An Approximate Truthfulness Motivated Spectrum Auction for Dynamic Spectrum Access

Qinhui Wang*, Baoliu Ye*, Tianyin Xu† and Sanglu Lu*
*State Key Lab. for Novel Software and Technology, Nanjing Univesity, China
†Institute of Computer Science, University of Goettingen, Germany

***Abstract*—Secondary Spectrum Auction (SSA) has been proposed as an effective approach to design spectrum sharing mechanism for dynamic spectrum access. However, due to the location-constrained spectrum interference among users, it is a great challenge to provide truthful auction with maximized spectrum utilization. Most previous SSA designs either fail in addressing truthfulness or cause loss on spectrum utilization. In this paper, we focus on providing truthful SSA with maximized spectrum utilization. In order to minimize the computational overhead involved in addressing location-constrained interference, we leverage the truthfulness by introducing approximate truthfulness. Moreover, we define a general spectrum auction model using linear programming. Based on this model, we further propose ETEX, a sealed-bid auction mechanism with approximate truthfulness. Theoretical analysis confirms that ETEX is able to achieve truthfulness in expectation with polynomial complexity. Extensive experimental results show that ETEX outperforms most popular truthful spectrum auctions in terms of social welfare, spectrum utilization and user satisfaction.**

I. Introduction

Radio Spectrum is a critical but scarce resource for wireless communications. Traditionally, to eliminate the interference between different wireless communication pairs, radio spectrum is divided into a set of disjoint sub-bands and authorized to licensed users in a static fashion. With this kind of spectrum allocation approach, each licensed user has exclusive right for the allocated spectrum, thus always leading to lower spectrum utilization [1]. In recent years, with the rapid deployment of wireless facilities as well as the explosive growth of novel wireless services, the demand for radio spectrum is constantly increasing. This makes previous static spectrum allocation unfeasible. How to satisfy the continuously increased spectrum requirements with limited spectrum resource becomes one of the key issues which have attracted great research attentions.

Recently, with the advances in cognitive radio, dynamic spectrum access [2], [3] was proposed to address the above mentioned challenge. Different from early static spectrum allocation scheme, dynamic spectrum access motivates a new paradigm where unlicensed users are allowed to use spectrum resources held by licensed users in an opportunistic way, which may significantly improve spectrum utilization. However, licensed users are reluctant to share out spectrum with others unless additional benefits are available. Hence how to effectively incentivize licensed users to open-up their unutilized spectrum for sharing becomes the key issue, which is important for significantly improving spectrum utilization. In recent years, SSA has been deemed as an effective as well as efficient approach to dynamically share spectrum between unlicensed users and licensed users [4], [5], [6], [7], [8].

Different from early FCC-style auction which includes only a few large corporations located over national regions, a typical SSA is made up of a lot of small users distributed over a relatively small geographic region. As a result, an efficient SSA is required to be quickly conducted. In addition, considering the selfishness of players and the local competition on the same channels between proximate players, there are two critical properties related to SSA, i.e., truthfulness and location-constrained interference. Truthfulness is used to ensure that buyers' utility could be maximized only by independently declaring their true valuation of the goods without knowing the other buyers' bids. Truthful auctions could avoid market manipulation which makes life easier for buyers. Otherwise, each buyer has to figure out the others' bidding strategies before being able to get an optimal bidding strategy for himself. Location-constrained interference means that spectrum interference is a local effect which only constrains spectrum sharing among buyers with close proximity. These properties make traditional auctions useless and make truthful SSA mechanisms either deduce to solving an NP-hard problem or fail in addressing interference constraint [6]. Although existing popular spectrum auction mechanisms, such as [6], [7], could avoid location-constrained interference effectively, they usually depend on local optimization and thus lose spectrum utilization.

