

Security Analysis of Proton Key Transparency

Thore Göbel
Master Thesis Final



Outline

1. Motivation

2. ProtonKT Architecture

3. Security Analysis

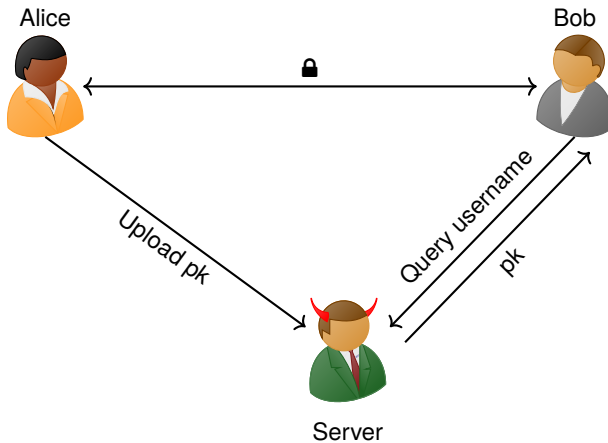
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Internet Messaging



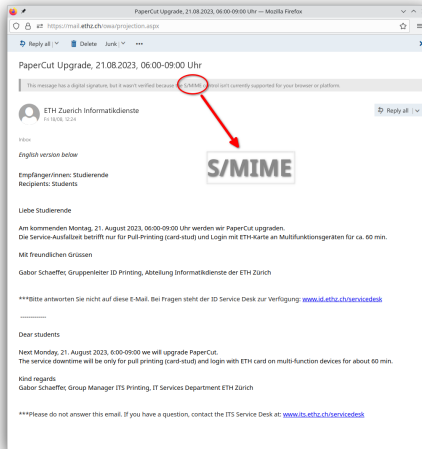
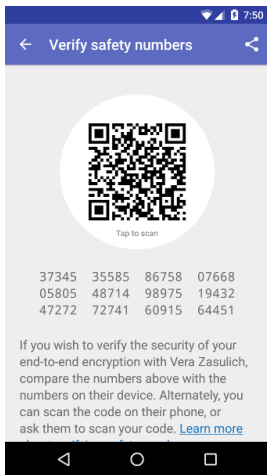
A Key Directory is just a Database



Username	Public Key
alice	pkA pkEve
bob	pkB

```
UPDATE table_keys SET pk="pkEve" WHERE username="alice";
```

Existing Solutions: Out-of-Band || Certificates



Key Transparency Goals

Goal 1: make key verification automatic

Goal 2: make server behaviour auditable (“transparent”)

Key Transparency in the Real World

- Keybase (docs [↗](#)), Zoom (Whitepaper [↗](#))
- WhatsApp (blog [↗](#), Stanford Security Seminar talk [↗](#))
- Apple iMessage (blog [↗](#))
- IETF Working Group [↗](#)
- Proton (this talk)

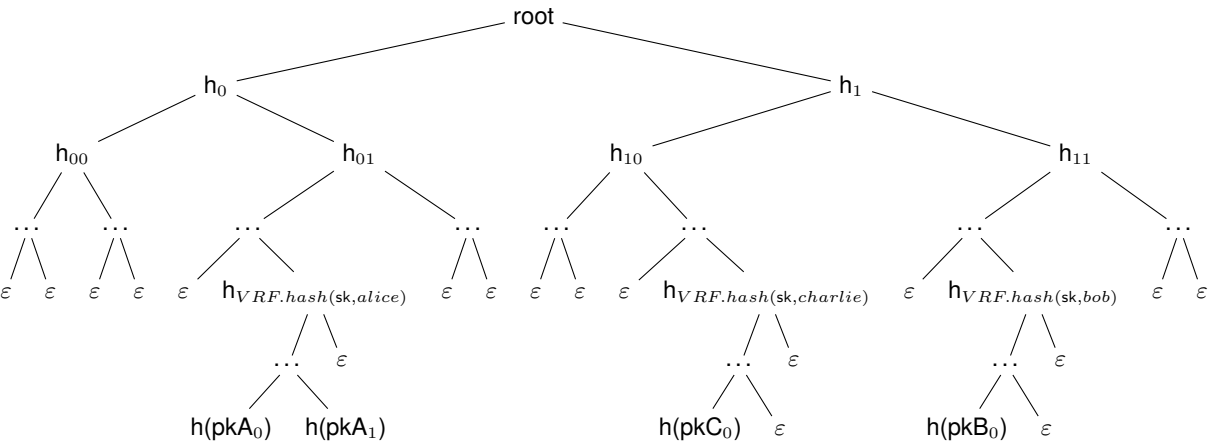
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Build a Merkle Hash Tree from the Key Directory



