மொறட்டுவை பல்கலைக்கழக பொறியியற் பீட தமிழ் மாணவர்கள் நடாத்தும் க.பொ.த உயர்தர மாணவர்களுக்கான 8வது முன்னோடிப் பரீட்சை - 2017

கல்விப் பொதுத் தராதரப் பத்திர(உயர் தர) முன்னோடிப் பரீட்சை - 2017

General Certificate of Education (Adv.Level) Pilot Examination - 2017

இணைந்த கணிதம் I
Combined Maths I

10 | E | I

மூன்று மணித்தியாலம்
Three hours

Instructions

* This question paper consists of two parts; **PART A** (Questions 1- 10) and **PART B** (Questions 11 – 17)
* **Part** ***A***

  Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
* **Part** ***B***

  Answer **five** questions only.
* At the end of the time allotted, tie the answers of the two parts together so that **PART A** is on top of **PART B** before handing them over to the supervisor.
* You are permitted to remove only **PART B** of the question paper from the Examination Hall.

(10) **Combined Mathematics I**

Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Total for Paper I		

Paper I	
Paper II	
Total	
Final Marks	

Combined Mathematics I

[see pg.2

Q1). Using **principle of mathematical induction**, show that $7^n - 2^n$ is divisible by 5 for all $n \in \mathbb{Z}^+$.

Q2). Find the number of arrangements that can be made using all the letters in the word **BANANA**. How many of these arrangements do not have the two **N**s next to each other?

[*see pg*.3

Q3). Find a such that $\left(\sqrt{3}+i\right)\left(a+i\right)=2\left(a-i\right)$. Find the modulus and argument of the complex number $\dfrac{2+\sqrt{3}-i}{2+\sqrt{3}+i}$.

Q4). Show that $\lim\limits_{x\to 1}\dfrac{x^3-x^2\ln x+\ln x-1}{x^2-1}=\dfrac{3}{2}$.

[***see pg.4***

Q5). Write the expansion of $(x-1)^n$. **Hence,** find the remainder when 2^{2003} is divided by 17.

Q6). Find the area of the shape formed by the intersection of the curve $y=x^2+3$, straight lines $y=4, y=6$ and $x=0$.

[see pg.5

Q7). Consider the curve $y = be^{-\frac{x}{a}}$. Show that it touches the straight line $\frac{x}{a}+\frac{y}{b}=1$ at the point of intersection of this curve with the y axis,.

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Q8). Show that the locus of foot of the perpendicular drawn from the origin to the straight line passing through the point $(2,3)$ is a circle. Write its center and radius.

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[*see pg*.6

Q9). A straight line passes through the point $\left(-\sqrt{8}, \sqrt{8}\right)$ and makes an angle of $135°$ with the positive direction of x axis. Show that this straight line intersect the circle $x = 5\cos\theta, y = 5\sin\theta$. Show that the length of this intersecting chord is 10. Here θ is a parameter.

Q10). Show that $2\tan^{-1}(-3) = \cos^{-1}\left(\dfrac{4}{5}\right) - \pi$.

[see pg.7

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இணைந்த கணிதம் I
Combined Maths I

10 | E | I

Part ***B*** *** Answer five questions only**

Q11) **a.** Three quadratic functions in x, $f(x), g(x), h(x)$ are given as follows.

$$f(x) = x^2 + x + 1$$
$$g(x) = 4x^2 + (m+3)x + 4$$
$$h(x) = 2x^2 + (3-m)x + 2$$

Here $m \in \mathbb{R}$.

$(\mathbf{i})$ Show that $f(x) > 0$ for all $x \in \mathbb{R}$.

$(\mathbf{ii})$ Show that $g(x) > 0$ for $-11 < m < 5$.

$(\mathbf{iii})$ Show that $h(x) > 0$ for $-1 < m < 7$.

Show that $-3 < \dfrac{x^2 + mx + 1}{x^2 + x + 1} < 3$ **if and only if** $-1 < m < 5$.

b. Let $f(x) = x^4 + 2x^3 - 3x^2 - 2x + 3$.

By using remainder theorem again and again, show that the remainder when $f(x)$ is divided by $(x-2)^2(x-3)$ is in the form $a(x-2)^2 + b(x-2) + c$. Here a, b, c are constants to be determined.

Q12) **a.** Simplify $\dfrac{1}{1+a^{n-1}} - \dfrac{1}{1+a^n}$. Here $a \in \mathbb{R}^+ - \{1\}$.

