Bayesian band-recovery model workshop

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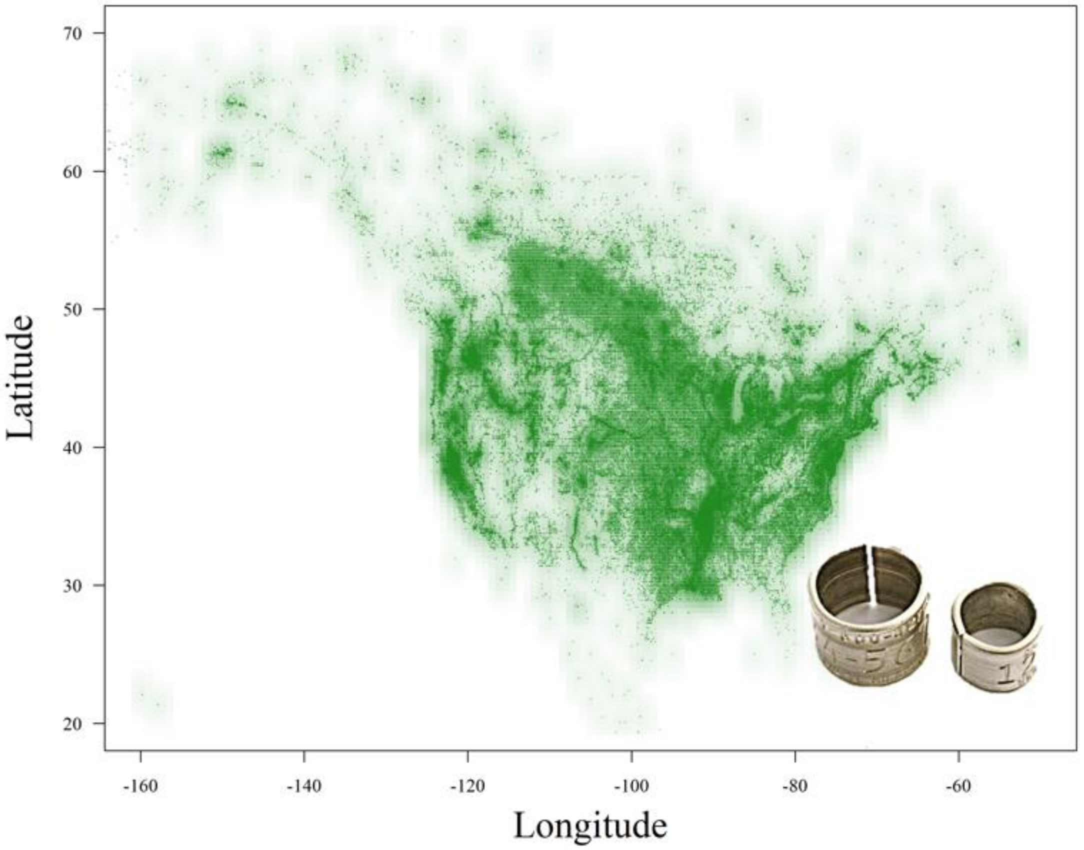

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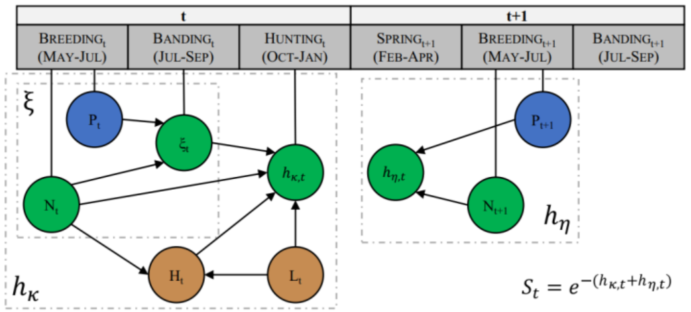

FIGURE 2 Directed acyclic graph demonstrating the hypothesized relationships among mallard breeding pair abundance (*N*), the number of ponds (*P*), harvest limits (*L*), the abundance of duck hunters (*H*), fecundity (ξ), harvest mortality hazard rate (h_κ), natural mortality hazard rate (h_η) and survival (*S*) for mallards marked and released in the Prairie Pothole Region of the United States and Canada, 1974–2016. Arrows represent covariate effects, grey dashed lines enclose separate generalized linear models and vertical solid lines denote the time period or interval when parameters were estimated, where survival (*S*) and natural mortality in year *t* are estimated from banding in year *t* to banding in year *t*+1. We estimated age-specific band recovery probabilities (*f*) as a function of age-specific harvest probability (κ), reporting rate (*r*) and crippling rate (c), $f = \kappa(1-c)r_t$. We note that we hypothesized the same relationships among demographic components for both juvenile and adult females.

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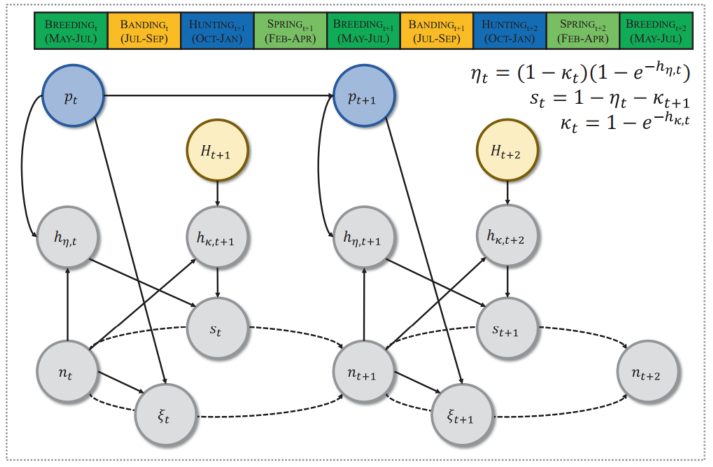
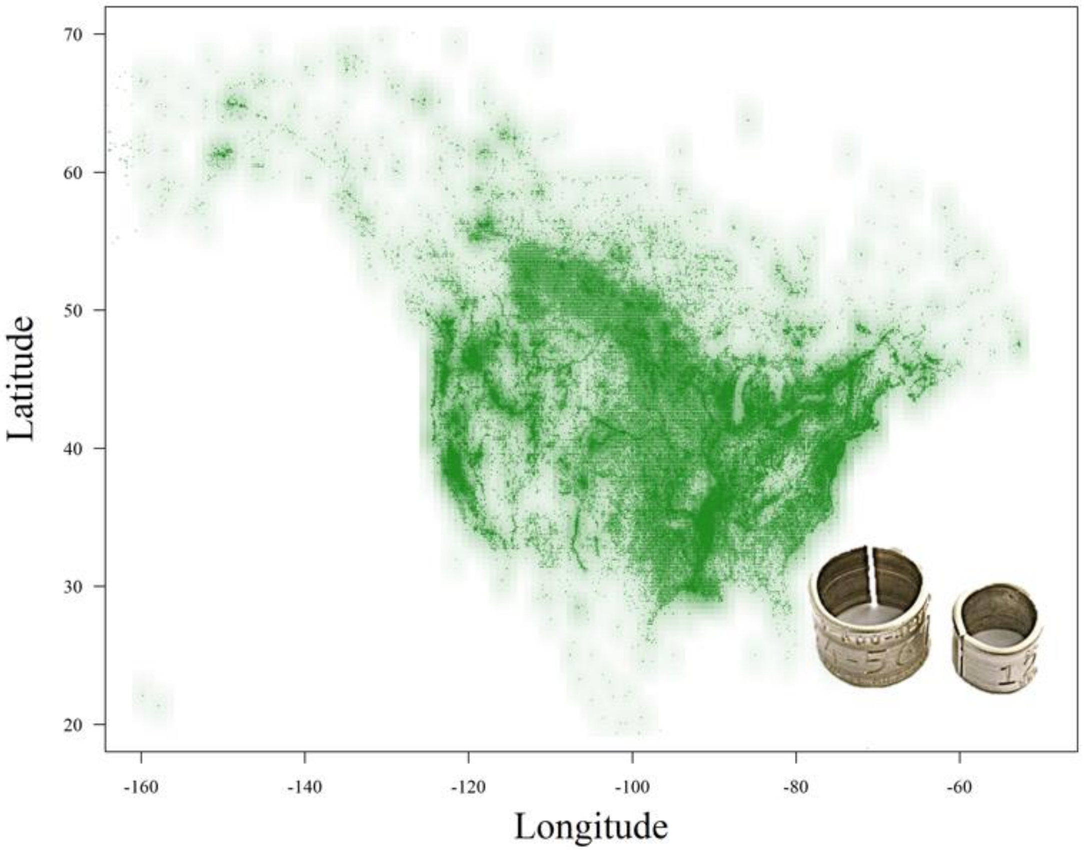


