

## NOTES AND CORRESPONDENCE

### Tidal Resonance in Juan de Fuca Strait and the Strait of Georgia

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#### ABSTRACT

The resonant period and quality factor  $Q$  are determined for the semienclosed sea comprising Juan de Fuca Strait, Puget Sound, and the Strait of Georgia. The observed tidal elevation gain and phase change, from the Pacific Ocean to this inland sea, are fitted to the predictions of simple analytic models, which give a resonant period of 17–21 h and a  $Q$  of about 2. The low  $Q$  value, indicative of a highly dissipative system, is consistent with the need for numerical models for the area to employ large bottom friction coefficients. These include the effects of form drag.

#### 1. Introduction

The response of coastal seas to tidal forcing depends on the properties of offshore tides, the details of the water depth and coastline, and the influence of friction. Discussing this response in terms of the proximity to resonance can be helpful in explaining particularly high tides, as in the Bay of Fundy (e.g., Garrett 1972). Knowing how close a system is to resonance provides an indication of the sensitivity of the local tidal regime to gradual changes in mean sea level and to changes in geometry caused by human activities.

It can, of course, be argued that the frequency-dependent response of a coastal region can be determined perfectly well using a numerical model. However, the uncertainties that are present in any numerical model are such that it is valuable if estimates of resonant frequencies can be made directly from data, exploiting the fact that the coastal region is being forced by nature at several different tidal frequencies and responds differently to each. This is particularly true for the imaginary part of any frequency response, which represents the damping by friction that may be difficult

to define precisely. In particular, the friction coefficients in a numerical model often need to be tuned extensively for the model to accurately simulate the tidal response for many of the constituents.

It is in this spirit that, in this paper, we examine the tides of the coastal sea comprising Juan de Fuca Strait, Puget Sound, and the Strait of Georgia (Fig. 1). In section 2 we examine the frequency dependence of the tidal response to offshore forcing, and in section 3 we use simple models to determine a resonant frequency and damping rate. The values thus obtained will be compared in section 4 with the results of numerical models, with particular attention to the determination of appropriate friction coefficients.

#### 2. Elevation gain and phase change

We base our analysis on the frequency dependence of the elevation gain and phase change, which is calculated by dividing the complex tidal height of the head of the system (northern Strait of Georgia) by the height forcing the system (just outside the entrance of Juan de Fuca Strait). The head of the system is represented by the complex mean of the tidal harmonics between Twin Islets and Little River (Table 1). Tidal harmonics outside the system were obtained from the analysis of Ocean Topography Experiment (TOPEX)/Poseidon

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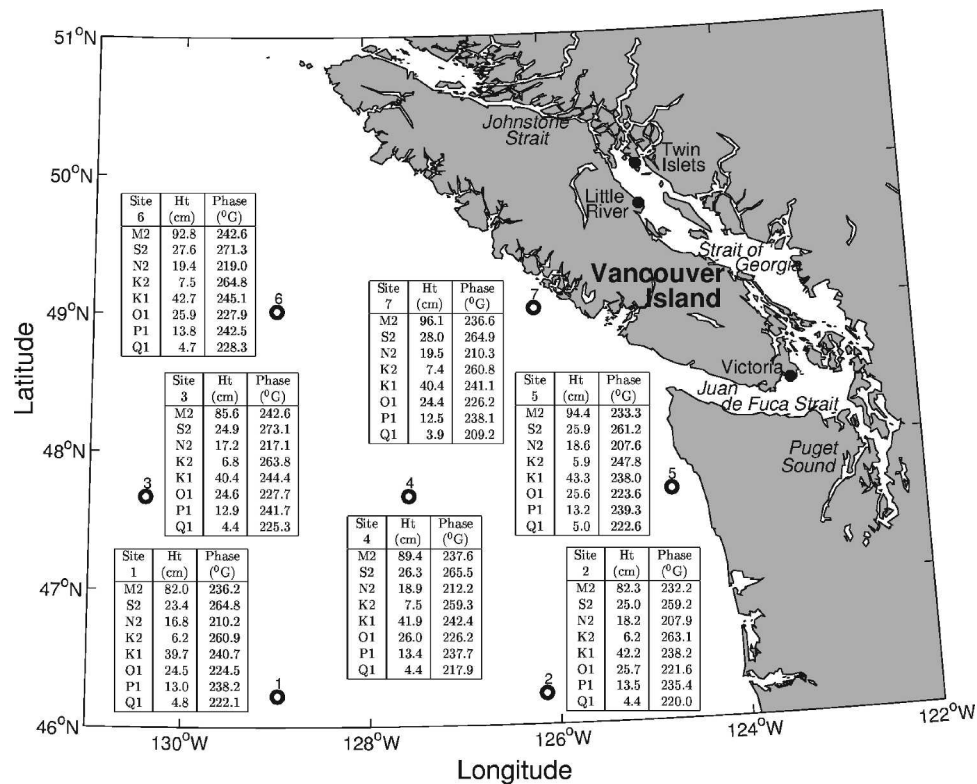


