

<p>Tamil medium, University of Madras, Faculty of Engineering, Chennai-600 025. கிழக்கு மதுரைப் பல்கலைக்கழகம், பொதுப் பரீட்சைத் துறை, கருவூர், சென்னை-600 025.</p> <p>தமிழ் மொழி, மதுரைப் பல்கலைக்கழகம், பொதுப் பரீட்சைத் துறை, கருவூர், சென்னை-600 025.</p> <p>Tamil medium, Tamil Nadu State University of Madras, Faculty of Engineering, Chennai-600 025. கிழக்கு மதுரைப் பல்கலைக்கழகம், பொதுப் பரீட்சைத் துறை, கருவூர், சென்னை-600 025.</p> <p>தமிழ் மொழி, தமிழ்நாடு மாநிலப் பல்கலைக்கழகம், பொதுப் பரீட்சைத் துறை, கருவூர், சென்னை-600 025.</p> <p>Tamil medium, Tamil Nadu State University of Madras, Faculty of Engineering, Chennai-600 025. கிழக்கு மதுரைப் பல்கலைக்கழகம், பொதுப் பரீட்சைத் துறை, கருவூர், சென்னை-600 025.</p> <p>தமிழ் மொழி, தமிழ்நாடு மாநிலப் பல்கலைக்கழகம், பொதுப் பரீட்சைத் துறை, கருவூர், சென்னை-600 025.</p>	<p>Combined Mathematics I இணைந்த கணிதம் I</p>	<p>10 E I</p>	<p>Three hours மூன்று மணித்தியாலம்</p>
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Q3). Find all the real vales of x satisfying the inequality $\frac{12}{x-3} < x+1$

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Q4). For $m, n \in \mathbb{Z}^+$ in the expansion of $(1+x)^m (1-x)^n$, the coefficients of x and x^2 are $3, -6$ respectively. Find m, n .

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Here t is a parameter. show that the locus of the centroid of this triangle is a circle and find its centre and radius.

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$$4(x^2 + y^2) - 4ax \pm 4ay + a^2 = 0$$

Q10). Let $f(x) = \frac{\sin x}{\sqrt{1-\cos^2 x}} + \frac{\cos x}{\sqrt{1-\sin^2 x}} + \frac{\tan x}{\sqrt{\sec^2 x - 1}} + \frac{\cot x}{\sqrt{\csc^2 x - 1}}$ Find the minimum value of $f(x)$ for $x \in \mathbb{R}$ find the value of x for which it is attained.

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மொறட்டுவைப் பல்கலைக்கழக பொறியியற்பீட தமிழ் மாணவர்கள் நபாத்தும்
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முன்னோடிப் பரீட்சை - 2015

கல்விப் பொதுத் தராதரப் பத்திர(உயர் தர) முன்னோடிப் பரீட்சை - 2015 General Certificate of Education (Adv. Level) Pilot Examination - 2015

Combined Mathematics I
இணைந்த கணிதம் I

10 E I

Part B * Auswer five questions only.

Q11) a. Let α, β be the roots of the equation $x^2 + 2px + q^2 = 0$ and γ, δ be the roots of the equation $x^2 + 2mx + n^2 = 0$ where $b, c, m, n \in \mathbb{R}$.

(i) If $\alpha + \gamma = \beta + \delta$ then $p^2 + n^2 = q^2 + m^2$

(ii) If $\alpha\gamma + \beta\delta = 0$ then $q^2n^2 = p^2n^2 + q^2m^2$

Let the points where the curve $y = x^2 + (2x + 3) - k$ cut the axis be A and B and the points where the curve $y = x^2 + 2(2x + k) - 3$ cut the x axis be P and Q .

