14.9 (a) Using Eq. (14.26), the power radiated is $P(t)=\frac{2}{3}\frac{q^2}{c}\left[|\dot{\vec\beta}|^2-|\vec\beta\times\dot{\vec\beta}|^2\right]\gamma^6$. Since the velocity is perpendicular to the acceleration, the power can be written as

$$P(t)=\frac{2}{3}\frac{q^2}{c}\gamma^6(1-\beta^2)|\dot{\vec\beta}|^2=\frac{2}{3}\frac{q^2}{c}\gamma^4|\dot{\vec\beta}|^2=\frac{2}{3}\frac{q^2}{c^3}\gamma^4|\dot{\vec v}|^2.$$

Using Eq. (12.1), the equation of motion for the particle in the magnetic field is given by $\frac{d\vec p}{dt}=e\vec\beta\times\vec B$. The magnetic field does no work, which means γ ($\vec p=\gamma m\vec v$) is constant in the magnetic field (ignoring the radiation effect). Then, $\dot{\vec v}=\frac{q\beta B}{\gamma m}$, where we have used the fact that magnetic field is perpendicular to velocity. Now, we can write the radiation power as

$$P(t)=\frac{2}{3}\frac{q^2}{c^3}\gamma^4\cdot\frac{q^2\beta^2B^2}{\gamma^2m^2}=\frac{2q^4B^2}{3m^2c^3}\beta^2\gamma^2=\frac{2q^4B^2}{3m^2c^3}(\gamma^2-1)$$

(b) The particle loses energy due to radiation,

$$\frac{dE}{dt}=-P(t)\quad\Rightarrow\quad\frac{d\gamma}{dt}=\frac{2q^4B^2}{3m^3c^5}(1-\gamma^2).$$

which leads to $\frac{1}{2}\log\left|\frac{1+\gamma}{1-\gamma}\right|\Big|_{\gamma_0}^{\gamma}=\frac{2q^4B^2}{3m^3c^5}t$

Notice that $\gamma,\gamma_0\gg1$, the L.H.S. of the above solution can be expanded to $O(1/\gamma)$ as $\frac{1}{\gamma}-\frac{1}{\gamma_0}$, which finally gives $t=\frac{3m^3c^5}{2q^4B^2}\left(\frac{1}{\gamma}-\frac{1}{\gamma_0}\right)$

(c) For non-relativistic particle, the energy loss due to radiation is small, and we can treat the radiant power as a constant. The acceleration in this case is

$\dot v=\frac{qvB}{mc}$, where $v=\sqrt{2T_0/m}$

Using Eq. (14.22), $P=\frac{2q^2}{3c^3}|\dot{\vec v}|^2=\frac{2q^2}{3c^2}\frac{q^2B^2}{m^2c^2}\cdot\frac{2T}{m}=\frac{4q^4B^2}{3m^2c^4}T_0.$

After time t, the energy of the particle becomes

$$T=T_0-Pt=T_0\left(1-\frac{4q^4B^2}{3m^2c^4}t\right)$$