

# Third order triangle shape functions and derivatives

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## 1 Shape functions

- **Shape function 1**

$$N_1(\zeta, \eta) = -\frac{9\eta^3}{2} - \frac{27\eta^2\zeta}{2} + 9\eta^2 - \frac{27\eta\zeta^2}{2} + 18\eta\zeta - \frac{11\eta}{2} - \frac{9\zeta^3}{2} + 9\zeta^2 - \frac{11\zeta}{2} + 1 \quad (1)$$

- **Shape function 2**

$$N_2(\zeta, \eta) = \frac{27\eta^2\zeta}{2} + 27\eta\zeta^2 - \frac{45\eta\zeta}{2} + \frac{27\zeta^3}{2} - \frac{45\zeta^2}{2} + 9\zeta \quad (2)$$

- **Shape function 3**

$$N_3(\zeta, \eta) = \frac{9\eta\zeta}{2} - \frac{9\zeta}{2} - \frac{27\eta\zeta^2}{2} + 18\zeta^2 - \frac{27\zeta^3}{2} \quad (3)$$

- **Shape function 4**

$$N_4(\zeta, \eta) = \frac{9\zeta^3}{2} - \frac{9\zeta^2}{2} + \zeta \quad (4)$$

- **Shape function 5**

$$N_5(\zeta, \eta) = \frac{27\eta\zeta^2}{2} - \frac{9\eta\zeta}{2} \quad (5)$$

- **Shape function 6**

$$N_6(\zeta, \eta) = \frac{27\eta^2\zeta}{2} - \frac{9\eta\zeta}{2} \quad (6)$$

- **Shape function 7**

$$N_7(\zeta, \eta) = \frac{9\eta^3}{2} - \frac{9\eta^2}{2} + \eta \quad (7)$$

- **Shape function 8**

$$N_8(\zeta, \eta) = \frac{9\eta\zeta}{2} - \frac{9\eta}{2} - \frac{27\eta^2\zeta}{2} + 18\eta^2 - \frac{27\eta^3}{2} \quad (8)$$

- Shape function 9

$$N_9(\zeta, \eta) = \frac{27\eta^3}{2} + 27\eta^2\zeta - \frac{45\eta^2}{2} + \frac{27\eta\zeta^2}{2} - \frac{45\eta\zeta}{2} + 9\eta \quad (9)$$

- Shape function 10

$$N_{10}(\zeta, \eta) = -27\eta^2\zeta - 27\eta\zeta^2 + 27\eta\zeta \quad (10)$$

## 2 $\zeta$ derivatives

- Shape function 1

$$\frac{\partial N_1(\zeta, \eta)}{\partial \zeta} = -\frac{27\eta^2}{2} - 27\eta\zeta + 18\eta - \frac{27\zeta^2}{2} + 18\zeta - \frac{11}{2} \quad (11)$$

- Shape function 2

$$\frac{\partial N_2(\zeta, \eta)}{\partial \zeta} = \frac{27\eta^2}{2} + 54\eta\zeta - \frac{45\eta}{2} + \frac{81\zeta^2}{2} - 45\zeta + 9 \quad (12)$$

- Shape function 3

$$\frac{\partial N_3(\zeta, \eta)}{\partial \zeta} = \frac{9\eta}{2} + 36\zeta - 27\eta\zeta - \frac{81\zeta^2}{2} - \frac{9}{2} \quad (13)$$

- Shape function 4

$$\frac{\partial N_4(\zeta, \eta)}{\partial \zeta} = \frac{27\zeta^2}{2} - 9\zeta + 1 \quad (14)$$

- Shape function 5

$$\frac{\partial N_5(\zeta, \eta)}{\partial \zeta} = 27\eta\zeta - \frac{9\eta}{2} \quad (15)$$

- Shape function 6

$$\frac{\partial N_6(\zeta, \eta)}{\partial \zeta} = \frac{27\eta^2}{2} - \frac{9\eta}{2} \quad (16)$$

- Shape function 7

$$\frac{\partial N_7(\zeta, \eta)}{\partial \zeta} = 0 \quad (17)$$

- Shape function 8

$$\frac{\partial N_8(\zeta, \eta)}{\partial \zeta} = \frac{9\eta}{2} - \frac{27\eta^2}{2} \quad (18)$$

- Shape function 9

$$\frac{\partial N_9(\zeta, \eta)}{\partial \zeta} = 27\eta\zeta - \frac{45\eta}{2} + 27\eta^2 \quad (19)$$

- Shape function 10

$$\frac{\partial N_{10}(\zeta, \eta)}{\partial \zeta} = 27\eta - 54\eta\zeta - 27\eta^2 \quad (20)$$

### 3 $\eta$ derivatives

- Shape function 1

$$\frac{\partial N_1(\zeta, \eta)}{\partial \eta} = -\frac{27\eta^2}{2} - 27\eta\zeta + 18\eta - \frac{27\zeta^2}{2} + 18\zeta - \frac{11}{2} \quad (21)$$

- Shape function 2

$$\frac{\partial N_2(\zeta, \eta)}{\partial \eta} = 27\eta\zeta - \frac{45\zeta}{2} + 27\zeta^2 \quad (22)$$

- Shape function 3

$$\frac{\partial N_3(\zeta, \eta)}{\partial \eta} = \frac{9\zeta}{2} - \frac{27\zeta^2}{2} \quad (23)$$

- Shape function 4

$$\frac{\partial N_4(\zeta, \eta)}{\partial \eta} = 0 \quad (24)$$

- Shape function 5

$$\frac{\partial N_5(\zeta, \eta)}{\partial \eta} = \frac{27 \zeta^2}{2} - \frac{9 \zeta}{2} \quad (25)$$

- Shape function 6

$$\frac{\partial N_6(\zeta, \eta)}{\partial \eta} = 27 \eta \zeta - \frac{9 \zeta}{2} \quad (26)$$

- Shape function 7

$$\frac{\partial N_7(\zeta, \eta)}{\partial \eta} = \frac{27 \eta^2}{2} - 9 \eta + 1 \quad (27)$$

- Shape function 8

$$\frac{\partial N_8(\zeta, \eta)}{\partial \eta} = 36 \eta + \frac{9 \zeta}{2} - 27 \eta \zeta - \frac{81 \eta^2}{2} - \frac{9}{2} \quad (28)$$

- Shape function 9

$$\frac{\partial N_9(\zeta, \eta)}{\partial \eta} = \frac{81 \eta^2}{2} + 54 \eta \zeta - 45 \eta + \frac{27 \zeta^2}{2} - \frac{45 \zeta}{2} + 9 \quad (29)$$

- Shape function 10

$$\frac{\partial N_{10}(\zeta, \eta)}{\partial \eta} = 27 \zeta - 54 \eta \zeta - 27 \zeta^2 \quad (30)$$