$$leafindex = VRF.verify(pk, label, \pi_{vrf}) \parallel rev$$

Verifiable Random Function (VRF)

$$(\mathbf{sk}, \mathbf{pk}) \leftarrow \mathit{VRF.kgen}()$$

$$\beta \leftarrow \mathit{VRF.hash}(\mathbf{sk}, \alpha)$$

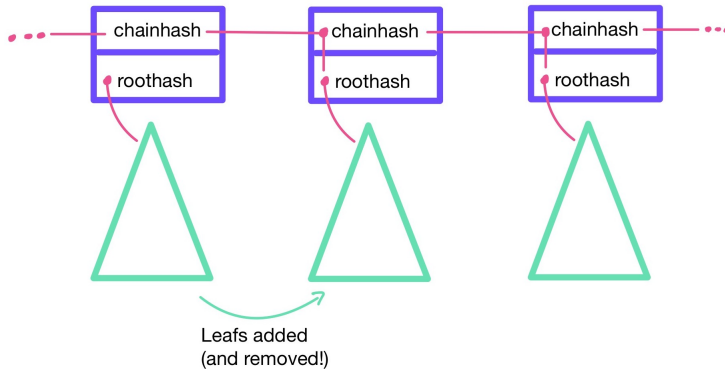
$$\pi \leftarrow \mathit{VRF.prove}(\mathbf{sk}, \alpha)$$

$$\beta/\perp \leftarrow \mathit{VRF.verify}(\mathbf{pk}, \alpha, \pi)$$

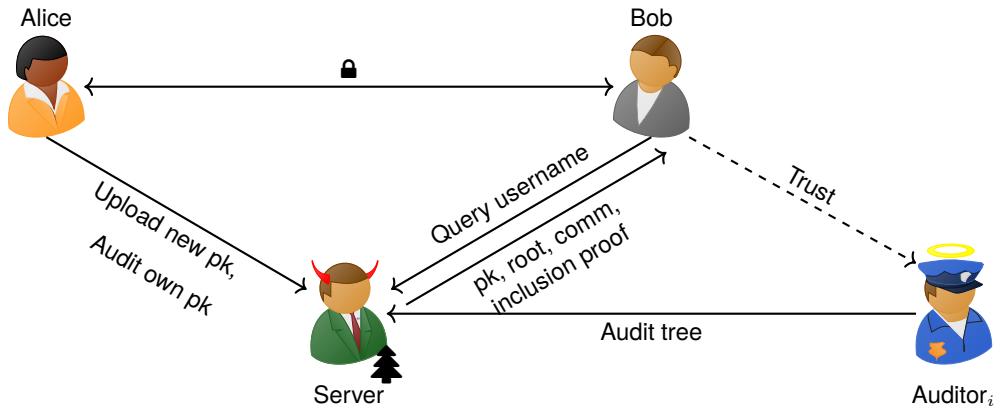
Properties: Pseudorandomness, Collision Resistance, Uniqueness

