2.20 Solution: (a) Using Eq. (1.42), the potential can be written as

$$\begin{aligned}
\Phi(\rho,\phi) &= \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} d\phi' \int_0^{\infty} \rho' d\rho' \cdot \sigma(\rho',\phi') G(\rho,\phi;\rho',\phi') \\
&= \frac{1}{4\pi\varepsilon_0}\int_0^{2\pi} d\phi' \int_0^{\infty} \rho' d\rho' \cdot \frac{\lambda}{a}\sum_{n=0}^{3}(-1)^n \delta(\rho'-a)\delta\left(\phi'-\frac{n\pi}{2}\right) \\
&\qquad \times\left(-\log\rho_>^2 + 2\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{\rho_<}{\rho_>}\right)^m \cos\left[m(\phi-\phi')\right]\right) \\
&= \frac{\lambda}{4\pi\varepsilon_0}\int_0^{2\pi} d\phi' \sum_{n=0}^{3}(-1)^n \delta\left(\phi'-\frac{n\pi}{2}\right) \\
&\qquad \times\left(-\log\rho_>^2 + 2\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{\rho_<}{\rho_>}\right)^m \cos\left[m(\phi-\phi')\right]\right) \\
&= \frac{\lambda}{2\pi\varepsilon_0}\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{\rho_<}{\rho_>}\right)^m \left(\cos(m\phi) - \cos\left(m\phi-\frac{m\pi}{2}\right)\right. \\
&\qquad\qquad \left. + \cos(m\phi - m\pi) - \cos\left(m\phi - \frac{3m\pi}{2}\right)\right) \\
&= \frac{\lambda}{2\pi\varepsilon_0}\sum_{m=1}^{\infty}\frac{1}{m}\left(\frac{\rho_<}{\rho_>}\right)^m \left[(1+(-1)^m)\cos(m\phi) - (1+(-1)^m)\cos\left(m\phi-\frac{m\pi}{2}\right)\right].
\end{aligned}$$

It can be easily shown that only $m = 4k+2$ terms with $k > 0$ contribute to the sum. The final result reads

$$\begin{aligned}
\Phi(\rho,\phi) &= \frac{\lambda}{2\pi\varepsilon_0}\sum_{k=0}^{\infty}\frac{1}{4k+2}\left(\frac{\rho_<}{\rho_>}\right)^m \cdot 4\cos[(4k+2)\phi] \\
&= \frac{\lambda}{\pi\varepsilon_0}\sum_{k=0}^{\infty}\frac{1}{2k+1}\left(\frac{\rho_<}{\rho_>}\right)^m \cos[(4k+2)\phi],
\end{aligned}$$

where $\rho_> = a$ ($\rho_< = a$) for potential inside (outside) the circle.

(b)