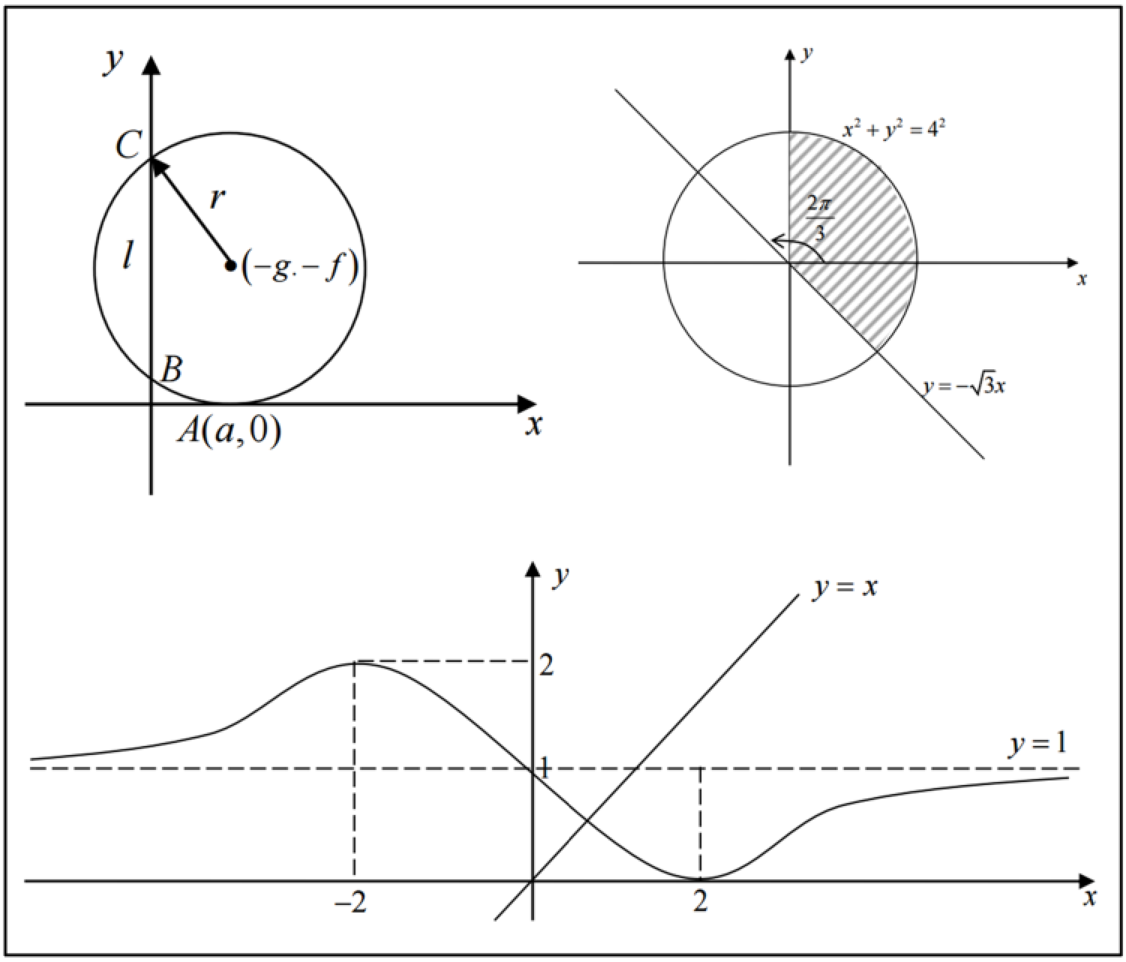
PILOT EXAMINATION FOR G.C.E A/L STUDENTS - 2017

CONDUCTED BY TAMIL STUDENTS OF FACULTY OF ENGINEERING
UNIVERSITY OF MORATUWA

10(I) - COMBINED MATHEMATICS I

ANSWERS



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Part - A

Q1). Let $f(n) = 7^n - 2^n$

When $n = 1$,

$f(1) = 7^1 - 2^1 = 5 = 5\times 1$

Therefore the above statement is true when $n = 1$

Assume that the above statement is true when $n = p \in \mathbb{Z}^+$

i.e. $f(p) = 7^p - 2^p = 5k \qquad (k \in \mathbb{Z}^+)$

When $n = p+1$

$f(p+1) = 7^{p+1} - 2^{p+1}$

$f(p+1) = 7.7^p - 2.2^p$

$f(p+1) = 7.(7^p - 2^p) + 5.2^p$

$f(p+1) = 7.5k + 5.2^p$

$f(p+1) = 5.(7k + 2^p) = 5m \qquad (m \in \mathbb{Z}^+)$

Therefore the above statement is true when $n = p+1$

Hence by principle of mathematical induction the above statement is true for all $n \in \mathbb{Z}^+$

Q2). $B-1,\ A-3,\ N-2$

The no. of arrangements that can be formed $= \dfrac{6!}{2!\times 3!} = 60$

Considering the two Ns as one, the no. of arrangements that can be formed $= \dfrac{5!}{3!}\times 2! = 40$

$\therefore$ The no. of arrangements that doesn't have two **N** s next to each other $= 60 - 40 = 20$

Q3). $\left(\sqrt{3}+i\right)\left(a+i\right) = 2\left(a-i\right)$

$\left(\sqrt{3}a-1\right) + i\left(a+\sqrt{3}\right) = 2a - 2i$

$\left(\sqrt{3}a-1\right) = 2a,\ i\left(a+\sqrt{3}\right) = -2i$

$a = \dfrac{1}{\sqrt{3}-2} = -\left(2+\sqrt{3}\right)$

$\left(\sqrt{3}+i\right)\left(-\left(2+\sqrt{3}\right)+i\right) = 2\left(-\left(2+\sqrt{3}\right)-i\right)$

$$\frac{\left(-\left(2+\sqrt{3}\right)+i\right)}{\left(-\left(2+\sqrt{3}\right)-i\right)} = \frac{\left(2+\sqrt{3}-i\right)}{\left(2+\sqrt{3}+i\right)} = \frac{2}{\left(\sqrt{3}+i\right)}$$

$$\frac{\left(2+\sqrt{3}-i\right)}{\left(2+\sqrt{3}+i\right)} = \frac{\left(\sqrt{3}-i\right)}{2} = 1.\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$$

$\therefore$ Modulus $= 1$, $Arg\ z = -\dfrac{\pi}{6}$

Q4). $\lim\limits_{x\to 1}\dfrac{x^3-x^2\ln x+\ln x-1}{x^2-1}$

$$=\lim_{x\to 1}\frac{(x-1)(x^2+x-x\ln x-\ln x+1)}{(x-1)(x+1)}$$

$$=\lim_{x\to 1}\frac{(x^2+x-x\ln x-\ln x+1)}{(x+1)}$$

$$=\frac{3}{2}$$

Q5). $\left(x-1\right)^n={}^nC_0x^n(-1)^0+{}^nC_1x^{n-1}(-1)^1+{}^nC_2x^{n-2}(-1)^2+........+{}^nC_rx^{n-r}(-1)^r+......+{}^nC_nx^0(-1)^n$

Substituting for $x=17,\ n=500$

$$\left(17-1\right)^{500}={}^{500}C_0 17^{500}(-1)^0+{}^{500}C_1 17^{499}(-1)^1+{}^{500}C_2 17^{498}(-1)^2+...........+{}^{500}C_r 17^{500-r}(-1)^r+...........$$
$$.......+{}^{500}C_{500}17^0(-1)^{500}$$

$$\left(16\right)^{500}=17.\left({}^{500}C_0 17^{499}(-1)^0+{}^{500}C_1 17^{498}(-1)^1+{}^{500}C_2 17^{497}(-1)^2+...........+{}^{500}C_r 17^{499-r}(-1)^r+..........\right.$$
$$\left.........+{}^{500}C_{499}17^0(-1)^{499}\right)+1$$

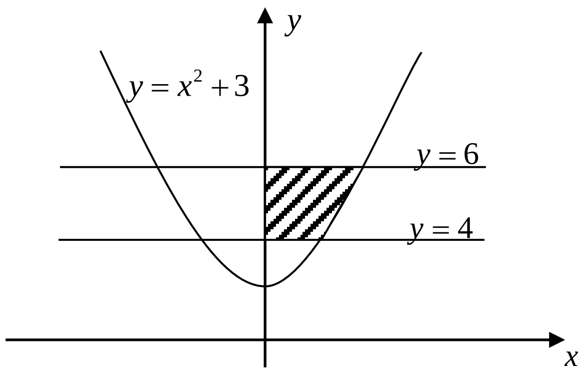
$$2^{2000}=17.\left({}^{500}C_0 17^{499}(-1)^0+{}^{500}C_1 17^{498}(-1)^1+{}^{500}C_2 17^{497}(-1)^2+.............+{}^{500}C_r 17^{499-r}(-1)^r+..........\right.$$
$$\left.........+{}^{500}C_{499}17^0(-1)^{499}\right)+1$$

Multiplying both side by 2^3 ,

$$2^{2003}=17.\left(8.\left({}^{500}C_0 17^{499}(-1)^0+{}^{500}C_1 17^{498}(-1)^1+{}^{500}C_2 17^{497}(-1)^2+...........+{}^{500}C_r 17^{499-r}(-1)^r+..........\right.\right.$$
$$\left.\left.........+{}^{500}C_{499}17^0(-1)^{499}\right)\right)+8$$

$\therefore$ The remainder when 2^{2003} is divided by $17=8$

Q6).



$$A=\int_4^6\sqrt{y-3}\,dy$$

$$=\left[\frac{\left(y-3\right)^{3/2}}{3/2}\right]_4^6$$

$$=\frac{2}{3}\left(3\sqrt{3}-1\right)$$ square units .

Q7). The point at which the curve $y = be^{-\frac{x}{a}}$ cuts y -axis $= (0, b)$

Gradient of the curve at that point $= \left.\dfrac{dy}{dx}\right|_{(0,b)}$

$$= -\frac{b}{a}e^{-\frac{x}{a}}\Big|_{(0,b)}$$

$$= -\frac{b}{a}$$

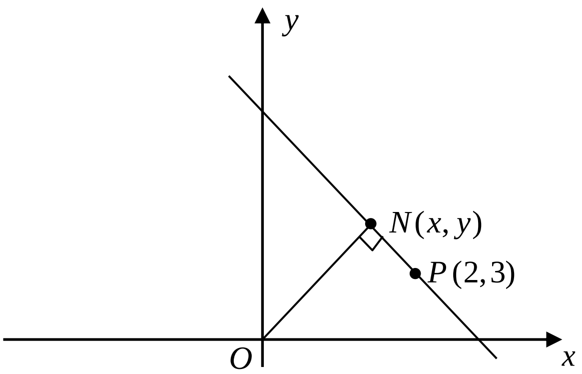
The equation of the curve drawn at that point $\Rightarrow$

$$y - b = -\frac{b}{a}(x - 0)$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$\therefore$ The line $\frac{x}{a} + \frac{y}{b} = 1$ touches the curve $y = be^{-\frac{x}{a}}$ at the point $(0, b)$.

Q8).



Gradient of $ON = \dfrac{y}{x}$

Gradient of $PN = \dfrac{y-3}{x-2}$

$ON \perp PN \Rightarrow \dfrac{y}{x} \times \dfrac{y-3}{x-2} = -1$

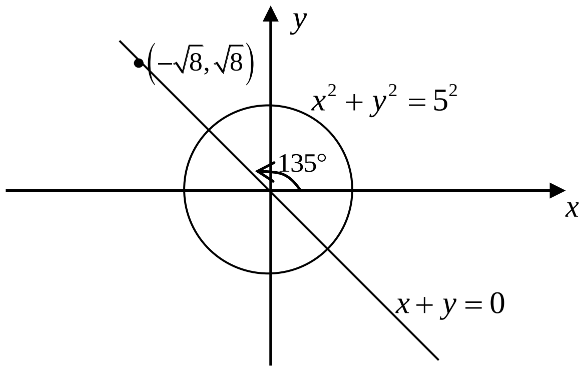
$$y(y-3) + x(x-2) = 0$$

$$(x-1)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{13}{4}$$

$\therefore$ The locus of N is a circle.

Center $= \left(1, \dfrac{3}{2}\right)$ Radius $= \dfrac{\sqrt{13}}{2}$

Q9). Equation of straight line $\Rightarrow$



$$y-\sqrt{8}=\tan(135°)\left(x-\left(-\sqrt{8}\right)\right)$$

$$y-\sqrt{8}=-\left(x+\sqrt{8}\right)$$

$$x+y=0$$

It is a line going through origin.

Equation of circle $\Rightarrow$

$$x^2+y^2=\left(5\cos\theta\right)^2+\left(5\sin\theta\right)^2$$

$$x^2+y^2=5^2$$

Center $=(0,0)$ Radius $=5$

$\therefore$ The circle and the straight line intersect each other.

The intersecting chord is the diameter of the circle $\Rightarrow$ length of the chord $=10$

Q10). Let $\alpha=\tan^{-1}(-3)$, $\beta=\cos^{-1}\left(\frac{4}{5}\right)$.

Here $-\frac{\pi}{2}<\alpha<-\frac{\pi}{4}, 0<\beta<\frac{\pi}{2}$.

$\therefore -\pi<2\alpha<-\frac{\pi}{2}, -\pi<\beta-\pi<-\frac{\pi}{2}$ ....(1)

$$\cos(2\alpha)=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}=\frac{1-(-3)^2}{1+(-3)^2}=-\frac{4}{5}$$

$$\cos(\beta-\pi)=\cos(\pi-\beta)=-\cos(\beta)=-\frac{4}{5}$$

$\therefore \cos(2\alpha)=\cos(\beta-\pi)$

From (1), $2\alpha=\beta-\pi$

$$\Rightarrow 2\tan^{-1}(-3)=\cos^{-1}\left(\frac{4}{5}\right)-\pi$$

Part – B

Q11). (*a*) (*i*) $f(x)=x^2+x+1=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

$\therefore\ f(x)>0$ for all x.

(*ii*) $g(x)=4x^2+(m+3)x+4$

For $g(x)>0$, $a>0 \,\&\, b^2-4ac<0 \Rightarrow$

$4>0\ \&\ (m+3)^2-4\times4\times4<0$

$(m+11)(m-5)<0$

$-11<m<5$

(*iii*) $h(x)=2x^2+(3-m)x+2$

For $h(x)>0$, $a>0 \,\&\, b^2-4ac<0 \Rightarrow$

$2>0\ \&\ (3-m)^2-4\times2\times2<0$

$(m+1)(m-7)<0$

$-1<m<7$

$$-3<\frac{x^2+mx+1}{x^2+x+1}<3$$

$\Leftrightarrow -3(x^2+x+1)<x^2+mx+1<3(x^2+x+1) \quad (\because x^2+x+1>0)$

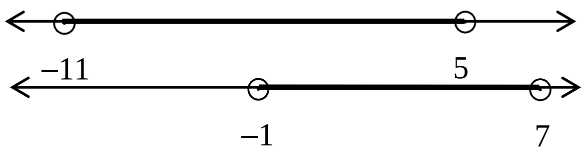
$\Leftrightarrow -3(x^2+x+1)<x^2+mx+1\ \&\ x^2+mx+1<3(x^2+x+1)$

$\Leftrightarrow 0<4x^2+(m+3)x+4\ \ \&\ 0<2x^2+(3-m)x+2$

$\Leftrightarrow g(x)>0\ \&\ h(x)>0$

$\Leftrightarrow -11<m<5\ \&\ -1<m<7$

$\Leftrightarrow -1<m<5$



(*b*) $f(x)=x^4+2x^3-3x^2-2x+3$

According to remainder theorem,

$f(x)=(x-2)\phi(x)+R$

$f(2)=R=16+16-12-4+3=19$

$\phi(x)=(x-2)\phi_1(x)+R_1$

$\phi_1(x)=(x-3)\phi_2(x)+R_2$

$f(x)=(x-2)\left((x-2)\phi_1(x)+R_1\right)+R$..............................(1)

$f(x)=(x-2)\left((x-2)\left((x-3)\phi_2(x)+R_2\right)+R_1\right)+R$

$f(x)=(x-2)^2(x-3)\phi_2(x)+(x-2)^2R_2+(x-2)R_1+R$ $\quad\therefore$ The remainder take that form

$(1)\Rightarrow f(x)=(x-2)^2\phi_1(x)+R_1(x-2)+R$

$f'(x)=(x-2)^2\phi_1'(x)+2(x-2)\phi_1(x)+R_1$

$f'(x)=4x^3+6x^2-6x-2$

$f'(2) = R_1 = 32 + 24 - 12 - 2 = 42$

$f(x) = (x-2)^2 (x-3)\phi_2(x) + (x-2)^2 R_2 + 42(x-2) + 19$

$f(3) = R_2 + 42 + 19 = 81 + 54 - 27 - 6 + 3$

$R_2 = 44$

$\therefore$ Remainder $= 44(x-2)^2 + 42(x-2) + 19$

$\therefore\ a = 44,\ b = 42,\ c = 19$

**Q12). ** (a) $\dfrac{1}{1+a^{n-1}} - \dfrac{1}{1+a^n} = \dfrac{a^n - a^{n-1}}{\left(1+a^{n-1}\right)\left(1+a^n\right)} = \dfrac{a^{n-1}(a-1)}{\left(1+a^{n-1}\right)\left(1+a^n\right)}$

$$\frac{a^{r-1}}{\left(1+a^{r-1}\right)\left(1+a^r\right)} = \left(\frac{1}{a-1}\right)\left(\frac{1}{1+a^{r-1}} - \frac{1}{1+a^r}\right) = \frac{1}{(a-1)\left(1+a^{r-1}\right)} - \frac{1}{(a-1)\left(1+a^r\right)}$$

$$\frac{a^{r-1}}{\left(1+a^{r-1}\right)\left(1+a^r\right)} = f(r-1) - f(r)$$

$$\therefore\ f(r) = \frac{1}{(a-1)\left(1+a^r\right)}$$

$$U_r = \frac{a^{r-1}}{\left(1+a^{r-1}\right)\left(1+a^r\right)} = f(r-1) - f(r)$$

$U_1 = f(0) - \cancel{f(1)}$

$U_2 = \cancel{f(1)} - \cancel{f(2)}$

$U_3 = \cancel{f(2)} - \cancel{f(3)}$

.....