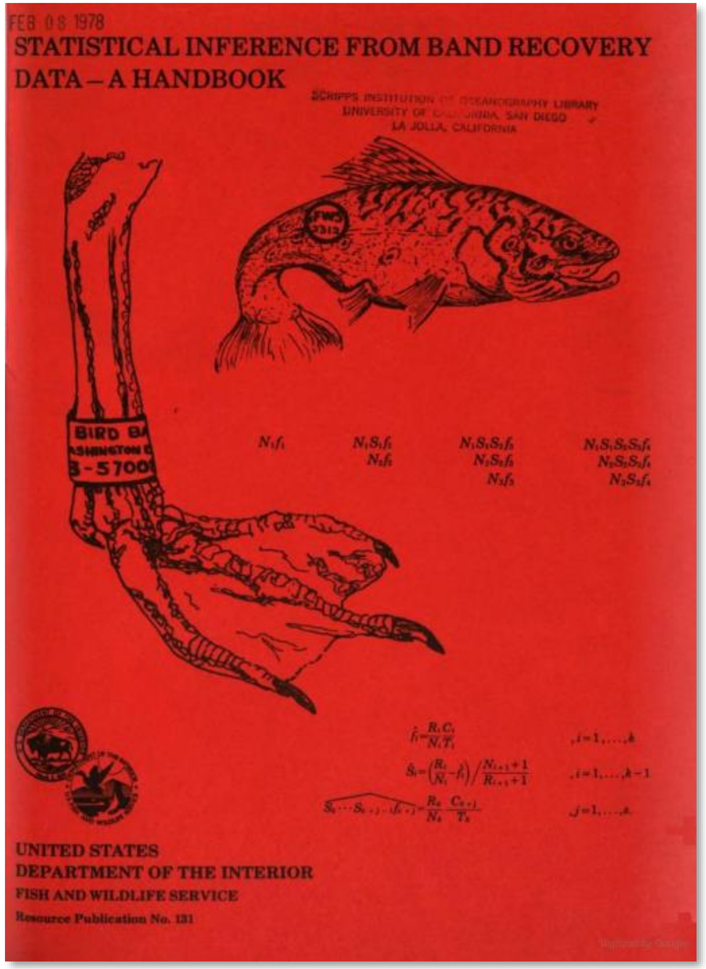
FIGURE 2 A directed acyclic graph demonstrating the relationships among abundance (***n***), ponds (***p***; blue), fecundity (ξ), hunting mortality hazard rate ($\boldsymbol{h}_\kappa$), natural mortality hazard rate ($\boldsymbol{h}_\eta$), survival (**s**) and the number of duck hunters (***H***; brown) for blue-winged teal breeding in the North American Prairie Pothole Region across the annual cycle (1973–3016). Solid arrows represent estimated directional relationships, and dashed arrows represent processes leading to changes in population abundance.

These data are the product of citizen-science



Cavell Brownie figured this out 50 years ago!

Dr. Cavell Brownie
PhD, Cornell University
Professor Emeritus, North Carolina State University



Going Bayesian

Dave Koons

Todd Arnold

Scott Boomer

Jim Sedinger

Facilitators

Thomas Riecke &
Ben Sedinger

Dan Gibson

Madeleine
Lohman

Caroline
Blommel

Jordan
Thompson

Workshop ‘tone’

1. The facilitators are the strength of the workshop.

2. We’re going to cover concepts at different levels: ask questions!

3. We’re going to do small group exercises: get to know folks!

4. This is complicated: be patient (and kind).

Outline

1. How band-recovery models work and some coding basics (1h)

2. Priors: estimating band-recovery and harvest probability (1h)

3. Survival models: random effects and covariates (1h)

4. Lincoln estimates and cross-seasonal models (1h)

5. Path analysis and acquiring data from the BBL (1h)

1. How band-recovery models work and coding basics

Year banded	Number banded	Recoveries by hunting season				
		1	2	3,	…	,k
1	N_1	R_{11}	R_{12}	R_{13},	…	,R_{1k}
2	N_2		R_{22}	R_{23},	…	,R_{2k}
3	N_3			R_{33},	…	,R_{3k}
.	.				.	.
.	.				.	.
.	.				.	.
k	N_k					R_{kk}

How do we format our data…?!

An example from a 2-year study

We format the data into m-arrays* and release vectors

$$M = \begin{bmatrix} & & \\ & & \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{1000} \\ 1000 \end{bmatrix}$$

We released 1000 birds in year 1

***m-array is short for multinomial array**

We format the data into m-arrays and release vectors

$$M = \begin{bmatrix} \quad \end{bmatrix} \qquad R = \begin{bmatrix} 1000 \\ \mathbf{1000} \end{bmatrix}$$

We released 1000 birds in year 2

We format the data into m-arrays and release vectors

$$M = \begin{bmatrix} \mathbf{100} & & & \\ & & & \end{bmatrix} \qquad R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

We had 100 <u>direct</u> recoveries in year 1

We format the data into m-arrays and release vectors

$$M = \begin{bmatrix} 100 & \mathbf{50} & \\ & & \end{bmatrix} \qquad R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

We had 50 <u>indirect</u> recoveries initially marked in year 1 recovered in year 2

We format the data into m-arrays and release vectors

$$M = \begin{bmatrix} 100 & 50 & \mathbf{850} \end{bmatrix} \qquad R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

850 birds from year 1 were never recovered

We format these data into m-arrays and release vectors

$$M = \begin{bmatrix} \mathbf{100} & \mathbf{50} & \mathbf{850} \\ & & \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{1000} \\ 1000 \end{bmatrix}$$

‘Fate(s)’ of 1000 birds released in year 1…

We format the data into m-arrays and release vectors

$$M = \begin{bmatrix} 100 & 50 & 850 \\ \mathbf{0} & & \end{bmatrix} \qquad R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

No birds released in year 2 were shot in year 1

We format the data into m-arrays and release vectors

$$M = \begin{bmatrix} 100 & 50 & 850 \\ 0 & \mathbf{100} & \end{bmatrix} \qquad R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

We had 100 direct recoveries in year 2

We format the data into m-arrays and release vectors

$$M = \begin{bmatrix} 100 & 50 & 850 \\ 0 & 100 & \mathbf{900} \end{bmatrix} \qquad R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

900 birds from year 2 were never recovered

We format these data into m-arrays and release vectors

$$M = \begin{bmatrix} 100 & 50 & 850 \\ \boxed{0 \quad 100 \quad 900} \end{bmatrix} \qquad R = \begin{bmatrix} 1000 \\ \mathbf{1000} \end{bmatrix}$$

‘Fate(s)’ of 1000 birds released in year 2…

We format these data into m-arrays and release vectors

$$M = \begin{bmatrix} \mathbf{100} & \mathbf{50} & \mathbf{850} \\ 0 & 100 & 900 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{1000} \\ 1000 \end{bmatrix}$$

Each row of M has to add up to that year's R

This starts to get complicated (?!)

