

For the jupyter notebook we use this octave kernel: https://github.com/Calysto/octave_kernel

```
pip install octave_kernel
cd <root of the project>
jupyter notebook
```

```
In [1]: format longE

# setup the path to include the 'utils' directory
directory = pwd
addpath(genpath(directory))
```

```
directory = /home/thodkatz/repos/personal/math-optimization
```

```
In [2]: # define rosenbrock function

function f = rosen_sym()
    syms x y
    f = 100*(y-x^2)^2 + (1-x)^2;
end
```

```
In [3]: search_x = -1.8:0.1:1.2;
search_y = -1.8:0.1:1.2;
[xmin, fmin] = newton(rosen_sym, [-1.8,-1.8]', 'bisection_wolfe_weak', search_x, search_y)
```

Symbolic pkg v3.1.1: Python communication link active, SymPy v1.11.1.

STARTED Line search using newton

ENDED Line search using newton

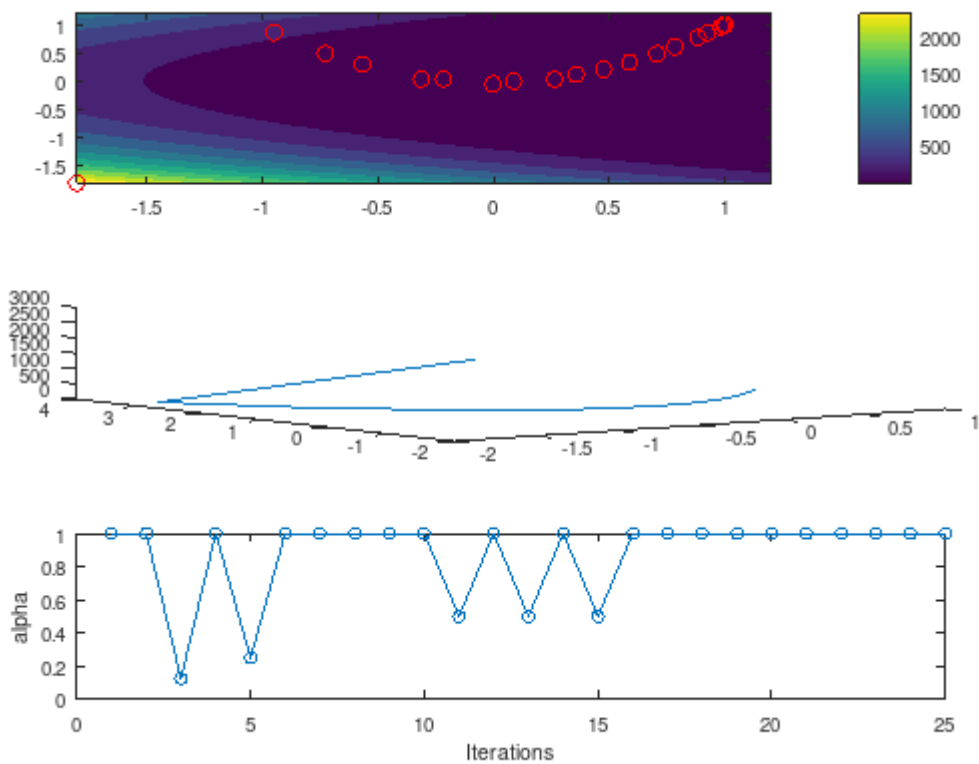
xmin =

9.999999999816617e-01

9.999999999576692e-01

fmin = 3.533228612015991e-21

iter = 2.500000000000000e+01

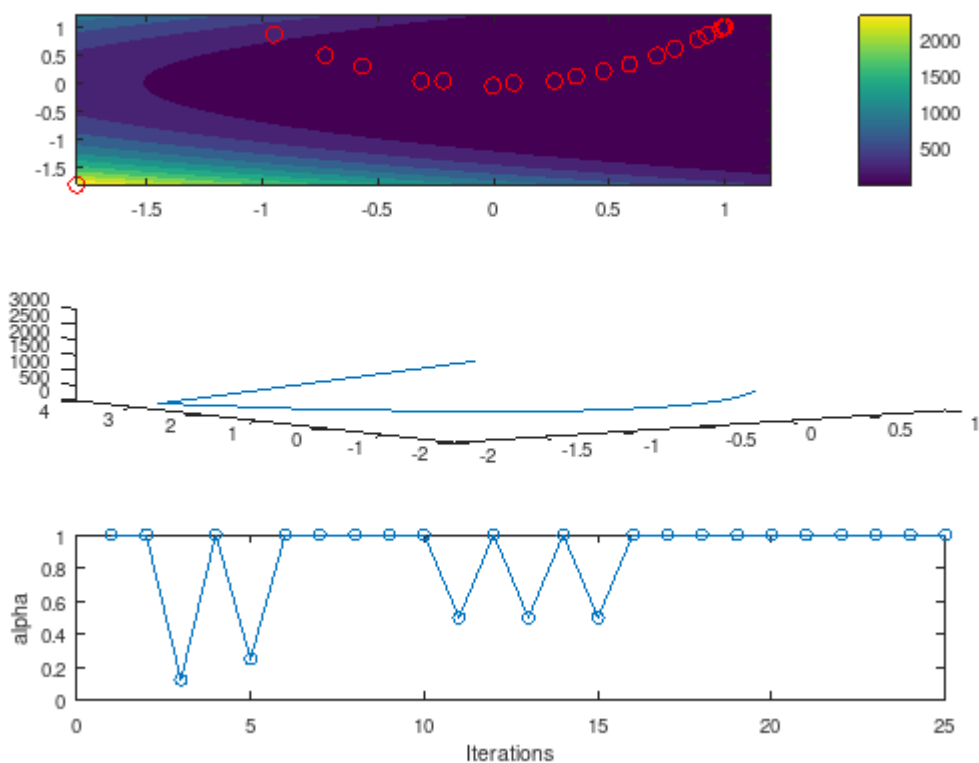


```
In [4]: search_x = -1.8:0.1:1.2;
search_y = -1.8:0.1:1.2;
[xmin, fmin] = newton(rosen_sym, [-1.8,-1.8]', 'backtracking_armijo', search_x, search_y)
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

9.999999999816617e-01
 9.999999999576692e-01

fmin = 3.533228612015991e-21
 iter = 2.500000000000000e+01

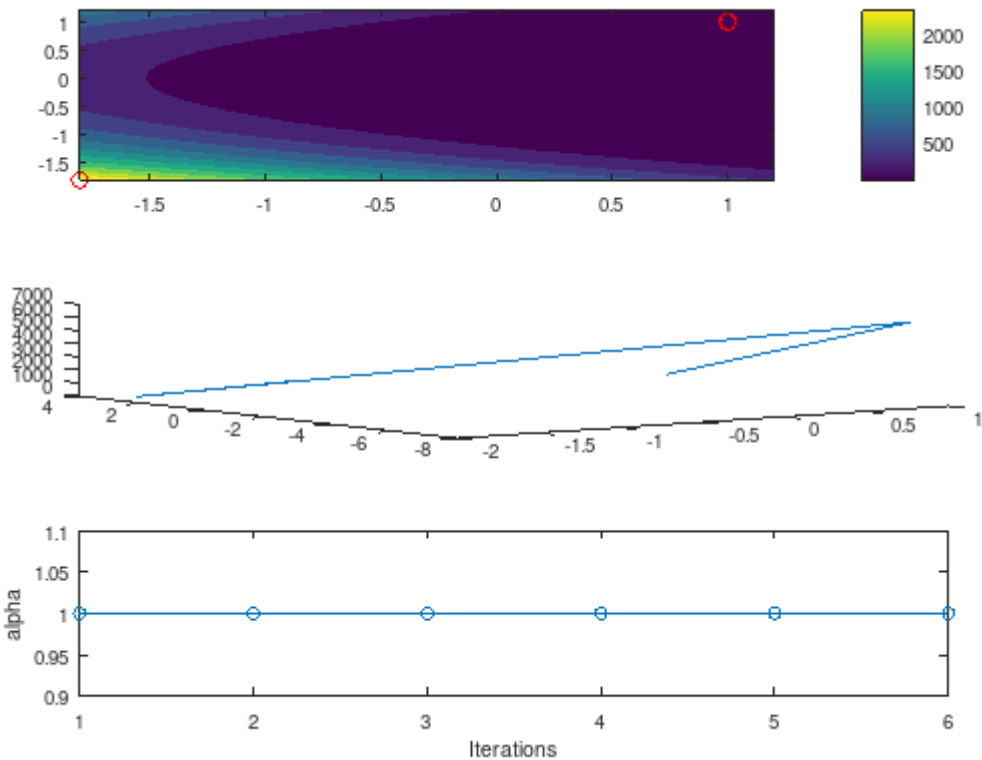


```
In [5]: search_x = -1.8:0.1:1.2;
search_y = -1.8:0.1:1.2;
[xmin, fmin] = newton(rosen_sym, [-1.8,-1.8]', 'none', search_x, search_y);
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

```
9.999999999999893e-01
9.999999999999787e-01
```

```
fmin = 1.135959703518257e-28
iter = 6.000000000000000e+00
```

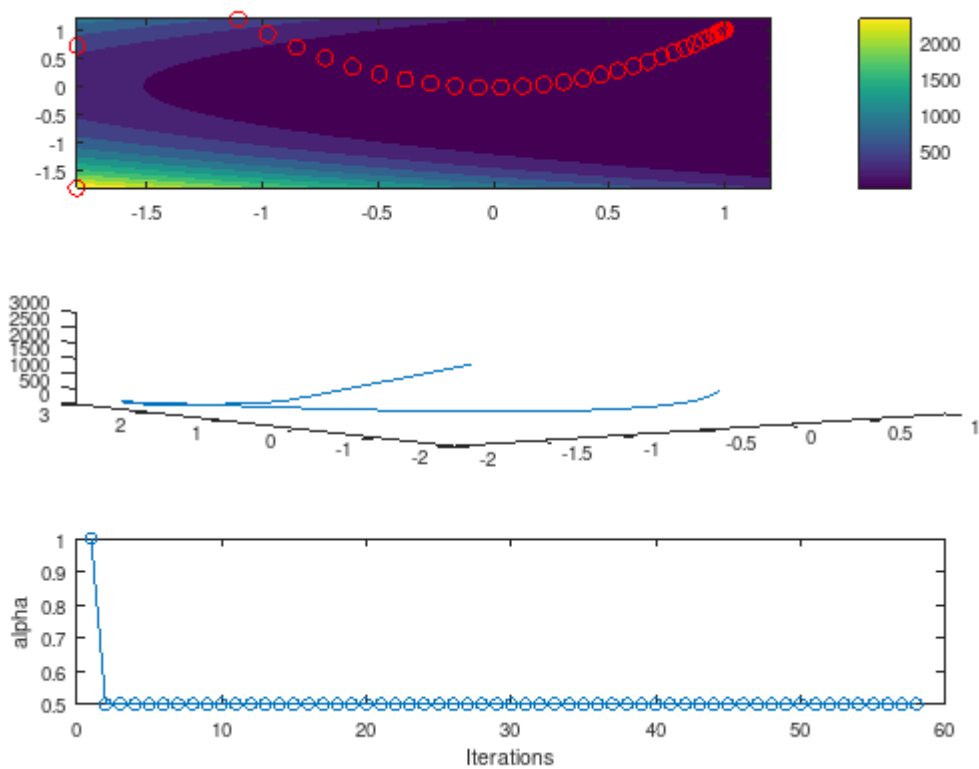


```
In [6]: search_x = -1.8:0.1:1.2;
search_y = -1.8:0.1:1.2;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = newton(rosen_sym, [-1.8,-1.8]', 'wolfe_strong', search_x, search_y, c,
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

```
9.999999547008681e-01
9.999999073667162e-01
```

```
fmin = 2.466142820222457e-15
iter = 5.800000000000000e+01
```

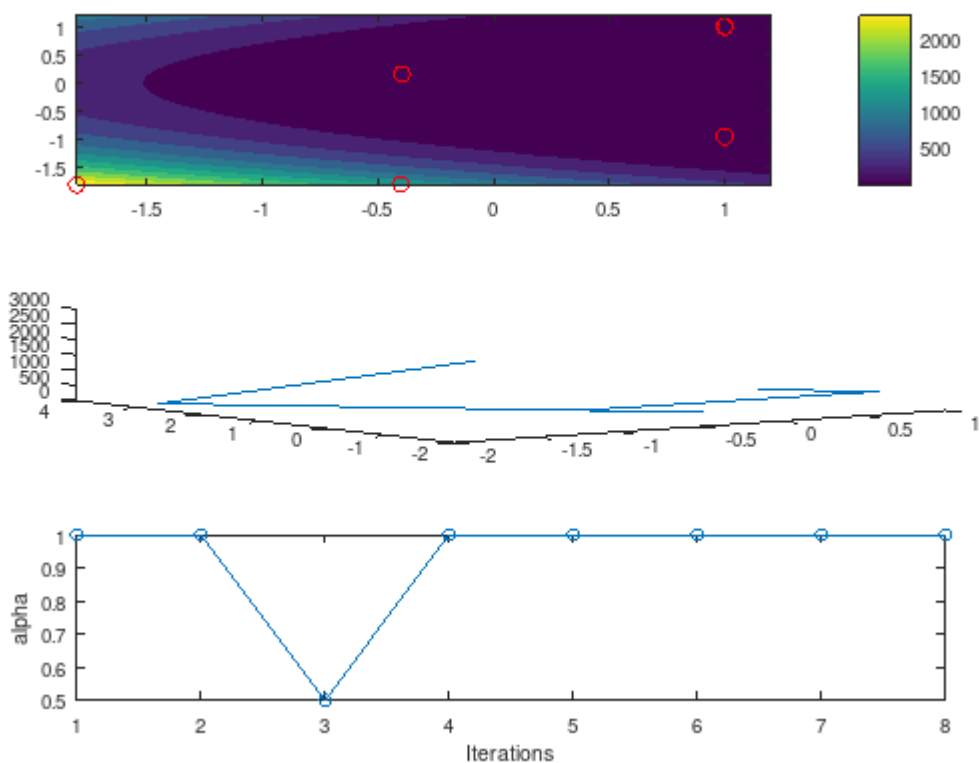


```
In [7]: search_x = -1.8:0.1:1.2;
search_y = -1.8:0.1:1.2;
[xmin, fmin] = newton(rosen_sym, [-1.8,-1.8]', 'nonmonotone_backtracking_armijo', sea
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

9.999999999998419e-01
 9.999999999996839e-01

fmin = 2.499550151618695e-26
 iter = 8.000000000000000e+00

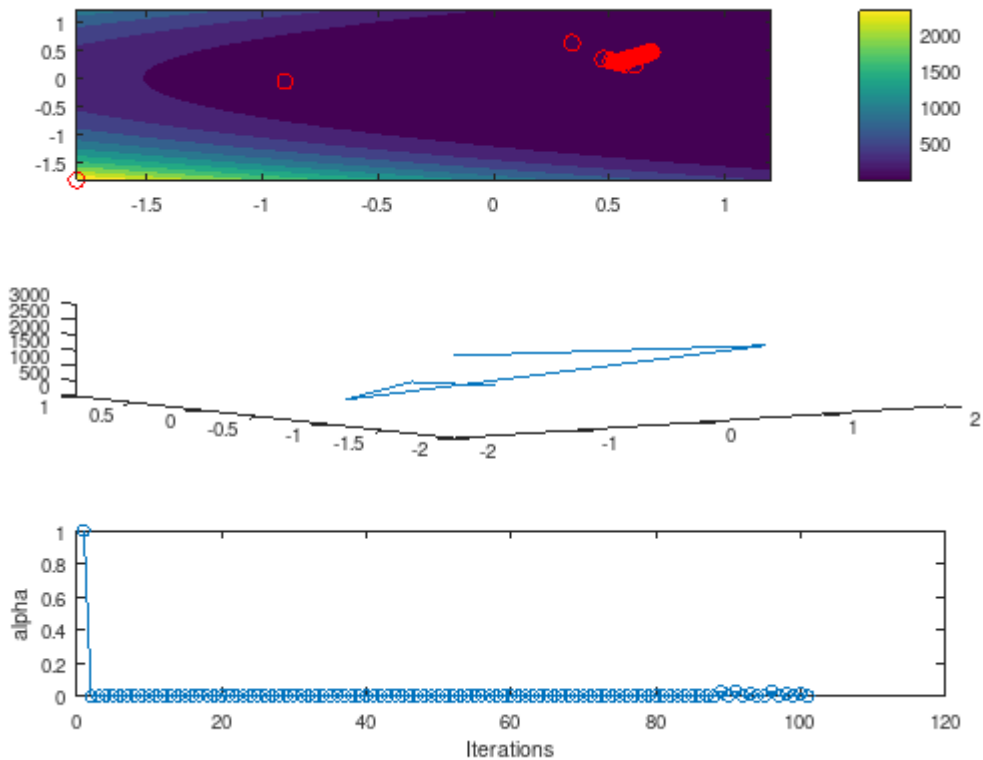


```
In [8]: search_x = -1.8:0.1:1.2;
search_y = -1.8:0.1:1.2;
[xmin, fmin] = steepest_descent(rosen_sym, [-1.8,-1.8]', 'bisection_wolfe_weak', sear
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

6.830308556342202e-01
 4.645442633745788e-01

fmin = 1.008642102262282e-01
 iter = 1.000000000000000e+02

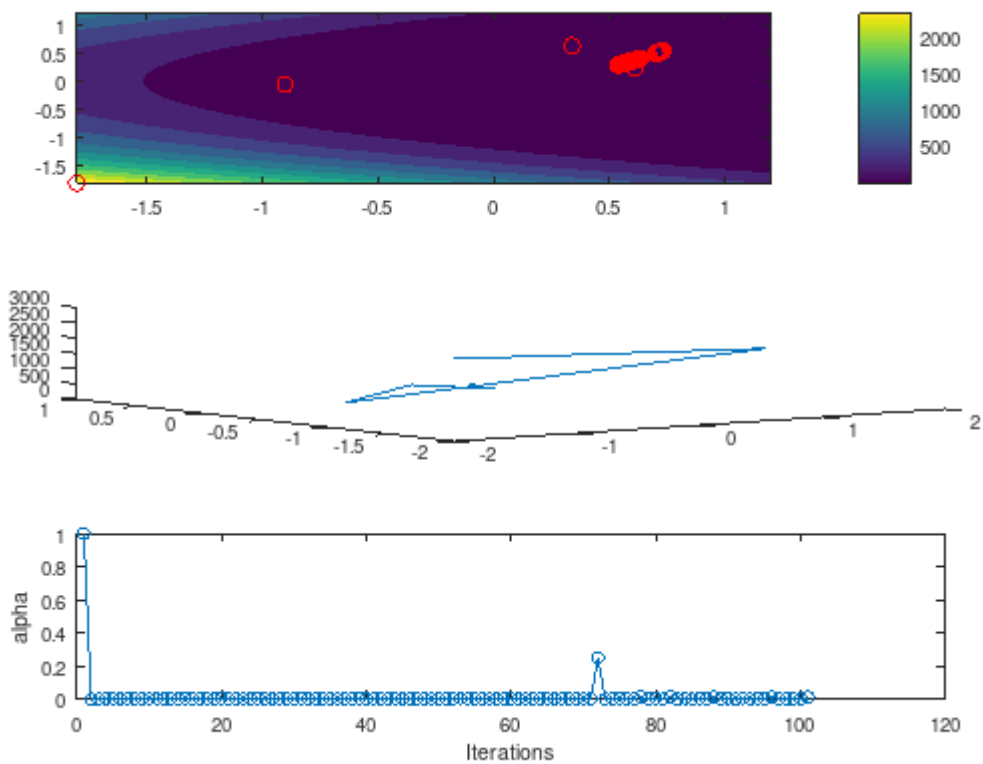


```
In [9]: search_x = -1.8:0.1:1.2;
search_y = -1.8:0.1:1.2;
[xmin, fmin] = steepest_descent(rosen_sym, [-1.8,-1.8]', 'backtracking_armijo', sear
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

7.314832480328023e-01
 5.367666412124242e-01

fmin = 7.238987188855454e-02
 iter = 1.000000000000000e+02



```
In [10]: search_x = -1.8:0.1:1.2;
search_y = -1.8:0.1:1.2;
[xmin, fmin] = steepest_descent(rosen_sym, [-1.8,-1.8]', 'none', search_x, search_y);
```

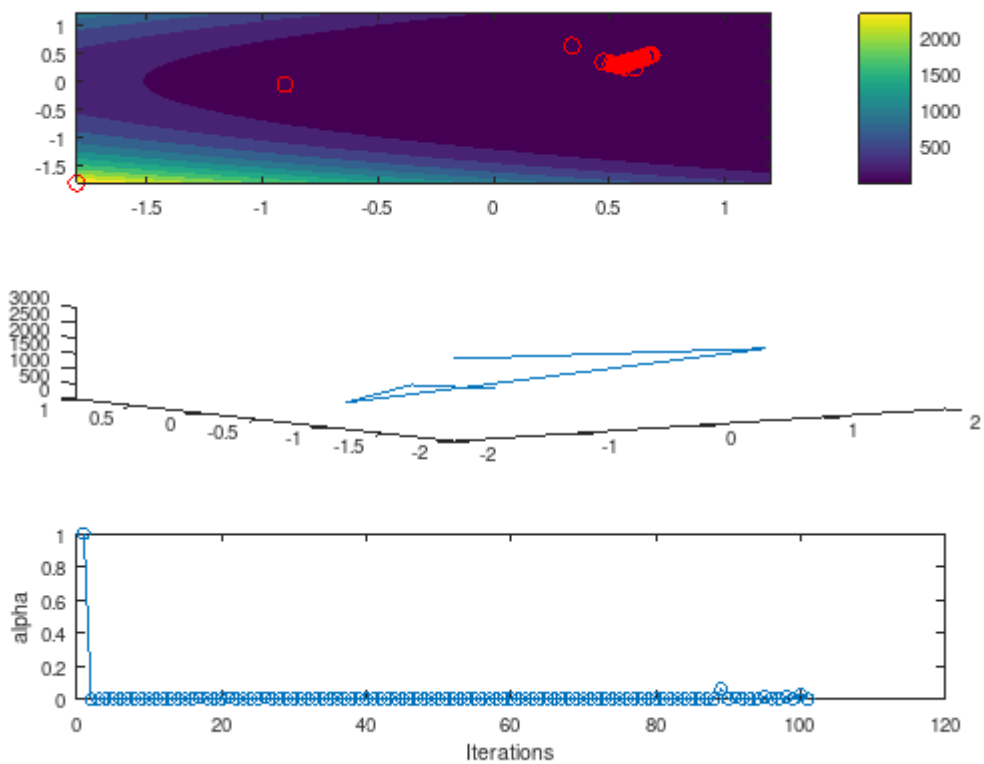
STARTED Line search using steepest descent
error: Failed to converge. Inf value reached
error: called from
steepest_descent at line 47 column 13

```
In [11]: search_x = -1.8:0.1:1.2;
search_y = -1.8:0.1:1.2;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = steepest_descent(rosen_sym, [-1.8,-1.8]', 'wolfe_strong', search_x, se
```

STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =

6.826021505276559e-01
4.636937784980137e-01

fmin = 1.012485080504488e-01
iter = 1.000000000000000e+02

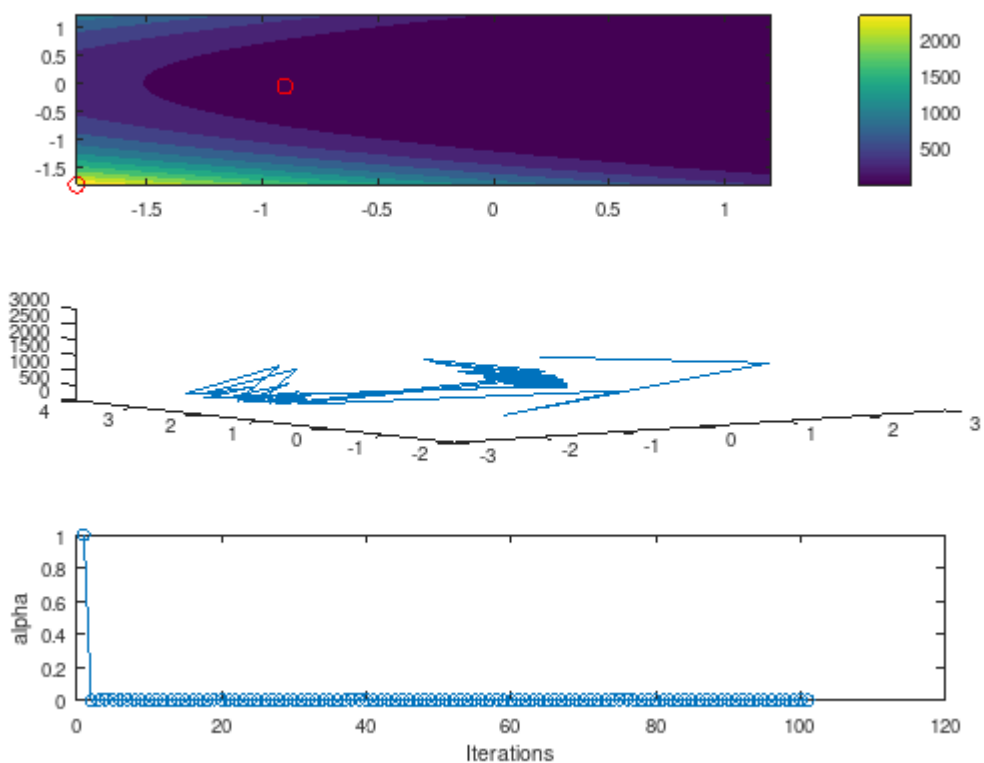


```
In [12]: search_x = -1.8:0.1:1.2;
search_y = -1.8:0.1:1.2;
[xmin, fmin] = steepest_descent(rosen_sym, [-1.8,-1.8]', 'nonmonotone_backtracking_ar
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

1.360838364594753e+00
 2.372879081831821e+00

fmin = 2.727409876821381e+01
 iter = 1.000000000000000e+02



In [3]: `% f(x,y) = x^2 + 4y^2 + 2xy -- fmin = 0 at x = 0, y = 0 -- Search domain: -3 ≤ x, y ≤`

```
function f = f1_sym()
    syms x y
    f = x^2 + 4*y^2 + 2*x*y;
end
```

In [4]: `search_x = -3:0.2:0.4;`
`search_y = -3:0.2:0.4;`
`[xmin, fmin] = newton(f1_sym, [-3,-3]', 'bisection_wolfe_weak', search_x, search_y)`

Symbolic pkg v3.1.1: Python communication link active, SymPy v1.11.1.

STARTED Line search using newton

ENDED Line search using newton

xmin =

-1.332267629550188e-15

8.881784197001252e-16

fmin = 2.563797941968288e-30

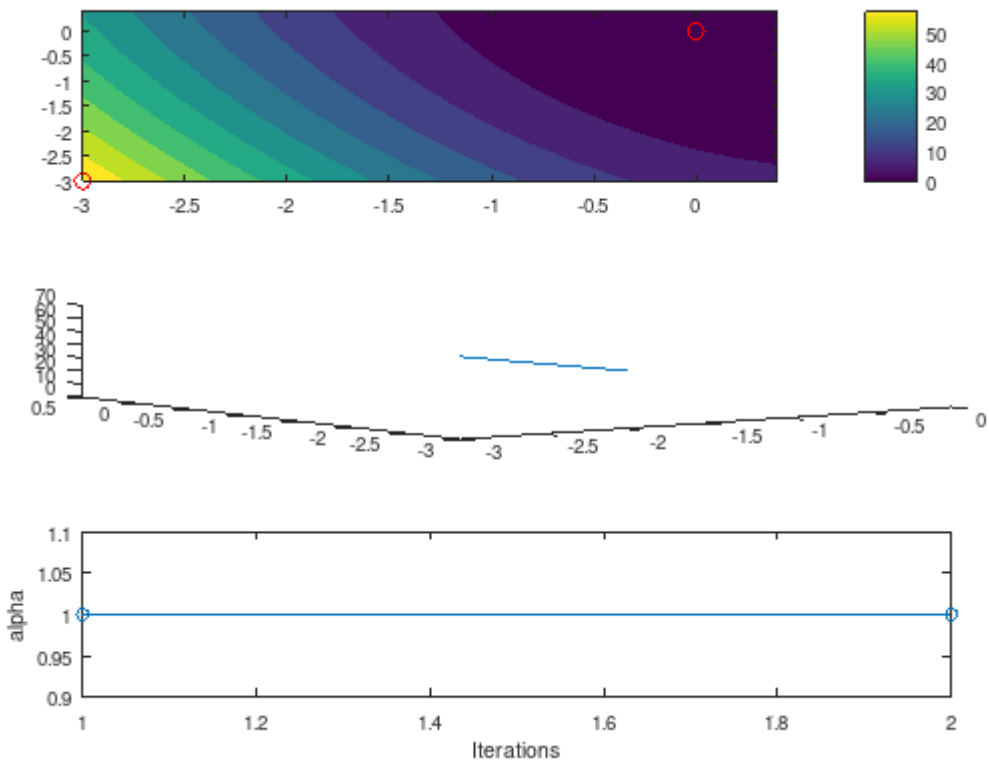
iter = 2.000000000000000e+00

xmin =

-1.332267629550188e-15

8.881784197001252e-16

fmin = 2.563797941968288e-30



In [5]: `search_x = -3:0.2:0.4;`
`search_y = -3:0.2:0.4;`
`[xmin, fmin] = newton(f1_sym, [-3,-3]', 'backtracking_armijo', search_x, search_y)`

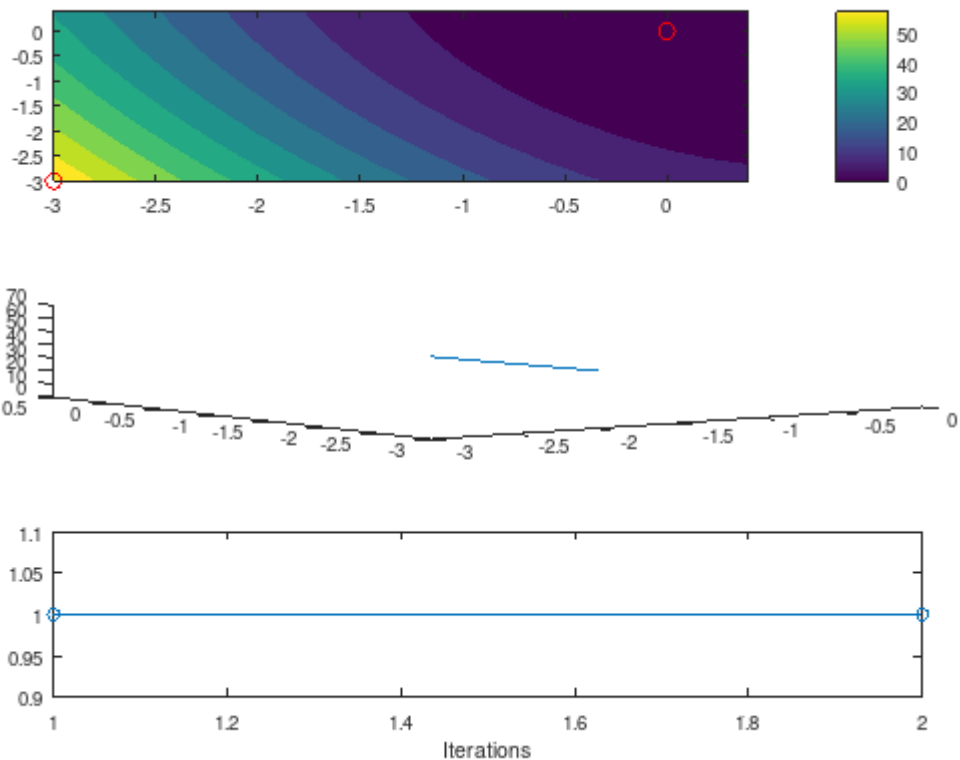
STARTED Line search using newton
ENDED Line search using newton
xmin =

-1.332267629550188e-15
8.881784197001252e-16

fmin = 2.563797941968288e-30
iter = 2.000000000000000e+00
xmin =

-1.332267629550188e-15
8.881784197001252e-16

fmin = 2.563797941968288e-30



```
In [6]: search_x = -3:0.2:0.4;  
search_y = -3:0.2:0.4;  
[xmin, fmin] = newton(f1_sym, [-3,-3]', 'none', search_x, search_y)
```

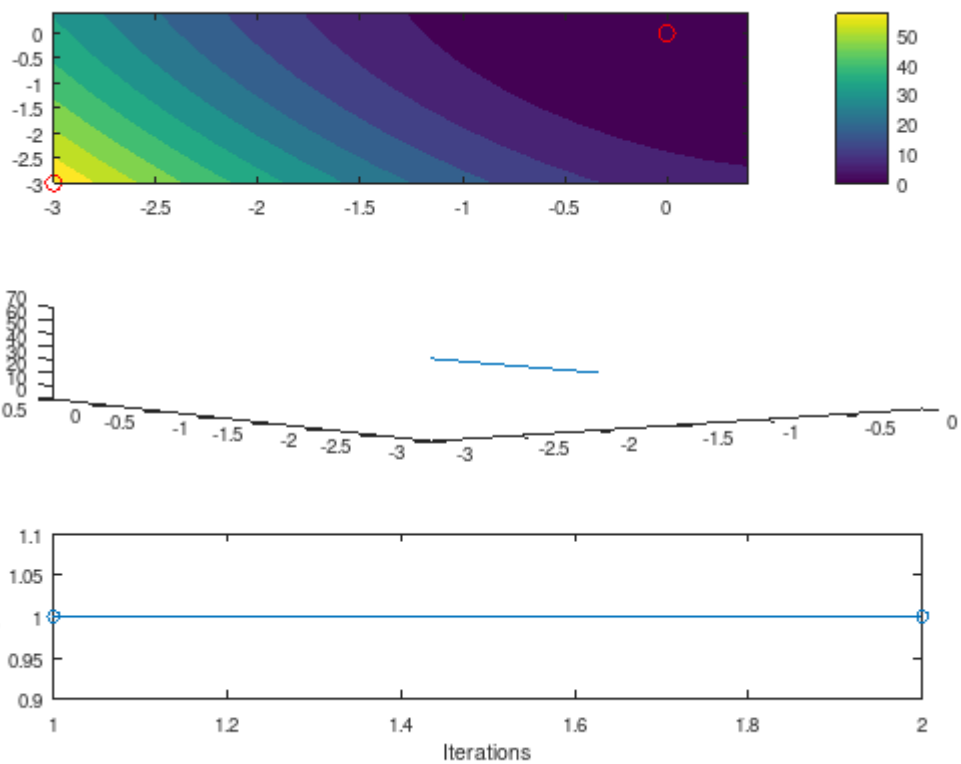
STARTED Line search using newton
ENDED Line search using newton
xmin =

-1.332267629550188e-15
8.881784197001252e-16

fmin = 2.563797941968288e-30
iter = 2.000000000000000e+00
xmin =

-1.332267629550188e-15
8.881784197001252e-16

fmin = 2.563797941968288e-30



```
In [7]: search_x = -3:0.2:0.4;
search_y = -3:0.2:0.4;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = newton(f1_sym, [-3,-3]', 'wolfe_strong', search_x, search_y, c, rho)
```

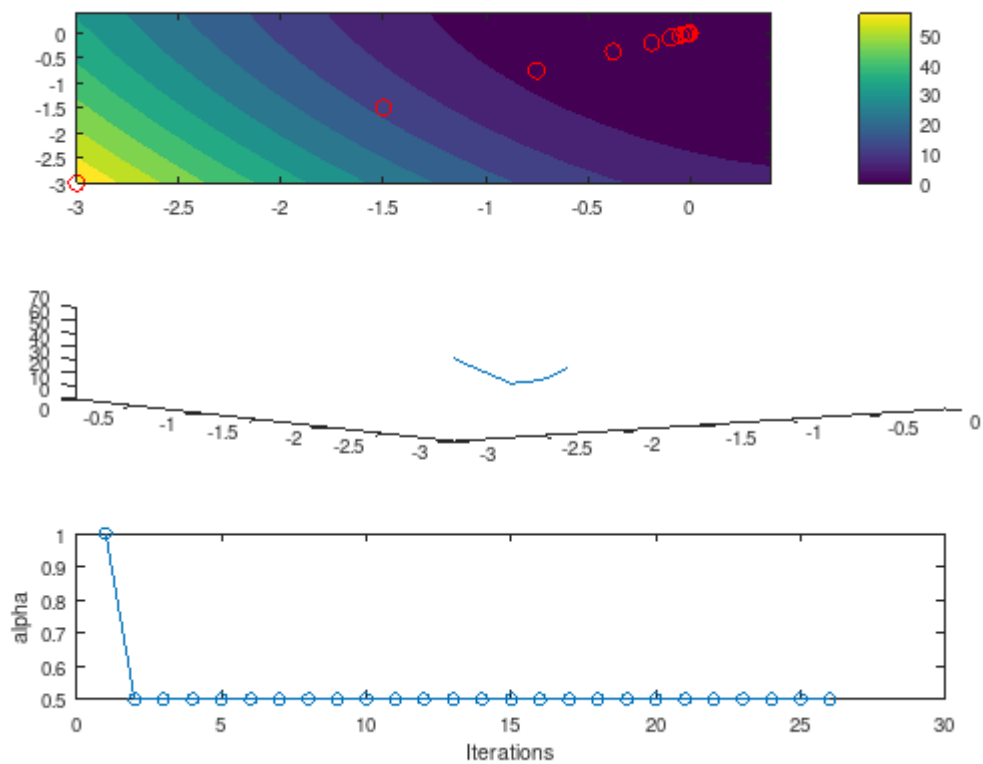
STARTED Line search using newton
 ENDED Line search using newton
 xmin =

-8.940696716308723e-08
 -8.940696716308500e-08

fmin = 5.595524044110751e-14
 iter = 2.600000000000000e+01
 xmin =

-8.940696716308723e-08
 -8.940696716308500e-08

fmin = 5.595524044110751e-14



```
In [8]: search_x = -3:0.2:0.4;
search_y = -3:0.2:0.4;
[xmin, fmin] = newton(f1_sym, [-3,-3]', 'nonmonotone_backtracking_armijo', search_x,
```

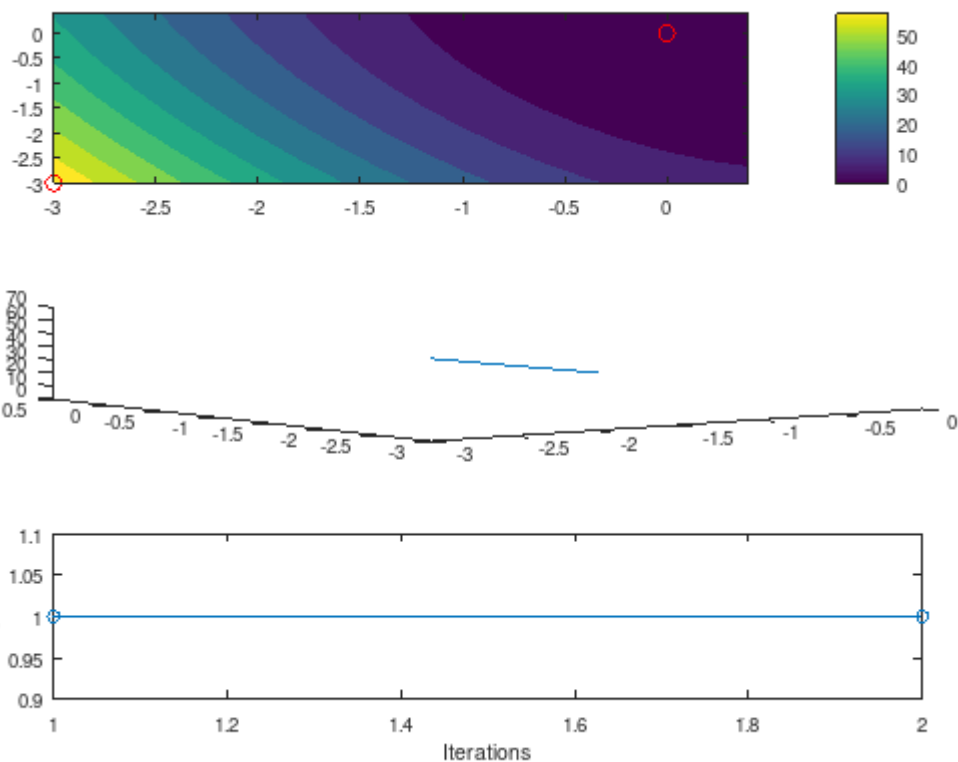
```
STARTED Line search using newton
ENDED Line search using newton
xmin =
```

```
-1.332267629550188e-15
8.881784197001252e-16
```

```
fmin = 2.563797941968288e-30
iter = 2.000000000000000e+00
xmin =
```

```
-1.332267629550188e-15
8.881784197001252e-16
```

```
fmin = 2.563797941968288e-30
```



```
In [9]: search_x = -3:0.2:0.4;
search_y = -3:0.2:0.4;
[xmin, fmin] = steepest_descent(f1_sym, [-3,-3]', 'bisection_wolfe_weak', search_x, s
```

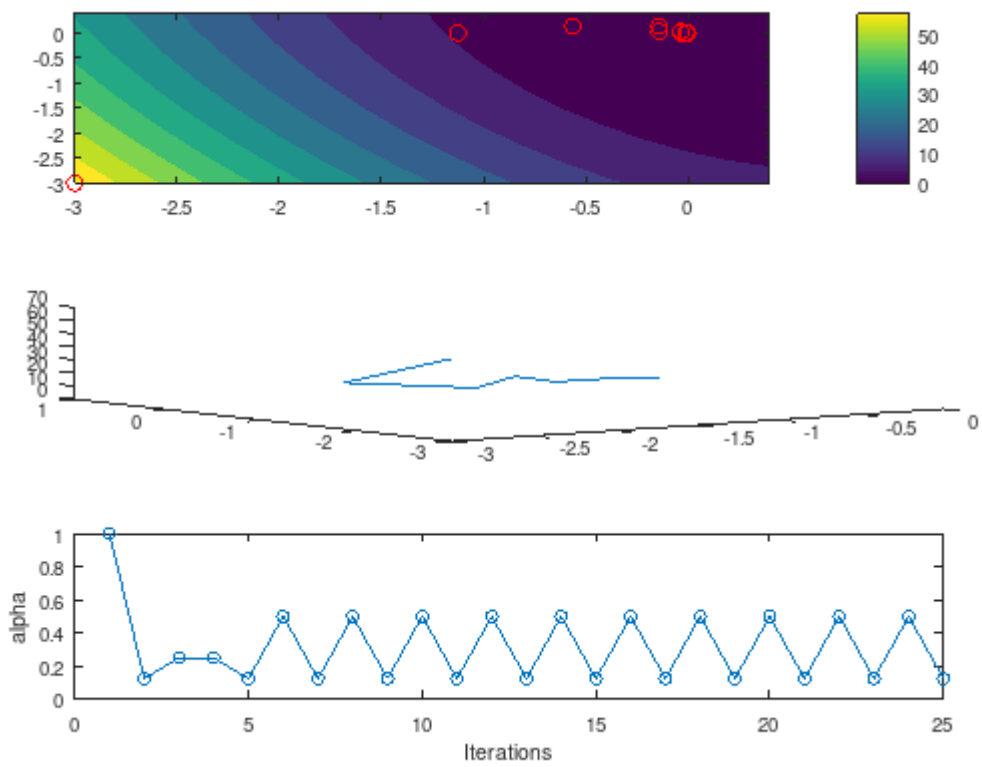
```
STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =
```

```
-5.364418029785156e-07
1.341104507446289e-07
```

```
fmin = 2.158273559871304e-13
iter = 2.500000000000000e+01
xmin =
```

```
-5.364418029785156e-07
1.341104507446289e-07
```

```
fmin = 2.158273559871304e-13
```

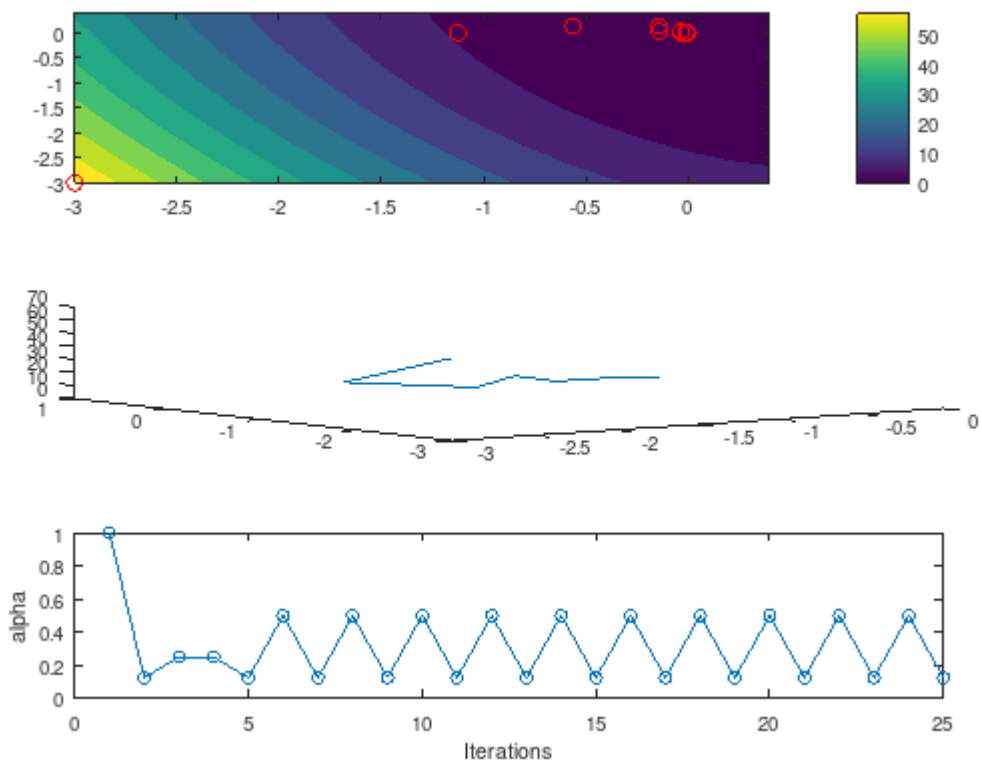


```
In [20]: search_x = -3:0.2:0.4;
search_y = -3:0.2:0.4;
[xmin, fmin] = steepest_descent(f1_sym, [-3,-3]', 'backtracking_armijo', search_x, se
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