$U_{n-1} = \cancel{f(n-2)} - \cancel{f(n-1)}$

$U_n = \cancel{f(n-1)} - f(n)$

$$\sum_{r=1}^{n} U_r = f(0) - f(n) = \frac{1}{2(a-1)} - \frac{1}{(a-1)\left(1+a^n\right)} = \frac{a^n - 1}{2(a-1)\left(a^n+1\right)}$$

$$a = 2 \Rightarrow \sum_{r=1}^{n} \frac{2^{r-1}}{\left(1+2^{r-1}\right)\left(1+2^r\right)} = \frac{2^n - 1}{2\left(2^n+1\right)} = \frac{1}{2} - \frac{1}{\left(2^n+1\right)}$$

$$\sum_{r=1}^{n} \frac{2^r}{\left(1+2^{r-1}\right)\left(1+2^r\right)} = 1 - \frac{2}{\left(2^n+1\right)}$$

For any $n \in \mathbb{Z}^+,\ 0 < \dfrac{2}{\left(2^n+1\right)} < 1$.

$$\Rightarrow 0 < 1 - \frac{2}{\left(2^n+1\right)} < 1$$

$$\therefore 0 < \sum_{r=1}^{n} \frac{2^r}{\left(1+2^{r-1}\right)\left(1+2^r\right)} < 1$$

$$a = 2017 \Rightarrow \sum_{r=1}^{n} \frac{2017^{r-1}}{\left(1+2017^{r-1}\right)\left(1+2017^{r}\right)} = \frac{2017^{n}-1}{2\times 2016\times\left(2017^{n}+1\right)}$$

$$\sum_{r=1}^{n} \frac{2017^{r}}{\left(1+2017^{r-1}\right)\left(1+2017^{r}\right)} = \frac{2017}{4032}\left(\frac{2017^{n}-1}{2017^{n}+1}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n} \frac{2017^{r}}{\left(1+2017^{r-1}\right)\left(1+2017^{r}\right)} = \frac{2017}{4032}\lim_{n\to\infty}\left(\frac{2017^{n}-1}{2017^{n}+1}\right) = \frac{2017}{4032}\lim_{n\to\infty}\left(\frac{1-{}^{1}\!/\!{}_{2017^{n}}}{1+{}^{1}\!/\!{}_{2017^{n}}}\right) = \frac{2017}{4032}$$

(*b*)

$y = \left|x^2 - 2x\right|$

$y = x(x-2) = (x-1)^2 - 1;\ x \le 0 \text{ or } x \ge 2$

$y = 1-(x-1)^2;\ 0 < x < 2$

$y = \left|1-2x\right|$

$y = 2x-1;\ x \ge \frac{1}{2}$

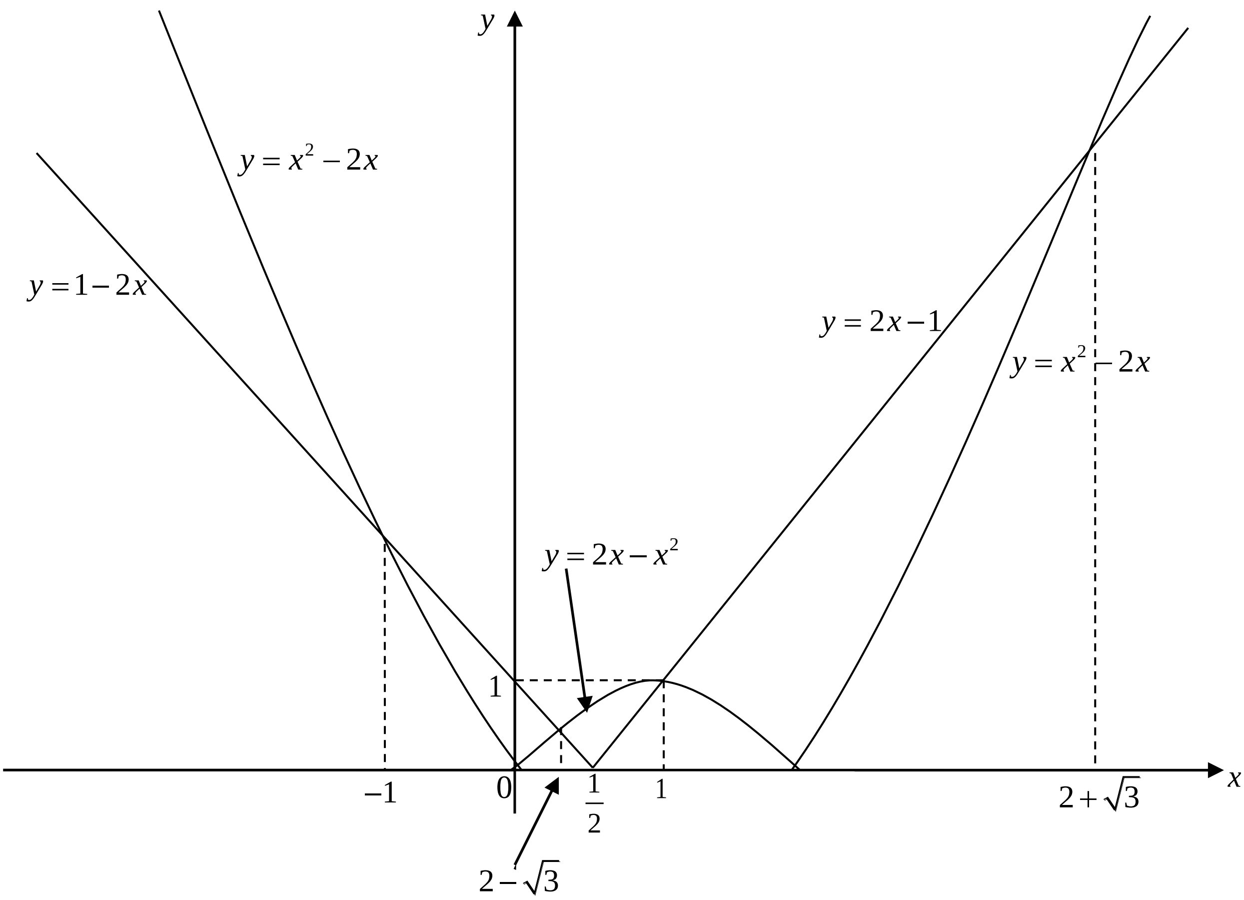
$y = 1-2x;\ x < \frac{1}{2}$

$x = 0 \Rightarrow y = 0$

$y = 0 \Rightarrow x = 0, 2$

Symmetrical axis $\Rightarrow x = 1$

$x \to \pm\infty \Rightarrow y \to \pm\infty$



Intersection points of the curves

$x^2-2x=1-2x$	$2x-x^2=1-2x$	$2x-x^2=2x-1$	$x^2-2x=2x-1$
$x=\pm 1$	$x=2\pm\sqrt{3}$	$x=\pm 1$	$x=2\pm\sqrt{3}$
$\Rightarrow x=-1$	$\Rightarrow x=2-\sqrt{3}$	$\Rightarrow x=1$	$\Rightarrow x=2+\sqrt{3}$

$$\left|x^2-2x\right|\le\left|1-2x\right|$$

The region where the curve $y=\left|1-2x\right|$ lies above $y=\left|x^2-2x\right|$

Solution set $=\left\{x\in\mathbb{R}\,\middle|\,-1\le x\le 2-\sqrt{3}\right\}\cup\left\{x\in\mathbb{R}\,\middle|\,1\le x\le 2+\sqrt{3}\right\}$

Q13). (a) $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}.\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}=\frac{1}{ad-bc}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$=\frac{1}{ad-bc}\begin{pmatrix} ad-bc & 0 \\ 0 & ad-bc \end{pmatrix}$$

$$=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\therefore\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1}=\frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

For existence of an inverse for a 2×2 matrix, $ad-bc\neq 0$

$$\mathbf{A}=\begin{pmatrix} 3 & 7 \\ 1 & 2 \end{pmatrix},\mathbf{B}=\begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}$$

$$\mathbf{AB}=\begin{pmatrix} 3 & 7 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}=\begin{pmatrix} -9 & 4 \\ -2 & 1 \end{pmatrix}$$

$$\left(\mathbf{AB}\right)^{-1}=\frac{1}{(-9+8)}\begin{pmatrix} 1 & -4 \\ 2 & -9 \end{pmatrix}=\begin{pmatrix} -1 & 4 \\ -2 & 9 \end{pmatrix}$$

$$\mathbf{A}^{-1}=\frac{1}{(6-7)}\begin{pmatrix} 2 & -7 \\ -1 & 3 \end{pmatrix}=\begin{pmatrix} -2 & 7 \\ 1 & -3 \end{pmatrix}$$

$$\mathbf{B}^{-1}=\frac{1}{(4-3)}\begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}=\begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}$$

(i) $\mathbf{A}^{-1}\mathbf{B}^{-1}=\begin{pmatrix} -2 & 7 \\ 1 & -3 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}=\begin{pmatrix} 19 & 26 \\ -8 & -11 \end{pmatrix}\Rightarrow\left(\mathbf{AB}\right)^{-1}\neq\mathbf{A}^{-1}\mathbf{B}^{-1}$

(ii) $\mathbf{B}^{-1}\mathbf{A}^{-1}=\begin{pmatrix} 1 & 1 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -2 & 7 \\ 1 & -3 \end{pmatrix}=\begin{pmatrix} -1 & 4 \\ -2 & 9 \end{pmatrix}\Rightarrow\left(\mathbf{AB}\right)^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$

(b) $A=\left\{z\in\mathbb{C}:|z|\le 4\right\}\cap\left\{z\in\mathbb{C}:\mathrm{Im}\left(\dfrac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right)\ge 0\right\}\cap\left\{z\in\mathbb{C}:\mathrm{Re}(z)\ge 0\right\}$

Let $z=x+iy$.

$|z|\le 4$

$\sqrt{x^2+y^2}\le 4$

$x^2+y^2\le 4^2$..................................(1)

$\mathrm{Im}\left(\dfrac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right)\ge 0$

$\mathrm{Im}\left(\dfrac{x+iy-1+\sqrt{3}i}{1-\sqrt{3}i}\right)\ge 0$

$\mathrm{Im}\left(\dfrac{x-\sqrt{3}y-4+i\left(y+\sqrt{3}x\right)}{4}\right)$

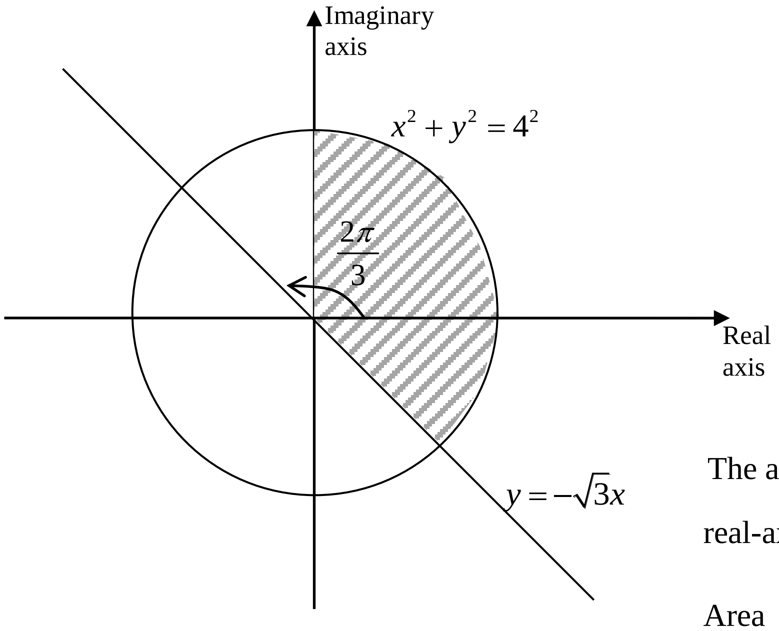
$\dfrac{\left(y+\sqrt{3}x\right)}{4}\ge 0$

$y\ge-\sqrt{3}x$..(2)

$\mathrm{Re}(z)\ge 0$

$x\ge 0$.............................(3)

$A=(1)\cap(2)\cap(3)$



The angle made by $y=-\sqrt{3}x$ with positive real-axis $=\pi-\tan^{-1}\left(\sqrt{3}\right)=\dfrac{2\pi}{3}$

Area $=\dfrac{1}{2}\times 4^2\times\dfrac{5\pi}{6}$

$=\dfrac{20\pi}{3}$

(c) Let $z = x + iy$.

$|z|^2 = x^2 + y^2$

$\overline{z} = x - iy$

$z.\overline{z} = (x+iy)(x-iy) = x^2 + y^2 = |z|^2$

$z + \overline{z} = (x+iy) + (x-iy) = 2x = 2\operatorname{Re} z$

$$|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)}$$
$$= (z_1 + z_2)\left(\overline{z_1} + \overline{z_2}\right)$$
$$= z_1\overline{z_1} + z_2\overline{z_2} + z_1\overline{z_2} + z_2\overline{z_1}$$
$$= |z_1|^2 + |z_2|^2 + z_1\overline{z_2} + \overline{z_1\overline{z_2}}$$
$$= |z_1|^2 + |z_2|^2 + 2\operatorname{Re}\left(z_1\overline{z_2}\right)$$

$$|z_1 - z_2|^2 = |z_1|^2 + |-z_2|^2 + 2\operatorname{Re}\left(z_1\overline{(-z_2)}\right)$$
$$= |z_1|^2 + |z_2|^2 - 2\operatorname{Re}\left(z_1\overline{z_2}\right)$$

$$\left|\frac{(z_1 - z_2)}{(z_1 + z_2)}\right| = 1$$

$|z_1 - z_2| = |z_1 + z_2|$

$|z_1 - z_2|^2 = |z_1 + z_2|^2$

$|z_1|^2 + |z_2|^2 + 2\operatorname{Re}\left(z_1\overline{z_2}\right) = |z_1|^2 + |z_2|^2 - 2\operatorname{Re}\left(z_1\overline{z_2}\right)$

$\operatorname{Re}\left(z_1\overline{z_2}\right) = 0$

$$\operatorname{Re}\left(\frac{z_1}{z_2}\right) = \operatorname{Re}\left(\frac{z_1\overline{z_2}}{z_2\overline{z_2}}\right) = \operatorname{Re}\left(\frac{z_1\overline{z_2}}{|z_2|^2}\right) = \frac{1}{|z_2|^2}\operatorname{Re}\left(z_1 z_2\right) = 0$$

$\therefore \dfrac{z_1}{z_2}$ is purely imaginary.

Q14). (a) $y = \dfrac{(x-2)^2}{x^2+4}$

$\left(x^2 + 4\right)y = x^2 - 4x + 4$

$(y-1)x^2 + 4x + 4y - 4 = 0$

$x \in \mathbb{R} \Rightarrow\ b^2 - 4ac \geq 0$

$4^2 - 4(y-1)(4y-4) \geq 0$

$y(y-2) \leq 0$

$0 \leq y \leq 2$

$$\frac{dy}{dx} = \frac{\left(x^2+4\right)2(x-2) - (x-2)^2 2x}{\left(x^2+4\right)^2} = \frac{4(x-2)(x+2)}{\left(x^2+4\right)^2}$$

$\dfrac{dy}{dx} = 0 \Rightarrow x = 2, -2$

Turning points : $(2,0), (-2,2)$

$+$	$-$	$+$	$\frac{dy}{dx}$
$-$	$-$	$+$	$x-2$
$-$	$+$	$+$	$x+2$
$-\infty < x < -2$	$-2 < x < 2$	$2 < x < \infty$	

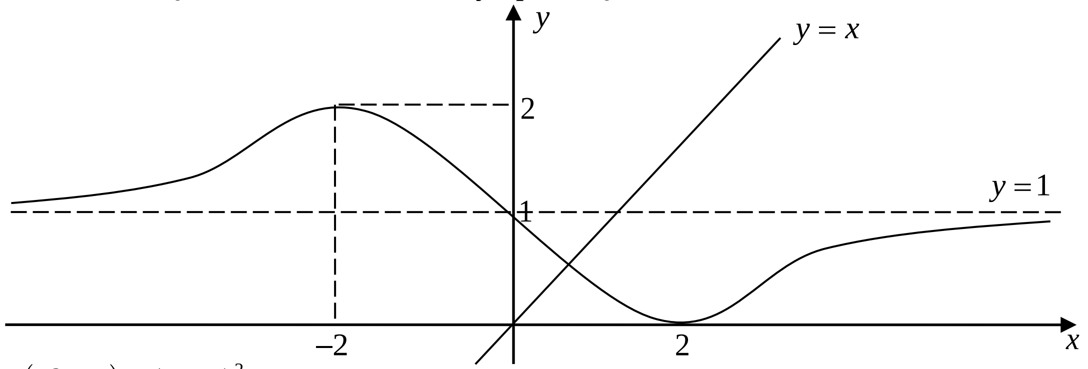
$\therefore (-2,2)$ is maximum point and $(2,0)$ is minimum point.

$x = 0 \Rightarrow y = 1,\ y = 0 \Rightarrow x = 2$

Points of intersection with the axes : $(0,1), (2,0)$

$$y = \frac{(x-2)^2}{x^2+4} = \frac{x^2-4x+4}{x^2+4} = \frac{1 - \frac{4}{x} + \frac{4}{x^2}}{1 + \frac{4}{x^2}}$$

As $x \to \pm\infty$, $y \to 1 \Rightarrow$ Horizontal asymptote : $y = 1$



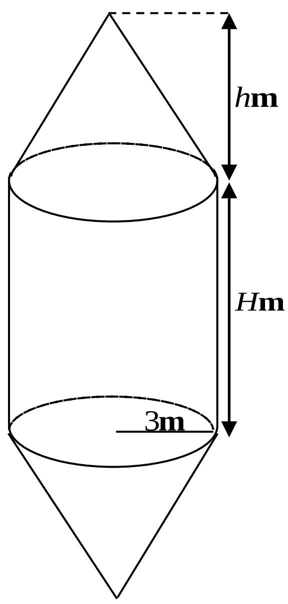
$x(x^2+4) = (x-2)^2$

$$x = \frac{(x-2)^2}{(x^2+4)} = y$$

The line $y = x$ cuts the curve at only one point.

$\therefore x(x^2+4) = (x-2)^2$ has only one real solution.

(*b*) $V = 2 \times \dfrac{1}{3}\pi R^2 h + \pi R^2 H$



$900 = 2 \times \dfrac{1}{3}\pi \times 3^2 \times h + \pi \times 3^2 \times H$

$H = \dfrac{100}{\pi} - \dfrac{2}{3}h$

$S = 2 \times \pi R l + 2\pi R H$

$S = 2\pi \times 3 \times \sqrt{h^2+3^2} + 2\pi \times 3 \times \left(\dfrac{100}{\pi} - \dfrac{2}{3}h\right)$

$S = 600 - 4\pi h + 6\pi\sqrt{9+h^2}$

$$\frac{dS}{dh}=-4\pi+6\pi\frac{h}{\sqrt{9+h^2}}$$

$$\frac{dS}{dh}=0\Rightarrow -4\pi+6\pi\frac{h}{\sqrt{9+h^2}}=0$$

$$5h^2=36$$

$$h=\frac{6}{\sqrt{5}}\ (\because h>0)$$

$$h<\frac{6}{\sqrt{5}}\Rightarrow\frac{dS}{dh}<0$$

$$h>\frac{6}{\sqrt{5}}\Rightarrow\frac{dS}{dh}>0$$

$\therefore$ Therefore S is minimum when $h=\frac{6}{\sqrt{5}}$.

