smoothmanifold.asy - v6.3.0

Diagrams in higher mathematics with Asymptote

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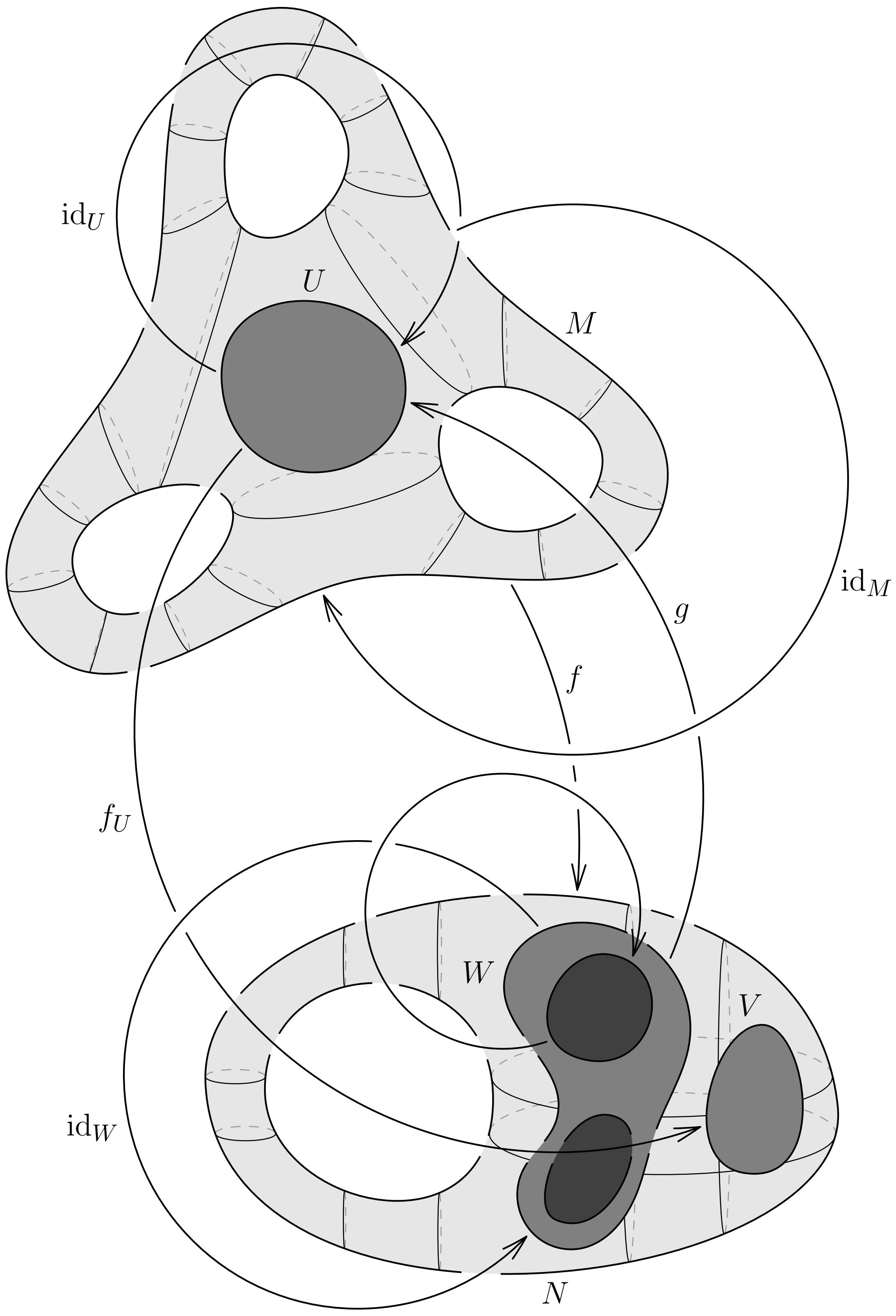


Abstract

This document contains the full description and user manual to the `smoothmanifold` Asymptote module, home page https://github.com/thornoar/smoothmanifold.