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16
1	49	31	22	14	8	15	7	5	1	2	1	2	1	1	0	841
2	0	52	33	22	14	10	4	9	5	2	3	2	1	0	0	843
3	0	0	50	26	24	21	17	8	9	7	5	6	2	5	0	820
4	0	0	0	43	37	16	11	8	12	7	5	4	1	1	1	854
5	0	0	0	0	47	41	19	22	4	8	11	3	1	5	2	837
6	0	0	0	0	0	61	45	24	16	10	13	8	5	2	4	812
7	0	0	0	0	0	0	47	36	31	13	23	9	7	3	2	829
8	0	0	0	0	0	0	0	50	43	33	19	13	9	6	2	825
9	0	0	0	0	0	0	0	0	53	36	30	24	8	11	5	833
10	0	0	0	0	0	0	0	0	0	51	33	28	18	8	14	848
11	0	0	0	0	0	0	0	0	0	0	50	28	22	20	12	868
12	0	0	0	0	0	0	0	0	0	0	0	58	37	26	13	866
13	0	0	0	0	0	0	0	0	0	0	0	0	49	36	20	895
14	0	0	0	0	0	0	0	0	0	0	0	0	0	54	28	918
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	52	948

Oh geez… we’ll learn some simple tricks!

Our first (back-of-the-envelope) survival estimate!

An example from a 2-year study

The basic idea: releases (R)

- Let's imagine we capture, mark, and release 1000 ad. male mallards (immediately prior to the hunting season)

$$R = \begin{bmatrix} \mathbf{1000} \\ \\ \end{bmatrix}$$

$$R_1 = 1000$$

$$M = \begin{bmatrix} & & \\ & & \end{bmatrix}$$

The basic idea: m-arrays (M: multinomial array)

- Let's imagine we capture and mark 1000 ad. male mallards

- 100 are shot and reported by hunters that hunting season
  - i.e., direct 'recoveries'

$$R = \begin{bmatrix} \mathbf{1000} \\ \end{bmatrix}$$

$$m_{1,1} = 100$$

$$M = \begin{bmatrix} \mathbf{100} & & & \\ & & & \end{bmatrix}$$

The basic idea: band recovery probability (*f*)

- Let's imagine we capture and mark 1000 ad. male mallards
- 100 are shot and reported by hunters that hunting season
  - i.e., 'direct' recoveries

P: cell probabilities

$$P = \begin{bmatrix} f_1 & \\ & \end{bmatrix}$$

$$R = \begin{bmatrix} \mathbf{1000} \\ \end{bmatrix}$$

$$M = \begin{bmatrix} \mathbf{100} & \\ & \end{bmatrix}$$

$$\boldsymbol{f_1 = \frac{100}{1000} = 0.1}$$

The basic idea

- The exact same thing happens next year (seems unlikely, but ok…)

$$P = \begin{bmatrix} f_1 & & \\ 0 & \color{red}{f_2} & \end{bmatrix}$$

$$R = \begin{bmatrix} 1000 \\ \color{red}{\mathbf{1000}} \end{bmatrix}$$

$$\color{red}{\boldsymbol{f_2 = \frac{100}{1000} = 0.1}}$$

$$M = \begin{bmatrix} 100 & & \\ 0 & \color{red}{\mathbf{100}} & 900 \end{bmatrix}$$

A note about subscripts

$$f_1 = \frac{100}{1000} = 0.1$$

This corresponds to year 1

$$f_2 = \frac{100}{1000} = 0.1$$

This corresponds to year 2

Imagine we also shoot some more birds marked in *t*=1 during the t=2 hunting season

- We also shoot 50 ducks marked in the first year in year 2…
  - i.e., 'indirect' recoveries

- <u>**What does the 50 tell us?***</u>

$$P = \begin{bmatrix} f_1 & \color{red}{?} & \\ 0 & f_2 & \end{bmatrix}$$

$$R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

$$M = \begin{bmatrix} 100 & \color{red}{\mathbf{50}} & 850 \\ 0 & 100 & 900 \end{bmatrix}$$

<u>***This is the key idea**</u>

If we know we're shooting 10% of the birds….

$$100 = 1000 \times f_2$$
$$f_2 = 0.1$$

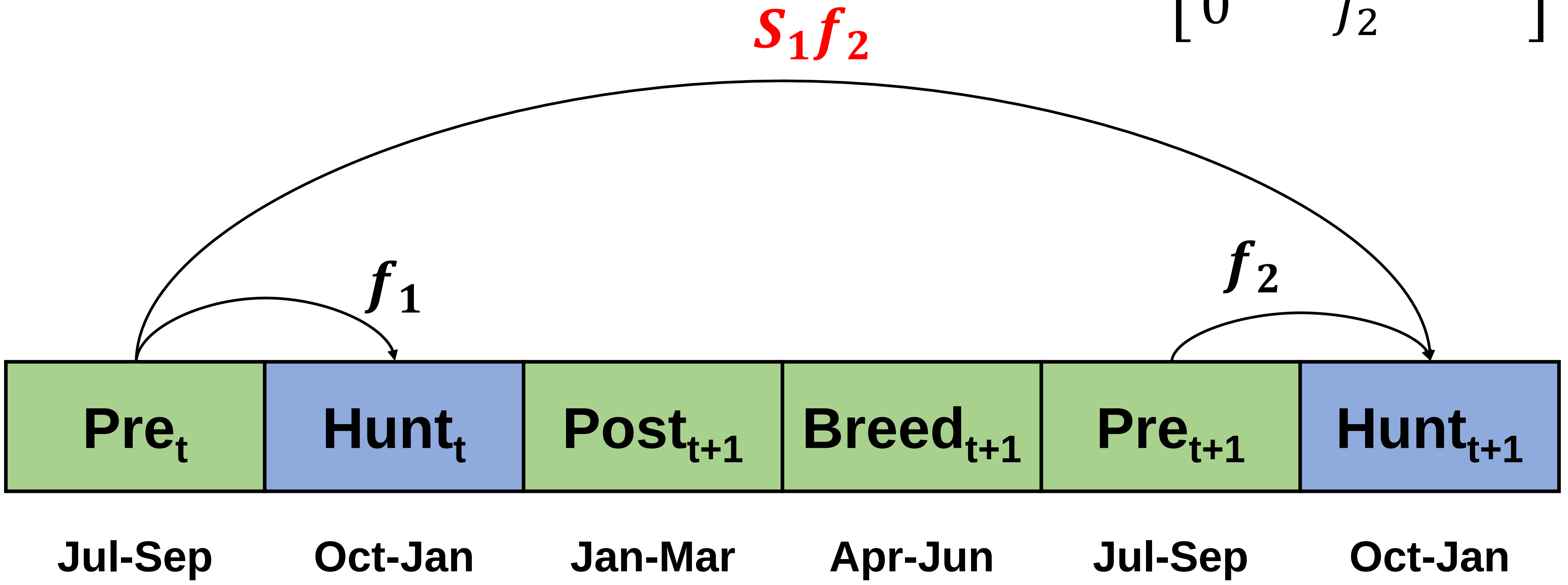
$$P = \begin{bmatrix} f_1 & & \\ 0 & f_2 & \end{bmatrix}$$

$$R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

$$M = \begin{bmatrix} 100 & \mathbf{50} & 850 \\ 0 & \mathbf{100} & 900 \end{bmatrix}$$

What is the probability a duck marked in 1 will be shot in 2?

$$P = \begin{bmatrix} f_1 & \mathbf{s_1 f_2} \\ 0 & f_2 \end{bmatrix}$$



What is the probability a duck marked in 1 will be shot in 2?

