14.10 (a) Using Eq. (14.39), we know $\frac{dP(t')}{d\Omega} = \frac{e^2 \dot\beta(t)^2}{4\pi c} \frac{\sin^2\theta}{(1-\beta(t')\cos\theta)^5}$.

Since $\dot\beta(t) = \beta/\Delta t$, and $\beta(t) = \beta(1 - t/\Delta t)$, the radiant energy emitted per unit solid angle is

$$\frac{dE}{d\Omega} = \int_0^{\Delta t} \frac{dP(t')}{d\Omega} dt' = \frac{e^2\beta^2}{4\pi c(\Delta t)^2} \int_0^{\Delta t} \frac{\sin^2\theta}{\left[1-\beta\cos\theta + \frac{\beta\cos\theta}{\Delta t} t'\right]^5} dt' = \frac{e^2\beta^2}{4\pi c(\Delta t)^2} \cdot \frac{\Delta t}{\beta\cos\theta} \left(-\frac{1}{4}\left(1-\beta\cos\theta + \frac{\beta\cos\theta}{\Delta t} t'\right)^{-4}\right)\Bigg|_0^{\Delta t}$$

$$= \frac{e^2\beta^2}{16\pi c\Delta t} \frac{1}{\beta\cos\theta}\left((1-\beta\cos\theta)^{-4} - 1\right) \sin^2\theta$$

$$= \frac{e^2\beta^2}{16\pi c\Delta t} \frac{(2-\beta\cos\theta)\left[1+(1-\beta\cos\theta)^2\right]}{(1-\beta\cos\theta)^4} \sin^2\theta$$

(b) For $\gamma \gg 1$, the radiation is peaked around $\theta \sim 0$. Using the expansion $\beta = (1-1/\gamma^2)^{1/2} \simeq 1 - \frac{1}{2\gamma^2}$,

$\cos\theta \simeq 1 - \theta^2/2$, $\sin\theta \simeq \theta$, we have

$$1-\beta\cos\theta = 1 - \left(1-\frac{1}{2\gamma^2}\right)\left(1-\frac{\theta^2}{2}\right) = \frac{1}{2\gamma^2}(1+\gamma^2\theta^2).$$

and $$\frac{dE}{d\Omega} = \frac{e^2\beta^2}{16\pi c\Delta t} \frac{\left[1+\frac{1}{2\gamma^2}(1+\gamma^2\theta^2)\right]\left[1+\frac{1}{4\gamma^4}(1+\gamma^2\theta^2)^2\right]\theta^2}{\frac{1}{16\gamma^8}(1+\gamma^2\theta^2)^4} = \frac{e^2\beta^2}{\pi c\Delta t}\gamma^6 \frac{(\gamma\theta)^2}{(1+(\gamma\theta)^2)^4}.$$

Let $\xi = (\gamma\theta)^2$, we then have

$$\frac{dE}{d(\cos\theta)} = \int d\phi \frac{dE}{d\Omega} = 2\pi \cdot \frac{dE}{d\Omega} = \frac{2e^2\beta^2}{c\Delta t}\gamma^6 \frac{\xi}{(1+\xi)^4}$$

and $$\frac{dE}{d\xi} = \frac{dE}{d(\cos\theta)}\left|\frac{d(\cos\theta)}{d\xi}\right| = \frac{1}{2\gamma^2}\frac{dE}{d(\cos\theta)} = \frac{e^2\beta^2}{c\Delta t}\gamma^4 \frac{\xi}{(1+\xi)^4}$$

Then, $$\langle\xi\rangle = \int_0^{+\infty} \xi \frac{dE}{d\xi} d\xi \Big/ \int_0^{+\infty} \frac{dE}{d\xi} d\xi = \int_0^{+\infty} \frac{\xi^2}{(1+\xi)^4} d\xi \Big/ \int_0^{+\infty} \frac{\xi}{(1+\xi)^4} d\xi = \frac{1/3}{1/6} = 2,$$

or $\langle\theta^2\rangle = 2/\gamma^2$, $\sqrt{\langle\theta^2\rangle} = \sqrt{2}/\gamma$.