An Efficient Approach to the Simultaneous Localisation and Mapping Problem

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Abstract

This paper presents a novel approach to the Simultaneous Localisation and Mapping (SLAM) algorithm that exploits the manner in which observations are fused into the global map of the environment to manage the computational complexity of the algorithm and improve the data association process. Rather than incorporating every observation directly into the global map of the environment, the Constrained Local Submap Filter (CLSF) relies on creating an independent, local submap of the features in the immediate vicinity of the vehicle. This local submap is then periodically fused into the global map of the environment using appropriately formulated constraints between the common feature estimates. This approach is shown to be effective in reducing the computational complexity of maintaining the global map estimates as well as improving the data association process.

1 Introduction

Simultaneous Localisation and Mapping (SLAM) is the process of concurrently building a feature based map of the environment and using this map to obtain estimates of the location of the vehicle. In essence, the vehicle relies on its ability to extract useful navigation information from the data returned by its sensors. The SLAM algorithm has recently seen a considerable amount of interest from the mobile robotics community as a tool to enable fully autonomous navigation [2][3][4][8][11][13][15]. The prospect of deploying a robotic vehicle that can build a map of its environment while simultaneously using that map to localise itself promises to allow these vehicles to operate autonomously for long periods of time in unknown environments. Much of this work has focused on the use of stochastic estimation techniques to build and maintain estimates of vehicle and map feature locations. In particular, the Extended Kalman Filter (EKF) has been proposed as a mechanism by which the information gathered by the vehicle can be consistently fused to yield bounded estimates of vehicle and landmark locations in a recursive fashion [3][7].

While the Kalman Filter approach to the SLAM problem has received considerable interest, alternative philosophies also appear in the literature. A number of research teams have tackled the problem of map building and localisation using batch estimation techniques [10][5][13]. Still other approaches to the problem of map building and localisation have done away with the rigorous mathematical models of the vehicle and sensing properties and have relied instead on more qualitative knowledge of the nature of the environment [1][6][9]. While all of these alternative approaches to the problem have their own particular strengths, this paper will be concerned primarily with a recursive, on-line approach to the problem and will rely on the EKF as the primary means of simultaneously building a map while localising the vehicle.

This paper presents a novel approach to SLAM that exploits the manner in which observations are fused into the global map of the environment to manage the computational complexity of the algorithm and improve the data association process. Section 2 begins by summarising the key characteristics of the proposed SLAM algorithm, touching on the estimation process using this representation and highlighting the differences between it and the usual single, global SLAM filter. Section 3 examines the computational savings that can be realised from this approach while Section 4 discusses benefits for data association. Section 5 presents results illustrating the application of the approach and highlighting the performance of the Constrained Local Submap Filter algorithm applied in simulation and to data collected using an Autonomous Underwater Vehicle. Finally, Section 6 summarises the paper and provides concluding remarks.

2 Constrained Local Submap Filter

The Constrained Local Submap Filter (CLSF) presents a novel scheme for addressing the computational complexity of the SLAM algorithm by allowing the update of the full covariance matrix to be scheduled at appropriate intervals. The method developed here maintains an independent, local submap estimate of the features in the immediate vicinity of the vehicle (see Figure 1). The observations are fused to create a local map of the environment referenced to a local frame of reference whose global position is known. At appropriate intervals, the information contained in the local map is transferred into the global map using appropriately formulated constraints between the feature estimates. Subject to the usual linearising assumptions, the resulting map and vehicle estimates are identical to those obtained using the single global map algorithm [14]. This approach to the SLAM algorithm allows the computationally expensive process of updating the cross-covariances in the global map to be scheduled at convenient intervals and