- It has to survive one year (S_1)
- <u>And</u> be recovered in the next (f_2)
- **<u>What does the 50 tell us?</u>**

$$P = \begin{bmatrix} f_1 & \boldsymbol{S_1 f_2} \\ 0 & f_2 \end{bmatrix}$$

$$R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

$$M = \begin{bmatrix} 100 & \mathbf{50} & 850 \\ 0 & 100 & 900 \end{bmatrix}$$

Imagine survival was 50%...

We released 1000 birds a year ago…

How many are still alive?

Imagine survival was 50%...

We released 1000 birds a year ago…

How many are still alive?
<u>500</u>

Now imagine we shoot 10% of those 500 birds...

How many would we shoot?

Now imagine we shoot 10% of those 500 birds...

How many would we shoot?
<u>50</u>

We estimate survival as a function of R and f

$$100 = R_2 \times f_2$$
$$100 = 1000 \times f_2$$
$$f_2 = 0.1$$

$$P = \begin{bmatrix} f_1 & \boldsymbol{S_1 f_2} & \\ 0 & \boldsymbol{f_2} & \end{bmatrix}$$

$$50 = R_1 \times S_1 \times f_2$$

$$R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

$$50 = 1000 * S_1 * 0.1$$

$$S_1 = \frac{50}{1000 \times 0.1} = 0.5$$

$$M = \begin{bmatrix} 100 & \mathbf{50} & 850 \\ 0 & \mathbf{100} & 900 \end{bmatrix}$$

50 is 10% of <u>500</u>

If **we shoot 50** birds banded in the first year in year two, and **we know we're shooting 10% of the ducks**, then how many were available to be shot?

If 500 are alive (available to be shot) now, and 1000 were alive one year ago, what is S?

$$S_1 = \frac{50}{1000 \times 0.1} = 0.5$$

$$f_2 = \frac{100}{1000} = 0.1$$

What would our survival estimate be if…

we’d had 60 indirect recoveries?

we’d had 70 indirect recoveries?

we’d had 70 recoveries, but $f_2 = 0.2$?

That’s the basic idea.

We know what proportion we shoot.

The number of indirect recoveries tell us number still alive

If we know the number still alive, we can estimate S!

This starts to get complicated (?!)

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

Oh geez…

But not really… there’s a few simple rules!

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

The rows correspond to year of release

But not really… there's a few simple rules!

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

This is what happened to birds released in the first year

But not really… there's a few simple rules!

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

This is what happened to birds released in the second year

But not really… there’s a few simple rules!

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

The rows correspond to year of release

The columns correspond to year of recovery

But not really… there's a few simple rules!

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

These are recoveries from year 4

But not really… there's a few simple rules!

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

The rows correspond to year of release
The columns correspond to year of recovery
These are individuals that were never encountered again

But not really… there's a few simple rules!

$$\begin{array}{cc} & \text{Year of recovery} \\ \text{Year of release} & M = \begin{bmatrix} 100 & 50 & \mathbf{850} \\ 0 & 100 & \mathbf{900} \end{bmatrix} \end{array}$$

The rows correspond to year of release

The columns correspond to year of recovery

These are individuals that were never encountered again

But not really… there's a few simple rules!

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16
1	49	31	22	14	8	15	7	5	1	2	1	2	1	1	0	841
2	0	52	33	22	14	10	4	9	5	2	3	2	1	0	0	843
3	0	0	50	26	24	21	17	8	9	7	5	6	2	5	0	820
4	0	0	0	43	37	16	11	8	12	7	5	4	1	1	1	854
5	0	0	0	0	47	41	19	22	4	8	11	3	1	5	2	837
6	0	0	0	0	0	61	45	24	16	10	13	8	5	2	4	812
7	0	0	0	0	0	0	47	36	31	13	23	9	7	3	2	829
8	0	0	0	0	0	0	0	50	43	33	19	13	9	6	2	825
9	0	0	0	0	0	0	0	0	53	36	30	24	8	11	5	833
10	0	0	0	0	0	0	0	0	0	51	33	28	18	8	14	848
11	0	0	0	0	0	0	0	0	0	0	50	28	22	20	12	868
12	0	0	0	0	0	0	0	0	0	0	0	58	37	26	13	866
13	0	0	0	0	0	0	0	0	0	0	0	0	49	36	20	895
14	0	0	0	0	0	0	0	0	0	0	0	0	0	54	28	918
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	52	948

The rows correspond to year of release

The columns correspond to year of recovery

These are individuals that were never encountered again

It's fairly simple to do this by hand with tiny datasets

$$P = \begin{bmatrix} f_1 & S_1 f_2 & 1 - f_1 - S_1 f_2 \\ 0 & f_2 & 1 - f_2 \end{bmatrix}$$

$$R = \begin{bmatrix} 1000 \\ 1000 \end{bmatrix}$$

$$M = \begin{bmatrix} 100 & 50 & 850 \\ 0 & 100 & 900 \end{bmatrix}$$

How do we deal with this amount of info (240 cells)?

	V1	V2	V3	V4	V5	V6	V7	V8	V9	V10	V11	V12	V13	V14	V15	V16
1	49	31	22	14	8	15	7	5	1	2	1	2	1	1	0	841
2	0	52	33	22	14	10	4	9	5	2	3	2	1	0	0	843
3	0	0	50	26	24	21	17	8	9	7	5	6	2	5	0	820
4	0	0	0	43	37	16	11	8	12	7	5	4	1	1	1	854
5	0	0	0	0	47	41	19	22	4	8	11	3	1	5	2	837
6	0	0	0	0	0	61	45	24	16	10	13	8	5	2	4	812
7	0	0	0	0	0	0	47	36	31	13	23	9	7	3	2	829
8	0	0	0	0	0	0	0	50	43	33	19	13	9	6	2	825
9	0	0	0	0	0	0	0	0	53	36	30	24	8	11	5	833
10	0	0	0	0	0	0	0	0	0	51	33	28	18	8	14	848
11	0	0	0	0	0	0	0	0	0	0	50	28	22	20	12	868
12	0	0	0	0	0	0	0	0	0	0	0	58	37	26	13	866
13	0	0	0	0	0	0	0	0	0	0	0	0	49	36	20	895
14	0	0	0	0	0	0	0	0	0	0	0	0	0	54	28	918
15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	52	948

The rows correspond to year of release

The columns correspond to year of recovery

These are individuals that were never encountered again

Understanding indexing is key to writing code…

Refuge[big shop, east wall, 3rd shelf from right, behind the nets, little case]

1-dimensional indexing!

$$R_1 = 1000 \qquad R = \begin{bmatrix} \mathbf{1000} \\ 950 \end{bmatrix}$$

R[1]=1000

data[#]

2-dimensional indexing!

As simple as a spreadsheet

m[1, 4]; $m_{1,4}$

	V1	V2	V3	V4	V5	V6
1	59	34	27	$\boxed{19}$	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

data[row, column]

M-array[year released, year recovered]

3-dimensional indexing!

