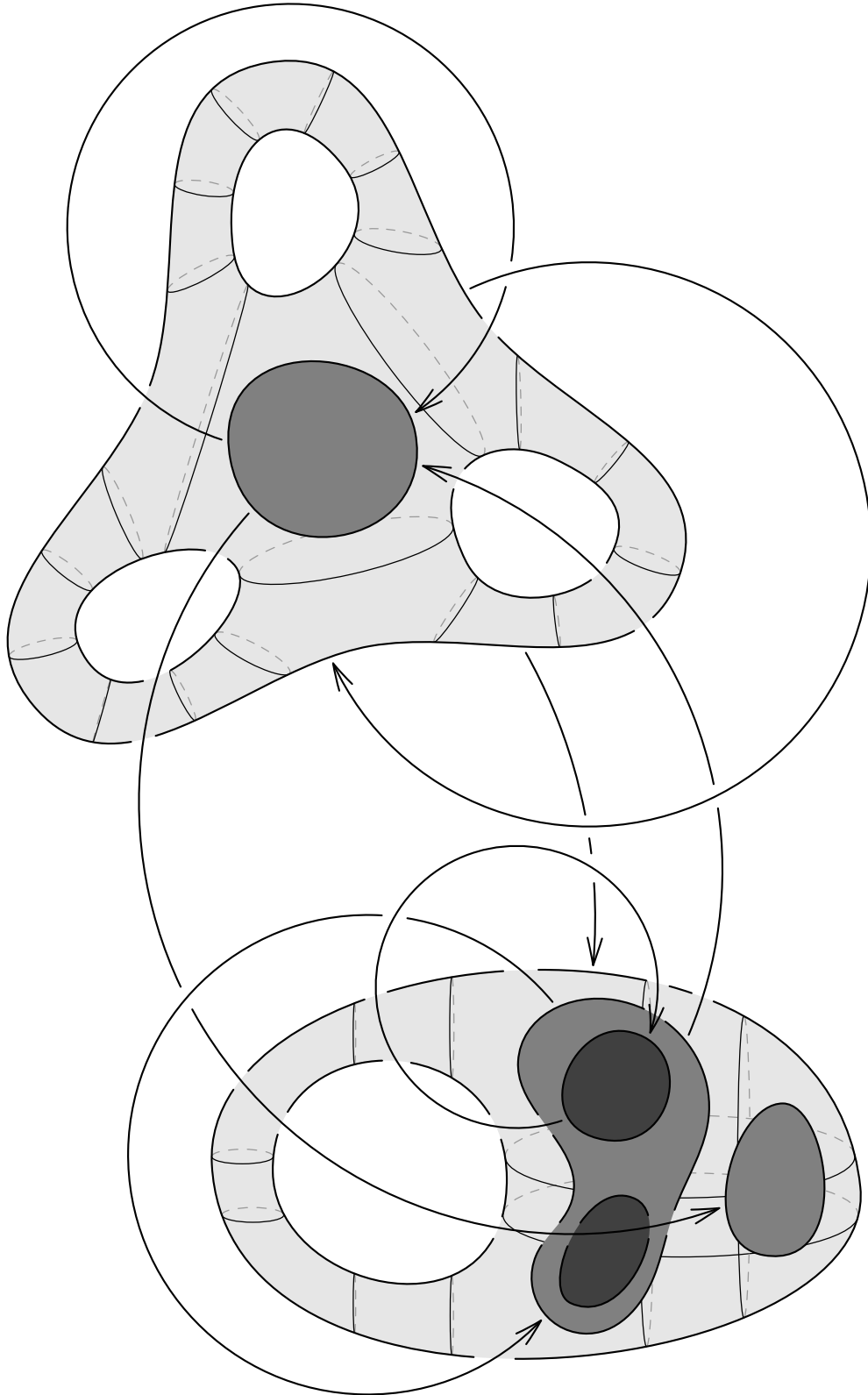# Diagrams in higher mathematics with Asymptote

by Roman Maksimovich

Abstract

This document contains the full description and user manual to the `smoothmanifold` Asymptote module, home page https://github.com/thornoar/smoothmanifold.

Contents

1. Introduction

In higher mathematics, diagrams often take the form of "blobs" (representing sets and their subsets) placed on the plane, connected with paths or arrows. This is particularly true for set theory and topology, but other more advanced fields inherit this style. In differential geometry and multivariate calculus, one draws spheres, tori, and other surfaces in place of these "blobs". In category theory, commutative diagrams are commonplace, where the "blobs" are replaced by symbols. Here are a couple of examples, all drawn with `smoothmanifold`:
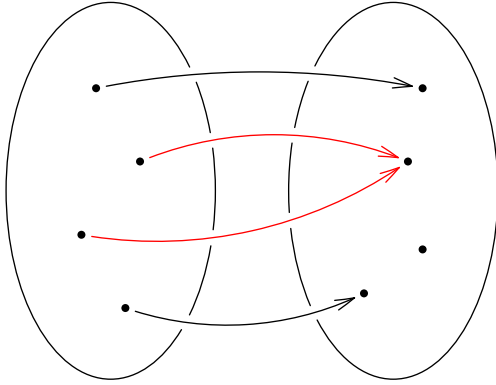

