Placement report :

Price production decisions with deterministic demand

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Academic year 2010/2011

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Aknowledgment

I want to thank the Høgskolen i molde which allow me to do this placement in there university.

And mostly Guillaume Lanquepin-Chesnais, my supervisor in this placement because of his help during all my trip in Norway and to have use so much time to help me all along my work.

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Introduction

As a second-year student in Polytech'Clermont-Ferrand engineering school, we have to perform a placement abroad to discover professional world and to possibly act up to what we have learned during the years of school.

I made my intership in the university of Molde in Norway : "Høgskolen i Molde". This university is specialized in logistics. During my work in Molde, I worked with Guillaume Lanquepin-Chesnais on lots sizing problem. Lot sizing is a classic issue of logistics which consists to satisfy the demand and minimize the producing cost. We need to balance setup's cost and hold's stock.

As this placement is more "research-type", it consisted in read and understand articles about the topic and to adapt what I have read for a new problem. An other purpose of this placement is to write an article which explain what we do and our results about theses problems. That's why this repport look like an article, I decided to use research standart for the write of this repport.

Hence, in this repport we can find information about the university, the goals of this placement and the different step of my placement whith all my study and results.

1 Høgskolen i Molde

1.1 The university

Molde University College - Specialized University in Logistics is located in the city of Molde, on the Western coast of Norway. Molde University College was established in 1994, after merging the Regional College of Molde and the Nursing School of Molde.

Figure 1: Høgskolen i Molde

The university of Molde is 2000 students, 180 employees and 32 study programs in 2 faculties:

- Faculty of Economics, Informatics and Social Sciences
- Faculty of Health and Social Care

1.2 Purpose of the placement

Lot sizing is a classic issue of logistics which consists to satisfy the demand and minimize the producing cost. We need to balance setup's cost and hold's stock.

Even if there are a lot of heuristics, exact's methods exists. In the 50's, Wagner and Whitin[WW04], show that no producing planning can be optimal if we product and hold in the same time it is the theorem horizon. There are an algorithm, based on the dynamic programming, which solve a lot sizing problem in quadratic complexity (for each period it compute all producing costs.) Then it's showed that the aggregate demand and costs make a convex space where its envelop is the optimal solution[WVK92]. This allow to search the lower cost on this planning horizon, and this search have a $logT$ complexity that's why we have an algorithm with a best complexity, even linear when all parameters are constant and the search occurred in constant complexity.

On an other side Thomas[Tho70] extends the minimization cost of Wagner and Whitin [WW04] by the maximization of profit. In fact in the monopolistic environment the demand depend only of the price that allow to it increase benefits. But this algorithm have a quadratic complexity

The first goald is to use the algorithm based on the dynamic programming of [WVK92] and to apply it in the monopolistic environment to adapt the algorithm of Thomas to have a best complexity.

The second goald of this placement is to add a marketing effect in the model and to study this model.

2 Monopolistic environment

2.1 The Thomas's algorithm

The model

$$\Pi = \sum_{t=0}^{T} d_t p_t - s_t \delta_t - h_{t-1} I_t - x_t X_t \tag{1}$$

subject to

$$\begin{aligned} &\forall t \in \{0..T\} \quad X_t + I_{t-1} - I_t = d_t \\ &\forall t \in \{0..T\} \quad I_t \geq 0, X_t \geq 0 \\ &\forall t \in \{0..T\} \quad \delta_t \in \{0,1\} \end{aligned} \tag{2}$$

d_t is the demand in the period t

p_t is the price in period t

s_t is the price of the setup in period t

$$\delta_t = \begin{cases} 1 & \text{when items is produced in period t} \\ 0 & \text{otherwise} \end{cases}$$

h_t is the price to hold in period t

I_t is the amount stored in period t

x_t is the price to produced in period t

X_t is the amount produced in period t

2.2 The planning horizon

In the Thomas's algorithm [Tho70] the only assumption on the demand is that $d(p_t)p_t$ take is maximum value for a finite value of p_t. This assumption allow to have a single maximum on each batch.

The theorem of the planning horizon is based on $X_t I_t = 0$. Concretely we can't have an optimal solution if we product and stock in the same time. This allow to consider only one state.