-5.364418029785156e-07
 1.341104507446289e-07

fmin = 2.158273559871304e-13
 iter = 2.500000000000000e+01



```
In [10]: search_x = -3:0.2:0.4;
search_y = -3:0.2:0.4;
[xmin, fmin] = steepest_descent(f1_sym, [-3,-3]', 'none', search_x, search_y);
```

```
STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =
```

```
-1.406335921346234e+88
-4.644812019489721e+88
```

```
fmin = 1.013392274910718e+178
iter = 1.000000000000000e+02
error: set: "cameraviewangle" must be finite
error: called from
    plot3 at line 371 column 10
    plot_line_search1 at line 40 column 9
    steepest_descent at line 79 column 9
```

Inline plot failed, consider trying another graphics toolkit

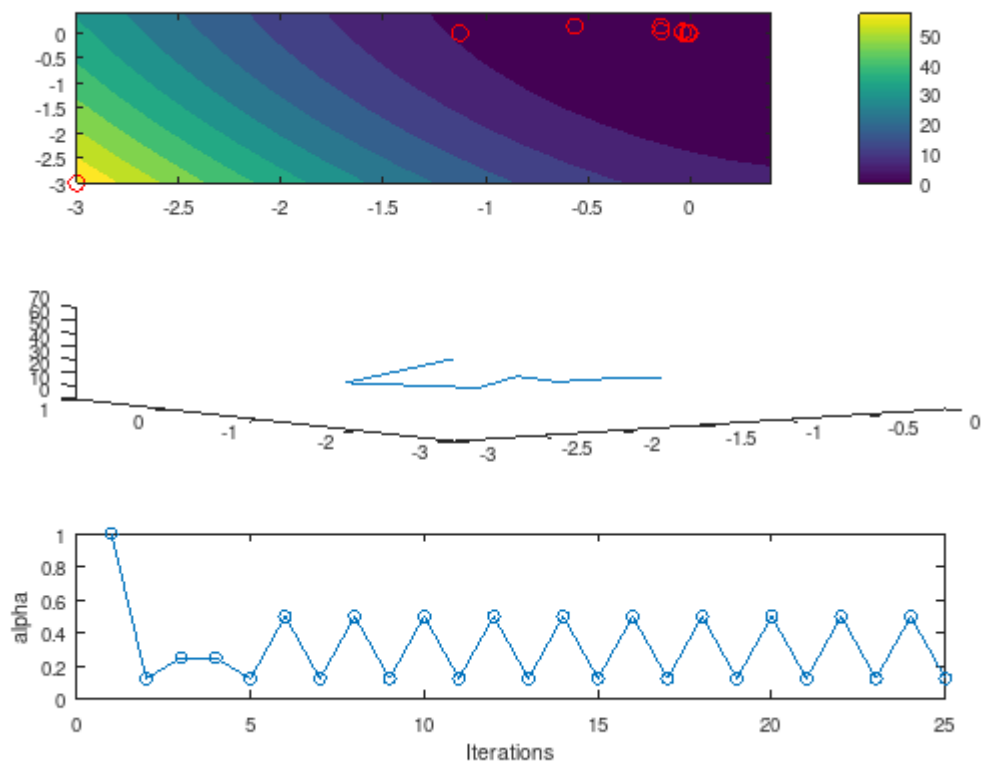
```
error: set: "cameraviewangle" must be finite
error: called from
    _make_figures>safe_print at line 125 column 7
    _make_figures at line 49 column 13
```

```
In [22]: search_x = -3:0.2:0.4;
search_y = -3:0.2:0.4;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = steepest_descent(f1_sym, [-3,-3]', 'wolfe_strong', search_x, search_y,
```

```
STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =
```

```
-5.374485741426969e-07
1.166874153963192e-07
```

```
fmin = 2.178878114452140e-13
iter = 2.500000000000000e+01
```

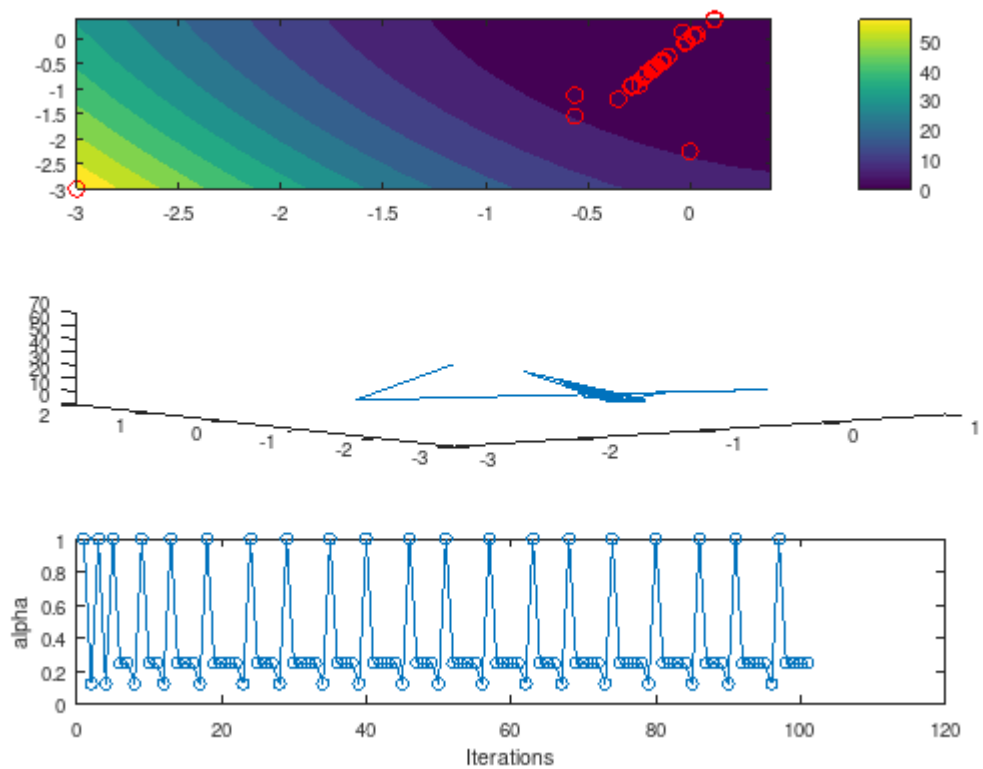


```
In [23]: search_x = -3:0.2:0.4;
search_y = -3:0.2:0.4;
[xmin, fmin] = steepest_descent(f1_sym, [-3,-3]', 'nonmonotone_backtracking_armijo',
```

```
STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =
```

```
-1.881054424659777e-01
-6.212700727014284e-01
```

```
fmin = 1.813018234255819e+00
iter = 1.000000000000000e+02
```



In [11]: `% f(x,y) = (x + 2y - 7)^2 + (2x + y - 5)^2 -- fmin = 0 at x = 1, y = 3 -- Search doma`

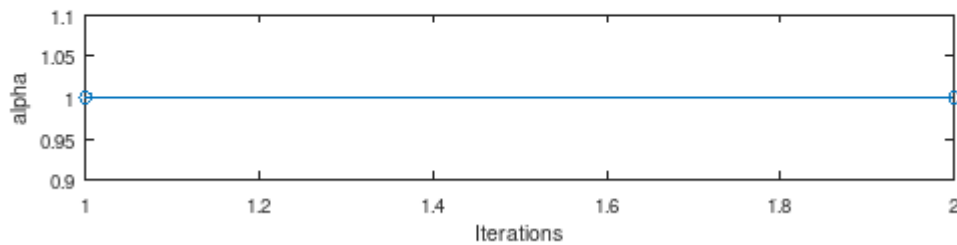
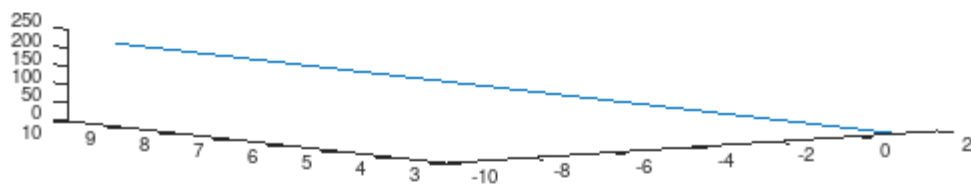
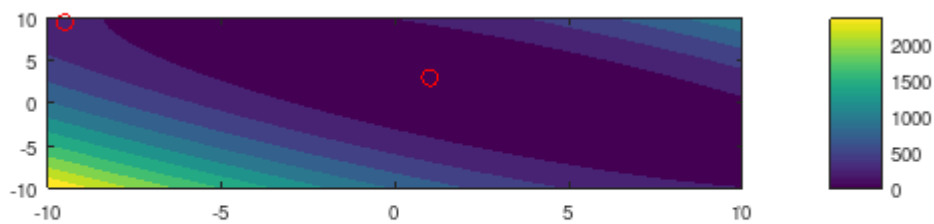
```
function f = f2_sym()
    syms x y
    f = (x+2*y-7)^2 + (2*x + y - 5)^2;
end
```

In [12]: `search_x = -10:0.5:10;`
`search_y = -10:0.5:10;`
`[xmin, fmin] = newton(f2_sym, [-9.5,9.5]', 'bisection_wolfe_weak', search_x, search_y`

STARTED Line search using newton
ENDED Line search using newton
xmin =

1.000000000000000e+00
3.000000000000000e+00

fmin = 0
iter = 2.000000000000000e+00

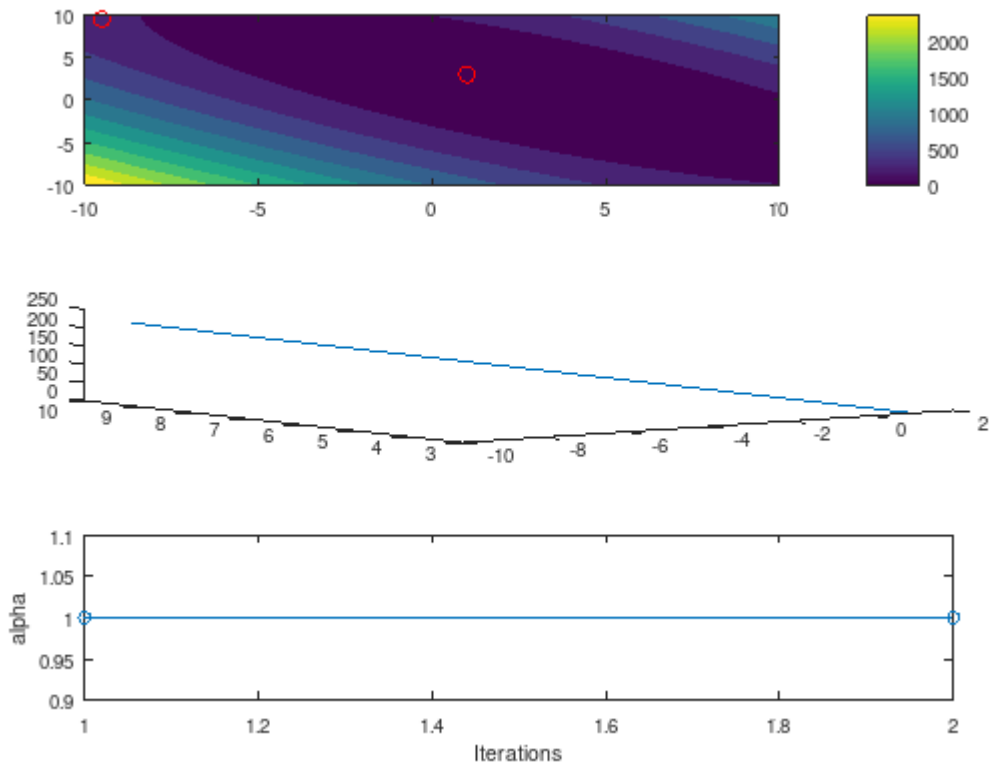


In [13]: `search_x = -10:0.5:10;`
`search_y = -10:0.5:10;`
`[xmin, fmin] = newton(f2_sym, [-9.5,9.5]', 'backtracking_armijo', search_x, search_y)`

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

1.0000000000000000e+00
 3.0000000000000000e+00

fmin = 0
 iter = 2.0000000000000000e+00

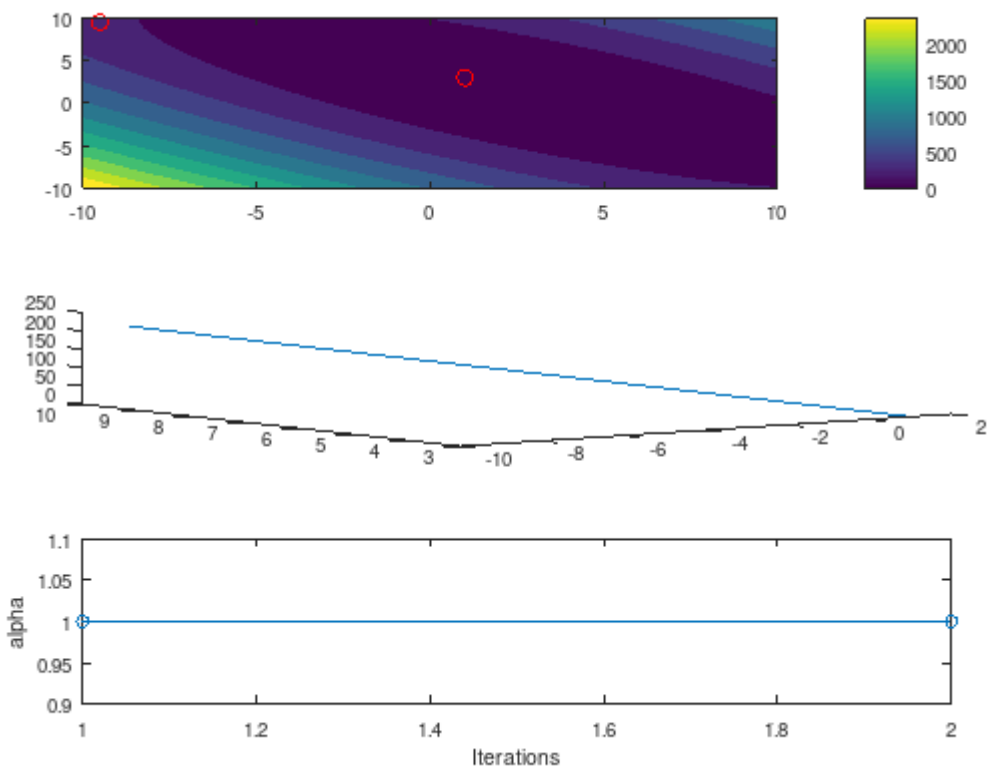


```
In [14]: search_x = -10:0.5:10;
search_y = -10:0.5:10;
[xmin, fmin] = newton(f2_sym, [-9.5,9.5]', 'none', search_x, search_y);
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

1.0000000000000000e+00
 3.0000000000000000e+00

fmin = 0
 iter = 2.0000000000000000e+00

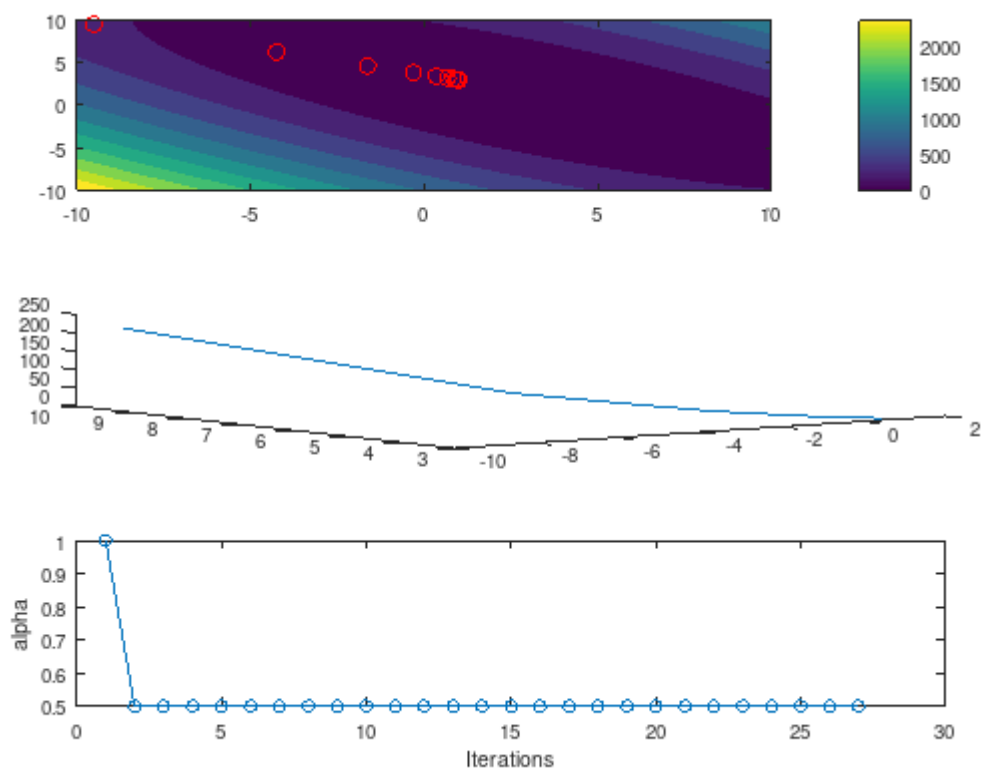


```
In [15]: search_x = -10:0.5:10;
search_y = -10:0.5:10;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = newton(f2_sym, [-9.5,9.5]', 'wolfe_strong', search_x, search_y, c, rho
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

9.999998435378075e-01
 3.000000096857548e+00

fmin = 4.807265696626928e-14
 iter = 2.700000000000000e+01

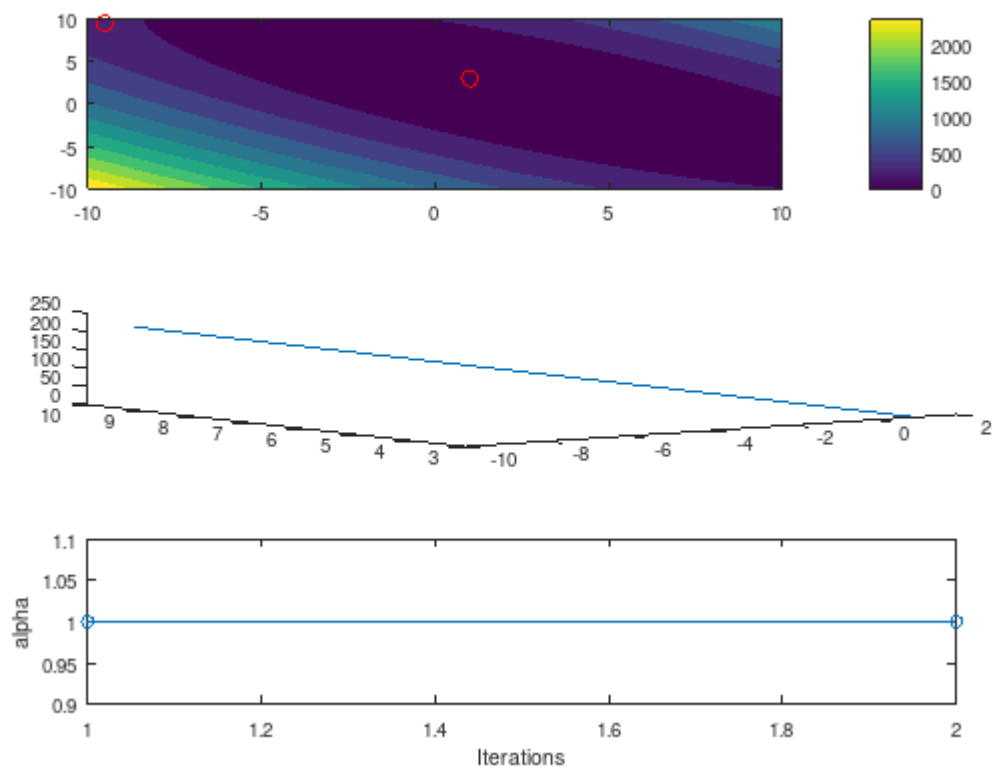


```
In [16]: search_x = -10:0.5:10;
search_y = -10:0.5:10;
[xmin, fmin] = newton(f2_sym, [-9.5,9.5]', 'nonmonotone_backtracking_armijo', search_
```

```
STARTED Line search using newton
ENDED Line search using newton
xmin =
```

```
1.0000000000000000e+00
3.0000000000000000e+00
```

```
fmin = 0
iter = 2.0000000000000000e+00
```