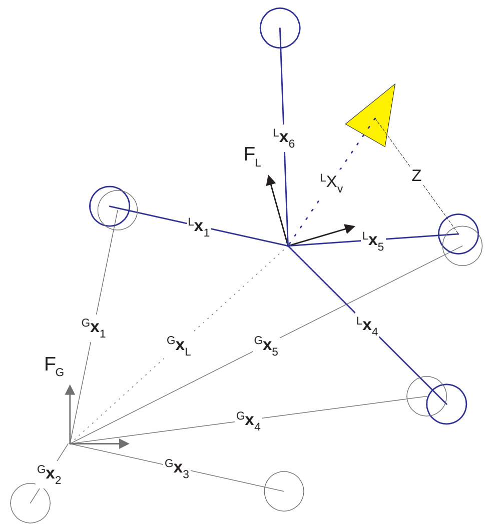


Fig. 1. Local submap state estimation. The vehicle maintains a local map of the features around it. At appropriate intervals, the local map features are fused into the global feature map using appropriately formulated constraints. This approach to map building significantly improves the computational complexity of SLAM.

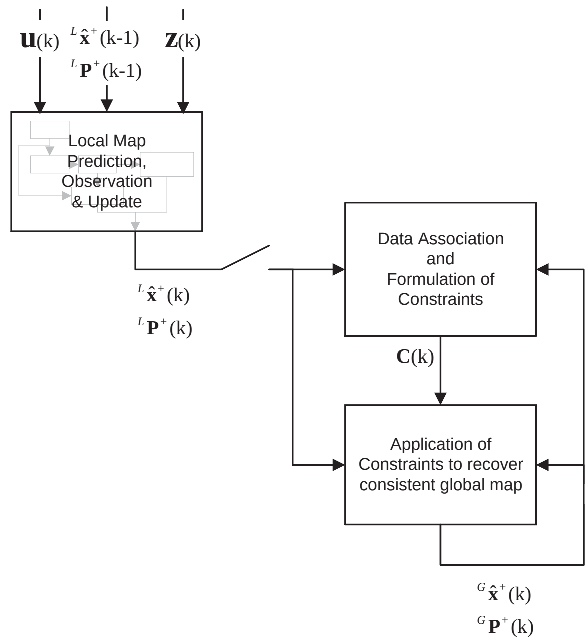


Fig. 2. Scheduling of the application of constraints using the Constrained Local Submap Filter. The vehicle operates in a local frame of reference, building an independent map of the features around it. At appropriate intervals, indicated by the switch, the local map is transformed into the global frame of reference and the information fused into the global map using constraints to produce the updated global map estimate.

for a potentially large number of observations to be fused consistently into the global map in a single step. It also aids in the data association problem as the uncertainties of the feature and vehicle estimates in the local frame of reference tend to be comparatively small. Furthermore, it allows data association decisions in the global frame to be deferred until an improved local map of the environment is available.

Figure 2 shows the basic steps in this approach to SLAM. At some time, a new frame of reference is defined at the current vehicle position. The estimate of the vehicle pose therefore represents an estimate of the pose of this frame of reference with respect to the global frame. The vehicle is now at the origin of this local reference frame with zero uncertainty at the instant the local frame is created. A new, local vehicle estimate is initialised relative to the new frame and the algorithm begins building a standard SLAM map with respect to the local frame. The estimates in this frame of reference will be shown to be independent of the estimates in the global frame of reference, implying that only a small state covariance matrix must be updated with each observation.

When the decision is made to transfer information contained in the local map into the global map, indicated by the switch in Figure 2, the state vector will contain both local and global position estimates of some of the landmarks. A data association strategy is used to identify the landmarks that have duplicate estimates in the state vector and a constraint based projection is used to yield a single, consistent estimate of these landmark states. This step effectively recovers all of the information available to the filter and allows the filter to generate the global map state estimate despite the fact that the full covariance matrix has not been updated with each observation. Once the local map has been fused into the global map, a new local map is instantiated at the updated vehicle position and the process of building a local map begins again. This approach blends the ideas of applying constraints presented in the Geometric Projection Filter (GPF) [11] with the intuitive nature of the global map algorithm.

