

மொறாட்டுவை பல்கலைக்கழகப் பொறியியல் பீடத்திற் மாணவர்கள்

General Certificate of Education (Adv. Level) Pilot Examination - 2018

10 E I

Index No :

- * This question paper consists of two parts;
- * **Part A** (Question 1-10) and **Part B** (Question 11-17)

Answer all 10 questions. Write your answers to each question in the space provided, You may use additional sheets if more space is needed.

Answer five questions only. Write your answer on the sheets provided.

- * At the end of the time allotted, tie the answer scripts of the two parts together so that Part A is on the top of Part B and hand them over to the supervisor.
- * You are permitted to remove **only Part B** of the question paper from the Examination Hall.

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper I	
Paper II	
Total	
Final Marks	

In Numbers	
In Letters	

Marking Examiner 1	
Marking Examiner 2	
Checked by	
Supervised by	

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- 07) The gradient of the normal at the point P of a curve given by $x = 3e^{2t} - t, y = e^{3t} - 2t$ is $-\frac{1}{2}$.

Here t is a parameter. Find the value of t corresponding to the point P . Give your answer in the form $t = \ln k$. Here k is a constant.

- 08) The center of the circle S having radius 4 is in the second quadrant. S touches the two axes x, y . Find the equation of the S . Also find the value of m such that the straight line $y = mx + 12$ is a tangent of S .

2; A circle having $(3, -1)$ as its center intersects a straight line $2x - 5y + 18 = 0$ at A, B , if $AB = 6$ then find the equation of the circle.

10) Solve $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$

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10 E I

Part B
Answer five questions only

11) (a) Let $f(x)$ be a quadratic function in the form $f(x) = ax^2 + bx + c$. Here a, b, c are real constants and $a \neq 0$.

Show that $\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2} + \frac{f(x)}{a}$.

Hence **deduce** that if the equation $ax^2 + bx + c = 0$ has real roots $b^2 - 4ac \geq 0$.

Show that **if and only if** $p \geq q$ then the quadratic equation $qx^2 - 2p\sqrt{p}x + p^2 = 0$ has real roots. Here $p, q \in \mathbb{R}$

(b) Let $f(x) = ax^4 + x^3 - x^2 - x - b$. Here a, b are real constants

It is given that $(x-1)$ is a factor of $f(x)$ and the remainder when $f(x)$ is divided by $(x-2)$ is 33. Find the values of a, b .

Show that $(x+1)$ is also a factor of $f(x)$

Express $f(x)$ as a product of two linear factors and a quadratic factor which is positive for all $x \in \mathbb{R}$

12) (a) Consider the binomial expansion $(10+3x)^{15}$ for $x > 0$.

Let T_{r+1} be the $(r+1)^{\text{th}}$ term in the above expansion. Here $r=1,2,3,\dots,14,15$.

Show that $\frac{T_{r+1}}{T_r} = \frac{3(16-r)}{10r}x$.

Show that the range of x is $\frac{10}{3} < x < \frac{30}{7}$ for T_9 to be the largest term

Hence deduce the largest term in the expansion $(10 + 3x)^{15}$ when $x = 4$.

(b) Let $f(r) = \frac{r+a}{r^2}$ for $r \in \mathbb{Z}^+$. Here $a \in \mathbb{R}$.

Find the value of a such that $f(r) - f(r+1) = \frac{r^2 + 3r + 1}{r^2(r+1)^2}$ for $r \in \mathbb{Z}^+$

If the r^{th} term of a infinite series is $U_r = \frac{r^2 + 3r + 1}{r^2(r+1)^2}$ find $\sum_{r=1}^n U_r$. Is $\sum_{r=1}^{\infty} U_r$ convergent? Justify your answer

13) (a) Let $\mathbf{A} = \begin{pmatrix} -1 & 0 & 0 \\ 1 & 2 & 1 \\ -2 & -3 & -2 \end{pmatrix}$

Find $\mathbf{A}^2 - \mathbf{I}$ then show that $\mathbf{A}(\mathbf{A}^2 - \mathbf{I}) = \mathbf{I} - \mathbf{A}^2$. Here \mathbf{I} is a identity matrix of order 3.

Hence find \mathbf{A}^{-1} . Find the matrix \mathbf{B} such that $\mathbf{AB} = \mathbf{I} + 2\mathbf{A}$.

(b) In the Argand diagram, the two points P_1, P_2 denotes the complex numbers z_1, z_2 respectively. Show that $|z_1 - z_2| = P_1P_2$

By considering the triangle OP_1P_2 show that $|z_1 - z_2| \geq ||z_1| - |z_2||$.

