A unified model of demographic time

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The Lexis diagram relates the age (chronological age), period, and birth cohort (APC) dimensions of demographic time, but it does not account for remaining years of life (thanatological age), and its related time indices. The thanatological counterpart to APC is an identity between thanatological age, period, and death cohort (TLD). Other identities also exist. For instance, within a birth cohort (C), chronological age (A), thanatological age (T), ultimate completed lifespan (L), period (P), and death cohort (D) are other temporal indices that together redundantly define the coordinates of a plane (the ATL plane, where P and D are determined by setting the birth cohort). Altogether, the relationship between these aspects of time and lifespan can be confusing to any demographer. These six dimensions have to my knowledge never been considered jointly. The only subset thereof that has received serious treatment is the APC framework. Many valid temporal relationships have been neglected outright.

In this paper I first state the relationships between all six dimensions of demographic time. The APC, TDL, and ATL planes are introduced as degenerate cases of the unified model. The case of the ATLC cross-sectional plane is introduced, followed by the full three-dimensional unified model, the APCT model. Finally, I demonstrate the use of this coordinate system for the case of end-of-life trajectories of some characteristics of morbidity. I explain the utility of this model by demonstrating a case where heterogeneity with respect to unaccounted-for time dimensions has caused serious misunderstandings in the scientific literature and in public policy.

Intersecting planes

The model can be introduced in terms of four planes intersecting in Euclidean 3d space. Two of the planes motivate the model, and the other two are in this case artifactual, although they may have demographic meaning. The first plane is the widely used APC plane. This plane omits remaining years of life (T).

APC

The Lexis diagram has long been used in demography to relate chronological age (A) with birth cohorts (C) and calendar years (P). Since the so-called Lexis diagram could have been named for others (Vandeschrick 2001, Keiding 2011), and since we compare with other temporal configurations, let us refer to it as the APC diagram, as seen in Figure 1 When a value (data) is structured by APC coordinates, we refer to it as an APC surface.

The APC diagram in Figure 1 represents years lived on the y axis, calendar years on the x axis, and birth cohorts as the right-ascending diagonals. This is the most common of several possible configurations of the APC dimensions. Individual lifelines are aligned in the cohort direction, starting with birth (filled circle) at chronological age zero, and death ()

Any APC surface can be interpreted along each of these three dimensions of temporal structure. Such interpretation is a descriptive task, and it does not succumb to problems of overidentification. Variation along these three dimensions can not be parsimoniously separated into the three effects of A, P, and C. This is the so-called APC problem, and it is not the concern of the present work.

It has long been noted (Zeuner 1869, Perozzo 1880) that the birth cohort dimension, as represented in Figure 1, is relatively longer than either the age or years axes. If a right angle and unity aspect ratio is forced between any two of the APC dimensions, the third dimension is always be stretched by $\sqrt{2}$. Another long-standing, but less common variant, is to represent

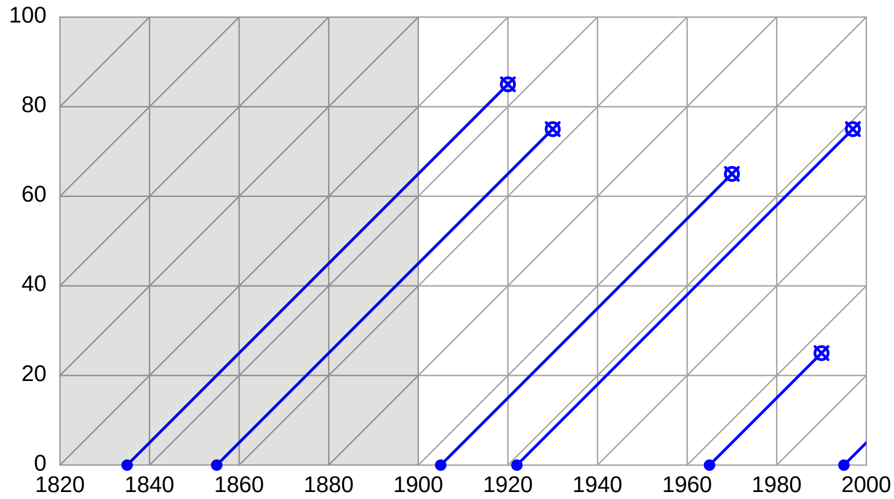


Figure 1: Lifelines in the APC diagram

TPD

Thanatological age (T), period (P) and death cohort (D) form a coordinate system best imagined as the opposite of APC. One may take the same lifelines from Figure 1 and realign them in descending fashion to create the diagram in Figure 2

ATL

The second plane is ATL, a valid coordinate system for processes that vary over the lifecourse, but not over time (P). Since the lifecourse belongs to the cohort perspective, it is best to think of the ATL plane as belonging to some particular birth cohort. Alternatively, an ATL triangle may be taken as a cross-section along through the period dimension, a sort of synthetic ATL plane.