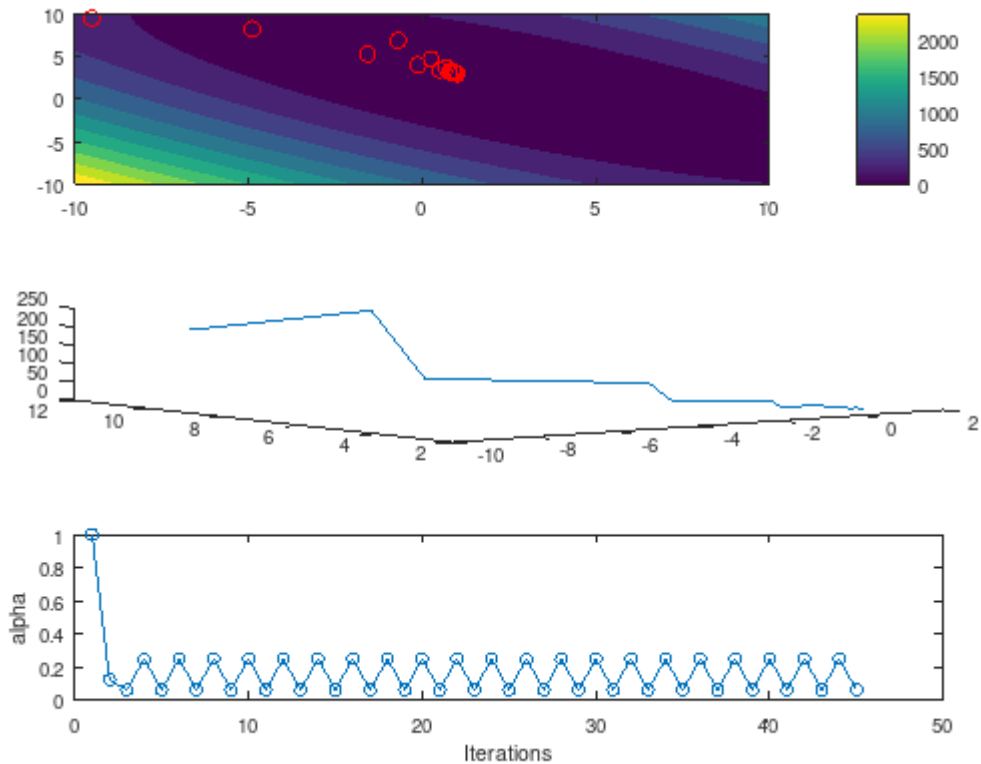


```
In [17]: search_x = -10:0.5:10;
search_y = -10:0.5:10;
[xmin, fmin] = steepest_descent(f2_sym, [-9.5,9.5]', 'bisection_wolfe_weak', search_x,
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

```
9.999998299014625e-01
3.000000152050947e+00
```

```
fmin = 5.335586537193396e-14
iter = 4.500000000000000e+01
```

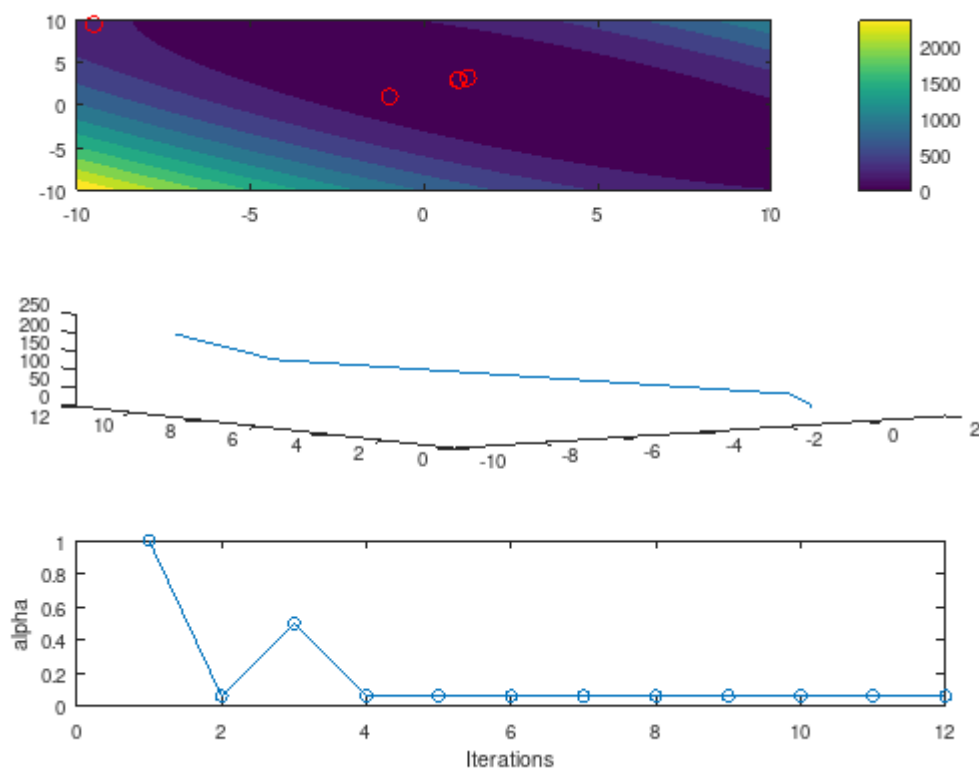


```
In [31]: search_x = -10:0.5:10;
search_y = -10:0.5:10;
[xmin, fmin] = steepest_descent(f2_sym, [-9.5,9.5]', 'backtracking_armijo', search_x,
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

```
1.000000014901161e+00
3.000000014901161e+00
```

```
fmin = 3.996802888650564e-15
iter = 1.200000000000000e+01
```



```
In [32]: search_x = -10:0.5:10;
search_y = -10:0.5:10;
[xmin, fmin] = steepest_descent(f2_sym, [-9.5,9.5]', 'none', search_x, search_y);
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

-2.217798745561566e+123
 -2.217798745561566e+123

fmin = 8.853536296466025e+247
 iter = 1.000000000000000e+02
 error: set: "cameraviewangle" must be finite
 error: called from
 plot3 at line 371 column 10
 plot_line_search1 at line 40 column 9
 steepest_descent at line 79 column 9

Inline plot failed, consider trying another graphics toolkit

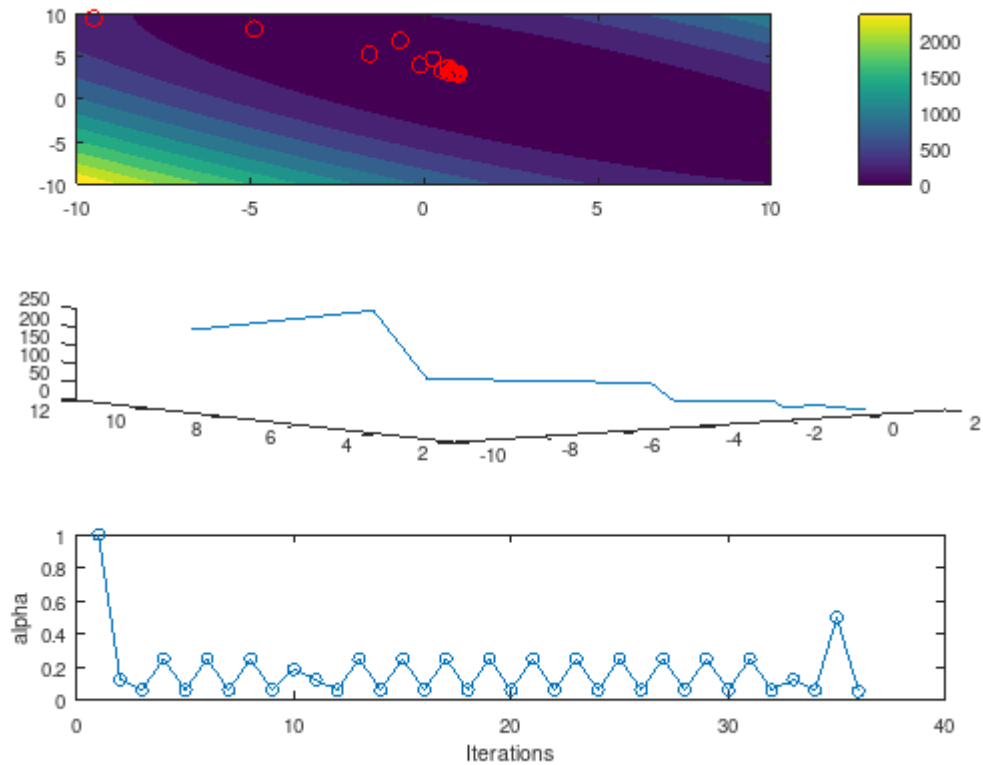
error: set: "cameraviewangle" must be finite
 error: called from
 _make_figures>safe_print at line 125 column 7
 _make_figures at line 49 column 13

```
In [33]: search_x = -10:0.5:10;
search_y = -10:0.5:10;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = steepest_descent(f2_sym, [-9.5,9.5]', 'wolfe_strong', search_x, search_y);
```

STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =

9.999999647537134e-01
2.999999964753711e+00

fmin = 2.236141486005015e-14
iter = 3.600000000000000e+01

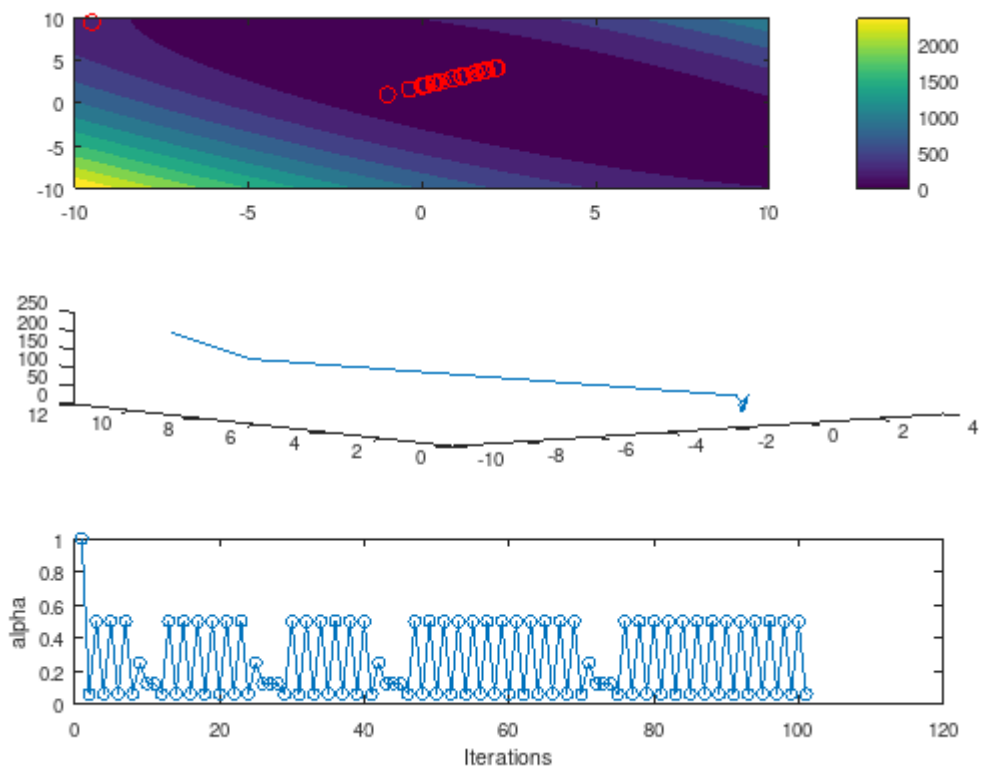


```
In [34]: search_x = -10:0.5:10;  
search_y = -10:0.5:10;  
[xmin, fmin] = steepest_descent(f2_sym, [-9.5,9.5]', 'nonmonotone_backtracking_armijo
```

STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =

8.933741807150000e-01
2.893374180715000e+00

fmin = 2.046431760875547e-01
iter = 1.000000000000000e+02



```
In [35]: % f(x,y) = 5x^4 + 6y^4 - 6x^2 + 2xy + 5y^2 +15x - 7y + 13 -- fmin = -6.4931 at x = -1

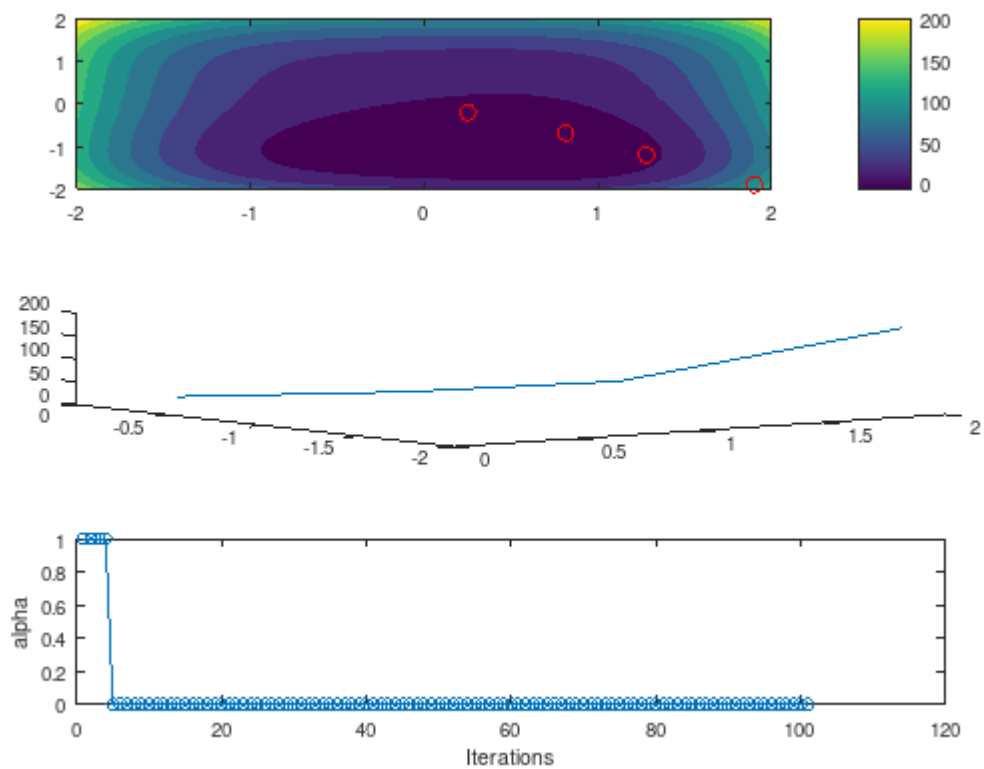
function f = f3_sym()
    syms x y
    f = 5*x^4 + 6*y^4 - 6*x^2 + 2*x*y + 5*y^2 + 15*x - 7*y + 13;
end
```

```
In [36]: search_x = -2:0.1:2;
search_y = -2:0.1:2;
[xmin, fmin] = newton(f3_sym, [1.9,-1.9]', 'bisection_wolfe_weak', search_x, search_y
```

```
STARTED Line search using newton
ENDED Line search using newton
xmin =
```

```
2.506762108304057e-01
-2.038861707803349e-01
```

```
fmin = 1.794605560958717e+01
iter = 1.000000000000000e+02
```

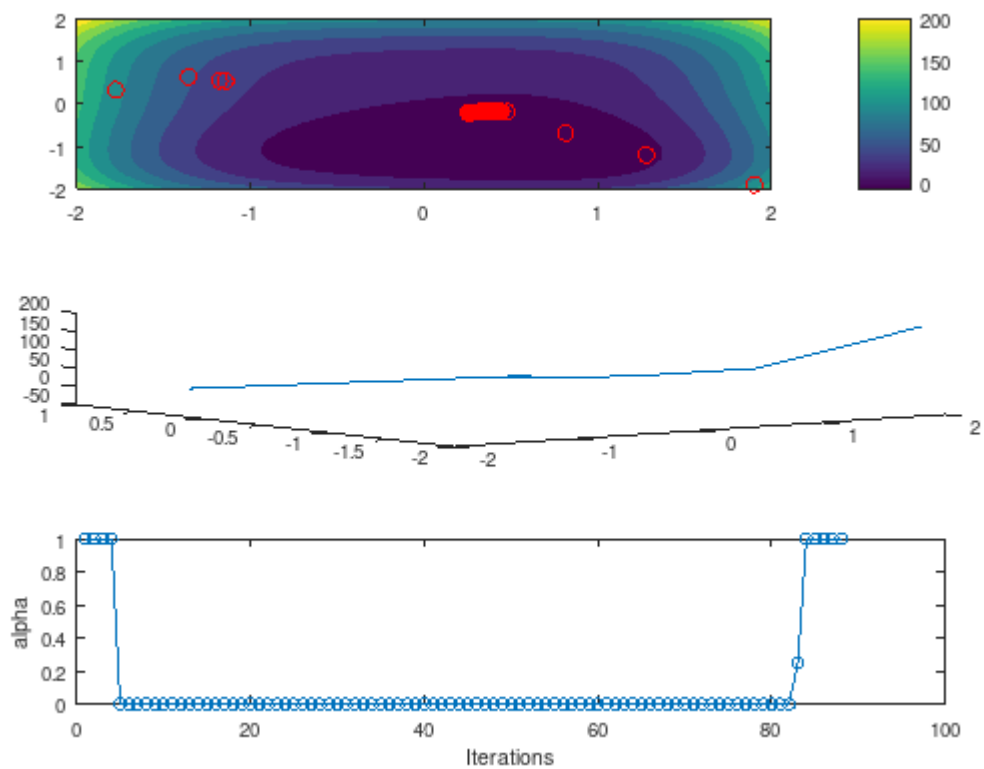


```
In [37]: search_x = -2:0.1:2;
search_y = -2:0.1:2;
[xmin, fmin] = newton(f3_sym, [1.9,-1.9]', 'backtracking_armijo', search_x, search_y)
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

-1.142054928372278e+00
 5.433724812053312e-01

fmin = -6.496118935491065e+00
 iter = 8.800000000000000e+01

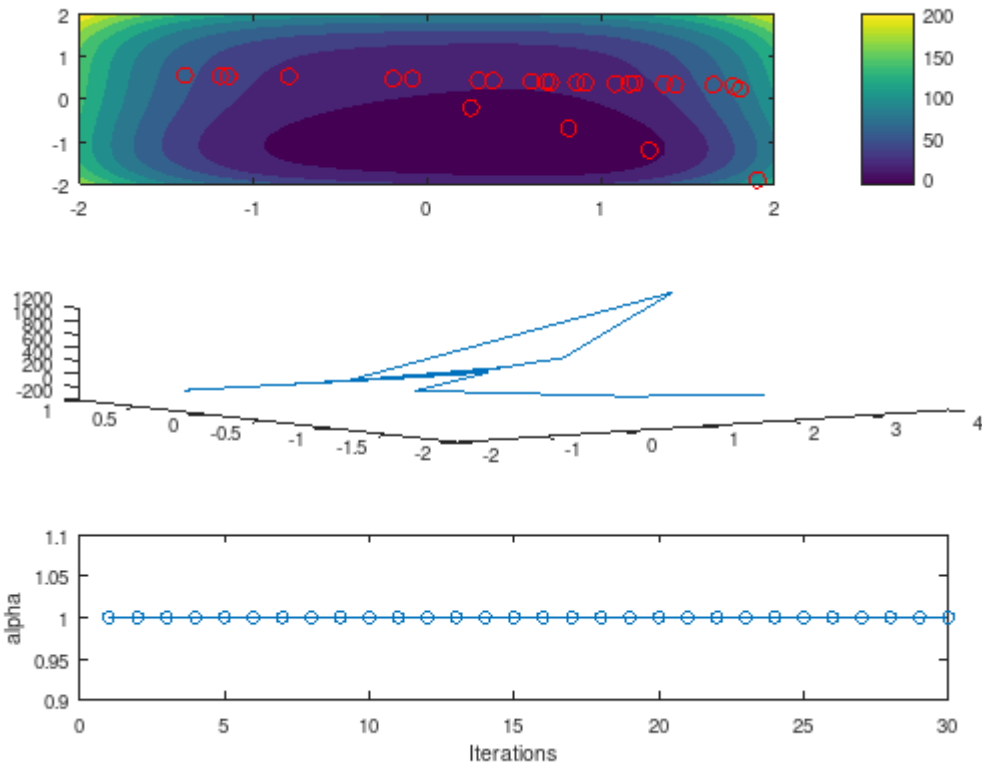



```
In [38]: search_x = -2:0.1:2;
search_y = -2:0.1:2;
[xmin, fmin] = newton(f3_sym, [1.9,-1.9]', 'none', search_x, search_y);
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

```
-1.142054928396837e+00
5.433724812070151e-01
```

```
fmin = -6.496118935491069e+00
iter = 3.000000000000000e+01
```

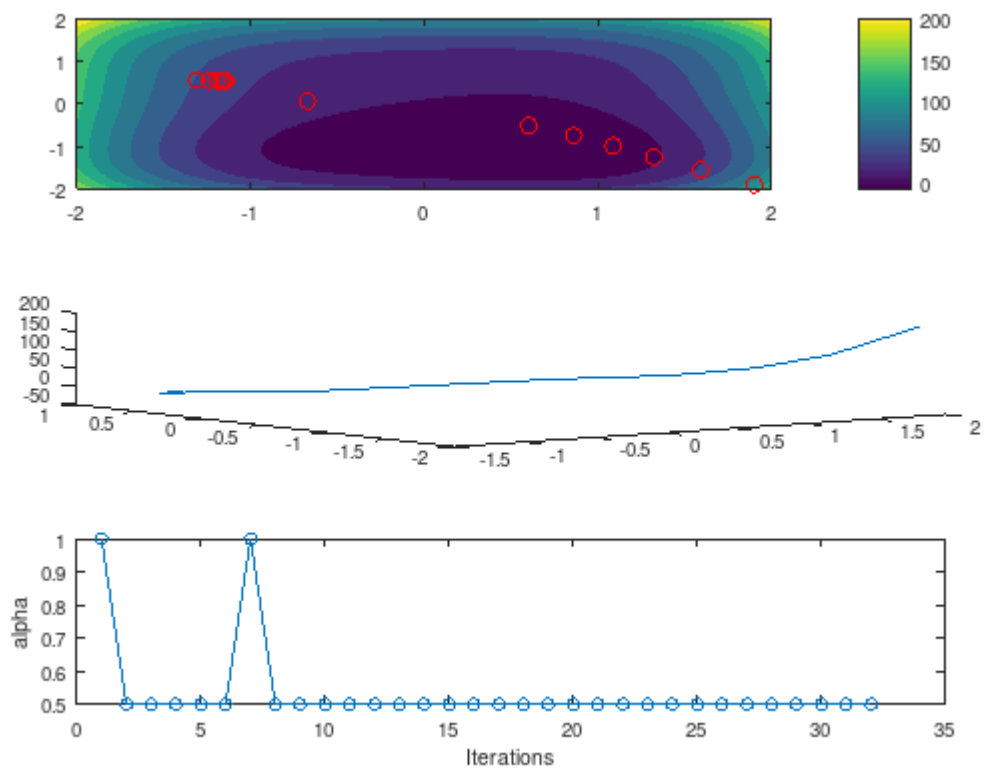


```
In [39]: search_x = -2:0.1:2;
search_y = -2:0.1:2;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = newton(f3_sym, [1.9,-1.9]', 'wolfe_strong', search_x, search_y, c, rho)
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

```
-1.142054942016023e+00
5.433724826032792e-01
```

```
fmin = -6.496118935491054e+00
iter = 3.200000000000000e+01
```