2.1 The Estimation Process

For the CLSF formulation of the SLAM algorithm, the EKF is used to estimate the pose of the vehicle $\mathbf{x}_v(k)$ along with the positions of the n_f observed features $\mathbf{x}_i(k), i = 1...n_f$. The major difference with this representation arises from the fact that only those feature estimates observed in the current local frame of reference must be updated during an observation. The remaining state estimates represent the feature estimates in the global map of the environment and are not updated until the information in the local map is fused into the global map. This leads to two distinct independent state estimates while the vehicle is operating in the local submap.

$$\hat{\mathbf{x}}^+_{cls}(k) = \begin{bmatrix} {}^G\hat{\mathbf{x}}^+_L(k) \\ {}^G\hat{\mathbf{x}}^+_m(k) \\ {}^L\hat{\mathbf{x}}^+_v(k) \\ {}^L\hat{\mathbf{x}}^+_m(k) \end{bmatrix} \tag{1}$$

The notation ${}^L\hat{\mathbf{x}}^+_i(k)$ indicates a state estimate of feature $\mathbf{x}_i(k)$ in the local frame of reference. Estimates taken in the global frame of reference will be referred to using the notation ${}^G\hat{\mathbf{x}}^+_i(k)$. The special case of the estimate of the position of the local frame of reference in the global frame is referred to as ${}^G\hat{\mathbf{x}}^+_L(k)$.

The covariance matrix takes on the usual form and contains estimates of the vehicle state covariances and map feature covariances together with the appropriate cross-covariance terms. As shown in [14], the local map estimates are decorrelated from the global estimates since an independent local frame of reference is instantiated relative to the current vehicle estimate. Prior feature estimates are not used while the vehicle is operating in this local frame and the covariance matrix therefore takes on a block diagonal structure with the global map representing one block and the local map the other. Only the local map estimates need to be updated during the estimation process as a result of this covariance structure.

$$\mathbf{P}^+(k) = \begin{bmatrix} {}^G\mathbf{P}^+(k) & 0 \\ 0 & {}^L\mathbf{P}^+(k) \end{bmatrix} \tag{2}$$

with

$$\begin{aligned} {}^G\mathbf{P}^+(k) &= \begin{bmatrix} {}^G\mathbf{P}^+_{LL}(k) & {}^G\mathbf{P}^+_{mL}(k) \\ {}^G\mathbf{P}^{+T}_{mL}(k) & {}^G\mathbf{P}^+_{mm}(k) \end{bmatrix} \\ {}^L\mathbf{P}^+(k) &= \begin{bmatrix} {}^L\mathbf{P}^+_{vv}(k) & {}^L\mathbf{P}^+_{vm}(k) \\ {}^L\mathbf{P}^{+T}_{vm}(k) & {}^L\mathbf{P}^+_{mm}(k) \end{bmatrix} \end{aligned}$$

where ${}^L\mathbf{P}^+_{vv}(k)$ represents the vehicle covariance in the local frame of reference, ${}^L\mathbf{P}^+_{mm}(k)$, represent the local landmark covariances, ${}^G\mathbf{P}^+_{mm}(k)$, represents the local landmark covariances and ${}^G\mathbf{P}^+_{LL}(k)$, represents the covariance of the estimate of the local frame of reference in the global frame.

2.2 Decorrelated Local State Estimates

A local submap as defined here consists of a frame of reference within which the vehicle and map states are estimated. This frame of reference is defined by an initial estimate of the position of the vehicle when the frame of reference is initialised. This approach results in the local estimates of vehicle and map states being fully decorrelated to the global map estimates

When the decision is made to build a new local map, a new coordinate frame, $\mathcal{F}_L$, is defined coincident with the estimate of the current vehicle location, ${}^G\mathbf{x}_v(k-1)$. The estimated position of the vehicle, ${}^G\hat{\mathbf{x}}^+_v(k-1)$, represents the transformation from the current frame of reference, $\mathcal{F}_G$, to the new frame of reference, $\mathcal{F}_L$ and will be replaced with the term ${}^G\hat{\mathbf{x}}^+_L(k-1)$. This indicates that the previous vehicle estimate is no longer a vehicle estimate but instead represents the estimate of the local frame within the global frame of reference. As the new frame of reference is centered at the current vehicle position, the new local position of the vehicle, ${}^L\hat{\mathbf{x}}^+_v(k-1)$, is at the origin of the local frame with no uncertainty. As the vehicle operates in the new frame of reference, the local vehicle state estimate will remain fully decorrelated with respect to the global map estimates.

