5.29 In the region outside of the wires, the Poisson equations for the electric and magnetic scalar potentials are

$$\nabla^2 \Phi_e = 0, \qquad \nabla^2 \Phi_m = 0$$

For Φ_e, the boundary conditions are $\vec{D}\cdot\vec{n} = \sigma$, $\vec{E}\times\vec{n} = 0$, or

$$-\varepsilon \frac{\partial \Phi_e}{\partial n} = \sigma.$$

For Φ_m, the boundary conditions for perfect conductor are $\vec{B}\cdot\vec{n} = 0$, $\vec{n}\times\vec{H} = \vec{K}$, or

$$-\frac{\partial \Phi_m}{\partial n} = K$$

since the magnetic field must be normal to the surface of the conductor. The current at the surface totally comes from the motion of charge density,

$$K = v\sigma$$

for some velocity v. Then, from these, we can infer, that

$$\Phi_m = v\varepsilon\Phi_e \qquad (*)$$

The electric energy is given by $W_e = \frac{\varepsilon}{2}\int |\nabla\Phi_e|^2 d^3x$, and the capacitance per unit length is given by

$$W_e = \frac{\sigma^2 l^2}{2C}$$

where l is the circumference of the wire. Similarly, the self induction per unit length is

$$W_m = \frac{1}{2} L K^2 l^2 = \frac{1}{2} L v^2 \sigma^2 l^2$$

From $(*)$,, $W_m = \frac{\mu}{2}\int |\nabla\Phi_m|^2 d^3x = \frac{\mu\varepsilon^2 v^2}{2}\int |\nabla\Phi_e|^2 d^3x = \mu\varepsilon v^2 W_e$

which is $\frac{1}{2} L v^2 \sigma^2 l^2 = \mu\varepsilon v^2 \cdot \frac{\sigma^2 l^2}{2C}$.

Then, we have

$$LC = \mu\varepsilon.$$