Triangle Lying on a Circle is Right

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This is one half of Thales' Theorem.

Theorem 7.4. If AB is the diameter of a circle and C lies on the circle, then angle ACB is a right angle.

Proof. Let center of the circle be point D. Construct segment DC. By definition of a circle, segments DA, DB, and DC are congruent. Consider isosceles triangles ACD and BCD within triangle ABC. By proposition 32, angles CAD, ACD, CBD, and BCD together form two right angles. By Proposition 5 angles CAD and ACD are congruent, as are angles BCD and CBD. Since twice the angle of ACD together with twice the angle of BCD form two right angles, angle ACD and BCD together form one right angle. Because angle ACB is angle ACD and BCD together, angle ACB is a right angle. □

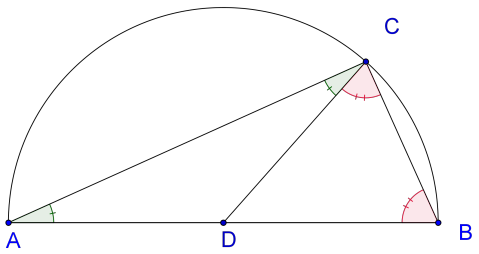

Figure 1: Triangles ADC, BDC within ABC