Figure 1: An illustration of non-injectivity (set theory)
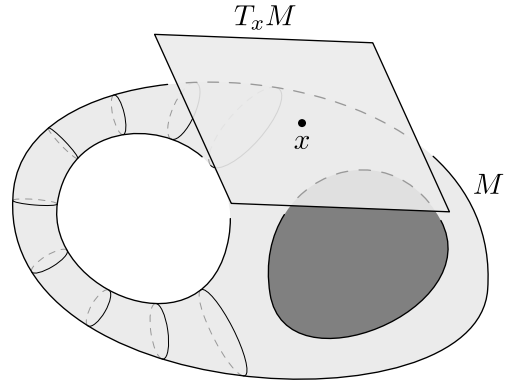
Figure 2: Tangent space at a point on a manifold (diff. geometry)
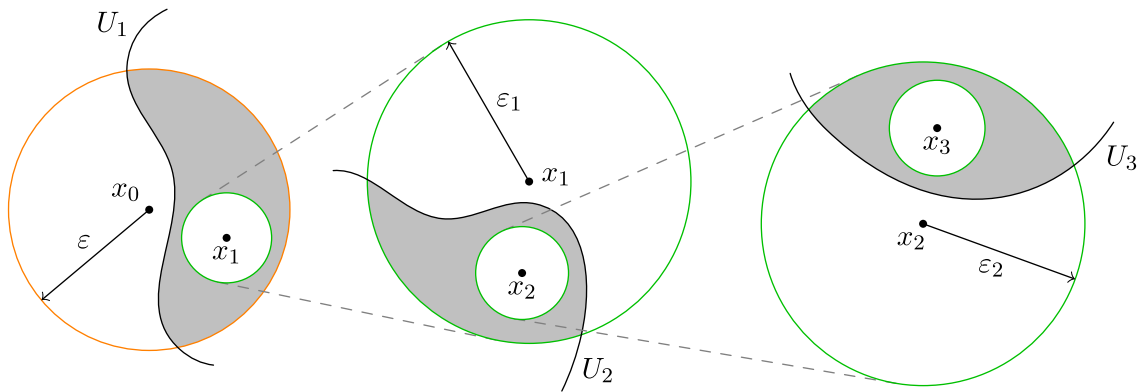
Figure 3: The proof of the Baire category theorem (topology)

Take special note of the gaps that arrows leave on the boundaries of the ovals on Figure 1. I find this feature quite hard to achieve in plain Asymptote, and module `smoothmanifold` uses some dark magic to implement it. Similarly, note the shaded areas on Figure 3. They represent intersections of areas bounded by two paths. Finding the bounding path of such an intersection is non-trivial and also implemented in `smoothmanifold`. Lastly, Figure 2 shows a three-dimensional surface, while the picture was fully drawn in 2D. The illusion is achieved through these cross-sectional "rings" on the left of the diagram.

To summarize, the most prominent features of module `smoothmanifold` are the following:

- **Gaps in overlapping paths**, achieved through a system of deferred drawing;
- **Set operations on paths bounding areas**, e.g. intersection, union, set difference, etc.;
- **Three-dimensional drawing**, achieved through an automatic (but configurable) addition of cross sections to smooth objects.

Do take a look at the source code for the above diagrams, to get a feel for how much heavy lifting is done by the module, and what is required from the user. We will now consider each of the above mentioned features (and some others as well) in full detail.

2. Deferred drawing and path overlapping

2.1. The general mechanism

In the `picture` structure, the paths drawn on a picture are not stored in an array, but rather indirectly stored in a `void` callback. That is, when the `draw` function is called, the *instruction to draw* the path is added to the picture, not the path itself. This makes it quite impossible to "modify the path after it is drawn". To go around this limitation, `smoothmanifold` introduces an auxiliary struct:

```
struct deferredPath {
    path[] g;
    pen p;
    int[] under;
    tarrow arrow;
    tbar bar;
}
```

It stores the path(s) to draw later, and how to draw them. Now, `smoothmanifold` executes the following steps to draw a "mutable" path `p` to a picture `pic` and then draw it for real:

1. Have a global two-dimensional array, say `arr`, of `deferredPath`'s;
2. Construct a `deferredPath` based on p, say `dp`;
3. Exploit the `nodes` field of the `picture` struct to store an integer. Retrieve this integer, say `n`, from `pic` (or create one if the `nodes` field doesn't contain it).
4. Store the deferred path `dp` in the one-dimensional array `arr[n]`;
5. Move on with the original code, perhaps modifying the deferred path `dp` in `arr` as needed, e.g. adding gaps;
6. At shipout time, when processing the picture `pic`, retrieve the index `n` from its `nodes` field and draw all `deferredPath` objects in the array `arr[n]`.

All these steps require no extra input from the user, since the shipout function is redefined to do them automatically. One only needs to use the `fitpath` function instead of `draw`.

2.2. The `tarrow` and `tbar` structures

Similarly to drawing paths to a picture, arrows and bars are implemented through a function type `bool(picture, path, pen, margin)`, typedef'ed as `arrowbar`. Moreover, when this arrowbar is called, it automatically draws not only itself, but also the path is was attached to. This makes it impossible to attach an arrowbar to a path and then mutate the path — the arrowbar will remember the path's original state. Hence, `smoothmanifold` implements custom arrow/bar implementations:

```
struct tarrow {
    arrowhead head;
    real size;
    real angle;
    filltype ftype;
    bool begin;
    bool end;
    bool arc;
}
```

```
struct tbar {
    real size;
    bool begin;
    bool end;
}
```

These structs store information about the arrow/bar, and are converted to regular arrowbars when the corresponding path is drawn to the picture. For creating new `tarrow`/`tbar` instances and converting them to arrowbars, the following functions are available:

```
tarrow DeferredArrow(                    tbar DeferredBar(
    arrowhead head = DefaultHead,            real size = 0,
    real size = 0,                           bool begin = false,
    real angle = arrowangle,                 bool end = false
    bool begin = false,                  )
    bool end = true,                     arrowbar convertbar(
    bool arc = false,                        tbar bar,
    filltype filltype = null                 bool overridebegin = false,
)                                            bool overrideend = false
arrowbar convertarrow(                   )
    tarrow arrow,
    bool overridebegin = false,
    bool overrideend = false
)
```

The `overridebegin` and `overrideend` options let the user force disable the arrow/bar at the beginning/end of the path.