Q15). (*a*) $I=\int e^{ax}\sin bx\,\mathrm{dx}$

$$=\frac{e^{ax}\sin bx}{a}-\int\frac{e^{ax}\cos\ bx.b}{a}\,\mathrm{d}x$$

$$=\frac{e^{ax}\sin bx}{a}-\frac{b}{a}\left(\frac{e^{ax}\cos bx}{a}-\int\frac{e^{ax}.-\sin\ bx.b}{a}\,\mathrm{d}x\right)$$

$$=\frac{e^{ax}\sin bx}{a}-\frac{be^{ax}\cos bx}{a^2}-\frac{b^2}{a^2}I$$

$$I=\frac{1}{a^2+b^2}\left(ae^{ax}\sin bx-be^{ax}\cos bx\right)+C$$ $C-$ Arbitrary constant

(*b*) $$\frac{11+3x-2x^2}{(x+3)(x-1)^2}=\frac{A}{(x+3)}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}$$

$$11+3x-2x^2=A(x-1)^2+B(x-1)(x+3)+C(x+3)$$

$$x=1\Rightarrow C=3$$
$$x=-3\Rightarrow A=-1$$

The co-efficient of $x^o\ \Rightarrow A-3B+3C=11\Rightarrow B=-1$

$$\frac{11+3x-2x^2}{(x+3)(x-1)^2}=\frac{-1}{(x+3)}+\frac{-1}{(x-1)}+\frac{3}{(x-1)^2}$$

$$\int\frac{11+3x-2x^2}{(x+3)(x-1)^2}dx=\int\frac{-1}{(x+3)}dx+\int\frac{-1}{(x-1)}dx+\int\frac{3}{(x-1)^2}dx$$

$$=-\ln(x+3)-\ln(x-1)-\frac{3}{(x-1)}+C$$

$C-$ Arbitrary constant

(c) $$\frac{d}{dx}\left(\frac{Ax+B}{ax^2+bx+c}\right)=\frac{1}{\left(ax^2+bx+c\right)^2}-\frac{C}{ax^2+bx+c}$$

$$\frac{\left(ax^2+bx+c\right)A-(Ax+B)(2ax+b)}{\left(ax^2+bx+c\right)^2}=\frac{1}{\left(ax^2+bx+c\right)^2}-\frac{C}{ax^2+bx+c}$$

$$\left(ax^2+bx+c\right)A-(Ax+B)(2ax+b)=1-C\left(ax^2+bx+c\right)$$

Co-efficient of $x^2 \Rightarrow -aA=-Ca \Rightarrow A=C$

Co-efficient of $x \Rightarrow -2aB=-Cb \Rightarrow bC=2aB$

Co-efficient of $x^0 \Rightarrow Ac-Bb=1-Cc \Rightarrow Cc-\frac{b^2}{2a}C=1-Cc$

$$C=\frac{2a}{4ac-b^2}\ ,\ A=\frac{2a}{4ac-b^2},\ B=\frac{b}{4ac-b^2}$$

$$a=1,\ b=4,\ c=1 \Rightarrow A=-\frac{1}{6}, B=-\frac{1}{3}, C=-\frac{1}{6}$$

$$\frac{d}{dx}\left(\frac{-\frac{1}{6}x+-\frac{1}{3}}{x^2+4x+1}\right)=\frac{1}{\left(x^2+4x+1\right)^2}+\frac{\frac{1}{6}}{x^2+4x+1}$$

$$\int_0^1\frac{d}{dx}\left(\frac{-\frac{1}{6}x+-\frac{1}{3}}{x^2+4x+1}\right)dx=\int_0^1\frac{1}{\left(x^2+4x+1\right)^2}dx+\frac{1}{6}\int_0^1\frac{1}{x^2+4x+1}dx$$

$$\int_0^1\frac{1}{\left(x^2+4x+1\right)^2}dx=\int_0^1\frac{d}{dx}\left(\frac{-\frac{1}{6}x-\frac{1}{3}}{x^2+4x+1}\right)dx-\frac{1}{6}\int_0^1\frac{1}{(x+2)^2-3}dx$$

$$\int_0^1\frac{1}{\left(x^2+4x+1\right)^2}dx=\left(\frac{-\frac{1}{6}x-\frac{1}{3}}{x^2+4x+1}\right)_0^1-\frac{1}{12\sqrt{3}}\int_0^1\frac{1}{(x+2-\sqrt{3})}-\frac{1}{(x+2+\sqrt{3})}dx$$

$$\int_0^1\frac{1}{\left(x^2+4x+1\right)^2}dx=\left(\frac{-\frac{1}{6}x-\frac{1}{3}}{x^2+4x+1}\right)_0^1-\frac{1}{12\sqrt{3}}\ln\left(\frac{x+2-\sqrt{3}}{x+2+\sqrt{3}}\right)\Bigg|_0^1$$

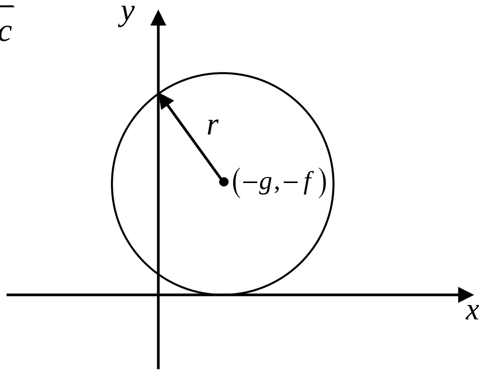
$$\int_0^1\frac{1}{\left(x^2+4x+1\right)^2}dx=\left(\frac{-\frac{1}{6}x-\frac{1}{3}}{x^2+4x+1}\right)_0^1-\frac{\sqrt{3}}{36}\ln\left(\frac{x+2-\sqrt{3}}{x+2+\sqrt{3}}\right)\Bigg|_0^1$$

$$=-\frac{1}{12}+\frac{1}{3}-\frac{\sqrt{3}}{36}\left(\ln\left(\frac{3-\sqrt{3}}{3+\sqrt{3}}\right)-\ln\left(\frac{2-\sqrt{3}}{2+\sqrt{3}}\right)\right)$$

$$=\frac{1}{3}-\frac{\sqrt{3}}{36}\ln\left(\frac{3-\sqrt{3}}{3+\sqrt{3}}\times\frac{2+\sqrt{3}}{2-\sqrt{3}}\right)=\frac{1}{4}-\frac{\sqrt{3}}{36}\ln\left(2+\sqrt{3}\right)$$

Q16). (a) $x^2+y^2+2gx+2fy+c=0$

Center $=(-g,-f)$ Radius $r=\sqrt{g^2+f^2-c}$



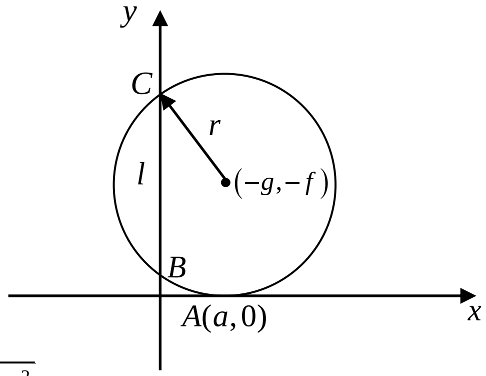
Circle touches x-axis $\Rightarrow$

$$r=|-f|$$
$$g^2+f^2-c=f^2$$
$$g^2=c$$

Circle cuts y-axis $\Rightarrow$

$$r>|-g|$$
$$g^2+f^2-c>g^2$$
$$f^2>c$$

Length of chord $=2\sqrt{r^2-(-g)^2}=2\sqrt{g^2+f^2-c-g^2}=2\sqrt{f^2-c}$



Circle touches x-axis at $(a,0)$ $\Rightarrow$

$$-g=a$$
$$c=g^2=a^2$$

Length of chord $=l\Rightarrow l=2\sqrt{f^2-c}\Rightarrow f=-\dfrac{\sqrt{l^2+a^2}}{2}$

(The circle cuts positive y-axis)

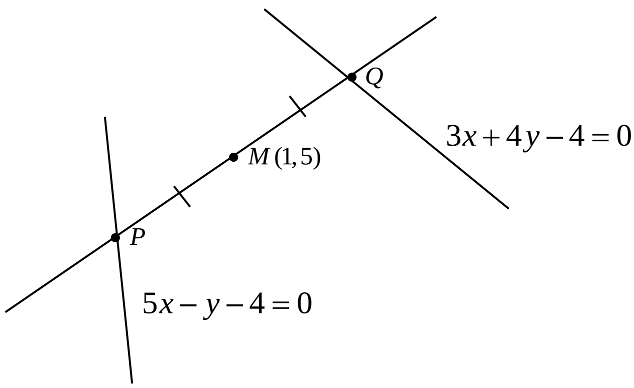
Center $=\left(a,\ \dfrac{\sqrt{l^2+a^2}}{2}\right)$ Radius $r=\dfrac{\sqrt{l^2+a^2}}{2}$

Equation of circle $\Rightarrow (x-a)^2+\left(y-\dfrac{\sqrt{l^2+4a^2}}{2}\right)^2=\dfrac{l^2+4a^2}{4}$

$a=12,\ l=10\Rightarrow \Delta ABC=\dfrac{1}{2}al=60$ square units

(b) P lies on $5x-y-4=0$ $\Rightarrow$

$5x-y-4=0$.................(1)



Gradient of PM is m $\Rightarrow$

$m=\dfrac{y-5}{x-1}$......................(2)

$(2)\Rightarrow y=5+m(x-1)$

$(1)\Rightarrow 5x-(5+m(x-1))-4=0$

$$x=\frac{9-m}{5-m},\quad y=\frac{25-m}{5-m}$$

$$\therefore P=\left(\frac{9-m}{5-m},\frac{25-m}{5-m}\right)$$

Q lies on $3x+4y-4=0 \Rightarrow$

$$3x+4y-4=0\text{.................}(1)$$

Gradient of QM is $m \Rightarrow$

$$m=\frac{y-5}{x-1}\text{......................}(2)$$

$$(2)\Rightarrow y=5+m(x-1)$$

$$(1)\Rightarrow 3x+4\left(5+m(x-1)\right)-4=0$$

$$x=\frac{4m-16}{4m+3},\quad y=\frac{m+15}{4m+3}$$

$$\therefore Q=\left(\frac{4m-16}{4m+3},\frac{m+15}{4m+3}\right)$$

Center of PQ is $M \Rightarrow$

$$\frac{1}{2}\left(\frac{9-m}{5-m}+\frac{4m-16}{4m+3}\right)=1$$

$$35m=83$$

$$m=\frac{83}{35}$$

or

$$\frac{1}{2}\left(\frac{25-m}{5-m}+\frac{m+15}{4m+3}\right)=5$$

$$m(35m-83)=0$$

$$m=\frac{83}{35}\quad (\because m\neq 0)$$

Equation of PQ:

$$y-5=\frac{83}{35}(x-1)$$

$$83x-35y+92=0$$

Q17). (a) $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

$$\tan 15^\circ=\tan(45^\circ-30^\circ)=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}=\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}$$

$$\tan\left(\frac{x}{2}\right)=\frac{\sqrt{1+\tan^2 x}-1}{\tan x}$$

$$R.H.S=\frac{\sqrt{1+\tan^2 x}-1}{\tan x}=\frac{\sqrt{\sec^2 x}-1}{\tan x}=\frac{\sec x-1}{\tan x}$$

$$=\frac{1-\cos x}{\sin x}$$

$$=\frac{2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}$$

$$=\tan\left(\frac{x}{2}\right)$$

$$=L.H.S$$

$$\tan 7\frac{1}{2}^\circ = \frac{\sqrt{1+\tan^2 15^\circ}-1}{\tan 15^\circ} = \frac{\sqrt{1+\left(2-\sqrt{3}\right)^2}-1}{2-\sqrt{3}}$$

$$= \frac{\sqrt{8-4\sqrt{3}}-1}{2-\sqrt{3}}$$

$$= \frac{\sqrt{\left(\sqrt{6}-\sqrt{2}\right)^2}-1}{2-\sqrt{3}}$$

$$= (\sqrt{6}-\sqrt{2}-1)(2+\sqrt{3})$$

$$= \sqrt{6}-2-\sqrt{3}+\sqrt{2}$$

$$= \left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{2}-1\right)$$

$$\cot 7\frac{1}{2}^\circ = \frac{1}{\tan 7\frac{1}{2}^\circ} = \frac{1}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{2}-1\right)}$$

$$= \left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{2}+1\right)$$

$$= \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$$

(b) $\sin^3 x+\cos^3 x+\sin x\cos x = 1$

$(\sin x+\cos x)(\sin^2 x+\cos^2 x-\sin x\cos x)+\sin x\cos x-1=0$

$(\sin x+\cos x)(1-\sin x\cos x)+\sin x\cos x-1=0$

$(\sin x+\cos x-1)(1-\sin x\cos x)=0$

$\sin x+\cos x = 1$ *or* $\sin x\cos x = 1$

$\sin 2x = 2$

$\therefore$ *No solution*

$$\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi}{4}\right)\cos x+\sin\left(\frac{\pi}{4}\right)\sin x = \cos\left(\frac{\pi}{4}\right)$$

$$\cos\left(x-\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$$

$$x-\frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4} \quad \left(n\in\mathbb{Z}\right)$$

$$x = 2n\pi \pm \frac{\pi}{4}+\frac{\pi}{4}$$

(c) Sine rule: In any ΔABC , $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{s\text{in}\, C}$

Let $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{s\text{in}\, C} = k$

(i) $$\frac{a-b}{c} = \frac{ks\text{in}\, A-k\sin B}{k\sin C}$$

$$= \frac{s\text{in}\, A-\sin B}{\sin C}$$

$$=\frac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$=\frac{\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)}{\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)} \qquad (\because A+B+C=\pi)$$

$$=\frac{\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{C}{2}\right)}{\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$=\frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{C}{2}\right)}$$

$$\therefore (a-b)\cos\frac{C}{2}=c\sin\left(\frac{A-B}{2}\right)$$

(ii) $$\frac{\tan A-\tan B}{\tan A+\tan B}=\frac{c-b}{c}$$

$$\frac{\frac{\sin A}{\cos A}-\frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}=\frac{k\sin C-k\sin B}{k\sin C}$$

$$\frac{\sin A\cos B-\sin B\cos A}{\sin A\cos B+\sin B\cos A}=\frac{\sin C-\sin B}{\sin C}$$

$$\frac{\sin(A-B)}{\sin(A+B)}=\frac{\sin C-\sin B}{\sin C}$$

$$\frac{\sin(A-B)}{\sin(\pi-C)}=\frac{\sin(\pi-(A+B))-\sin B}{\sin C} \qquad (\because A+B+C=\pi)$$

$$\sin(A-B)=\sin(A+B)-\sin B$$

$$\sin B=\sin(A+B)-\sin(A-B)$$

$$\sin B=2\sin B\cos A$$

$$\sin B(2\cos A-1)=0$$

$$\cos A=\frac{1}{2} \quad (\because \sin B\neq 0)$$

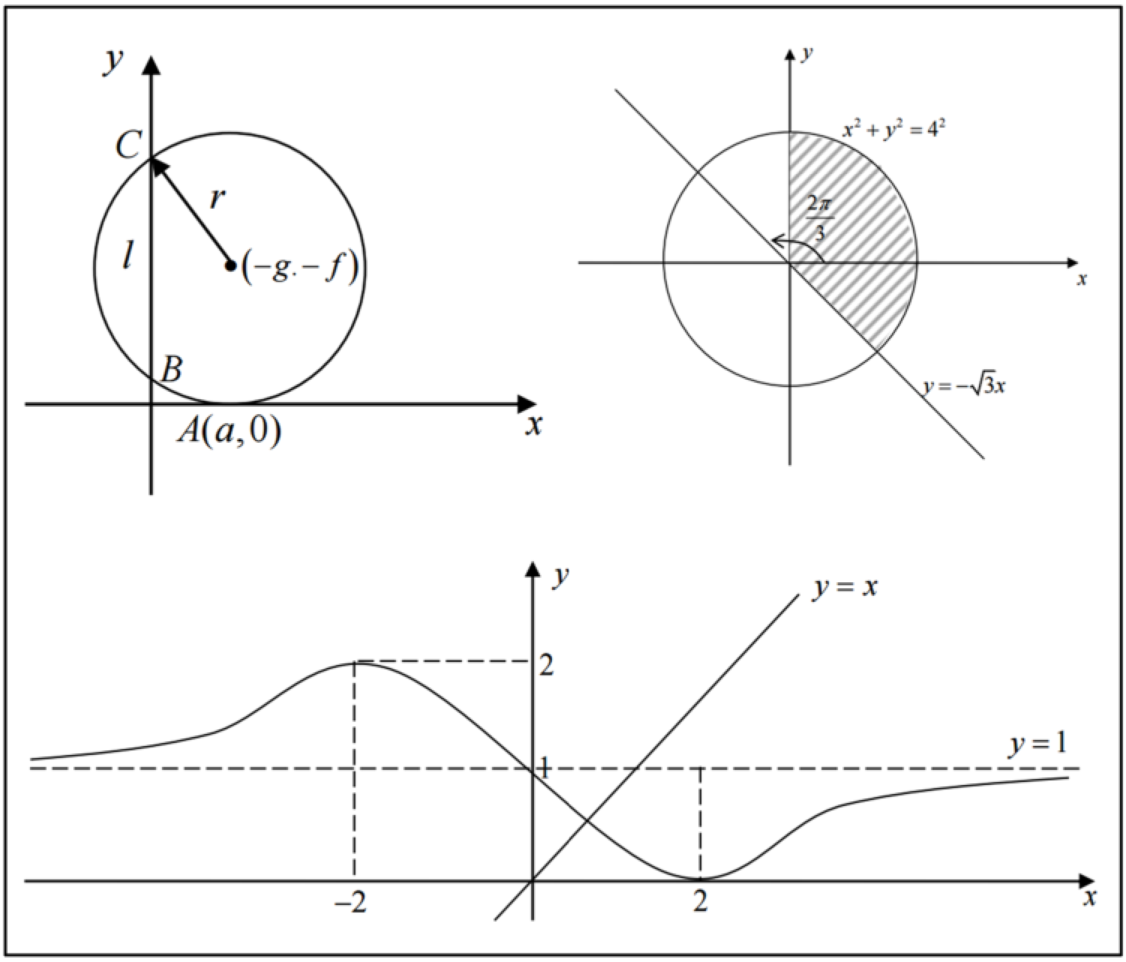
$$A=60^\circ$$

மொறட்டுவை பல்கலைக்கழக பொறியியற் பீட தமிழ் மாணவர்கள்
நடாத்தும் க.பொ.த உயர்தர மாணவர்களுக்கான 8 வது

முன்னோடிப் பரீட்சை - 2017

10(I) - இணைந்தகணிதம் I

விடைகள்



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பகுதி - **A**

Q1). $f(n)=7^n-2^n$ என்க.

$n=1$ இற்கு

$f(1)=7^1-2^1=5=5\times1$

$\therefore n=1$ இற்கு முடிவு உண்மை.

$n=p\in\mathbb{Z}^+$ இற்கு முடிவு உண்மை என்க.

$f(p)=7^p-2^p=5k \qquad (k\in\mathbb{Z}^+)$

$n=p+1$ இற்கு

$f(p+1)=7^{p+1}-2^{p+1}$

$f(p+1)=7.7^p-2.2^p$

$f(p+1)=7.(7^p-2^p)+5.2^p$

$f(p+1)=7.5k+5.2^p$

$f(p+1)=5.(7k+2^p)=5m \qquad (m\in\mathbb{Z}^+)$

$\therefore n=p+1$ இற்கு முடிவு உண்மை.

கணித தொகுத்தறிவு கோட்பாட்டின் படி எல்லா $n\in\mathbb{Z}^+$ இற்கும் முடிவு உண்மை.

Q2). $B-1,\ A-3,\ N-2$

ஆக்கக்கூடிய ஒழுங்கமைப்புகளின் எண்ணிக்கை $=\dfrac{6!}{2!\times3!}=60$

இரண்டு N ஐயும் ஒன்றாக கருதுக.