As simple as a notebook

m[1, 4, 1]; $m_{1,4,1}$

	V1	V2	V3	V4	V5	V6
1	59	34	27	$\boxed{19}$	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

m[3, 3, 2]; $m_{3,3,2}$

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	$\boxed{42}$	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

data[row, column, page]

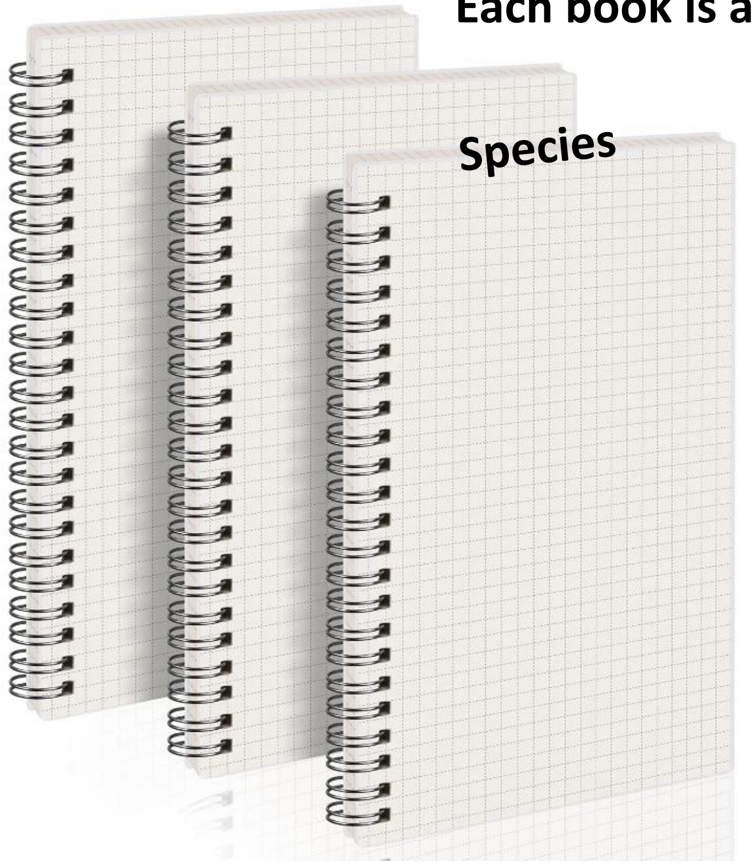
M-array[year released, year recovered, sex]

4-dimensional indexing! (a stack of notebooks)

imagine surveys at twenty locations (L) for fifteen species (S) that are performed five times per breeding season (J) across six years (T)

```
row: location [1,2,3,...,L]
col: species [1,2,3,...,S]
page: survey [1,2,3,...,J]
book: year [1,2,3,...,T]
```

data[row, column, page, data book]
data[location, species, survey, year]



7-dimensional indexing! (walmart)

Back-to-school sale

Walmart[row, column, page, book, shelf row, shelf column, aisle]

Walmart[1, 1, 1, 1, 1, 1, end cap aisle 24/25]

19 ducks were released in year 1 and shot in year 4

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

$M[1,4] = 19$

49 ducks were released in year 5 and shot in year 5

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

$M[5,5] = 49$

8 ducks were released in year 1 and shot in year 5

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

$M[1,5] = 8$

895 ducks were released in year 3 and never recovered

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

$M[3,6] = 895$

Why are these all 0's

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

M[2:5,1] = [0,0,0,0]

In R, 2:5 is the same as 2,3,4,5

Why are these all 0's?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

You can't be shot in year 1 if you were released after t=1…

Why are these the biggest numbers (typically)?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

M[1:5,6] = [853,885,895,918,951]

Why are these the biggest numbers (typically)?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

Most ducks aren’t shot, retrieved, and reported by a hunter

Why do these numbers (generally) decline?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

$M[1,1:5] = [59,34,27,19,8]$

Why are these the 2nd biggest numbers (typically)?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

M[1,1] = 39, M[2,2] = 51, M[3,3] = 42

Why are these the 2nd biggest numbers (typically)?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

These are recoveries immediately after release (no mortality)

What are the probabilities of these outcomes?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

What is the probability of this outcome?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	$\boxed{34}$	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

Released in year 1, shot in year 2…

What is the probability of this outcome?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

You have to survive year 1, and be shot in year 2 ($S_1 f_2$)

What is the probability of this outcome?

$$P = \begin{bmatrix} f_1 & \boxed{S_1 f_2} & S_1 S_2 f_3 & S_1 S_2 S_3 f_4 & S_1 S_2 S_3 S_4 f_5 & 1 - \sum P_{1,1:5} \\ 0 & f_2 & S_2 f_3 & S_2 S_3 f_4 & S_2 S_3 S_4 f_5 & 1 - \sum P_{2,1:5} \\ 0 & 0 & f_3 & S_3 f_4 & S_3 S_4 f_5 & 1 - \sum P_{3,1:5} \\ 0 & 0 & 0 & f_4 & S_4 f_5 & 1 - \sum P_{4,1:5} \\ 0 & 0 & 0 & 0 & f_5 & 1 - \sum P_{5,1:5} \end{bmatrix}$$

What is the probability of this outcome?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	$\boxed{19}$	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

Released in year 1, shot in year 4…

What is the probability of this outcome?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	$\boxed{19}$	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

You have to survive 3 years, and be shot in year 4 ($S_1S_2S_3f_4$)

What is the probability of this outcome?

$$P = \begin{bmatrix} f_1 & S_1 f_2 & S_1 S_2 f_3 & \boxed{S_1 S_2 S_3 f_4} & S_1 S_2 S_3 S_4 f_5 & 1 - \sum P_{1,1:5} \\ 0 & f_2 & S_2 f_3 & S_2 S_3 f_4 & S_2 S_3 S_4 f_5 & 1 - \sum P_{2,1:5} \\ 0 & 0 & f_3 & S_3 f_4 & S_3 S_4 f_5 & 1 - \sum P_{3,1:5} \\ 0 & 0 & 0 & f_4 & S_4 f_5 & 1 - \sum P_{4,1:5} \\ 0 & 0 & 0 & 0 & f_5 & 1 - \sum P_{5,1:5} \end{bmatrix}$$

Released in year 1, shot in year 4…

What is the probability of this outcome?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	13	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

What is the probability of this outcome?

V1-V5: Year of recovery

V6: never reencountered

1-5: Year of release

	V1	V2	V3	V4	V5	V6
1	59	34	27	19	8	853
2	0	51	27	24	$\boxed{13}$	885
3	0	0	42	43	20	895
4	0	0	0	43	39	918
5	0	0	0	0	49	951

You have to survive 3 years, and be shot in year 5 ($S_2S_3S_4f_5$)

What is the probability of this outcome?