FIG. 1. Map of Vancouver Island. The solid dots denote tide gauge locations, and the circles are TOPEX/Poseidon satellite altimetry track cross-over locations.

satellite altimetry data (Cherniawsky et al. 2001) at track crossover locations near Vancouver Island (Fig. 1, circles).

Sensitivity to variations in the forcing was obtained from three trial sets of TOPEX/Poseidon locations.

Sites 1, 2, 3, 4, and 6 were used in the first trial, sites 5 and 7 in the second trial, and all seven sites were used in the third trial.

The forcing via Johnstone Strait has not been considered in the analysis, because the volume of the tidal

TABLE 1. Tidal constituent data for two locations in the northern Strait of Georgia obtained through a harmonic analysis (Foreman 1977) of a 1-yr time series (Department of Fisheries and Oceans 1994) at each respective location. The phase lag is relative to Greenwich mean time.

Site	Lat (°N)	Lon (°W)	Constituent	Amplitude (cm)	Phase lag (°)
Little River	49.741	124.923	$M_2$	99.36	32.87
			$S_2$	25.02	61.60
			$N_2$	21.64	5.42
			$K_2$	6.80	62.56
			$K_1$	90.19	287.03
			$O_1$	49.26	263.94
			$P_1$	28.62	285.67
			$Q_1$	8.38	257.20
Twin Islets	50.030	124.936	$M_2$	101.29	35.37
			$S_2$	25.82	64.80
			$N_2$	21.82	9.82
			$K_2$	6.92	63.66
			$K_1$	90.37	287.53
			$O_1$	49.29	264.24
			$P_1$	28.62	286.97
			$Q_1$	7.89	258.59

flow through it is only one-fifteenth of that through Juan de Fuca Strait (Thomson 1981).

### 3. Simple models

Near resonance, the frequency response of a system is given reasonably well by the equation (e.g., Garrett 1972)

$$\zeta(\mathbf{x}, \omega) = \zeta(\mathbf{x})a \left(1 - \frac{\omega}{\omega_0} - \frac{1}{2}iQ^{-1}\right)^{-1}, \quad (1)$$

where  $\text{Re}[\zeta(\mathbf{x}, \omega)e^{-i\omega t}]$  is the height of the tide,  $\zeta(\mathbf{x})$  is the shape of the normal mode,  $a$  is the (complex) height of the tide forcing the system,  $\omega$  is the frequency of the tide,  $\omega_0$  is the resonant frequency of the system, and the fraction of energy dissipated per cycle is defined as  $2\pi Q^{-1}$ . Fitting elevation ratios proved to be very effective in determining the resonant period and  $Q$  for the Bay of Fundy and Gulf of Maine (Garrett 1972) because the resonant period of 13.3 h is close enough to the periods of the semidiurnal tides that it is reasonable to use (1). Moreover, the constituents have significantly different gains that well-defined values of  $\omega_0$  and  $Q$  could be obtained using (1).

However, in the Juan de Fuca Strait/Strait of Georgia system, the dominant resonant period is located in between the semidiurnal and diurnal frequencies (Crean et al. 1988), rendering (1) a dubious approximation. Also, the amplitude response varies little over a single tidal band. Therefore, this method must be further developed to cover a larger range of frequencies than just those close to resonance.

#### a. Rectangular bay model

The Juan de Fuca/Strait of Georgia system is first approximated as a rectangular bay of length  $L$  and depth  $h$ . The Rossby radius of deformation is much greater than the channel width, allowing the Coriolis parameter to be neglected and a one-dimensional model to be adopted. An analytic solution for the tidal height,  $\text{Re}(\zeta e^{-i\omega t})$ , and current,  $\text{Re}(ue^{-i\omega t})$ , is easily obtained from the linearized momentum and continuity equations

$$-i\omega u + g \frac{\partial \zeta}{\partial x} + \lambda u = 0 \quad \text{and} \quad (2)$$

$$-i\omega \zeta + h \frac{\partial u}{\partial x} = 0, \quad (3)$$

where  $\omega$  is the forcing frequency,  $g$  is the acceleration due to gravity,  $h$  is the uniform depth of the bay, and  $\lambda$  is a linear friction coefficient that crudely approximates the more common quadratic expression with

$$\lambda = \frac{C_D |u|}{h}, \quad (4)$$

where  $C_D$  is the quadratic bottom friction coefficient.

