

5.17 With image current, the magnetic field for $z > 0$ is

$$\vec{B}_> = \frac{\mu_0}{4\pi} \int \left[\frac{\vec{J}(x,y,z) \times \vec{R}_1}{R_1^3} + \frac{\vec{J}'(x,y,-z) \times \vec{R}_2}{R_2^3} \right] d^3x$$

and for $z < 0$,

$$\vec{B}_< = \frac{\mu}{4\pi} \int \frac{\vec{J}''(x,y,z) \times \vec{R}_1}{R_1^3} d^3x,$$

where $\vec{R}_1 = (u-x, v-y, w-z)$ and $\vec{R}_2 = (u-x, v-y, w+z)$. At the boundary,

$w = 0$, we must have

$$B_{>,z}(x,y,0) = B_{<,z}(x,y,0), \quad H_{>,xy}(x,y,0) = H_{<,xy}(x,y,0)$$

Since $\vec{J}(x,y,z) \times \vec{R}_1 \Big|_{w=0} = \hat{i}(-j_y z - j_z(v-y)) + \hat{j}(j_z(u-x) + j_x z) + \hat{k}(j_x(v-y) - j_y(u-x))$

$$\vec{J}'(x,y,-z) \times \vec{R}_2 \Big|_{w=0} = \hat{i}(j_y' z - j_z'(v-y)) + \hat{j}(j_z'(u-x) - j_x' z) + \hat{k}(j_x'(v-y) - j_y'(u-x))$$

$$\vec{J}''(x,y,z) \times \vec{R}_1 \Big|_{w=0} = \hat{i}(-j_y'' z - j_z''(v-y)) + \hat{j}(j_z''(u-x) + j_x'' z) + \hat{k}(j_x''(v-y) - j_y''(u-x))$$

We have the conditions, which leads to

$$\left\{ \begin{array}{l} j_x - j_x' = j_x'' \\ j_y - j_y' = j_y'' \\ j_z + j_z' = j_z'' \\ j_x + j_x' = \frac{\mu}{\mu_0} j_x'' \\ j_y + j_y' = \frac{\mu}{\mu_0} j_y'' \end{array} \right. \quad \begin{array}{l} j_x' = \frac{\mu_r - 1}{\mu_r + 1} j_x \\ j_y' = \frac{\mu_r - 1}{\mu_r + 1} j_y \\ j_x'' = \frac{-2}{\mu_r + 1} j_x \\ j_y'' = \frac{-2}{\mu_r + 1} j_y \end{array}$$

with $\mu_r = \mu/\mu_0$.

To determine the z components of the image currents, we can invoke the continuity condition. $\nabla \cdot \vec{J} = 0$.

Notice that for \vec{J}' , $z < 0$. We then have

$$\hat{j}_z' = -\frac{\mu_r - 1}{\mu_r + 1} \hat{j}_z, \quad \hat{j}_z'' = \frac{2}{\mu_r + 1} \hat{j}_z$$