2.3. The `fitpath` function

This is a substitute for the plain `draw` function. The `fitpath` function implements steps 1-4 of the deferred drawing system describes above.

```
void fitpath (picture pic, path gs, bool overlap, int covermode, bool drawnow, Label L,
pen p, tarrow arrow, tbar bar)
```

Arguments:
- `pic` — the picture to fit the path to;
- `gs` — the path to fit;
- `overlap` — whether to let the path overlap the previously fit paths. A value of `false` will lead to gaps being left in all paths that `gs` intersects;
- `covermode` — if the path `gs` is cyclic, this option lets you decide what happens to the parts of previously fit paths that fall "inside" of `gs`. Suppose a portion `s` of another path falls inside the cyclic path `gs`. Then
  ‣ `covermode == 2`: The portion `s` will be erased completely;
  ‣ `covermode == 1`: The portion `s` will be "demoted to the background" — either temporarily removed or drawn with dashes;
  ‣ `covermode == 0`: The portion `s` will be drawn like the rest of the path;
  ‣ `covermode == -1`: If the portion `s` is "demoted", it will be brought "back to the surface", i.e. drawn with solid pen. Otherwise, it will be draw as-is.

Consider the following example:
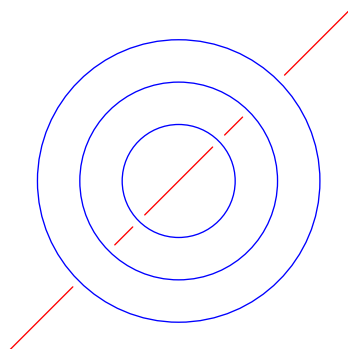

```
import smoothmanifold;
path l = (-1.2,-1.2) -- (1.2,1.2);
path c1 = unitcircle;
path c2 = scale(.7) * unitcircle;
path c3 = scale(.4) * unitcircle;
fitpath(l, red);
fitpath(c1, blue, covermode = 1);
fitpath(c2, blue, covermode = -1);
fitpath(c3, blue, covermode = 0);
```

Figure 4: A showcase of the `fitpath` function

- `drawnow` — whether to draw the path `gs` immediately to the picture. When `drawnow == true`, the path `gs` leaves gaps in other paths, but is immutable itself, i.e. later fit paths will not leave any gaps in it.

When `drawnow == false`, the path `gs` is not immediately drawn, but rather saved to be mutated and finally drawn at shipout time;

- `L` — the label to attach to `gs`. This label is drawn to `pic` immediately on call of `fitpath`, unlike `gs`;
- `p` — the pen to draw `gs` with;
- `arrow` — the arrow to attach to the path. Note that the type is `tarrow`, not `arrowbar`;
- `bar` — the bar to attach to the path. Note that the type is `tbar`, not `arrowbar`.

Apart from different types of the `arrow`/`bar` arguments, the `fitpath` function is identical to `draw` in type signature, and they can be used interchangeably. Moreover, there are overloaded versions of `fitpath`, where parameters are given default values (one of these versions is used in the example above):

```
void fitpath (
    picture pic = currentpicture,
    path g,
    bool overlap = config.drawing.overlap,
    int covermode = 0,
    Label L = "",
    pen p = currentpen,
    bool drawnow = config.drawing.drawnow,
    tarrow arrow = null,
    tbar bar = config.arrow.currentbar
)
```

```
void fitpath (
    picture pic = currentpicture,
    path[] g,
    bool overlap = config.drawing.overlap,
    int covermode = 0,
    Label L = "",
    pen p = currentpen,
    bool drawnow = config.drawing.drawnow
)
```

Here, `config` is the global configuration structure, see Section 5. Furthermore, there are corresponding `fillfitpath` functions that serve the same purpose as `filldraw`.

2.4. Other related routines

```
int extractdeferredindex (picture pic)
```

Inspect the `nodes` field of `pic` for a string in a particular format, and, if it exists, extract an integer from it.

```
deferredPath[] extractdeferredpaths (picture pic, bool createlink)
```

Extract the deferred paths associated with the picture `pic`. If `createlink` is set to `true` and `pic` has no integer stored in its `nodes` field, the routine will find the next available index and store it in `pic`.

```
path[] getdeferredpaths (picture pic = currentpicture)
```

A wrapper around `extractdeferredpaths`, which concatenates the `path[]` `g` fields of the extracted deferred paths.

```
void purgedeferredunder (deferredPath[] curdeferred)
```

For each deferred path in `curdeferred`, delete the segments that are "demoted" to the background (i.e. going under a cyclic path, drawn with dashed lines).

```
void drawdeferred (
    picture pic = currentpicture,
    bool flush = true
)
```

Render the deferred paths associated with `pic`, to the picture `pic`. If `flush` is `true`, delete these deferred paths.

```
void flushdeferred (picture pic = currentpicture)
```

Delete the deferred paths associated with `pic`.

```
void plainshipout (...) = shipout;
shipout = new void (...)
{
    drawdeferred(pic = pic, flush = false);
    draw(pic = pic, debugpaths, red+1);
    plainshipout(prefix, pic, orntn, format, wait, view, options, script, lt, P);
};
```

A redefinition of the `shipout` function to automatically draw the deferred paths at shipout time. For a definition of `debugpaths`, see Section 4.4.

The functions `erase`, `add`, `save`, and `restore` are redefined to automatically handle deferred paths.