Solving (2) and (3) for  $\zeta$  and  $u$ , with boundary conditions  $\zeta(0) = a$  and  $u(L) = 0$  at the entrance and the head of the system, respectively, gives

$$\zeta(x) = \frac{a \cos k(x - L)}{\cos kL} \quad \text{and} \quad (5)$$

$$u(x) = \frac{i\omega a \sin k(x - L)}{kh \cos kL}, \quad (6)$$

where

$$k^2 = \frac{\omega^2}{gh} \left(1 + \frac{i\lambda}{\omega}\right)$$

and  $a$  is the complex amplitude of the tide forcing the bay. Assuming  $\omega \gg \lambda$ , the product  $kL$  (to first order) is

$$kL \approx \frac{\omega L}{\sqrt{gh}} \left(1 + \frac{i\lambda}{2\omega}\right) = \frac{\omega}{\omega_0} \frac{\pi}{2} + \frac{i\lambda}{2\omega_0} \frac{\pi}{2} = \theta + i\epsilon, \quad (7)$$

where  $\theta$  and  $\epsilon$  are defined as

$$\theta = \frac{\omega}{\omega_0} \frac{\pi}{2} \quad \text{and} \quad (8)$$

$$\epsilon = \frac{\lambda}{2\omega_0} \frac{\pi}{2}. \quad (9)$$

Here  $\omega_0$ , the quarter-wavelength resonance frequency for a rectangular bay of constant depth, is given by

$$\omega_0 = \frac{\sqrt{gh} \pi}{L}. \quad (10)$$

Utilizing (5), along with the above approximation for  $kL$ , the theoretical amplitude gain is calculated by evaluating  $\zeta(L)/\zeta(0)$ ; that is,

$$Ae^{i\phi} = \text{seck}L \quad (11)$$

$$\approx (\cos\theta - i\epsilon \sin\theta)^{-1}, \quad (12)$$

where  $A$  is the amplitude gain and  $\phi$  is the phase difference of the elevation at the head of the system compared to that at the entrance.

For frequencies near resonance (i.e., as  $\omega \rightarrow \omega_0$ ), the amplitude gain of (12) behaves as

$$Ae^{i\phi} = \left(1 - \frac{\omega}{\omega_0} - \frac{1}{2}i \frac{\lambda}{\omega_0}\right)^{-1}, \quad (13)$$

which is consistent with (1) using the definition for the quality factor

$$Q = \frac{\omega_0}{\lambda}. \quad (14)$$

If the friction is quadratic and if the current speed of a particular tidal constituent is much greater than the other constituents, the dominant constituent will experience less friction than weaker constituents. It has been shown (e.g., Jeffreys 1970) that for  $u = \cos\omega_1 t +$