Different from previous work, in this paper we focus on providing truthful SSA with maximized spectrum utilization. In order to minimize the computational overhead involved in addressing location-constrained interference while improving spectrum utilization, we leverage the truthfulness by introducing the concept of approximate truthfulness [10] to replace the previous strong truthfulness. Approximate truthfulness is introduced to replace strong truthfulness to reduce the computation overhead. In this paper, by employing approximate truthfulness to SSA, computation overhead is reduced and spectrum utilization is also significantly improved (by up to 150%). In this paper, we begin by defining a general spectrum auction model using linear programming. Based on this model, we propose ETEX, an efficient auction mechanism achieving truthfulness in expectation. ETEX is a sealed-bid auction where bidders submit their bids privately to the auctioneer. Both our theoretical analysis and simulation results show that ETEX provides a computationally efficient spectrum auction

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with expected truthfulness. It also has a near-optimal allocation algorithm with maximizing social welfare and could generate more social welfare and higher spectrum utilization. To the best of our knowledge, it is the first work that introduce approximate truthfulness into dynamic spectrum access. The main contributions of our work in this paper are as follows:

1) We introduce the idea of approximate truthfulness to dynamic spectrum access and provide an approximate truthfulness auction model for spectrum auction via linear programming. Various approximate truthful auctions can be designed based on the model.
2) We propose ETEX, an efficient spectrum auction mechanism which can significantly improve the spectrum utilization by global optimization. Our theoretical analysis show that ETEX could achieve truthfulness in expectation with polynomial complexity.

The rest of the paper is organized as follows. Sec. II reviews related work. Sec. III introduces the spectrum auction model for dynamic spectrum access. In Sec.IV, we presents ETEX. Experimental results are shown in Sec. V. Sec. VI concludes the paper.

II. Related Work

Nowadays, efficient spectrum allocation for dynamic spectrum systems is a challenge problem and attracts great research interests [12], [13], [14], [15], [16]. In particular, there has been a lot of work on designing SSA mechanisms recently [4], [6], [7], [8]. There are multiple complementary ways to design SSAs, each applicable to different scenario. Gandhi *et al.* [4] propose a real-time spectrum auctions framework addressing interference constraint. To obtain computationally efficient mechanisms, Gandhi *et al.* [4] choose to linearize the interference constraint. Subramanian *et al.* [5] propose approximate algorithms to approach the optimal revenue. However, these auctions don't take truthfulness into account.

Zhou *et al.* [6] firstly investigate truthful SSA mechanism design issue and propose VERITAS which is a truthful spectrum auction with polynomial complexity. VERITAS, whose objective is to maximize social welfare (sum of total winners' bids), greedily allocates buyers sequentially based on sorted bids from high to low. In order to maximize total revenue, Jia *et al.* [7] adopt the concept of virtual valuation instead of the buyers' original value. Then maximizing total virtual valuation is equivalent to maximizing expected revenue (sum of total winners' charged prices). With the objective of maximizing revenue, Jia *et al.* [7] present a non-computationally efficient yet optimal truthful auction based on maximizing virtual valuation, and propose a computationally efficient suboptimal truthful auction, where allocation algorithm is also greedy on buyer's virtual valuation to determine winner vector. These works only consider the scenario of one seller and multi-buyers. Considering the scenario of multi-sellers and multi-buyers, Zhou *et al.* [8] then introduced double truthful spectrum auction to ensure the utility of both auctioneer and the buyer to be maximized by truth-telling.

However, all the allocation algorithms of the above truthful mechanisms are greedy-like which means considering only local effect on maximizing objective. Different from these work, our mechanism is based on global optimization which will generate much more valuation and higher spectrum utilization.

III. Auction Model for Dynamic Spectrum Access

Now we focus on the spectrum auction model for dynamic spectrum access. We first describe the notations and assumptions. We assume the spectrum to be auctioned is located in a geographic region and consists of m orthogonal channels which have uniform characteristics. There are totally n buyers (unlicensed users) biding for the requested channels. Each buyer submits its bid b_i to the auctioneer to request d_i channels, and each buyer has a true valuation v_i for the requested channels which is private information. After collecting bids from the bidders, the auctioneer determines winners according to some rules like maximizing social welfare. We use binary vector $\mathbf{X} = (x_1, \cdots, x_n)$ to indicate which users win in the auction, where $x_i = 1$ means buyer i wins and its channel request can be satisfied, while $x_i = 0$ means the opposite. For simplicity, we use $\mathbf{b_{-i}}$ to denote the vector of bids by all buyers except i. After winner determination, the auctioneer charge each winner with price p_i. The profit, also called as utility, obtained by each buyer can be denoted as $u_i(b_i, \mathbf{b_{-i}}) = v_i x_i(b_i, \mathbf{b_{-i}}) - p_i$. Note that the utility obtained by buyers is no less than 0. This ensures buyers will voluntarily participation the auction.