Thank's to (2) we reformulated the model :

$$\Pi = \sum_{t=0}^{T} \left[d(p_t) + V_t + \sum_{i=0}^{t} h_i \right] p_t - s_t \delta_t \tag{3}$$

$$\Pi = \sum_{t=0}^{T} \pi_t, \text{ where } \pi_t = \max_p \left(\left[d(p_t) V_t + \sum_{i=0}^{t} h_i \right] p_t - s_t \delta_t \right) \tag{4}$$

We know that we have a single maximum and with this reformulation it is easy to compute this maximum. The Thomas's algorithm [Tho70] can be explain as a short path problem[1] and it is easier to understand :

[1] For one product, it would be the same for several products

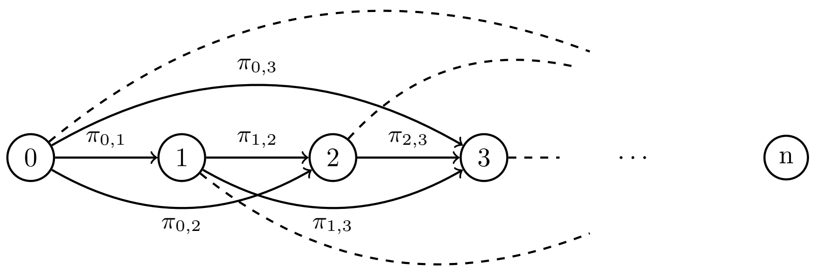

Figure 2: Illustration of the short path problem

Where states are setup period, and $\pi_{t,\tau}$ the profit to go from a setup at the period t to one at the period τ. We can then write the dynamic program above.

$$\Pi(t) = \max_{\tau}[\pi_{t,\tau} + \Pi(\tau)] \tag{5}$$

With this diagram we can easily understand the idea of the planning horizon. In fact a period between two setup is a planning horizon and we only have to maximize the profit on this period. We can understand that for each period Thomas's algorithm [Tho70] compute all previous profits, in the example, at the period 3 we compute $\pi_{2,3}, \pi_{1,3}$ and $\pi_{0,3}$. That's why we have a complexity in T^2

2.3 Idea to perform the algorithm

We have seen that the complexity of Thomas's algorithm [Tho70] is T^2. To decrease this complexity we want to adapt the search of Wagelmans and Kolen [WVK92] and to apply it on the case of Thomas.

The Wagner and Whitin algorithm[WW04] is to minimize the production costs, for each period they compute all the previous production costs that's lead to a complexity in T^2 too. The idea is to use aggregate demands.

Wagelmans and Kolen model Their first model is similar to the Wagner and Whitin one [WW04] with an aggregate demand.

$$G = \sum_{t=0}^{T} p_t X_t + s_t \delta_t + h_t I_t \tag{6}$$

subject to

$$X_t + I_{t-1} - I_t = d_t \tag{7}$$
$$d_{t,T}\, \delta_t - x_t \geq 0 \tag{8}$$
$$I_0 = I_T = 0 \tag{9}$$
$$d_{j,t} = \sum_{k=j}^{t} d_k$$

Then they reformulated the model to add the aggregate number of unit and thank's to that they can work on period's interval.

$$G = \sum_{t=0}^{T} \left(s_t \delta_t + c_t \sum_{k=t}^{T} x_{t,k} \right) \tag{10}$$

subject to

$$\sum_{k=t}^{T} x_{t,k} = d_t \tag{11}$$

$$d_{t,T}\,\delta_t - x_{t,k} \geq 0 \tag{12}$$

$$x_{t,k} \geq 0$$

$$c_t = p_t + \sum_{k=t}^{T} h_k \tag{13}$$

$x_{i,j}$ is the number of units produced in period i to satisfy demand of period j $\geq$ i

To solve this problem they use a convex function with become the tool of search for the lower cost.

$$g(t) = \left\{ \begin{array}{l} \min_{t \leq i \leq T+1} [s_t + c_t d_{t,i-1} + g(i)] \text{ if } d_t \geq 0 \\ \min\left[g(t+1), \min_{t \leq i \leq T+1} s_t + c_t d_{t,i-1} + g(i)\right] \text{ if } d_t = 0 \end{array} \right. \tag{14}$$

It is the built of this function which is the search of the minimize production costs. Instead of compute all previous production costs as in the Wagner and Whitin algorithm [WW04] they compare different slope of this function g. This lead to a $logT$ complexity.