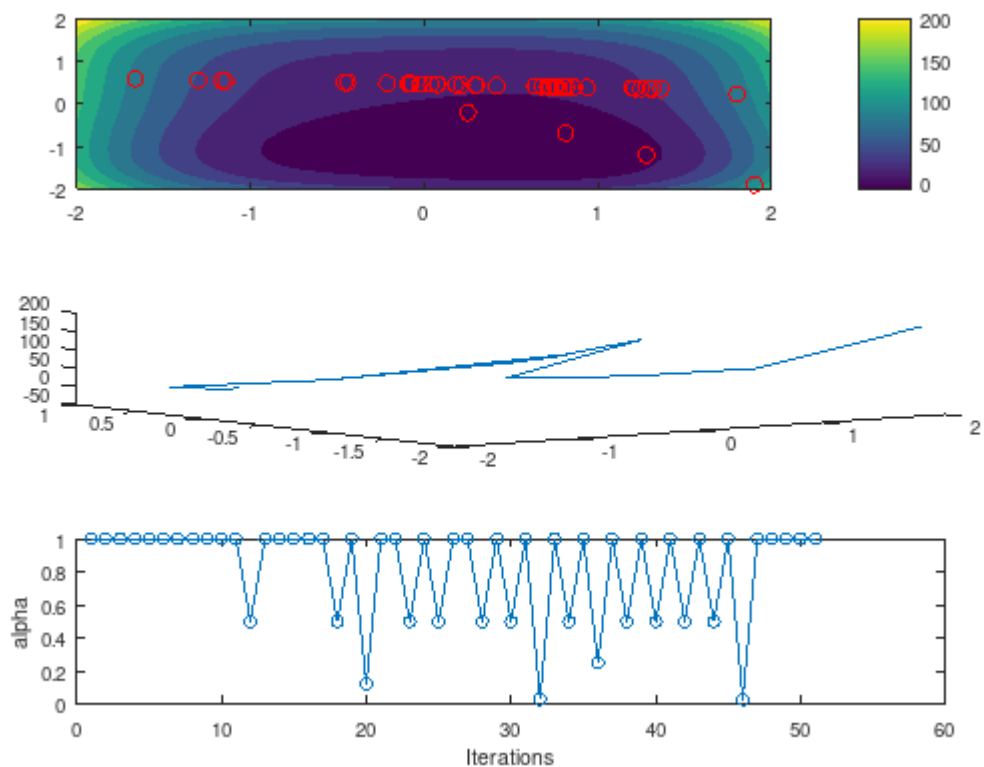


```
In [40]: search_x = -2:0.1:2;
search_y = -2:0.1:2;
[xmin, fmin] = newton(f3_sym, [1.9,-1.9]', 'nonmonotone_backtracking_armijo', search_
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

-1.142054928369279e+00
 5.433724812050896e-01

fmin = -6.496118935491069e+00
 iter = 5.100000000000000e+01



```
In [41]: search_x = -2:0.1:2;
search_y = -2:0.1:2;
[xmin, fmin] = steepest_descent(f3_sym, [1.9,-1.9]', 'bisection_wolfe_weak', search_x,
```

STARTED Line search using steepest descent

ENDED Line search using steepest descent

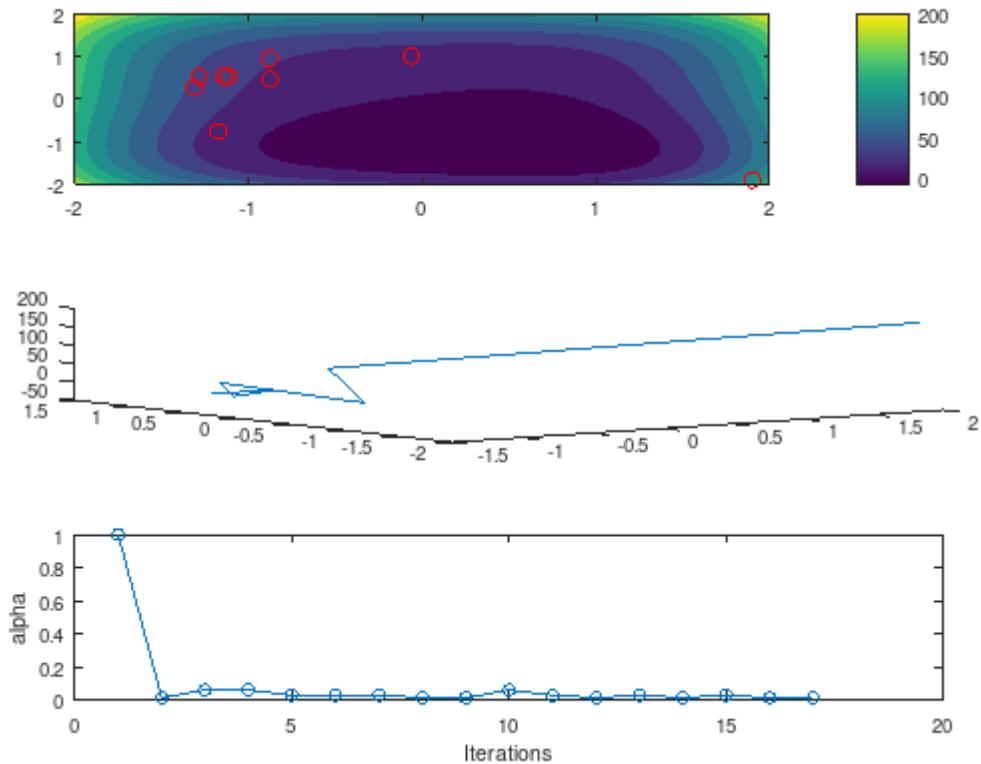
xmin =

-1.142054930103961e+00

5.433724860137112e-01

fmin = -6.496118935491065e+00

iter = 1.700000000000000e+01



```
In [42]: search_x = -2:0.1:2;
search_y = -2:0.1:2;
[xmin, fmin] = steepest_descent(f3_sym, [1.9,-1.9]', 'backtracking_armijo', search_x,
```

STARTED Line search using steepest descent

ENDED Line search using steepest descent

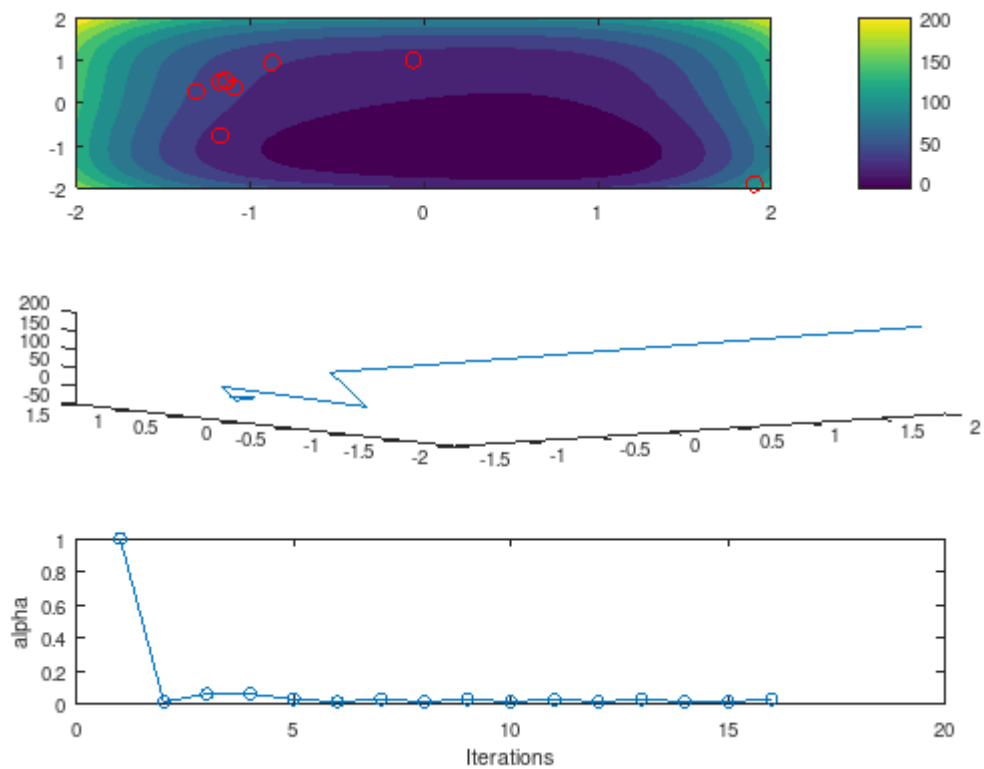
xmin =

-1.142054921038707e+00

5.433724802876780e-01

fmin = -6.496118935491069e+00

iter = 1.600000000000000e+01



```
In [43]: search_x = -2:0.1:2;
search_y = -2:0.1:2;
[xmin, fmin] = steepest_descent(f3_sym, [1.9,-1.9]', 'none', search_x, search_y);
```

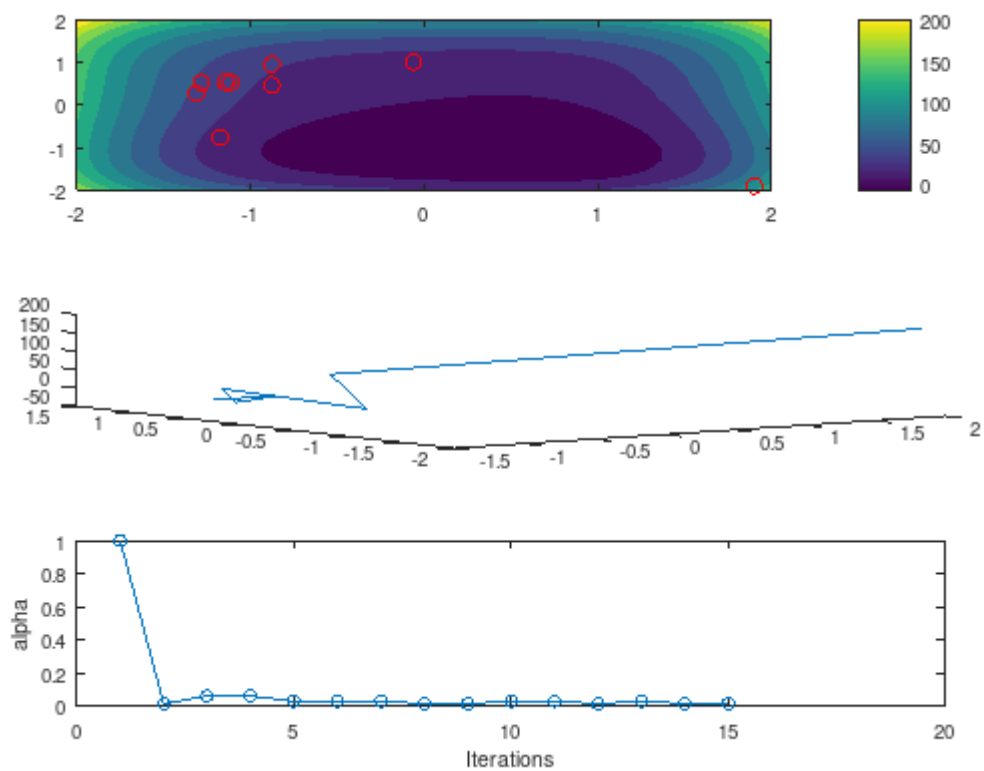
STARTED Line search using steepest descent
error: Failed to converge. Inf value reached
error: called from
 steepest_descent at line 47 column 13

```
In [44]: search_x = -2:0.1:2;
search_y = -2:0.1:2;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = steepest_descent(f3_sym, [1.9,-1.9]', 'wolfe_strong', search_x, search_y);
```

STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =

-1.142054939773352e+00
5.433724777523828e-01

fmin = -6.496118935491062e+00
iter = 1.500000000000000e+01

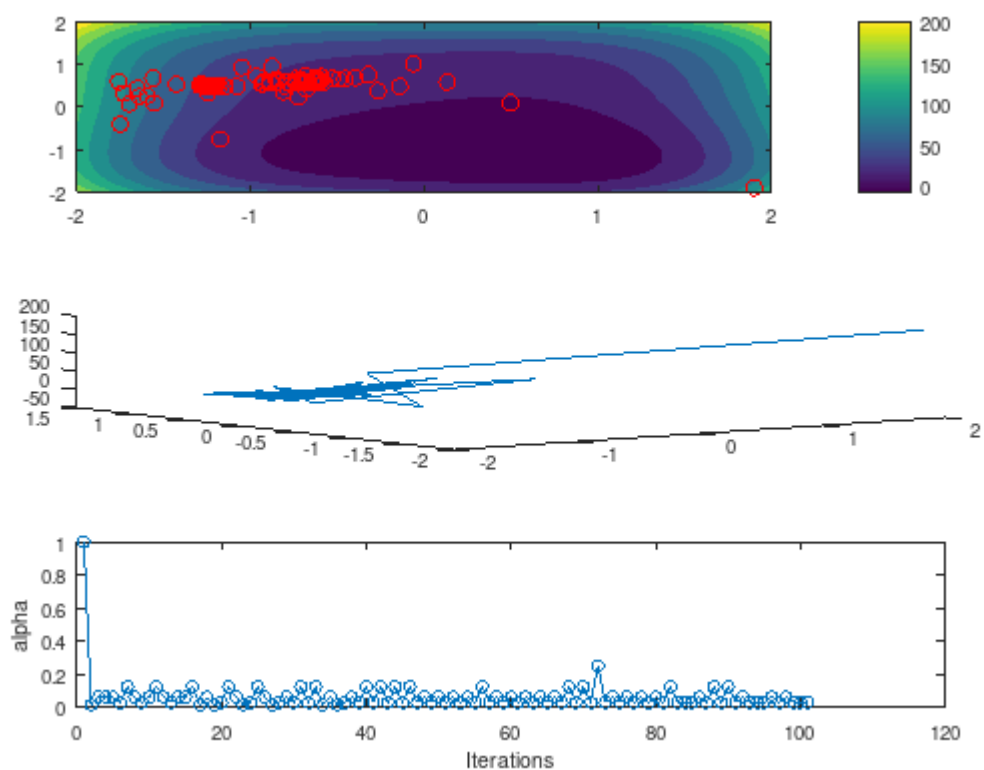


```
In [45]: search_x = -2:0.1:2;
search_y = -2:0.1:2;
[xmin, fmin] = steepest_descent(f3_sym, [1.9, -1.9]', 'nonmonotone_backtracking_armijo
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

-1.277655501102711e+00
 5.306173638659559e-01

fmin = -5.822348174526592e+00
 iter = 1.000000000000000e+02



In [46]: `% f(x,y) = (x^2)^(y^2 + 1) + (y^2)^(x^2+1) -- fmin = 0 at x = 0, y = 0 Search domain:`

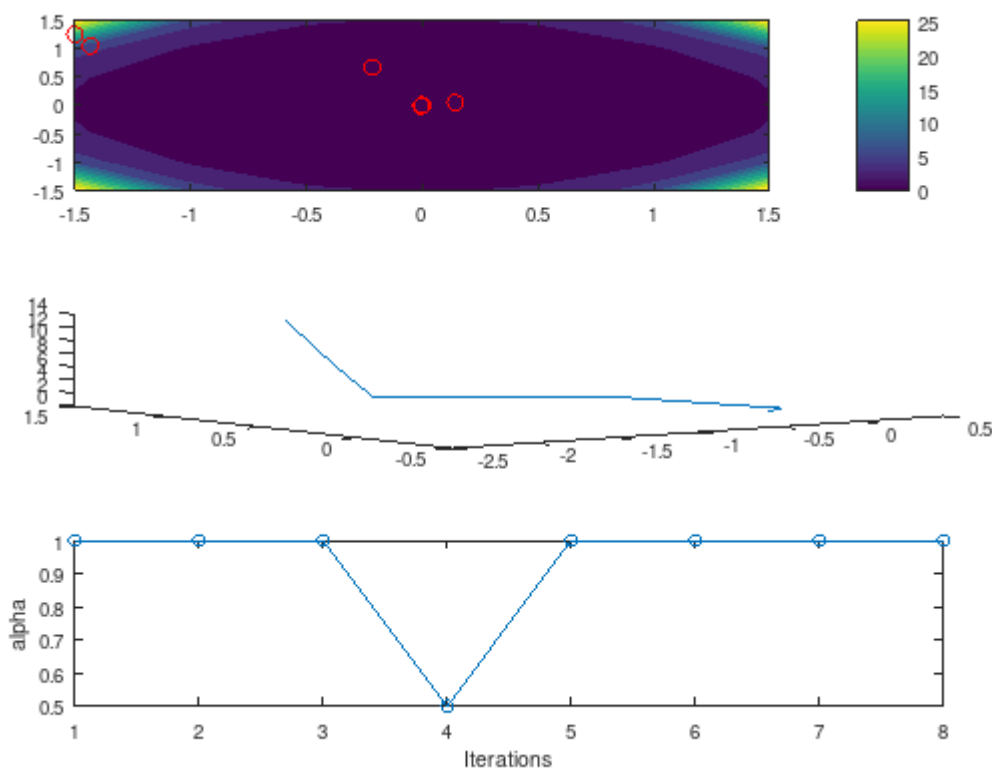
```
function f = f4_sym()
    syms x y
    f = (x^2)^(y^2 + 1) + (y^2)^(x^2 + 1);
end
```

In [47]: `search_x = -1.5:0.5:1.5;
search_y = -1.5:0.5:1.5;
[xmin, fmin] = newton(f4_sym, [-1.5,1.25]', 'bisection_wolfe_weak', search_x, search_y`

STARTED Line search using newton
ENDED Line search using newton
xmin =

-4.898290974220121e-13
-1.411519032285999e-12

fmin = 2.232318523186867e-24
iter = 8.000000000000000e+00

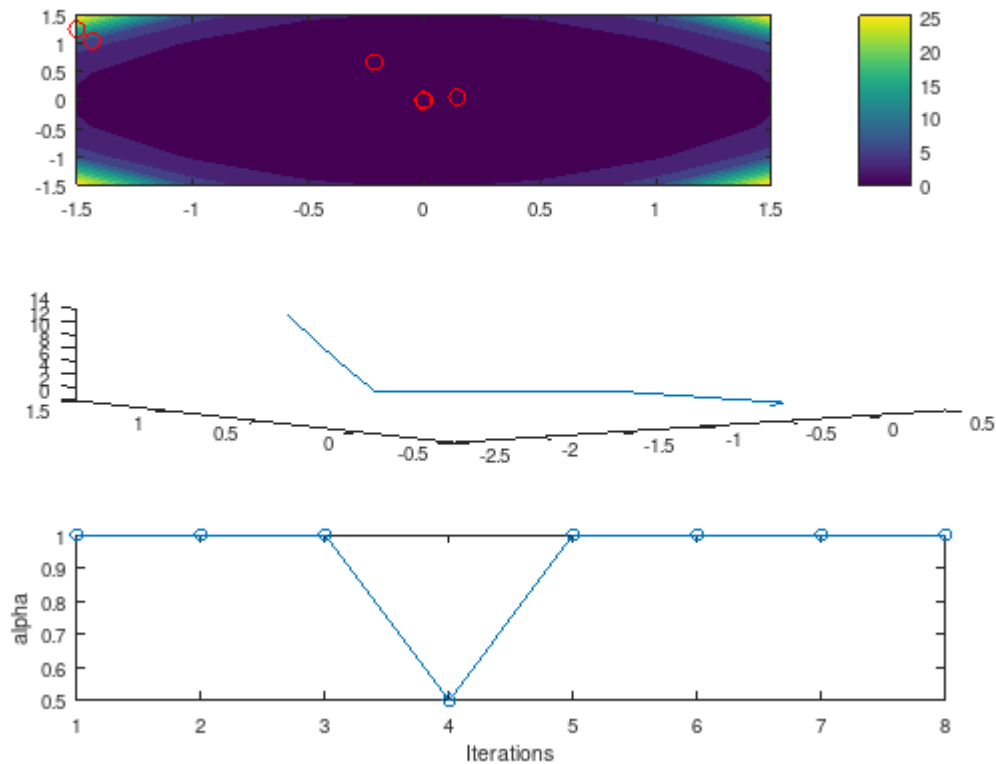


In [48]: `search_x = -1.5:0.5:1.5;
search_y = -1.5:0.5:1.5;
[xmin, fmin] = newton(f4_sym, [-1.5,1.25]', 'backtracking_armijo', search_x, search_y`

STARTED Line search using newton
ENDED Line search using newton
xmin =

-4.898290974220121e-13
-1.411519032285999e-12

fmin = 2.232318523186867e-24
iter = 8.000000000000000e+00

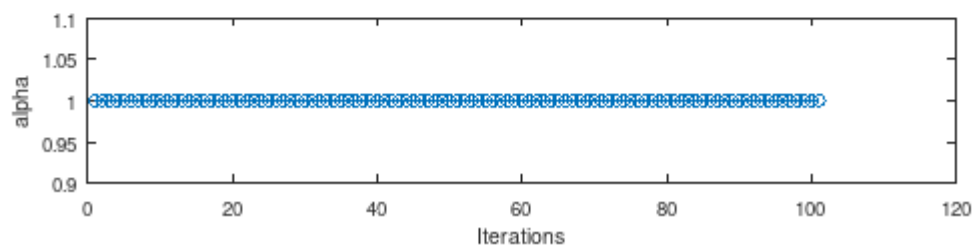
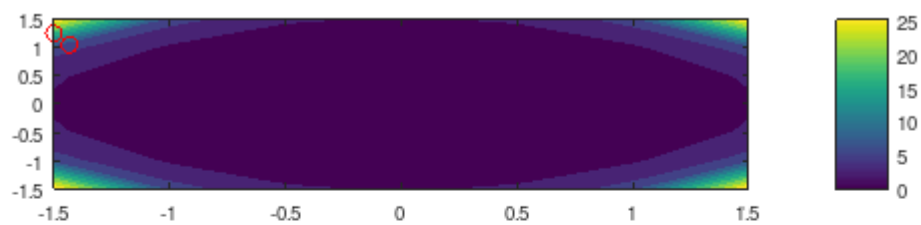


```
In [49]: search_x = -1.5:0.5:1.5;  
search_y = -1.5:0.5:1.5;  
[xmin, fmin] = newton(f4_sym, [-1.5,1.25]', 'none', search_x, search_y);
```