When the vehicle takes observations in the local submap, features are initialised relative to the current frame of reference using the vehicle estimate, ${}^L\hat{\mathbf{x}}^+_v(k)$. This implies that the new features will also be independent of the previous features in the global map. The update of the local covariance estimates will therefore be a function of the number of features in the local submap and not of the entire map.

2.3 Transforming to The Global Frame

The estimate of the pose of the local frame, ${}^G\hat{\mathbf{x}}^+_L(k)$, represents the relative transformation between the initial frame of reference, $\mathcal{F}_G$, and the current frame of reference, $\mathcal{F}_L$ with associated covariance, ${}^G\mathbf{P}^+_{LL}(k)$. The transformation matrix $\mathbf{T}(\hat{\mathbf{x}}^+_{cls}(k))$ transforms the local CLSF state estimates into the global frame of reference. This allows the vehicle and map estimates from local frame $\mathcal{F}_L$ to be transformed to the global frame of reference at any time using the estimated relationships between the frames of reference. The covariance estimate for the transformed states can be generated by also projecting through the transformation matrix

$$\begin{aligned} {}^G\hat{\mathbf{x}}^+_{cls}(k) &= \mathbf{T}(\hat{\mathbf{x}}^+_{cls}(k))\hat{\mathbf{x}}^+_{cls}(k) \quad (3) \\ {}^G\mathbf{P}^+_{cls}(k) &= \nabla\mathbf{T}(k)\mathbf{P}^+_{cls}(k)\nabla\mathbf{T}^T(k) \quad (4) \end{aligned}$$

2.4 Constraining the Independent Feature Estimates

In the CLSF approach to SLAM, an independent estimate of the vehicle state is used while the vehicle is operating with respect to the local frame of reference. This implies that the global estimate of a landmark state cannot be used for observations arising in the local frame of reference without introducing correlation between the states. Information about the landmark states may therefore be distributed between the global and local frames of reference. This may result in multiple estimates associated with a single feature once the local map feature estimates are transformed to the global frame. These independent estimates of the landmark location must be fused together to recover all of the information available to the filter.

At any time, consistent estimates of the feature states can be recovered by applying constraints to recover the known relationship between common states. The constraint is used to fuse the independent estimates of the feature state to produce a single, consistent estimate of the state by enforcing the known relationship between the common states. Constraints are applied to ensure that the estimates are consistent and to recover all of the information available to the filter. The constraint operation can be considered a weighted projection of the estimates onto the space spanned by the constraints [11]. The weighting factors are functions of the variance of the prior estimates.

Given an estimate of the position of landmark i in the global frame, ${}^G\hat{\mathbf{x}}^+_i(k)$, an estimate of the pose of the local frame with respect to the global frame, ${}^G\hat{\mathbf{x}}^+_L(k)$, and an estimate of the landmark position in the local frame, ${}^L\hat{\mathbf{x}}^+_i(k)$, the following constraint must hold

$${}^G\hat{\mathbf{x}}^+_i(k) - ({}^G\hat{\mathbf{x}}^+_L(k) \oplus {}^L\hat{\mathbf{x}}^+_i(k)) = 0. \tag{5}$$

where, following the compact notation given in [12], $\oplus$ denotes a compounding operator used to calculate the resultant relationship from addition between different frames of reference. Note that these transformations may involve rotations between the frames of reference and are therefore not equivalent to simple vector addition.

3 Computational Complexity

The computational savings that can be realised using this method arise during the update step of an observation. Assume that there are n_f map features. Furthermore, assume that there are n_G features in the global map and n_L features in the local map in which the vehicle is currently operating with $n_L \ll n_G$. Some of the states may be estimates in both the global and local maps such that $n_L + n_G \geq n_f$.

With each observation in the global SLAM case the full covariance update matrix must be calculated. This step requires at best $O(n^2)$ operations to compute the matrix update. For the CLSF, however, the update requires only $O(n_L^2)$ operations - a considerable saving for each individual observation. The computationally intensive step in the filter of updating the full covariance matrix is deferred until the constraints are applied.

