A unified model of demographic time

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The Lexis diagram relates the age (chronological age), period, and birth cohort (APC) dimensions of demographic time, but it does not account for remaining years of life (thanatological age), and its related time indices. The thanatological counterpart to APC is an identity between thanatological age, period, and death cohort (TLD). Other identities also exist. For instance, within a birth cohort (C), chronological age (A), thanatological age (T), ultimate completed lifespan (L), period (P), and death cohort (D) are other temporal indices that together redundantly define the coordinates of a plane (the ATL plane, where P and D are determined by setting the birth cohort). Altogether, the relationship between these aspects of time and lifespan can be confusing to any demographer. These six dimensions have to my knowledge never been considered jointly. The only subset thereof that has received serious treatment is the APC framework. Many valid temporal relationships have been neglected outright.

In this paper I first state the relationships between all six dimensions of demographic time. The APC, TDL, and ATL planes are introduced as degenerate cases of the unified model. The case of the ATLC cross-sectional plane is introduced, followed by the full three-dimensional unified model, the APCT model. Finally, I demonstrate the use of this coordinate system for the case of end-of-life trajectories of some characteristics of morbidity. I explain the utility of this model by demonstrating a case where heterogeneity with respect to unaccounted-for time dimensions has caused serious misunderstandings in the scientific literature and in public policy.

Four planes

There are at least two ways to think about the model presented here: Either we situate the model in a ternary 3d space, similar to the tetrahedral simplex, or we stay in the more familar euclidean 3d space. Since the latter is more generally intuitive, we opt to describe the intersection of four planes passing through euclidean space. We begin with the Lexis diagram, and then build out from there. Two of these four planes motivate the present model, and the other two are artifacts, with potential demographic meaning.

APC

The Lexis diagram has long been used in demography to relate chronological age (A) with birth cohorts (C) and calendar years (P). Since the so-called Lexis diagram could have been named for others, and since we'll be comparing with other temporal configurations, let us refer to it as the APC diagram. When a value (data) is structured by APC coordinates, we refer to it as an APC surface.

Any APC surface can be interpreted along each of these three dimensions of temporal structure. Such interpretation is a descriptive task, and it does not succumb to problems of overidentification. Variation along these three dimensions can not be parsimonsiously separated into effects of A, P, and C. This is the so-called APC problem, and it is not the concern of the present work.

APCT

I propose a geometric identity that unifies all such temporal notions into a single (simple) spatial relationship that serves as an omnibus conceptual aid to demographers, much as the Lexis diagram does for APC relationships. The full result is a three dimensional space that can be disected by any of four different planes, each of which is parallel to the faces of a regular tetrahedron (see Figure 2 for a first mock-up of the model). Each dissecting plane relates three indices of demographic time in proportion to one another (1:1:1 ternary aspect ratio). The complete space can be described in geometry nomenclature as the tetrahedral-