Here O is the origin of the Argand diagram

If z is a varying complex number such that $|z - 2 + i| \leq 2$ then using the above result deduce that $\sqrt{5} - 2 \leq |z| \leq \sqrt{5} + 2$. When z varies in the Argand plane, shade the region S such that $|z - 2 + i| \leq 2$ and show that $\sqrt{5} - 2 \leq |z| \leq \sqrt{5} + 2$.

14) (a) Let $f(x) = \frac{x^2}{(x+1)(x-2)}$ for $x \neq -1, 2$

Show that the derivative of $f(x)$, $f'(x)$ is given by $f'(x) = -\frac{x(4+x)}{(x+1)^2(x-2)^2}$ for $x \neq -1, 2$.

Sketch the graph of $y = f(x)$ indicating the asymptotes and the turning points.

Solve $\frac{x^2}{(x+1)(x-2)} \leq 0$ using the graph

(b)

A wire of given length 36 metre is cut into two parts. One part is made into a equilateral triangle and the other part made into a square. Show that the sum of the area of the equilateral triangle and square $A(x)$ is given by $A(x) = x^2 + \frac{4\sqrt{3}}{9}(9-x)^2$ square units.

Here $4x, (0 < x < 9)$ is a length of the part of wire which made into a square.

Hence show that the lengths of the two parts of the wires when the area $A(x)$ is minimum are

$$\left(\frac{324}{9+4\sqrt{3}} \right), \left(\frac{144\sqrt{3}}{9+4\sqrt{3}} \right) \text{ metre.}$$

15) (a) Using integration by parts find $\int \frac{1}{x^2} \ln(1+x^2) dx$

(b) Show that $\frac{d}{dx} \left(x^{n-1} \sqrt{16-x^2} \right) = \frac{16(n-1)x^{n-2}}{\sqrt{16-x^2}} - \frac{nx^n}{\sqrt{16-x^2}}$

Hence find $\int_0^2 \frac{x^2}{\sqrt{16-x^2}} dx$.

(c) Express $\frac{x^3+3x^2+8x+26}{(x+1)(x^2+9)}$ in the form $a + \frac{b}{(x+1)} + \frac{cx+d}{(x^2+9)}$. Here a, b, c, d are the

equupcpw'q"dg"f gvgto kpgf

Show that $\int_0^3 \frac{x^3+3x^2+8x+26}{(x+1)(x^2+9)} dx = 3 + 4 \ln 2 - \frac{\pi}{12}$

16) (a) Show that the origin does not lie on the straight line $ax + by + c = 0$ if $c \neq 0$. A diagonal of a square $OABC$ is straight line $ax + by + c = 0$ such that O is origin. Here $c \neq 0, a \neq b$.

Show that the equations of the four sides of the square are $y = \left(\frac{b-a}{b+a} \right) x$, $y = \left(\frac{b+a}{a-b} \right) x$,

$$y + \frac{2bc}{a^2+b^2} = \left(\frac{b+a}{a-b} \right) \left(x + \frac{2ac}{a^2+b^2} \right), y + \frac{2bc}{a^2+b^2} = \left(\frac{b-a}{a+b} \right) \left(x + \frac{2ac}{a^2+b^2} \right)$$

Show that the area of the square is $\frac{2c^2}{a^2+b^2}$

(b) Show that **if and only if** $r^2(m^2+1) = (q-mp-c)^2$ then the straight line $y = mx + c$ touches the circle $(x-p)^2 + (y-q)^2 = r^2$.

Let $k \in \mathbb{R}$. It is given that the straight line $x + y = k$ touches the circle $x^2 + y^2 - 4x - 2y - 13 = 0$. Find the two values of k .

17) (a) Find the general solution of the equation $8 \sin x = \frac{\sqrt{3}}{\cos x} + \frac{1}{\sin x}$

(b) For a triangle ABC in usual notation

$$\text{Show that } \frac{\cos^2 \left(\frac{B-C}{2} \right)}{(b+c)^2} + \frac{\sin^2 \left(\frac{B-C}{2} \right)}{(b-c)^2} = \frac{1}{a^2}.$$

(c) For $0 \leq \theta \leq 2\pi$

(i) Write the solutions such that $\sin \theta = 0$

(ii) Write the solutions such that $\sin 5\theta = 0$.

By considering $\sin 5\theta = \sin(3\theta + 2\theta)$ show that $\sin 5\theta = \sin \theta (16 \sin^4 \theta - 20 \sin^2 \theta + 5)$.