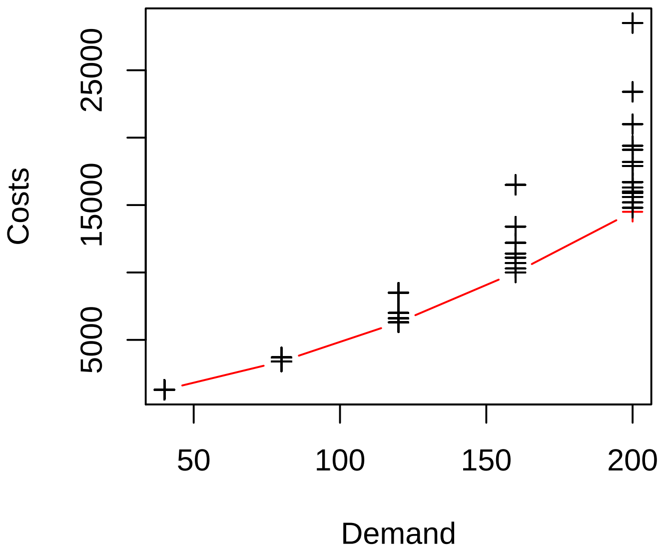

Figure 3: Illustration of g

As we can see on this graphics, the comparison of the slope of g allow to determine the lower cost. We replace the computation of all previous costs by the search of the lowest slope. This is this idea which lead to a $logT$ complexity.

Problem to adapt for Thomas's algorithm The main difference between the two model is that price are not constant in the Thomas's algorithm [Tho70] and of course if the prices are not constant the demand isn't too. The idea is to built a function as Wagelmans and Kolen [WVK92] to maximize the profit but this function isn't convex because of the profit and the demand depend of the price. Thus we can't compare slope of the function and consequently can't use the search in $logT$. To illustrate the problem we make a program which compute all demands for all profit.

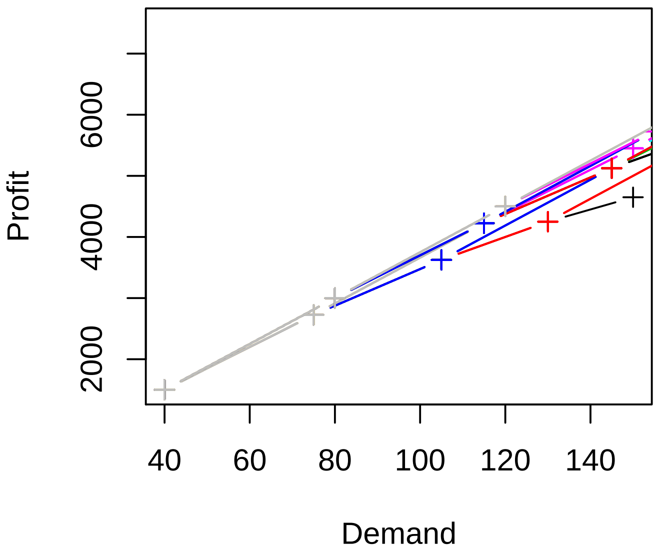

Figure 4: Compute of the profit switch the demand

3 Add of the marketing effect

A new parameter can modify the demand, switch the marketing effect the demand increase differently. With your previous model we can't model some situation like sale moreover marketing is really part of commercial strategy, that's why we try to add in your model the marketing effect.

3.1 The discrete marketing

For adding this marketing effect we add a binary coefficient Δ which is equal to 1 when there is marketing and 0 otherwise, a coefficient M to quantify the marketing effect and of course the cost of this marketing effect c. This allow to change just a little the model and to use the Thomas 's algorithm [Tho70] and the main asset of this discrete model is to have a lag effect. In fact the marketing have effect on several period that's why we try thank's to this discrete model to characterize this aspect.

$$\Pi = \sum_{t=0}^{T} D(p_t)p_t - s_t\delta_t - h_{t-1}I_t - x_tX_t - \Delta_t c_t \tag{15}$$

subject to

$$\forall t \in \{0..T\} \quad D_t(p_t) = d_t(p_t) + \Delta_t M_t + \Delta_{t-1} M_{t-1} \tag{16}$$
$$\tag{17}$$

Here we focus on two period t and t-1 but it is easy to extend this to more periods.