If $AB = PQ$ find k

b. Let $f(x) = x^4 - 2x^2 + 6$. using remainder theorem, show that $f(x)$ has no factor in the form $(x - \alpha)$ Find the range of x such that $f(x) \geq 30$

b, c are real costants such that $g(x) = 3f(x) + bx^3 + cx$ If $(x - 1)$ and $(x - 2)$ are the factors of $g(x)$. Find the values of b and c . Find the range of x such that $g(x) \geq 0$

Q12) a. Let the r^{th} term of the series $\frac{4}{1.2.3}\left(\frac{1}{3}\right) + \frac{5}{2.3.4}\left(\frac{1}{3}\right)^2 + \frac{6}{3.4.5}\left(\frac{1}{3}\right)^3 + \dots$ be U_r and

for $r \in \mathbb{Z}^+$ let $S_n = \sum_{r=1}^n U_r$ for $r \in \mathbb{Z}^+$. find the values of the constants A, B such that

$$\frac{U_r}{\left(\frac{1}{3}\right)^r} = \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$$

Hence, for $n \in \mathbb{Z}^+$ show that $S_n = \frac{1}{4} - \frac{1}{(n+1)(n+2)}\left(\frac{1}{3}\right)^n$

the infinite series $\sum_{r=1}^{\infty} U_r$ convergence? Justify your answer.

- b. Clearly stating all the axioms used draw the graph of $y = x^2 - b$, where $b > 0$. **Hence** draw the graph of $y = |x^2 - b|$, draw the graphs of $y = |x^2 - 1|$, $y = |x^2 - 7|$ on same diagram. Shade the region $\{(x, y) : |x^2 - 7| \geq y \geq |x^2 - 1|\}$ and find the area of shaded region.

Q13) a. Let $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$ be a 2×2 matrix.

For $a, b \in \mathbb{R}$ find the values of a, b such that $a\mathbf{I} + b\mathbf{A} + \mathbf{A}^2 = \mathbf{O}$. Here \mathbf{I} is 2×2 identity matrix and \mathbf{O} is 2×2 zero matrix.

Hence, find \mathbf{A}^{-1} . Let $\mathbf{B} = \begin{pmatrix} 3 & 2 \\ 5 & -3 \end{pmatrix}$ be a 2×2 matrix. By considering that \mathbf{B}^{-1} is in the form

of $\mathbf{B}^{-1} = \begin{pmatrix} p & q \\ r & s \end{pmatrix}$ find \mathbf{B}^{-1} . Find the 2×2 matrix \mathbf{C} such that $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix} \cdot \mathbf{C} \cdot \begin{pmatrix} 3 & 2 \\ 5 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 3 & -1 \end{pmatrix}$

- b. Using the **principle of mathematical induction**, show that $(1+i)^n = 2^{\frac{n}{2}} \left(\cos \frac{n\pi}{4} + i \sin \frac{n\pi}{4} \right)$

for a non zero positive integer n . By considering the binomial expansion of $(1+x)^n$. Find

$$\operatorname{Re}\{(1+i)^n\}, \operatorname{Im}\{(1+i)^n\}$$

Hence, for $n \in \mathbb{Z}^+$ deduce that $\tan \frac{n\pi}{4} = \frac{{}^nC_1 - {}^nC_3 + {}^nC_5 - {}^nC_7 + \dots}{1 - {}^nC_2 + {}^nC_4 - {}^nC_6 + \dots}$

Q14) a. Let $f(x) = \frac{x^2 - 6x + 4}{x^2 + 2x + 4}$ Show that $f'(x) = \frac{8(x-2)(x+2)}{(x^2 + 2x + 4)^2}$ and deduce that the curve has

turning points at $(-2, 5)$ and $\left(2, -\frac{1}{3}\right)$. Draw a rough sketch of the curve $y = f(x)$

showing the turning points and asymptotes.

Hence, find the number of solutions of the equation $(x^2 - 6x + 4) = (x^2 + 2x + 4) \cdot (e^x - e^{-x})$

- b. The total surface area of a closed right circular cylinder is $2\pi \mathbf{m}^2$. show that its volume is

$V = \pi(r - r^3) \mathbf{m}^3$ Here r is the radius of the cylinder. When r varies, show that the maximum

volume of the cylinder is $\frac{2\pi}{3\sqrt{3}} \mathbf{m}^3$

Q15) a. using a suitable substitution, show that $\int_0^{\pi} \frac{1}{4-3\sin x} dx = \frac{\pi}{\sqrt{7}} + \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{3}{\sqrt{7}} \right)$

b. Let $f(x) = \frac{Ax+B}{x^2+1}$ where $A, B \in \mathbb{R}$ find A, B such that $f(x) + f'(x) = \frac{x^3 - x + 2}{(x^2 + 1)^2}$