Contents

1. Introduction

In higher mathematics, diagrams often take the form of "blobs" (representing sets and their subsets) placed on the plane, connected with paths or arrows. This is particularly true for set theory and topology, but other more advanced fields inherit this style. In differential geometry, one draws spheres, tori, and other surfaces in place of these "blobs". In category theory, commutative diagrams are commonplace, where the "blobs" are replaced by symbols. Here are a couple of examples, all drawn with `smoothmanifold`:

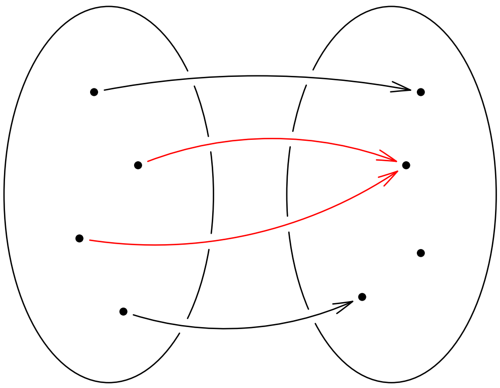


Figure 1: An illustration of non-injectivity (set theory)

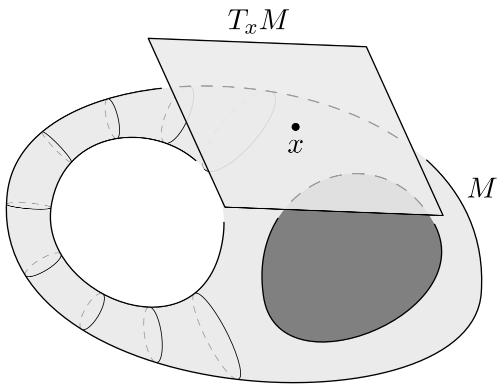


Figure 2: Tangent space at a point on a manifold (diff. geometry)

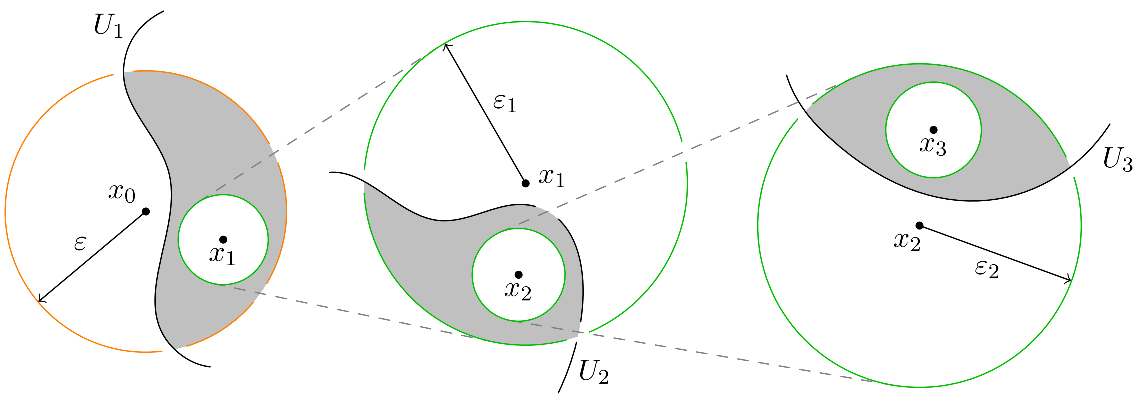


Figure 3: The proof of the Baire category theorem (topology)

Take special note of the gaps that paths leave on already drawn paths upon every intersection. I find this feature quite hard to achieve in plain Asymptote, and module `smoothmanifold` uses some dark magic to implement it. Similarly, note the shaded areas on page 2. They represent *intersections* of areas bounded by

two paths. Finding the bounding path of such an intersection is non-trivial and also implemented in `smoothmanifold`. Lastly, page 2 shows a three-dimensional surface, while the picture was fully drawn in 2D. The illusion is achieved through these cross-sectional "rings" on the left of the diagram.
To summarize, the most prominent features of module `smoothmanifold` are the following:

- **Gaps in overlapping paths**, achieved through a system of deferred drawing;
- **Set operations on paths bounding areas**, e.g. intersection, union, set difference, etc.;
- **Three-dimensional drawing**, achieved through an automatic (but configurable) addition of cross sections to smooth objects.

Do take a look at the source code for the above diagrams, to get a feel for how much heavy lifting is done by the module, and what is required from the user. We will now consider each of the above mentioned features (and some others as well) in full detail.