To describe the interference constraint, we introduce $G = (V, E)$ to represent the conflict graph where V is the collection of the buyers and E is the collection of edges where two buyers are in proximity conflict when sharing the same channels. Let $e_{ij} \in E$ and $e_{ij} \in \{0, 1\}$, where $e_{ij} = 1$ indicates that user i and j share an edge while $e_{ij} = 0$ means the opposite. The winner determination vector $\mathbf{X}$ is associated with a feasible allocation $A = \{a_{ij} | a_{ij} \in \{0, 1\}\}_{n \times m}$, where $a_{ij} = 1$ means channel j is assigned to user i and $a_{ij} = 0$ means the opposite. Given the above notations, the constraints for a feasible winner determination and its allocation vector can be formally expressed as follows:

$$\sum_{k=1}^{m} a_{ik} = x_i d_i, i = 1, ..., n \tag{1}$$

$$a_{ik} + a_{jk} \leq 1, \forall e_{ij} = 1, k = 1, ..., m, i, j = 1, ..., n \tag{2}$$

$$x_i, a_{ij} \in \{0, 1\}, i = 1, ..., n, j = 1, ..., m \tag{3}$$

Where (1) and (3) ensure the request is strict, which means the buyer will get either all d_i channels or nothing, and (2) represents the interference constraint.

In this paper, we follow the risk-neutral setting in economics which means buyers will have no preference between risk aversion and risk seeking. We assume the exact valuation v_i is private information and auctions are sealed bid auctions.

An auction mechanism consists of an allocation algorithm along with a pricing scheme. Let $M = (A, P)$ denotes the mechanism, where A denotes the allocation algorithm and P denotes the pricing scheme. The mechanism could be randomized, in which case A or P are randomize variables. A truthful auction is strongly related to certain monotonicity conditions on the allocation algorithm, which was formalized by Mu'alem *et al.* [11]. In other words, if A is monotone, then there must exist a single critical value c such that if i's bid $b_i > c$, then b_i is a winning declaration, while $b_i < c$ is a losing declaration. Moreover, if the payment P is based on critical value, then the auction(A, P) is truthful.

Strong truthfulness means to find the single critical value, which always invove complex computations. Strong truthfulness is so restrictive that a weaker but still useful concept is adopted. Approximate truthfulness is related to a randomized mechanism, and there have been various attempts to find it. One approach is to guarantee *truthfulness in expectation*, i.e., truthful bidding maximizes a buyer's expected profit [18]. Another is to guarantee *truthfulness with high probability*, i.e., a buyer may lie with small probability. In this paper, we pursue the former approach.

***Definition** 1 (Truthful in Expectation):* A randomized mechanism is truthful in expectation if for any buyer i, we have $E[u_i(v_i, \mathbf{b_{-i}})] \geq E[u_i(b_i, \mathbf{b_{-i}})]$ for any fixed bids $\mathbf{b_{-i}}$.

IV. ETEX Auction Design

In designing auctions, there are two natural goals either maximizing the revenue or maximizing the social welfare (also called as total valuation). The former is to maximize the revenue obtained by the seller, that is maximizing $\sum_{i=1}^{n} p_i$. However, the clearing price p_i is related to the true valuations which is private in most cases. As a result, we usually focus on the latter objective, maximizing the social welfare $\sum_{i=1}^{n} x_i b_i$, which is referred to as efficiency in the economics terminology. In this paper, we try to maximize the social welfare because v_i is private, and more important is that the auctioneer is willing to sell the spectrum to the users who could best use it, and the social welfare a strong indicator of how well positioned the buyer is to make good use of the sold spectrum bands. With the objective of maximizing social welfare, an optimal allocation algorithm can be given:

$$\max_{\mathbf{X}} \sum_{i=1}^{n} b_i x_i \tag{4}$$

s.t. (1)(2)(3)

The social welfare returned by the above allocation method is optimal. In fact, the above optimization problem is actually an Integer Linear Programming(ILP) problem. Its easy to check that the optimal allocation is NP-hard, since it's a SET PACKING problem for a simple case that there is stricter constraint where each channel could only be allocated to one user and the optimization version is a MAXIMUM SET PACKING problem.