STARTED Line search using newton
ENDED Line search using newton
xmin =

-3.930613690245791e+00
2.178050831894393e+00

fmin = 1.325174444770471e+11
iter = 1.000000000000000e+02

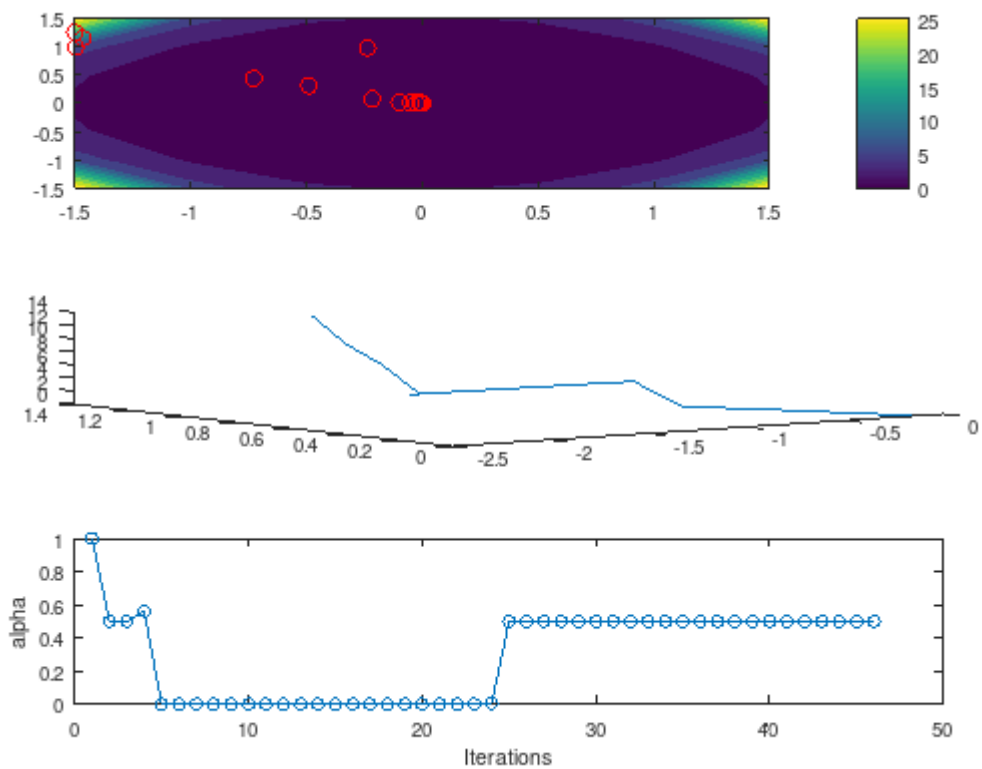


```
In [50]: search_x = -1.5:0.5:1.5;
search_y = -1.5:0.5:1.5;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = newton(f4_sym, [-1.5,1.25]', 'wolfe_strong', search_x, search_y, c, rh
```

```
STARTED Line search using newton
ENDED Line search using newton
xmin =
```

```
-3.779140906340462e-07
3.798078947649028e-08
```

```
fmin = 1.442616002690029e-13
iter = 4.600000000000000e+01
```

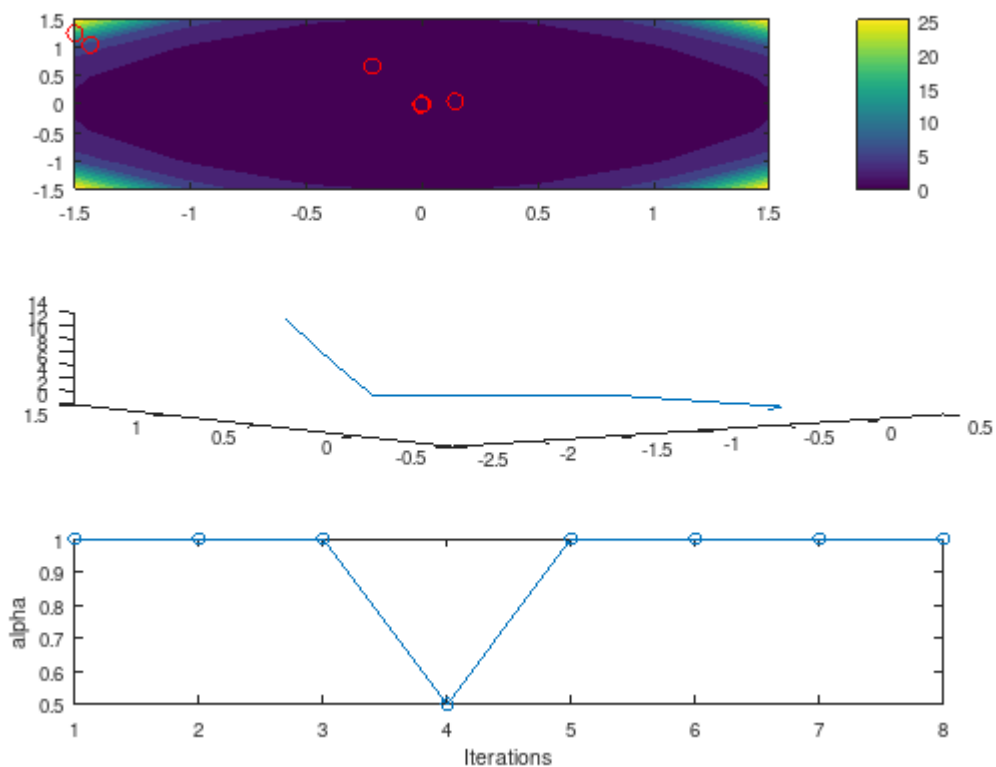



```
In [51]: search_x = -1.5:0.5:1.5;
search_y = -1.5:0.5:1.5;
[xmin, fmin] = newton(f4_sym, [-1.5,1.25]', 'nonmonotone_backtracking_armijo', search
```

STARTED Line search using newton
 ENDED Line search using newton
 xmin =

-4.898290974220121e-13
 -1.411519032285999e-12

fmin = 2.232318523186867e-24
 iter = 8.000000000000000e+00

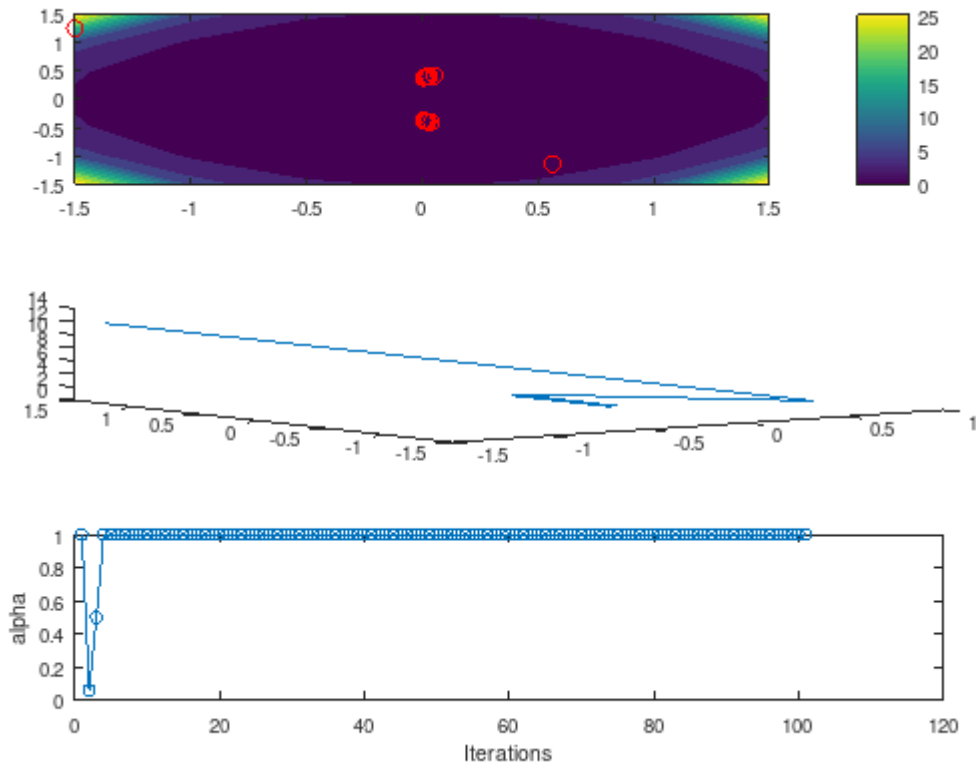


```
In [52]: search_x = -1.5:0.5:1.5;
search_y = -1.5:0.5:1.5;
[xmin, fmin] = steepest_descent(f4_sym, [-1.5,1.25]', 'bisection_wolfe_weak', search_x
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

6.611447956945881e-03
 3.750924354722853e-01

fmin = 1.406929221864008e-01
 iter = 1.000000000000000e+02

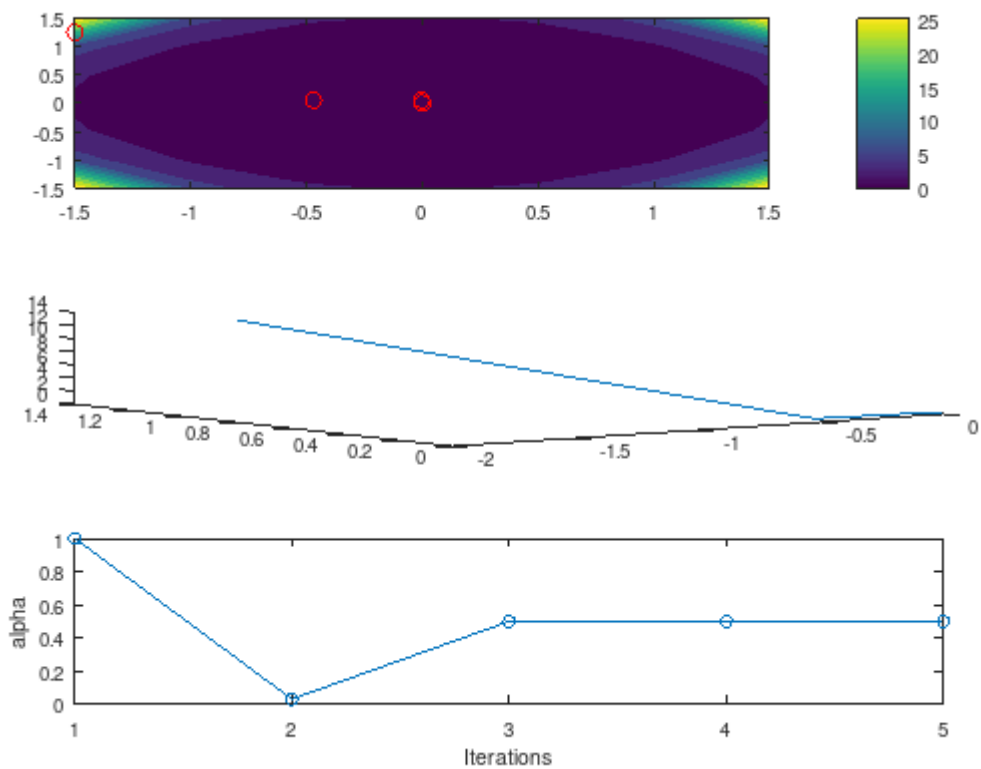


```
In [53]: search_x = -1.5:0.5:1.5;
search_y = -1.5:0.5:1.5;
[xmin, fmin] = steepest_descent(f4_sym, [-1.5,1.25]', 'backtracking_armijo', search_x
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

-1.442387198178838e-13
 2.888507155359744e-12

fmin = 8.364278394859144e-24
 iter = 5.000000000000000e+00



```
In [54]: search_x = -1.5:0.5:1.5;
search_y = -1.5:0.5:1.5;
[xmin, fmin] = steepest_descent(f4_sym, [-1.5,1.25]', 'none', search_x, search_y);
```

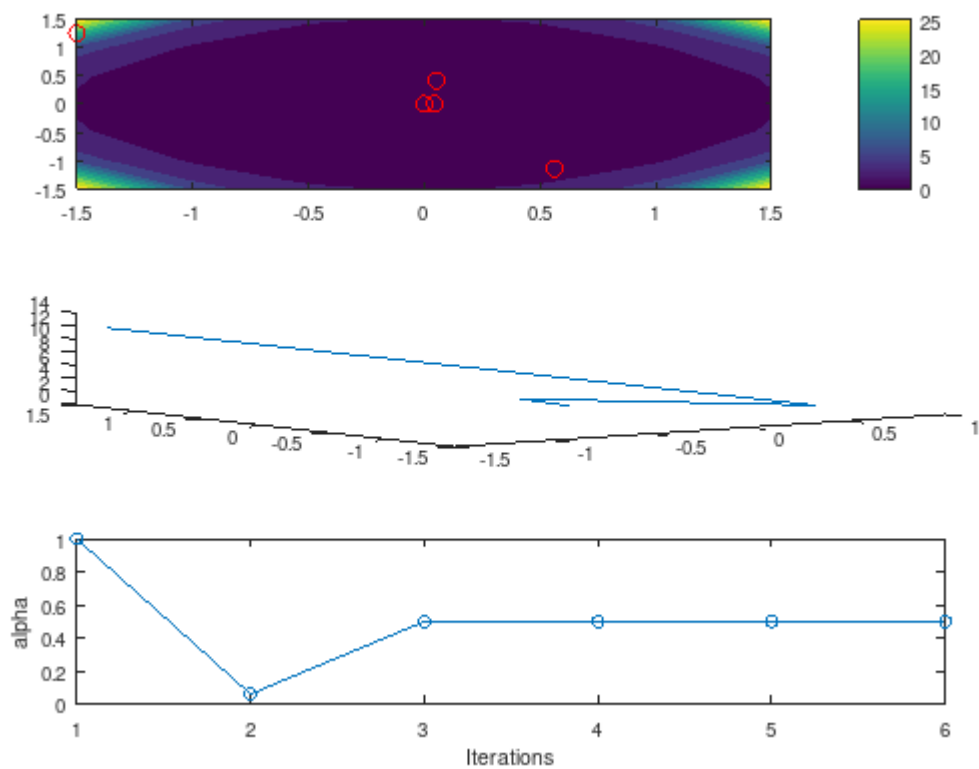
STARTED Line search using steepest descent
error: Failed to converge. Inf value reached
error: called from
steepest_descent at line 47 column 13

```
In [55]: search_x = -1.5:0.5:1.5;
search_y = -1.5:0.5:1.5;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = steepest_descent(f4_sym, [-1.5,1.25]', 'wolfe_strong', search_x, search_y);
```

STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =

4.147330097959410e-12
3.015130327485224e-13

fmin = 1.729125705035722e-23
iter = 6.000000000000000e+00

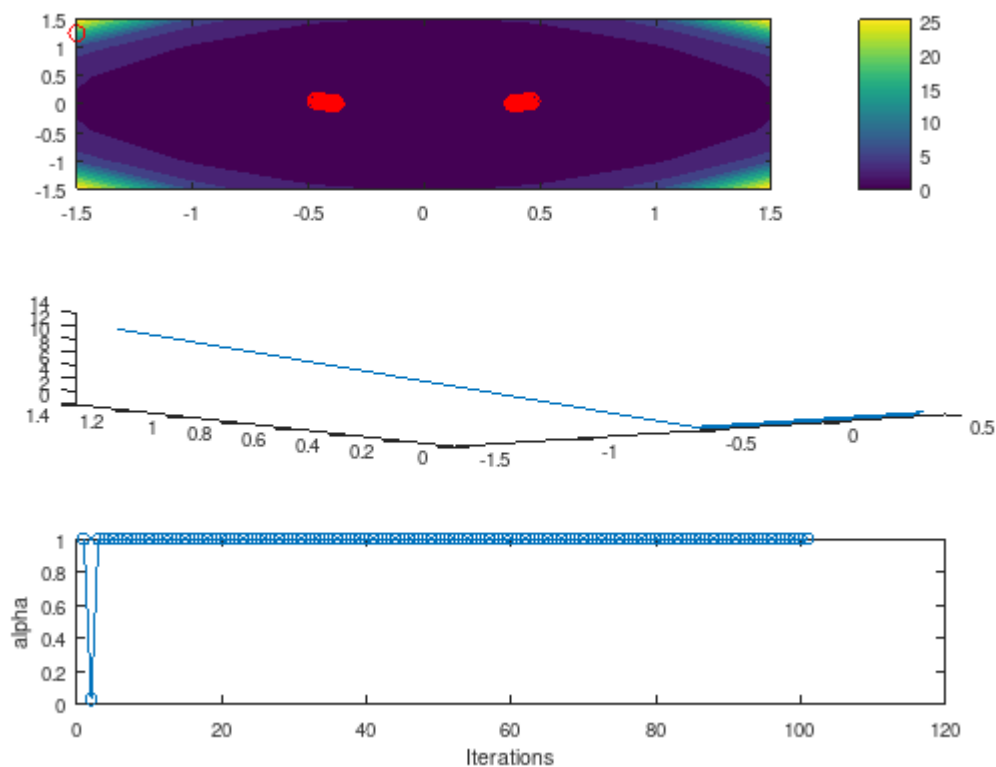


```
In [56]: search_x = -1.5:0.5:1.5;
search_y = -1.5:0.5:1.5;
[xmin, fmin] = steepest_descent(f4_sym, [-1.5,1.25]', 'nonmonotone_backtracking_armij
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

3.814861157367117e-01
 8.229065413614869e-03

fmin = 1.455294104707443e-01
 iter = 1.000000000000000e+02



In [57]: `% f(x,y,z)=(x^2+y^3-z^4)^2+(2*x*y*z)^2+(2*x*y-3*y*z+x*z)^2 -init x=y=z=10-obj =0.0000`

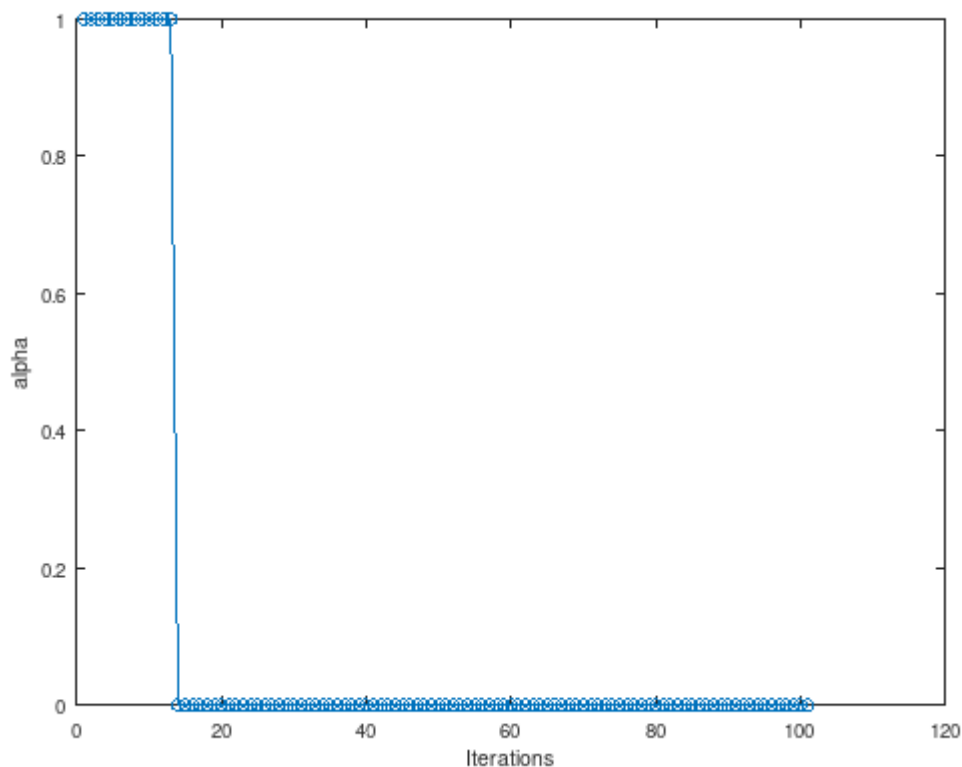
```
function f = f5_sym()
    syms x y z
    f = (x^2 + y^3 - z^4)^2 + (2*x*y*z)^2 + (2*x*y-3*y*z+x*z)^2;
end
```

In [58]: `search_x = -1:1:1;
search_y = -1:1:1;
[xmin, fmin] = newton(f5_sym, [10,10,10]', 'bisection_wolfe_weak', search_x, search_y`

STARTED Line search using newton
ENDED Line search using newton
xmin =

8.983670709317353e-01
1.278653983182028e+00
1.626970300034634e+00

fmin = 3.701670260120608e+01
iter = 1.000000000000000e+02

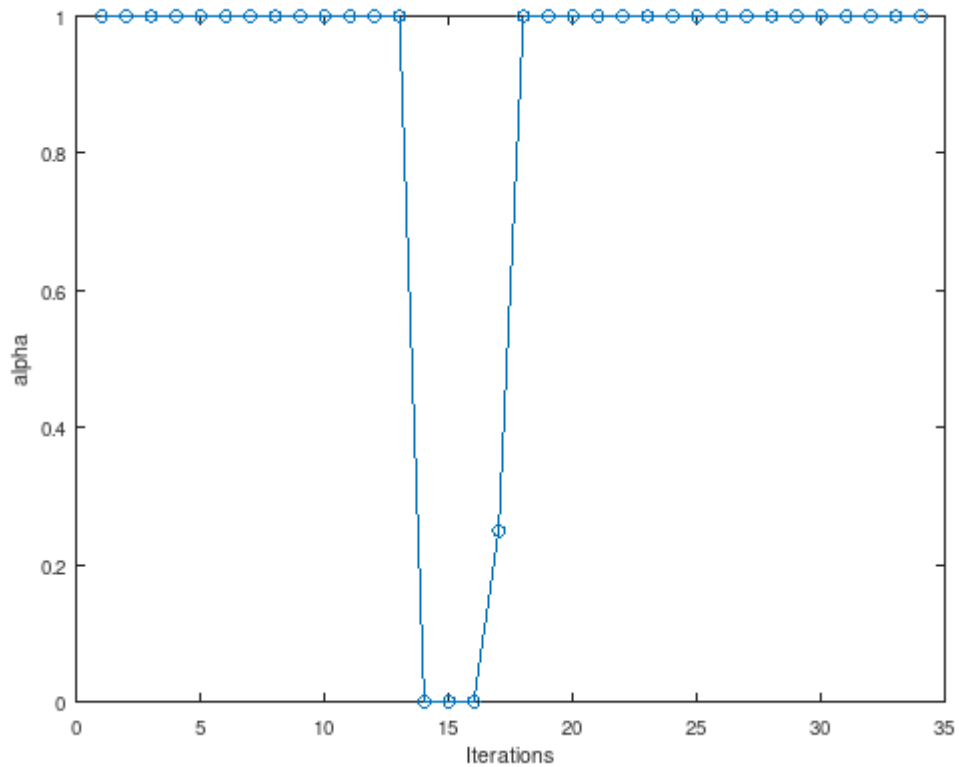


In [59]: `search_x = -1:1:1;
search_y = -1:1:1;
[xmin, fmin] = newton(f5_sym, [10,10,10]', 'backtracking_armijo', search_x, search_y)`