2. Deferred drawing and path overlapping

2.1. The general mechanism

In the `picture` struct, the paths drawn to a picture are not stored in an array, but rather indirectly stored in a `void` callback. That is, when the `draw` function is called, the *instruction to draw* the path is added to the picture, not the path itself. This makes it quite impossible to "modify the path after it is drawn". To go around this limitation, `smoothmanifold` introduces an auxiliary struct:

```
struct deferredPath {
    path[] g;
    pen p;
    int[] under;
    tarrow arrow;
    tbar bar;
}
```

It stores the path(s) to draw later, and how to draw them. Now, `smoothmanifold` executes the following steps to draw a "mutable" path `p` to a picture `pic` and then draw it for real:

1. Construct a `deferredPath` based on `p`, say `dp`;
2. Exploit the `nodes` field of the `picture` struct to store an integer. Retrieve this integer, say `n`, from `pic`.
3. Store the deferred path `dp` in a global two-dimensional array, under index `n`;
4. Modify the deferred path `dp` as needed, e.g. add gaps;
5. At shipout time, when processing the picture `pic`, retrieve the index `n` from its `nodes` field and draw all `deferredPath` objects in the two-dimensional array at index `n`.

All these steps require no extra input from the user, since the shipout function is redefined to do them automatically. One only needs to use the `fitpath` function instead of `draw`.

2.2. The `tarrow` and `tbar` structures

Similarly to drawing paths to a picture, arrows and bars are implemented through a function type `bool(picture, path, pen, margin)`, typedef'ed as `arrowbar`. Moreover, when this arrowbar is called, it automatically draws not only itself, but also the path is was attached to. This makes it impossible to attach an arrowbar to a path and then mutate the path — the arrowbar will remember the path's original state. Hence, `smoothmanifold` implements custom arrow/bar implementations:

```
struct tarrow {
    arrowhead head;
    real size;
    real angle;
    filltype ftype;
    bool begin;
    bool end;
    bool arc;
}
```

```
struct tbar {
    real size;
    bool begin;
    bool end;
}
```

These structs store information about the arrow/bar, and are converted to regular arrowbars when the corresponding path is drawn to the picture. For creating new `tarrow`/`tbar` instances and converting them to arrowbars, the following functions are available:

```
tarrow DeferredArrow(
    arrowhead head = DefaultHead,
    real size = 0,
    real angle = arrowangle,
    bool begin = false,
    bool end = true,
    bool arc = false,
    filltype filltype = null
)

arrowbar convertarrow(
    tarrow arrow,
    bool overridebegin = false,
    bool overrideend = false
)
```

```
tbar DeferredBar(
    real size = 0,
    bool begin = false,
    bool end = false
)

arrowbar convertbar(
    tbar bar,
    bool overridebegin = false,
    bool overrideend = false
)
```

The `overridebegin` and `overrideend` options let the user force disable the arrow/bar at the beginning/end of the path.

2.3. The `fitpath` function

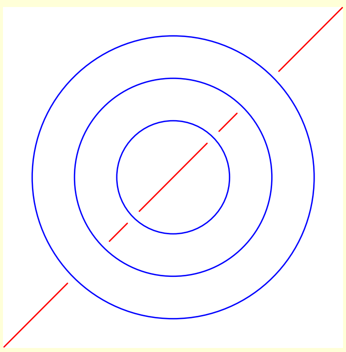
This is a substitute for the plain `draw` function. The `fitpath` function implements steps 1-4 of the deferred drawing system describes above.

```
void fitpath (picture pic, path gs, bool overlap, int covermode, bool drawnow, Label L, pen p,
tarrow arrow, tbar bar)
```

Arguments:

- `pic` — the picture to fit the path to;
- `gs` — the path to fit;
- `overlap` — whether to let the path overlap the previously fit paths. A value of `false` will lead to gaps being left in all paths that `gs` intersects;
- `covermode` — if the path `gs` is cyclic, this option lets you decide what happens to the parts of previously fit paths that fall "inside" of `gs`. Suppose a portion `s` of another path falls inside the cyclic path `gs`. Then
  - `covermode == 2`: The portion `s` will be erased completely;
  - `covermode == 1`: The portion `s` will be "demoted to the background" — either temporarily removed or drawn with dashes;
  - `covermode == 0`: The portion `s` will be drawn like the rest of the path;
  - `covermode == -1`: If the portion `s` is "demoted", it will be brought "back to the surface", i.e. drawn with solid pen. Otherwise, it will be draw as-is.

