REAL ROOTS

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Abstract. The Macualay2 package `RealRoots` contains methods for studying real roots of univariate polynomials and real solutions of multivariate systems as well as @@@@@. It updates and expands the capabilities of the package "RealRoots" given by @@@@@. For univariate polynomials, counting roots in intervals, isolating roots symbolically, and stability. For multivariate systems there are general methods of elimination for zero dimensional systems or univariate eliminant of zero-dimensional systems, this includes the rational univariate representative. For real multivariate, we have the trace form.

Introduction

1. Real Univariate Systems

Theorem 1 (Budan-Fourier)**.** *Let $f \in \mathbb{R}[t]$ be a univariate polynomial and $a < b$ two numbers in $\mathbb{R} \cup \{\pm\infty\}$. Then*

$$var(\delta f, a) - var(\delta f, b) \geq r(f, a, b),$$

and the difference is even.

Theorem 2 (Sylvester)**.** *Let $f, g \in \mathbb{R}[t]$ and $Syl(f, f'g)$ denote the Sylvester Sequence of f and $f'g$. The number of roots of f in $(a, b]$ where g is positive minus the number of roots of f in $[a, b)$ where g is negative is counted by the difference in variation*

$$var(Syl(f, f'g), a) - var(Syl(f, f'g), b).$$

Theorem 3 (Sturm)**.** *Let f be a univariate polynomial and $a, b \in \mathbb{R} \cup \{\pm\infty\}$ with $a < b$ and $f(a), f(b) \neq 0$. Then the number of zeroes of f in the interval $(a, b]$ is the difference*

$$var(F, a) - var(F, b),$$

where F is the Sturm sequence of f.

Theorem 4. *Let $f(x) = \sum_{j=0}^{n} a_j x^j$ with $n \geq 1$ and $a_n > 0$. Then f is Hurwitz stable if and only if all the Hurwitz determinants $\delta_1, \ldots, \delta_n$ are all positive.*

grants?

2. Eliminations

3. Real Multivariate Systems

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