```
STARTED Line search using newton
ENDED Line search using newton
xmin =
```

```
-2.126482131637632e-05
-7.097574163884356e-06
9.006911586906940e-02
```

```
fmin = 4.331130106466260e-09
iter = 3.400000000000000e+01
```

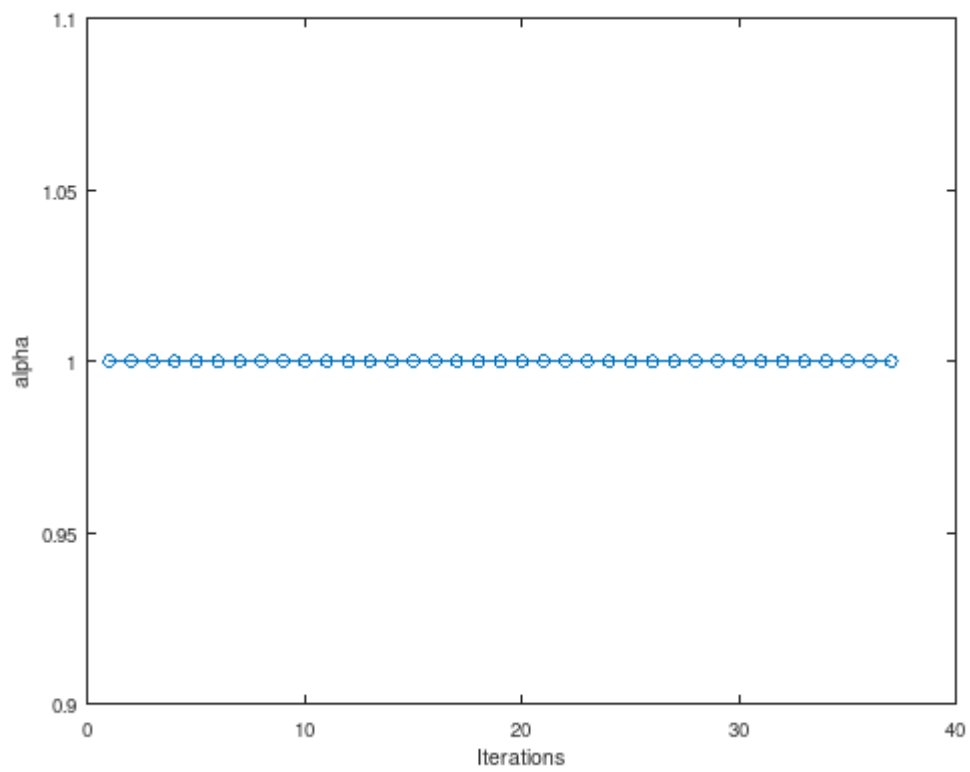


```
In [60]: search_x = -1:1:1;
search_y = -1:1:1;
[xmin, fmin] = newton(f5_sym, [10,10,10]', 'none', search_x, search_y);
```

```
STARTED Line search using newton
ENDED Line search using newton
xmin =
```

```
2.525601052450414e-04
3.792558884056361e-02
1.684963639464983e-04
```

```
fmin = 2.982697279578865e-09
iter = 3.700000000000000e+01
```

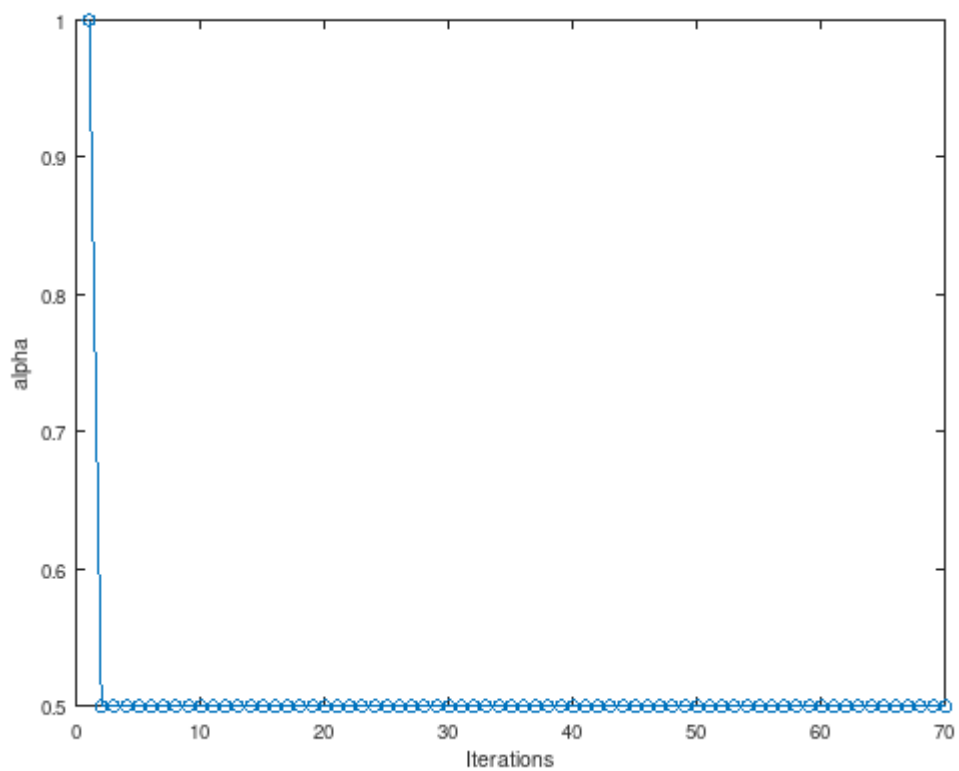


```
In [61]: search_x = -1:1:1;
search_y = -1:1:1;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = newton(f5_sym, [10,10,10]', 'wolfe_strong', search_x, search_y, c, rho
```

```
STARTED Line search using newton
ENDED Line search using newton
xmin =

    1.471639574257892e-02
    5.329826606075574e-03
    1.221230745097386e-01

fmin = 4.015195365725640e-10
iter = 7.000000000000000e+01
```

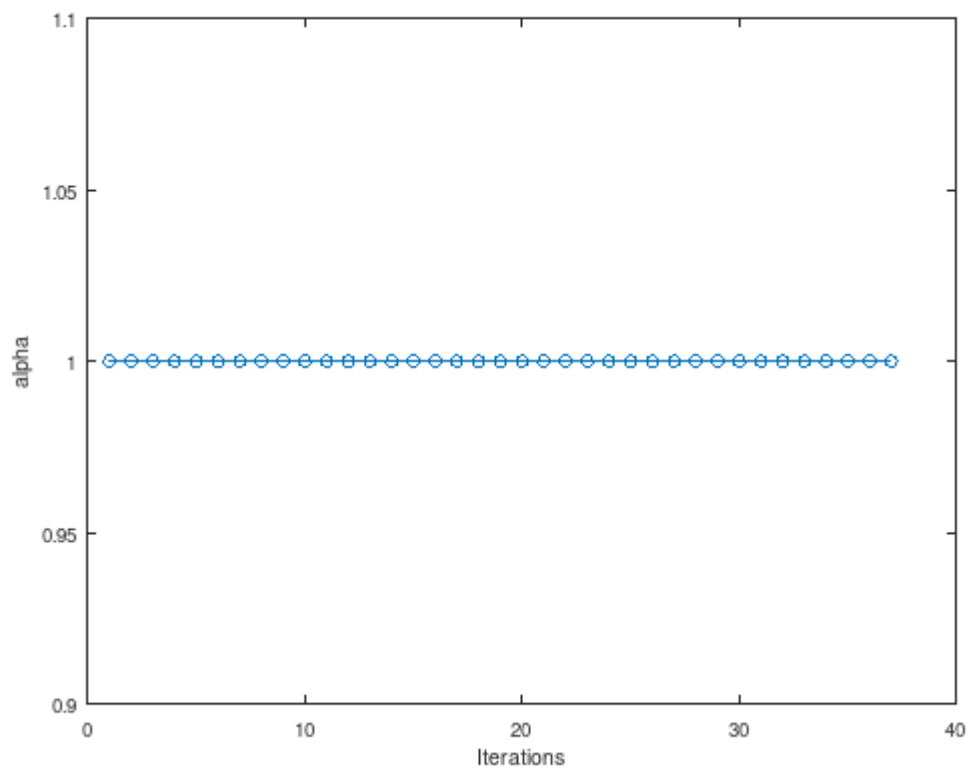


```
In [62]: search_x = -1:1:1;
search_y = -1:1:1;
[xmin, fmin] = newton(f5_sym, [10,10,10]', 'nonmonotone_backtracking_armijo', search_
```

```
STARTED Line search using newton
ENDED Line search using newton
xmin =
```

```
2.525601052450414e-04
3.792558884056361e-02
1.684963639464983e-04
```

```
fmin = 2.982697279578865e-09
iter = 3.700000000000000e+01
```

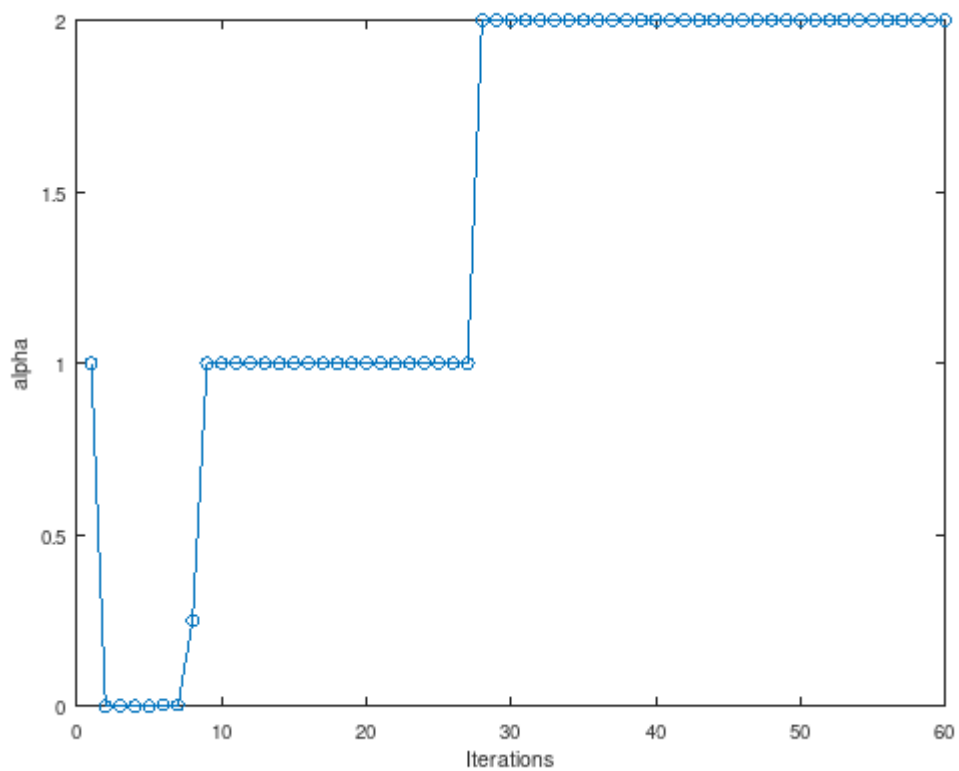



```
In [63]: search_x = -1:1:1;  
search_y = -1:1:1;  
[xmin, fmin] = steepest_descent(f5_sym, [10,10,10]', 'bisection_wolfe_weak', search_x
```

```
STARTED Line search using steepest descent  
ENDED Line search using steepest descent  
xmin =
```

```
-3.345935060119946e-04  
-1.228037158510390e-04  
-6.617411756693116e-02
```

```
fmin = 3.680758575969366e-10  
iter = 6.000000000000000e+01
```

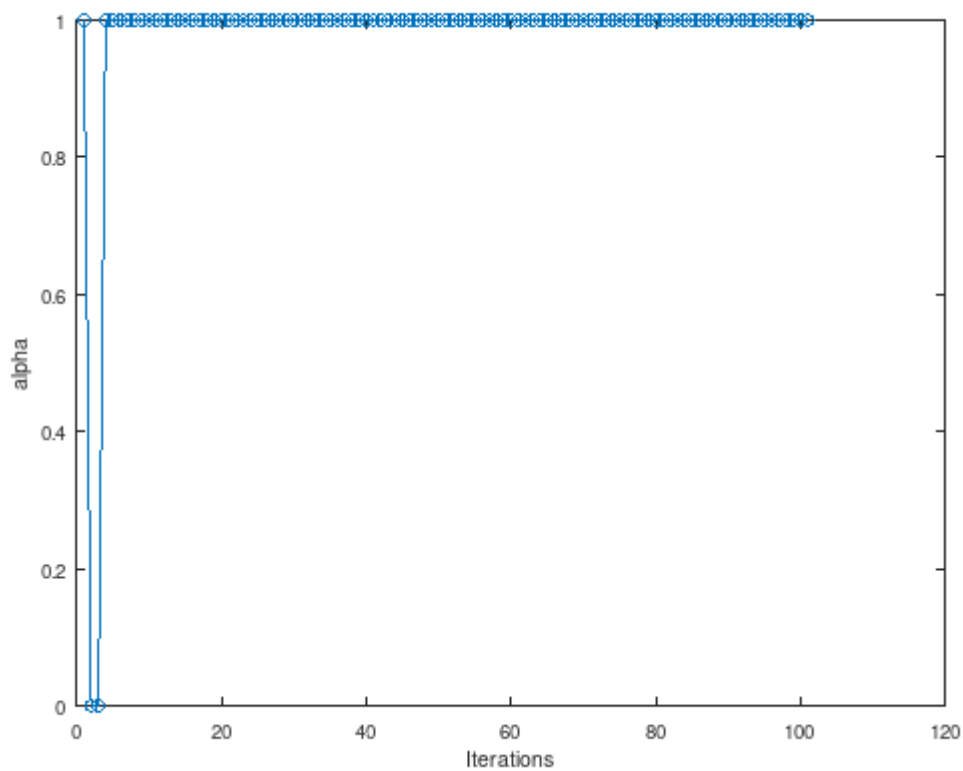


```
In [64]: search_x = -1:1:1;
search_y = -1:1:1;
[xmin, fmin] = steepest_descent(f5_sym, [10,10,10]', 'backtracking_armijo', search_x,
```

```
STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =
```

```
NaN
NaN
NaN
```

```
fmin = NaN
iter = 1.0000000000000000e+02
```



```
In [65]: search_x = -1:1:1;
search_y = -1:1:1;
[xmin, fmin] = steepest_descent(f5_sym, [10,10,10]', 'none', search_x, search_y);
```

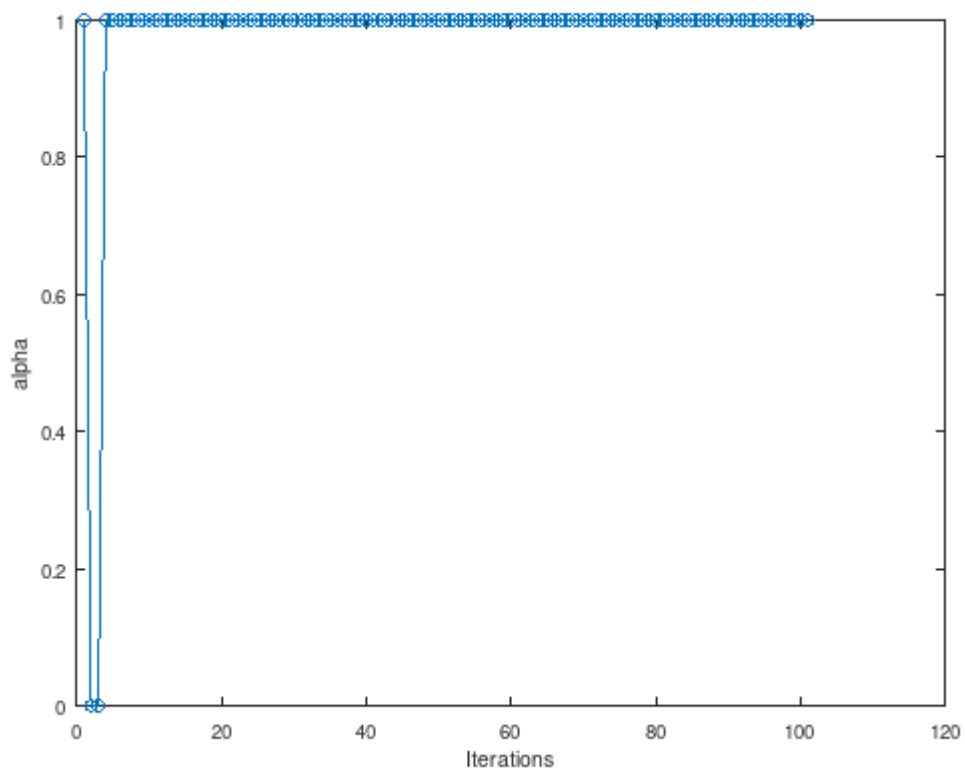
STARTED Line search using steepest descent
error: Failed to converge. Inf value reached
error: called from
steepest_descent at line 47 column 13

```
In [66]: search_x = -1:1:1;
search_y = -1:1:1;
[xmin, fmin] = steepest_descent(f5_sym, [10,10,10]', 'nonmonotone_backtracking_armijo');
```

STARTED Line search using steepest descent
ENDED Line search using steepest descent
xmin =

NaN
NaN
NaN

fmin = NaN
iter = 1.0000000000000000e+02

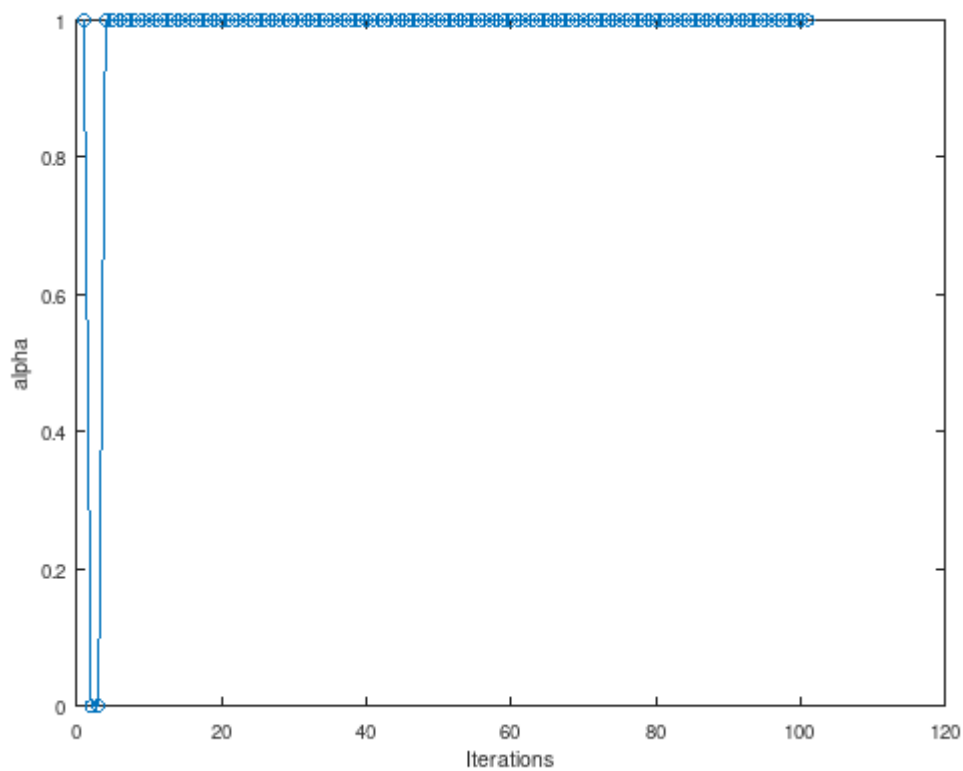


```
In [67]: search_x = -1:1:1;
search_y = -1:1:1;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = steepest_descent(f5_sym, [10,10,10]', 'wolfe_strong', search_x, search_y, c, rho);
```

STARTED Line search using steepest descent
 ENDED Line search using steepest descent
 xmin =

NaN
 NaN
 NaN

fmin = NaN
 iter = 1.0000000000000000e+02



```
In [2]: % f(x,y,z,k)=(x-1)^2+ (x-y)^2 + (y-z)^2 + (z-k)^2 -init x=y=z=k=0.1 -obj =1.13719e-10

function f = f6_sym()
    syms x y z k
    f = (x-1)^2 + (x-sqrt(y))^2 + (y-sqrt(z))^2 + (z-sqrt(k))^2;
end
```

```
In [69]: search_x = -1:1:1;
search_y = -1:1:1;
[xmin, fmin] = newton(f6_sym, [0.1,0.1,0.1,0.1]', 'bisection_wolfe_weak', search_x, s
```

STARTED Line search using newton

a = 1.375000000000000e+00

ENDED Line search using newton

xmin =

-1.138581225566442e+00

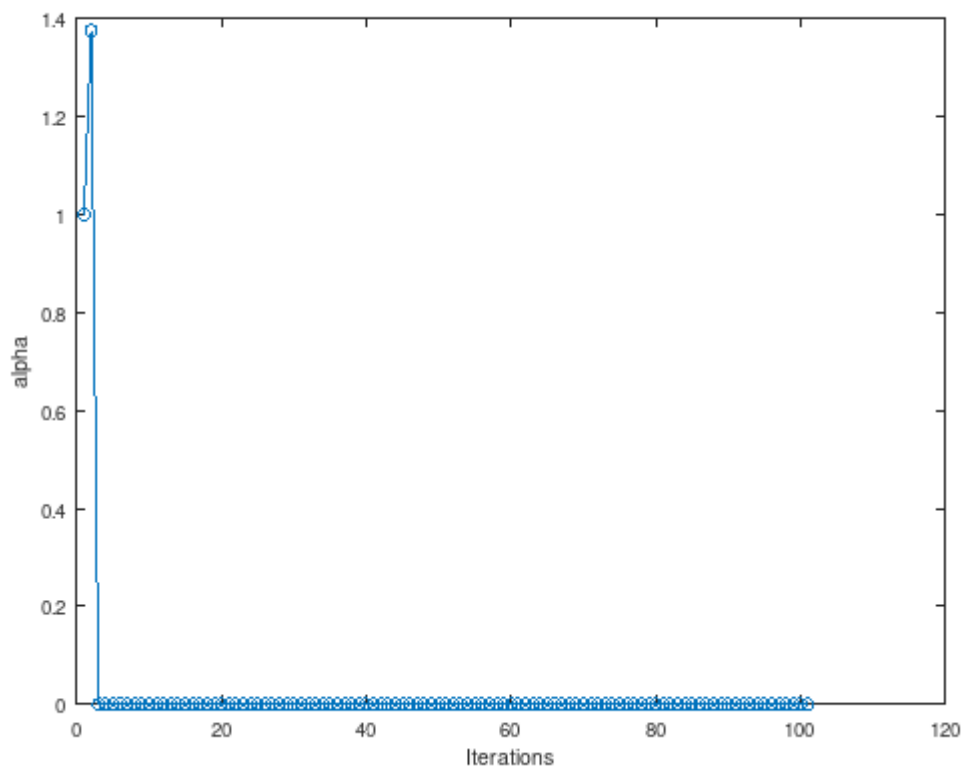
1.644719667126171e+00

1.083231912593286e+00

-2.219774345100689e-01

fmin = 6.425081016625709e-01 - 5.469991617284283e-01i

iter = 1.000000000000000e+02