Trees Across Epochs

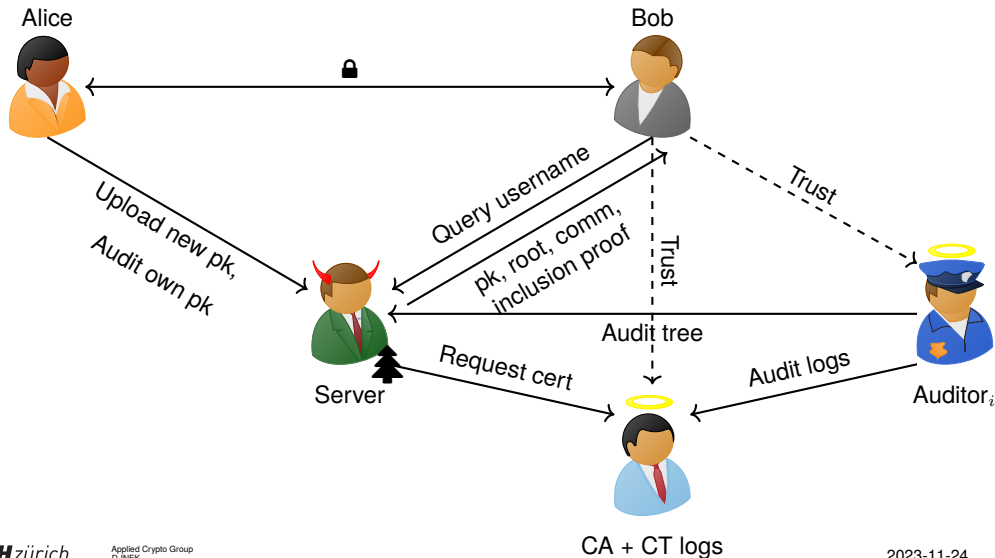
$$chainhash_i = h(chainhash_{i-1} || roothash_i)$$



System Overview and Roles



System Overview and Roles



Committing to the Tree Root

`chainhash[0:32].chainhash[32:64].timestamp.epochid.1.keytransparency.ch.`

crt.sh Identity Search

Criteria: Type: Identity Match: LIKE Search: '321.1.keytransparency.ch'

Certificates	crt.sh ID	Logged At	Not Before	Not After	Common Name	Matching Identities	Issuer Name
	10232277493	2023-08-13	2023-08-13	2023-11-11	epoch.321.1.keytransparency.ch	a78f116c473f70399a9ec6bae84f2f84.e152ba803dd34b231ad9ccd389003f03.1691888383.321.1.keytransparency.ch	C=AT, O=ZeroSSL, CN=ZeroSSL, RSA
	10232277526	2023-08-13	2023-08-13	2023-11-11	epoch.321.1.keytransparency.ch	a78f116c473f70399a9ec6bae84f2f84.e152ba803dd34b231ad9ccd389003f03.1691888383.321.1.keytransparency.ch	Domain Secure Site CA

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Tree Leaves

- $leafindex = VRF.verify(pk, label, \pi_{vrf}) \parallel rev$
- $val_{abs} = \emptyset$
- $val_{incl} = \{keylist, minEpochId\}$
- $val_{obs} = \{ObsolenceToken, minEpochId\}$

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- $val_{abs} = \emptyset$
- $val_{incl} = \{keylist, minEpochId\}$
- $val_{obs} = \{ObsolenceToken, minEpochId\}$
- $leafhash_{abs} = \varepsilon$
- $leafhash_{incl} = h(h(keylist) \parallel minEpochId)$
- $leafhash_{obs} = h(h(ObsolenceToken) \parallel minEpochId)$

Deletions

Deletion of leaf rev allowed \iff leaf rev + 1 inserted \geq 90 days ago

$\{keylist_1, minEpochId_1\}, \{ObsolenceToken_2, minEpochId_2\}, \{keylist_3, minEpochId_3\}, \dots$

ProtonKT Subprotocols (simplified)

- $\text{ProtonKT.RequestInsertion}(\text{label}, \text{keylist})$
- $\text{ProtonKT.Publish}(\{\text{label}_i, \text{keylist}_i\}_i)$

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- $\text{ProtonKT.QueryEpoch}(t)$
- $\text{ProtonKT.QueryValue}(\text{roothash}_t, \text{label})$

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- $\text{ProtonKT.QueryValue}(\text{roothash}_t, \text{label})$
- $\text{ProtonKT.SelfAudit}(\text{roothash}_t, \text{label}, \text{keylist})$
- $\text{ProtonKT.PromiseAudit}(\text{roothash}_t, \text{promises})$
- $\text{ProtonKT.ExtAudit}()$

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Security Properties

Consistency: for a given (label, rev), we agree on (τ, val) .