During the period between the application of constraints there will be some number of observations that have been fused into the filter. These n_{obs} observations will have resulted in update calculations of $O(n_{obs} \times n^2)$ for the global SLAM case. When the decision is made to apply the constraints to the CLSF map to fuse the local estimates into the global map there will be some n_C common states between the global and local map estimates. The constraints can either be applied in a single step or individually. Applying the constraints in a single step requires the inversion of the constraint covariance matrix, $\mathbf{S}_c(k)$. This leads to a complexity of $O(n_C^2 \times n^2)$. However, since the constraints themselves are assumed independent, they can be applied sequentially yielding an update complexity of $O(n_C \times n^2)$. In general, given the nature of the SLAM problem the number of constraints, n_C, will be significantly less than the number of observations, n_{obs}, taken between applications of the constraints, resulting in a much smaller computational cost over the period between the application of constraints.

The vehicle is physically constrained to move within its environment and so will only observe those features currently within range of its sensors. The rate at which sensor readings are taken is typically much higher than the rate at which new features will be observed as the vehicle moves through its environment. The computationally intensive update of the global map is therefore not dependent on the number of times a particular feature is observed but instead is dependent on the number of common features between the maps. This will largely be a function of the rate with which new features are observed and thus will be somewhat application dependent. Additionally, it is possible to further manage the computational effort of the constraint application by selective application of the constraints. By applying only a small subset of the available constraints, the computational effort can be further reduced.

4 Data Association

The CLSF representation of the map presents a mechanism for generating consistent, high accuracy feature sets. Data association is simplified by maintaining an accurate local map of the features surrounding the vehicle. The local map of the environment generated using this approach is then fused periodically into the global map. This approach simplifies the data association problem in two significant ways. Firstly, when a new observation is received, it must only be matched against the limited number of features in the local submap. This can lead to significant computational savings given that the new estimate does not need to be compared against every estimate in the global map. Secondly, when the local map is fused into the global map a more robust association can be performed between the two feature sets. This allows the distribution of features in the environment to be taken into account when performing the association. Using multiple features to establish correspondence can aid in the data association problem by allowing a more informed association to be established between the local and global map features.

5 Results

This section presents results of the application of the CLSF techniques to the SLAM problem.

5.1 Simulation

This section presents simulation results of the application of the CLSF techniques to the SLAM problem. The landmarks are randomly distributed throughout the environment and the vehicle takes noisy range/bearing observation using an on-board sensor. The vehicle trajectory is approximately circular.

Figure 3 shows a comparison between the error in the vehicle estimate for the global SLAM case versus the error committed by the CLSF along with the 2σ confidence bounds for both cases. In this instance, the constraint application is scheduled to happen at fixed intervals. As can be seen, the global covariance of the CLSF vehicle estimate increases when the vehicle is operating relative to the local submap. When the constraints are applied, however, the covariance estimate generated is identical to the covariance generated by the global SLAM algorithm using the same series of observations.

Figure 4 compares the floating point operations required by each algorithm to build and maintain the map. It is clear that for the case of the global SLAM algorithm, the computational burden rises quadratically until all of the observable features have been incorporated into the map. The complexity of each update is then maintained. For the CLSF, on the other hand, each observation incurs only a small computational cost associated with the update of the local submap estimates. The application of the constraint, however, requires a computationally intensive update of whole the map. By proper management of the update, however, this approach can yield considerable computational savings when compared with the global approach, as can be seen in Figure 4 (c).