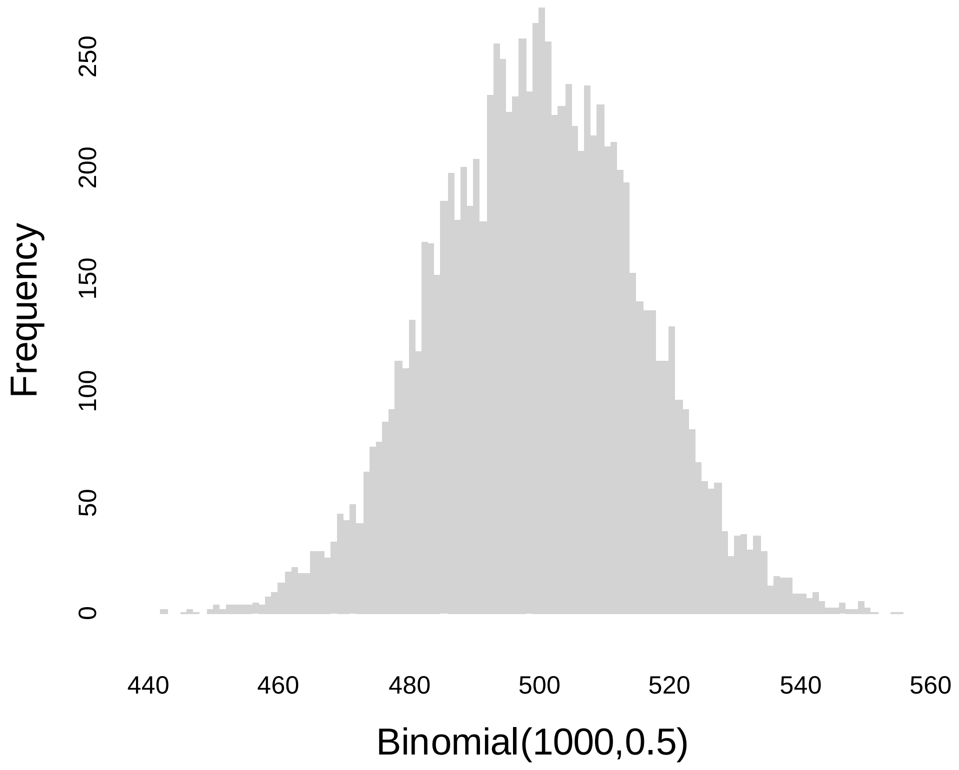
$$P = \begin{bmatrix} f_1 & S_1 f_2 & S_1 S_2 f_3 & S_1 S_2 S_3 f_4 & S_1 S_2 S_3 S_4 f_5 & 1 - \sum P_{1,1:5} \\ 0 & f_2 & S_2 f_3 & S_2 S_3 f_4 & \boxed{S_2 S_3 S_4 f_5} & 1 - \sum P_{2,1:5} \\ 0 & 0 & f_3 & S_3 f_4 & S_3 S_4 f_5 & 1 - \sum P_{3,1:5} \\ 0 & 0 & 0 & f_4 & S_4 f_5 & 1 - \sum P_{4,1:5} \\ 0 & 0 & 0 & 0 & f_5 & 1 - \sum P_{5,1:5} \end{bmatrix}$$

Released in year 2, shot in year 5…

How do we model that?

The binomial distribution: flip a coin 1000 times

$$\boldsymbol{y} \sim \text{binomial}(\mathbf{1000}, \theta)$$
$$\theta = 0.5$$



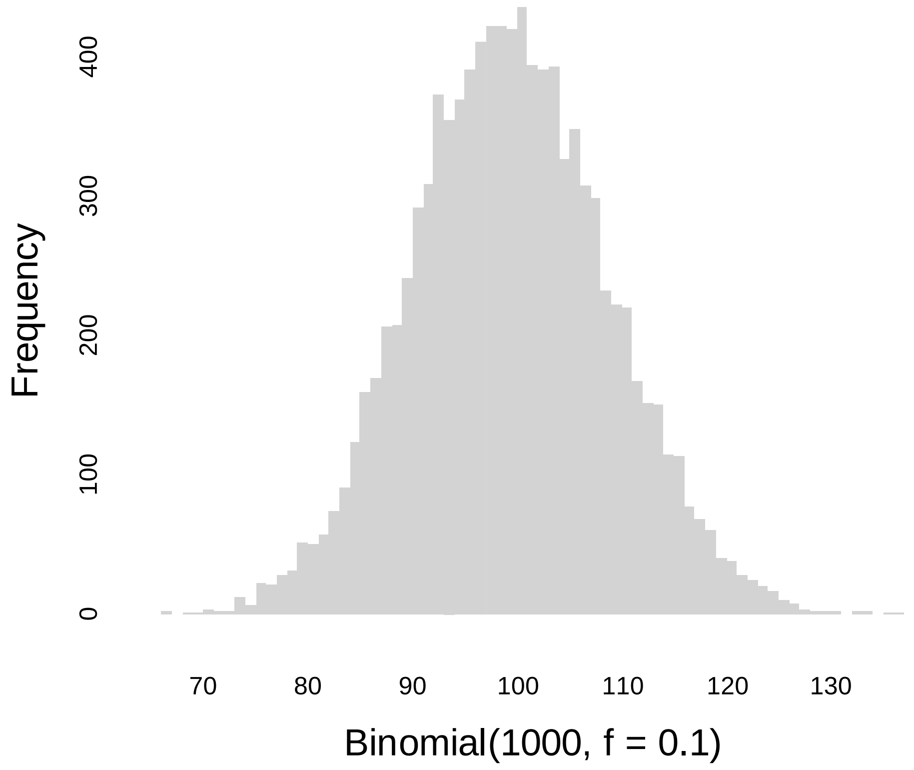
Binomial outcomes for band-recovery models

How many heads from 1000 flips?

How many direct recoveries from 1000 releases?

The binomial distribution: band 1000 mallards, recover 10%

$$y \sim \text{binomial}(1000, f)$$
$$f = 0.1$$

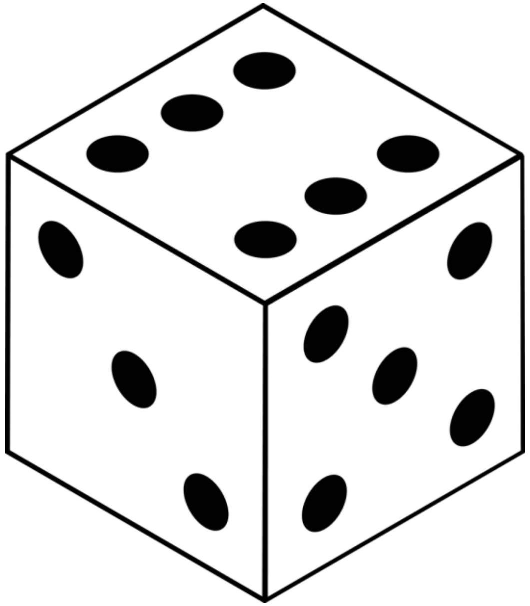


The multinomial distribution: roll a die 1000 times

$$y \sim \mathrm{multinomial}(\mathbf{1000}, \boldsymbol{\theta})$$

$$\boldsymbol{\theta} = \left[\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right]$$

```
> y <- rmultinom(1, 1000, rep(1/6, 6))
> y
     [,1]
[1,]  162
[2,]  155
[3,]  155
[4,]  181
[5,]  187
[6,]  160
> 
```



The multinomial distribution : roll a die 1000 times

$$\boldsymbol{y} \sim \text{multinomial}(1000, \boldsymbol{\theta})$$

$$\boldsymbol{\theta} = \left[\frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}\right]$$

```
> y <- rmultinom(1, 1000, rep(1/20, 20))
> as.vector(y)
 [1] 47 59 59 42 47 53 49 53 50 49 54 53 47 42 53 59 52 37 53 42
> 1:20
 [1]  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20
>
```

Multinomial outcomes for band-recovery models

Did we roll a 1, or a 2, or a 3, or a 4, ….. , or a 20

Shot in year 1, or shot in year 2, … or shot in year 15, or never shot

The multinomial distribution: roll a die 1000 times

$$\boldsymbol{y} \sim \text{multinomial}(1000, \boldsymbol{\theta})$$

$$\boldsymbol{\theta} = \left[\frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}\right]$$

For released ducks, the probabilities are different

$$R = \begin{bmatrix} \mathbf{1000} \\ 1000 \end{bmatrix}$$

$$\boldsymbol{m_1} \sim \text{multinomial}(\mathbf{1000}, \boldsymbol{p_1})$$

$$\boldsymbol{p_1} = [f_1, \quad S_1 f_2, \quad 1 \; - (f_1 + S_1 f_2)]$$

$$M = \begin{bmatrix} \mathbf{100} & \mathbf{50} & \mathbf{850} \\ 0 & 100 & 900 \end{bmatrix}$$

For released ducks, the probabilities are different

$$\boldsymbol{m_2} \sim \text{multinomial}(\mathbf{1000}, \boldsymbol{p_2})$$

$$\boldsymbol{p_2} = [0, \quad f_{2,} \quad 1 - (f_2)]$$

$$R = \begin{bmatrix} 1000 \\ \mathbf{1000} \end{bmatrix}$$

$$M = \begin{bmatrix} 100 & 50 & 850 \\ \mathbf{0} & \mathbf{100} & \mathbf{900} \end{bmatrix}$$

How do we write code to simulate and model this…?!

