10.1 (a) The crucial identity for this problem is that, for a unit vector $\vec{n}$ and the two polarization vectors associated with it, $\vec{\epsilon}_1$ and $\vec{\epsilon}_2$, we have

$$\sum_{\lambda=1,2} \epsilon^*_{\lambda,i}\,\epsilon_{\lambda,j} = \delta_{ij} - n_i n_j .$$

With this knowledge, and also using Einstein summation convention, the differential scattering cross section before averaging over outgoing polarization, is

$$\frac{d\sigma}{d\Omega}(\vec{n}_0, \vec{\epsilon}_0; \vec{n}, \vec{\epsilon}) = k^4 a^6 \left| \vec{\epsilon}^*\cdot\vec{\epsilon}_0 - \frac{1}{2}(\vec{n}\times\vec{\epsilon}^*)\cdot(\vec{n}_0\times\vec{\epsilon}_0)\right|^2 .$$

$$= k^4 a^6 \left| \vec{\epsilon}^*\cdot\vec{\epsilon}_0 - \frac{1}{2}(\vec{n}\cdot\vec{n}_0)(\vec{\epsilon}^*\cdot\vec{\epsilon}_0) + \frac{1}{2}(\vec{n}\cdot\vec{\epsilon}_0)(\vec{\epsilon}^*\cdot\vec{n}_0)\right|^2$$

$$= k^4 a^6 \left| \left[1 - \frac{1}{2}(\vec{n}\cdot\vec{n}_0)\right](\vec{\epsilon}^*\cdot\vec{\epsilon}_0) + \frac{1}{2}(\vec{n}\cdot\vec{\epsilon}_0)(\vec{\epsilon}^*\cdot\vec{n}_0)\right|^2$$

$$= k^4 a^6 \left\{ \left[1 - \frac{1}{2}(\vec{n}\cdot\vec{n}_0)\right]^2 |\vec{\epsilon}^*\cdot\vec{\epsilon}_0|^2 + \left[1 - \frac{1}{2}(\vec{n}\cdot\vec{n}_0)\right](\vec{n}\cdot\vec{\epsilon}_0)(\vec{\epsilon}^*\cdot\vec{\epsilon}_0)(\vec{\epsilon}\cdot\vec{n}_0) \right.$$

$$\left. + \frac{1}{4}|\vec{n}\cdot\vec{\epsilon}_0|^2 |\vec{\epsilon}^*\cdot\vec{n}_0|^2 \right\}.$$

Using the above identity, we can find that

$$\sum_\lambda (\vec{\epsilon}_\lambda\cdot\vec{a})(\vec{\epsilon}^*_\lambda\cdot\vec{b}) = \sum_\lambda \epsilon_{\lambda i}\epsilon^*_{\lambda j} a_i b_j = (\delta_{ij} - n_i n_j) a_i b_j = \vec{a}\cdot\vec{b} - (\vec{n}\cdot\vec{a})(\vec{n}\cdot\vec{b}).$$

Applying to the differential scattering cross section, we can get

$$\frac{d\sigma}{d\Omega}(\vec{\epsilon}_0, \vec{n}_0, \vec{n}) = \sum_\lambda \frac{d\sigma}{d\Omega}(\vec{n}_0, \vec{\epsilon}_0; \vec{n}, \vec{\epsilon}_\lambda)$$

$$= k^4 a^6 \left\{ \left[1 - \frac{1}{2}(\vec{n}\cdot\vec{n}_0)\right]^2 \left(1 - |\vec{n}\cdot\vec{\epsilon}_0|^2\right) + \left[1 - \frac{1}{2}(\vec{n}\cdot\vec{n}_0)\right](\vec{n}\cdot\vec{\epsilon}_0)\left[(\vec{n}_0\cdot\vec{\epsilon}_0) - (\vec{n}\cdot\vec{n}_0)(\vec{n}\cdot\vec{\epsilon}_0)\right] \right.$$

$$\left. + \frac{1}{4}|\vec{n}\cdot\vec{\epsilon}_0|^2\left(1 - |\vec{n}\cdot\vec{n}_0|^2\right) \right\}$$