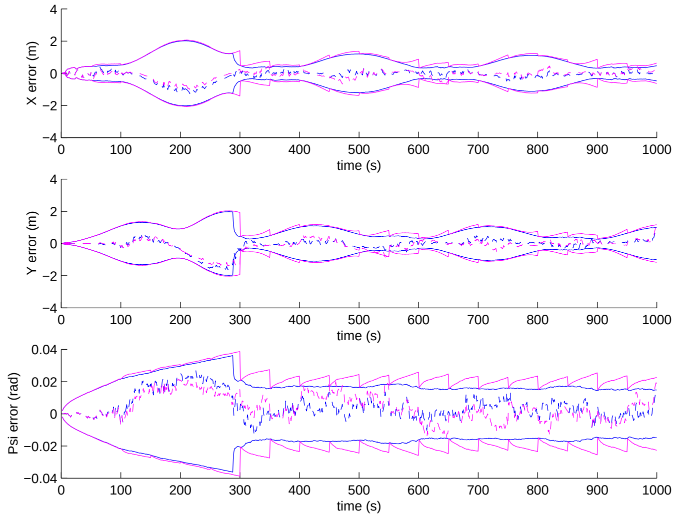
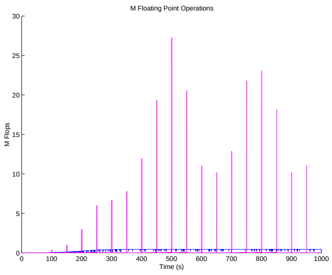
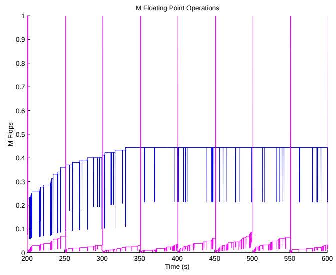


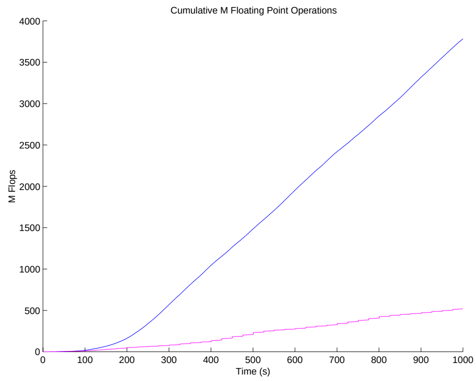
Fig. 3. The vehicle estimate errors with the 2σ covariance bounds. The global SLAM covariance estimates (dark) are shown together with the CLSF case (light). The global vehicle uncertainty grows for the CLSF case between applications of the constraints but the full covariance estimate is recovered when constraints are applied.



(a) MFlops



(b) Close-up of MFlops



(c) Sum of MFlops

Fig. 4. The floating point operations required for the prediction and update stages of the filters. The CLSF (light) has significantly less computational burden while operating in the local submap with a large burden imposed when constraints are applied. This can be significantly reduced by selective application of constraints.

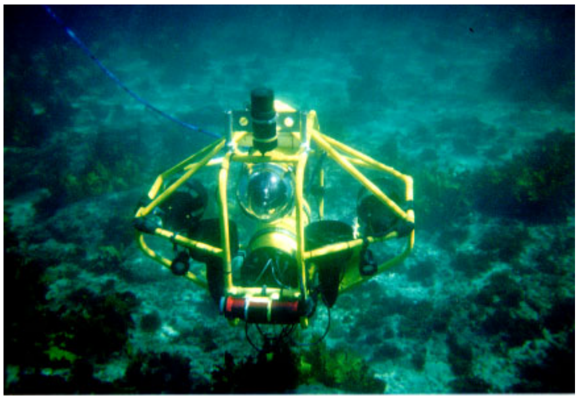


Fig. 5. Oberon at Sea

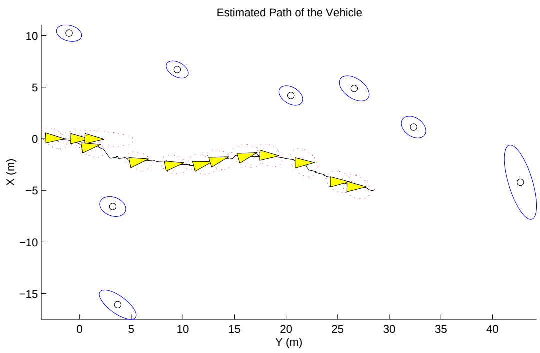
5.2 Subsea Deployment

The experimental platform used for the work reported in this paper is a mid-size submersible robotic vehicle called Oberon designed and built at the Australian Centre for Field Robotics (see Figure 5). This vehicle is used to demonstrate the methods and algorithms proposed herein. The vehicle is equipped with two scanning low frequency terrain-aiding sonars and a colour CCD camera, together with bathyometric depth sensors, a fiber optic gyroscope and a magneto-inductive compass with integrated 2-axis tilt sensor [15].