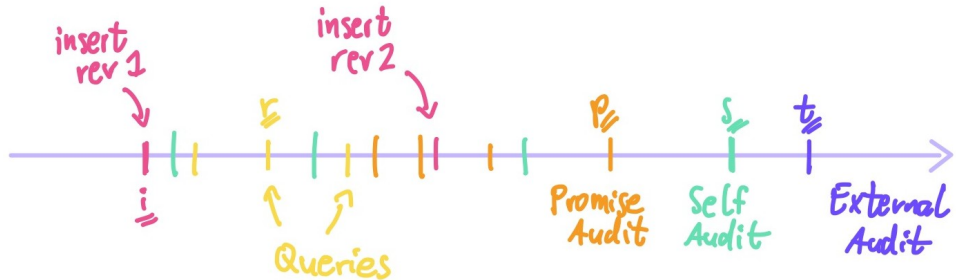
Consistency between queries and Self Audits \implies correctness of keys

Security Property: Query-to-SelfAudit Consistency

We say that ProtonKT provides *Query-to-SelfAudit Consistency*, if

- whenever there was a successful External Audit of epoch t
- and client A runs a successful Self Audit SA for its label at epoch $s \leq t$ and SA passes with $latestRev \geq rev$,
- and prior to epoch t A has run a successful Self Audit at least once every DeletionParam (e.g. every 90 days),
- and a query Q for label in epoch $r \leq t$ returned outcome $O = (\tau, rev, val)$,
- and – if Q returned O as a promise P – there was a successful Promise Audit that sees P at an epoch p with $r < p \leq t$,
- then client A agrees that (τ, rev, val) is the expected outcome for rev .

Security Property: Query-to-SelfAudit Consistency



Adversary Model

The adversary can:

- Control the network (active network adversary, Dolev-Yao). Reorder, replay, drop, insert, modify messages.
- Corrupt the KT server. Insert, modify, delete leaves in the Merkle tree.

The adversary cannot:

- Break SHA-256 collision resistance, break ECVRF uniqueness.
- Prevent External Auditors from seeing all CT log entries.

Manual Analysis of Query-to-SelfAudit Consistency

Analysis goal 1: server cannot equivocate on root hash.

- Assume two executions Q, U of $\text{ProtonKT.QueryEpoch}(t)$ accepted $\text{roothash}_t^Q \neq \text{roothash}_t^U$. Also assume an External Audit passed.
- Then there must exist $\text{chainhash}_t^Q = h(\text{chainhash}_{t-1}^Q || \text{roothash}_t^Q)$ and cert^Q , and same for U .

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- *Case 1* ($\text{chainhash}_t^Q \neq \text{chainhash}_t^U$): Then $\text{cert}^Q \neq \text{cert}^U$. But these certs must be in CT logs. Then the External Audit finds the equivocation. If it doesn't, then a CT log was malicious or the auditor does not have a global view of CT.

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- *Case 2* ($\text{chainhash}_t^Q = \text{chainhash}_t^U$): Then we have $(pch || rh) \neq (pch' || rh')$ such that $h(pch || rh) = h(pch' || rh')$. Contradiction to SHA-256 collision resistance.

CA/CT can Discredit the Server

Problem:

```
chainhash[0:32].chainhash[32:64].timestamp.epochid.1.keytransparency.ch.
```

```
chainhash'[0:32].chainhash'[32:64].timestamp.epochid.1.keytransparency.ch.
```



CA + CT logs

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`chainhash'[0:32].chainhash'[32:64].timestamp.epochid.1.keytransparency.ch.`



CA + CT logs

Possible solution:

`sig[0:32].sig[32:64].sig[64:96].sig[96:128].epochid.1.keytransparency.ch.`

Manual Analysis of Query-to-SelfAudit Consistency

Analysis goal 2: Query-to-SelfAudit (part of it)

- Assume a query and a Self Audit disagree on the outcome for rev:

$$O^Q = (\tau^Q, \text{rev}, \text{val}^Q) \neq (\tau^A, \text{rev}, \text{val}^A) = O^A$$

Also assume an External Audit passed.

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- *Case 1* ($\text{idx}^Q \neq \text{idx}^A$):

Let leaf index $\text{idx}^Q = \text{VRF.verify}(\text{sk}, \text{label}, \pi^Q) \parallel \text{rev}$, and
 $\text{idx}^A = \text{VRF.verify}(\text{sk}, \text{label}, \pi^A) \parallel \text{rev}$.