D_t is the demand in the period t with the marketing effect
M_t is the marketing effect in period t
c_t is the cost of marketing in period t

In this model the idea is that if we have marketing we add a quantity to the demand and of course a marketing's cost. As in the Thomas's algorithm [Tho70] the function Π is concave for all Δ. This model allow to have the theorem of the planning horizon even if we have a link between period because of the marketing. In fact as we just add a binary and a parameter which don't depend of the price we have the same assumption, we just have to know if we have marketing in the last batch, for this we add an indicator. To illustrated this we can use again the short path model :

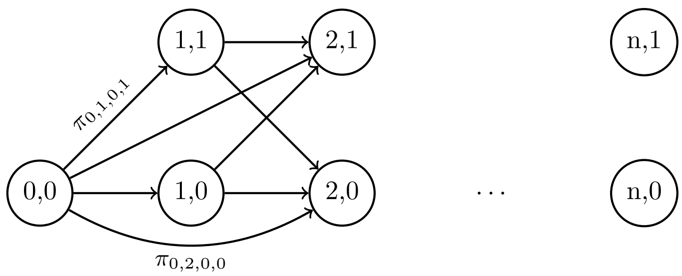

Figure 5: Short path for discrete marketing model

Now we must add indicator to know if we have marketing before or not, the indicator of $\pi_{i,j,k,l}$ mean :

(i, j) Period during which the profit is compute

$$k = \begin{cases} 1 \text{ if } \Delta_i = 1 \\ 0 \text{ if } \Delta_i = 0 \end{cases}$$

$$l = \begin{cases} 1 \text{ if } \Delta_j = 1 \\ 0 \text{ if } \Delta_j = 0 \end{cases}$$

For example in the figure (5) we can see $\pi_{0,1,0,1}$ it the profit between the period 0 and the period 1 knowing that there is no marketing for period 0 and there is marketing for the period 1.

To solve this model we have made an algorithm based on the Thomas's algorithm [Tho70] which compute the profit with and without marketing thank's to Thomas's method and choose the higher profit for each period. This algorithm need more computation but it have the same complexity.

The problem of this model is that we have no solver which allow to solve a no linear problem with binary variable, thus we have an algorithm but we can test it effectively.

3.2 The continue marketing

Using binary variable is very binding and in order to have a link between the price and the marketing effect we have add a continue variable q which represent the marketing effect in the period t per unit, this allow us to insert the effect and it cost in the demand.

$$\Pi = \sum_{t=0}^{T} d(p_t, q_t)p_t - s_t\delta_t - h_{t-1}I_t - x_tX_t \tag{18}$$

$$d(p_t, q_t) = d_0 - \alpha p_t + \beta q_t + \gamma q_t p_t \tag{19}$$

$$d_0, \alpha, \beta \geq 0 \tag{20}$$

For this model we built the demand in order to have :

When the price increase the demand must decrease $\Rightarrow -\alpha p_t$
When the marketing increase the demand must increase $\Rightarrow \beta q_t$

We want to characterize the sale effect, in fact when the price decrease and the marketing increase at the same time the demand should increase a lot, more than when just the price decrease or the marketing increase independatly. To have this effect we must have $\frac{\partial d^2}{\partial p \partial q} \neq 0$

We assume that to be coherent with the idea of sale $\frac{\partial d^2}{\partial p \partial q} \leq 0$. To illustrated your model we use a linear regression which compute the coefficient α, β and γ in the sale case.

	1	2	3	4	5	6	7	8	9	10
p	100	90	80	70	60	50	40	30	20	10
q	1	2	3	4	5	6	10	28	40	50
d	12	20	30	40	50	60	70	80	90	100

Table 1: Data for the linear regression

We can see in the table (3.2) that p decrease and q increase at the same time so we are in the case of sale, the demand should increase too and we can see that the coefficient of interaction p:q (with represent γ) is negative.