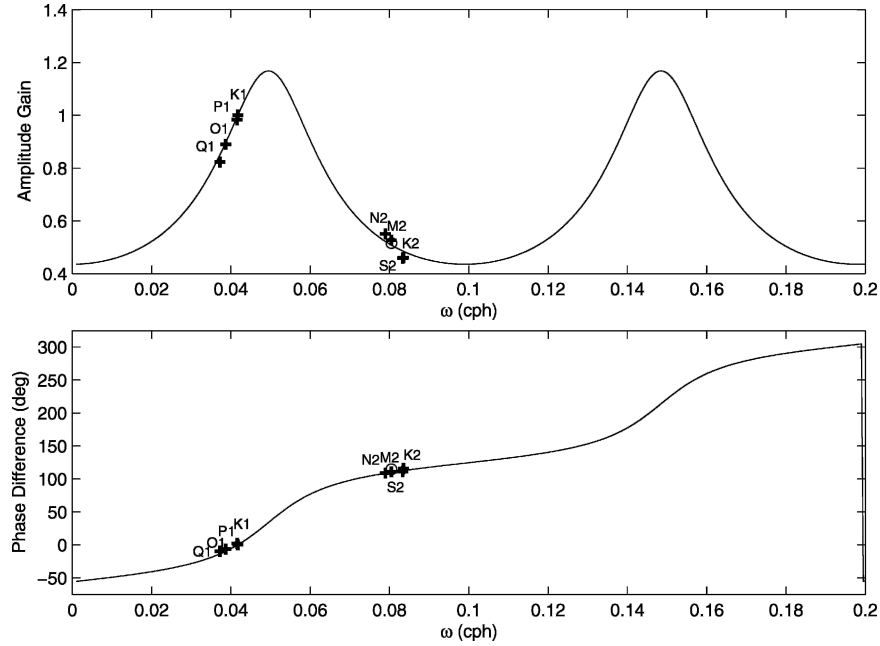


FIG. 2. Resonance fitting of (15) for the Juan de Fuca/Strait of Georgia system. The circle denotes the  $M_2$  response of (15), which is slightly different than the rest of the curve because of this constituent experiencing less friction than the other constituents. The amplitude gain and phase difference are normalized to the  $K_1$  constituent, and the outside forcing is represented by the first trial.

$\nu \cos \omega_2 t$ , where  $\nu < 1$  is the ratio of the current speed for the weaker  $\omega_2$  constituent to that of the stronger  $\omega_1$  constituent, the Fourier component of  $u|u|$  at frequency  $\omega_2$  is  $1.5\nu + O(\nu^3)$  times the Fourier component at frequency  $\omega_1$ . In short, weaker constituents feel 50% more damping than the dominant constituent. This is associated with quadratic damping being greater than average for large tides and less than average for small tides. This effect was taken into account by Garrett (1972) in his analysis of the Bay of Fundy and Gulf of Maine. Although the tidal elevation amplitudes of  $M_2$  and  $K_1$  are similar for most of Juan de Fuca Strait and the Strait of Georgia, the  $M_2$  constituent transports the same volume of water in one-half of the time, causing the tidal currents associated with  $M_2$  to be larger than those of the other constituents (Thomson 1981).

To account for this, (12) is replaced with

$$Ae^{i\phi} = (\cos \theta - ic\epsilon \sin \theta)^{-1}, \quad (15)$$

where  $c = 2/3$  for  $M_2$  and  $c = 1$  for all the other constituents. In all subsequent calculations the  $Q$  value obtained will be the  $Q$  experienced by all the constituents other than  $M_2$ . The  $Q$  for  $M_2$  is assumed to be 50% greater than that of the other constituents, though this may be too big an increase as  $\nu$  is not particularly small here.

To determine  $\omega_0$  and  $\lambda$ , a least squares approach was

used to fit the data with (15) for the three different trials, described earlier, of the tidal elevation forcing the system. All of the fits were achieved with both the data and (15) normalized to the complex elevation of either the  $K_1$  or  $M_2$  tidal constituent. The regression was done by varying  $\omega_0$  and  $\lambda$  to minimize the function

$$\sum_j \sigma_j^2 = \sum_j \left| \frac{A_j e^{i\phi_j}}{A_n e^{i\phi_n}} - \frac{(\cos \theta_j - ic\epsilon_j \sin \theta_j)^{-1}}{(\cos \theta_n - ic\epsilon_n \sin \theta_n)^{-1}} \right|^2, \quad (16)$$

where the subscript  $n$  denotes the constituent for which (15) was normalized. The  $j$  subscript denotes the tidal constituents other than  $n$ .

The resonance curve for trial 1 normalized to  $K_1$  is shown in Fig. 2. The computed resonant period and  $Q$  varied very little with the choice of normalizing constituent or trial number for the different choices of tidal forcing (Table 2). The only thing that varied was the

TABLE 2. Results from the resonance fit to the rectangular bay model for the Strait of Georgia and Juan de Fuca Strait.

Trial	Normalized to $K_1$			Normalized to $M_2$		
	$T_0$ (h)	$Q$	$\Sigma_j \sigma_j^2$	$T_0$ (h)	$Q$	$\Sigma_j \sigma_j^2$
1	20.1	2.1	0.013	20.5	1.9	0.047
2	20.1	2.2	0.022	21.0	2.0	0.107
3	20.1	2.2	0.013	20.6	1.9	0.054