Our approach to solve this complex optimization problem is using Linear Programming Relaxation. An NP-hard optimization problem (Integer Programming, IP) can be transformed into a related problem that is solvable in polynomial time (Linear Programming, LP) by the relaxation technique. That's replacing the constraint that each variable must be 0 or 1 by a weaker constraint, that each variable belong to the interval [0,1]. We set $\mathbf{X}^*(x_1^*, \cdots, x_n^*)$ be the fractional solution of the integer programming relaxation, where $x_i^* \in [0, 1]$. The relaxation allocation is given as follows:

$$\max_{\mathbf{X}^*} \sum_{i=1}^{n} b_i x_i^* \tag{5}$$

s.t.

$$\sum_{k=1}^{m} a_{ik}^* = x_i^* d_i, i = 1, ..., n \tag{6}$$

$$a_{ik}^* + a_{jk}^* \leq 1, \forall e_{ij} = 1, i, j = 1, ..., n, k = 1, ..., m \tag{7}$$

$$0 \leq a_{ik}^*, x_i^* \leq 1, i = 1, ..., n, k = 1, ..., m \tag{8}$$

It's easy to show that for any fixed $\mathbf{b_{-i}}$, x_i^* is a non-decreasing function of b_i. We omit the proof in this paper.

***Lemma** 1:* For any fixed $\mathbf{b_{-i}}$, if $b_i' > b_i$, then $x_i^*(b_i', \mathbf{b_{-i}}) \geq x_i^*(b_i, \mathbf{b_{-i}})$.

Various approximate truthful auctions can be designed based on the fractional solution. Next we first give a simple truthful auction and an auction achieving truthfulness with error probability, then we present ETEX. We'll compare ETEX with the following simple auctions to demonstrate the efficiency of ETEX.

A. A Simple Truthful Auction

Based on the fractional allocation solution, a simple allocation algorithm can be given as follows:

1) Solve (5) to get an optimal solution $\mathbf{X}^*$.
2) Satisfy all buyers i for which $x_i^* = 1$.

The following lemma(see lemma 2) will guarantee the simple allocation algorithm is monotone.

***Lemma** 2:* In simple allocation algorithm, if any buyer i is allocated by bidding b_1, then it will also be allocated if it bids b_2, where $b_2 > b_1$.

Proof: Let x_1 and x_2 be the corresponding solution of LP with bidding b_1 and b_2. If buyer i wins with bidding b_1, then $x_i = 1$. If $b_2 > b_1$, then by lemma 1, we get $x_2 \geq x_1$, so x_2 will also equals 1, as a result the buyer will also win in the auction. ■

Therefore, if the payment pricing is based on critical value, we can easily demonstrate the simple allocation algorithm along with the 'threshold' payment is truthful.

Furthermore, we can easily obtain another naive auction achieving truthfulness with error probability via randomized rounding. We first obtain the optimal solution $\mathbf{X}^*$, then we randomly rounding each x_i' to 1 with probability x_i^* and to 0 with probability $1 - x_i^*$. We satisfy buyers i with $x_i' = 1$ if the available channels of i $Avai(i)$ is greater than the request d_i. It's easy to demonstrate the naive auction will be universally truthful if no conflicts happened (according to lemma 1), as a result, it'll be truthful with error probability and we have tested the error probability in section V.

B. ETEX Auction Design

However the social welfare returned by the simple auction may not be good enough since not all users whose requests can be satisfied will be computed with $x_i^* = 1$, and the error probability caused by the naive approximate truthful auction may not be low. Therefore we present ETEX.

1) ETEX Allocation Rules: Let $B = \{x_1^*, \cdots, x_n^*\}$, we first sort the optimal solution set B. Let $B^{'} = Sort(B)$, we then sequentially allocate channels to buyers from high to low according to the solution ranking $B^{'}$. In ETEX, strict request is adopted, which means the buyer will get either all d_i channels or nothing. For each buyer i, if the available set of channels $Avai(i)$ is greater than the request d_i, we then use $Assign(i, d_i)$ to assign i d_i channels that are in $Avai(i)$. After allocating, we use $Update(Avai(N(i)))$ to remove the allocated channels from the available set of channels of the neighbors $N(i)$ of user i. The detailed allocation algorithm is given as follows:

Algorithm 1 ETEX Allocation Algorithm

$BFS(G)$
Solve (5) to get the determining set B
$B^{'} = Sort(B)$
while $B^{'} \neq \phi$ **do**
  $i = Top(B^{'})$
  if $d_i \leq Avai(i)$ **then**
    $Assign(i, d_i)$
    $Update(Avai(N(i)))$
  end if
  $B^{'} = B^{'} \backslash \{b_i\}$
end while

To guarantee satisfying the buyer i's channel request with optimal solution $x_i^* = 1$, we first use Breadth-First Search $BFS(G)$ to sort the topology $G(V, E)$, which ensures the update process is executed sequentially. It's easy to check that the total valuation generated by ETEX allocation is no less than that generated by the simple allocation algorithm, since the buyer i with $x_i^* = 1$ will also be satisfied in ETEX allocation. Moreover, the following lemma guarantees the ETEX allocation algorithm is monotone.

***Lemma** 3:* The ETEX allocation algorithm is monotone

Proof: Consider two sorted lists of solution of LP, B_1 and B_2 in Figure 1. In B_1, the buyer i is allocated by bidding b_1, and the corresponding solution is x_1, and in B_2, the buyer i bids b_2, where $b_2 > b_1$. Let $x_k^1, k \in N(i)$ be the corresponding LP solution of the neighbors of buyer i, and Y_1 denotes the sorted set of $\{x_k^1, k \in N(i)\} \cup x_1$, then the position of buyer i in Y_1 is $pos(b_1)$. Similarly, let $x_k^2, k \in N(i)$ be the corresponding LP solution of the neighbors of buyer i, and Y_2 denotes the sorted set of $\{x_k^2, k \in N(i)\} \cup x_1$, then the position of buyer i in Y_2 is $pos(b_2)$. As $b_2 > b_1$, then $x_2 \geq x_1$ by lemma 1, and each $x_k^2 \leq x_k^1, \forall k \in N(i)$ by the interference constraint. Therefore we get $pos(b_2) \geq pos(b_1)$. As a result, if buyer i is allocated with position $pos(b_1)$, then it also be allocated with position $pos(b_2)$, because no neighbors of i will cause conflicts to user i as the same with position $pos(b_1)$. ■

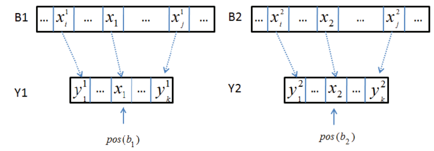


Fig. 1. Two sorted lists of solution of LP where only i's bid is different, $pos(b_1)$ and $pos(b_2)$ denotes the position in the sorted list of Y_1 and Y_2

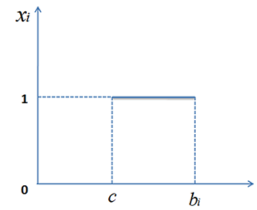


Fig. 2. The graph shows the each buyer i's winning probability as a function of different bids b_i fixing b_{-i}, where c is critical value

2) ETEX Pricing Scheme: There must exist a threshold payment c to ensure truthfulness due to the monotonicity of the ETEX allocation algorithm. Let x_i denotes the overall probability that i wins the desired channels. As show in Figure 2, $x_i(b_i, \mathbf{b_{-i}})$ is a step function of b_i. When bidding $b_i > c$ then i will be allocated with the requested channels, where c is the critical value of buyer i. Therefore, if we charge each winner c, the ETEX will guarantee truthfulness. However, it's hard to find the exact threshold payment value. Instead, we randomize the price for winner i. Let random variable Z denote the integral part of the payment. Select some $u \in [0, b_i]$ uniformly at random, and run the efficient allocation algorithm once with buyer i bidding u. If the buyer wins, then set $Z = b_i$, else set $Z = 0$. It's easy to compute the expectation of the payment $E[P(b_i, \mathbf{b_{-i}})]$ for winner i is:

$$E[P(b_i, \mathbf{b_{-i}})] = b_i - (\frac{c_i}{b_i} * 0 + (1 - \frac{c_i}{b_i}) * b_i) = c_i \qquad (9)$$