ஆக்கக்கூடிய ஒழுங்கமைப்புகளின் எண்ணிக்கை $=\dfrac{5!}{3!}\times2!=40$

இரு **N** உம் அடுத்தடுத்து இல்லாத ஒழுங்கமைப்புகள் $=60-40=20$

Q3). $\left(\sqrt{3}+i\right)\left(a+i\right)=2\left(a-i\right)$

$\left(\sqrt{3}a-1\right)+i\left(a+\sqrt{3}\right)=2a-2i$

$\left(\sqrt{3}a-1\right)=2a,\ \left(a+\sqrt{3}\right)=-2$

$a=\dfrac{1}{\sqrt{3}-2}=-\left(2+\sqrt{3}\right)$

$\left(\sqrt{3}+i\right)\left(-\left(2+\sqrt{3}\right)+i\right)=2\left(-\left(2+\sqrt{3}\right)-i\right)$

$\dfrac{\left(-\left(2+\sqrt{3}\right)+i\right)}{\left(-\left(2+\sqrt{3}\right)-i\right)}=\dfrac{\left(2+\sqrt{3}-i\right)}{\left(2+\sqrt{3}+i\right)}=\dfrac{2}{\left(\sqrt{3}+i\right)}$

$\dfrac{\left(2+\sqrt{3}-i\right)}{\left(2+\sqrt{3}+i\right)}=\dfrac{\left(\sqrt{3}-i\right)}{2}=1.\left(\cos\left(-\dfrac{\pi}{6}\right)+i\sin\left(-\dfrac{\pi}{6}\right)\right) \qquad \therefore$ மட்டு $=1$; $Arg\ z=-\dfrac{\pi}{6}$

Q4). $\lim_{x\to 1}\frac{x^3-x^2\ln x+\ln x-1}{x^2-1}$

$$=\lim_{x\to 1}\frac{(x-1)(x^2+x-x\ln x-\ln x+1)}{(x-1)(x+1)}$$

$$=\lim_{x\to 1}\frac{(x^2+x-x\ln x-\ln x+1)}{(x+1)}$$

$$=\frac{3}{2}$$

Q5). $\left(x-1\right)^n={}^nC_0x^n(-1)^0+{}^nC_1x^{n-1}(-1)^1+{}^nC_2x^{n-2}(-1)^2+........+{}^nC_rx^{n-r}(-1)^r+......+{}^nC_nx^0(-1)^n$

$x=17,\ n=500$ என பிரதியிட

$$\left(17-1\right)^{500}={}^{500}C_0 17^{500}(-1)^0+{}^{500}C_1 17^{499}(-1)^1+{}^{500}C_2 17^{498}(-1)^2+...........+{}^{500}C_r 17^{500-r}(-1)^r+...........$$
$$.......+{}^{500}C_{500}17^0(-1)^{500}$$

$$\left(16\right)^{500}=17.\left({}^{500}C_0 17^{499}(-1)^0+{}^{500}C_1 17^{498}(-1)^1+{}^{500}C_2 17^{497}(-1)^2+...........+{}^{500}C_r 17^{499-r}(-1)^r+..........\right.$$
$$\left.........+{}^{500}C_{499}17^0(-1)^{499}\right)+1$$

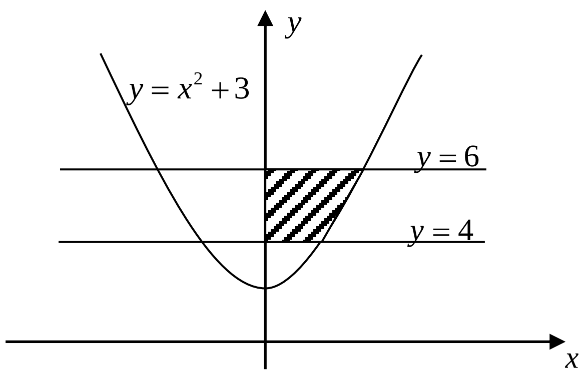
$$2^{2000}=17.\left({}^{500}C_0 17^{499}(-1)^0+{}^{500}C_1 17^{498}(-1)^1+{}^{500}C_2 17^{497}(-1)^2+.............+{}^{500}C_r 17^{499-r}(-1)^r+..........\right.$$
$$\left.........+{}^{500}C_{499}17^0(-1)^{499}\right)+1$$

இருபுறமும் 2^3 இனால் பெருக்க

$$2^{2003}=17.\left(8.\left({}^{500}C_0 17^{499}(-1)^0+{}^{500}C_1 17^{498}(-1)^1+{}^{500}C_2 17^{497}(-1)^2+...........+{}^{500}C_r 17^{499-r}(-1)^r+..........\right.\right.$$
$$\left.\left.........+{}^{500}C_{499}17^0(-1)^{499}\right)\right)+8$$

$\therefore 2^{2003}$ இனை 17 இனால் வகுக்க வரும் மீதி $=8$

Q6).



$$A=\int_4^6\sqrt{y-3}\,dy$$

$$=\left[\frac{(y-3)^{3/2}}{3/2}\right]_4^6$$

$$=\frac{2}{3}\left(3\sqrt{3}-1\right)$$ சது.அலகு

Q7). வளையி $y = be^{-\frac{x}{a}}$ ஆனது y அச்சை வெட்டும் புள்ளி $=(0,b)$

அப்புள்ளியில் வளையியின் படித்திறன் $= \left.\dfrac{dy}{dx}\right|_{(0,b)}$

$$= -\left.\frac{b}{a}e^{-\frac{x}{a}}\right|_{(0,b)}$$

$$= -\frac{b}{a}$$

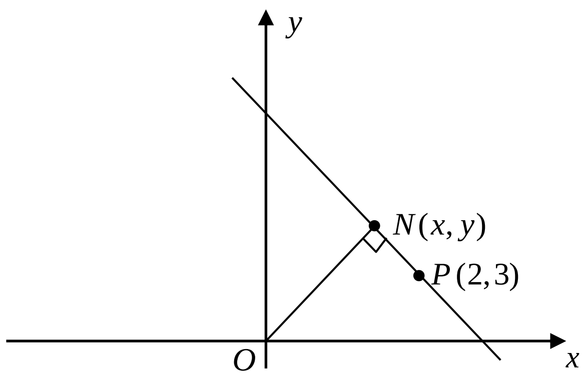
அப்புள்ளியில் வளையிக்கு வரையப்படும் தொடலியின் சமன்பாடு $\Rightarrow$

$$y - b = -\frac{b}{a}(x-0)$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

$\therefore \dfrac{x}{a} + \dfrac{y}{b} = 1$ எனும் கோடு வளையி $y = be^{-\frac{x}{a}}$ ஐ $(0,b)$ இல் தொடுகின்றது.

Q8).



ON இன் படித்திறன் $= \dfrac{y}{x}$

PN இன் படித்திறன் $= \dfrac{y-3}{x-2}$

$ON \perp PN \Rightarrow \dfrac{y}{x} \times \dfrac{y-3}{x-2} = -1$

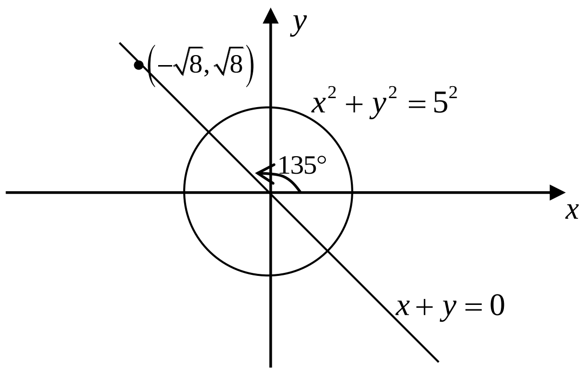
$$y(y-3) + x(x-2) = 0$$

$$(x-1)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{13}{4}$$

$\therefore N$ இன் ஒழுக்கு ஒரு வட்டம்

மையம் $= \left(1, \dfrac{3}{2}\right)$ ஆரை $= \dfrac{\sqrt{13}}{2}$

Q9). நேர் கோட்டின் சமன்பாடு $\Rightarrow$



$$y-\sqrt{8}=\tan(135°)\left(x-\left(-\sqrt{8}\right)\right)$$

$$y-\sqrt{8}=-\left(x+\sqrt{8}\right)$$

$$x+y=0$$

இது உற்பத்தியினூடாக செல்லும் ஒரு நேர்கோடு

வட்டத்தின் சமன்பாடு $\Rightarrow$

$$x^2+y^2=\left(5\cos\theta\right)^2+\left(5\sin\theta\right)^2$$

$$x^2+y^2=5^2$$

மையம் $=(0,0)$ ஆரை $=5$

$\therefore$ நேர்கோடும் வட்டமும் இடைவெட்டும்

இடைவெட்டும் நாண் வட்டத்தின் விட்டம் $\Rightarrow$ நாணின் நீளம் $=10$

Q10). $\alpha=\tan^{-1}(-3),\ \beta=\cos^{-1}\left(\frac{4}{5}\right)$ என்க.

இங்கு $-\frac{\pi}{2}<\alpha<-\frac{\pi}{4}, 0<\beta<\frac{\pi}{2}$ ஆகும்.

$\therefore -\pi<2\alpha<-\frac{\pi}{2}, -\pi<\beta-\pi<-\frac{\pi}{2}$ ....(1)

$$\cos(2\alpha)=\frac{1-\tan^2\alpha}{1+\tan^2\alpha}=\frac{1-(-3)^2}{1+(-3)^2}=-\frac{4}{5}$$

$$\cos(\beta-\pi)=\cos(\pi-\beta)=-\cos(\beta)=-\frac{4}{5}$$

$$\therefore \cos(2\alpha)=\cos(\beta-\pi)$$

(1) இலிருந்து $2\alpha=\beta-\pi$

$$\Rightarrow 2\tan^{-1}(-3)=\cos^{-1}\left(\frac{4}{5}\right)-\pi$$

பகுதி – B

Q11). (a) (i) $f\left(x\right)=x^2+x+1=\left(x+\frac{1}{2}\right)^2+\frac{3}{4}$

$\therefore$ எல்லா x இற்கும் $f(x)>0$ ஆகும்.

(ii) $g\left(x\right)=4x^2+\left(m+3\right)x+4$

$g\left(x\right)>0$ ஆக இருக்க $a>0\,\&\,b^2-4ac<0\Rightarrow$

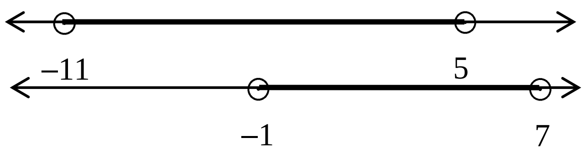
$$4>0\ \ \&\ \left(m+3\right)^2-4\times4\times4<0$$
$$\left(m+11\right)\left(m-5\right)<0$$
$$-11<m<5$$

(iii) $h\left(x\right)=2x^2+\left(3-m\right)x+2$

$h\left(x\right)>0$ ஆக இருக்க $a>0\,\&\,b^2-4ac<0\Rightarrow$

$$2>0\ \ \&\ \left(3-m\right)^2-4\times2\times2<0$$
$$\left(m+1\right)\left(m-7\right)<0$$
$$-1<m<7$$

$$-3<\frac{x^2+mx+1}{x^2+x+1}<3$$
$$\Leftrightarrow -3\left(x^2+x+1\right)<x^2+mx+1<3\left(x^2+x+1\right)\quad\left(\because x^2+x+1>0\right)$$
$$\Leftrightarrow -3\left(x^2+x+1\right)<x^2+mx+1\ \&\ x^2+mx+1<3\left(x^2+x+1\right)$$
$$\Leftrightarrow 0<4x^2+\left(m+3\right)x+4\ \ \&\ 0<2x^2+\left(3-m\right)x+2$$
$$\Leftrightarrow g(x)>0\ \ \&\ h(x)>0$$
$$\Leftrightarrow -11<m<5\ \ \&\ -1<m<7$$
$$\Leftrightarrow -1<m<5$$



(b) $f\left(x\right)=x^4+2x^3-3x^2-2x+3$

மீதித் தேற்றத்தின் படி

$$f\left(x\right)=(x-2)\phi(x)+R$$
$$f(2)=R=16+16-12-4+3=19$$

$$\phi\left(x\right)=(x-2)\,\phi_1(x)+R_1$$
$$\phi_1\left(x\right)=(x-3)\,\phi_2(x)+R_2$$
$$f\left(x\right)=(x-2)\left((x-2)\,\phi_1(x)+R_1\right)+R\text{..............................}(1)$$
$$f\left(x\right)=(x-2)\left((x-2)\left((x-3)\,\phi_2(x)+R_2\right)+R_1\right)+R$$
$$f(x)=(x-2)^2(x-3)\,\phi_2(x)+(x-2)^2R_2+(x-2)R_1+R$$

$\therefore$ மீதி அவ்வடிவத்தை எடுக்கும்.

$$(1)\Rightarrow\ f\left(x\right)=(x-2)^2\,\phi_1(x)+R_1(x-2)+R$$
$$f'\left(x\right)=(x-2)^2\,\phi_1'(x)+2(x-2)\,\phi_1(x)+R_1$$
$$f'(x)=4x^3+6x^2-6x-2$$

$$f'(2) = R_1 = 32 + 24 - 12 - 2 = 42$$

$$f(x) = (x-2)^2(x-3)\phi_2(x) + (x-2)^2 R_2 + 42(x-2) + 19$$

$$f(3) = R_2 + 42 + 19 = 81 + 54 - 27 - 6 + 3$$

$$R_2 = 44$$

$\therefore$ மீதி $= 44(x-2)^2 + 42(x-2) + 19$

$\therefore\ a = 44,\ b = 42,\ c = 19$

Q12). (a) $$\frac{1}{1+a^{n-1}} - \frac{1}{1+a^n} = \frac{a^n - a^{n-1}}{\left(1+a^{n-1}\right)\left(1+a^n\right)} = \frac{a^{n-1}(a-1)}{\left(1+a^{n-1}\right)\left(1+a^n\right)}$$

$$\frac{a^{r-1}}{\left(1+a^{r-1}\right)\left(1+a^r\right)} = \left(\frac{1}{a-1}\right)\left(\frac{1}{1+a^{r-1}} - \frac{1}{1+a^r}\right) = \frac{1}{(a-1)\left(1+a^{r-1}\right)} - \frac{1}{(a-1)\left(1+a^r\right)}$$

$$\frac{a^{r-1}}{\left(1+a^{r-1}\right)\left(1+a^r\right)} = f(r-1) - f(r)$$

$$\therefore\ f(r) = \frac{1}{(a-1)\left(1+a^r\right)}$$

$$U_r = \frac{a^{r-1}}{\left(1+a^{r-1}\right)\left(1+a^r\right)} = f(r-1) - f(r)$$

$$U_1 = f(0) - \cancel{f(1)}$$
$$U_2 = \cancel{f(1)} - \cancel{f(2)}$$
$$U_3 = \cancel{f(2)} - \cancel{f(3)}$$
$$\cdots\cdots$$
$$U_{n-1} = \cancel{f(n-2)} - \cancel{f(n-1)}$$
$$U_n = \cancel{f(n-1)} - f(n)$$

$$\sum_{r=1}^{n} U_r = f(0) - f(n) = \frac{1}{2(a-1)} - \frac{1}{(a-1)\left(1+a^n\right)} = \frac{a^n - 1}{2(a-1)\left(a^n+1\right)}$$

$$a = 2 \Rightarrow \sum_{r=1}^{n} \frac{2^{r-1}}{\left(1+2^{r-1}\right)\left(1+2^r\right)} = \frac{2^n - 1}{2\left(2^n+1\right)} = \frac{1}{2} - \frac{1}{\left(2^n+1\right)}$$

$$\sum_{r=1}^{n} \frac{2^r}{\left(1+2^{r-1}\right)\left(1+2^r\right)} = 1 - \frac{2}{\left(2^n+1\right)}$$

எல்லா $n \in \mathbb{Z}^+$ இற்கும் $0 < \dfrac{2}{\left(2^n+1\right)} < 1$ ஆகும்.

$$\Rightarrow 0 < 1 - \frac{2}{\left(2^n+1\right)} < 1$$

$$\therefore 0 < \sum_{r=1}^{n} \frac{2^r}{\left(1+2^{r-1}\right)\left(1+2^r\right)} < 1$$

$$a = 2017 \Rightarrow \sum_{r=1}^{n} \frac{2017^{r-1}}{\left(1+2017^{r-1}\right)\left(1+2017^{r}\right)} = \frac{2017^{n}-1}{2\times 2016\times\left(2017^{n}+1\right)}$$

$$\sum_{r=1}^{n} \frac{2017^{r}}{\left(1+2017^{r-1}\right)\left(1+2017^{r}\right)} = \frac{2017}{4032}\left(\frac{2017^{n}-1}{2017^{n}+1}\right)$$

$$\lim_{n\to\infty}\sum_{r=1}^{n} \frac{2017^{r}}{\left(1+2017^{r-1}\right)\left(1+2017^{r}\right)} = \frac{2017}{4032}\lim_{n\to\infty}\left(\frac{2017^{n}-1}{2017^{n}+1}\right) = \frac{2017}{4032}\lim_{n\to\infty}\left(\frac{1-\frac{1}{2017^{n}}}{1+\frac{1}{2017^{n}}}\right) = \frac{2017}{4032}$$

(b)

$y = \left|x^2 - 2x\right|$

$y = x(x-2) = (x-1)^2 - 1;\ x \le 0 \ or\ x \ge 2$

$y = 1-(x-1)^2;\ 0 < x < 2$

$y = \left|1-2x\right|$

$y = 2x-1;\ x \ge \frac{1}{2}$

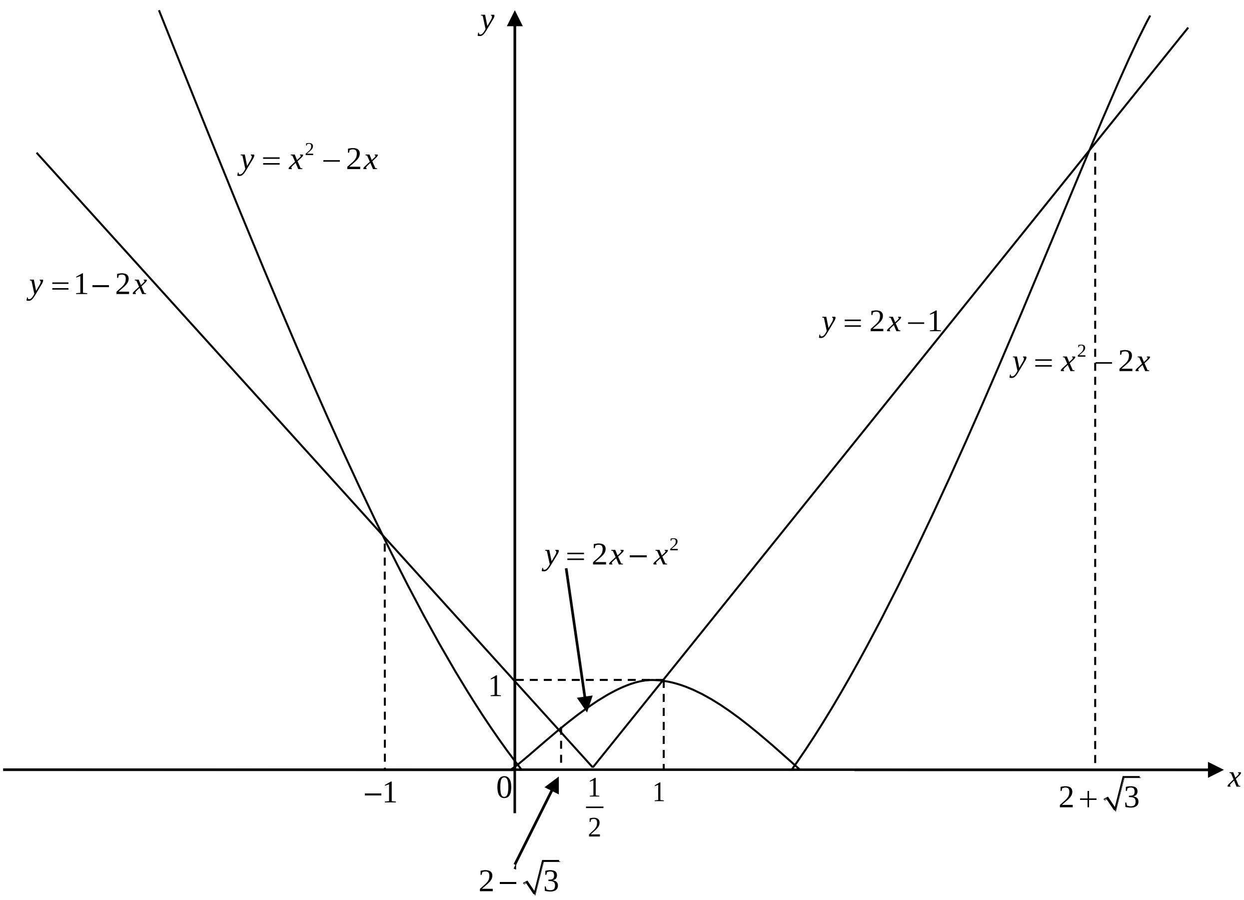
$y = 1-2x;\ x < \frac{1}{2}$

$x = 0 \Rightarrow y = 0$

$y = 0 \Rightarrow x = 0, 2$

சமச்சீர் அச்சு $\Rightarrow x = 1$

$x \to \pm\infty \Rightarrow y \to \pm\infty$



வரைபுகள் இடைவெட்டும் புள்ளிகள்

$x^2-2x=1-2x$	$2x-x^2=1-2x$	$2x-x^2=2x-1$	$x^2-2x=2x-1$
$x=\pm 1$	$x=2\pm\sqrt{3}$	$x=\pm 1$	$x=2\pm\sqrt{3}$
$\Rightarrow x=-1$	$\Rightarrow x=2-\sqrt{3}$	$\Rightarrow x=1$	$\Rightarrow x=2+\sqrt{3}$

$$\left|x^2-2x\right|\le\left|1-2x\right|$$

$y=\left|x^2-2x\right|$ இன் வரைபுக்கு மேல் $y=\left|1-2x\right|$ இன் வரைபு இருக்கும் பிரதேசம்