  Consider the following example:



```
import smoothmanifold;
config.drawing.gaplength = .12;

path l = (-1.2,-1.2) -- (1.2,1.2);
path c1 = unitcircle;
path c2 = scale(.7) * unitcircle;
path c3 = scale(.4) * unitcircle;

fitpath(l, red);
fitpath(c1, blue, covermode = 1);
fitpath(c2, blue, covermode = -1);
fitpath(c3, blue, covermode = 0);
```

- `drawnow` — whether to draw the path `gs` immediately to the picture. When `drawnow == true`, the path `gs` leaves gaps in other paths, but is immutable itself, i.e. later fit paths will not leave any gaps in it. When `drawnow == false`, the path `gs` is not immediately drawn, but rather saved to be mutated and finally drawn at shipout time;
- `L` — the label to attach to `gs`. This label is drawn to `pic` immediately on call of `fitpath`, unlike `gs`;
- `p` — the pen to draw `gs` with;
- `arrow` — the arrow to attach to the path. Note that the type is `tarrow`, not `arrowbar`;
- `bar` — the bar to attach to the path. Note that the type is `tbar`, not `arrowbar`.

Apart from different types of the `arrow`/`bar` arguments, the `fitpath` function is identical to `draw` in type signature, and they can be used interchangeably. Moreover, there are overloaded versions of `fitpath`, where parameters are given default values (one of these versions is used above):

```
void fitpath (
    picture pic = currentpicture,
    path g,
    bool overlap = config.drawing.overlap,
    int covermode = 0,
    Label L = "",
    pen p = currentpen,
    bool drawnow = config.drawing.drawnow,
    tarrow arrow = null,
    tbar bar = config.arrow.currentbar
)
```

```
void fitpath (
    picture pic = currentpicture,
    path[] g,
    bool overlap = config.drawing.overlap,
    int covermode = 0,
    Label L = "",
    pen p = currentpen,
    bool drawnow = config.drawing.drawnow
)
```

Here, `config` is the global configuration structure, see TODO. Furthermore, there are corresponding `fillfitpath` functions that serve the same purpose as `filldraw`.

2.4. Other related routines

```
int extractdeferredindex (picture pic)
```

Inspect the `nodes` field of `pic` for a string in a particular format, and, if it exists, extract an integer from it.

```
deferredPath[] extractdeferredpaths (picture pic, bool createlink)
```

Extract the deferred paths associated with the picture `pic`. If `createlink` is set to `true` and `pic` has no integer stored in its `nodes` field, the routine will find the next available index and store it in `pic`.

```
path[] getdeferredpaths (picture pic = currentpicture)
```

A wrapper around `extractdeferredpaths`, which concatenates the `path[] g` fields of the extracted deferred paths.

```
void purgedeferredunder (deferredPath[] curdeferred)
```

For each deferred path in `curdeferred`, delete the segments that are “demoted” to the background (i.e. going under a cyclic path, drawn with dashed lines).

```
void drawdeferred (
    picture pic = currentpicture,
    bool flush = true
)
```

Render the deferred paths associated with `pic`, to the picture `pic`. If `flush` is `true`, delete these deferred paths.

```
void flushdeferred (picture pic = currentpicture)
```

Delete the deferred paths associated with `pic`.

```
void plainshipout (...) = shipout;
shipout = new void (...)
{
    drawdeferred(pic = pic, flush = false);
    draw(pic = pic, debugpaths, red+1);
    plainshipout(prefix, pic, orntn, format, wait, view, options, script, lt, P);
};
```

A redefinition of the `shipout` function to automatically draw the deferred paths at shipout time. For a definition of `debugpaths`, see TODO.

The functions `erase`, `add`, `save`, and `restore` are redefined to automatically handle deferred paths.

3. Operations on paths

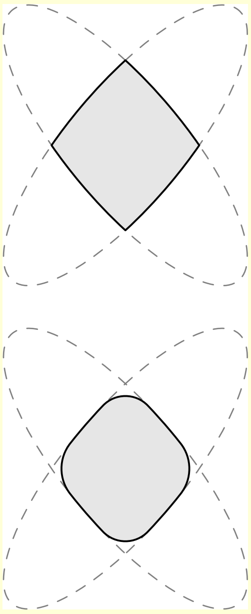
3.1. Combination of bounded regions

Module `smoothmanifold` defines a routine called `combination` which, given two *cyclic* paths `p` and `q`, calculates a result path which encloses a region that is a combination of the regions `p` and `q`:

```
path[] combination (path p, path q, int mode, bool round, real roundcoeff)
```