```
In [70]: search_x = -1:1:1;
search_y = -1:1:1;
[xmin, fmin] = newton(f6_sym, [0.1,0.1,0.1,0.1]', 'backtracking_armijo', search_x, se
```

STARTED Line search using newton

ENDED Line search using newton

xmin =

-1.275648809119522e+00 + 1.948144597017928e-01i

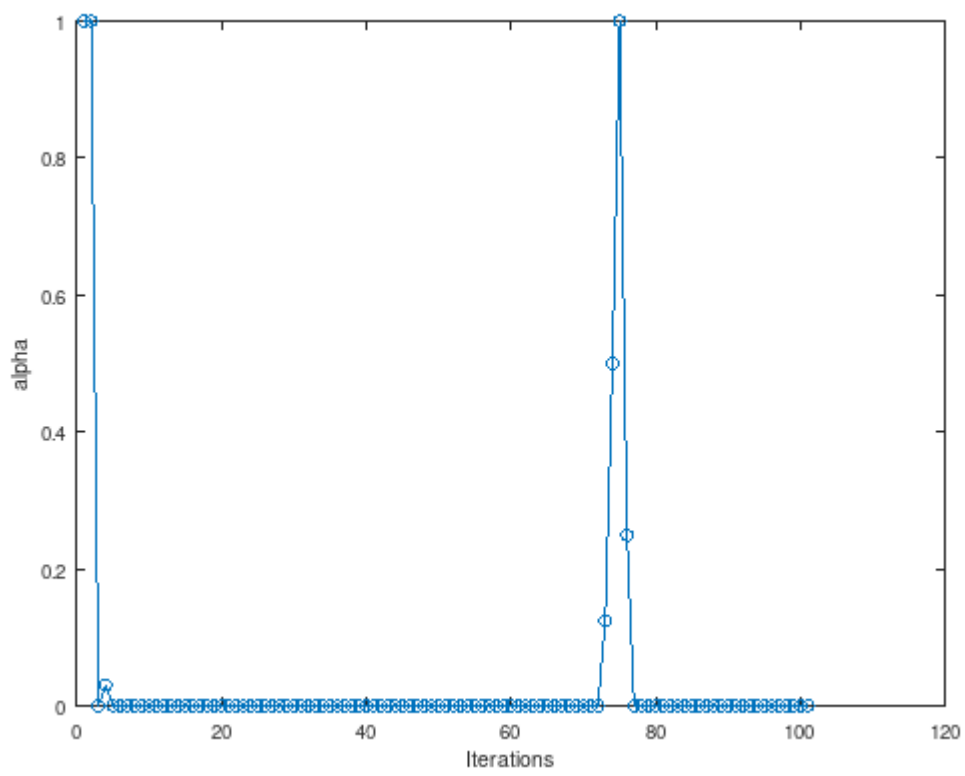
9.616769665903562e-01 + 2.034956822374417e-01i

7.897232587945412e-01 + 7.155710473173078e-01i

-1.999978995807842e-02 + 1.211620234828867e+00i

fmin = -6.761901558860429e-02 - 3.428204759301078e-02i

iter = 1.000000000000000e+02



```
In [71]: search_x = -1:1:1;  
search_y = -1:1:1;  
[xmin, fmin] = newton(f6_sym, [0.1,0.1,0.1,0.1]', 'none', search_x, search_y);
```

STARTED Line search using newton
warning: matrix singular to machine precision, rcond = 3.84378e-17
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 1.05562e-17
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 9.22692e-17
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 1.09759e-17
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 2.51442e-18
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 4.85082e-17
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 1.38253e-19
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 1.39665e-20
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 5.11049e-21
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 5.79531e-19
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 6.55108e-20
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 1.68254e-20
warning: called from
 linsolve at line 113 column 7
 newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 3.50783e-18
warning: called from

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 7.31482e-20

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 3.24632e-20

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 1.11058e-17

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 2.44158e-20

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 9.92179e-21

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 4.61339e-18

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 8.36135e-21

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 3.97565e-21

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 2.15292e-18

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 3.62005e-21

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 2.14544e-21

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 8.59479e-19

warning: called from

linsolve at line 113 column 7

newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 1.27107e-21

warning: called from

linsolve at line 113 column 7

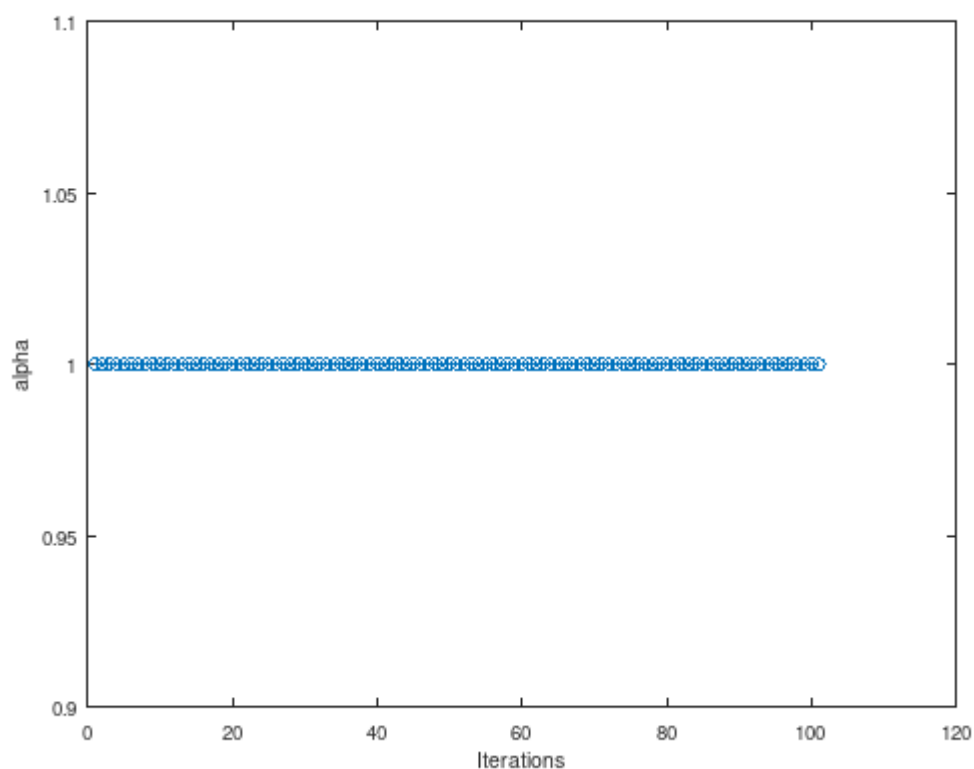
```
warning: matrix singular to machine precision, rcond = 1.28246e-21
warning: called from
    linsolve at line 113 column 7
    newton at line 65 column 12

warning: matrix singular to machine precision, rcond = 4.15527e-22
warning: called from
    linsolve at line 113 column 7
    newton at line 65 column 12

ENDED Line search using newton
xmin =

-4.322675667954263e+11 - 8.000222987957930e+10i
1.381465220186992e+01 - 5.163037414030744e+00i
6.017609396478031e+02 - 5.492879863319638e+02i
6.058424879083266e+04 - 6.602560246354424e+05i

fmin = 2.746306516696158e+02 - 2.645557012175036e+02i
iter = 1.000000000000000e+02
```



```
In [72]: search_x = -1:1:1;
search_y = -1:1:1;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = newton(f6_sym, [0.1,0.1,0.1,0.1]', 'wolfe_strong', search_x, search_y,
```

STARTED Line search using newton

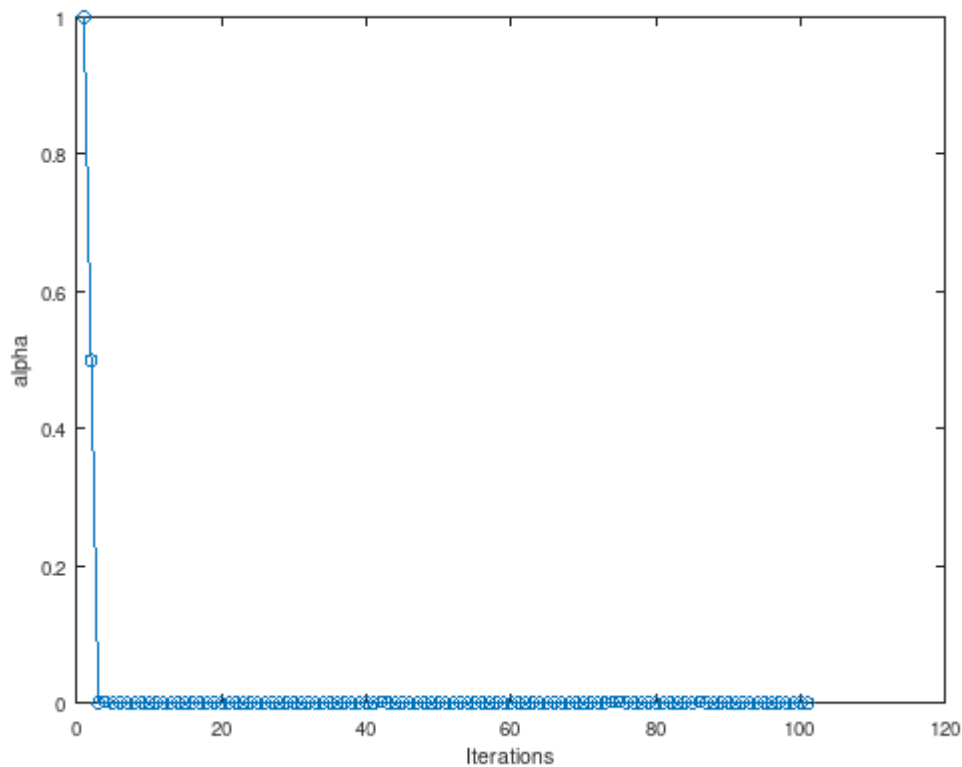
ENDED Line search using newton

xmin =

-1.361385425368038e+00 + 3.885754775529124e-01i
6.695765460675781e-01 + 9.140101024038358e-03i
4.521181075312456e-01 + 2.454475619419351e-02i
-3.192301867236078e-02 + 5.948072978256743e-03i

fmin = -1.061127406857631e+00 + 3.205596677020798e-01i

iter = 1.000000000000000e+02



```
In [73]: search_x = -1:1:1;  
search_y = -1:1:1;  
[xmin, fmin] = newton(f6_sym, [0.1,0.1,0.1,0.1]', 'nonmonotone_backtracking_armijo',
```

STARTED Line search using newton

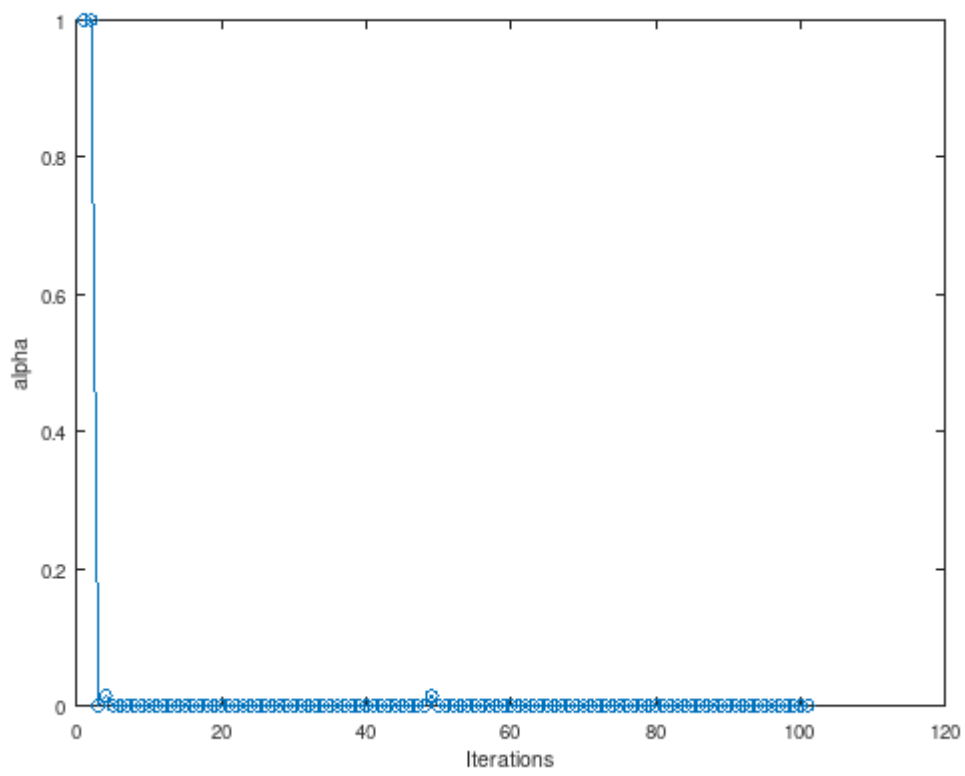
ENDED Line search using newton

xmin =

-1.186260517860390e+00 + 5.675876850936620e-02i
1.167717436927453e+00 + 2.355065436150665e-03i
7.318938876553883e-01 + 8.241635586688709e-03i
-1.456374021543944e-01 + 3.849595251286136e-02i

fmin = -6.263764011833950e-01 - 1.535302585731935e-01i

iter = 1.000000000000000e+02



```
In [4]: search_x = -1:1:1;  
search_y = -1:1:1;  
[xmin, fmin] = steepest_descent(f6_sym, [0.1,0.1,0.1,0.1]', 'bisection_wolfe_weak', s
```

[illegible]

STARTED Line search using steepest descent

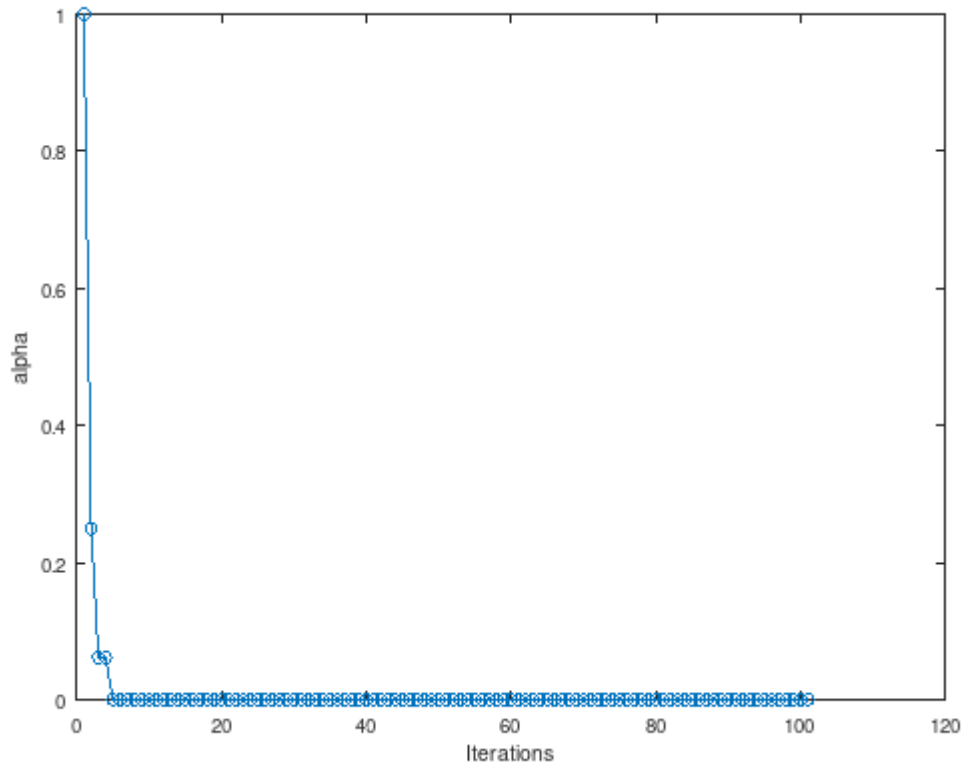
ENDED Line search using steepest descent

xmin =

```
-2.814318502399638e-01 - 2.506790378829603e-02i  
6.881560267709074e-01 - 1.231516695056739e-03i  
2.405134243476587e-01 - 3.186031048226507e-02i  
-9.074265140294489e-02 - 9.102092987613261e-02i
```

fmin = -1.245083923308999e-01 - 2.590845474129049e-02i

iter = 1.000000000000000e+02



```
In [6]: search_x = -1:1:1;  
search_y = -1:1:1;  
[xmin, fmin] = steepest_descent(f6_sym, [0.1,0.1,0.1,0.1]', 'none', search_x, search_y)
```

STARTED Line search using steepest descent

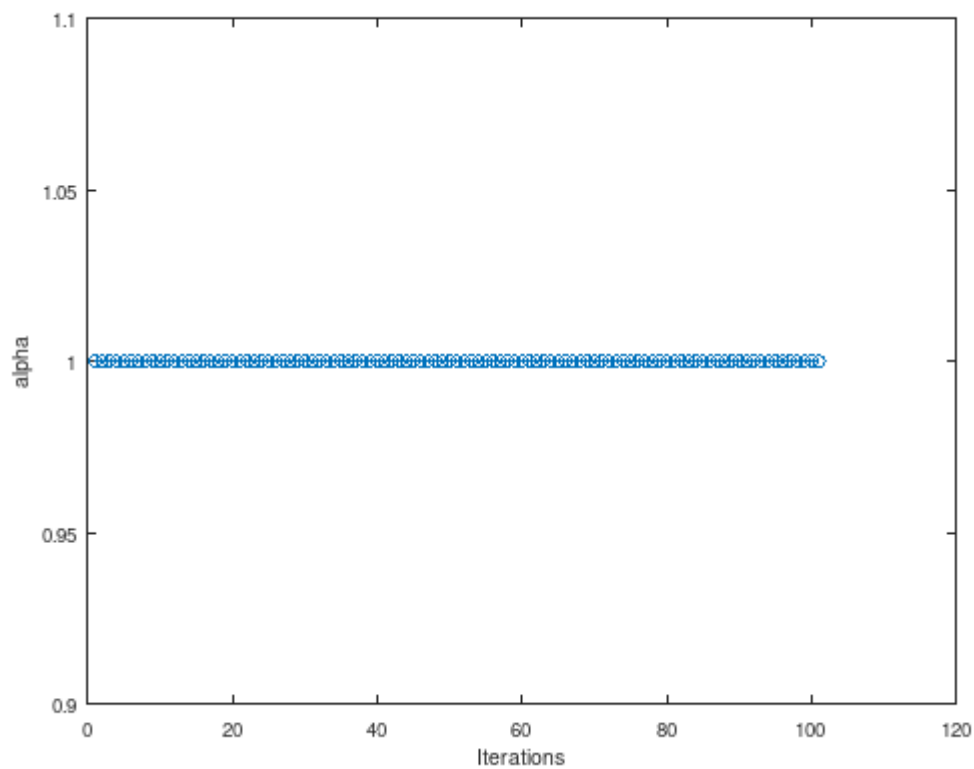
ENDED Line search using steepest descent

xmin =

```
-5.228331177714992e+13 - 1.021207628216148e+14i  
-3.642509764244053e+47 + 1.564103102087386e+47i  
7.217456164812621e+30 - 3.448496448072096e+31i  
-1.328467272463260e+21 + 7.265725877934436e+19i
```

fmin = 2.164291773730777e+95 - 2.278904328655086e+95i

iter = 1.000000000000000e+02



```
In [8]: search_x = -1:1:1;
search_y = -1:1:1;
c = [1e-4 0.9];
rho = 2;
[xmin, fmin] = steepest_descent(f6_sym, [0.1,0.1,0.1,0.1]', 'wolfe_strong', search_x,
```

STARTED Line search using steepest descent

ENDED Line search using steepest descent

xmin =

-1.249155617953189e+00 - 5.574090517722542e-01i

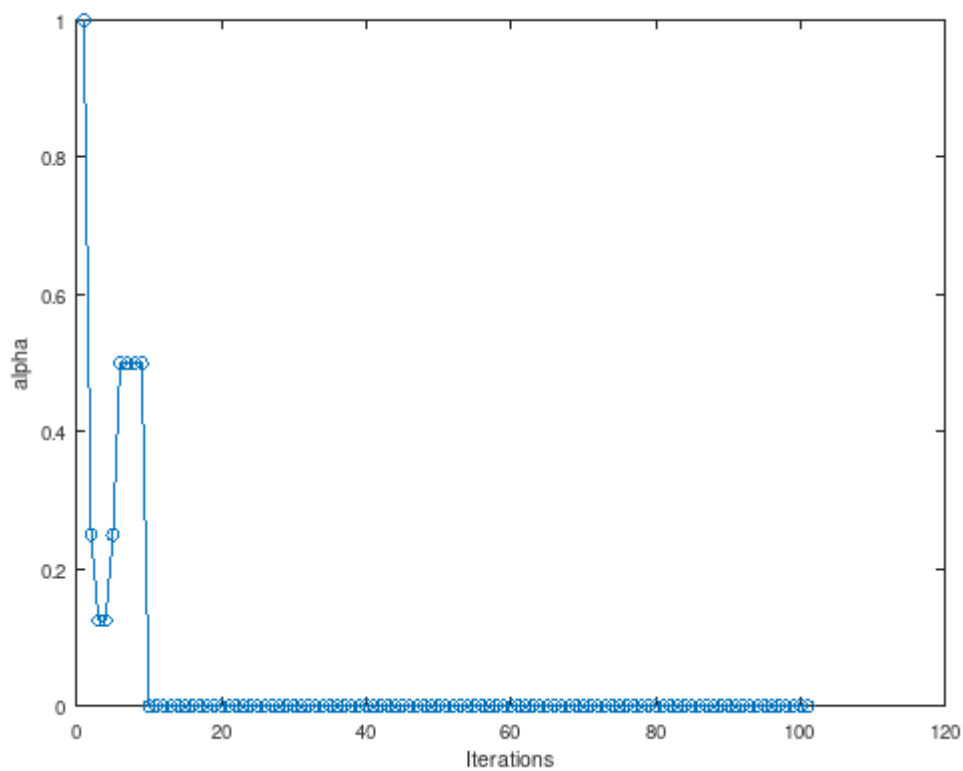
8.667395391229523e-01 - 1.210548444392310e-01i

7.919154692250872e-01 - 6.093374437211777e-01i

1.392880907222363e-01 - 1.114475031123164e+00i

fmin = -2.935628698360437e-02 - 6.335250053537342e-03i

iter = 1.000000000000000e+02



```
In [9]: search_x = -1:1:1;
search_y = -1:1:1;
[xmin, fmin] = steepest_descent(f6_sym, [0.1,0.1,0.1,0.1]', 'nonmonotone_backtracking
```

STARTED Line search using steepest descent

ENDED Line search using steepest descent

xmin =

```
-2.132964834432622e+00 - 1.278702393007991e+00i
1.060685232342657e+00 - 7.261229595966293e-02i
9.546959319230055e-01 - 7.464273282522021e-01i
3.783332994429612e-01 - 1.486521003339685e+00i
```

fmin = -8.194429697305859e-02 - 8.979835730716325e-04i

iter = 1.000000000000000e+02