தீர்வுத்தொடை $=\left\{x\in\mathbb{R}\,\middle|\,-1\le x\le 2-\sqrt{3}\right\}\cup\left\{x\in\mathbb{R}\,\middle|\,1\le x\le 2+\sqrt{3}\right\}$

Q13). (a) $\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}^{-1}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$

$$\begin{pmatrix}a & b\\ c & d\end{pmatrix}.\frac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}=\frac{1}{ad-bc}\begin{pmatrix}a & b\\ c & d\end{pmatrix}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$$

$$=\frac{1}{ad-bc}\begin{pmatrix}ad-bc & 0\\ 0 & ad-bc\end{pmatrix}$$

$$=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

$$\therefore\begin{pmatrix}a & b\\ c & d\end{pmatrix}^{-1}=\frac{1}{ad-bc}\begin{pmatrix}d & -b\\ -c & a\end{pmatrix}$$

2×2 தாயத்திற்கு நேர்மாறு இருப்பதற்கு $ad-bc\neq 0$

$$\mathbf{A}=\begin{pmatrix}3 & 7\\ 1 & 2\end{pmatrix},\mathbf{B}=\begin{pmatrix}4 & -1\\ -3 & 1\end{pmatrix}$$

$$\mathbf{AB}=\begin{pmatrix}3 & 7\\ 1 & 2\end{pmatrix}\begin{pmatrix}4 & -1\\ -3 & 1\end{pmatrix}=\begin{pmatrix}-9 & 4\\ -2 & 1\end{pmatrix}$$

$$(\mathbf{AB})^{-1}=\frac{1}{(-9+8)}\begin{pmatrix}1 & -4\\ 2 & -9\end{pmatrix}=\begin{pmatrix}-1 & 4\\ -2 & 9\end{pmatrix}$$

$$\mathbf{A}^{-1}=\frac{1}{(6-7)}\begin{pmatrix}2 & -7\\ -1 & 3\end{pmatrix}=\begin{pmatrix}-2 & 7\\ 1 & -3\end{pmatrix}$$

$$\mathbf{B}^{-1}=\frac{1}{(4-3)}\begin{pmatrix}1 & 1\\ 3 & 4\end{pmatrix}=\begin{pmatrix}1 & 1\\ 3 & 4\end{pmatrix}$$

(i) $\mathbf{A}^{-1}\mathbf{B}^{-1}=\begin{pmatrix}-2 & 7\\ 1 & -3\end{pmatrix}\begin{pmatrix}1 & 1\\ 3 & 4\end{pmatrix}=\begin{pmatrix}19 & 26\\ -8 & -11\end{pmatrix}\Rightarrow(\mathbf{AB})^{-1}\neq\mathbf{A}^{-1}\mathbf{B}^{-1}$

(ii) $\mathbf{B}^{-1}\mathbf{A}^{-1}=\begin{pmatrix}1 & 1\\ 3 & 4\end{pmatrix}\begin{pmatrix}-2 & 7\\ 1 & -3\end{pmatrix}=\begin{pmatrix}-1 & 4\\ -2 & 9\end{pmatrix}\Rightarrow(\mathbf{AB})^{-1}=\mathbf{B}^{-1}\mathbf{A}^{-1}$

(b) $A=\left\{z\in\mathbb{C}:|z|\le 4\right\}\cap\left\{z\in\mathbb{C}:\operatorname{Im}\left(\dfrac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right)\ge 0\right\}\cap\left\{z\in\mathbb{C}:\operatorname{Re}(z)\ge 0\right\}$

$z=x+iy$ என்க.

$|z|\le 4$

$\sqrt{x^2+y^2}\le 4$

$x^2+y^2\le 4^2$..................................(1)

$\operatorname{Im}\left(\dfrac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right)\ge 0$

$\operatorname{Im}\left(\dfrac{x+iy-1+\sqrt{3}i}{1-\sqrt{3}i}\right)\ge 0$

$\operatorname{Im}\left(\dfrac{x-\sqrt{3}y-4+i\left(y+\sqrt{3}x\right)}{4}\right)$

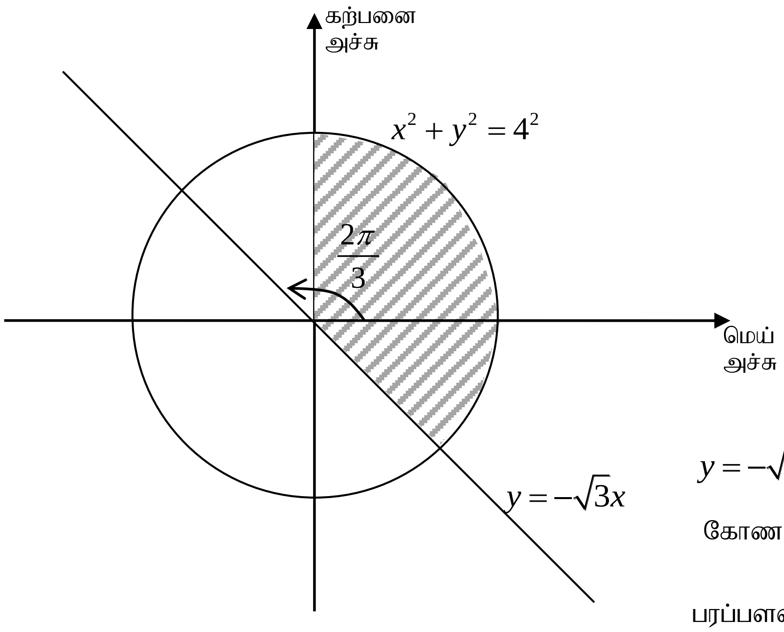
$\dfrac{\left(y+\sqrt{3}x\right)}{4}\ge 0$

$y\ge -\sqrt{3}x$..(2)

$\operatorname{Re}(z)\ge 0$

$x\ge 0$.............................(3)

$A=(1)\cap(2)\cap(3)$



$y=-\sqrt{3}x$ ஆனது மெய் அச்சுடன் அமைக்கும்

கோணம் $=\pi-\tan^{-1}\left(\sqrt{3}\right)=\dfrac{2\pi}{3}$

பரப்பளவு $=\dfrac{1}{2}\times 4^2\times\dfrac{5\pi}{6}$

$=\dfrac{20\pi}{3}$

(c) $z = x + iy$ என்க.

$|z|^2 = x^2 + y^2$

$\bar{z} = x - iy$

$z.\bar{z} = (x+iy)(x-iy) = x^2 + y^2 = |z|^2$

$z + \bar{z} = (x+iy) + (x-iy) = 2x = 2\,\mathrm{Re}\,z$

$$|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)}$$
$$= (z_1 + z_2)(\overline{z_1} + \overline{z_2})$$
$$= z_1\overline{z_1} + z_2\overline{z_2} + z_1\overline{z_2} + z_2\overline{z_1}$$
$$= |z_1|^2 + |z_2|^2 + z_1\overline{z_2} + \overline{z_1\overline{z_2}}$$
$$= |z_1|^2 + |z_2|^2 + 2\,\mathrm{Re}\left(z_1\overline{z_2}\right)$$

$$|z_1 - z_2|^2 = |z_1|^2 + |-z_2|^2 + 2\,\mathrm{Re}\left(z_1\overline{(-z_2)}\right)$$
$$= |z_1|^2 + |z_2|^2 - 2\,\mathrm{Re}\left(z_1\overline{z_2}\right)$$

$$\left|\frac{(z_1 - z_2)}{(z_1 + z_2)}\right| = 1$$

$|z_1 - z_2| = |z_1 + z_2|$

$|z_1 - z_2|^2 = |z_1 + z_2|^2$

$|z_1|^2 + |z_2|^2 + 2\,\mathrm{Re}\left(z_1\overline{z_2}\right) = |z_1|^2 + |z_2|^2 - 2\,\mathrm{Re}\left(z_1\overline{z_2}\right)$

$\mathrm{Re}\left(z_1\overline{z_2}\right) = 0$

$$\mathrm{Re}\left(\frac{z_1}{z_2}\right) = \mathrm{Re}\left(\frac{z_1\overline{z_2}}{z_2\overline{z_2}}\right) = \mathrm{Re}\left(\frac{z_1\overline{z_2}}{|z_2|^2}\right) = \frac{1}{|z_2|^2}\mathrm{Re}\left(z_1 z_2\right) = 0$$

$\therefore \dfrac{z_1}{z_2}$ அறக்கற்பனை.

Q14). (a) $y = \dfrac{(x-2)^2}{x^2+4}$

$(x^2 + 4)y = x^2 - 4x + 4$

$(y-1)x^2 + 4x + 4y - 4 = 0$

$x \in \mathbb{R} \Rightarrow \quad b^2 - 4ac \geq 0$

$4^2 - 4(y-1)(4y-4) \geq 0$

$y(y-2) \leq 0$

$0 \leq y \leq 2$

$$\frac{dy}{dx} = \frac{(x^2+4)2(x-2) - (x-2)^2\,2x}{(x^2+4)^2} = \frac{4(x-2)(x+2)}{(x^2+4)^2}$$

$\dfrac{dy}{dx} = 0 \Rightarrow \; x = 2, -2$

திரும்பல் புள்ளிகள் $:(2,0),(-2,2)$

$+$	$-$	$+$	$\frac{dy}{dx}$
$-$	$-$	$+$	$x-2$
$-$	$+$	$+$	$x+2$
$-\infty<x<-2$	$-2<x<2$	$2<x<\infty$	

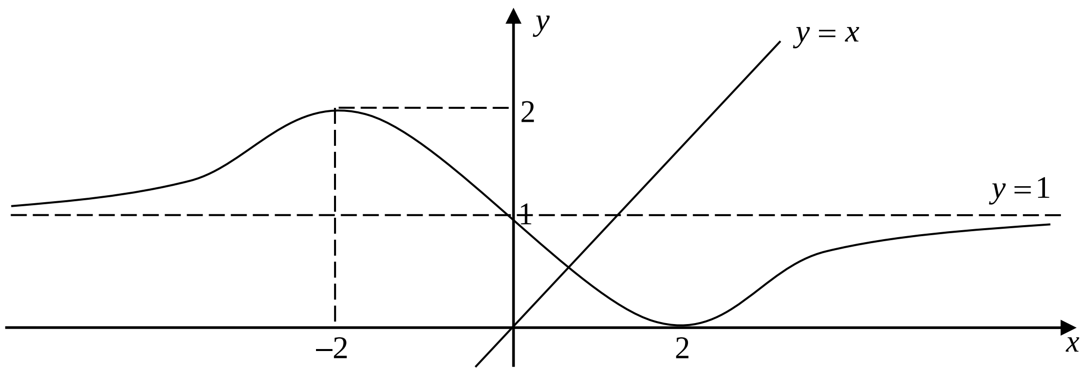
$\therefore (-2,2)$ உயர்வுப்புள்ளி, $(2,0)$ இழிவுப்புள்ளி

$x=0 \Rightarrow y=1,\ y=0 \Rightarrow x=2$

அச்சுக்களை இடைவெட்டும் புள்ளிகள் $:(0,1),(2,0)$

$$y=\frac{(x-2)^2}{x^2+4}=\frac{x^2-4x+4}{x^2+4}=\frac{1-\frac{4}{x}+\frac{4}{x^2}}{1+\frac{4}{x^2}}$$

$x \to \pm\infty$ ஆக $y \to 1 \Rightarrow$ கிடை அணுகுகோடு $: y=1$

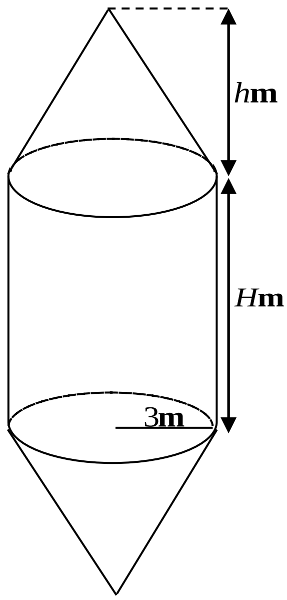


$$x\left(x^2+4\right)=(x-2)^2$$

$$x=\frac{(x-2)^2}{\left(x^2+4\right)}=y$$

$y=x$ எனும் கோடு வளையியை ஒரு புள்ளியில் மாத்திரம் இடைவெட்டுகிறது.

$\therefore x\left(x^2+4\right)=(x-2)^2$ இற்கு ஒரு மெய்த்தீர்வு மட்டும் உள்ளது.



(b) $$V=2\times\frac{1}{3}\pi R^2h+\pi R^2H$$

$$900=2\times\frac{1}{3}\pi\times3^2\times h+\pi\times3^2\times H$$

$$H=\frac{100}{\pi}-\frac{2}{3}h$$

$$S=2\times\pi Rl+2\pi RH$$

$$S=2\pi\times3\times\sqrt{h^2+3^2}+2\pi\times3\times\left(\frac{100}{\pi}-\frac{2}{3}h\right)$$

$$S=600-4\pi h+6\pi\sqrt{9+h^2}$$

$$\frac{dS}{dh}=-4\pi+6\pi\frac{h}{\sqrt{9+h^2}}$$

$$\frac{dS}{dh}=0\Rightarrow -4\pi+6\pi\frac{h}{\sqrt{9+h^2}}=0$$

$$5h^2=36$$

$$h=\frac{6}{\sqrt{5}}\ (\because h>0)$$

$$h<\frac{6}{\sqrt{5}}\Rightarrow\frac{dS}{dh}<0$$

$$h>\frac{6}{\sqrt{5}}\Rightarrow\frac{dS}{dh}>0$$

$\therefore h=\dfrac{6}{\sqrt{5}}$ இல் S இழிவாகும்.

Q15). (a) $I=\int e^{ax}\sin bx\,\mathrm{dx}$

$$=\frac{e^{ax}\sin bx}{a}-\int\frac{e^{ax}\cos\,bx.b}{a}\,\mathrm{d}x$$

$$=\frac{e^{ax}\sin bx}{a}-\frac{b}{a}\left(\frac{e^{ax}\cos bx}{a}-\int\frac{e^{ax}.-\sin\,bx.b}{a}\,\mathrm{d}x\right)$$

$$=\frac{e^{ax}\sin bx}{a}-\frac{be^{ax}\cos bx}{a^2}-\frac{b^2}{a^2}I$$

$$I=\frac{1}{a^2+b^2}\left(ae^{ax}\sin bx-be^{ax}\cos bx\right)+C$$ $C-$ தொகையீட்டு மாறிலி

(b) $$\frac{11+3x-2x^2}{(x+3)(x-1)^2}=\frac{A}{(x+3)}+\frac{B}{(x-1)}+\frac{C}{(x-1)^2}$$

$$11+3x-2x^2=A(x-1)^2+B(x-1)(x+3)+C(x+3)$$

$$x=1\Rightarrow C=3$$
$$x=-3\Rightarrow A=-1$$

x^o இன் குணகம் $\Rightarrow A-3B+3C=11\Rightarrow B=-1$

$$\frac{11+3x-2x^2}{(x+3)(x-1)^2}=\frac{-1}{(x+3)}+\frac{-1}{(x-1)}+\frac{3}{(x-1)^2}$$

$$\int\frac{11+3x-2x^2}{(x+3)(x-1)^2}dx=\int\frac{-1}{(x+3)}dx+\int\frac{-1}{(x-1)}dx+\int\frac{3}{(x-1)^2}dx$$

$$=-\ln(x+3)-\ln(x-1)-\frac{3}{(x-1)}+C$$

$C-$ தொகையீட்டு மாறிலி

(c) $$\frac{d}{dx}\left(\frac{Ax+B}{ax^2+bx+c}\right)=\frac{1}{\left(ax^2+bx+c\right)^2}-\frac{C}{ax^2+bx+c}$$

$$\frac{\left(ax^2+bx+c\right)A-(Ax+B)(2ax+b)}{\left(ax^2+bx+c\right)^2}=\frac{1}{\left(ax^2+bx+c\right)^2}-\frac{C}{ax^2+bx+c}$$

$$\left(ax^2+bx+c\right)A-(Ax+B)(2ax+b)=1-C\left(ax^2+bx+c\right)$$

x^2 இன் குணகம் $\Rightarrow -aA=-Ca\Rightarrow A=C$

x இன் குணகம் $\Rightarrow -2aB=-Cb\Rightarrow bC=2aB$

x^0 இன் குணகம் $\Rightarrow Ac-Bb=1-Cc\Rightarrow Cc-\frac{b^2}{2a}C=1-Cc$

$$C=\frac{2a}{4ac-b^2}\ ,\ A=\frac{2a}{4ac-b^2},\ B=\frac{b}{4ac-b^2}$$

$$a=1,\ b=4,\ c=1\Rightarrow A=-\frac{1}{6},B=-\frac{1}{3},C=-\frac{1}{6}$$

$$\frac{d}{dx}\left(\frac{-\frac{1}{6}x+-\frac{1}{3}}{x^2+4x+1}\right)=\frac{1}{\left(x^2+4x+1\right)^2}+\frac{\frac{1}{6}}{x^2+4x+1}$$

$$\int_0^1\frac{d}{dx}\left(\frac{-\frac{1}{6}x+-\frac{1}{3}}{x^2+4x+1}\right)dx=\int_0^1\frac{1}{\left(x^2+4x+1\right)^2}dx+\frac{1}{6}\int_0^1\frac{1}{x^2+4x+1}dx$$

$$\int_0^1\frac{1}{\left(x^2+4x+1\right)^2}dx=\int_0^1\frac{d}{dx}\left(\frac{-\frac{1}{6}x-\frac{1}{3}}{x^2+4x+1}\right)dx-\frac{1}{6}\int_0^1\frac{1}{(x+2)^2-3}dx$$

$$\int_0^1\frac{1}{\left(x^2+4x+1\right)^2}dx=\left(\frac{-\frac{1}{6}x-\frac{1}{3}}{x^2+4x+1}\right)_0^1-\frac{1}{12\sqrt{3}}\int_0^1\frac{1}{(x+2-\sqrt{3})}-\frac{1}{(x+2+\sqrt{3})}dx$$

$$\int_0^1\frac{1}{\left(x^2+4x+1\right)^2}dx=\left(\frac{-\frac{1}{6}x-\frac{1}{3}}{x^2+4x+1}\right)_0^1-\frac{1}{12\sqrt{3}}\ln\left(\frac{x+2-\sqrt{3}}{x+2+\sqrt{3}}\right)\Bigg|_0^1$$

$$\int_0^1\frac{1}{\left(x^2+4x+1\right)^2}dx=\left(\frac{-\frac{1}{6}x-\frac{1}{3}}{x^2+4x+1}\right)_0^1-\frac{\sqrt{3}}{36}\ln\left(\frac{x+2-\sqrt{3}}{x+2+\sqrt{3}}\right)\Bigg|_0^1$$

$$=-\frac{1}{12}+\frac{1}{3}-\frac{\sqrt{3}}{36}\left(\ln\left(\frac{3-\sqrt{3}}{3+\sqrt{3}}\right)-\ln\left(\frac{2-\sqrt{3}}{2+\sqrt{3}}\right)\right)$$

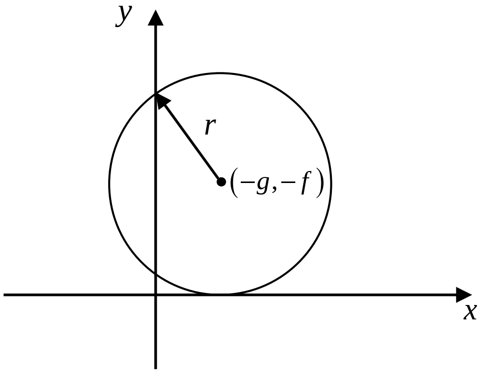
$$=\frac{1}{3}-\frac{\sqrt{3}}{36}\ln\left(\frac{3-\sqrt{3}}{3+\sqrt{3}}\times\frac{2+\sqrt{3}}{2-\sqrt{3}}\right)=\frac{1}{4}-\frac{\sqrt{3}}{36}\ln\left(2+\sqrt{3}\right)$$