3. Operations on paths

3.1. Set operations on bounded regions

Module `smoothmanifold` defines a routine called `combination` which, given two *cyclic* paths `p` and `q`, calculates a result path which encloses a region that is a combination of the regions `p` and `q`:

```
path[] combination (path p, path q, int mode, bool round, real roundcoeff)
```

This function returns an array of paths because the combination of two bounded regions may be bounded by multiple paths. Rundown of the arguments:

- `p` and `q` — cyclic paths bounding the regions to combine;
- `mode` — an internal parameter which allows to specialize `combination` for different purposes;
- `round` and `roundcoeff` — whether to round the sharp corners of the resulting bounding path(s).
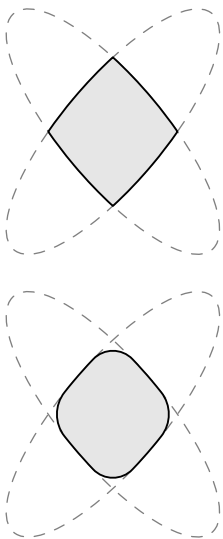

```
<...>
filldraw(
    combination(p, q, 1, false, 0), // <- no rounding
    drawpen = linewidth(.7),
    fillpen = palegrey
);
<...>
<...>
filldraw(
    combination(p, q, 1, true, .04), // <- yes rounding
    drawpen = linewidth(.7),
    fillpen = palegrey
);
<...>
```

Figure 5: A showcase of the `round` and `roundcoeff` parameters

Based on different values for the `mode` parameter, the module defines the following specializations:

```
path[] difference (
  path p,
  path q,
  bool correct = true,
  bool round = false,
  real roundcoeff = config.paths.roundcoeff
)
```

```
path[] operator - (path p, path q)
{ return difference(p, q); }
```

Calculate the path(s) bounding the set difference of the regions bounded by `p` and `q`. The `correct` parameter determines whether the paths should be "corrected", i.e. oriented clockwise.

```
path[] symmetric (
  path p,
  path q,
  bool correct = true,
  bool round = false,
  real roundcoeff = config.paths.roundcoeff
)
```

```
path[] operator :: (path p, path q)
{ return symmetric(p, q); }
```

Calculate the path(s) bounding the set symmetric difference of the regions bounded by p and q.

```
path[] intersection (
  path p,
  path q,
  bool correct = true,
  bool round = false,
  real roundcoeff = config.paths.roundcoeff
)
```

```
path[] operator ^ (path p, path q)
{ return intersection(p, q); }
```

Calculate the path(s) bounding the set intersection of the regions bounded by p and q. The following array versions are also available:

```
path[] intersection (
  path[] ps,
  bool correct = true,
  bool round = false,
  real roundcoeff = config.paths.roundcoeff
)
```

```
path[] intersection (
  bool correct = true,
  bool round = false,
  real roundcoeff = config.paths.roundcoeff
  ... path[] ps
)
```

Inductively calculate the total intersection of an array of paths.

```
path[] union (
  path p,
  path q,
  bool correct = true,
  bool round = false,
  real roundcoeff = config.paths.roundcoeff
)
```

```
path[] operator | (path p, path q)
{ return union(p, q); }
```

Calculate the path(s) bounding the set union of the regions bounded by p and q. The corresponding array versions are available:

```
path[] union (
  path[] ps,
  bool correct = true,
  bool round = false,
  real roundcoeff = config.paths.roundcoeff
)
```

```
path[] union (
  bool correct = true,
  bool round = false,
  real roundcoeff = config.paths.roundcoeff
  ... path[] ps
)
```

Inductively calculate the total union of an array of paths. Here is an illustration of the specializations:
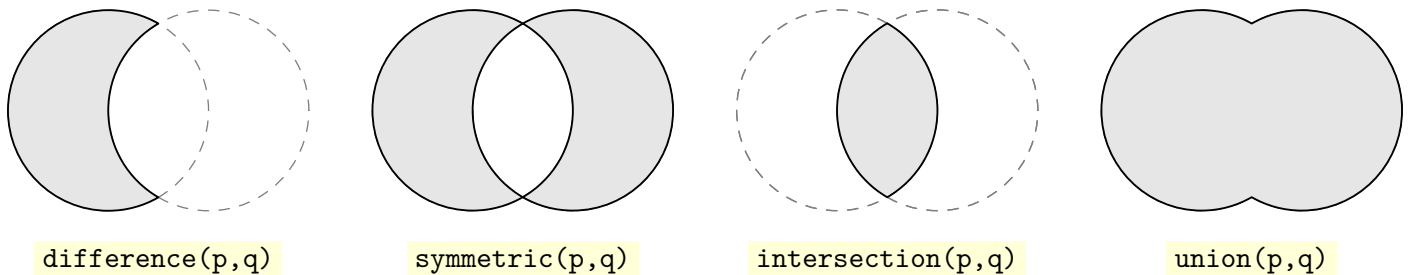

difference(p,q)   symmetric(p,q)   intersection(p,q)   union(p,q)

Figure 6: Different specializations of the combination function

3.2. Other path utilities

Module `smoothmanifold` features dozens of useful auxiliary path utilities, all of which are listed below.

```
path[] convexpaths = { ... }
path[] concavepaths = { ... }
```

Predefined collections of convex and concave paths (14 and 7 paths respectively), added for user convenience.

```
path randomconvex ()
path randomconcave ()
```

Allows the user to sample a random path from the above arrays.

```
path ucircle = reverse(unitcircle);
path usquare = (1,1) -- (1,-1) -- (-1,-1) -- (-1,1) -- cycle;
```

Slightly changed versions of the `unitcircle` and `unitsquare` paths. Most notably, these are *clockwise,* since most of this module-s functionality prefers to deal with clockwise paths.

```
pair center (path p, int n = 10, bool arc = true, bool force = false)
```

Calculate the center of mass of the region bounded by the cyclic path `p`. If `force` is `false` and the center of mass is outside of `p`, the routine uses a heuristic to return another point, inside of `p`.

```
bool insidepath (path p, path q)
```

