5.36 (a) From $\vec{E} = -\partial\vec{A}/\partial t$, we have

$$E_\phi = \frac{3B_0 a}{\pi} \sin\theta \int_0^{+\infty} \nu k^2 e^{-\nu t k^2} j_1(k) j_1(kr/a)\, dk.$$

In the limit of $t \to 0^+$, the electric field becomes

$$E_\phi = \frac{3B_0 a\nu}{\pi} \sin\theta \int_0^{+\infty} k^2 j_1(k) j_1(kr/a)\, dk = \frac{3B_0 a\nu}{\pi} \sin\theta \, \frac{\pi}{2} \delta\left(1 - \frac{r}{a}\right) = \frac{3B_0}{2} \sin\theta \frac{a^2}{\mu\sigma a^2} \delta(r-a)$$

$$= \frac{3B_0}{2\mu\sigma} \sin\theta\, \delta(r-a)$$

and the current density is $\vec{J} = \sigma\vec{E} = \frac{3B_0}{2\mu} \sin\theta\, \delta(r-a)\hat{\phi}$. This agrees with prob. 5.35 (a).

For $\nu t \gg 1$, applying the same approximation as in Prob. 5.35 (c), we have

$$E_\phi = \frac{3B_0 a}{\pi} \sin\theta \frac{\nu}{(\nu t)^{3/2}} \int_0^{+\infty} e^{-u^2} u^2 \cdot j_1\left(\frac{u}{\sqrt{\nu t}}\right) j_1\left(\frac{u}{\sqrt{\nu t}} \frac{r}{a}\right) du$$

$$= \frac{3B_0 a}{\pi} \sin\theta \frac{\nu}{(\nu t)^{5/2}} \int_0^{+\infty} e^{-u^2} \cdot u^2 \cdot \frac{u}{3} \cdot \frac{u}{3} \frac{r}{a}\, du$$

$$= \frac{3B_0 r}{\pi} \sin\theta \frac{\nu}{(\nu t)^{5/2}} \cdot \frac{3\sqrt{\pi}}{72} = \frac{B_0 \nu}{8\sqrt{\pi}(\nu t)^{5/2}} r \sin\theta.$$

(b) The power dissipated is given by

$$P = \int \vec{J}\cdot\vec{E}\, d^3x = \frac{9\sigma B_0^2 a^2}{\pi^2} \int d^3x \sin^2\theta \int_0^{+\infty} \nu k_1^2 e^{-\nu t k_1^2} j_1(k_1) j_1(k_1 r/a)\, dk_1$$
$$\int_0^{+\infty} \nu k_2^2 e^{-\nu t k_2^2} j_1(k_2) j_1(k_2 r/a)\, dk_2$$

$$= \frac{9\sigma B_0^2 a^2}{\pi^2} \nu^2 \int_0^{+\infty} dk_1 \int_0^{+\infty} dk_2\, k_1^2 e^{-\nu t k_1^2} k_2^2 e^{-\nu t k_2^2} j_1(k_1) j_1(k_2)$$
$$\int d\Omega \sin^2\theta \int_0^{+\infty} r^2 j_1(k_1 r/a) j_1(k_2 r/a)\, dr$$

$$= \frac{9\sigma B_0^2 a^2}{\pi^2} \nu^2 \cdot \frac{8\pi}{3} \int_0^{+\infty} dk_1 \int_0^{+\infty} dk_2\, k_1^2 e^{-\nu t k_1^2} j_1(k_1) k_2^2 e^{-\nu t k_2^2} j_1(k_2) \frac{\pi}{2(k_1/a)^2} \delta\left(\frac{k_1}{a} - \frac{k_2}{a}\right)$$

$$= \frac{9\sigma B_0^2 a^2}{\pi^2} \nu^2 \frac{8\pi}{3} \cdot \frac{\pi}{2} a^3 \cdot \int_0^{+\infty} e^{-2\nu t k^2} [k j_1(k)]^2 dk$$

$$= \frac{12 B_0^2 a^3 \nu}{\mu} \int_0^{+\infty} e^{-2\nu t k^2} [k j_1(k)]^2 dk.$$

Obviously, $P = -dW_m/dt$.

(c) Let $k = u/\sqrt{2\nu t}$, then

$$P = \frac{12 B_0^2 a^3 \nu}{\mu} \frac{1}{(2\nu t)^{3/2}} \int_0^{+\infty} e^{-u^2} \left[u j_1\left(\frac{u}{\sqrt{2\nu t}}\right)\right]^2 du$$

For $\nu t \gg 1$, we have

$$P = \frac{12 B_0^2 a^3 \nu}{\mu} \frac{1}{(2\nu t)^{5/2}} \int_0^{+\infty} e^{-u^2} \frac{u^4}{9} du$$

which decays asymptotically as $(\nu t)^{-5/2}$ and is the same order as the induced current.