Q16). (a) $x^2+y^2+2gx+2fy+c=0$

மையம் $=(-g,-f)$ ஆரை $r=\sqrt{g^2+f^2-c}$

x அச்சை தொடுகின்றது $\Rightarrow$

$$r=|-f|$$
$$g^2+f^2-c=f^2$$
$$g^2=c$$



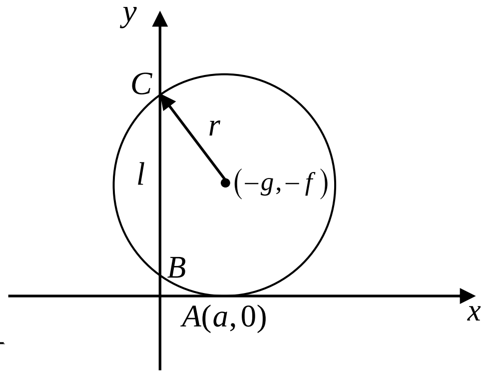
y அச்சை வெட்டுகின்றது $\Rightarrow$

$$r>|-g|$$
$$g^2+f^2-c>g^2$$
$$f^2>c$$

நாணின் நீளம் $=2\sqrt{r^2-(-g)^2}=2\sqrt{g^2+f^2-c-g^2}=2\sqrt{f^2-c}$

x அச்சை $(a,0)$ இல் தொடுகின்றது $\Rightarrow$

$$-g=a$$
$$c=g^2=a^2$$



நாணின் நீளம் $=l\Rightarrow l=2\sqrt{f^2-c}\Rightarrow f=-\dfrac{\sqrt{l^2+a^2}}{2}$

(வட்டம் நேர் y அச்சை வெட்டுகிறது)

மையம் $=\left(a,\dfrac{\sqrt{l^2+a^2}}{2}\right)$ ஆரை $r=\dfrac{\sqrt{l^2+a^2}}{2}$

வட்டத்தின் சமன்பாடு $\Rightarrow (x-a)^2+\left(y-\dfrac{\sqrt{l^2+4a^2}}{2}\right)^2=\dfrac{l^2+4a^2}{4}$

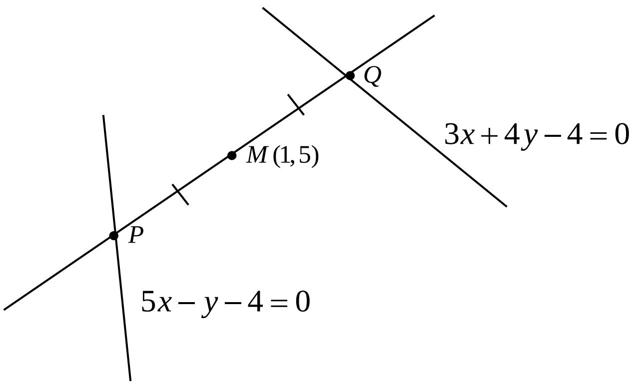
$a=12,\ l=10\Rightarrow \Delta ABC=\dfrac{1}{2}al=60$ சது.அலகு

(b) P ஆனது $5x-y-4=0$ மீது கிடக்கும் $\Rightarrow$

$5x-y-4=0$................(1)

PM இன் படித்திறன் m $\Rightarrow$

$m=\dfrac{y-5}{x-1}$......................(2)



$(2)\Rightarrow y=5+m(x-1)$

$(1)\Rightarrow 5x-(5+m(x-1))-4=0$

$$x=\frac{9-m}{5-m},\quad y=\frac{25-m}{5-m}$$

$$\therefore P=\left(\frac{9-m}{5-m},\frac{25-m}{5-m}\right)$$

Q ஆனது $3x+4y-4=0$ மீது கிடக்கும் $\Rightarrow$

$$3x+4y-4=0.................(1)$$

QM இன் படித்திறன் m $\Rightarrow$

$$m=\frac{y-5}{x-1}......................(2)$$

$$(2)\Rightarrow y=5+m(x-1)$$

$$(1)\Rightarrow 3x+4\big(5+m(x-1)\big)-4=0$$

$$x=\frac{4m-16}{4m+3},\quad y=\frac{m+15}{4m+3}$$

$$\therefore Q=\left(\frac{4m-16}{4m+3},\frac{m+15}{4m+3}\right)$$

PQ இன் நடுப்புள்ளி M $\Rightarrow$

$$\frac{1}{2}\left(\frac{9-m}{5-m}+\frac{4m-16}{4m+3}\right)=1$$

$$35m=83$$

$$m=\frac{83}{35}$$

அல்லது

$$\frac{1}{2}\left(\frac{25-m}{5-m}+\frac{m+15}{4m+3}\right)=5$$

$$m(35m-83)=0$$

$$m=\frac{83}{35}\ (\because m\neq 0)$$

PQ இன் சமன்பாடு;

$$y-5=\frac{83}{35}(x-1)$$

$$83x-35y+92=0$$

Q17). (a) $\tan(A-B)=\dfrac{\tan A-\tan B}{1+\tan A\tan B}$

$$\tan 15^\circ=\tan(45^\circ-30^\circ)=\frac{\tan 45^\circ-\tan 30^\circ}{1+\tan 45^\circ\tan 30^\circ}=\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}$$

$$\tan\left(\frac{x}{2}\right)=\frac{\sqrt{1+\tan^2 x}-1}{\tan x}$$

$$R.H.S=\frac{\sqrt{1+\tan^2 x}-1}{\tan x}=\frac{\sqrt{\sec^2 x}-1}{\tan x}=\frac{\sec x-1}{\tan x}$$

$$=\frac{1-\cos x}{\sin x}$$

$$=\frac{2\sin^2\left(\frac{x}{2}\right)}{2\sin\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right)}$$

$$=\tan\left(\frac{x}{2}\right)$$

$$=L.H.S$$

$$\tan 7\frac{1}{2}^\circ = \frac{\sqrt{1+\tan^2 15^\circ}-1}{\tan 15^\circ} = \frac{\sqrt{1+\left(2-\sqrt{3}\right)^2}-1}{2-\sqrt{3}}$$

$$= \frac{\sqrt{8-4\sqrt{3}}-1}{2-\sqrt{3}}$$

$$= \frac{\sqrt{\left(\sqrt{6}-\sqrt{2}\right)^2}-1}{2-\sqrt{3}}$$

$$= (\sqrt{6}-\sqrt{2}-1)(2+\sqrt{3})$$

$$= \sqrt{6}-2-\sqrt{3}+\sqrt{2}$$

$$= \left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{2}-1\right)$$

$$\cot 7\frac{1}{2}^\circ = \frac{1}{\tan 7\frac{1}{2}^\circ} = \frac{1}{\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{2}-1\right)}$$

$$= \left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{2}+1\right)$$

$$= \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$$

(*b*) $\sin^3 x + \cos^3 x + \sin x \cos x = 1$

$(\sin x + \cos x)(\sin^2 x + \cos^2 x - \sin x \cos x) + \sin x \cos x - 1 = 0$

$(\sin x + \cos x)(1 - \sin x \cos x) + \sin x \cos x - 1 = 0$

$(\sin x + \cos x - 1)(1 - \sin x \cos x) = 0$

$\sin x + \cos x = 1$ *or* $\sin x \cos x = 1$

$\sin 2x = 2$

$\therefore$ தீர்வு இல்லை

$$\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x = \frac{1}{\sqrt{2}}$$

$$\cos\left(\frac{\pi}{4}\right)\cos x + \sin\left(\frac{\pi}{4}\right)\sin x = \cos\left(\frac{\pi}{4}\right)$$

$$\cos\left(x - \frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right)$$

$$x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4} \quad \left(n \in \mathbb{Z}\right)$$

$$x = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{4}$$

(*c*) சைன் விதி: யாதாயினும் ஒரு ΔABC யில் $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{s\text{in}\, C}$ ஆகும்

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{s\text{in}\, C} = k$ என்க.

(*i*) $$\frac{a-b}{c} = \frac{ks\text{in}\, A - k\sin B}{k\sin C}$$

$$= \frac{s\text{in}\, A - \sin B}{\sin C}$$

$$= \frac{2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)}{2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$= \frac{\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)}{\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)} \qquad (\because A+B+C=\pi)$$

$$= \frac{\sin\left(\frac{A-B}{2}\right)\sin\left(\frac{C}{2}\right)}{\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)}$$

$$= \frac{\sin\left(\frac{A-B}{2}\right)}{\cos\left(\frac{C}{2}\right)}$$

$$\therefore (a-b)\cos\frac{C}{2} = c\sin\left(\frac{A-B}{2}\right)$$

(ii) $$\frac{\tan A-\tan B}{\tan A+\tan B} = \frac{c-b}{c}$$

$$\frac{\frac{\sin A}{\cos A}-\frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}} = \frac{k\sin C-k\sin B}{k\sin C}$$

$$\frac{\sin A\cos B-\sin B\cos A}{\sin A\cos B+\sin B\cos A} = \frac{\sin C-\sin B}{\sin C}$$

$$\frac{\sin(A-B)}{\sin(A+B)} = \frac{\sin C-\sin B}{\sin C}$$

$$\frac{\sin(A-B)}{\sin(\pi-C)} = \frac{\sin(\pi-(A+B))-\sin B}{\sin C} \qquad (\because A+B+C=\pi)$$

$$\sin(A-B) = \sin(A+B)-\sin B$$

$$\sin B = \sin(A+B)-\sin(A-B)$$

$$\sin B = 2\sin B\cos A$$

$$\sin B(2\cos A-1) = 0$$

$$\cos A = \frac{1}{2} \quad (\because \sin B \neq 0)$$

$$A = 60^\circ$$

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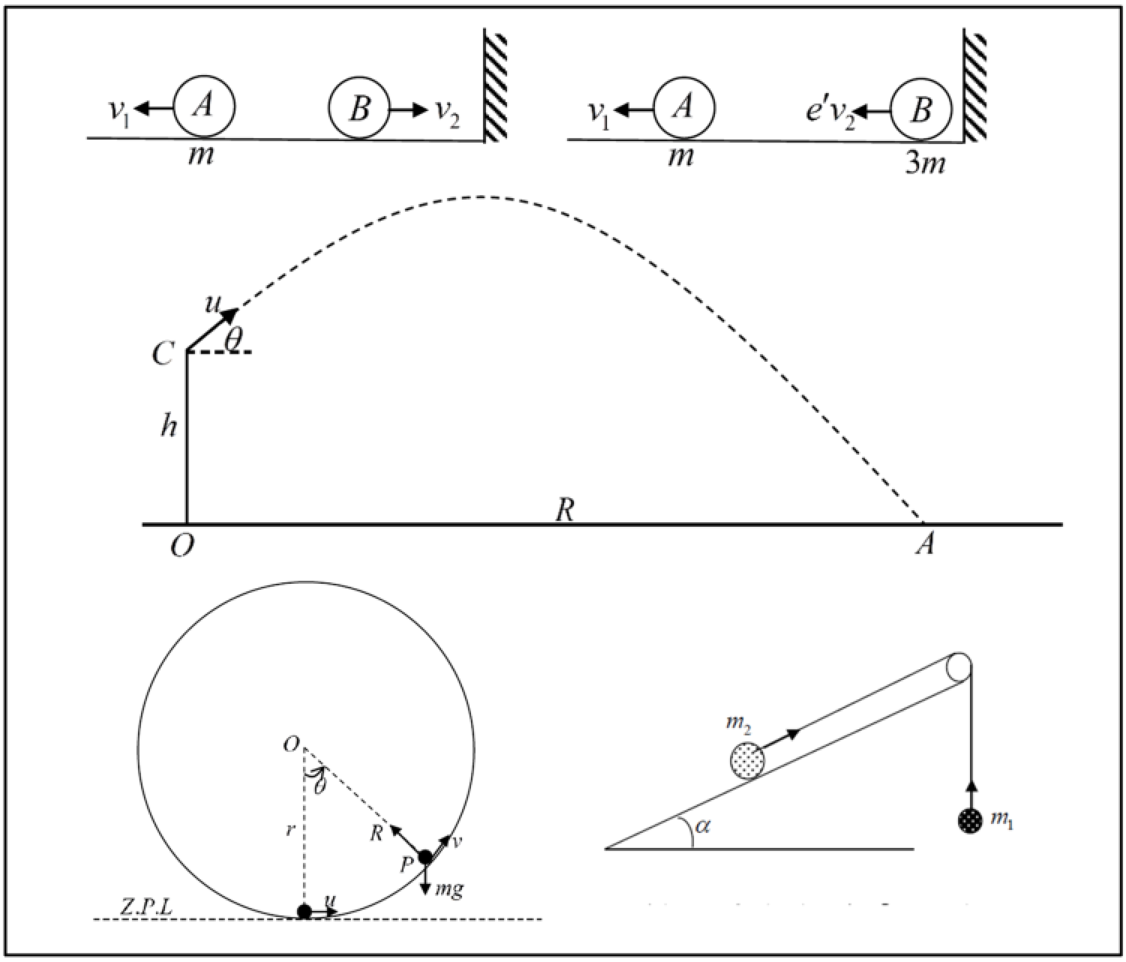
மொறட்டுவை பல்கலைக்கழக பொறியியற் பீட தமிழ் மாணவர்கள்
நடாத்தும் க.பொ.த உயர்தர மாணவர்களுக்கான 8 வது

முன்னோடிப் பரீட்சை - 2017

10(II) - இணைந்தகணிதம் II

விடைகள்



Prepared By

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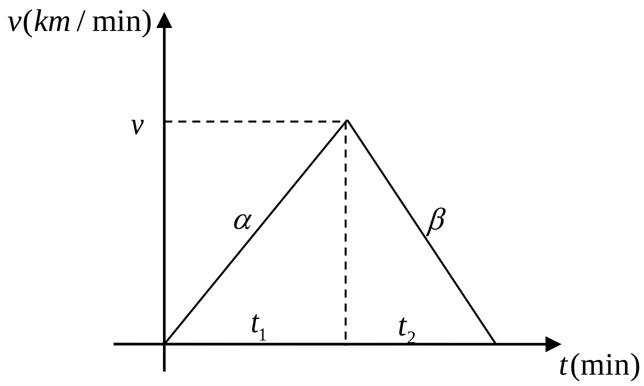
பகுதி - A

Q1). $t_1 + t_2 = 4\,\text{min}........(1)$



$$\frac{1}{2}\times 4\times v = 4$$

$$v = 2$$

$$t_1 = \frac{v}{\alpha} = \frac{2}{\alpha}$$

$$t_1 = \frac{v}{\beta} = \frac{2}{\beta}$$

$$(1) \Rightarrow \frac{2}{\alpha} + \frac{2}{\beta} = 4$$

$$\frac{1}{\alpha} + \frac{1}{\beta} = 2$$

Q2). $x = a\cos nt + b\sin nt$

$$\dot{x} = -an\sin nt + bn\cos nt$$

$$\ddot{x} = -an^2\cos nt - bn^2\sin nt$$

$$\ddot{x} = -n^2 x$$

$$\ddot{x} = -\omega^2 x \quad (\omega = n)$$

$\therefore$ துணிக்கை எளிமை இசை இயக்கத்தை ஆற்றும்.

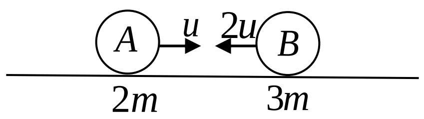
$$x = \sqrt{a^2+b^2}\left(\frac{a}{\sqrt{a^2+b^2}}\cos nt + \frac{b}{\sqrt{a^2+b^2}}\sin nt\right)$$

$$x = \sqrt{a^2+b^2}\left(\sin\alpha\cos nt + \cos\alpha\sin nt\right) \qquad \left(\alpha = \sin^{-1}\left(\frac{a}{\sqrt{a^2+b^2}}\right) = \cos^{-1}\left(\frac{a}{\sqrt{a^2+b^2}}\right)\right)$$

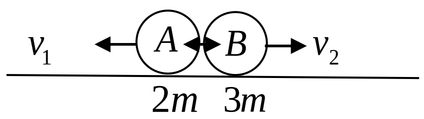
$$x = \sqrt{a^2+b^2}\sin(nt+\alpha)$$

$\therefore$ வீச்சம் $= x_{\max} = \sqrt{a^2+b^2}$

Q3).



மோத முன்



மோதிய பின்

உந்தக்காப்பு தத்துவத்தின் படி

$$\rightarrow 2m.u - 3m.2u = 3mv_2 - 2mv_1$$

$$-4u = 3v_2 - 2v_1.................(1)$$

நியூட்டனின் பரிசோதனை விதிப்படி

$$\frac{v_1+v_2}{2u+u} = \frac{1}{3}$$

$$u = v_1 + v_2............(2)$$

$$(1) + 2\times(2) \Rightarrow v_2 = -\frac{2u}{5},\ v_1 = \frac{7u}{5}$$

$2m$ இற்கு $\leftarrow I = \Delta mv \Rightarrow I = 2m\left(\frac{7u}{5} - (-u)\right) = \frac{24mu}{5}$

Q4). $\overrightarrow{OA} = 3\mathbf{a} + 2\mathbf{b}$, $\overrightarrow{OB} = 2\mathbf{a} - \mathbf{b}$

$$\overrightarrow{OA}.\overrightarrow{OB} = (3\mathbf{a} + 2\mathbf{b})(2\mathbf{a} - \mathbf{b})$$
$$= 6\mathbf{a.a} - 3\mathbf{b.a} + 4\mathbf{b.a} - 2\mathbf{b.b}$$
$$= 6|\mathbf{a}|^2 + \mathbf{a.b} - 2|\mathbf{b}|^2$$

$\mathbf{a} = 2\mathbf{i} + \mathbf{j} \Rightarrow |\mathbf{a}| = \sqrt{5}$

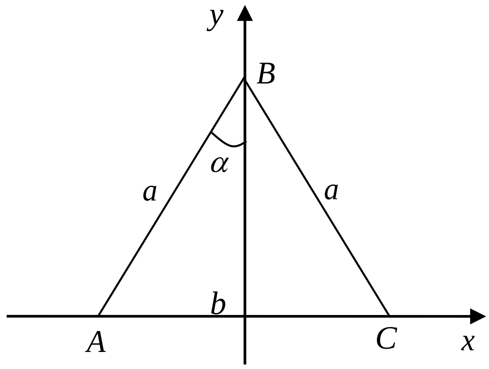
$\mathbf{b} = 2\mathbf{i} - \mathbf{j} \Rightarrow |\mathbf{b}| = \sqrt{5}$

$\mathbf{a.b} = (2\mathbf{i} + \mathbf{j})(2\mathbf{i} - \mathbf{j}) = 3$

$$\overrightarrow{OA}.\overrightarrow{OB} = 6|\mathbf{a}|^2 + \mathbf{a.b} - 2|\mathbf{b}|^2$$
$$= 30 + 3 - 10 = 23$$

Q5).