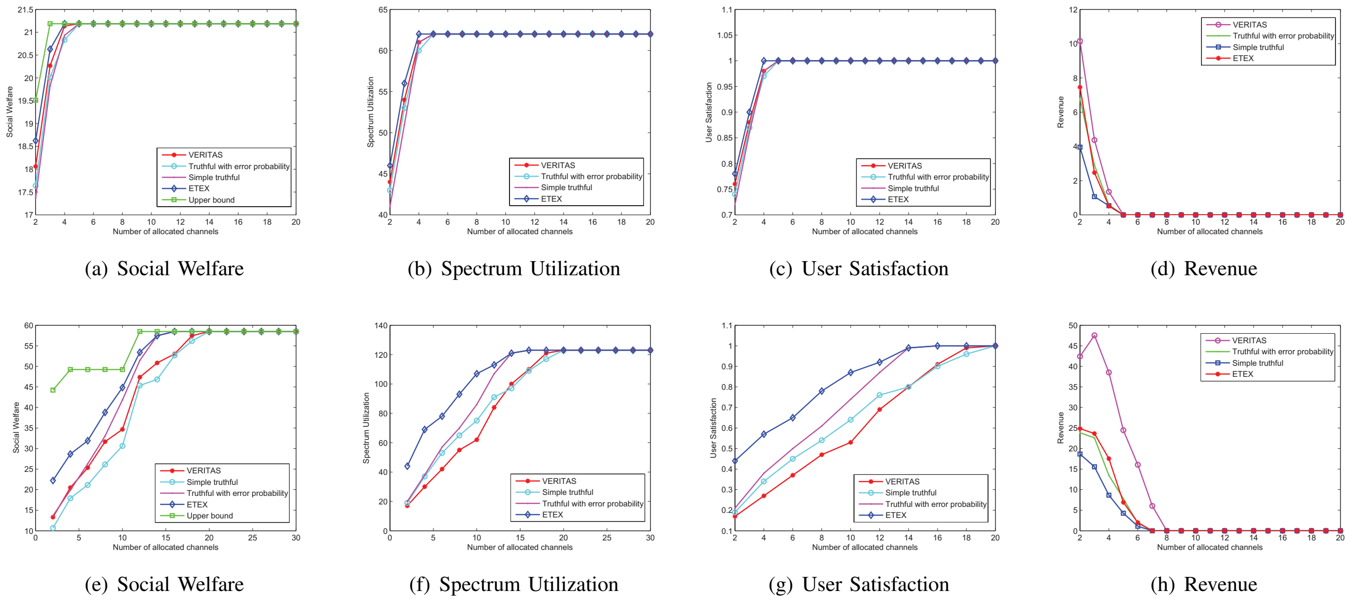
3) Properties: We evaluate the properties of ETEX in terms of truthfulness and computational complexity. With the allocation rules and the pricing shceme, we obtain:

***Theorem** 1:* The ETEX is truthful in expectation.

Proof: Let v_i be the truthful bidding, and b_i be the bidding such that $b_i \neq v_i$.

1) CASE 1: i loses by bidding b_i, then i will also lose by bidding v_i by lemma 3, and $u_i(v_i) = u_i(b_i) = 0$.
2) CASE 2: i wins by bidding b_i. If i also wins by bidding v_i, because the payment of critical value is same, then $E[u_i(b_i)] = v_i - c = E[u_i(v_i)]$. If i loses by bidding v_i, then the critical value c must be greater than v_i. Therefore the utility by bidding b_i is $E[u_i(b_i)] = v_i - c < 0 = E[u_i(v_i)]$.

In bothe cases, we have $E[u_i(b_i)] \leq E[u_i(v_i)]$, this completes the proof. ■



(a) Social Welfare (b) Spectrum Utilization (c) User Satisfaction (d) Revenue

(e) Social Welfare (f) Spectrum Utilization (g) User Satisfaction (h) Revenue

Fig. 3. Evaluate the performance of truthful auction mechanisms. Top 4 figures assume scatter conflict graphs and bottom 4 figures assume cluster conflict graphs

We now analyze the computational complexity of ETEX. ETEX runs the ETEX allocation twice. Despite the process of LP, the ETEX-allocation takes $O(n+|E|)$ to sort the topology, and $O(nlogn)$ time to sort B. For each process of updating the available channels of the neighbors of i, it takes at most $O(n)$ time. Then the ETEX runs in time $O(|E|+n^2)$ plus double times of the time consumed by the LP process with bounded $m \times n$ parameters, where $|E| \leq \frac{n(n-1)}{2}$.