Check if path `q` is completely inside the cyclic path `p` (directions of `p` and `q` do not matter).

```
real xsize (path p) { return xpart(max(p)) - xpart(min(p)); }
real ysize (path p) { return ypart(max(p)) - ypart(min(p)); }
```

Calculate the horizontal and vertical size of a path.

```
real radius (path p) { return (xsize(p) + ysize(p))*.25; }
```

Calculate the approximate radius of the region enclosed by `p`.

```
real arclength (path g, real a, real b) { return arclength(subpath(g, a, b)); }
```

A more general version of `arclength`.

```
real relarctime (path g, real t0, real a)
```

Calculate the time at which arclength `a` will be traveled along the path `g`, starting from time `t0`.

```
path arcsubpath (path g, real arc1, real arc2)
```

Calculate the subpath of `g`, starting from arclength `arc1`, and ending with arclength `arc2`.

```
real intersectiontime (path g, pair point, pair dir)
```

Calculate the time of the intersection of `g` with a beam going from `point` in direction `dir`

```
pair intersection (path g, pair point, pair dir)
```

Same as `intersectiontime`, but returns the point instead of the intersection time.

```
path reorient (path g, real time)
```

Shift the starting point of the cyclic path `g` by time `time`. The resulting path will be same as `g`, but will start from time `time` along `g`.

```
path turn (path g, pair point, pair dir)
{ return reorient(g, intersectiontime(g, point, dir)); }
```

A combination of `reorient` and `intersectiontime`, that shifts the starting point of the cyclic path `g` to its intersection with the ray cast from `point` in the direction `dir`.

```
path subcyclic (path p, pair t)
```

Calculate the subpath of the *cyclic* path p, from time `t.x` to time `t.y`. If `t.y < t.x`, the subpath will still go in the direction of `g` instead of going backwards.

```
bool clockwise (path p)
```

Determine if the cyclic path `p` is going clockwise.

```
bool meet (path p, path q) { return (intersect(p, q).length > 0); }
bool meet (path p, path[] q) { ... }
bool meet (path[] p, path[] q) { ... }
```

A shorthand function to determine if two (or more) paths have an intersection point.

```
pair range (path g, pair center, pair dir, real ang, real orient = 1)
```

Calculate the begin and end times of a subpath of `g`, based on `center`, `dir`, and `angle`, as such:
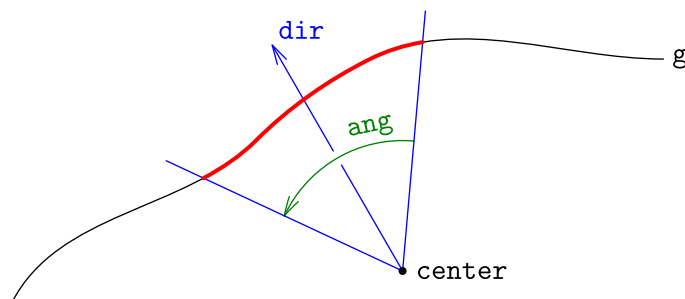

Figure 7: An illustration of the `range` function

```
bool outsidepath (path p, path q)
```

Check if `q` is completely outside (that is, inside the complement) of the region enclosed by `p`.

```
path ellipsepath (pair a, pair b, real curve = 0, bool abs = false)
```

Produce half of an ellipse connecting points `a` and `b`. Curvature may be relative or absolute.

```
path curvedpath (pair a, pair b, real curve = 0, bool abs = false)
```

Constuct a curved path between two points. Curvature may be relative (from 0 to 1) or absolute.

```
path cyclepath (pair a, real angle, real radius)
```

A circle of radius `radius`, starting at `a` and turned at `angle`.

```
path midpath (path g, path h, int n = 20)
```

Construct the path going "between" `g` and `h`. The parameter `n` is the number of sample points, the more the more precise the output.
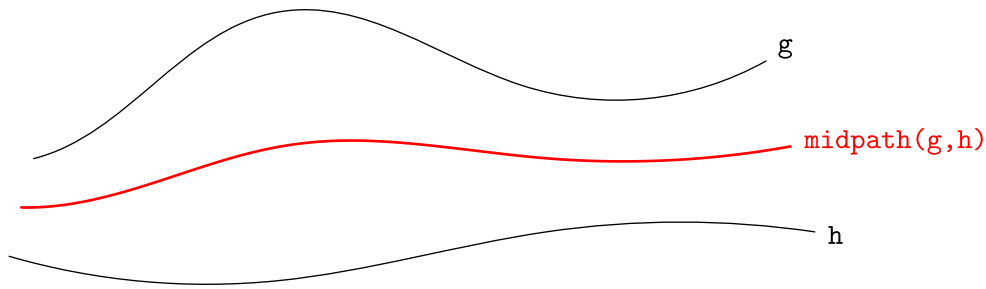
Figure 8: An illustration of the `midpath` function

```
path connect (pair[] points)
path connect (... pair[] points)
```

Connect an array of points with a path.

```
path wavypath (real[] nums, bool normaldir = true, bool adjust = false)
path wavypath (... real[] nums)
```

Generate a clockwise cyclic path around the point (`0`,`0`), based on the `nums` parameter. If `normaldir` is set to `true`, additional restrictions are imposed on the path. If `adjust` is `true`, then the path is shifted and scaled such that its `center` [page 9] is (`0`,`0`), and its `radius` [page 9] is `1`. Consider the following example:
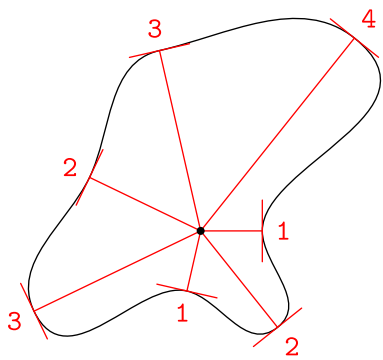

```
real[] nums = {1,2,1,3,2,3,4};
bool normaldir = true;

draw(wavypath(nums, normaldir));

for (int i = 0; i < nums.length; ++i) {
  <...> // draw numbers
}

dot((0,0));
```