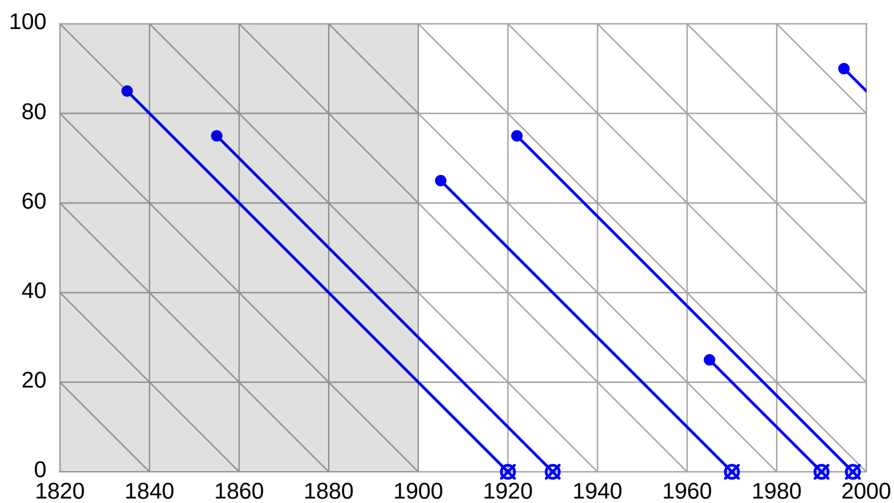
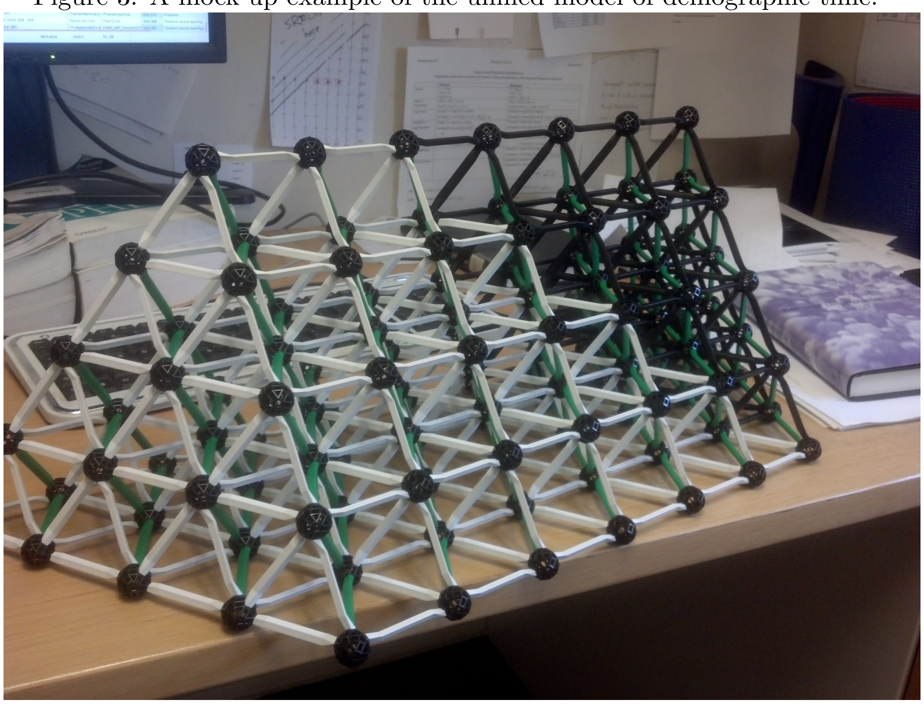


Figure 2: Lifelines in the TPD diagram

APCT

I propose a geometric identity that unifies all such temporal notions into a single (simple) spatial relationship that serves as an omnibus conceptual aid to demographers, much as the Lexis diagram does for APC relationships. The full result is a three dimensional space that can be disected by any of four different planes, each of which is parallel to the faces of a regular tetrahedron (see Figure 3 for a first mock-up of the model). Each dissecting plane relates three indices of demographic time in proportion to one another (1:1:1 ternary aspect ratio). The complete space can be described in geometry nomenclature as the tetrahedral-octahedral honeycomb, which is a kind of space-filling tessellation.[1] One of these planes is the familar Lexis plane (horizontal planes in Figure 3, and the other three will be new surprises for demographers. This three dimensional space is not only useful for the sake of formalizing observed temporal relationships, but also for encolosing demographic time in the past and future (e.g., before the first census and after the most recent census).

Figure 3: A mock-up example of the unified model of demographic time.[2]



[1]Constructs following this geometry exist both in nature and in man-made structures.

A property of the geometry that I propose is that the time units in every direction (with respect to each index) are proportional. The Lexis diagram based on right angles and 45° birth cohort lines does not have this property, whereas Lexis diagrams and surfaces based on equilateral triangles, such as some early proposals (inter alia, Lexis 1875, Lewin 1876), the masterful stereogram of Perozzo (1880), or the more recent APC diagram of Ryder (1980), do have this property. The disecting planes of the model I propose are likewise composed of equilateral triangles. In Lexis nomenclature, the 3d projections of an AP square, and AC or PC parallelograms are all congruent shapes known as regular trigonal trapezohedra (RTT). The orientation of a given RTT uniquely defines the Lexis shape in question. Similar constructs exist in the other time dimensions, and these will also be described.

References

N. Keiding. Age–period–cohort analysis in the 1870s: Diagrams, stereograms, and the basic differential equation. *Canadian Journal of Statistics*, 39(3): 405–420, 2011.

J. Lewin. Rapport sur la détermination et le recueil des données relatives aux tables de mortalité. *Programme de la neuvième session du Congrès International de statistique à Budapest I*, pages 295–361, 1876.

W.H.R.A. Lexis. *Einleitung in die Theorie der Bevölkerungsstatistik*. KJ Trübner, 1875.

L. Perozzo. Della rappresentazione graphica di una collettivita di individui nella successione del tempo. *Annali di Statistica*, 12:1–16, 1880.

Norman B Ryder. *The cohort approach: Essays in the measurement of temporal variations in demographic behavior*. PhD thesis, Princeton University, 1980.

C. Vandeschrick. The lexis diagram, a misnomer. *Demographic Research*, 4(3): 97–124, 2001.

Gustav Zeuner. *Abhandlungen aus der mathematischen statistik*. Verlag von Arthur Felix, 1869.

[2] This and other figures to be replaced with vector graphics, although I may bring this model to the presentation, since it helps explain concepts.