2.24 Solution: After separation of variables, the differential equation in the ϕ direction becomes

$$\frac{\partial^2\Phi}{\partial\phi^2} = -\nu^2\Phi,$$

where ν is a constant that needs to be fixed by the boundary conditions. Solutions to the above differential equations must be linear combinations of $e^{i\nu\phi}$ and $e^{-i\nu\phi}$, or equivalently, $\sin(\nu\phi)$ and $\cos(\nu\phi)$. The Dirichlet boundary condition at $\phi = 0$ for the vanishing of the solution picks the $\sin(\nu\phi)$ function. At $\phi = \beta$, we have $\sin(\nu\beta) = 0$, which means that $\nu\beta = m\pi$, or,

$$\nu = \frac{m\pi}{\beta}$$

for $m > 0$. Notice that

$$\int_0^\beta \sin(m\pi\phi/\beta)\sin(n\pi\phi/\beta)d\phi = \frac{\beta}{2}\delta_{mn},$$

then

$$\sqrt{\frac{2}{\beta}}\sin(m\pi\phi/\beta)$$

with $m > 0$ are orthonormal functions.

Given an arbitrary function $f(\phi)$ on the interval of $[0, \beta]$, it can be expanded with the above orthonormal functions,

$$f(\phi) = \sqrt{\frac{2}{\beta}}\sum_{m=1}^{\infty} A_m \sin(m\pi\phi/\beta), \tag{1}$$

where the coefficients A_m can be determined as

$$A_m = \sqrt{\frac{2}{\beta}}\int_0^\beta f(\phi)\sin(m\pi\phi/\beta)d\phi. \tag{2}$$

Put Eq. (2) back into Eq. (1), we have

$$\begin{aligned} f(\phi) &= \frac{2}{\beta}\sum_{m=1}^{\infty}\left(\int_0^\beta f(\phi')\sin(m\pi\phi'/\beta)d\phi'\right)\sin(m\pi\phi/\beta) \\ &= \int_0^\beta f(\phi')\left(\frac{2}{\beta}\sum_{m=1}^{\infty}\sin(m\pi\phi/\beta)\sin(m\pi\phi'/\beta)\right)d\phi', \end{aligned}$$

from which we can see that

$$\delta(\phi - \phi') = \frac{2}{\beta}\sum_{m=1}^{\infty}\sin(m\pi\phi/\beta)\sin(m\pi\phi'/\beta).$$