But $\text{VRF.verify}(\text{sk}, \text{label}, \pi^Q) \neq \text{VRF.verify}(\text{sk}, \text{label}, \pi^A)$ contradicts uniqueness of ECVRF.

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But $\text{VRF.verify}(\text{sk}, \text{label}, \pi^Q) \neq \text{VRF.verify}(\text{sk}, \text{label}, \pi^A)$ contradicts uniqueness of ECVRF.

- Case 2 (Q and SA at same epoch)*: By non-equivocation, Q and SA agree on roothash_t .
(Case for different epochs: omitted.)

Manual Analysis of Query-to-SelfAudit Consistency

Analysis goal 2: Query-to-SelfAudit (part of it, continued)

$$O^Q = (\tau^Q, \text{rev}, \text{val}^Q) \neq (\tau^A, \text{rev}, \text{val}^A) = O^A$$

So far: same tree root hash, same leaf index.

- *Case 3* ($\text{leafhash}_{idx}^Q \neq \text{leafhash}_{idx}^A$): Hash collision on the path to the root.

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 - *Case 4.2* ($\text{val}^Q = \text{val}^A$):

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 - *Case 4.2* ($\text{val}^Q = \text{val}^A$):
 - ▶ *Case 4.1.0* ($\tau^Q = \text{abs}, \tau^A = \text{abs}$): $O^Q = O^A$, done.
 - ▶ *Case 4.1.1* ($\tau^Q = \text{abs}, \tau^A \neq \text{abs}$): Absence has $\text{val}_{\text{abs}} = \varepsilon$ but inclusion/absence have values.
 - ▶ *Case 4.1.2* ($\tau^Q \neq \text{abs}, \tau^A = \text{abs}$): same.

Manual Analysis of Query-to-SelfAudit Consistency

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 - ▶ *Case 4.1.2* ($\tau^Q \neq \text{abs}, \tau^A = \text{abs}$): same.
 - ▶ *Case 4.1.3* ($\tau^Q = \text{incl}, \tau^A = \text{obs}$): Then $\text{val}^Q \stackrel{\text{Def}}{=} \{\text{keylist}, \text{minEpochId}\} = \{\text{ObsolenceToken}, \text{minEpochId}\} \stackrel{\text{Def}}{=} \text{val}^A$.
I.e. Q interprets the first field as a keylist and A as an ObsolenceToken.
This is a contradiction to the fact that the algorithms check that the keylist is JSON-encoded and that ObsolenceToken is a non-empty hex value.
 - ▶ *Case 4.1.4* ($\tau^Q = \text{obs}, \tau^A = \text{incl}$): same.

Better Leaf Hashes

- $leafhash_{incl} = h(h(keylist) || minEpochId)$
- $leafhash_{obs} = h(h(ObsolescenceToken) || minEpochId)$

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- $leafhash_{incl} = h(h(keylist) || minEpochId)$
- $leafhash_{obs} = h(h(ObsolescenceToken) || minEpochId)$
- $leafhash_{incl}^{proposed} = h(h("1" || keylist) || minEpochId)$
- $leafhash_{obs}^{proposed} = h(h("2" || ObsolescenceToken) || minEpochId)$

Server can Delay Promise Audit

```
if (promise.expectedMinEpochID > currentEpoch.EpochID) {  
    return LocalStorageAuditStatus.RetryLater;  
}
```

```
/* ... only further down the Maximum Merge Delay is checked ... */  
if (isTimestampTooOld(promise.creationTimestamp)) {  
    throwKTErrror('promise was ignored beyond MMD');  
}
```

Formal Analysis of Query-to-SelfAudit Consistency



- Used new Tamarin features (subterm, natural numbers).
- Tried to prove Query-to-SelfAudit Consistency but ran into a limitation of how Tamarin handles induction.
- Still a useful exercise to understand the protocol better, to find gaps in your understanding.

Conclusion

- ProtonKT Specification
- Adversary Model, Security Property
- Manual Analysis
- Formal Analysis with dead-end
- Recommendations: sign chainhash in CT to prevent discrediting, make type explicit in leaf hash, server could delay Promise Audit forever

Questions?

