2.2 Solution: (a) The result for a point charge located *outside* of the hollow, grounded, conducting sphere equally applies to the case when the point charge is located inside the sphere. Therefore, the potential inside the sphere at $\mathbf{x}$ for a point charge at $\mathbf{y}$ is given by

$$\begin{aligned}\Phi(\mathbf{x}) &= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{|\mathbf{x}-\mathbf{y}|} - \frac{a}{y\left|\mathbf{x}-\dfrac{a^2}{y^2}\mathbf{y}\right|}\right)\\ &= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y^2-2xy\cos\gamma}} - \frac{1}{\sqrt{\dfrac{x^2y^2}{a^2}+a^2-2xy\cos\gamma}}\right),\end{aligned}$$

where γ is the angle between x and y.

(b) The charge density on the inner surface is

$$\begin{aligned}\sigma &= -\varepsilon_0\left.\frac{\partial\Phi}{\partial n}\right|_{x=a} = \varepsilon_0\left.\frac{\partial\Phi}{\partial x}\right|_{x=a}\\ &= \frac{q}{4\pi}\left.\left(\frac{y\cos\gamma - x}{(x^2+y^2-2xy\cos\gamma)^{3/2}} + \frac{\dfrac{y^2}{a}-y\cos\gamma}{\left(\dfrac{x^2y^2}{a^2}+a^2-2xy\cos\gamma\right)^{3/2}}\right)\right|_{x=a}\\ &= \frac{q}{4\pi a}\frac{y^2-a^2}{(a^2+y^2-2ay\cos\gamma)^{3/2}}\\ &= \frac{q}{4\pi a^2}\left(\frac{a}{y}\right)\frac{1-\dfrac{a^2}{y^2}}{\left(1+\dfrac{a^2}{y^2}-2\dfrac{a}{y}\cos\gamma\right)^{3/2}}.\end{aligned}$$

A simple surface integration will show that the total induced charge is $-q$. In fact,

$$\begin{aligned}Q &= \int\sigma ds = \frac{q}{2}\frac{a}{y}\int_{-1}^{1}\frac{1-\dfrac{a^2}{y^2}}{\left(1+\dfrac{a^2}{y^2}-2\dfrac{a}{y}\cos\gamma\right)^{3/2}}d(\cos\gamma)\\ &= \frac{q}{2}\left.\frac{1-\dfrac{a^2}{y^2}}{\left(1+\dfrac{a^2}{y^2}-2\dfrac{a}{y}u\right)^{1/2}}\right|_{u=-1}^{1}\\ &= \frac{q}{2}\left(1-\frac{a^2}{y^2}\right)\left(\frac{1}{|1-a/y|}-\frac{1}{|1+a/y|}\right)\\ &= \frac{q}{2}\left(1-\frac{a^2}{y^2}\right)\left(\frac{1}{a/y-1}-\frac{1}{a/y+1}\right) \qquad \text{(since } a>y\text{)}\\ &= \frac{q}{2}\left(1-\frac{a^2}{y^2}\right)\frac{2}{a^2/y^2-1}\end{aligned}$$

$$= \quad -q. \tag{1}$$

(c) The total force acted on the charge q can be calculated with its image charge, which is oppositely charged with $-aq/y$ and located at a^2/y. Thus, using Coulomb's law, the total force is

$$F = \frac{q^2}{4\pi\varepsilon_0}\frac{a}{y}\frac{1}{(a^2/y-y)^2} = \frac{q^2}{4\pi\varepsilon_0}\frac{ay}{(a^2-y^2)^2},$$

pointing from the charge to the sphere, along the direction of $\mathbf{y}$.