Figure 9: A showcase of the `wavypath` function

```
path connect (path p, path q)
```

Connect the paths `p` and `q` smoothly.

```
pair randomdir (pair dir, real angle)
{ return dir(degrees(dir) + (unitrand()-.5)*angle); }
path randompath (pair[] controlpoints, real angle)
```

Create a pseudo-random path passing through the `controlpoints`. The `angle` parameter determines the "spread" of randomness. Here's an example:
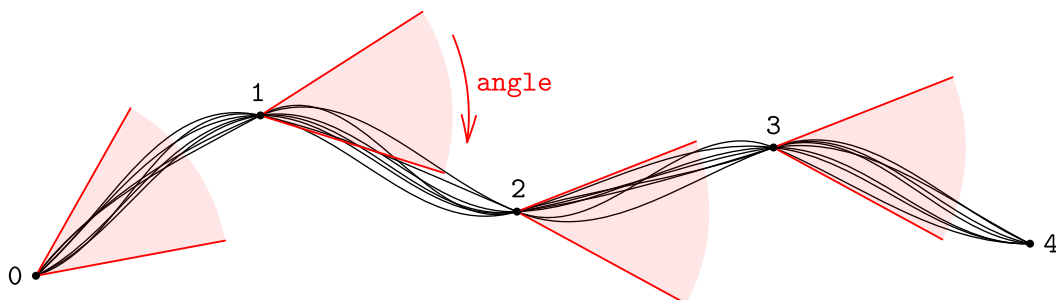

Figure 10: A showcase of the `randompath` function

```
path neigharc (
    real x = 0,
    real h = config.paths.neighheight,
    int dir = 1,
    real w = config.paths.neighwidth
)
```

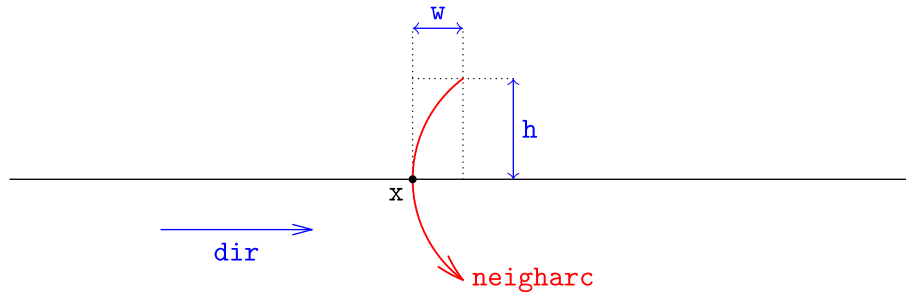Draw an "open neighborhood bracket" on the real line, like so:

Figure 11: A showcase of the `neigharc` function

4. Smooth objects

4.1. Definition of the `smooth`, `hole`, `subset`, and `element` structures

The `smoothmanifold` module's original purpose was to introduce a suitable abstraction to simplify drawing blobs on the plane. The `smooth` structure is, perhaps, the oldest part of `smoothmanifold`, that has persisted through countless updates and changes. In its current form, here is how it's defined:

```
struct smooth {
    path contour;
    pair center;

    string label;
    pair labeldir;
    pair labelalign;

    hole[] holes;
    subset[] subsets;
    element[] elements;

    transform unitadjust;

    real[] hratios;
    real[] vratios;

    bool isderivative;

    smooth[] attached;

    void postdraw (dpar, smooth);

    static smooth[] cache;
}
```

```
struct hole {
    path contour;
    pair center;
    real[][] sections;
    int scnumber;
}

struct subset {
    path contour;
    pair center;
    string label;
    pair labeldir;
    pair labelalign;
    int layer;
    int[] subsets;
    bool isderivative;
    bool isonboundary;
}

struct element {
    pair pos;
    string label;
    pair labelalign;
}
```

Every `smooth` object has a:
- `contour` — the clockwise cyclic path that serves as a boundary of the object;
- `center` — the center of the object, usually inferred automatically with `center`(`contour`) [page 9];
- `label` — a string label, e.g. `"$A$"` or `"$S$"`, that will be displayed when drawing the smooth object;

- `labeldir` and `labelalign` — they determine where the label is to be drawn, namely at `intersection(contour, center, labeldir)` [page 9], with `labelalign` as `align`.
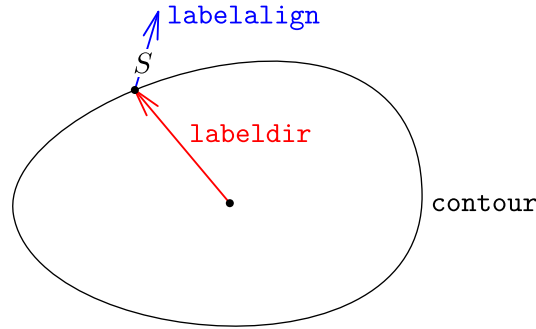

