15.4 (a) For multiple particles, the radiation electric field is

$$\vec{E}_{rad} = \frac{1}{c}\sum_j e_j \left[\frac{\vec{n}\times\{(\vec{n}-\vec{\beta}_j)\times\dot{\vec{\beta}}_j\}}{(1-\vec{\beta}_j\cdot\vec{n})^3 R} \right]_{ret}$$

Then, following the same argument leading to Eq. (14.63), we can see that the equivalent expression in the multi-particle case is

$$\frac{d^2I}{d\omega d\Omega} = \frac{1}{4\pi^2 c}\left| \vec{\varepsilon}^* \cdot \int_{-\infty}^{+\infty} \sum_j e_j \frac{\vec{n}\times[(\vec{n}-\vec{\beta}_j)\times\dot{\vec{\beta}}_j]}{(1-\vec{\beta}_j\cdot\vec{n})^2} \exp\{i\omega(t-\vec{n}\cdot\vec{r}_j(t)/c)\}\, dt \right|^2$$

$$= \frac{1}{4\pi^2 c}\left| \vec{\varepsilon}^* \cdot \sum_j e_j \int_{-\infty}^{+\infty} \frac{d}{dt}\left[\frac{\vec{n}\times(\vec{n}\times\vec{\beta})}{1-\vec{\beta}\cdot\vec{n}} \right] \exp\{i\omega(t-\vec{n}\cdot\vec{r}_j(t)/c)\}\, dt \right|^2$$

For $\omega\tau \ll 1$, we can treat $e^{i\omega t} \sim 1$, and $\vec{r}_j(t)$ can be approximated with $\vec{r}_j(0)$. Then, we will get

$$\frac{d^2I}{d\omega d\Omega} = \frac{1}{4\pi^2 c}\left| \vec{\varepsilon}^* \cdot \sum_j e_j \left(\frac{\vec{\beta}_j'}{1-\vec{\beta}_j'\cdot\vec{n}} - \frac{\vec{\beta}_j}{1-\vec{\beta}_j\cdot\vec{n}} \right) e^{-i\omega\vec{n}\cdot\vec{r}_j(0)/c} \right|^2$$

(b) Before decay, both particles of the pair have 0 velocity. After decay, the particles have opposite velocity and charge. Also, their initial position is at the origin. Then,

$$\frac{d^2I}{d\omega d\Omega} = \frac{1}{4\pi^2 c}\left| \vec{\varepsilon}^* \cdot e\left(\frac{\vec{\beta}}{1-\vec{\beta}\cdot\vec{n}} - \frac{-\vec{\beta}}{1+\vec{\beta}\cdot\vec{n}} \right) \right|^2 = \frac{e^2}{\pi^2 c}\left| \vec{\varepsilon}^* \cdot \frac{\vec{\beta}}{1-(\vec{n}\cdot\vec{\beta})^2} \right|^2.$$

Assuming $\vec{\beta}$ is in the z-direction, and $\vec{n} = (\sin\theta, 0, \cos\theta)$. It is easy to see that only polarization perpendicular to $\vec{n}$ and in the same plane as $\vec{n}$ and $\vec{\beta}$ will have non-zero contribution, i.e. $\vec{\varepsilon} = (-\cos\theta, 0, \sin\theta)$. Therefore,

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{\pi^2 c}\frac{\beta^2\sin^2\theta}{(1-\beta^2\cos^2\theta)^2}$$

Performing the angular integration,

$$\frac{dI}{d\omega} = \int \frac{d^2I}{d\omega d\Omega} d\Omega = \frac{2e^2}{\pi c} \int_{-1}^{1} \frac{\beta^2 \sin^2\theta}{(1-\beta^2\cos^2\theta)^2} d(\cos\theta) = \frac{e^2}{\pi c}\left[\frac{1+\beta^2}{\beta} \log\left(\frac{1+\beta}{1-\beta}\right) - 2 \right].$$

For $\beta \to 1$, $\frac{1+\beta^2}{\beta} \to 2$, and

$$\frac{1+\beta}{1-\beta} = \frac{1+\left(1-\frac{1}{\gamma^2}\right)^{1/2}}{1-\left(1-\frac{1}{\gamma^2}\right)^{1/2}} = \frac{2-\frac{1}{2\gamma^2}}{\frac{1}{2\gamma^2}} = 4\gamma^2.$$

Also, as the meson decays, most of its energy goes into the decay products, i.e., $M_\omega c^2 = 2\gamma mc^2$, or $\frac{M_\omega}{m} = 2\gamma$. Then,

$$\frac{dI}{d\omega} = \frac{e^2}{\pi c}\left(2\log(4\gamma^2) - 2\right) = \frac{4e^2}{\pi c}\left(\log(2\gamma) - \frac{1}{2}\right) = \frac{4e^2}{\pi c}\left[\log\left(\frac{M_\omega}{m}\right) - \frac{1}{2}\right].$$

Following Eq. (15.68) and (15.69),

$$\frac{E_{rad}}{E} = \frac{4}{\pi}\frac{e^2}{\hbar c}\left[\log\left(\frac{M_\omega}{m}\right) - \frac{1}{2}\right].$$