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# SYNCHROTRON RADIATION FROM ELECTRONS IN HELICAL ORBITS

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ABSTRACT

This Letter constitutes a correction of some of the commonly used results of synchrotron radiation theory. The relevance of these revisions to the interpretation of astronomical observations is also discussed.

The widely accepted treatment of the radiation from ultra-relativistic electrons in helical orbits is due to Westfold (1959). That calculation, however, leads to incorrect results for the radiation received by a fixed observer because of an improper spectral analysis. The fundamental harmonic of the radiation emitted by an electron of energy $\gamma m_0c^2$ spiraling with pitch angle $\alpha$ about a magnetic field of strength $H$ is not $\omega_H = eH/\gamma m_0 c$ as was used, but $\omega_H(\sin\alpha)^{-2}$.

This result can be understood most easily by considering the frequency with which a fixed observer sees pulses of radiation from a highly relativistic electron moving along a helical trajectory. The electron emits radiation mainly in the direction of its instantaneous motion. A distant fixed observer who is located in the field of the radiation will see a pulse of radiation corresponding to each orbit of the electron about the magnetic field. Since the electron does not complete each orbit at the same point of space, the frequency of these pulses will be shifted by the classical Doppler factor $(1 - \beta_{||} \cos\zeta)^{-1}$, where $\beta_{||} = v/c \cos\alpha$ and $\zeta = \alpha + O(\gamma^{-1})$ is the angle between the observer and the magnetic field. Therefore, the fundamental harmonic of the radiation is

$$\omega_F = \omega_H \left(1 - \frac{v}{c}\cos\alpha\cos\zeta\right)^{-1} .$$

For $\gamma \gg 1$ and $\alpha \gg \gamma^{-1}$ this becomes

$$\omega_F = \omega_H(\sin\alpha)^{-2} .$$

This result was derived by LeRoux (1961); however, he failed to use it consistently throughout his analysis.

Pursuing the same formalism as Westfold, but spectrally analyzing the emission in harmonics of $\omega_F$, one obtains the correct expressions for the intensity of the radiation $P(\nu,\gamma,\alpha,\zeta)$ seen by a fixed observer in the usual limits $\gamma \gg 1$, $\alpha \gg 1/\gamma$. The spectral distribution of the power is

$$P(\nu,\gamma,\alpha) = \int P(\nu,\gamma,\alpha,\zeta)\, 2\pi \sin\zeta\, d\zeta = \frac{\sqrt{3}\, e^3 H}{m_0 c^2 \sin\alpha} \frac{\nu}{\nu_c} \int_{\nu/\nu_c}^{\infty} K_{5/3}(\eta)\, d\eta , \quad (1)$$

where

$$\nu_c = \frac{3eH\gamma^2}{4\pi m_0 c} \sin\alpha . \quad (2)$$

The total power is

$$P(\gamma,\alpha) = \int_0^{\infty} P(\nu,\gamma,\alpha)\, d\nu = \frac{2e^4H^2\gamma^2}{3m_0^2c^3}, \quad (3)$$

and the spectral distributions of the total power for two orthogonal polarizations (along and perpendicular to the projections of the magnetic field on the image plane, respectively) are

$$P^{(1)}(\gamma,\nu,\alpha)=\frac{\sqrt{3}\,e^3 H}{m_0 c^2\ \sin\ \alpha}\,\frac{\nu}{2\nu_c}\left[\int_{\nu/\nu_c}^{\infty}K_{5/3}(\eta)\,d\eta-K_{2/3}(\nu/\nu_c)\right], \tag{4}$$

$$P^{(2)}(\gamma,\nu,\alpha)=\frac{\sqrt{3}\,e^3 H}{m_0 c^2\ \sin\ \alpha}\,\frac{\nu}{2\nu_c}\left[\int_{\nu/\nu_c}^{\infty}K_{5/3}(\eta)\,d\eta+K_{2/3}(\nu/\nu_c)\right]. \tag{5}$$

It is perhaps necessary to comment on the expression for the total power $P(\gamma,\alpha)$. Westfold thought his results could be verified by comparison with the well-known covariant expression for the total power radiated by an electron in an arbitrary trajectory (Landau and Lifshitz 1962). However, simple considerations of simultaneity make it clear that the power which would be measured by a set of Lorentz observers is not necessarily the power received by distant, isolated observers.

Much research has been done employing the incorrect relations for synchrotron radiation; however, not all of it will need significant modification. Fortuitously, the critical frequency $\nu_c$ is unchanged, and the spectral distribution of the power $P(\nu,\gamma,\alpha)$ is only changed by a factor $(\sin\,\alpha)^{-2}$. Nevertheless, phenomena associated with inhomogeneous magnetic fields, such as the degree of polarization of the radiation and the relaxation of electron distributions, will have to be reinterpreted. Similarly, any theories which are dependent on the anisotropy of the pitch-angle distribution of ultra-relativistic electrons will have to be reconsidered.

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It has just been brought to the attention of the authors that a similar calculation has been performed by Ginzburg, Sazonov, and Syrovatskii (1967).

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