Figure 12: A showcase of the `labeldir` and `labelalign` fields

- `holes` — an array of `hole` structures, each of which has a:
  - `contour` — the clockwise cyclic boundary of the hole;
  - `center` — the center of the hole, typically calculated automatically by `center(contour)` [page 9];
  - `sections` — a two-dimensional array that determines where to draw the cross sections seen on Figure 2. For a detailed description, see Section 4.6.2;
  - `scnumber` — the maximum amount of cross sections that the hole allows other holes to share with it. These are sections *between holes,* not ones between a hole and the contour of the `smooth` object. For details, see Section 4.6.2;
- `subsets` — an array of `subset` structures, each of which has a:
  - `contour` — the clockwise cyclic boundary of the subset;
  - `center` — the center of the subset, likewise usually determined by `center(contour)` [page 9];
  - `label`, `labeldir`, `labelalign` — serve the same purpose as the respective fields of the `smooth` object;
  - `layer` — an integer determining the "depth" of the subset. A toplevel subset will have `layer == 0`, its subsets will have `layer == 1`, their subsets will have `layer == 2`, etc. This way, a hierarchy of subsets is established. For details, see Section 4.4;
  - `subsets` — an index `i` is an element of this array if and only if the subset `subsets[i]` (taken from the `subsets` field of the parent `smooth` object) is a subset of the current subset. For details, see Section 4.4;
  - `isderivative` — a flag that marks all automatically created subsets (i.e. those that represent intersections of existing subsets). For details, see Section 4.4;
  - `isonboundary` — a flag that marks if the current subset touches the boundary of another subset.
- `elements` — an array of `element` structures, each of which has a:
  - `pos` — the position of the element;
  - `label` — the label attached to the element, e.g. `"$x$"` or `"$y_0$"`;
  - `labelalign` — how to align the label when drawing the element;
- `unitadjust` — a transform that converts from unit coordinates of the smooth object to the global user coordinates (see Section 4.5);
- `hratios` and `vratios` — two arrays that determine where to draw cross sections in the `cartesian` mode. For details, see Section 4.6.3;
- `isderivative` — similarly to `subset`, this field marks those `smooth` objects which are obtained from preexisting objects through operations of intersection, union, etc.;
- `attached` — this field allows to bind an array of smooth objects to the current one. Drawing the current object will trigger drawing all of its `attached` objects. For example, the tangent space seen on Figure 2 is `attached` to the main object;
- `postdraw` — a callback to be executed *after the `smooth` object is drawn.* It takes as parameters a drawing configuration of type `dpar` (see Section 5.5) and the current `smooth` object;
- `static cache` — a global array of all `smooth` objects constructed so far. It is used mainly to search for smooth objects by label. See Section 4.9.

4.2. Construction

Each of the four structures is equipped with a sophisticated `void` `operator` `init` that will infer as much information as possible. To construct a `smooth`, `hole`, or `subset`, it is only necessary to pass a `contour`. All other fields can be set in the constructor, but they are optional. To construct an `element`, it is only necessary to pass a `pos`, the `label` and `labelalign` fields have default values.

4.3. Query and mutation

Already constructed structures can be queried and modified in a plethora of ways. Most methods return `this` at the end of execution for convenience.

4.3.1. Methods of `smooth` objects

```
real xsize () { return xsize(this.contour); }
real ysize () { return ysize(this.contour); }
```

Calculate the vertical and horizontal size of `this`.

```
bool inside (pair x)
```

Check if `x` lies inside the contour of `this`, but not inside any of its `holes`.

```
smooth move (                          smooth shift (explicit pair shift)
    pair shift = (0,0),
    real scale = 1,                     smooth shift
    real rotate = 0,                    (real xshift, real yshift = 0)
    pair point = this.center,
    bool readjust = true,               smooth scale (real scale)
    bool drag = true
)                                       smooth rotate (real rotate)
```

Scale `this` by `scale` (with center at `point`), rotate by `rotate` around `point`, and then shift by `shift`. If `readjust` is `true`, also recalculate the `unitadjust` field. If `drag` is `true`, also apply the `move` to all smooth objects `attached` to `this`. In the end return `this`. The `shift`, `scale` and `rotate` methods on the right are all specializations of the `move` method.

```
void xscale (real s)
```

Scale `this` by `s` along the x-axis.

```
smooth dirscale (pair dir, real s)
```

Scale `this` by `s` in the direction `dir`. Return `this`.

```
smooth setcenter (
    int index = -1,
    pair center = config.system.dummypair,
    bool unit = config.smooth.unit
)
```

Set the center of `this` if `index == -1`, and the center of `this.subsets[index]` otherwise. If `unit` is `true`, interpret `center` in the unit coordinates of `this` (i.e. apply `this.unitadjust` to `center`). For the definition of `config.system.dummypair`, see Section 5.1.

```
smooth setlabel (
    int index = -1,
    string label = config.system.dummystring,
    pair dir = config.system.dummypair,
    pair align = config.system.dummypair
)
```

Set the `label`, `labeldir` and `labelalign` of `this` if `index == -1`, and set these fields of `this.subsets[index]` otherwise. For the definition of `config.system.dummystring`, see Section 5.1.

```
smooth addelement (
    pair pos,
    string label = "",
    pair align = 1.5*S,
    int index = -1,
    bool unit = config.smooth.unit
)
```

```
smooth addelement (
    element elt,
    int index = -1,
    bool unit = config.smooth.unit
)
```

Add a new element to `this`. Return `this`. If `index` `>=` `0`, then the function will additionally check if the `element` is contained within the contour of `this.subsets[index]`. If `unit` is true, then the position of the `element` is interpreted in `this` object's unit coordinates.

```
smooth setelement (
    int index,
    pair pos = config.system.dummypair,
    string label = config.system.dummystring,
    pair labelalign = config.system.dummypair,
    int sbindex = -1,
    bool unit = config.smooth.unit
)
```

Change the fields of the element of `this` at index `index`. Return `this`. Slightly different versions of this routine are also defined in the source code, feel free to peruse it.

```
smooth rmelement (int index)
```

Delete an element at index `index`. Return `this`.

```
smooth movelement (int index, pair shift)
```

