M427L: Calculus review

January 18, 2022

There are probably too many problems here for you to finish all of them—that is intentional, since I am *not* grading this sheet. Feel free to skip around.

If you cannot remember how to solve a problem, that is ok! I am happy to work through any of this material (and more) in my office hours this week.

0.1 Computing derivatives

Find the derivative of the function with respect to x:

1. $f(x) = x^3 - 2\sqrt{x} + 1$
2. $g(x) = 2\sin(x)\cos(4x) + \exp(x-2)$
3. $f(x) = \dfrac{x+1}{\sqrt{x^2+1}}$

0.2 Differentiability

Which of the following functions are differentiable at $x = 0$? (You may want to use the limit definition of a derivative.)

1. $f(x) = \sqrt{|x|}$
2. $f(x) = |x|^3$
3. $g(x) = \begin{cases} x^2 \sin(1/x), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Write down an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is differentiable at $x = 0$, but whose *second* derivative does *not* exist at $x = 0$.

0.3 Sketching graphs

Sketch the graph of the function $f(x) = \dfrac{x}{(x-2)^2} + 1$. Find all of the local maxima and minima, and any horizontal and vertical asymptotes.