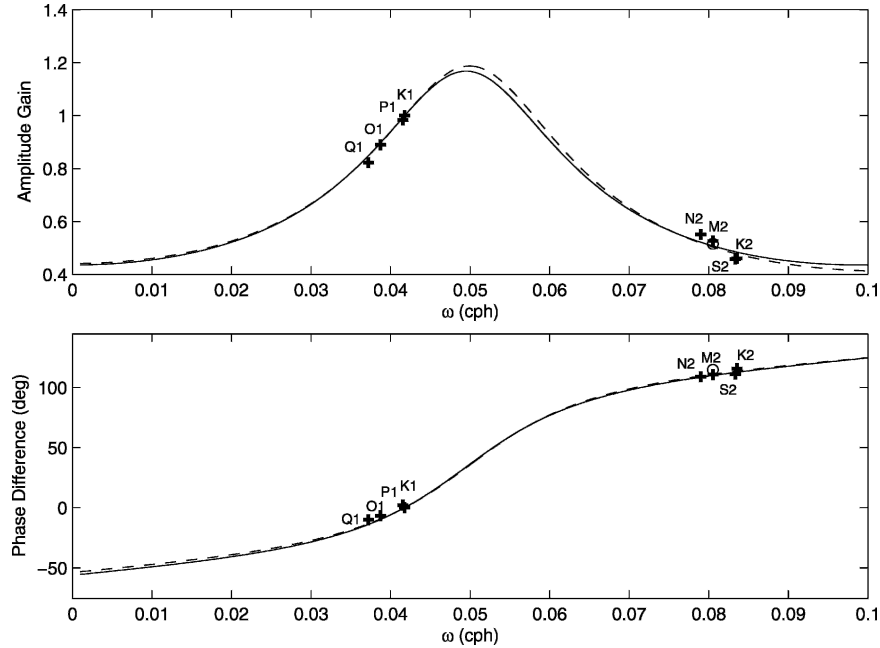


FIG. 3. Resonance fitting for the Juan de Fuca/Strait of Georgia system. The best fits for (11) and (12) are denoted by the dashed and solid lines, respectively. The circle denotes the  $M_2$  response of (11). The amplitude gain and phase difference are normalized to the  $K_1$  constituent, and the outside forcing is represented by the first trial.

accuracy of the fit with normalization to  $K_1$  being more accurate than normalization to  $M_2$  and trial 1 being more accurate than the other trials.

The low  $Q$  of about 2 causes some concern about the derivation of (7) and (12), which assumed that  $\epsilon \ll 1$ , or  $Q \gg 1$ . However, we may use the exact expression for the gain (11) together with the expression

$$kL = \frac{\omega}{\omega_0} \frac{\pi}{2} \sqrt{1 + \frac{ic\lambda}{\omega}}. \quad (17)$$

Now (11), rather than (15), is used in (16) to fit the data and obtain optimal values for  $\omega_0$  and  $\lambda$ , with  $\lambda$  reduced by 2/3 for  $M_2$ . The major difference now is that (11) is no longer a maximum at exactly  $\omega = \omega_0$ , but we retain (14) as the definition for  $Q$  since this is really just a way of expressing the inverse of the friction coefficient  $\lambda$ .

The least squares fit of (11) is shown in Fig. 3 (dashed line), with the results for the resonant period (obtained from the peak response, not  $\omega_0$ ) and  $Q$  displayed in Table 3. The responses are nearly identical to those obtained with the first-order approximation with the resonant period being a little over 20 h and a  $Q$  of about 2.

The low  $Q$  value, and hence broad resonance peak, is not unexpected given the observed phase shift of  $120^\circ$  between the diurnals and semidiurnals (shown in Figs. 2 and 3). A high  $Q$ , associated with a narrow resonance

peak, would cause the phase shift to be much closer to  $180^\circ$ . Furthermore, the phase shift, being between  $90^\circ$  and  $180^\circ$ , ensures the resonant period must be between the two tidal bands.

If the resonant period is 20 h, then  $|1 - \omega/\omega_0|$  in (1) is about 0.16 for  $K_1$ , less than the value of 0.25 for  $(1/2)Q^{-1}$ . Thus the diurnal tidal response is limited more by friction than by distance from resonance. For  $M_2$ , on the other hand,  $|1 - \omega/\omega_0|$  is about 0.6, considerably greater than 0.16 for  $(1/2)Q^{-1}$ . Thus the response at  $M_2$ , and for the other semidiurnal constituents with  $(1/2)Q^{-1} = 0.25$ , is limited more by distance from resonance than by friction. This fact is why the predicted response at  $M_2$ , shown by the circles in Fig. 3, does not noticeably depart from the trend for the other constituents, even though these have a lower  $Q$ .

While the frequency dependence of the observed el-

TABLE 3. Results from the resonance fit for the rectangular bay model using the exact expression for  $kL$ . Here  $T_0$  is the period at which (11) is a maximum.