V. Experimental Results

In this section, we use extensive experiments to evaluate the performance of ETEX. We compare ETEX with VERITAS which is proposed by Zhou *et al.* [6] and other approximate truthful auctions mentioned above: the simple truthful auction and the truthful auction with error probability. Similarly, we use the following performance metrics:

1) Social Welfare: The sum of all winners' bids, that's $\sum_{i=1}^{n} x_i b_i$.
2) Renevue: The sum of total charging payments from winners, that's $\sum_{i=1}^{n} x_i p_i$.
3) Spectrum Utilization: The sum of allocated channels of winners, that's $\sum_{i=1}^{n} x_i d_i$.
4) User Satisfactory: The percentage of winners.

We assume a number of buyers are randomly deployed in a square 1×1 area, and if the distance between any two buyers is less than 0.1, then there is an edge between them. That's to say they will interfere with each other if they share the same channel at the same time. We assume each buyer's channel request is randomly equal 1 or 2, which is less than the number of total available channels m, and each buyer's true valuation to the required channels set is uniformly distributed over $(0, 1]$.

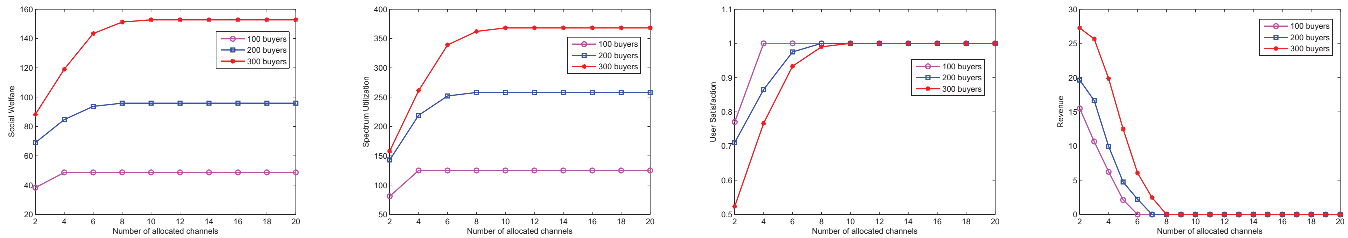
We first evaluate the performance of ETEX by comparing with VERITAS, also we test the simple truthful auction and the truthful auction with error probability mentioned above. We evaluate the performance with different topologies respectively.

Scatter Topologies The number of deployed buyers is 50. The results are show in Figure 3(a)-(d).

Cluster Topologies We randomly deploy 100 buyers in the square, but the interference distance is set to be 0.3. The results are show in Figure 3(e)-(h).

From Figure 3(a)-(h), we found: (1) ETEX is similar with VERITAS and other approximate truthful auctions when the bidders(buyers) are sparse, while ETEX outperforms other auctions when the bidders are intensive, such as ETEX outperforms VERITAS by up to 150% in spectrum utilization. This is because more mutual interferences cause less of buyer's requests are satisfied; (2) The revenue generated by ETEX is less than that generated by VERITAS when in intensive situation. Therefore, the spectrum holder's total valuation maximization is at the cost of degradation of revenue generation. (3) VERITAS generates more social welfare but with less user satisfaction due to the greedy on bids. Intuitively, much more buyers won't be charged when using global optimization allocation method, so the revenue obtained from winning buyers decreased.

Then we evaluate ETEX independently with 100 bidders, 200 bidders and 300 bidders respectively. Figure 4(a)-(d) show the results. The social welfare, spectrum utilization returned by ETEX is increasing with the growth of number of bidders because more buyers' request will be satisfied, Revenue is also increasing because bidders will be charged more to obtain the requested channels when the conflicting neighbors increases.



(a) Social Welfare (b) Spectrum Utilization (c) User Satisfaction (d) Revenue

Fig. 4. Evaluate the performance of ETEX with different bidders.

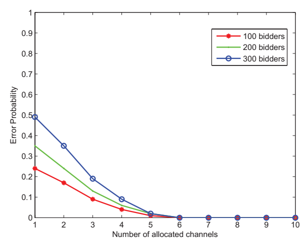


Fig. 5. The error probability of the truthful auction with error probability

We also test the error probability of the truthful auction with error probability. We evaluate the auction with 100 bidders, 200 bidders and 300 bidders respectively, and we test the conflict probability by running the allocation algorithm 3 * 100 times for each situation. The result is averaged over the 3 group experiments. As show in Figure 5, the error probability increases with the number of bidders where the conflicting edges increases. Therefore we concludes that although the truthful auction with error probability performs well yet it won't be truthful with big probability in intensive cases.