For-loops

```
> for (i in 1:10){
+     print(i)
+ }
[1] 1
[1] 2
[1] 3
[1] 4
[1] 5
[1] 6
[1] 7
[1] 8
[1] 9
[1] 10
```

For-loops

```
> for (i in 1:10){
+   print(i+3)
+ }
[1] 4
[1] 5
[1] 6
[1] 7
[1] 8
[1] 9
[1] 10
[1] 11
[1] 12
[1] 13
```

For-loops

```
> x <- rep(NA, 10)
> x
 [1] NA NA NA NA NA NA NA NA NA NA
> for (i in 1:10){
+   x[i] <- paste0('f',i)
+ }
> x
 [1] "f1"  "f2"  "f3"  "f4"  "f5"  "f6"  "f7"  "f8"  "f9"  "f10"
> 
```

These are our cell probabilities

$$P = \begin{bmatrix} f_1 & S_1 f_2 & S_1 S_2 f_3 & S_1 S_2 S_3 f_4 & S_1 S_2 S_3 S_4 f_5 & 1 - \sum P_{1,1:5} \\ 0 & f_2 & S_2 f_3 & S_2 S_3 f_4 & S_2 S_3 S_4 f_5 & 1 - \sum P_{2,1:5} \\ 0 & 0 & f_3 & S_3 f_4 & S_3 S_4 f_5 & 1 - \sum P_{3,1:5} \\ 0 & 0 & 0 & f_4 & S_4 f_5 & 1 - \sum P_{4,1:5} \\ 0 & 0 & 0 & 0 & f_5 & 1 - \sum P_{5,1:5} \end{bmatrix}$$

How could I write this in code using loops?

How could I write this in code?

$$P = \begin{bmatrix} f_1 & S_1 f_2 & S_1 S_2 f_3 & S_1 S_2 S_3 f_4 & S_1 S_2 S_3 S_4 f_5 & 1 - \sum P_{1,1:5} \\ 0 & f_2 & S_2 f_3 & S_2 S_3 f_4 & S_2 S_3 S_4 f_5 & 1 - \sum P_{2,1:5} \\ 0 & 0 & f_3 & S_3 f_4 & S_3 S_4 f_5 & 1 - \sum P_{3,1:5} \\ 0 & 0 & 0 & f_4 & S_4 f_5 & 1 - \sum P_{4,1:5} \\ 0 & 0 & 0 & 0 & f_5 & 1 - \sum P_{5,1:5} \end{bmatrix}$$

Let’s look for patterns

Constructing cell probabilities

$$P = \begin{bmatrix} f_1 & S_1 f_2 & S_1 S_2 f_3 & S_1 S_2 S_3 f_4 & S_1 S_2 S_3 S_4 f_5 & 1 - \sum P_{1,1:5} \\ 0 & f_2 & S_2 f_3 & S_2 S_3 f_4 & S_2 S_3 S_4 f_5 & 1 - \sum P_{2,1:5} \\ 0 & 0 & f_3 & S_3 f_4 & S_3 S_4 f_5 & 1 - \sum P_{3,1:5} \\ 0 & 0 & 0 & f_4 & S_4 f_5 & 1 - \sum P_{4,1:5} \\ 0 & 0 & 0 & 0 & f_5 & 1 - \sum P_{5,1:5} \end{bmatrix}$$

p[1,1] = f_1
p[2,2] = f_2
p[3,3] = f_3
p[4,4] = f_4
p[5,5] = f_5

Main diagonal

Constructing cell probabilities (a five-year study)

	V1	V2	V3	V4	V5	V6
1	NA	NA	NA	NA	NA	NA
2	NA	NA	NA	NA	NA	NA
3	NA	NA	NA	NA	NA	NA
4	NA	NA	NA	NA	NA	NA
5	NA	NA	NA	NA	NA	NA

$$f = [0.1, 0.1, 0.1, 0.1, 0.1]$$

Constructing cell probabilities

```
for (t in 1:nT){
  p[t,t] <- f[t]
}
```

	V1	V2	V3	V4	V5	V6
1	0.1	NA	NA	NA	NA	NA
2	NA	0.1	NA	NA	NA	NA
3	NA	NA	0.1	NA	NA	NA
4	NA	NA	NA	0.1	NA	NA
5	NA	NA	NA	NA	0.1	NA

$$p[1,1] = f_1$$
$$p[2,2] = f_2$$
$$p[3,3] = f_3$$
$$p[4,4] = f_4$$
$$p[5,5] = f_5$$

$$f = [0.1, 0.1, 0.1, 0.1, 0.1]$$

Constructing cell probabilities

$$P = \begin{bmatrix} f_1 & S_1 f_2 & S_1 S_2 f_3 & S_1 S_2 S_3 f_4 & S_1 S_2 S_3 S_4 f_5 & 1 - \sum P_{1,1:5} \\ 0 & f_2 & S_2 f_3 & S_2 S_3 f_4 & S_2 S_3 S_4 f_5 & 1 - \sum P_{2,1:5} \\ 0 & 0 & f_3 & S_3 f_4 & S_3 S_4 f_5 & 1 - \sum P_{3,1:5} \\ 0 & 0 & 0 & f_4 & S_4 f_5 & 1 - \sum P_{4,1:5} \\ 0 & 0 & 0 & 0 & f_5 & 1 - \sum P_{5,1:5} \end{bmatrix}$$

p[2,1] = 0
p[3,1] = 0
p[3,2] = 0
p[4,1] = 0
p[4,2] = 0
p[4,3] = 0
p[5,1] = 0
p[5,2] = 0
p[5,3] = 0
p[5,4] = 0

Below the main diagonal = 0

Constructing cell probabilities

```
for (t in 2:nT){
  for (j in 1:(t-1)){
    p[t,j] <- 0
  }
}
```

	V1	V2	V3	V4	V5	V6
1	0.1	NA	NA	NA	NA	NA
2	0.0	0.1	NA	NA	NA	NA
3	0.0	0.0	0.1	NA	NA	NA
4	0.0	0.0	0.0	0.1	NA	NA
5	0.0	0.0	0.0	0.0	0.1	NA

t = 2

p[2,1] = 0
p[3,1] = 0
p[3,2] = 0
p[4,1] = 0
p[4,2] = 0
p[4,3] = 0
p[5,1] = 0
p[5,2] = 0
p[5,3] = 0
p[5,4] = 0

Constructing cell probabilities

```
for (t in 2:nT){
   for (j in 1:(t-1)){
      p[t,j] <- 0
   }
}
```

	V1	V2	V3	V4	V5	V6
1	0.1	NA	NA	NA	NA	NA
2	0.0	0.1	NA	NA	NA	NA
3	0.0	0.0	0.1	NA	NA	NA
4	0.0	0.0	0.0	0.1	NA	NA
5	0.0	0.0	0.0	0.0	0.1	NA

t = 3

p[2,1] = 0
p[3,1] = 0
p[3,2] = 0
p[4,1] = 0
p[4,2] = 0
p[4,3] = 0
p[5,1] = 0
p[5,2] = 0
p[5,3] = 0
p[5,4] = 0