உருவம்	திணிவு	$x-$ அச்சிலிருந்து தூரம்
/	λa	$\frac{a}{2}\cos\alpha$
\	λa	$\frac{a}{2}\cos\alpha$
___	λb	0
Δ	$\lambda(2a+b)$	$\bar{y}$



சமச்சீரின் படி $\bar{x} = 0$

அலகு நீள திணிவு $= \lambda$

$$\lambda a \times \frac{a}{2}\cos\alpha + \lambda a \times \frac{a}{2}\cos\alpha = \lambda(2a+b)\bar{y}$$

$$\bar{y} = \frac{a^2}{2a+b} \times \frac{\sqrt{4a^2 - b^2}}{2a}$$

$$= \frac{a}{2}\sqrt{\frac{2a-b}{2a+b}}$$

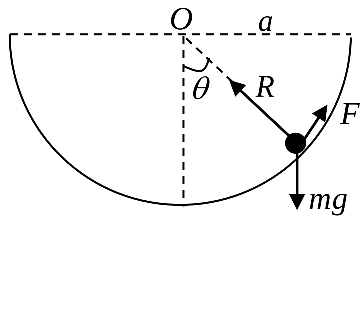
$$\sin\alpha = \frac{b}{2a}$$

Q6). O பற்றித் திருப்பம் $\Rightarrow$

$$amg\sin\theta - aF = 0$$
$$F = mg\sin\theta$$

O இனை நோக்கி $\nwarrow R - mg\cos\theta = 0$

$$R = mg\cos\theta$$



சமநிலைக்கு $\frac{F}{R} \le \mu = \tan\lambda$

$$\tan\theta \le \tan\lambda \Rightarrow \theta \le \lambda$$

துணிக்கை இருக்கும் உயரம் $h \Rightarrow \cos\theta = \frac{a-h}{a} \ge \cos\lambda \Rightarrow h \le a(1-\cos\lambda)$

துணிக்கை சமநிலையில் இருக்கும் அதி உயர் உயரம் $= a(1-\cos\lambda)$

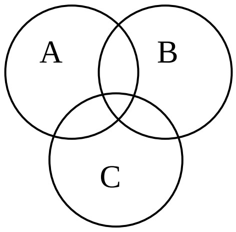
Q7). $0 < P(A) \le 1, 0 \le P(B) < 1$

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')}$$

$$= \frac{P(A \cup B)'}{P(B')}$$

$$= \frac{1 - P(A \cup B)}{P(B')}$$

Q8). $P(B) = \frac{3}{4}, P(A \cap B \cap C') = \frac{1}{3}, P(A' \cap B \cap C') = \frac{1}{3}$



$$P(B \cap C) = P(B) - P(A \cap B \cap C') - P(A' \cap B \cap C')$$

$$= \frac{3}{4} - \frac{1}{3} - \frac{1}{3}$$

$$= \frac{1}{12}$$

Q9). ஆகார வகுப்பு $= 20 - 30$

ஆகாரம் $= L + \frac{f_m - f_{m-1}}{(f_m - f_{m-1}) + (f_m - f_{m-1})} \times w$

$$24 = 20 + \frac{27 - x}{(27 - x) + (27 - y)} \times 10$$

$$3x - 2y = 27 \text{.......................(1)}$$

$$56 + x + y = 100$$

$$x + y = 44 \text{........................(2)}$$

$$(1), (2) \Rightarrow x = 23, \quad y = 21$$

இடையம் $= \frac{100}{2} = 50$ வது ஈட்டு

இடைய வகுப்பு $= 20 - 30$

இடையம் $= L + \frac{n/2 - B}{f} \times w$

$$= 20 + \frac{50 - 37}{27} \times 10$$

$$= 24.81$$

Q10). $\dfrac{1+2+6+\lambda+\mu}{5}=4.4$

$\lambda+\mu=13$.......................(1)

$8.24=\dfrac{1^2+2^2+6^2+\lambda^2+\mu^2}{5}-4.4^2$

$\lambda^2+\mu^2=97$.................(2)

$(1),(2)\Rightarrow$

$(13-\mu)^2+\mu^2=97$

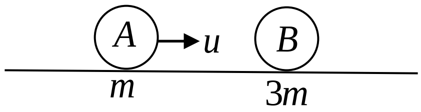
$(\mu-4)(\mu-9)=0$

$\mu=4\Rightarrow\lambda=9$

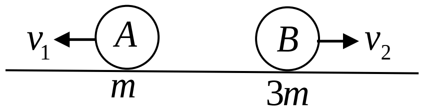
$\mu=9\Rightarrow\lambda=4$

பகுதி – **B**

Q11). $(a)(i)$



மோத முன்



மோதிய பின்

உந்தக்காப்பு தத்துவத்தின் படி

$\rightarrow mu = 3mv_2 - mv_1$

$u = 3v_2 - v_1$.................(1)

நியூட்டனின் பரிசோதனை விதிப்படி

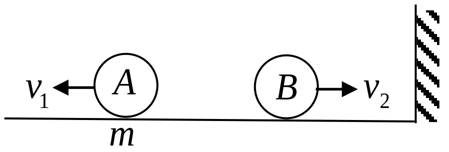
$\dfrac{v_2 + v_1}{u} = e$

$eu = v_2 + v_1$..................(2)

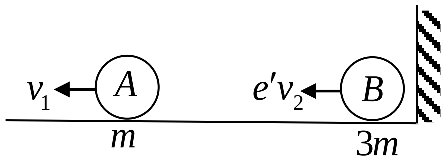
$(1)+(2) \Rightarrow v_2 = \dfrac{(e+1)u}{4}$

(ii) $v_1 = \dfrac{(3e-1)u}{4}$

(iii)



சுவரை மோத முன்



சுவரை மோதிய பின்

B சுவரை மோதிய பின் அதன் வேகம் $= e'v_2$

B ஆனது A இனை மோத $e'v_2 > v_1$

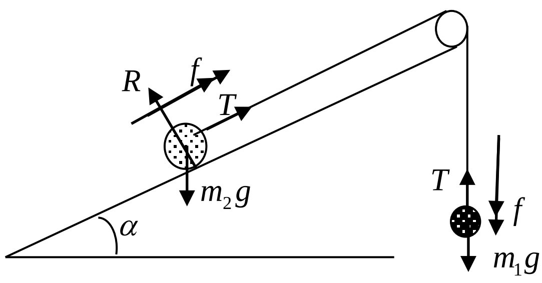
$e' = \dfrac{1}{2} \Rightarrow \dfrac{1}{2} \times \dfrac{(e+1)u}{4} > \dfrac{(3e-1)u}{4}$

$e < \dfrac{3}{5}$............................(3)

அதேவேளை A பின்னடிப்பதால் $v_1 > 0 \Rightarrow e > \dfrac{1}{3}$...................(4)

$(3),(4) \Rightarrow \dfrac{1}{3} < e < \dfrac{3}{5}$

(b)



(i) m_1 இற்கு $\downarrow F = ma$ இட

$$m_1 g - T = m_1 f \text{..................}(1)$$

m_2 இற்கு தளம் வழியே $\nearrow F = ma$ இட

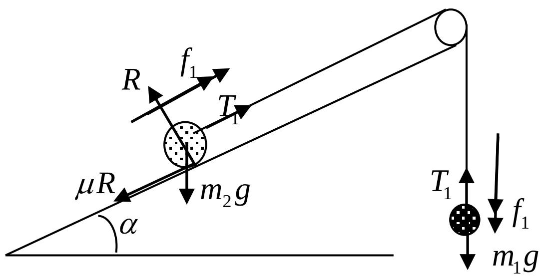
$$T - m_2 g\sin\alpha = m_2 f \text{..................}(2)$$

$$(1)+(2) \Rightarrow f = \frac{m_1 - m_2\sin\alpha}{m_1 + m_2} g$$

$$T = m_1 g - m_1 \frac{m_1 - m_2\sin\alpha}{m_1 + m_2} g$$

$$T = \frac{m_1 m_2 g(1+\sin\alpha)}{m_1 + m_2}$$

(ii)



உராய்வு விசை $= \mu R$

m_1 இற்கு $\downarrow F = ma$ இட

$$m_1 g - T_1 = m_1 f_1 \text{..................}(1)$$

m_2 இற்கு தளத்திற்கு செங்குத்தாக $\nwarrow F = ma$ இட

$$R - m_2 g\cos\alpha = 0$$

$$R = m_2 g\cos\alpha$$

m_2 இற்கு தளம் வழியே $\nearrow F = ma$ இட

$$T_1 - m_2 g\sin\alpha - \mu R = m_2 f_1$$

$$T_1 - m_2 g\sin\alpha - \mu m_2 g\cos\alpha = m_2 f_1 \text{..................}(2)$$

$$(1)+(2) \Rightarrow f_1 = \frac{m_1 - m_2(\sin\alpha + \mu\cos\alpha)}{m_1 + m_2} g$$

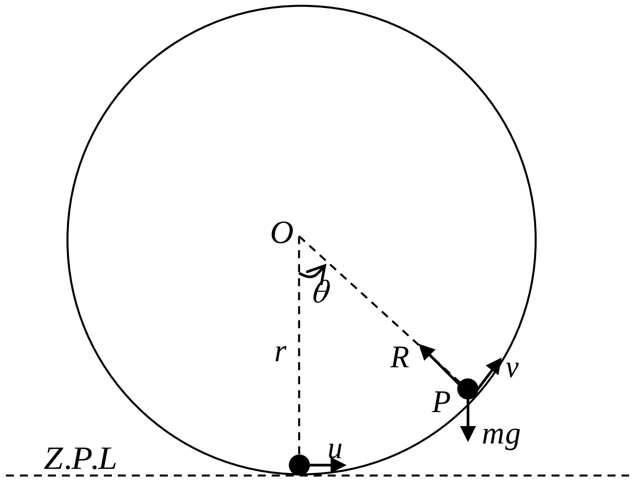
$$T_1 = m_1 g - m_1 \frac{m_1 - m_2(\sin\alpha + \mu\cos\alpha)}{m_1 + m_2} g$$

$$T_1 = \frac{m_1 m_2 g(1+\sin\alpha + \mu\cos\alpha)}{m_1 + m_2}$$

(iii) $$f - f_1 = \frac{m_1 - m_2\sin\alpha}{m_1 + m_2} g - \frac{m_1 - m_2(\sin\alpha + \mu\cos\alpha)}{m_1 + m_2} g$$

$$= \frac{\mu m_2 \cos\alpha}{m_1 + m_2} g$$

Q12). (a)



சக்திக்காப்பு விதிப்படி

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + mgr(1-\cos\theta)$$

$$v^2 = u^2 - 2gr(1-\cos\theta)$$

$$\omega = \frac{1}{r}\sqrt{u^2 - 2gr(1-\cos\theta)} \qquad (\because v = r\omega)$$

P யிற்கு $\nwarrow F = ma \Rightarrow$

$$R - mg\cos\theta = \frac{mv^2}{r}$$

$$R = mg\cos\theta + \frac{m}{r}\left(u^2 - 2gr(1-\cos\theta)\right)$$

$$R = \frac{m}{r}\left(u^2 - gr(2-3\cos\theta)\right)$$

$\theta = \pi$ இல் $R \geq 0$ எனின் P பூரண வட்ட இயக்கத்தை ஆற்றும்.

$$\frac{m}{r}\left(u^2 - gr(2-3\cos\pi)\right) \geq 0$$

$$u^2 - 5gr \geq 0$$

$$u \geq \sqrt{5gr}$$

$$\omega^2 = \frac{1}{r^2}\left(u^2 - 2gr(1-\cos\theta)\right)$$

$$\cos\theta = 1 \Rightarrow \omega_1^2 = \frac{u^2}{r^2}$$

$$\cos\theta = -1 \Rightarrow \omega_2^2 = \frac{1}{r^2}\left(u^2 - 4gr\right)$$

$$\omega_1^2\cos^2\frac{1}{2}\theta + \omega_2^2\sin^2\frac{1}{2}\theta = \frac{u^2}{r^2}\cos^2\frac{1}{2}\theta + \frac{1}{r^2}\left(u^2 - 4gr\right)\sin^2\frac{1}{2}\theta$$

$$= \frac{1}{r^2}\left(u^2 - 4gr\sin^2\frac{1}{2}\theta\right)$$

$$= \frac{1}{r^2}\left(u^2 - 2gr(1-\cos\theta)\right) = \omega^2$$

$$\therefore \omega = \sqrt{\omega_1^2 \cos^2 \frac{1}{2}\theta + \omega_2^2 \sin^2 \frac{1}{2}\theta}$$

$$R = \frac{m}{r}\left(u^2 - gr(2 - 3\cos\theta)\right)$$

$$\cos\theta = 1 \Rightarrow R_1 = \frac{m}{r}\left(u^2 + gr\right)$$

$$\cos\theta = -1 \Rightarrow R_2 = \frac{m}{r}\left(u^2 - 5gr\right)$$

$$R_1 \cos^2 \frac{1}{2}\theta + R_2 \sin^2 \frac{1}{2}\theta = \frac{m}{r}\left(u^2 + gr\right)\cos^2 \frac{1}{2}\theta + \frac{m}{r}\left(u^2 - 5gr\right)\sin^2 \frac{1}{2}\theta$$

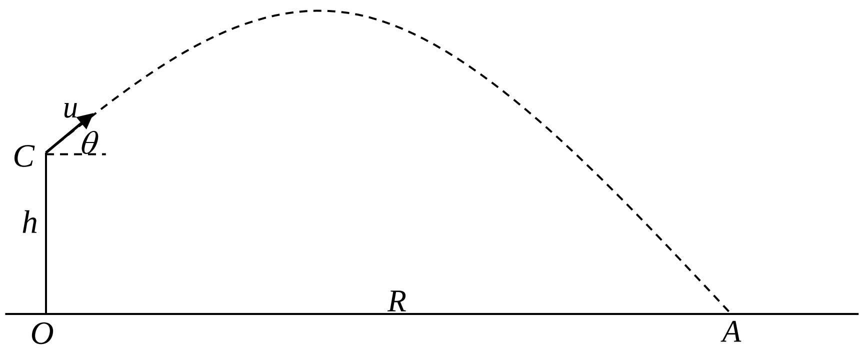
$$= \frac{m}{r}\left(u^2 + gr\left(\cos^2 \frac{1}{2}\theta - 5\sin^2 \frac{1}{2}\theta\right)\right)$$

$$= \frac{m}{r}\left(u^2 + gr\left(1 - 6\sin^2 \frac{1}{2}\theta\right)\right)$$

$$= \frac{m}{r}\left(u^2 + gr\left(1 - 3(1 - \cos\theta)\right)\right)$$

$$= \frac{m}{r}\left(u^2 - gr\left(2 - 3\cos\theta\right)\right) = R$$

(*b*)



$$C \to A \quad \to s = ut$$

$$R = u\cos\theta t$$

$$t = \frac{R}{u\cos\theta}\text{......................}(1)$$

$$C \to A \quad \uparrow s = ut + \frac{1}{2}at^2$$

$$-h = u\sin\theta t - \frac{1}{2}gt^2\text{.................}(2)$$

$$(1),(2) \Rightarrow -h = u\sin\theta \times \frac{R}{u\cos\theta} - \frac{1}{2}g\left(\frac{R}{u\cos\theta}\right)^2$$

$$-h = R\tan\theta - \frac{gR^2}{2u^2}\sec^2\theta$$

$$-h = R\tan\theta - \frac{gR^2}{2u^2}(1 + \tan^2\theta)$$

$$R^2\tan^2\theta - \frac{2u^2}{g}R\tan\theta + R^2 - \frac{2hu^2}{g} = 0$$

$\tan\theta$ இன் மெய்த்தீர்வுகளுக்கு $b^2 - 4ac \geq 0$

$$\Rightarrow \left(-\frac{2u^2}{g}R\right)^2 - 4R^2\left(R^2 - \frac{2hu^2}{g}\right) \geq 0$$

$$R^2\left(R^2 - \frac{2hu^2}{g} - \frac{u^4}{g^2}\right) \leq 0$$

$$\left(R^2 - \frac{2hu^2}{g} - \frac{u^4}{g^2}\right) \leq 0 \quad \left(\because R^2 > 0\right)$$

$$R \leq \sqrt{\frac{u^4}{g^2} + \frac{2hu^2}{g}}$$

R இன் உயர் வீச்சு $= \sqrt{\frac{u^4}{g^2} + \frac{2hu^2}{g}}$

உயர் வீச்சு $R' \Rightarrow R' = \sqrt{\frac{u^4}{g^2} + \frac{2hu^2}{g}}$

$\theta \to \alpha$ ஆக $R \to R' \Rightarrow R'^2 \tan^2\alpha - \frac{2u^2}{g}R'\tan\alpha + R'^2 - \frac{2hu^2}{g} = 0$

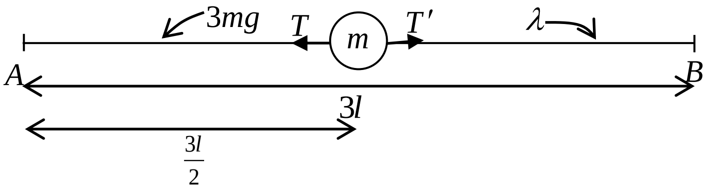
$$\left(R'\tan\alpha - \frac{u^2}{g}\right)^2 - \frac{u^4}{g^2} + R'^2 - \frac{2hu^2}{g} = 0$$

$$\left(R'\tan\alpha - \frac{u^2}{g}\right)^2 - \frac{u^4}{g^2} + \left(\frac{u^4}{g^2} + \frac{2hu^2}{g}\right) - \frac{2hu^2}{g} = 0$$

$$\tan\alpha = \frac{u^2}{gR'}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{2\left(\frac{u^2}{gR'}\right)}{1-\left(\frac{u^2}{gR'}\right)^2} = \frac{2R'\left(\frac{u^2}{g}\right)}{R'^2 - \left(\frac{u^4}{g^2}\right)} = \frac{2R'\left(\frac{u^2}{g}\right)}{\frac{2hu^2}{g}} = \frac{R'}{h}$$

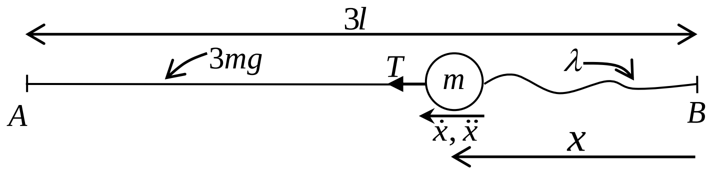
Q13).



துணிக்கையின் நாப்ப நிலைக்கு $T = T'$

$$\frac{3mg \times l/2}{l} = \frac{\lambda \times l/2}{l}$$

$$\lambda = 3mg$$



இழை இறுகமுன் $\leftarrow F = ma$ இட

$$\frac{3mg(2l-x)}{l} = m\ddot{x}$$

$$\ddot{x} = \frac{3g(2l-x)}{l}$$

$$X = 2l - x \Rightarrow \ddot{X} = -\ddot{x}$$

$$\ddot{X} = -\frac{3gX}{l}$$

$$\ddot{X} = -\omega^2 X \qquad \left(\omega^2 = \frac{3g}{l}\right)$$

$X = 0 \Rightarrow x = 2l$ இல் அலைவு மையம்.