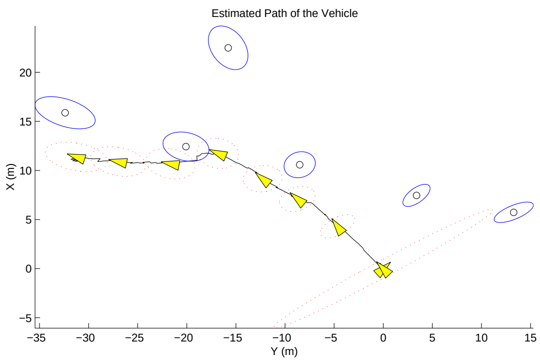
The SLAM algorithms have been tested in a natural environment off the coast of Sydney, Australia. The submersible was deployed in a natural inlet with the sonar targets positioned in a straight line at intervals of 10m [15]. The vehicle controls were set to maintain a constant heading and altitude during the run. Once the vehicle had reached the end of its tether (approximately $50m$) it was turned around and returned along the line of targets. The slope of the inlet in which the vehicle was deployed meant that the depth of the vehicle varied between approximately $1m$ and $5m$ over the course of the run.

The data collected by the vehicle during these trials is presented here to illustrate some of the properties of the Constrained Local Submap Filter in a real world setting. In this instance, the outward journey is used as the original, global map. When the vehicle is turned around to return along the line of sonar targets, a new map is initialised and a new local map is generated. This local map is relative to the final position of the vehicle in the outward leg of the run. When the vehicle reaches the end of its journey, the local map is transformed to the global frame of reference, associations between the feature estimates in the two maps are established and the final map of the environment is generated using the constrained map estimates. Figure 6 shows the two maps generated by this approach.There is a clear one-to-one mapping between the global map estimates and the local estimates transformed to the global frame of reference. This makes the data association problem relatively straightforward.

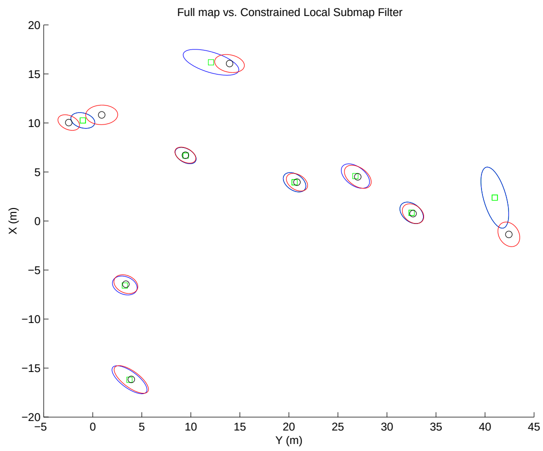
Finally, Figure 6 (c) shows the resulting constrained map. It is plotted relative to the map generated by the



(a) The first leg global map



(b) The second leg local map



(c) The fused maps

Fig. 6. Paths of the robot against the final map of the environment shown for (a) the outward leg, considered the global map, and (b) the return leg, considered the local map. (c) The fused maps of the environment generated by applying consistency constraints to the estimated features. The map is plotted on top of the original map generated by the global SLAM algorithm. There is a very good match between the two maps, as should be expected.

global algorithm. As would be expected, there is a good correspondence between the two maps.

6 Conclusions

This paper has presented the CLSF. This novel approach to the construction of the SLAM map generates a local map of the features in the immediate vicinity of the vehicle. This local map is then periodically fused into the global map to recover the full global map estimate. This approach has been shown to perform very well in a simulated environment. Simulation allows the performance of the filter to be checked to verify that it is, in fact, generating consistent estimates of the map and vehicle states. It is clear from the results presented that the approach yields nearly identical results to the global SLAM algorithm despite the fact that the entire global map covariance matrix is not updated with each observation. The approach has also been demonstrated to perform well using data collected using an underwater vehicle equipped with scanning sonar.

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