13.2. The equation of motion for the charge is

$$m\frac{d^2\vec{x}}{dt^2} + \Gamma\frac{d\vec{x}}{dt} + m\omega_0^2\vec{x} = e\left(\vec{E}(\vec{x},t) + \frac{\vec{v}}{c}\times\vec{B}(\vec{x},t)\right)$$

where $\vec{v} = d\vec{x}/dt$. Since magnetic field does not do work, we can ignore it in the above equation, for energy transfer calculation. Introduce the Fourier transform,

$$\vec{x}(t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\vec{x}(\omega)e^{-i\omega t}d\omega, \quad \vec{E}(\vec{x},\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\vec{E}(\vec{x},t)e^{-i\omega t}dt,$$

the equation of motion for frequency ω becomes

$$\left(m(\omega_0^2-\omega^2) - i\omega\Gamma\right)\vec{x}(\omega) = e\vec{E}(\vec{x},\omega).$$

Since the charge's motion is small in amplitude, to a crude approximation, we can replace $\vec{E}(\vec{x},\omega)$ with its value at origin. $\vec{E}(\omega) \equiv \vec{E}(0,\omega)$. Then,

$$\vec{x}(\omega) = \frac{e}{m}\frac{\vec{E}(\omega)}{\omega_0^2-\omega^2-i\omega\gamma}, \quad \text{where } \gamma = \Gamma/m.$$

The energy transferred to the particle is

$$\Delta E = e\int_{-\infty}^{+\infty}\vec{v}\cdot\vec{E}\,dt = 2e\,\mathrm{Re}\int_0^{+\infty}(-i\omega)\vec{x}(\omega)\,\vec{E}^*(\omega)\,d\omega$$

$$= \frac{2e^2}{m}\mathrm{Re}\int_0^{+\infty}\frac{-i\omega}{\omega_0^2-\omega^2-i\omega\gamma}|\vec{E}(\omega)|^2d\omega.$$

For small damping, $\gamma \ll 1$, we have

$$[\omega_0^2-\omega^2-i\omega\gamma]^{-1} = (\omega_0^2-\omega^2)^{-1} + i\pi\delta(\omega_0^2-\omega^2).$$

Then, $$\Delta E = \frac{2e^2}{m}\int_0^{+\infty}\omega\pi\,\delta(\omega_0^2-\omega^2)|\vec{E}(\omega)|^2d\omega = \frac{\pi e^2}{m}|\vec{E}(\omega_0)|^2,$$

where we have used the fact that

$$\delta(\omega_0^2-\omega^2) = \frac{1}{2\omega_0}\left[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)\right]$$

and also the integration w.r.t. Dirac δ-function.