Formal Analysis of Query-to-SelfAudit Consistency



```
rule CT_Insert:
  [ In(<epoch_id, chainhash>) ]
  --[ CtInsertChainhash(epoch_id, chainhash) ]->
  [ !CT(epoch_id, chainhash) ]
```

Tamarin Prover

Model 1: Tree as Persistent Facts.

```
!TreeLeaf($label, val, %rev, %min_epoch_id)
```

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Model 2: Trees as Terms.

```
roothash = h( <'head', h(ut_0), h(ut_1), ..., h(ut_n), 'tail'> )  
ut = < $label, <%n, val_n>, ..., <%3, val_3>, <%2, 'empty'>, 'rest' >
```

⇒ Only 3 levels, not binary, but more tree-ish.

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Model 1: Tree as Persistent Facts.

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```

⇒ Only 3 levels, not binary, but more tree-ish.

We didn't find a proof, we ran into a limitation of Tamarin's induction mechanism.

Proving Query-to-SelfAudit Consistency with Tamarin

Problem: Query and Self Audit can happen in different epochs $r \neq s$, w.l.o.g. $r < s$.

Recall: $chainhash_i = h(chainhash_{i-1} || roothash_i)$

Thus: $chainhash_r \sqsubset chainhash_s$

To reason about the leaves, we need to reason about how the tree(s) evolved, thus we need to reason about the chainhashes.

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Lemma:

"All $ch1 \ ch2 \ #i \ #j. Ch(ch1)@i \ \& \ Ch(ch2)@j \ ==> \ ch1 \ << \ ch2$ "

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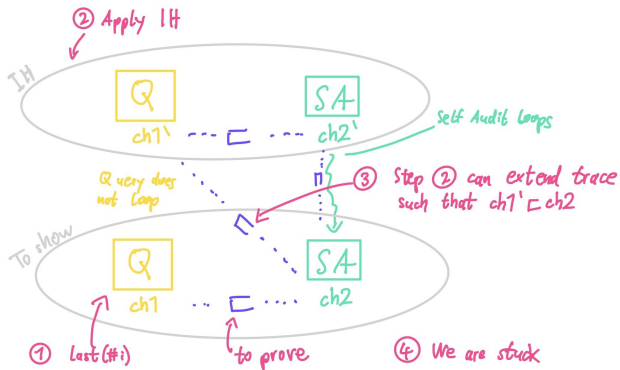
Induction hypothesis:

"All $ch1\ ch2\ \#i\ \#j. Ch(ch1)@i \ \&\ Ch(ch2)@j \implies ch1 \ll ch2 \mid last(\#i) \mid last(\#j)$ "

We cannot avoid induction because Self Audit and External Audit loop (over revisions and over epochs).

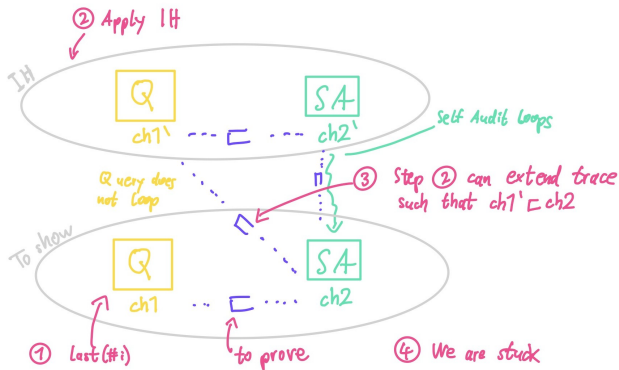
Case Split on Induction

IH: "All $ch1\ ch2\ \#i\ \#j. Ch(ch1)@i \ \& \ Ch(ch2)@j \implies ch1 \ll ch2 \mid last(\#i) \mid last(\#j) "$



Case Split on Induction

IH: "All ch1 ch2 #i #j. Ch(ch1)@i & Ch(ch2)@j ==> ch1 << ch2 | last(#i) | last(#j)"



We would like:

"All ch1 ch2 #j. Ch(ch2)@j & ch1 << ch2 ==> Ex #i. Ch(ch1)@i"

But ch1 is not guarded, i.e. invalid Tamarin syntax.