Analyze the above results and **deduce** that $\sin \left(\frac{\pi}{5} \right), \sin \left(\frac{2\pi}{5} \right), \sin \left(\frac{6\pi}{5} \right), \sin \left(\frac{7\pi}{5} \right)$ are the solutions of equation $16x^4 - 20x^2 + 5 = 0$

Substitute $y = x^2$ and **deduce** that the roots of the equation $16y^2 - 20y + 5 = 0$ are

$$\sin^2 \left(\frac{\pi}{5} \right), \sin^2 \left(\frac{2\pi}{5} \right). \quad \text{Hence show that } \sin \left(\frac{\pi}{5} \right) \sin \left(\frac{2\pi}{5} \right) = \frac{\sqrt{5}}{4} \text{ and } \cos \left(\frac{2\pi}{5} \right) = \frac{\sqrt{5}-1}{4}$$

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General Certificate of Education (Adv. Level) Pilot Examination - 2018

10 E II

Index No :.....

- * This question paper consists of two parts;
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- * In this question paper, g denotes the accelerations due to gravity.

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(10) Combined Mathematics II		
Part	Question No.	Marks
A	1	
	2	
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	8	
	9	
	10	
B	11	
	12	
	13	
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Total	
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Code Numbers

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Marking Examiner 2	
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If the resistance to motion is C Newton per 1 kg , show that the maximum speed of the car is $\frac{1000Hn}{M(Cn+g)}\text{ms}^{-1}$

04) A particle A of mass $3m$ is moving on a horizontal table in a straight line and collides with another particle B of mass $2m$. The speeds of them before the collision are $2u$ and u respectively towards each other. They collide and coalesce to form a particle C. What is the loss in kinetic energy due to this collision?

- 05) The position vectors of A, B with respect to origin O are \mathbf{a}, \mathbf{b} respectively. Here $|\mathbf{a}| = 1, |\mathbf{b}| = 3, \angle AOB = \frac{\pi}{3}$.

Show that $\left| \frac{\sqrt{3}}{9}(3\mathbf{a} - 2\mathbf{b}) \right| = 1$ Show that $\frac{\sqrt{3}}{9}(3\mathbf{a} - 2\mathbf{b})$ is a unit vector perpendicular to \overrightarrow{OA} .

- 06) A thin smooth hemispherical bowl of radius r is fixed such that its brim is uppermost and horizontal. A smooth uniform rod AB of weight w and length l ($2r < l < 4r$) is placed at rest such that its end A touching the inside surface of the bowl and a point C on it in contact with the brim. If the rod makes an angle θ with horizontal at equilibrium show that $\cos \theta = \frac{l}{16r} + \sqrt{\left(\frac{l}{16r}\right)^2 + \frac{1}{2}}$.

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x_i	1	2	3	4	5
f_i	x	11	y	8	9

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10 E II

Part B

Answer five questions only

- 11) a. Two particles P, Q are moving from A to B on a straight path such that $AB = 2s$. P starts its motion from A with initial velocity u and travels with constant acceleration f at the same time Q starts its motion from A with initial velocity $u' (\neq u)$ and constant acceleration $f' (\neq f)$.

Draw the velocity – time graph of both particles separately. Using these graphs

(i) If both pass the midpoint of AB at the same time show that the time they pass is $t = \frac{2(u-u')}{(f'-f)}$

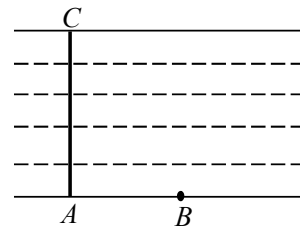
. Show that $f' > f$ if and only if $u > u'$,

(ii) In the following motion if the velocities of both particles when they pass B are equal show

$$\text{that } s = \frac{(u^2 - u'^2)}{4(f' - f)}.$$

Hence show that $(u + u')(f - f') = 8(fu' - f'u)$.

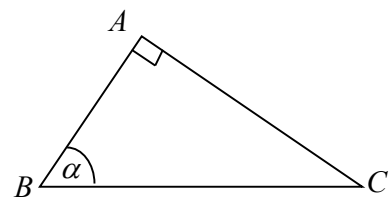
- b.** As shown in the figure, a river with breadth d flows parallel to bank along \overline{AB} with steady speed u with respect to ground. Here A, B are the two points on one side of bank such that $AB = d$. If for a man who can swim with speed v relative to river, time taken to swim from A to B and from B to A are



t, t' respectively then show that $u = \frac{d(t'-t)}{2tt'}$. Further show

that the time taken for the man to pass the river by swimming to a point C which is directly opposite to the point A is $\sqrt{tt'}$