Trial	Normalized to $K_1$			Normalized to $M_2$		
	$T_0$ (h)	$Q$	$\Sigma_j \sigma_j^2$	$T_0$ (h)	$Q$	$\Sigma_j \sigma_j^2$
1	20.0	2.0	0.012	20.6	1.8	0.040
2	20.2	2.1	0.021	20.8	2.0	0.106
3	20.0	2.0	0.012	20.6	1.9	0.049

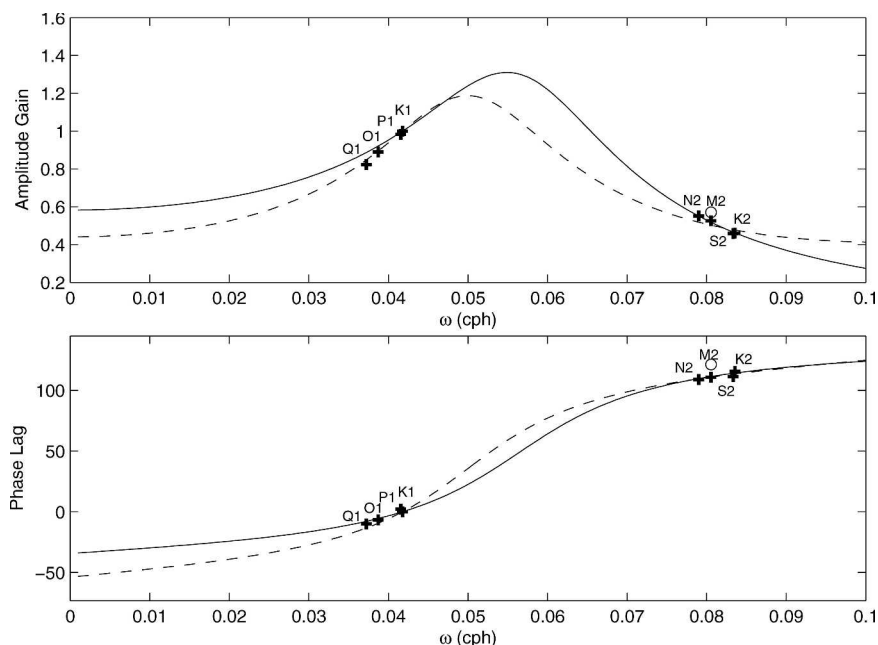


FIG. 4. Resonance fitting for the Juan de Fuca Strait of Georgia system. The best fits for (11) and (19) are denoted by the dashed and solid lines, respectively. The circle denotes the  $M_2$  response of (19). The amplitude gain and phase difference are normalized to the  $K_1$  constituent, and the outside forcing is represented by the first trial.

evation gain does seem to be well accounted for by this simple model, the Juan de Fuca Strait/Strait of Georgia system is neither rectangular nor uniform in depth. In particular, there is a major restriction of the tidal flow through the narrow passages east of Victoria (see Fig. 1) as it enters the Strait of Georgia, which can cause a Helmholtz resonance to occur. We explore this next as an alternative model.

#### b. Helmholtz model

When a bay is separated from the open ocean forcing by a narrow passage, its lowest mode will have a period longer than that for a quarter-wavelength oscillation. This causes Helmholtz resonance with a spatially uniform elevation inside the bay.

Here, the continuity equation is

$$-i\omega a' \mathcal{A} = \mathcal{E}u, \quad (18)$$

where  $a'$  is the complex height inside the bay,  $\mathcal{A}$  is the surface area of the bay, and  $\mathcal{E}$  is the cross-sectional area of the narrow strait through which a current  $u$  flows between the bay to the ocean. The governing momentum equation is the same as (2), but the connecting strait is considered short enough to assume the slope of the tidal height along the strait is linear. The complex elevation gain is

$$Ae^{i\phi} = \frac{1}{1 - \frac{\omega^2}{\omega_0^2} - ic \frac{\omega\lambda}{\omega_0^2}}, \quad (19)$$

where  $\omega_0$ , the resonant frequency for a Helmholtz oscillator, is defined as

$$\omega_0 = \sqrt{\frac{g\mathcal{E}}{\mathcal{A}l}}. \quad (20)$$

Here  $l$  is the length of the strait connecting the bay to the outside ocean. This model response, (19), also reduces to a form with the frequency dependence given by (1) as  $\omega \rightarrow \omega_0$ , assuming the same definition for  $Q$  as in (14).