Find $f(r)$ such that $\dfrac{a^{r-1}}{(1+a^{r-1})(1+a^r)} = f(r-1) - f(r)$.

Show that $\displaystyle\sum_{r=1}^{n} \frac{a^{r-1}}{(1+a^{r-1})(1+a^r)} = \frac{a^n - 1}{2(a-1)(a^n+1)}$ and **deduce** $\displaystyle\sum_{r=1}^{n} \frac{2^r}{(1+2^{r-1})(1+2^r)} < 1$.

Find $\displaystyle\lim_{n\to\infty} \sum_{r=1}^{n} \frac{2017^r}{(1+2017^{r-1})(1+2017^r)}$.

b. Draw the two graphs $y = |x^2 - 2x|$ and $y = |1 - 2x|$ on same diagram.

Hence, find the set of real values of x satisfying the inequality $|x^2 - 2x| \le |1 - 2x|$.

[*see pg.8*

Q13) **a.** Show that $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. Here $a,b,c,d \in \mathbb{R}$.

Hence, deduce the condition for the existence of inverse for a 2×2 real number matrix.

Let $\mathbf{A} = \begin{pmatrix} 3 & 7 \\ 1 & 2 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 4 & -1 \\ -3 & 1 \end{pmatrix}$. Find $\mathbf{AB}$ and $(\mathbf{AB})^{-1}$.

Show that,

(**i**) $(\mathbf{AB})^{-1} \neq \mathbf{A}^{-1}\mathbf{B}^{-1}$

(**ii**) $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

b. Shade the region $\{z\in\mathbb{C} : |z| \le 4\} \cap \left\{z \in \mathbb{C} : \mathrm{Im}\left(\dfrac{z-1+\sqrt{3}i}{1-\sqrt{3}i}\right) \ge 0\right\} \cap \{z\in\mathbb{C} : \mathrm{Re}(z) \ge 0\}$.

Show that the area of this region is $\dfrac{20\pi}{3}$.

c. Let z be a complex number. Show that $|z|^2 = z\bar{z}$.

While z_1, z_2 are two non-zero complex numbers, show that

$|z_1+z_2|^2 = |z_1|^2 + |z_2|^2 + 2\,\mathrm{Re}(z_1\bar{z}_2)$ and write down the expression for $|z_1-z_2|^2$.

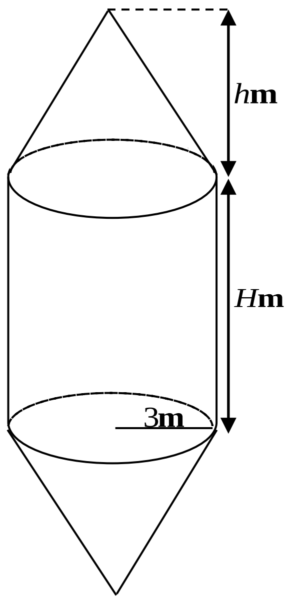
If the modulus value of $\dfrac{(z_1-z_2)}{(z_1+z_2)}$ is 1, show that $\dfrac{z_1}{z_2}$ is purely imaginary.

Q14) **a.** Let $y = \dfrac{(x-2)^2}{x^2+4}$ for $x \in \mathbb{R}$. Show that $0 \le y \le 2$.

Draw the curve $y = \dfrac{(x-2)^2}{x^2+4}$, showing the turning points and asymptotes.

The equation $x(x^2+4) = (x-2)^2$ has only one real root. Explain why?

b. A container is made by rigidly joining two hollow cones of radius 3 meters and height h meters with a hollow right circular cylinder of same radius and height H meters as shown in the diagram. The total volume of the container is $900\mathbf{m}^3$. Show that $H = \dfrac{100}{\pi} - \dfrac{2}{3}h$.



If the total surface area of the container is $S\mathbf{m}^2$, show that $S = 600 - 4\pi h + 6\pi\sqrt{9+h^2}$.

Find the value of h for which S is minimum.