VI. Conclusion and Future Work

In this paper, we define a general spectrum auction model for dynamic spectrum access, and we introduce the concept of approximate truthfulness to design spectrum auctions. We show that various approximate truthful auctions can be defined based on the model. Moreover, we design ETEX, an efficient dynamic spectrum auction mechanism to serve many small players. We prove analytically ETEX is truthful in expectation and has polynomial complexity, which outperforms some popular spectrum auctions in terms of social welfare, spectrum utilization and user satisfaction. The extension of the auction model to the other spectrum request formats, such as range-based request, will be included in the future work.

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References

[1] M. A. McHenry, P. A. Tenhula, D. McCloskey, D. A. Roberson, and C. S. Hood, "Chicago Spectrum Occupancy Measurements & Analysis and a Long-term Studies Proposal ", In *Proc. of International Workshop on Technology and Policy for Accessing Spectrum (TAPAS)*, Aug. 2006.

[2] R. J. Berger, "Open Spectrum: A Path to Ubiquitous Connectivity", *ACM Queue Magazine*, 1(3):60-68, Jul. 2003.

[3] XG working group RFC, "The XG Vision and The XG Architecture", http:// www.darpa.mil/ato/programs/XG.

[4] S. Gandhi, C. Buragohain, L. Cao, H. Zheng, and S. Suri, "A General Framework for Wireless Spectrum Auction", *In Proc. of IEEE DySPAN*, Apr. 2007.

[5] A.P. Subramanian, M. Al-Al-Ayyoub, H. Gupta, S.R. Das, and M.M. Buddhikot, "Near-Optimal Dynamic Spectrum Allocation in Cellular Networks", *In Proc. of IEEE DySPAN*, Oct. 2008.

[6] X. Zhou, S. Gandhi, S. Suri, and H. Zheng, "eBay in the Sky: Strategy-Proof Wireless Spectrum Auctions", *In Proc. of ACM MobiCom*, Sep. 2008.

[7] J. Jia, Q. Zhang, Qin Zhang, and M. Liu, "Revenue Generation for Truthful Spectrum Auction in Dynamic Spectrum Access", *In Proc. of ACM MobiHoc*, May 2009.

[8] X. Zhou and H. Zheng, "TRUST: A General Framework for Truthful Double Spectrum Auctions", *In Proc. of IEEE INFOCOM*, Apr. 2009.

[9] T. Groves, "Incentive in terms", *Econometrica*, 41(4):617-631, Jul. 1973.

[10] A. Archer, C. H. Papadimitriou, K. Talwar, and Éva Tardos, "An Approximate Truthful Mechanism for Combinatorial Auctions with Single Parameter Agents", *In Proc. of SODA*, Jan. 2003.

[11] A. Mu'alem and N. Nisan, "Truthful Approximate Mechanisms for Restricted Combinatorial Auctions", *In Proc. of AAAI*, Mar. 2002.

[12] C. Peng, H. Zheng, and B. Y. Zhao, "Utilization and Fairness in Spectrum Assignment for Opportunistic Spectrum Access", *ACM/Springer Mobile Networks and Applications (MONET)*, 11(4):555-576, Aug. 2006.

[13] H. Zheng and C. Peng, "Collaboration and Fairness in Opportunistic Spectrum Access", *In Proc. of IEEE ICC*, May 2005.

[14] L. Cao and H. Zheng, "Stable and Efficient Spectrum Access in Next Generation Dynamic Spectrum Networks", *In Proc. IEEE INFOCOM*, Apr. 2008.

[15] L. Cao and H. Zheng, "Spectrum Allocation in Ad Hoc Networks via Local Bargaining", *In Proc. of SECON*, Sep. 2005.

[16] N. Nie and C. Comaniciu, "Adaptive Channel Allocation Spectrum Etiquette for Cognitive Radio Networks", *In Proc. of IEEE DySPAN*, Nov. 2005.

[17] R. Motwani and P. Raghavan, "Randomized Algorithms", *Cambridge University Press*, 1995.

[18] A. Archer and Éva Tardos, "Truthful Mechanisms for One-Parameter Agents", *In Proc. of FOCS*, Oct. 2001.