Using intergraion by parts, find the value of $\int_0^{\pi} e^x \cdot \frac{x^3 - x + 2}{(x^2 + 1)^2} dx$

c. Let $I = \int_0^1 \frac{1}{\sqrt{4-x^2-x^3}} dx$

For $x \in (0,1)$. Show that $4-2x^2 < 4-x^2-x^3 < 4-x^2$. and deduce that $\frac{\pi}{4\sqrt{2}} > I > \frac{\pi}{6}$

Q16) a. Show that the equation of a straight line passing through a fined point (x_1, y_1) can be

expressed in the parametric form $\frac{x-x_1}{\cos \theta} = \frac{y-y_1}{\sin \theta} = r$ Here $\tan \theta$ is the gradient r of that

line. r is parameter. identify this parameter.

This line posses through the point $A(-5, -4)$ and meet the straight lines $x+3y+2=0$, $2x+y+4=0$, $x-y-5=0$ at the poins B, C, D respectively. Show that

$\left(\frac{15}{AB} \right) = \cos \theta + 3 \sin \theta$ and find $\left(\frac{10}{AC} \right)$ and $\left(\frac{6}{AD} \right)$ in terms of θ

Hence, If $\left(\frac{15}{AB} \right)^2 + \left(\frac{10}{AC} \right)^2 = \left(\frac{6}{AD} \right)^2$ find the equation of that straight line.

b. If the two circles $x^2 + y^2 + 2gx + 2fy + c = 0$, and $x^2 + y^2 + 2g'x + 2f'y + c' = 0$ intersect orthogonally, show that $2gg' + 2ff' = c + c'$

Let $S \equiv x^2 + y^2 - 8x - 6y + 21 = 0$ and $S' \equiv x^2 + y^2 - 2y - 15 = 0$ Show that $S = 0$ and $S' = 0$ intersect orthogonally. Find the equation of the circle which posses through the poins of intersection of these two circles the centre of $S = 0$

Q17) a. Let $f(x) = \frac{1 + \tan x}{\cos x + \tan x \cdot \sin x}$ for $-\frac{5\pi}{4} \leq x \leq \frac{3\pi}{4}$

Express $f(x)$ in the form $A \sin(x + \alpha)$ Here $A(>0)$ and $\alpha\left(0 < \alpha < \frac{\pi}{2}\right)$ are constants to

be determined. Draw the rough sketch of the graph $y = f(x)$

Hence, solve the equation $\sin x + \cos x = \frac{4\sqrt{2}}{\pi} x$

b. If $x = a^2 \cos^2 \theta + b^2 \sin^2 \theta$ show that $x = \frac{1}{2}(a^2 + b^2) + \frac{1}{2}(a^2 - b^2) \cos 2\theta$ While a, b are positive constants and θ varies in the range $0 \leq \theta \leq \frac{\pi}{4}$,

Let $y = \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$. using the above result or otherwise,

show that $y^2 = a^2 + b^2 + 2\sqrt{\frac{1}{4}(a^2 + b^2)^2 - \left\{\frac{1}{2}(a^2 + b^2) - x\right\}^2}$

Hence, show that $(a + b) \leq y \leq \sqrt{2(a^2 + b^2)}$

Find the range of $\sqrt{1 + \sin^2 \theta} + \sqrt{1 + \cos^2 \theta}$

END OF QUESTIONS


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முன்னோடிப் பரீட்சை -2015

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A diagram showing a horizontal ray OA and another ray OB originating from point O . The angle between them is labeled 30° .

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முன்னோடிப் பரீட்சை - 2015

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General Certificate of Education (Adv. Level) Pilot Examination - 2015

Combined Mathematics II
இணைந்த கணிதம் II

10 E II

Part B * Answer five questions only.

