8.18 (a) For TM waves, $(\nabla_t^2+\gamma_\lambda^2)E_{z\lambda}=0$, $(\nabla_t^2+\gamma_\mu^2)E_{z\mu}=0$, for λ and μ modes, with γ_λ^2 and γ_μ^2 as the corresponding eigenvalues. From the Green's theorem,

$$\int_A (\psi_\lambda \nabla_t^2 \psi_\mu - \psi_\mu \nabla_t^2 \psi_\lambda)\, da = \oint_C \left(\psi_\lambda \frac{\partial \psi_\mu}{\partial n} - \psi_\mu \frac{\partial \psi_\lambda}{\partial n}\right) dl$$

For TM wave, the right hand side is zero, due to the boundary condition, $\psi|_C=0$. Using the eigen equation, the left hand side is $(\gamma_\lambda^2-\gamma_\mu^2)\int_A E_{z\lambda}E_{z\mu}\,da=0$. For $\lambda\neq\mu$, $\gamma_\lambda\neq\gamma_\mu$, and we must have $\int_A E_{z\lambda}E_{z\mu}\,da=0$

For TE waves, repeat the same procedure for H_z, and the corresponding boundary condition now is $\left.\frac{\partial\psi}{\partial n}\right|_C=0$.

(b) Assuming Eq. (5.131) holds, noting that $\vec{H}_\lambda=\frac{\pm 1}{Z}\hat{z}\times\vec{E}_\lambda$, the normalization condition for the transverse magnetic field is

$$\int_A \vec{H}_\lambda\cdot\vec{H}_\mu\, da = \frac{1}{Z_\lambda Z_\mu}\int_A (\hat{z}\times\vec{E}_\lambda)\cdot(\hat{z}\times\vec{E}_\mu)\,da = \frac{1}{Z_\lambda Z_\mu}\int_A \vec{E}_\lambda\cdot\vec{E}_\mu\,da = \frac{1}{Z_\lambda^2}\delta_{\lambda\mu},$$

where we have used the fact that $(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D})=(\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D})-(\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})$.

For the power flow,

$$\frac{1}{2}\int_A (\vec{E}_\lambda\times\vec{H}_\mu)\cdot\hat{z}\,da = \frac{1}{2Z_\mu}\int_A [\vec{E}_\lambda\times(\hat{z}\times\vec{E}_\mu)]\cdot\hat{z}\,da$$

$$=\frac{1}{2Z_\mu}\int_A (\hat{z}\times\vec{E}_\lambda)\cdot(\hat{z}\times\vec{E}_\mu)\,da = \frac{1}{2Z_\mu}\int_A \vec{E}_\lambda\cdot\vec{E}_\mu\,da = \frac{1}{2Z_\lambda}\delta_{\lambda\mu}$$

Finally, notice that $\nabla_t\cdot(E_{z\lambda}\nabla_t E_{z\mu})=(\nabla_t E_{z\lambda})\cdot(\nabla_t E_{z\mu})+E_{z\lambda}\nabla_t^2 E_{z\mu}$, we have

$$\int_A \vec{E}_\lambda\cdot\vec{E}_\mu\,da = -\frac{k_\lambda}{\gamma_\lambda^2}\cdot\frac{k_\mu}{\gamma_\mu^2}\int(\nabla_t E_{z\lambda})\cdot(\nabla_t E_{z\mu})\,da = -\frac{k_\lambda}{\gamma_\lambda^2}\frac{k_\mu}{\gamma_\mu^2}\left[\oint E_{z\lambda}\frac{\partial E_{z\mu}}{\partial n}dl - \int_A E_{z\lambda}\nabla_t^2 E_{z\mu}\,da\right]$$

$$=\frac{k_\lambda}{\gamma_\lambda^2}\cdot\frac{k_\mu}{\gamma_\mu^2}\int_A E_{z\lambda}\nabla_t^2 E_{z\mu}\,da = -\frac{k_\lambda k_\mu}{\gamma_\lambda^2}\int_A E_{z\lambda}E_{z\mu}\,da$$

where we have dropped the surface integral and used the eigen equations. The L.H.S. is $\delta_{\lambda\mu}$ by orthogonality condition. Therefore,

$$\int_A E_{z\lambda}E_{z\mu}\,da = -\frac{\gamma_\lambda^2}{k_\lambda k_\mu}\delta_{\lambda\mu} = -\frac{\gamma_\lambda^2}{k_\lambda^2}\delta_{\lambda\mu}.$$

We can prove the similar result for H_z. For mixed modes, the argument stays the same.