This function returns an array of paths because the combination of two bounded regions may be bounded by multiple paths. Rundown of the arguments:

- `p` and `q` — cyclic paths bounding the regions to combine;
- `mode` — an internal parameter which allows to specialize `combination` for different purposes;
- `round` and `roundcoeff` — whether to round the sharp corners of the resulting bounding path(s).



```
...
filldraw(
    combination(p, q, 1, false, 0), // <- no rounding
    drawpen = linewidth(.7),
    fillpen = palegrey
);

...
p = shift((0,-5)) * p; q = shift((0,-5)) * q;
...

filldraw(
    combination(p, q, 1, true, .04), // <- rounding with coeff. 0.04
    drawpen = linewidth(.7),
    fillpen = palegrey
);
...
```

Based on different values for the `mode` parameter, the module defines the following specializations:

```
path[] difference (
    path p,
    path q,
    bool correct = true,
    bool round = false,
    real roundcoeff = config.paths.roundcoeff
)
```

```
path[] operator - (path p, path q)
{ return difference(p, q); }
```

Calculate the path(s) bounding the set difference of the regions bounded by `p` and `q`. The `correct` parameter determines whether the paths should be "corrected", i.e. oriented clockwise.

```
path[] symmetric (
    path p,
    path q,
    bool correct = true,
    bool round = false,
    real roundcoeff = config.paths.roundcoeff
)
```

```
path[] operator :: (path p, path q)
{ return symmetric(p, q); }
```

Calculate the path(s) bounding the set symmetric difference of the regions bounded by `p` and `q`.

```
path[] intersection (
    path p,
    path q,
    bool correct = true,
    bool round = false,
    real roundcoeff = config.paths.roundcoeff
)
```

```
path[] operator ^ (path p, path q)
{ return intersection(p, q); }
```

Calculate the path(s) bounding the set intersection of the regions bounded by `p` and `q`. The following array versions are also available:

```
path[] intersection (
    path[] ps,
    bool correct = true,
    bool round = false,
    real roundcoeff = config.paths.roundcoeff
)
```

```
path[] intersection (
    bool correct = true,
    bool round = false,
    real roundcoeff = config.paths.roundcoeff
    ... path[] ps
)
```

Inductively calculate the total intersection of an array of paths.

```
path[] union (
    path p,
    path q,
    bool correct = true,
    bool round = false,
    real roundcoeff = config.paths.roundcoeff
)
```

```
path[] operator | (path p, path q)
{ return union(p, q); }
```

Calculate the path(s) bounding the set union of the regions bounded by `p` and `q`. The corresponding array versions are available:

```
path[] union (
    path[] ps,
    bool correct = true,
    bool round = false,
    real roundcoeff = config.paths.roundcoeff
)
```

```
path[] union (
    bool correct = true,
    bool round = false,
    real roundcoeff = config.paths.roundcoeff
    ... path[] ps
)
```

Inductively calculate the total union of an array of paths. Here is an illustration of the specializations:

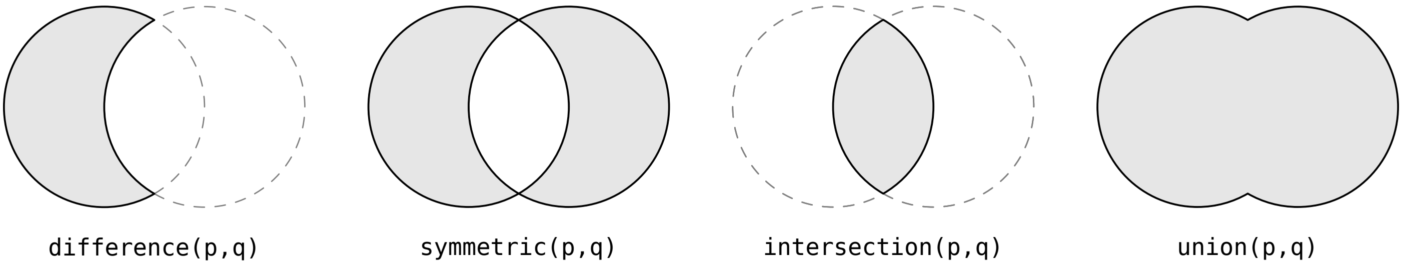


Figure 4: Different specializations of the `combination` function