Induced Charge on Capacitor Plates

Ching Yao Fong and C. Kittel

Citation: Am. J. Phys. 35, 1091 (1967); doi: 10.1119/1.1973741
View online: http://dx.doi.org/10.1119/1.1973741
View Table of Contents: http://ajp.aapt.org/resource/1/AJPIAS/v35/i11
Published by the American Association of Physics Teachers

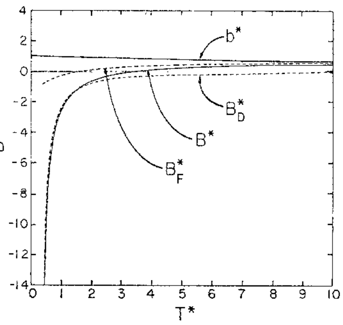

FIG. 2. The reduced second virial coefficient B^* and its analysis into B_F^* and B_D^* as a function of the reduced temperature T^*.

It is probably unwise to consider these results too quantitatively, however, since the relative contributions of bound and orbiting pairs depend very much on the exact form of the interaction potential. It is interesting to note that for a "square-well" potential there is no contribution from Thoburn's curved trajectories, and B_F^* actually increases as the temperature is lowered.

The reason we feel that it is important to emphasize the presence of bound double molecules in gases is that recently these have been directly observed by a number of experimental techniques[5-7] and are probably important in a variety of energy–transfer processes. Though the traditional van der Waals equation ignores the question by considering only a bulk interaction, we feel that any detailed model of nonideal gases should emphasize the presence of bound double molecules.

* National Research Council Senior Research Fellow at the University of Amsterdam, Laboratory for Physical Chemistry, 1966–1967.

1 W. C. Thoburn, Am. J. Phys. **34**, 136 (1966).

2 D. E. Stogryn and J. O. Hirschfelder, J. Chem. Phys. **31**, 1531 (1959).

3 J. O. Hirschfelder, C. F. Curtiss, and R. B. Bird, *Molecular Theory of Gases and Liquids* (John Wiley & Sons, Inc., New York, 1954), pp. 30–32, 162, 552–556, 1114.

4 D. E. Stogryn and J. O. Hirschfelder, J. Chem. Phys. **33**, 942 (1960).

5 A. Watanabe and H. L. Welsh, Phys. Rev. Letters **13**, 810 (1964).

6 E. J. Robbins and R. E. Lechenby, Nature **206**, 1253 (1965).

7 E. A. Ogryzlo and B. C. Sanctuary, J. Phys. Chem. **69**, 4422 (1965).

Induced Charge on Capacitor Plates

Ching Yao Fong and C. Kittel*
Department of Physics, University of California, Berkeley, California

(Received 9 June 1967)

A charge q at a position z_o within a grounded parallel plate capacitor in vacuum with plates at $z = 0$, L induces a charge $-q(L - z_o)/L$ in one plate and $-qz_o/L$ in the opposite plate. This simple and important result is very widely used in electrostatics, dielectric theory, and noise theory, yet its solution as a boundary value problem does not seem to have found its way into the standard references on electromagnetic theory. Indeed, several obvious ways of setting up the problem lead to series which appear not to converge, although by artifices they may be made to converge.

The elegant general argument due to Shockley[1] justifies the result without having to solve the potential theory problem. In this note we give a convergent solution, and also another general argument.

We need a solution of the Poisson equation which corresponds to a point charge q at $(0,0,z_o)$ and which vanishes on the planes $z = 0$ and $z = L$. The desired solution is

$$\varphi(\rho,z) = (4q/L)\sum_{n=1}^{\infty} \sin(n\pi z/L) \sin(n\pi z_o/L)\, K_0(n\pi\rho/L), \quad (1)$$

in cylindrical coordinates. Here K_0 is a modified Bessel function. (It requires some manipulation[2] to show that φ/q is the Green's function of the problem in cylindrical coordinates.)

The surface charge density on the plate $z = L$ is

$$\sigma(\rho,L) = (1/4\pi)\,(\partial\varphi/\partial z)_{z=L}$$

$$= (q/\pi L) \sum_n (n\pi/L)\,(-1)^n \sin(n\pi z_o/L)\, K_0(n\pi\rho/L). \quad (2)$$

We integrate over the area of the plate to obtain the the induced charge Q:

$$Q(L) = 2\pi\int_0^\infty \sigma(\rho,L)\rho d\rho = \left(\frac{2q}{L}\right)\sum_n (-1)^n \frac{L}{n\pi}\sin\frac{n\pi z_o}{L}\int_0^\infty xK_0(x)dx. \quad (3)$$

The integral on the right is unity, whence

$$Q(L) = \frac{2q}{\pi}\sum_{n=1}^{\infty}(-1)^n\frac{1}{n}\sin\frac{n\pi z_o}{L}. \quad (4)$$