Now we know the sign of γ we can have an other condition because we must have $\frac{\partial d}{\partial p} \leq 0$ and $\frac{\partial d}{\partial q} \geq 0$

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	108.5427	1.3621	79.69	0.0000
p	-0.9759	0.0151	-64.80	0.0000
q	0.0288	0.0234	1.23	0.2657
p:q	-0.0002	0.0014	-0.17	0.8712

Table 2: Illustration with a linear regression for the model

$$\frac{\partial d}{\partial q} = \beta + \gamma p \qquad \frac{\partial d}{\partial q} \geq 0 \Rightarrow q \geq \frac{\alpha}{\gamma}$$

This condition is already true because q is always positive

$$\frac{\partial d}{\partial p} = -\alpha + \gamma q \qquad \frac{\partial d}{\partial p} \leq 0 \Rightarrow p \leq \frac{-\beta}{\gamma} \tag{21}$$

In practice (21) can be understand by a price limit to don't have a negative demand. In fact we have : $p \leq \frac{d_0}{\alpha} \leq \frac{-\beta}{\gamma}$ We can see with the numerical result of the linear regression that $\frac{d_0}{\alpha} = 112$ and $\frac{-\beta}{\gamma} = 144$

Now we know all sign of the coefficient we can study the extremum of the profit function. To study this profit function Π is the same to study the function:

$$\phi(p, q) = d_0 p - \alpha p^2 + \beta pq + \gamma p^2 q$$

This function is affine for q and concave for p :

We can write ϕ as a polynom of degree 2 and as its first coefficient is negative this function is concave :

$$\phi(p, q) = \phi_q(p) = Ap^2 + Bp \quad \text{where } A = \gamma q - \alpha \leq 0 \quad and \quad B = \beta q + d_0$$

Thank's to this, and the fact that p and q is in a compact, we have a q_{max} and for each q we have a parabol which have a maximum which depend of q. So if we compute this maximum with q_{max} we have the maximum of ϕ and the only one.

So this profit function have one and only one maximum and we can compute it, theses results allow us to use the Thomas's algorithm [Tho70].

To illustrated the demand and is evolution switch the price and the marketing effect :

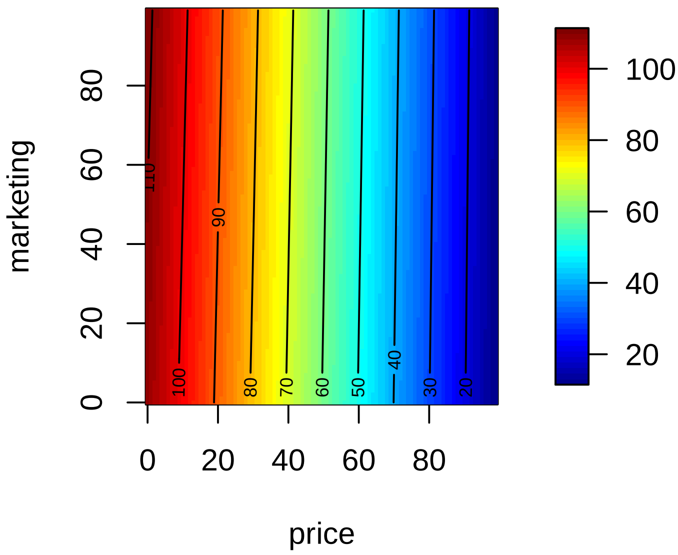

Figure 6: Linear model

As we can see on this figure this model isn't so representative, a linear diagram isn't the best way to model. That's why we try with an exponential model :

$$d(p,q) = d_0 - \alpha logp + \beta logq + \gamma log(pq) \tag{22}$$
$$d'(p,q) = \kappa p^{\alpha} q^{\beta} exp(\gamma log(pq)) \tag{23}$$

But the last term $exp(\gamma log(pq))$ have no sens for your model, moreover if we use the numerical result of the linear regression we can out that this term is negligible

$$1 \leq p \leq 100 \quad 1 \leq q \leq 100 \Rightarrow \gamma \leq \gamma log(pq) \leq 4\gamma$$
$$\gamma = 0.0002 \Rightarrow 0,0005 \leq exp(\gamma log(pq)) \leq 0,0008$$
$$exp(\gamma log(pq)) \approx 0$$