- Q11) a.** A particle dropped from rest, from a height h above the ground at $t = 0$ falls under gravity. At the same time, another particle B is thrown vertically upward from a point on the ground with velocity u . Draw the velocity – time graphs for the motion of each particle. Using the velocity – time graph, show that the particulars are at same height from ground at time $\frac{h}{u}$. If

this height is $h\left(\frac{n-1}{n}\right)$ deduce that $u = (kgh)^{\frac{1}{k}}$ here k is a constant to be determined

- b.** A smooth wedge of mass M is placed as a smooth horizontal table. A particle of mass m is placed on the smooth face of the wedge inclined at an angle α with the horizontal and thrown upward with the speed v along the highest slope of that face. Show that the acceleration of the wedge is $\frac{mg \sin \alpha \cos \alpha}{M + m \sin^2 \alpha}$ and the acceleration of the particle relative to the

wedge is $\frac{(m + M)g \sin \alpha}{M + m \sin^2 \alpha}$

- (i) If the particle rises to a height h above the point of projection, show that

$$V = \left(2gh \cdot \frac{M + m}{M + m \sin^2 \alpha} \right)^{\frac{1}{2}}$$

- (ii) Show that the particle returns to the point of projection after time $\frac{2V}{(M + m)g} \cdot \left(\frac{M + m \sin^2 \alpha}{\sin \alpha} \right)$

- Q12)** A particle P of mass m is thrown horizontally inside a smooth fixed spherical shell with centre O and radius a at $t = 0$ with a speed of $\sqrt{4ga}$ from the lowest point. Show that the speed of the particle when it makes an acute angle θ with the downward vertical is $v = 2\sqrt{ga}\sqrt{\frac{1+\cos\theta}{2}}$ and the reaction exerted by the spherical shell on the particle is $R = mga(2+3\cos\theta)$. Find the velocity of the particle when it leaves the circular path.

After leaving from circular motion, this particle P moves in a trajectory path under gravity. By taking the horizontal and vertically upward axes through the point of leaving L as x and y axes, show that the equation of trajectory path is $y = \frac{\sqrt{5}}{2}x - \frac{27}{16a}x^2$.

If the particle reaches the horizontal level through the centre O of the sphere at point S , show that $OS = \frac{(4\sqrt{23}-5\sqrt{5})}{27}a$ and the time taken is $\left(\frac{a}{g}\right)^{\frac{1}{2}}\left(\frac{\sqrt{10}+\sqrt{46}}{3\sqrt{3}} + \ln(\sqrt{5}+\sqrt{6})\right)$

- Q13)** A particle P of mass m is attached to the midpoint of a light elastic string of natural length $2l$ and elastic modulus mg . The two ends of the string are attached to two fixed points A, B on a smooth horizontal table at a separation $4l$. Initially the particle is thrown with speed $\sqrt{7gl}$ in the direction of \overrightarrow{AB} from its equilibrium point O on the table. If $OP = x$ at time t , show that $\ddot{x} = -\frac{2g}{l}x$ for $0 \leq x \leq l$ and find the centre of oscillation and amplitude. Show that its velocity when $x = l$ is $\sqrt{5gl}$.

Show that $\ddot{x} = -\frac{g}{l}(x+l)$ for $l \leq x \leq 2l$ and show that time taken by it to come to rest for the first

time is $\left(\frac{l}{g}\right)^{\frac{1}{2}}\left(\cos^{-1}\left(\frac{2}{3}\right) + \frac{1}{\sqrt{2}}\cos^{-1}\left(\sqrt{\frac{5}{7}}\right)\right)$

- Q14) a.** A rectangle $ABCD$ is defined by points $A(0,0), B(5,0), C(5,3), D(0,3)$. Here lengths are in m . The forces of magnitude $6N, 8N, 4N, 2N$ along.
- Find the resultant of this system of forces.
 - Find the angle between the line of action of this resultant force and the x -axis. This line of action intersects the x -axis at the point $(a,0)$.
 - By taking moment about point A , find a . Hence, deduce the equation of line of action.

