C and D.

Step 3: Construct line m through C and D. This line will be perpendicular to ℓ .

Proof. Since we have constructed circles AC and BC, we can construct segments AC, CB, BD, and DA. Note that this will create a kite. This kite will be convex if our selections of A and B were on opposite sides of our line m but will be non-convex if our selections were on the same side.

Case 1: A and B are on opposite sides of line m . Note that line ℓ is the extension of diagonal AB and line m is the extensions of diagonal CD. By the proof for conjecture 2.5, we know that the diagonals of a kite cross at a right angle, which makes ℓ perpendicular to m .

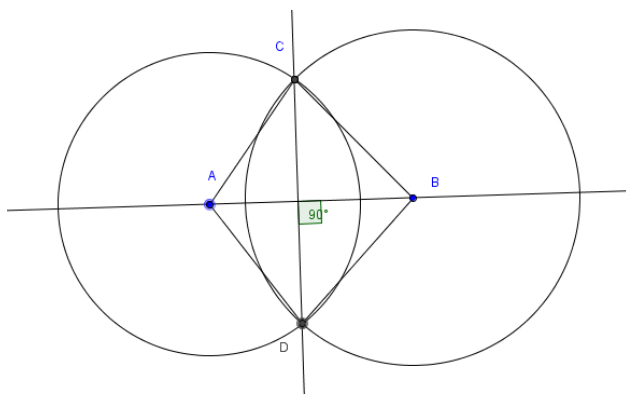


Figure 1:

Case 2: A and B are on the same side of line m . Note that we have non-convex kite ADBC with sides AC and AD being congruent and sides BC and BD being congruent. Note that line ℓ is the extension of diagonal AB and line m is the extension of diagonal CD. By the theorem for Conjecture J, we know that the diagonals of a kite cross at a right angle, which makes ℓ perpendicular to m .

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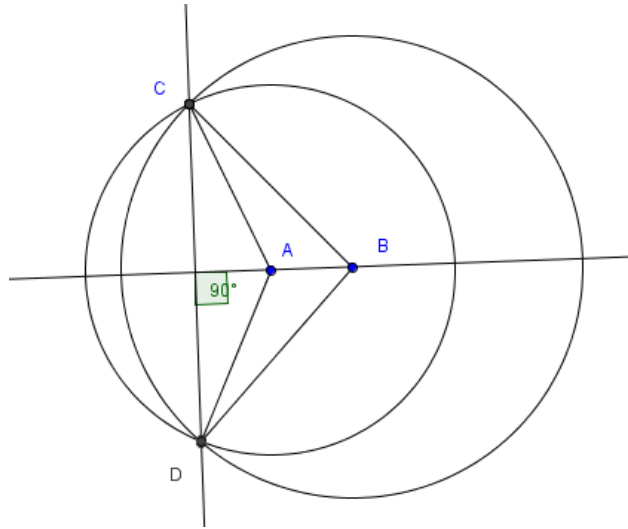


Figure 2:

Refereed by Ms. Sladana Bulic and Ms. Charlotte Brandenburg