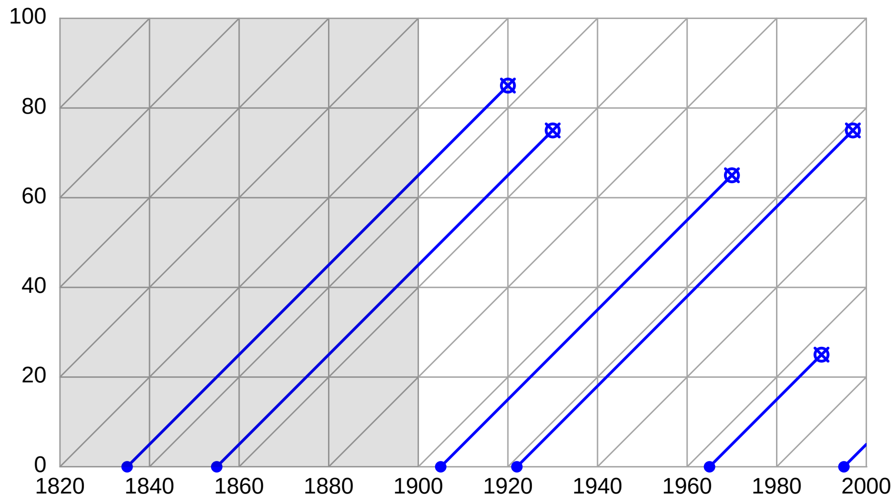


Figure 1: Lifelines in the APC diagram

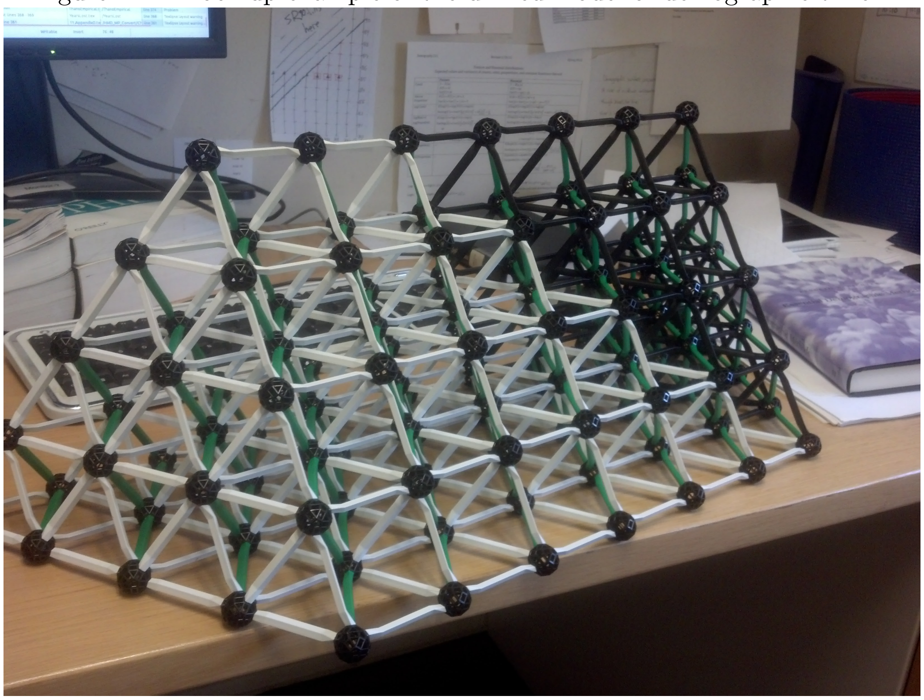
octahedral honeycomb, which is a kind of space-filling tessellation.[1] One of these planes is the familar Lexis plane (horizontal planes in Figure 2, and the other three will be new surprises for demographers. This three dimensional space is not only useful for the sake of formalizing observed temporal relationships, but also for encolosing demographic time in the past and future (e.g., before the first census and after the most recent census).

A property of the geometry that I propose is that the time units in every direction (with respect to each index) are proportional. The Lexis diagram based on right angles and 45° birth cohort lines does not have this property, whereas Lexis diagrams and surfaces based on equilateral triangles, such as some early proposals (inter alia, Lexis 1875, Lewin 1876), the masterful stereogram of Perozzo (1880), or the more recent APC diagram of Ryder (1980), do have this property. The disecting planes of the model I propose are likewise composed of equilateral triangles. In Lexis nomenclature, the 3d projections of an AP square,

[1]Constructs following this geometry exist both in nature and in man-made structures.

[2]This and other figures to be replaced with vector graphics, although I may bring this model to the presentation, since it helps explain concepts.

Figure 2: A mock-up example of the unified model of demographic time.[2]



and AC or PC parallelograms are all congruent shapes known as regular trigonal trapezohedra (RTT). The orientation of a given RTT uniquely defines the Lexis shape in question. Similar constructs exist in the other time dimensions, and these will also be described.

References

J. Lewin. Rapport sur la détermination et le recueil des données relatives aux tables de mortalité. *Programme de la neuvième session du Congrès International de statistique à Budapest I*, pages 295–361, 1876.

W.H.R.A. Lexis. *Einleitung in die Theorie der Bevölkerungsstatistik*. KJ Trübner, 1875.

L. Perozzo. Della rappresentazione graphica di una collettivita di individui nella successione del tempo. *Annali di Statistica*, 12:1–16, 1880.

Norman B Ryder. *The cohort approach: Essays in the measurement of temporal variations in demographic behavior.* PhD thesis, Princeton University, 1980.