That's why now we will study this new model, furthermore this model have already use by Won and DaeSoo [Won93]

$$\Pi = \sum_{t=0}^{T} d(p_t, q_t)(p_t - q_t) - s_t \delta_t - h_{t-1} I_t - x_t X_t \tag{24}$$

subject to

$$d(p_t, q_t) = \kappa p_t^{\alpha} q_t^{\beta} \tag{25}$$
$$\tag{26}$$

An important property of this profit function is to have one and only one maximum in order to be abble to use the Thomas's algorithm [Tho70] To study this function we just have to study the function :

$$\phi(p, q) = p^{\alpha} q^{\beta} (p - q)$$

As it model a price and a marketing it is in a compact so there is a maximum and to have a condition to have an unique maximum we just have to solve :

$$\begin{aligned} &\nabla\phi = 0 \\ &\Leftrightarrow \begin{cases} (\alpha+1)p - \alpha q = 0 \\ \beta p - (\beta+1)q = 0 \end{cases} \end{aligned}$$

This system have one and only one solution if its determinant is non null. Thus this function profit admit one and only one maximum if $\alpha + \beta \neq -1$

To illustrated the demand and is evolution switch the price and the marketing effect :

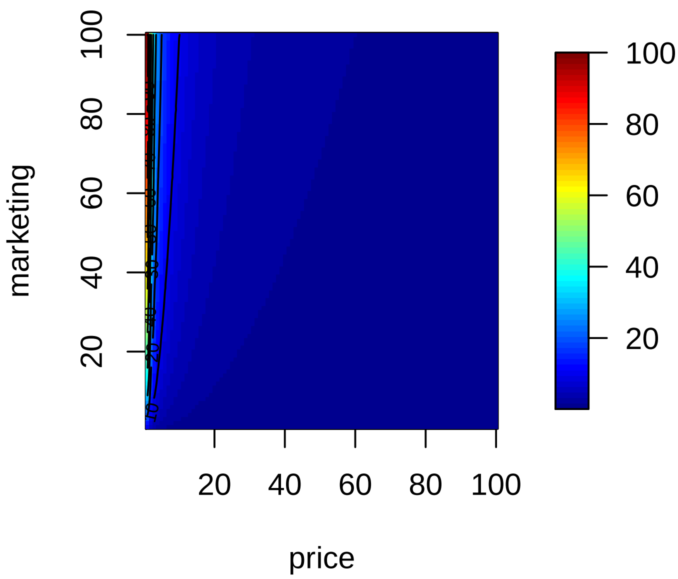

Figure 7: Exponential model

3.3 The delay effect

$$\Pi = \sum_{t=0}^{T} d(p_t, q_t, q_{t-1}, q_{t-2}, ...)p_t - s_, t\delta_t - h_{t-1}I_t - x_t X_t \tag{27}$$

subject to

$$d(p_t, q_t, q_{t-1}, q_{t-2}, ...) = \alpha p_t - \beta q_t - \gamma q_t p_t + \sum_{i=0}^{t-1} \theta^{t-i} q_i \tag{28}$$

$$X_t + I_{t-1} - I_t = d_t \tag{29}$$

$$I_t \geq 0 \quad X_t \geq 0 \quad \delta_t \in \{0, 1\}$$

Conclusion

To use the Wagelmans and Kolen [WVK92] idea to perform the Thomas's algorithm [Tho70] isn't possible or in fact the asset of this method can't balance the difficult to compare the different slope and thus it don't allow to decrease the complexity of the algorithm which is our purpose.

We have study different model for the adding of the marketing effect, each model have its own assets and disavantages, no one is really perfect but may be they can lead to a better model. For example the discret model can add a delay effect but it use a binary variable which is very binding and it is further of the reality contrary to the continue models whoses can't easily add a delay effect but they can add the sale effect which can close theses models to the reality.

For a personnel effect this placement allowed me to understand a bit more about the research and its way of proceed, where the most important is not really the results but the understanding of the methods and that it is such important to know if our idea is feasible or not. It is an idea that we don't have the habit. Moreover this placement is a way to introduced some tools usefull for the research such as R, Latex, Git or Python.

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