9.7 (a) From Problem 9.6(b), we can see that the radiating electric and magnetic field components that decay as r^{-1} are given by

$$\vec{E}_{rad} = \frac{1}{4\pi\varepsilon_0 c^2 r}\,\vec{n}\times\left(\vec{n}\times\frac{\partial^2\vec{p}_{ret}}{\partial t^2}\right), \quad \vec{B}_{rad} = -\frac{\mu_0}{4\pi c^2 r}\,\vec{n}\times\frac{\partial^2\vec{p}_{ret}}{\partial t^2},$$

Then, $$\frac{dP}{d\Omega} = r^2\vec{n}\cdot\vec{E}\times\vec{H} = -\frac{1}{16\pi^2\varepsilon_0 c^4}\,\vec{n}\cdot\left(\vec{n}\times\left(\vec{n}\times\frac{\partial^2\vec{p}_{ret}}{\partial t^2}\right)\right)\times\left(\vec{n}\times\frac{\partial^2\vec{p}_{ret}}{\partial t^2}\right)$$

$$= -\frac{Z_0}{16\pi^2 c^2}\left(\vec{n}\times\left(\vec{n}\times\frac{\partial^2\vec{p}_{ret}}{\partial t^2}\right)\right)\cdot\left(\left(\vec{n}\times\frac{\partial^2\vec{p}_{ret}}{\partial t^2}\right)\times\vec{n}\right)$$

$$= \frac{Z_0}{16\pi^2 c^2}\left|\left[\vec{n}\times\frac{\partial^2\vec{p}(t')}{\partial t'^2}\right]\times\vec{n}\right|^2,$$

where we have replaced $\partial^2\vec{p}_{ret}/\partial t^2$ with retarded time $t' = t - r/c$. Also, since we can working with real dipole moment, we do not need to perform time average with extra $1/2$ factor.

Similarly, we can handle the magnetic dipole as in Problem 9.6. Expand current to the next order,

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi}\int\left(\vec{J}_{ret}(\vec{x}') + \frac{\partial\vec{J}_{ret}(\vec{x}')}{\partial t}\frac{\vec{n}\cdot\vec{x}'}{c} + \cdots\right)\left(\frac{1}{r}+\cdots\right)d^3x'$$

$$= (\text{electric dipole contribution}) + \frac{\mu_0}{4\pi c r}\frac{\partial}{\partial t}\int(\vec{n}\cdot\vec{x}')\,\vec{J}_{ret}(\vec{x}')\,d^3x'$$

$$= (\cdots) + \frac{\mu_0}{4\pi c r}\frac{\partial}{\partial t}\int\left\{\frac{1}{2}\left[(\vec{n}\cdot\vec{x}')\vec{J}_{ret}(\vec{x}') + (\vec{n}\cdot\vec{J}_{ret}(\vec{x}'))\,\vec{x}'\right] + \frac{1}{2}\left(\vec{x}'\times\vec{J}_{ret}(\vec{x}')\right)\times\vec{n}\right\}d^3x'$$

Define $\vec{m} = \frac{1}{2}\int(\vec{x}'\times\vec{J})\,d^3x'$, then the contribution from the last antisymmetric term is

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi c r}\left(\frac{\partial\vec{m}_{ret}}{\partial t}\times\vec{n}\right).$$ Comparing to result in Problem 9.6(a), we can see that the result for magnetic dipole can be obtained by replacing $\vec{p}_{ret}$ with $\frac{1}{c}\vec{m}_{ret}\times\vec{n}$, and

$$\frac{dP}{d\Omega} = \frac{Z_0}{16\pi^2 c^2}\left|\left[\vec{n}\times\frac{1}{c}\left(\frac{d^2\vec{m}(t')}{dt'^2}\times\vec{n}\right)\right]\times\vec{n}\right|^2$$

$$= \frac{Z_0}{16\pi^2 c^2}\left|\left[\frac{1}{c}\frac{d^2\vec{m}(t')}{dt'^2} - \vec{n}\left(\frac{1}{c}\frac{d^2\vec{m}(t')}{dt'^2}\cdot\vec{n}\right)\right]\times\vec{n}\right|^2$$

$$= \frac{Z_0}{16\pi^2 c^2}\left|\frac{1}{c}\frac{d^2\vec{m}(t')}{dt'^2}\times\vec{n}\right|^2.$$

(b) Taking the symmetric part in (a), we have

$$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi c r}\frac{\partial}{\partial t}\int \frac{1}{2}\left[(\vec{n}\cdot\vec{x}')\vec{J}_{ret}(\vec{x}') + (\vec{n}\cdot\vec{J}_{ret}(\vec{x}'))\vec{x}'\right]d^3x'$$

$$= \frac{\mu_0}{8\pi c r}\frac{\partial^2}{\partial t^2}\int \vec{x}'(\vec{n}\cdot\vec{x}')\rho_{ret}(\vec{x}')d^3x' = \frac{\mu_0}{24\pi c r}\frac{\partial^2 \vec{Q}_{ret}(\vec{n})}{\partial t^2}$$

where we have used (9.37). This result can be obtained by replacing $\vec{P}_{ret}$ with $\frac{1}{6c}\frac{\partial \vec{Q}_{ret}(\vec{n})}{\partial t}$.

Then, for quadrupole radiation,

$$\frac{dP}{d\Omega} = \frac{Z_0}{16\pi^2 c^2}\cdot\frac{1}{36c^2}\left|\left[\vec{n}\times\frac{d^3\vec{Q}_{ret}(\vec{n})}{dt^3}\right]\times\vec{n}\right|^2 = \frac{Z_0}{576\pi^2 c^4}\left|\left[\vec{n}\times\frac{d^3\vec{Q}(\vec{n},t')}{dt'^3}\right]\times\vec{n}\right|^2$$