[see pg.9

Q15) **a.** Using integration by parts, find $\int e^{ax} \sin bx \, \mathrm{d}x$.

b. Using **partial fractions**, find $\int \dfrac{11+3x-2x^2}{(x+3)(x-1)^2} \, \mathrm{d}x$.

c. When a, b, c are constants and $b^2 - 4ac \neq 0$, find A, B, C such that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{Ax+B}{ax^2+bx+c}\right) = \frac{1}{\left(ax^2+bx+c\right)^2} - \frac{C}{ax^2+bx+c}.$$

Hence, show that $\displaystyle\int_0^1 \frac{\mathrm{d}x}{\left(x^2+4x+1\right)^2} = \frac{1}{4} - \frac{\sqrt{3}}{36}\ln\left(2+\sqrt{3}\right)$.

Q16) **a.** If the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ touches the x axis, show that $g^2 = c$. If $f^2 > c$, show that it intersect y axis and show that the length of this intersecting chord is $2\sqrt{f^2 - c}$.

A circle touches the x axis at point $A(a, 0)$ and intersect the positive y axis at the points B and C. If $BC = l$, show that the equation of this circle is given by

$$(x-a)^2 + \left(y - \frac{\sqrt{l^2+4a^2}}{2}\right)^2 = \frac{l^2+4a^2}{4}$$

If $a = 12$ and $l = 10$, find the area of the triangle ABC.

b. A straight line intersect the line $5x - y - 4 = 0$ at point P and the line $3x + 4y - 4 = 0$ at point Q. The mid point of PQ is $M(1, 5)$. If the gradient of the line PQ is m, show that

$P = \left(\dfrac{9-m}{5-m}, \dfrac{25-m}{5-m}\right)$, $Q = \left(\dfrac{4m-16}{4m+3}, \dfrac{m+15}{4m+3}\right)$ and find the equation of PQ.

Q17) **a.** Using the expansion for $\tan(A-B)$, show that $\tan 15^\circ = 2 - \sqrt{3}$.

Show that $\tan\left(\dfrac{x}{2}\right) = \dfrac{\sqrt{1+\tan^2 x} - 1}{\tan x}$ for $0 < x < \dfrac{\pi}{2}$.

Show that $\tan 7\dfrac{1}{2}^\circ = \left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{2} - 1\right)$ and **deduce** that $\cot 7\dfrac{1}{2}^\circ = \sqrt{2} + \sqrt{3} + \sqrt{4} + \sqrt{6}$.

b. Find the general solutions of the equation $\sin^3 x + \cos^3 x + \sin x \cos x = 1$.

c. State the sine rule for a triangle.

For ΔABC, in usual notations,

(i) Show that $(a-b)\cos\dfrac{C}{2} = c\sin\left(\dfrac{A-B}{2}\right)$.

(ii) If $\dfrac{\tan A - \tan B}{\tan A + \tan B} = \dfrac{c-b}{c}$, show that $A = 60^\circ$.

**** END OF QUESTIONS ****

[*see pg.*10

No Question in this page

மொறட்டுவை பல்கலைக்கழக பொறியியற் பீட தமிழ் மாணவர்கள்
நடாத்தும் க.பொ.த உயர்தர மாணவர்களுக்கான 8வது
முன்னோடிப் பரீட்சை - 2017

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General Certificate of Education (Adv.Level) Pilot Examination - 2017

இணைந்த கணிதம் II Combined Maths II	10	E	II	மூன்று மணித்தியாலம் *Three hours*

Stream [ ] District No [ ][ ] Index No [ ][ ][ ][ ]

Instructions

- ⁎ This question paper consists of two parts; **PART A** (Questions 1- 10) and **PART B** (Questions 11 – 17)
- ⁎ **Part *A***

  Answer **all** questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
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(10) **Combined Mathematics II**		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
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	Total	
B	11	
	12	
	13	
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	15	
	16	
	17	
	Total	
Total for Paper I		

Paper I	
Paper II	
Total	
Final Marks	

[***see pg.2***

Q1). A train starting from rest at station A, accelerates with an acceleration of α for a certain distance and decelerates with a deceleration of β until it comes to rest at B. $AB = 4km$ and the travelling time is 4 minutes. Show that $\frac{1}{\alpha}+\frac{1}{\beta}=2$. Here α and β are in *km/minutes*.

Q2). If a particle P satisfies the equation $x = a\cos nt + b\sin nt$ when it is at a distance x from a fixed point, show that the motion of this particle is a simple harmonic motion and its amplitude is $\sqrt{a^2+b^2}$. Here a, b and n are constants.

[see pg.3

Q3). Two smooth spheres of masses $2m$ and $3m$ travels with respective velocities u and $2u$ in opposite directions and collide directly with each other. If the co-efficient of restitution between the spheres is $\frac{1}{3}$ find the velocities of the spheres after the collision and the impulse created by the collision.

