13.3 (a) From (11.152), $E_{\parallel}(t) = -\dfrac{q\gamma v t}{(b^2+\gamma^2v^2t^2)^{3/2}}$, with $q = ze$. Then

$$E_{\parallel}(\omega) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty} E_{\parallel}(t)\, e^{i\omega t}\, dt = -\frac{q\gamma v}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\frac{t e^{i\omega t}}{(b^2+\gamma^2v^2t^2)^{3/2}}\,dt$$

$$= \frac{1}{\sqrt{2\pi}}\frac{q}{\gamma v}\int_{-\infty}^{+\infty} e^{i\omega t}\, d\left([b^2+\gamma^2v^2t^2]^{-1/2}\right)$$

$$= -\frac{1}{\sqrt{2\pi}}\frac{i\omega q}{\gamma v}\int_{-\infty}^{+\infty}\frac{e^{i\omega t}}{(b^2+\gamma^2v^2t^2)^{1/2}}\,dt = -\frac{i}{\sqrt{2\pi}}\frac{\omega q}{\gamma^2v^2}\int_{-\infty}^{+\infty}\frac{\exp(i\frac{\omega b}{\gamma v}t)}{(1+t^2)^{1/2}}\,dt$$

$$= -\frac{i}{\sqrt{2\pi}}\frac{\omega q}{\gamma^2v^2}\, 2K_0\left(\frac{\omega b}{\gamma v}\right).$$

Define $\xi = \omega b/\gamma v$, the final result becomes

$$E_{\parallel}(\omega) = -i\frac{ze}{\gamma b v}\left(\frac{2}{\pi}\right)^{1/2}\xi K_0(\xi).$$

Similarly, $E_{\perp}(\omega) = \dfrac{1}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{+\infty} E_{\perp}(t)\, e^{i\omega t}\,dt = \dfrac{\gamma q b}{\sqrt{2\pi}}\displaystyle\int_{-\infty}^{+\infty}\dfrac{e^{i\omega t}}{(b^2+\gamma^2v^2t^2)^{3/2}}\,dt$

$$= \frac{1}{\sqrt{2\pi}}\frac{q}{bv}\int_{-\infty}^{+\infty}\frac{\exp\{i\frac{\omega b}{\gamma v}t\}}{(1+t^2)^{3/2}}\,dt = \frac{q}{bv}\left(\frac{2}{\pi}\right)^{1/2}\frac{\omega b}{\gamma v}K_1\left(\frac{\omega b}{\gamma v}\right)$$

$$= \frac{ze}{bv}\left(\frac{2}{\pi}\right)^{1/2}\xi K_1(\xi).$$

The above Fourier transforms are performed by the following identity.

$$K_\nu(xz) = \frac{\Gamma(\nu+\frac{1}{2})\,(2z)^\nu}{x^\nu\sqrt{\pi}}\int_0^{+\infty}\frac{\cos(xt)}{(t^2+z^2)^{\nu+1/2}}\,dt.$$

(b) From Prob. 13.2,

$$\Delta E = \frac{\pi e^2}{m}|\vec{E}(\omega)|^2 = \frac{\pi e^2}{m}\cdot\frac{2}{\pi}\frac{z^2e^2}{b^2v^2}\xi_0^2\left[\frac{1}{\gamma^2}K_0(\xi_0)^2 + K_1(\xi_0)^2\right].$$

$$= \frac{2z^2e^4}{mv^2}\cdot\frac{\xi_0^2}{b^2}\left[\frac{1}{\gamma^2}K_0(\xi_0)^2 + K_1(\xi_0)^2\right],$$

with $\xi_0 = b\omega_0/\gamma v$.

For small impact parameter b, $\xi_0 \ll 1$, then

$$\xi_0 K_0(\xi_0) \to 0, \qquad \text{and} \qquad \xi_0 K_1(\xi_0) \to 1,$$

The energy transfer becomes

$$\Delta E = \frac{2z^2 e^4}{m v^2} \frac{1}{b^2},$$

which diverges and requires some regularization from physical consideration. Then it will agree with Prob. 13.1.

On the other hand, for large impact parameter b, $\xi_0 \gg 1$. Since $K_\nu(x) \to 0$ for $x \gg 1$, there will be no energy transfer for too large impact parameter. Therefore, only those particle trajectory with $b \lesssim \frac{\gamma v}{\omega_0}$ will contribute.