- b. A straight line intersect the sides CA, AB of a triangle ABC internally at E, F respectively and produced BC at D let $\overrightarrow{AB} = \mathbf{b}, \overrightarrow{AC} = \mathbf{c}$. If $\frac{BD}{CD} = p, \frac{CE}{EA} = q, \frac{AF}{FB} = r$

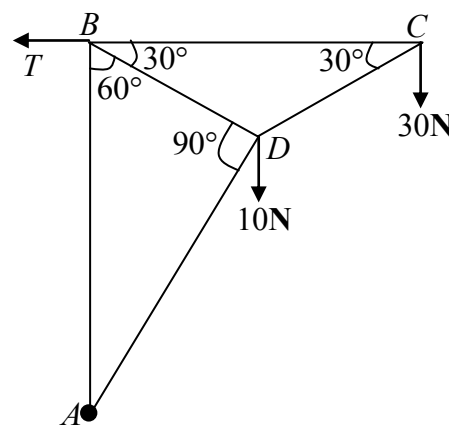
Show that $\overrightarrow{EF} = \frac{r}{(r+1)}\mathbf{b} - \frac{1}{(q+1)}\mathbf{c}$ and $\overrightarrow{DF} = \frac{pr+1}{(p-1)(r+1)}\mathbf{b} - \frac{p}{(p-1)}\mathbf{c}$ hence,

deduce that $pqr = 1$

- Q15) a.** Two uniform rods AB, AC of equal lengths and weight w, w' are smoothly jointed at A and Hanged from two hinges B, C at same level in a vertical plane. Prove that the horizontal component of the reaction at hinge A is $\frac{1}{4} \frac{(w+w')a}{h}$ Here $2a$ is the distance BC and h is the depth of A below BC . Find the vertical component of the reaction.

- b. Five light rods AB, BC, CD, DA and BD are smoothly jointed at their ends to form a framework as shown in the diagram. Here $\hat{DBC} = \hat{BCD} = 30^\circ, \hat{ABD} = 60^\circ, \hat{BDA} = 90^\circ$ The frame work is smoothly hinged at A and carries 30N at C and 10N at D . The frame work is kept in a vertical plane such that BC is horizontal.

using Bow's notation, draw a stress diagram for the framework and find the stress in all rods distinguishing whether they are tensions or thrusts.

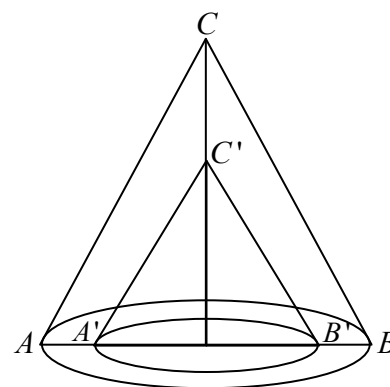


- Q16)** Using integration, show that centre of gravity of a uniform right circular solid cone of height h is at a distance $\frac{1}{4}h$ from the base of cone, on its symmetrical axis

A uniform right circular solid cone $A'B'C'$ of height h' and semi vertex angle α is removed from a uniform right circular solid cone ABC of height h and semi vertex angle α and the remaining part is shown in the diagram. Show that the centre of gravity of the remaining part S is at a distance

$$\frac{1}{4} \cdot \frac{(h+h')(h^2+h'^2)}{h^2+h'^2+hh'}$$
 from AB .

Hence, deduce the centre of gravity of a uniform hollow cone.



This object S is hanged from a point on the surface $A'C'$ at a distance $\frac{h'}{4}$ from AB by an inelastic string. In this state, If this string make an angle β with AB such that the other parts of it don't touch the cone, show that $3 \tan \alpha \tan \beta = \frac{h^3}{h^2 h' + h h'^2 + h'^3}$

- Q17) a.** A boy watches **shakthi** channel or **Ten sports** channel every day evening. The probability of watching **Ten sports** channel is $\frac{4}{5}$. If he watches **shakthi** channel, the probability for sleeping is $\frac{3}{4}$. If he watches **Ten Sports** channel the probability for sleeping is $\frac{1}{4}$. In an evening if he sleeps while watching channel find the probability of the event that he was watching **shakhi** channel.
- b.** The mass of pigs in an agricultural farm are measured to nearest kilogram. The data obtained are shown in the frequency table.

Mass Range	Number of Pigs
65 – 70	3
75 – 85	f_1
85 – 95	20
95 – 105	f_2
105 – 115	7

In the table the frequencies of mass ranges 75 – 85, and 95 – 105 are missing. But the median and mode of the distribution are found to be 90Kg and 87.5 kg.

Find the missing frequencies in the table and hence find the mean and standard deviation of this frequencies distribution.

END OF QUESTIONS