14.11 (a) From Eqs. (12.1) and (12.2), we know

$$\frac{d\vec{p}}{dt} = ze\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right), \quad \frac{dE}{dt} = ze\vec{v}\cdot\vec{E}.$$

Notice that $p^\mu = (E/c, \vec{p})$, Eq. (14.24) can be written as

$$P = -\frac{2}{3}\frac{z^2e^2}{m^2c^3}\frac{dp_\mu}{d\tau}\frac{dp^\mu}{d\tau} = -\frac{2}{3}\frac{z^2e^2}{m^2c^3}\gamma^2\frac{dp_\mu}{dt}\frac{dp^\mu}{dt}$$

$$= -\frac{2}{3}\frac{z^2e^2}{m^2c^3}\gamma^2\cdot z^2e^2\left(\left(\frac{\vec{v}}{c}\cdot\vec{E}\right)^2 - \left(\vec{E}+\frac{\vec{v}}{c}\times\vec{B}\right)^2\right)$$

$$= \frac{2}{3}\frac{z^4e^4}{m^2c^3}\gamma^2\left[\left(\vec{E}+\vec{\beta}\times\vec{B}\right)^2 - (\vec{\beta}\cdot\vec{E})^2\right]$$

(b) We can also write the result in part (a) as

$$P = \frac{2}{3}\frac{z^4e^4}{m^4c^5}(\gamma mc)^2\left[\left(\vec{E}+\vec{\beta}\times\vec{B}\right)^2 - (\vec{\beta}\cdot\vec{E})^2\right] = \frac{2}{3}\frac{z^4r_0^2}{m^2c}\left[\left(p^0\vec{E}+\vec{p}\times\vec{B}\right)^2 - (\vec{p}\cdot\vec{E})^2\right]$$

Since

$$F^{\alpha\beta}p_\beta = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{pmatrix}\begin{pmatrix} p^0 \\ -p_x \\ -p_y \\ -p_z \end{pmatrix} = \begin{pmatrix} \vec{p}\cdot\vec{E} \\ p^0E_x + (\vec{p}\times\vec{B})_x \\ p^0E_y + (\vec{p}\times\vec{B})_y \\ p^0E_z + (\vec{p}\times\vec{B})_z \end{pmatrix},$$

then it is obvious that

$$g_{\alpha\mu}F^{\alpha\beta}p_\beta F^{\mu\nu}p_\nu = (\vec{p}\cdot\vec{E})^2 - \left(p^0\vec{E}+\vec{p}\times\vec{B}\right)^2.$$

The above identity can also be written as $F^{\alpha\beta}p_\beta F_{\alpha\nu}p^\nu$. Using the anti-symmetry of the field strength tensor, $F^{\alpha\beta} = -F^{\beta\alpha}$, we will arrive at the desired result

$$P = \frac{2z^4r_0^2}{3m^2c}F^{\mu\nu}p_\nu p^\lambda F_{\lambda\mu}.$$