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Q4). If $\overrightarrow{OA} = 3\mathbf{a} + 2\mathbf{b}$ and $\overrightarrow{OB} = 2\mathbf{a} - \mathbf{b}$, show that $\overrightarrow{OA}.\overrightarrow{OB} = 6|\mathbf{a}|^2 + \mathbf{a}.\mathbf{b} - 2|\mathbf{b}|^2$.

If $\mathbf{a} = 2\mathbf{i} + \mathbf{j}, \mathbf{b} = 2\mathbf{i} - \mathbf{j}$, find $\overrightarrow{OA}.\overrightarrow{OB}$. Here $\mathbf{i}$ and $\mathbf{j}$ have usual meanings.

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[see pg.4

Q5). A thin uniform rod is made in the form of an isosceles triangle ABC. Here $AB = BC = a, AC = b, 2a > b$. Show that the the center of gravity of this triangle ABC lies at a distance $\frac{a}{2}\sqrt{\frac{2a-b}{2a+b}}$ from AC on the perpendicular bisector of AC. If this triangle is hung from A, find the angle made by AC with the vertical.

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Q6). A rough hollow sphere of radius a is fixed. Show that a particle cannot be in equilibrium on the inner surface of the sphere above a height of $a(1-\cos\lambda)$ from the lowest point of the sphere. Here λ is the angle of friction between the sphere and the particle.

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[***see pg*.5**

Q7). A and B are two events in a sample space. If $0 < P(A) \leq 1, 0 \leq P(B) < 1$, show that

$$P(A'/B') = \frac{1 - P(A \cup B)}{P(B')} .$$

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Q8). If $P(B) = \frac{3}{4}, P(A \cap B \cap C') = \frac{1}{3}, P(A' \cap B \cap C') = \frac{1}{3}$ in a sample space, find $P(B \cap C)$.

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[see pg.6

Q9). The mode of 100 observations in the following frequency distribution is 24.

$0-10$	$10-20$	$20-30$	$30-40$	$40-50$
14	x	27	y	15

Find the values of x, y and determine the median of the distribution.

Q10). The mean and variance of five observations $1, 2, 6, \lambda, \mu$ are 4.4 and 8.24 respectively. Find λ, μ.

[see pg.7

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General Certificate of Education (Adv.Level) Pilot Examination - 2017

இணைந்த கணிதம் II
Combined Maths II

10	E	II

Part-B * Answer five questions only

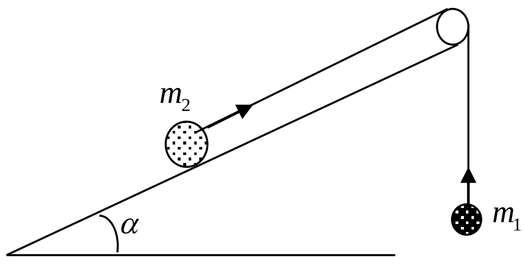
Q11) a. A and B are two spheres of masses $m, 3m$ respectively. On a smooth horizontal table, while B is at rest, A collides directly with B with a velocity of u. The co-efficient of restitution between A and B is e. If A bounce back after the collision.

(i) Show that the velocity of B after collision is $\frac{u}{4}(1+e)$.

(ii) What is the velocity of A after the collision?

(iii) B collides with a vertical wall in the subsequent motion. The co-efficient of restitution between the wall and the particle is $\frac{1}{2}$. If B collides with A again, show that $\frac{1}{3} < e < \frac{3}{5}$.

b. Two masses m_1 and m_2 are joined by a light inelastic string which goes over a smooth pulley as shown in the figure. The pulley is fixed to the top of an inclined plane inclined at an angle α to the horizontal.



(i) If the inclined plane is smooth, find the acceleration at which m_1 descends and the tension in the string.

(ii) If the inclined plane is rough and the co-efficient of friction is μ, find the acceleration at which m_1 descends and the tension in the string.

(iii) Show that the difference between the acceleration when the plane is smooth and the acceleration when the plane is rough is $\frac{\mu m_2 \cos\alpha}{m_1 + m_2} g$.