In fitting the model to our data, the same least squares approach was used as for the rectangular bay model. The best fits are shown in Fig. 4, with the results from the regression shown in Table 4. The resonant period is now a bit more than 17 h, rather than the 20 h of our first model, but the  $Q$  is still close to 2.

A resonant period of 17 h would change our conclusions about the relative importance of distance from resonance and friction in limiting the response at diurnal and semidiurnal frequencies. For  $K_1$ ,  $|1 - \omega/\omega_0|$  is now about 0.29, a little larger than 0.25 for  $(1/2)Q^{-1}$ . Thus the distance from resonance and friction are of comparable importance. For  $M_2$ ,  $|1 - \omega/\omega_0|$  is now



TABLE 4. Results from the resonance fit to the Helmholtz model.

Trial	Normalized to $K_1$			Normalized to $M_2$		
	$T_0$ (h)	$Q$	$\Sigma_j \sigma_j^2$	$T_0$ (h)	$Q$	$\Sigma_j \sigma_j^2$
1	17.3	2.2	0.009	17.1	2.2	0.026
2	17.5	2.3	0.007	17.5	2.2	0.033
3	17.3	2.2	0.006	17.2	2.2	0.021

about 0.37 as compared with 0.16 for  $(1/2)Q^{-1}$ . For the other semidiurnal constituents,  $(1/2)Q^{-1}$  is 0.25. Distance from resonance still dominates the response, but less than if the resonant period were 20 h. As a consequence, the predicted response to  $M_2$  (the circles in Fig. 4) now departs noticeably from the trend of the other constituents. It also differs from the observed response at  $M_2$ , which falls in with the trend of other constituents. This discrepancy either suggests that the period is actually longer than 17 h or that the  $M_2$  currents are not as dominant as we have supposed. Previous results (Crean et al. 1988) suggest a resonant period of close to 17 h, so it appears more appropriate to assume that  $M_2$  has the same  $Q$  as the other constituents. This would scarcely change the model fits, implying a  $Q$  of about 2 for  $M_2$  as well as for the other constituents.

Another possible explanation is that the damping is not all associated with quadratic friction within the system, but has a linear radiative component associated with loss of energy back into the Pacific Ocean. This seems unlikely though, because the radiative  $Q$  for a rectangular bay with a constant depth equal to that of the exterior ocean is approximately length/width [e.g., from a simple analysis of the results in Garrett (1975)]. This ratio is at least 20 here and the radiative  $Q$  is further increased if the exterior ocean is deeper than the strait.

#### c. Model limitations

Representative values of the Juan de Fuca/Strait of Georgia system of 150 m and 350 km for  $h$  and  $L$ , respectively, would imply an expected resonance period, calculated with (10), of roughly 10 h. Even with the Helmholtz model, (20), and representative values of 9000 km<sup>2</sup> for the surface area  $\mathcal{A}$  of the Strait of Georgia, 0.6 km<sup>2</sup>, and 25 km for the cross-sectional area  $\mathcal{E}$  and length  $l$ , respectively, of the narrow channels east of Victoria, a resonant period of 11 h is calculated. Both of these estimated periods are significantly shorter than implied by our resonance fits, or indeed, by the phase jump of 120° between the responses of the semidiurnal and diurnal bands.

Observations suggest that the response is most likely a composite of the two models presented here, with the

response in Juan de Fuca Strait forcing a Helmholtz response in the Strait of Georgia. If, as an extreme, we take the connecting strait to consist of both the Juan de Fuca Strait and the small narrow channels east of Victoria, values for  $\mathcal{E}$  and  $l$  would now be 1.8 km<sup>2</sup> and 150 km, respectively. This, along with (20), would increase the expected resonant period to be roughly 16 h. This is consistent with our results (Table 4) in addition to previous calculations (Crean et al. 1988), suggesting the Helmholtz oscillator may be the better simple model.

#### d. Model friction

Estimates of the quadratic bottom friction coefficient,  $C_D$ , can be made directly from (4) using our values for  $\lambda$ . For a current  $u = u_0 \cos \omega t$ , the Fourier component of  $u|u|$  at the frequency  $\omega$  is  $[8/(3\pi)]u_0^2 \cos \omega t$ , that is,  $[8/(3\pi)]u_0 u$ . Thus the coefficient of  $u$  is  $[8/(3\pi)]u_0$ . Then (4) should really be replaced by  $\lambda = (8/3\pi)C_D u_0/h$ , giving

$$C_D = \frac{3\pi}{8} \frac{\omega_0 h}{Q u_0}. \quad (21)$$

Multiplying (4) by this factor of  $8/(3\pi)$  and using values for  $u_0$ ,  $h$ ,  $Q$ , and  $\omega_0$  of 1 m s<sup>-1</sup>, 150 m, 2, and  $2\pi/(17.3 \times 3600)$  rad s<sup>-1</sup> gives a drag coefficient of 0.009. However, these estimates are an average for the entire area and do not reflect spatial variability in frictional forces.