Notice that $\vec{n}_0\cdot\vec{\epsilon}_0 = 0$, the above equation can be slightly simplified as

$$\frac{d\sigma}{d\Omega}(\vec{\epsilon}_0, \vec{n}_0, \vec{n}) = k^4 a^6 \left\{ \left(1 - (\vec{n}\cdot\vec{n}_0) + \frac{1}{4}|\vec{n}\cdot\vec{n}_0|^2\right)\left(1 - |\vec{n}\cdot\vec{\epsilon}_0|^2\right) - \left(1 - \frac{1}{2}(\vec{n}\cdot\vec{n}_0)\right)(\vec{n}\cdot\vec{n}_0)|\vec{n}\cdot\vec{\epsilon}_0|^2 \right.$$

$$\left. + \frac{1}{4}|\vec{n}\cdot\vec{\epsilon}_0|^2\left(1 - |\vec{n}\cdot\vec{n}_0|^2\right) \right\}$$

$$= k^4a^6 \Big\{ 1 - |\vec{n}\cdot\vec{\varepsilon}_0|^2 - |\vec{n}\cdot\vec{n}_0| + (\vec{n}\cdot\vec{n}_0)(\vec{n}\cdot\vec{\varepsilon}_0)^2 + \frac{1}{4}|\vec{n}\cdot\vec{n}_0|^2 - \frac{1}{4}|\vec{n}\cdot\vec{n}_0|^2|\vec{n}\cdot\vec{\varepsilon}_0|^2$$

$$- |\vec{n}\cdot\vec{n}_0||\vec{n}\cdot\vec{\varepsilon}_0|^2 + \frac{1}{2}|\vec{n}\cdot\vec{n}_0|^2|\vec{n}\cdot\vec{\varepsilon}_0|^2$$

$$+ \frac{1}{4}|\vec{n}\cdot\vec{\varepsilon}_0|^2 - \frac{1}{4}|\vec{n}\cdot\vec{\varepsilon}_0|^2|\vec{n}\cdot\vec{n}_0|^2 \Big\}$$

$$= k^4a^6 \Big\{ 1 - \vec{n}\cdot\vec{n}_0 - \frac{3}{4}|\vec{n}\cdot\vec{\varepsilon}_0|^2 + \frac{1}{4}|\vec{n}\cdot\vec{n}_0|^2 \Big\}$$

This can be further manipulated to the desired form by noticing that

$$|\vec{n}\cdot\vec{\varepsilon}_0|^2 + |\vec{n}\cdot\vec{n}_0|^2 + |\vec{n}\cdot(\vec{n}_0\times\vec{\varepsilon}_0)|^2 = 1,$$

as $\vec{n}_0, \vec{\varepsilon}_0$, and $\vec{n}_0\times\vec{\varepsilon}_0$ form a complete set of base. Then,

$$\frac{d\sigma}{d\Omega}(\vec{\varepsilon}_0, \vec{n}_0, \vec{n}) = k^4a^6 \Big\{ 1 - \vec{n}\cdot\vec{n}_0 - |\vec{n}\cdot\vec{\varepsilon}_0|^2 + \frac{1}{4}\left(|\vec{n}\cdot\vec{\varepsilon}_0|^2 + |\vec{n}\cdot\vec{n}_0|^2\right) \Big\}$$

$$= k^4a^6 \Big\{ 1 - \vec{n}\cdot\vec{n}_0 - |\vec{n}\cdot\vec{\varepsilon}_0|^2 + \frac{1}{4}\left(1 - |\vec{n}\cdot(\vec{n}_0\times\vec{\varepsilon}_0)|^2\right) \Big\}$$

$$= k^4a^6 \Big[\frac{5}{4} - |\vec{\varepsilon}_0\cdot\vec{n}|^2 - \frac{1}{4}|\vec{n}\cdot(\vec{n}_0\times\vec{\varepsilon}_0)|^2 - \vec{n}\cdot\vec{n}_0 \Big]$$

(b) For linearly polarized incident radiation, we can choose $\vec{n}_0 = (0, 0, 1)$, $\vec{n} = (\sin\theta, 0, \cos\theta)$, and $\vec{\varepsilon}_0 = (\cos\phi, \sin\phi, 0)$. Then,

$$\vec{n}\cdot\vec{n}_0 = \cos\theta, \quad \vec{n}\cdot\vec{\varepsilon}_0 = \sin\theta\cos\phi, \quad \vec{n}\cdot\vec{n}_0 = \cos\theta,$$

and

$$\frac{d\sigma}{d\Omega}(\vec{\varepsilon}_0, \vec{n}_0, \vec{n}) = k^4a^6 \Big\{ 1 - \cos\theta - \frac{3}{4}\sin^2\theta\cos^2\phi + \frac{1}{4}\cos^2\theta \Big\}$$

$$= k^4a^6 \Big\{ 1 - \cos\theta - \frac{3}{4}\sin^2\theta\cdot\frac{1}{2}(\cos2\phi + 1) + \frac{1}{4}\cos^2\theta \Big\}$$

$$= k^4a^6 \Big\{ 1 - \cos\theta - \frac{3}{8}\sin^2\theta\cos2\phi - \frac{3}{8}(1 - \cos^2\theta) + \frac{1}{4}\cos^2\theta \Big\}$$

$$= k^4a^6 \Big\{ \frac{5}{8}(1 + \cos^2\theta) - \cos\theta - \frac{3}{8}\sin^2\theta\cos2\phi \Big\}$$