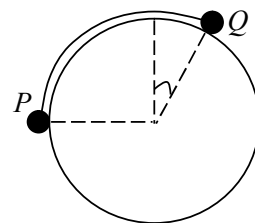
- 12) a. In the given figure, triangle ABC denotes the vertical cross section through center of gravity of a uniform smooth wedge of mass M . It is placed on smooth horizontal ground. It is given that AB, AC are the lines of greatest slope of respective faces and $\hat{BAC} = 90^\circ, \hat{ABC} = \alpha$. Two particles with mass m_1, m_2 are



released to move on AB, AC respectively from the vertex A of wedge at the same time. Write enough equations to determine the accelerations of the particles and the acceleration of wedge. If both particle reach the horizontal ground at the same time then

show that $\tan \alpha = \sqrt{\frac{M+m_2}{M+m_1}}$

- b.** A smooth cylinder of radius r is fixed. Two particles P, Q of mass $m, 3m$ are attached to the ends of a light inextensible string. The system is kept at rest such that string taut, particle P of mass m is at the end of the horizontal radius touching the cylinder and Particle Q of mass $3m$ is at the end of a radius which makes an angle α with upward vertical touching the cylinder as shown in the figure. Then when the system is released slowly, If $\sin \alpha > \frac{1}{3}$ then show that the particles P, Q starts to move in the clockwise sense on the surface of the cylinder. Show that in the continuing motion the speed v of Q is given by $v^2 = \frac{gr}{2}(3\cos \alpha - \sin \beta - 3\cos(\alpha + \beta))$ after it turned through an angle β using the principle of conservation of energy and if at that instance the particle Q leaves the cylinder then show that $(5\sin \alpha - 1)\sin \beta = (5\cos \beta - 3)\cos \alpha$.



- 13)** One end of a light elastic string of natural length a is attached to a fixed point O while other end of the string is attached to a particle P of mass m . When the particle hangs in equilibrium at point E , the length of the string is $\frac{3a}{2}$. Show that the modulus of elasticity of the string is $2mg$. Now the particle is kept at O and released from rest at O . In the following motion after the string becomes taut using the principle of conservation of energy show that its velocity at that instance \dot{x} is given by $\dot{x}^2 = \frac{2g}{a}\left(\frac{5a^2}{4} - x^2\right)$ where x is the displacement of the particle measured downwards from E .
- Deduce that $-\frac{a}{2} \leq x \leq \frac{\sqrt{5}a}{2}$ and show that the maximum extension of the string is $\frac{a}{2}(\sqrt{5} + 1)$.
- Show that there is a simple harmonic motion in the range $-\frac{a}{2} \leq x \leq \frac{\sqrt{5}a}{2}$.
- Consider a solution $x = A \cos \omega t + B \sin \omega t$ and find constants A, B, ω .
- Hence show that the time taken to reach the point of amplitude is $\sqrt{\frac{2g}{a}}(\pi - \tan^{-1} 2)$.

- 14) a.** Let $OABC$ be a parallelogram where O is the origin. The position vectors of the points A, C with respect to O are \mathbf{a}, \mathbf{c} $\left(\mathbf{c} > \frac{\mathbf{a}}{3}\right)$ respectively.
- Point E is on CB such that $CE : EB = 1 : 2$. Line AE meets the bisector of the angle $\angle OAC$ at P . Extended CP meets AB at F .
- (i) Show that the position vector of E is $\frac{\mathbf{a} + 3\mathbf{c}}{3}$.
- (ii) Show that the position vector of the point P can be expressed in the form $\lambda \left(\frac{\mathbf{a}}{|\mathbf{a}|} + \frac{\mathbf{c}}{|\mathbf{c}|} \right)$. Here λ is a constant.
- (iii) Consider $EP : PA = \mu : 1$ and find the position vector of point P in terms of $\mathbf{a}, \mathbf{c}, \mu$. Hence show that $\overrightarrow{OP} = \frac{3|\mathbf{a}||\mathbf{c}|}{3|\mathbf{c}| + 2|\mathbf{a}|} \left(\frac{\mathbf{a}}{|\mathbf{a}|} + \frac{\mathbf{c}}{|\mathbf{c}|} \right)$ and $AF : FB = 3|\mathbf{c}| : 3|\mathbf{c}| - |\mathbf{a}|$.