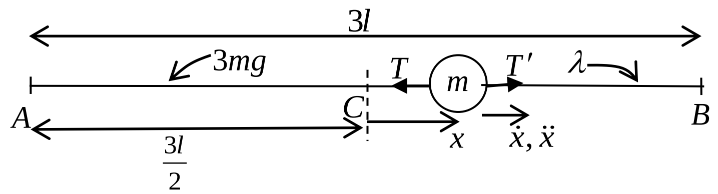
$$\therefore A = 2l$$

இழை இறுகும் போது $x = l \Rightarrow X = l$

$$V^2 = \omega^2\left(A^2 - X^2\right)$$

$$V^2 = \frac{3g}{l}\left((2l)^2 - l^2\right)$$

$$V = 3\sqrt{gl}$$



$\rightarrow F = ma$ இட

$$T' - T = m\ddot{x}$$

$$\frac{3mg\left(l/2 - x\right)}{l} - \frac{3mg\left(l/2 + x\right)}{l} = m\ddot{x}$$

$$\ddot{x} = -\frac{6g}{l}x$$

$$\frac{d^2x}{dt^2} + \frac{6g}{l}x = 0$$

தீர்வு: $x = A\cos\omega t + B\sin\omega t$

$t = 0$ இல் $x = l/2 \Rightarrow A = l/2$

$$\dot{x} = -A\omega\sin\omega t + B\omega\cos\omega t$$

$$\ddot{x} = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$$

$$\ddot{x} + \omega^2 x = 0$$

$$\omega = \sqrt{\frac{6g}{l}}$$

$t=0$ இல் $\dot{x}=-3\sqrt{gl} \Rightarrow B\omega=-3\sqrt{gl}$

$$B=-\sqrt{\frac{3}{2}}l$$

$$x=\frac{l}{2}\cos\sqrt{\frac{6g}{l}}t-\sqrt{\frac{3}{2}}l\sin\sqrt{\frac{6g}{l}}t$$

$$x=-\frac{l}{2}\Rightarrow -\frac{l}{2}=\frac{l}{2}\cos\omega t-\sqrt{\frac{3}{2}}l\sin\omega t$$

$$\frac{1}{2}(1+\cos\omega t)=\sqrt{\frac{3}{2}}\sin\omega t$$

$$\cos^2\left(\frac{\omega t}{2}\right)=\sqrt{6}\sin\left(\frac{\omega t}{2}\right)\cos\left(\frac{\omega t}{2}\right)$$

$$\tan\left(\frac{\omega t}{2}\right)=\frac{1}{\sqrt{6}} \quad \left(\because 0<\frac{\omega t}{2}<\frac{\pi}{2}\right)$$

$$\operatorname{cosec}^2\left(\frac{\omega t}{2}\right)=1+\cot^2\left(\frac{\omega t}{2}\right)$$

$$\operatorname{cosec}^2\left(\frac{\omega t}{2}\right)=7$$

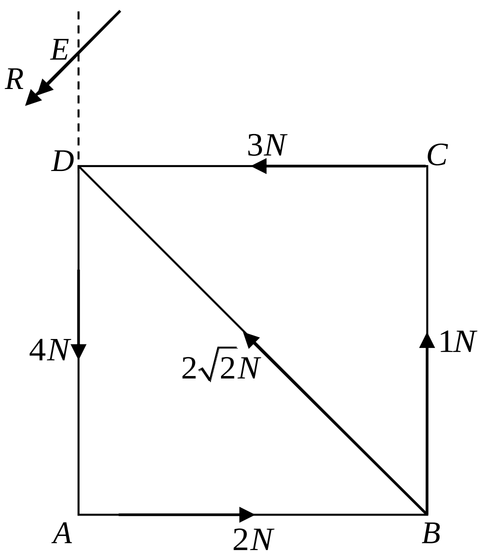
$$\operatorname{cosec}\left(\frac{\omega t}{2}\right)=\sqrt{7} \quad \left(\because 0<\frac{\omega t}{2}<\frac{\pi}{2}\right)$$

$$\sin\left(\frac{\omega t}{2}\right)=\frac{1}{\sqrt{7}}$$

$$\frac{\omega t}{2}=\sin^{-1}\left(\frac{1}{\sqrt{7}}\right) \quad \left(\because 0<\frac{\omega t}{2}<\pi\right)$$

$$t=\left(\frac{2l}{3g}\right)^{\frac{1}{2}}\sin^{-1}\left(\frac{1}{\sqrt{7}}\right)$$

Q14). (a)

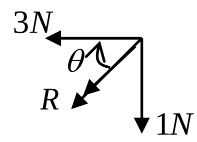


$\leftarrow X = -2 + 2\sqrt{2}\cos 45^\circ + 3$

$X = 3N$

$\downarrow Y = -1 - 2\sqrt{2}\sin 45^\circ + 4$

$Y = 1N$



விளையுள் $R = \sqrt{10}N$

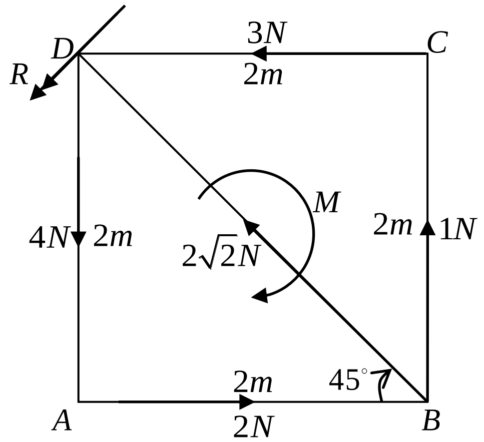
$\theta = \tan^{-1}\left(\frac{1}{3}\right)$

D பற்றி தொகுதியின் திருப்பம் $=$ D பற்றி விளையுளின் திருப்பம்

$\circlearrowleft\ 2N \times 2m + 1N \times 2m = \sqrt{10}\cos\theta \times DE$

$DE = 2m \Rightarrow AE = 4m$

(i) E இல் $\measuredangle\theta$ திசையில் $\sqrt{10}N$ சேர்க்க தொகுதி சமநிலை அடையும்.

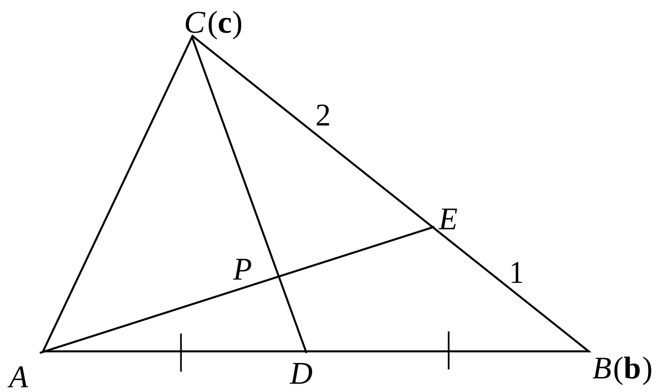


(ii) இணை சேர்த்த பின் D பற்றிய திருப்பம் $=0$

$\circlearrowright\ M - 2N \times 2m - 1N \times 2m = 0$

$M = 6Nm$ வலஞ்சுழிப் போக்கு

(b)



$\overrightarrow{AB} = \mathbf{b},\ \overrightarrow{AC} = \mathbf{c}$

$\overrightarrow{AD} = \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}\mathbf{b}$

$\overrightarrow{BC} = \mathbf{c}\text{-}\mathbf{b}$

$\overrightarrow{BE} = \frac{1}{3}\overrightarrow{BC} = \frac{1}{3}(\mathbf{c}\text{-}\mathbf{b})$

$\overrightarrow{CD} = \left(\frac{1}{2}\mathbf{b} - \mathbf{c}\right)$

$\overrightarrow{AE} = \overrightarrow{AB} + \overrightarrow{BE} = \frac{1}{3}(2\mathbf{b} + \mathbf{c})$

$\overrightarrow{AP} = \lambda\overrightarrow{AE}, \overrightarrow{CP} = \mu\overrightarrow{CD}$ என்க.

$\overrightarrow{AC} = \overrightarrow{AP} + \overrightarrow{PC}$

$= \frac{\lambda}{3}(2\mathbf{b} + \mathbf{c}) - \mu\left(\frac{1}{2}\mathbf{b} - \mathbf{c}\right)$

$\mathbf{c} = \left(\frac{2\lambda}{3} - \frac{\mu}{2}\right)\mathbf{b} + \left(\frac{\lambda}{3} + \mu\right)\mathbf{c}$

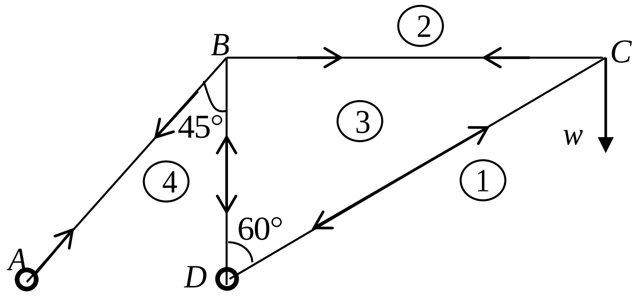
$\left(\frac{2\lambda}{3} - \frac{\mu}{2}\right)\mathbf{b} + \left(\frac{\lambda}{3} + \mu - 1\right)\mathbf{c} = 0$

$\therefore \frac{2\lambda}{3} - \frac{\mu}{2} = 0, \ \frac{\lambda}{3} + \mu - 1 = 0 \ \left(\because \mathbf{b} \not\parallel \mathbf{c}\right)$

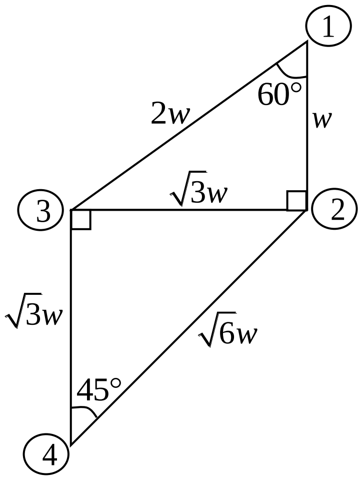
$\Rightarrow \lambda = \frac{3}{5}, \mu = \frac{4}{5}$

$\therefore \frac{AP}{PE} = \frac{3}{2}, \ \frac{CP}{PD} = \frac{4}{1}$

Q15). (*a*)

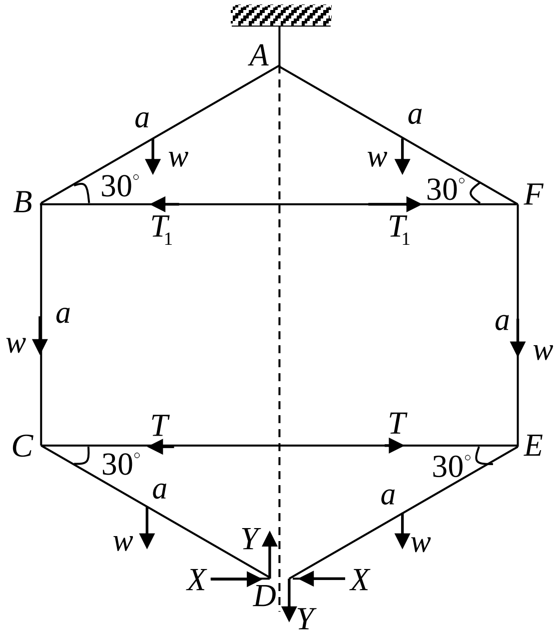


தகைப்பு வரிப்படம்



கோல்	தகைப்பு	இழுவை	உதைப்பு
AB	②④	$\sqrt{6}w$	–
BC	②③	$\sqrt{3}w$	–
CD	①③	–	$2w$
BD	③④	–	$\sqrt{3}w$

(b)



சமச்சீரின் படி $Y = 0$

CD யின் சமநிலைக்கு,

$$\overset{\curvearrowleft}{C} \quad X\,a\sin 30^\circ - w\frac{a}{2}\cos 30^\circ = 0$$

$$X = \frac{\sqrt{3}w}{2}$$

$BC,\ CD$ யின் சமநிலைக்கு,

$$\overset{\curvearrowleft}{B} \quad X\left(a + a\sin 30^\circ\right) - w\frac{a}{2}\cos 30^\circ - Ta = 0$$

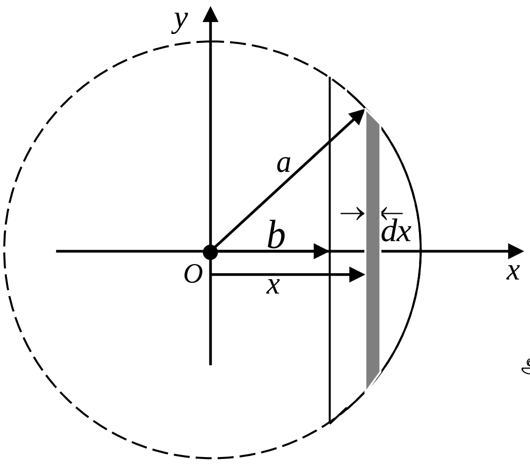
$$T = \frac{\sqrt{3}w}{2}$$

$AB,\ BC,\ CD$ யின் சமநிலைக்கு,

$$\overset{\curvearrowleft}{A} \quad X\left(a + 2a\sin 30^\circ\right) + 2\times w\frac{a}{2}\cos 30^\circ + w\,a\cos 30 - T\left(a + a\sin 30^\circ\right) - T_1 a\sin 30^\circ = 0$$

$$T_1 = \frac{5\sqrt{3}w}{2}$$

Q16).



அலகுக் கனவளவுக்கான திணிவு $= \rho$

சமச்சீரின் படி $\bar{y}=0$

$$\left(\int_b^a \pi(a^2-x^2)\rho dx\right)\bar{x}=\int_b^a \pi(a^2-x^2)x\rho dx$$

$$\left(a^2x-\frac{x^3}{3}\right)_b^a \bar{x}=\left(\frac{a^2x^2}{2}-\frac{x^4}{4}\right)_b^a$$

$$\left(\left(a^3-\frac{a^3}{3}\right)-\left(a^2b-\frac{b^3}{3}\right)\right)\bar{x}=\left(\frac{a^4}{2}-\frac{a^4}{4}\right)-\left(\frac{a^2b^2}{2}-\frac{b^4}{4}\right)$$

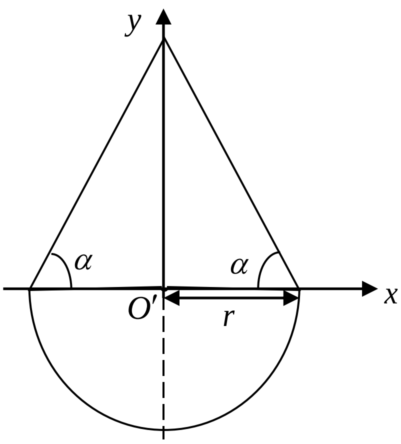
$$\left(\frac{2a^3+b^3-3a^2b}{3}\right)\bar{x}=\left(\frac{a^4+b^4-2a^2b^2}{4}\right)$$

$$\bar{x}=\frac{3\left(a^2+b^2-2ab\right)\left(a^2+b^2+2ab\right)}{4\left(2a+b\right)\left(a^2+b^2-2ab\right)}$$

$$\bar{x}=\frac{3}{4}\frac{\left(a+b\right)^2}{\left(2a+b\right)}$$

திண்ம அரைக்கோளத்திற்கு $b\to 0 \Rightarrow$

$$\bar{x}=\frac{3}{4}\frac{\left(a+0\right)^2}{\left(2a+0\right)}=\frac{3a}{8}$$



அலகுக் கனவளவுக்கான திணிவு $=\rho$

சமச்சீரின் படி $\bar{x}=0$

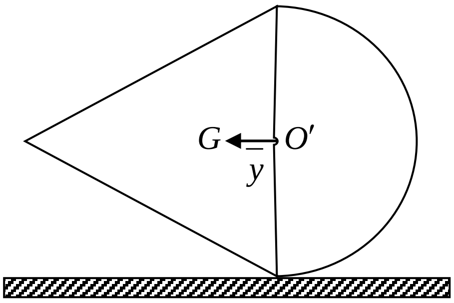
கூம்பின் உயரம் $h=r\tan\alpha$

$$\frac{1}{3}\pi r^2h\rho\times\frac{h}{4}-\frac{2}{3}\pi r^3\rho\times\frac{3r}{8}=\left(\frac{1}{3}\pi r^2h\rho+\frac{2}{3}\pi r^3\rho\right)\bar{y}$$

$$\frac{(r\tan\alpha)^2}{4}-\frac{6r^2}{8}=\left(r\tan\alpha+2r\right)\bar{y}$$

$$\bar{y}=\frac{r\left(\tan^2\alpha-3\right)}{4\tan\alpha+8}$$

$\therefore O'$ இலிருந்து தூரம் $=\dfrac{r\left|\tan^2\alpha-3\right|}{4\tan\alpha+8}$



$(i)\ \alpha < \tan^{-1}\left(\sqrt{3}\right) \Rightarrow \bar{y} < 0$

$\Rightarrow G$ அரைக்கோளத்தினுள் கிடக்கும்

$\therefore$ பொருள் வலஞ்சுழியாக திரும்பி நிலைக்குத்தாக சமநிலை அடையும்

$(ii)\ \alpha > \tan^{-1}\left(\sqrt{3}\right) \Rightarrow \bar{y} > 0$

$\Rightarrow G$ கூம்பினுள் கிடக்கும்

$\therefore$ பொருள் இடஞ்சுழியாக திரும்பும். (கவிண்டு விழும்)

$(iii)\ \alpha = \tan^{-1}\left(\sqrt{3}\right) \Rightarrow \bar{y} = 0$

$\Rightarrow G$ ஆனது O' மீது கிடக்கும்

$\therefore$ பொருள் அந்நிலையிலேயே சமநிலை அடையும்

Q17). (a) $P(A) = P(A/B) = \dfrac{1}{4}, P(B/A) = \dfrac{1}{2}$

$(i)\ P(B/A) = \dfrac{P(B \cap A)}{P(A)} = \dfrac{1}{2}$

$P(B \cap A) = \dfrac{1}{8} \neq 0$

$\therefore A, B$ தம்முள் புறநீங்கும் நிகழ்ச்சிகள் அல்ல.

$(ii)\ P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{1}{4}$

$P(B) = \dfrac{1}{2}$

$P(A)\,P(B) = \dfrac{1}{4} \times \dfrac{1}{2} = \dfrac{1}{8} = P(A \cap B)$

$\therefore A, B$ சாரா நிகழ்ச்சிகள்.

$(iii)\ P(A'/B) = 1 - P(A/B) = \dfrac{3}{4}$

$(iv)\ P(A'/B') = \dfrac{P(A' \cap B')}{P(B')} = \dfrac{P(A \cup B)'}{P(B')}$

$$= \frac{1 - \left(P(A) + P(B) - P(A \cap B)\right)}{P(B')}$$

$$= \frac{1 - \left(\frac{1}{4} + \frac{1}{2} - \frac{1}{8}\right)}{\frac{1}{2}} = \frac{3}{4}$$

(b) $P(A)=\frac{1}{2},\ P(B)=\frac{3}{4},\ P(C)=\frac{1}{4}$

$P(A\cap B)=P(A)P(B)=\frac{3}{8}$ $\quad\because A, B$ சாரா நிகழ்ச்சிகள்

$P(B\cap C)=P(B)P(C)=\frac{3}{16}$ $\quad\because B, C$ சாரா நிகழ்ச்சிகள்

$P(A\cap C)=P(A)P(C)=\frac{1}{8}$ $\quad\because A, C$ சாரா நிகழ்ச்சிகள்

$P(A\cap B\cap C)=P(A)P(B)P(C)=\frac{3}{32}$ $\quad\because A, B, C$ சாரா நிகழ்ச்சிகள்

வினா தீர்க்கப்படுவதற்கான நிகழ்தகவு $=P(A\cup B\cup C)$

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap B)+P(A\cap B\cap C)$$
$$=\frac{1}{2}+\frac{3}{4}+\frac{1}{4}-\frac{3}{8}-\frac{3}{16}-\frac{1}{8}+\frac{3}{32}$$
$$=\frac{29}{32}$$

(c) ஆகாரம் $=L+\dfrac{f_m-f_{m-1}}{(f_m-f_{m-1})+(f_m-f_{m-1})}\times w$

$$38=30+\frac{(f-24)-18}{(f-24-18)+((f-24)-(67-f))}\times 10$$

$f=46$

புள்ளிகள்	நடுப்பெறுமானம் x	எண்ணிக்கை	திரள் மீடிறன்	$x'=\frac{x-45}{10}$	fx'	fx'^2
$0-10$	5	4	4	-4	-16	64
$10-20$	15	2	6	-3	-6	18
$20-30$	25	18	24	-2	-36	72
$30-40$	35	22	46	-1	-22	22
$40-50$	45	21	67	0	0	0
$50-60$	55	19	86	1	19	19
$60-70$	65	10	96	2	20	40
$70-80$	75	4	100	3	12	36
$80-90$	85	1	101	4	4	16
		101			-25	287

$$x' = \frac{x-45}{10}$$

$$x = 10x' + 45$$

$$\overline{x} = 10\overline{x'} + 45$$

$$\overline{x'} = \frac{\sum f_i x_i'}{\sum f_i} = \frac{-25}{101} \simeq -0.25$$

$$\overline{x} = 42.5$$

$$\sigma'^2 = \frac{\sum f_i x_i'^2}{\sum f_i} - \overline{x'}^2 = \frac{287}{101} - (0.25)^2 = 2.78$$

$$\sigma^2 = 100\sigma'^2 = 278$$

இடையம் $= \frac{101}{2} = 51.5$ வது ஈட்டு

இடைய வகுப்பு $= 40 - 50$

இடையம் $= L + \frac{n/2 - B}{f} \times w$

$$= 40 + \frac{50.5 - 46}{21} \times 10$$

$$= 42.14$$