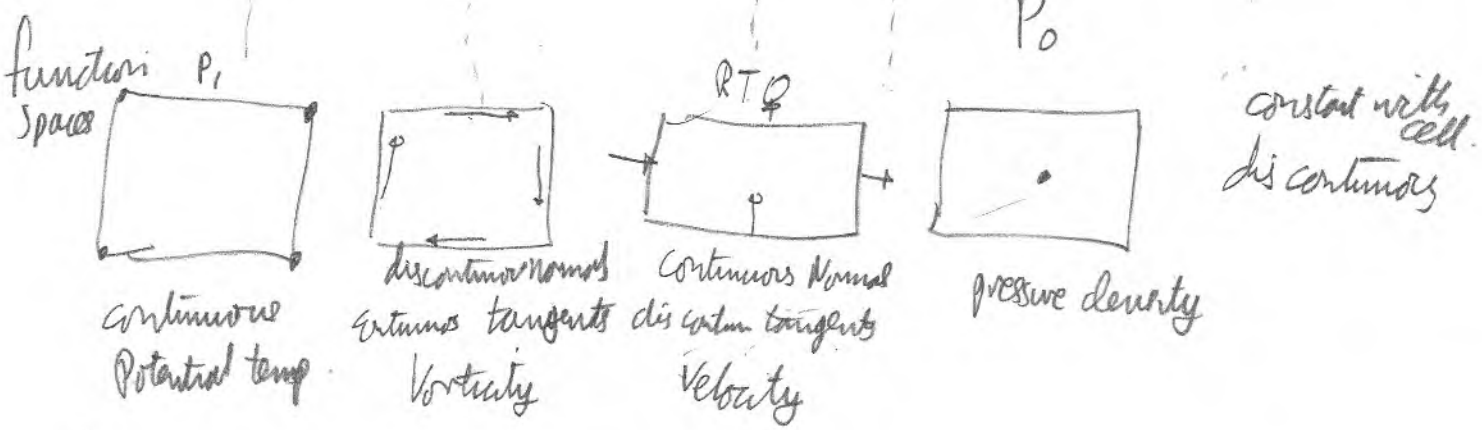
DynaMO – a PSyclone-lite test case 12/2/2014

Galerkin projection ~~for~~ on a bi periodic plane.
RT0 Quads – extruded in the vertical to make a cuboid

in 3D we have 4 spaces (2+1)D

$$V_0 \xrightarrow{\nabla} V_1 \xrightarrow{\nabla\times} V_2 \xrightarrow{\nabla\cdot} V_3$$



Galerkin projection

Take analytic function and project them into an FE space
function space means where dof for field lives on element.
each FS has basis test functions

Chose basis test functions,

$V_0 : \gamma$ – continuous linear only in this cell — tri-linear in 3D. 8 values.

$V_1 : \underset{\sim}{\varepsilon}$ – bi-linear $\times$ constant. (probably)

$V_2 : \chi$ ~ linear in normal $\times$ const $\times$ const.

$V_3 : \rho$ ~ constant

$F(\xi) \to \pm \frac{\xi}{\Delta\xi}$  , $G=1 \to$ 

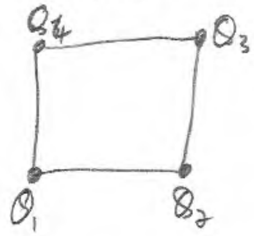
$\gamma = F(x)\,F(y)\,F(z)$ for example.

$\underset{\sim}{v} = \left(F(x).G(y).G(z),\ 0,\ 0 \right)$ 

in this low order case $G=1$, but ~~who~~ want to keep this structure because in higher order it isn't!

$\rho = G(x).G(y).G(z) = 1$

Consider a variable in the V_0 space.