Move the element at index `index` by `shift`. Return `this`.

```
smooth addhole (
    path contour,
    pair center = config.system.dummypair,
    real[][] sections = {},
    pair shift = (0,0),
    real scale = 1,
    real rotate = 0,
    pair point = center(contour),
    bool clip = config.smooth.clip,
    bool unit = config.smooth.unit
)
```

```
smooth addhole (
    hole hl,
    int insertindex = this.holes.length,
    bool clip = config.smooth.clip,
    bool unit = config.smooth.unit
)
```

Add a new hole to `this`. If `clip` is `true` and the hole's contour is *not* contained within `this` object's contour, then clip `this` smooth object's contour using `difference` [page 7], like so:
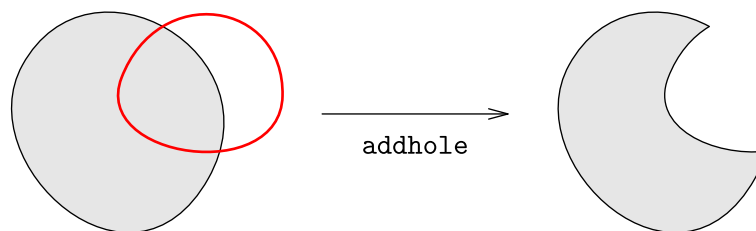

Figure 13: A showcase of the `clip` parameter

If `unit` is `true`, then the hole contour will be treated in the unit coordinates, i.e. the `unitadjust` transform will be applied to it.

```
smooth addholes (                      smooth addholes (
    hole[] holes,                          path[] contours,
    bool clip = config.smooth.clip,        bool clip = config.smooth.clip,
    bool unit = config.smooth.unit         bool unit = config.smooth.unit
)                                      )
smooth addholes (                      smooth addholes (
    bool clip = config.smooth.clip,        bool clip = config.smooth.clip,
    bool unit = config.smooth.unit         bool unit = config.smooth.unit
    ... hole[] holes                       ... path[] contours
)                                      )
```

Auxiliary routines made for conveniently adding multiple holes at the same time.

```
smooth rmhole (int index = this.holes.length-1)
```

Delete the hole under index `index` from `this`.

```
smooth rmholes (int[] indices)
smooth rmholes (... int[] indices)
```

Delete a number of holes from `this`.

```
smooth movehole (
    int index,
    pair shift = (0,0),
    real scale = 1,
    real rotate = 0,
    pair point = this.holes[index].center,
    bool movesections = false
)
```

Move the hole at index `index` by scaling and rotating it around `point`, and shifting it by `shift`. If `movesections` is `true`, the `sections` values of the hole are updated accordingly with the transform. This is only really necessary when `rotate` is non-zero.

```
smooth addsection (        smooth setsection (        smooth rmsection (
    int index,                 int index,                 int index =
    real[] section = {}        int scindex = 0,              this.holes.length-1,
)                              real[] section = {}        int scindex = 0
                           )                              )
```

Add, set or remove a section under `scindex` in the hole under `index`.

4.4. The subset hierarchy

4.5. Unit coordinates in a `smooth` object

```
transform selfadjust ()
{ return shift(this.center)*scale(radius(this.contour)); }
```

Calculate the unit coordinates of `this`. See `unitadjust` [page 12] and Section 4.5 for reference.

```
transform adjust (int index)
```

Calculate the unit coordinates of the subset of `this` at index `index`. If `index` is set to `-1`, the `unitadjust` field of `this` is used instead.

```
pair relative (pair point)
```

4.6. The modes of cross section drawing

4.6.1. `plain` mode

4.6.2. `free` mode

4.6.3. `cartesian` mode

```
real getyratio (real y)
real getxratio (real x)
real getypoint (real y)
real getxpoint (real x)
```

Convert to and from relative lengths.

```
smooth setratios (real[] ratios, bool horiz)
```

Set the horizontal/vertical cartesian ratios of `this`.

4.6.4. `combined` mode

4.7. The `dpar` drawing configuration structure

4.8. The `draw` function

4.9. Reference by label

```
static bool repeats (string label)
```

Check if `label` already exists as a label of some `smooth`, `subset`, or `element` object.

```
int findlocalsubsetindex (string label)
```

Locate a subset of `this` by its label and return its index.

```
smooth setcenter (
    string destlabel,
    pair center,
    bool unit = config.smooth.unit
) { return this.setcenter(findlocalsubsetindex(destlabel), center, unit); }
```

An alternative to `setcenter` [page 14], but finds the subset by label.

```
smooth setlabel (
    string destlabel,
    string label,
    pair dir = config.system.dummypair,
    pair align = config.system.dummypair
) { return this.setlabel(findlocalsubsetindex(destlabel), label, dir, align); }
```

An alternative to `setlabel` [page 14], but finds the subset by label.

```
smooth setelement (
    string destlabel,
    pair pos = config.system.dummypair,
    string label = config.system.dummystring,
    pair labelalign = config.system.dummypair,
    bool unit = config.smooth.unit
) { return this.setelement(findlocalelementindex(destlabel), pos, label, labelalign,
unit); }
```

An alternative to `setelement` [page 15], but finds the element by label.

```
smooth rmelement (string destlabel)
```

An alternative to `rmelement` , but finds the element by label.

```
smooth movelement (string destlabel, pair shift)
```

An alternative to `movelement` , but finds the element by label.

5. Global configuration

5.1. System variables

5.2. Path variables

5.3. Cross section variables

5.4. Smooth object variables

5.5. Drawing-related variables

5.6. Help-related variables

5.7. Arrow variables

6. Debugging capabilities

6.1. Errors

Should you perform an erroneous calculation step (like adding an out-of-bounds subset to a smooth object, or referring to a non-existent label), `smoothmanifold` will crash with an error message.

6.2. Warnings

7. Miscellaneous auxiliary routines

7.1. Useful array functions

8. The `export.asy` auxiliary module

8.1. The `export` routine

8.2. Animations

8.3. Configuration

9. Index