### 4. Comparisons with previous results

Numerical models have the potential for determining the resonant response because of their ability to include the complex shapes and depths that characterize the waters of Juan de Fuca and Georgia Strait. Where these models often come up short is in their ability to model frictional forces. Generally, all the friction in a 2D modeled system tends to be lumped into one parameter in the form of bottom friction. Previous models for this region have either required a variable bottom friction coefficient that varies by an order of magnitude over highly dissipative areas (Crean et al. 1988) or required a higher friction coefficient over a subdomain of this region (Foreman et al. 1995).

These frictional increases have been chosen to compensate for poor resolution and to allow for missing physics such as form drag (as described by MacCready et al. 2003). Crean et al. (1988), in their 2D barotropic model, employed a friction coefficient of 0.003 throughout most of the domain but increased that by a factor of 10 in Haro Strait, the highly dissipative region that connects Juan de Fuca Strait and the Strait of Georgia. By analyzing the modeled amplitude gain at Point Atkinson (49.3°N, 123.3°W) at various forcing frequencies,

Crean et al. (1988) also calculated the resonant period of the system to be around 16 h.

Foreman et al. (1995) were able to accurately reproduce the tides with their 3D barotropic finite-element model for eastern Juan de Fuca Strait and the southern Strait of Georgia by implementing a bottom friction coefficient of 0.01 throughout the whole model domain. Three-dimensional models generally have higher friction coefficients since they are applied to the bottom current 1 m above the seabed rather than the depth-averaged currents. In terms of the larger depth-averaged currents, an approximate drag coefficient for the Foreman et al. (1995) model would be approximately 0.006 rather than 0.01.

Incidentally, the same numerical model was unsuccessful in producing resonance curves analogous to those shown in Figs. 2–4. This is because the frequency-domain calculations in that model require division by a term similar to  $\omega^2 - f^2$ , which in this case approaches zero near the resonant frequency. Specifically, the inertial period for this system is approximately 16 h, the same value that Crean et al. (1988) estimated to be the resonant period with their 2D model.

## 5. Summary

The nonproximity of the diurnal and semidiurnal frequencies to resonance necessitated the generalization of the simple formula applied to the Bay of Fundy (Garrett 1972). We have fitted the simple models for a rectangular bay and a Helmholtz oscillator using regression analysis for the elevation gain and phase. These models produced a resonant period of 17 and 21 h for the Helmholtz and rectangular bay model, respectively. However, both models estimated  $Q$  to be close to 2.

The Helmholtz model appears to be the more appropriate model of the two used here. The regression errors are smaller (Table 4), and the rough calculation of the resonant period, from (20) and assuming Juan de Fuca Strait is the connecting strait, is consistent with results from our model. Furthermore, the observed phases of both the diurnal and semidiurnal tides in the Strait of Georgia are nearly constant (Dohler 1966), indicative of a Helmholtz oscillator. The one argument against this simple interpretation was that the response at the  $M_2$  frequency does not show the expected small departure from the trend connecting the other semidiurnal constituents, but we have attributed this to  $M_2$  currents not being as dominant as required for the  $Q$  for  $M_2$  to be 50% greater than that for the other constituents. While not strictly correct, it seems more appropriate to take the same  $Q$  for all the constituents,

and we have concluded that the value of this is approximately 2.

Estimating  $C_D$  from  $Q$  is also difficult given the high variability of the current speed and water depth. Although the models presented here are unable to show the spatial variability in  $C_D$ , they do support the need for higher friction coefficients required for numerical models of the area (Crean et al. 1988; Foreman et al. 1995) to more accurately reproduce the tides. Smoothly varying seabed topography in the area causes a form drag that leads to an increase in the quadratic friction coefficient by a factor of 2–5 (MacCready et al. 2003), and flow separation at the exit point from narrow channels can also appear as a head loss that is quadratic in the current. It is clear that there is still much to be understood in accurately modeling how energy is dissipated. We await the development of more sophisticated models that are capable of validating the resonant period and  $Q$  estimated here and identifying both the nature and subregions of particularly high dissipation.

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