Q12) a. A particle P of mass m is thrown horizontally from the lowest point of a fixed smooth spherical shell with center O and radius r with a velocity u. When OP makes an angle θ with the downward vertical, show that the the angular velocity of the particle, $\omega = \frac{1}{r}\sqrt{u^2 - 2gr(1-\cos\theta)}$ and the reaction given to the particle by the shell,

[see pg.8

$R = \frac{m}{r}\left\{u^2 - gr\left(2 - 3\cos\theta\right)\right\}$. Prove that the particle will execute complete circular motion if $u \geq \sqrt{5gl}$. For $u \geq \sqrt{5gl}$, if ω_1 and ω_2 are the maximum and minimum angular velocities of the particle and, R_1 and R_2 are maximum and minimum reactions on the particle, show that $\omega = \sqrt{\omega_1^2 \cos^2 \frac{1}{2}\theta + \omega_2^2 \sin^2 \frac{1}{2}\theta}$ and $R = R_1 \cos^2 \frac{1}{2}\theta + R_2 \sin^2 \frac{1}{2}\theta$.

b. A particle is thrown under gravity from a point C, which is at a height h from a point O with a velocity u making an upward angle θ with horizontal in a vertical plane. If the horizontal range of the particle on the horizontal plane through O is R, show that $R^2 \tan^2\theta - \frac{2u^2}{g} R\tan\theta + R^2 - \frac{2hu^2}{g} = 0$ C. **Hence**, deduce the maximum range of this particle on the horizontal plane through O for this velocity u is $\sqrt{\frac{u^4}{g^2} + \frac{2hu^2}{g}}$. If the maximum horizontal range of the particle is R' and the corresponding angle of projection is α , deduce that $\tan 2\alpha = \frac{R'}{h}$.

Q13) A and B are two points on a smooth horizontal table separated by a distance $3l$. A particle P of mass m is placed at a point between A and B on the line AB. The particle is attached to A by an elastic string of length l and elastic modulus $3mg$ and to B by another elastic string of length l and elastic modulus λ. If the particle P is in equilibrium at point C, where $AC = \frac{3}{2}l$, find λ.

The particle P is kept at B and released from rest. Show that the velocity of P when the string BP becomes taut is $3\sqrt{gl}$.

In the subsequent motion, when both the strings are taut, if the displacement of P measured from the initial point of equilibrium C along CB is x, show that $\frac{d^2x}{dt^2} + \frac{6g}{l}x = 0$.

Here $-\frac{l}{2} \leq x \leq \frac{l}{2}$. Assuming that the solution of the above equation is in the form $x = A\cos\omega t + B\sin\omega t$, find the values of the constants A, B and ω . Show that the time taken from the instance when BP became taut until AP becomes slack is $\left(\frac{2l}{3g}\right)^{\frac{1}{2}} \sin^{-1}\left(\frac{1}{\sqrt{7}}\right)$.

[see pg.9

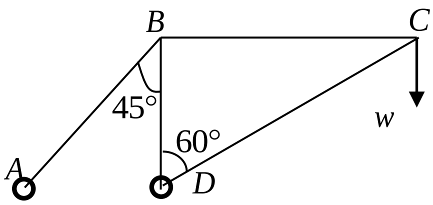
Q14) **a.** Forces $2, 1, 3, 4, 2\sqrt{2}$ Newtons act along the sides $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{DA}, \overrightarrow{BD}$ of a square $ABCD$ of side length 2**m** respectively in the direction indicated by the order of letters. Find the magnitude and direction of the resultant and the length AE if E is the point where the line of action of the resultant meets AD.

(i) Find the magnitude, direction and the line of action of the force to be added to the system to keep it in equilibrium.

(ii) Find the magnitude and sense of the couple required to shift the resultant force to point D.