This expression is just the Fourier series for $-qz_o/L$ in the interval $0 < z_o < L$. Thus the charge on the plate is

$$Q(L) = -qz_o/L. \quad (5)$$

The surface charge density on the plane $z = 0$ is

$$\sigma(\rho,0) = -\frac{1}{4\pi}\left(\frac{\partial\varphi}{\partial z}\right)_{z=0}$$

$$= -\frac{q}{\pi L}\sum_n \frac{n\pi}{L}\sin\left(\frac{n\pi z_o}{L}\right)K_0\frac{n\pi\rho}{L}, \quad (6)$$

and the charge is

$$Q(0) = -(2q/\pi)\sum(1/n)\sin(n\pi z_o/L), \quad (7)$$

or

$$Q(0) = -q(1 - z_o/L). \quad (8)$$

Equations (5) and (8) are the expected results. The extension to a dipole of moment $p = q\delta$ follows by adding a charge $-q$ at $z_o - \delta$. Then

$$Q(L) = -q(z_o/L) + q(z_o - \delta)/L = -p/L \quad (9)$$

and

$$Q(0) = -q(1-z_o/L) + q[1-(z_o-\delta)/L] = p/L. \quad (10)$$

Thus the dipole moment of the induced charges is

$$[Q(L)-Q(0)]L = -p. \quad (11)$$

The result (11) follows also from a general argument: The plates in effect form a closed grounded surface containing the dipole p. Because $\varphi = 0$ everywhere on this surface, $\varphi = 0$ everywhere outside the surface. This is possible only if all moments of the total charge distribution vanish. Hence the dipole moment of the charge on the plates must be equal but opposite to the dipole moment p. This argument is independent of the shape of the closed container.

* Work supported by the National Science Foundation.
[1] W. Shockley, J. Appl. Phys. **9**, 635 (1938); see also W. Smythe, *Static and dynamic Electricity* (McGraw–Hill Book Co., New York, 1950), second ed., p. 36.
[2] J. D. Jackson, *Classical Electrodynamics* (John Wiley & Sons, Inc., New York, 1962), pp. 84–86 and problem 3.13.

Instant Holograms

J. C. Albergotti
University of San Francisco, San Francisco, California

(Received 26 April 1967)

Some modern students view the use of the physics department darkroom in the same way they view the measurement of gamma-ray energy by means of the absorber technique: that it is of historical interest only. Another contribution to the demise of the photographic darkroom may be the use of Polaroid 55 P/N film packets to make simple holograms in the optics laboratory. This film has several advantages over the customary Kodak 649F emulsion plates used for this purpose. It is cheaper, relatively fast (A.S.A. index 50 as compared to 0.003 for the Kodak plates), and of course it features automatic 20-sec development. On the other hand the resolution[1] is not as high as the Kodak product.

To take advantage of the characteristics of this film the laser is used in a convenient geometry (transmission rather than reflection) to give direct illumination of the object and film. This allows short exposure times and minimizes the problem of vibration. No special precautions were taken to insure freedom from vibration other than those usually taken in ordinary photography. This fact and the ease of development make this a good experiment for the undergraduate optics laboratory.

After the laser, the next optical element is a microscope objective to spread out the beam to a convenient size. About two meters away the beam falls on a mask with two openings placed as close as possible to each other. One opening holds the object transparency which may be a piece of plate glass with block letters glued on it. The other opening contains a lens to bend the unobstructed beam to fall on the same area of film that the beam from the object transparency covers. About 50 cm behind the mask is the Polaroid #500 4-in. × 5-in. film holder which is essential to the automatic development process. With a low-power laser (Electro Optics Associates type LAS-201, a He–Ne device of about 1-mW power output) an exposure time of about ½ sec gives good results, and a hand can be used as a shutter.

After development the Polaroid film is not light sensitive, so a darkroom safelight need not be used. The positive print is separated from the negative transparency and discarded. Some of the chemicals used in development cling to the negative so they must be washed off in an 18% sodium sulfite solution. The instructions with the film indicate that this should be followed by an acid bath and fresh water rinse. Immediately after drying, the negative may be inspected for holographic images.

The simple hologram thus produced exhibits the following characteristics:

(a) If the unexpanded laser beam is allowed to pass through the hologram and fall on a nearby screen, three spots of light are seen. The central and brightest one is due to the zero-order beam. The outer ones are due to first-order diffracted beams and contain the picture information. Only those parts of the negative where the object beam overlapped the unobstructed beam produce this effect.

(b) Upon looking at the expanded laser beam through the hologram as if it were a window, one sees the reconstructed scene in the first-order diffracted beams with the unaided eye.

(c) The reconstructed wavefront exhibits three-dimensional effects when viewed as in (b) to the extent that as one's head is moved the first-order diffracted beam appears to move also and is obscured by opaque objects in the reconstructed scene.

(d) It is found that if the hologram is cut in half, each half reproduces the entire image.

(e) The image produced by the hologram is a positive even though the hologram itself would normally be considered a negative.

(f) A microscopic examination of the hologram reveals fringe patterns which diffract the light. Students may measure the average distance between fringes with a comparator.

The more original students can be encouraged to try variations such as using a prism instead of a lens, or inserting a diffusing element between source and object.[2] This makes an interesting laboratory experiment although it is somewhat qualitative.

[1] G. W. Stroke, D. Brumm, A. Funkhouser, J. Opt. Soc. Am. **55**, 1327 (1965).
[2] E. N. Leith and J. Upatnieks, J. Opt. Soc. Am. **54**, 1295 (1964).