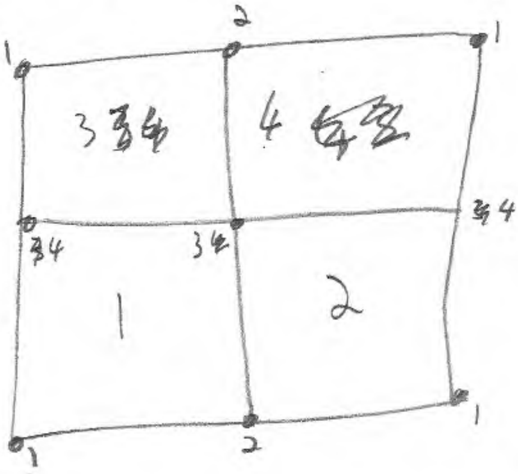


$$\Theta = \Theta_1\gamma_1 + \Theta_2\gamma_2 + \Theta_3\gamma_3 + \Theta_4\gamma_4 \quad \text{(in 2D)}$$

$$\Theta = \sum_{i=1}^{ndf} \Theta_i \gamma_i$$

in full model $\Theta_1\left[\text{dofmap}V_0(i,1)\right]\gamma_1 + \Theta_2\left[\text{dofmap}V_0(i,2)\right]\gamma_2 + \dots$

Consider 2 element Bi periodic plane



$$\text{dofmap}V_0 = \begin{array}{c} \text{cell} \rightarrow \\ \left(\begin{array}{cccc} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 4 & 3 & 2 & 1 \\ 3 & 4 & 1 & 2 \end{array} \right) \end{array} \begin{array}{c} \\ \text{dof} \end{array}$$

in this case 'dofmapV_2' is the same, but not true in general, eg. cube!

Want to solve the following

$$\Theta = \Theta^*$$

$$\int \gamma \Theta \, d\underset{\sim}{x} = \int \gamma \Theta^* \, d\underset{\sim}{x}$$ this produces a Matrix vector system.

$M_\Theta \hat{\Theta} = R_\Theta \leftarrow$ known vector which is our "analytic" function

$M_\Theta\left[\#\Theta_{dofs},\ \Theta\#_{dofs}\right]$, $R_\Theta\left[\#\Theta_{dofs}\right]$, $\hat{\Theta}$'s the thing

we want to know this $\hat{\Theta} = M_\Theta^{-1} R_\Theta$

Rather than determine M and invert compute M^{e} ie per element and assemble M^{-1} from M^{ec}

Will also assemble R, element (actually, cell).

$$\left(M_{\Theta}^{c}\right)^{df=1} = \int \gamma_1 \left[\Theta_1 \gamma_1 + \Theta_2 \gamma_2 + \Theta_3 \gamma_3 + \Theta_4 \gamma_4 \cdots \right] d\underset{\sim}{x}_c$$

this is one row of the local matrix.

$$\left(M_{\Theta}\right)^{jk} = \int \gamma_j \gamma_k \, d\underset{\sim}{x}_c$$

choose some quadrature rule to evaluate.

j^{th} dof, k^{th} element expansion

$$= \sum_{i}^{nquad} W_i \gamma_j(x_i) \gamma_k(x_i)$$

Weights are determined once at the start as are x_i. $\gamma(x_i)$ are known and precomputed.

thus have a (#df, #df) Matrix of values for each cell

Also need R.

$$\left(R_{\Theta}^{c}\right)^{j} = \int \gamma_j \Theta^{*} d\underset{\sim}{x}_c$$

$$= \sum_{i}^{nquad} W_i \gamma_j(x_i) \Theta^{*}(x_i)$$

back to global

compute R_Θ from R_Θ^c

```
Do i = 1, ncell  <- R_Θ^c compute
  Do j = 1, ndf
    ij = dofmapv0(i,j)
    R_Θ(ij) = R_Θ(ij) + R_Θ^c(j)
  enddo
end do.
```

a kernel!

now compute LHS.

Do i = 1, ncell

compute M_Q^c for this cell.

Do j = ndf

ij = dofmap Vo (ij)

$Q^c_{(j)} = Q(ij)$

end do.

$L_Q^c = \text{matmut}(M_Q^c, Q_c)$

Do j = ndf.

ij = dofmapVo (ij)

$L_o(ij) = L_o(ij) + L^c_o(j)$

enddo

end do.

some intial guess for Q.

this is mat ax of eg bicgstab solver

$L_o = R_o$ + err

done ✓.