Constructing cell probabilities

```
for (t in 2:nT){
  for (j in 1:(t-1)){
    p[t,j] <- 0
  }
}
```

	V1	V2	V3	V4	V5	V6
1	0.1	NA	NA	NA	NA	NA
2	0.0	0.1	NA	NA	NA	NA
3	0.0	0.0	0.1	NA	NA	NA
4	0.0	0.0	0.0	0.1	NA	NA
5	0.0	0.0	0.0	0.0	0.1	NA

t = 4

p[2,1] = 0
p[3,1] = 0
p[3,2] = 0
p[4,1] = 0
p[4,2] = 0
p[4,3] = 0
p[5,1] = 0
p[5,2] = 0
p[5,3] = 0
p[5,4] = 0

Constructing cell probabilities

```
for (t in 2:nT){
  for (j in 1:(t-1)){
    p[t,j] <- 0
  }
}
```

	V1	V2	V3	V4	V5	V6
1	0.1	NA	NA	NA	NA	NA
2	0.0	0.1	NA	NA	NA	NA
3	0.0	0.0	0.1	NA	NA	NA
4	0.0	0.0	0.0	0.1	NA	NA
5	0.0	0.0	0.0	0.0	0.1	NA

t = 5

p[2,1] = 0
p[3,1] = 0
p[3,2] = 0
p[4,1] = 0
p[4,2] = 0
p[4,3] = 0
p[5,1] = 0
p[5,2] = 0
p[5,3] = 0
p[5,4] = 0

Constructing cell probabilities

$$P = \begin{bmatrix} f_1 & \boxed{S_1 f_2 \quad S_1 S_2 f_3 \quad S_1 S_2 S_3 f_4 \quad S_1 S_2 S_3 S_4 f_5} & 1 - \sum P_{1,1:5} \\ 0 \quad f_2 & \boxed{S_2 f_3 \quad S_2 S_3 f_4 \quad S_2 S_3 S_4 f_5} & 1 - \sum P_{2,1:5} \\ 0 \quad 0 \quad f_3 & \boxed{S_3 f_4 \quad S_3 S_4 f_5} & 1 - \sum P_{3,1:5} \\ 0 \quad 0 \quad 0 \quad f_4 & \boxed{S_4 f_5} & 1 - \sum P_{4,1:5} \\ 0 \quad 0 \quad 0 \quad 0 & f_5 & 1 - \sum P_{5,1:5} \end{bmatrix}$$

p[1,2] = $S_1 f_2$
p[1,3] = $S_1 S_2 f_3$
p[1,4] = …
p[1,5] = …
p[2,3] = $S_2 f_3$
p[2,4] = $S_2 S_3 f_4$
p[2,5] = …
p[3,4] = $S_3 f_4$
p[3,5] = $S_3 S_4 f_5$
p[4,5] = $S_4 f_5$

Constructing cell probabilities

```
for (t in 1:(nT-1)){
  for (j in (t+1):nT){
    p[t,j] <- prod(S[t:(j-1)]) * f[j]
  }
}
```

	V1	V2	V3	V4	V5	V6
1	0.1	0.07	0.049	0.0343	0.02401	NA
2	0.0	0.10	0.070	0.0490	0.03430	NA
3	0.0	0.00	0.100	0.0700	0.04900	NA
4	0.0	0.00	0.000	0.1000	0.07000	NA
5	0.0	0.00	0.000	0.0000	0.10000	NA

$f = [0.1, 0.1, 0.1, 0.1, 0.1]$

$S = [0.7, 0.7, 0.7, 0.7]$

$p[1,2] = S_1 f_2$
$p[1,3] = S_1 S_2 f_3$

Constructing cell probabilities

$$P = \begin{bmatrix} f_1 & S_1 f_2 & S_1 S_2 f_3 & S_1 S_2 S_3 f_4 & S_1 S_2 S_3 S_4 f_5 & 1 - \sum P_{1,1:5} \\ 0 & f_2 & S_2 f_3 & S_2 S_3 f_4 & S_2 S_3 S_4 f_5 & 1 - \sum P_{2,1:5} \\ 0 & 0 & f_3 & S_3 f_4 & S_3 S_4 f_5 & 1 - \sum P_{3,1:5} \\ 0 & 0 & 0 & f_4 & S_4 f_5 & 1 - \sum P_{4,1:5} \\ 0 & 0 & 0 & 0 & f_5 & 1 - \sum P_{5,1:5} \end{bmatrix}$$

Constructing cell probabilities

```
for (t in 1:nT){
  p[t,nT+1] <- 1 - sum(m[t,1:nT])
}
```

	V1	V2	V3	V4	V5	V6
1	0.1	0.07	0.049	0.0343	0.02401	0.72269
2	0.0	0.10	0.070	0.0490	0.03430	0.74670
3	0.0	0.00	0.100	0.0700	0.04900	0.78100
4	0.0	0.00	0.000	0.1000	0.07000	0.83000
5	0.0	0.00	0.000	0.0000	0.10000	0.90000

Simulating data

	V1	V2	V3	V4	V5	V6
1	0.1	0.07	0.049	0.0343	0.02401	0.72269
2	0.0	0.10	0.070	0.0490	0.03430	0.74670
3	0.0	0.00	0.100	0.0700	0.04900	0.78100
4	0.0	0.00	0.000	0.1000	0.07000	0.83000
5	0.0	0.00	0.000	0.0000	0.10000	0.90000

The multinomial distribution : roll a die 1000 times

$$\boldsymbol{y} \sim \text{multinomial}(1000, \boldsymbol{\theta})$$

$$\boldsymbol{\theta} = \left[\frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}, \frac{1}{20}\right]$$

```
> y <- rmultinom(1, 1000, rep(1/20, 20))
> as.vector(y)
 [1] 47 59 59 42 47 53 49 53 50 49 54 53 47 42 53 59 52 37 53 42
> 1:20
 [1]  1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20
> 
```

Constructing cell probabilities

$$m_1 \sim \text{multinomial}(1000, \boxed{p_1})$$

	V1	V2	V3	V4	V5	V6
1	0.1	0.07	0.049	0.0343	0.02401	0.72269
2	0.0	0.10	0.070	0.0490	0.03430	0.74670
3	0.0	0.00	0.100	0.0700	0.04900	0.78100
4	0.0	0.00	0.000	0.1000	0.07000	0.83000
5	0.0	0.00	0.000	0.0000	0.10000	0.90000

Constructing cell probabilities

$$\boldsymbol{m_2} \sim \text{multinomial}(\mathbf{1000}, \boxed{\boldsymbol{p_2}})$$

	V1	V2	V3	V4	V5	V6
1	0.1	0.07	0.049	0.0343	0.02401	0.72269
2	0.0	0.10	0.070	0.0490	0.03430	0.74670
3	0.0	0.00	0.100	0.0700	0.04900	0.78100
4	0.0	0.00	0.000	0.1000	0.07000	0.83000
5	0.0	0.00	0.000	0.0000	0.10000	0.90000

Constructing cell probabilities

```
for (t in 1:nT){
  m[t,1:(nT+1)] <- rmultinom(1, 1000, p[t,1:(nT+1)])
}
```

	V1	V2	V3	V4	V5	V6
1	88	72	39	35	25	741
2	0	106	66	47	41	740
3	0	0	85	90	53	772
4	0	0	0	115	67	818
5	0	0	0	0	88	912