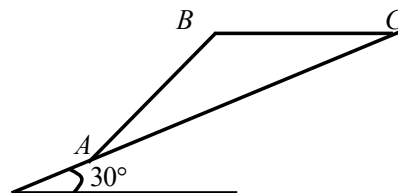
- d. A system of 3 forces on a plane xy in which distance is measured in metre and force measured in Newton is given.

If the system is in equilibrium find a, b, c .

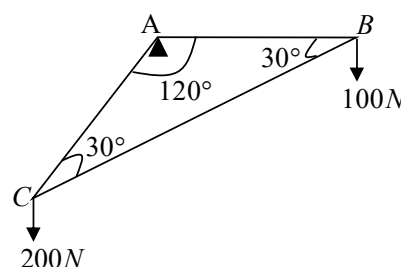
Now the force F_3 is inversed and a couple of magnitude $21Nm$ in the clockwise sense is added. Find the magnitude, direction and the equation of the line of action of the resultant of the new system.

Coordinate of the point of action	Components of force in the directions Ox, Oy
$A(c, 1)$	$F_1 = (5, 6)$
$B(2, -1)$	$F_2 = (a, -4)$
$C(3, 2)$	$F_3 = (-6, b)$

- 15) a. Two equal uniform rods AB, BC each of weight w are jointed freely at B and placed at equilibrium in a vertical plane on a fixed rough inclined plane inclined at an angle 30° with horizontal such that A, C on the plane as shown in the figure. The rod BC is horizontal. Find the normal reaction and friction on A, C and state their directions. If the equilibrium is disturbed when the coefficient of friction on A, C are equal show that it is disturbed due to the slipping of A at the same time C is remaining at rest. Deduce the minimum value of the coefficient of friction for equilibrium to be possible.



- d. The given figure ABC is a framework which has three smoothly jointed light rods AB, BC, CA . It is supported by a smooth peg at A such that AB is horizontal. Draw a stress diagram using Bow's notation and find the stresses in all rods.



- 16) Show that the centre of gravity of a uniform lamina in the shape of a sector of a circle of radius a , subtending an angle 2α at the centre is on the axis of symmetry, at distance $\frac{2a \sin \alpha}{3\alpha}$ from its centre.

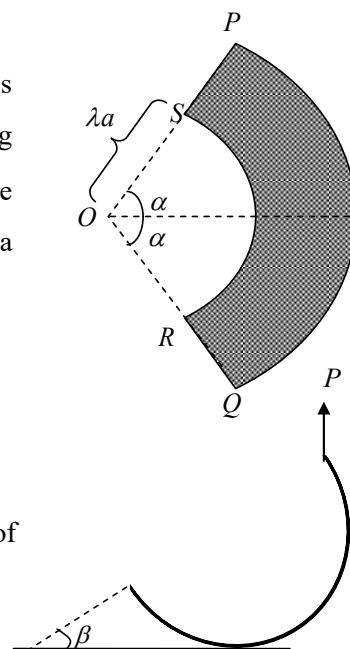
A sector OSR of radius λa and subtending 2α at the center O is cut and removed from a lamina OPQ in the shape of a sector having radius a and subtending 2α at the same center O . Show that the center of gravity of the remaining part $SPQR$ is at a

distance $\frac{2a \sin \alpha}{3\alpha} \left(\frac{\lambda^2 + \lambda + 1}{\lambda + 1} \right)$ from the axis of symmetry.

Hence deduce the center of mass of a circular arc of radius a and subtending 2α at the center.

A uniform semicircular wire is placed on a smooth horizontal table. When a vertical force P is applied on one end of the semicircular wire as shown in the figure, it is at equilibrium such that its diameter makes an angle

$\beta \left(0 < \beta < \frac{\pi}{2} \right)$ with table. If the reaction on wire by table is R show that $\frac{R}{P} = \frac{\pi}{2 \tan \beta} - 1$



- 17) a. In a box there are 3 blue marbles and 2 red marbles . In another box there are 2 blue marbles and 3 red marbles. A marble that is taken from one of these boxes is blue color. What is the probability of that marble coming from the first box ? Consider that both boxes are identical.
- b. A trader states that the bulb he sells has an average lifetime of 4000 hours. A company which uses these bulbs in large amount expressed a favor to test the statement of that trader using its old records. A table of frequency of lifetime of bulb is given below according to the past time records. 1 unit = 1000 hours

- (i) Find the mean of lifetime of sample?
- (ii) which is the modal class?
- (iii) Estimate the median and the mode of the lifetime
- (iv) Also find the standard deviation and state the skewness of the distribution?

Class intervals (In units)	frequency
0 – 2	10
2 – 4	55
4 – 6	30
6 – 8	05

*** END OF QUESTIONS ***