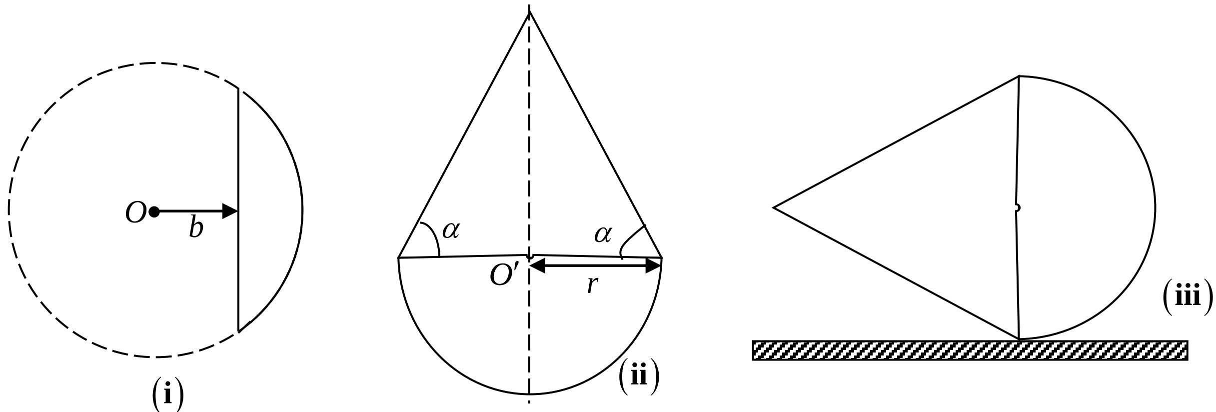
b. ABC is a triangle. D is the mid point of AB. Point E lies on BC such that $BE:EC = 1:2$. The lines AE and CD meet at P. By taking the position vectors of points B and C with respect to point A as **b** and **c** respectively, show that $\dfrac{AP}{PE} = \dfrac{3}{2}$ and $\dfrac{CP}{PD} = \dfrac{4}{1}$.

Q15) **a.** Four light rods AB, BC, CD and BD are smoothly joined to form a framework as shown in the figure. The rod BD is vertical while the rod BC is horizontal. The framework is hinged to the horizontal ground at A and D, while a weight w is hung at C. By using Bow's notation, find the stresses in each rod and distinguish whether they are tension or thrust.



b. Six identical rods of weight w and length a are smoothly joined to form a regular hexagon $ABCDEF$. This system is hung from A and kept in the shape of hexagon by the rod BF joining B and F and the rod CE joining C and E. When the system hangs in equilibrium show that the stresses in the rods BF, CE are $\dfrac{5\sqrt{3}}{2}w$ and $\dfrac{\sqrt{3}}{2}w$ respectively.

Q16)



A solid frustum is made by removing the larger part from a solid sphere of radius a at a distance b from O as shown in the figure **(i)**. By using integration, show that the center of gravity of the frustum lies at a distance $\dfrac{3}{4}\dfrac{(a+b)^2}{(2a+b)}$ from O.

see pg.10

Hence, deduce the center of gravity of a uniform solid hemisphere of radius a. A composite body formed by joining a uniform solid hemisphere of radius r and a right circular solid cone of radius r, such that their planar faces coincide as shown in the figure $(\mathbf{ii})$. Assuming that the densities of the bodies are same, show that the center of mass of the composite body is at a distance $\dfrac{r\left|\tan^2\alpha-3\right|}{8+4\tan\alpha}$ from O' along the symmetrical axis. The composite body is kept on a horizontal ground as shown in figure $(\mathbf{iii})$ and released from rest. Explain what would happen under following conditions.

$(\mathrm{a})\ \alpha<\tan^{-1}\left(\sqrt{3}\right)$ $(\mathrm{b})\ \alpha>\tan^{-1}\left(\sqrt{3}\right)$ $(\mathrm{c})\ \alpha=\tan^{-1}\left(\sqrt{3}\right)$

Q17) **a.** A and B are any two events such that $P(A)=P(A/B)=\dfrac{1}{4},P(B/A)=\dfrac{1}{2}$. State whether the following statements are true or false with reasons.

$(\mathbf{i})$ A and B are mutually exclusive

$(\mathbf{ii})$ A and B are independent.

$(\mathbf{iii})$ $P(A'/B)=\dfrac{3}{4}$

$(\mathbf{iv})$ $P(A'/B')=\dfrac{1}{2}$

b. A problem in combined mathematics is given to three students A,B and C separately to be solved. The probabilities for A,B and C to solve the problem correctly are $\dfrac{1}{2},\dfrac{3}{4}$ and $\dfrac{1}{4}$ respectively. Find the probability for the problem to be solved correctly.

c. The details of the marks obtained by students in a school for a particular exam are given in the table below·

Marks	count
$0-10$	4
$10-20$	2
$20-30$	18
$30-40$	$f-24$
$40-50$	$67-f$
$50-60$	19
$60-70$	10
$70-80$	4
$80-90$	1

If the mode of the the distribution is 38 find f. Further, find the mean, median and variance of the distribution.

∗ ***END OF QUESTIONS*** ∗