

13.13 We can not ignore the exponential prefactor in the equation after (13.80), for $z = a$

Then, when taking the modulus of \vec{F} , there is an extra factor of $\sin\left[\frac{a}{2}\left(\frac{\omega}{v} - k \cos\theta\right)\right]$.

It is essentially equivalent to show how to obtain the approximate equation (13.83) from (13.82) under the extreme condition $\omega \gg \omega_p$, $r \gg 1$, and $\theta \ll 1$. For these conditions,

$$\frac{\omega}{v} - k \cos\theta = k \left(\frac{\omega}{kv} - 1 + \frac{\theta^2}{2} \right) = k \left(\frac{1}{\beta \sqrt{\epsilon(\omega)}} - 1 + \frac{\theta^2}{2} \right) = k \left(\left[\left(1 - \frac{1}{r^2}\right) \left(1 - \frac{\omega_p^2}{\omega^2}\right) \right]^{-1/2} - 1 + \frac{\theta^2}{2} \right)$$

$$= k \left(1 + \frac{1}{2r^2} + \frac{\omega_p^2}{2\omega^2} - 1 + \frac{\theta^2}{2} \right) = \frac{k}{2r^2} \left(1 + \frac{2\omega_p^2}{\omega^2} + r^2\theta^2 \right) = \frac{\omega}{2rc} \left(1 + \frac{1}{r^2} + \eta \right)$$

$$= \frac{\omega_p}{2rc} \cdot \frac{\omega}{\omega_p} \left(1 + \frac{1}{r^2} + \eta \right) = \frac{v}{2D} \left(1 + \frac{1}{r^2} + \eta \right)$$

Therefore, $\mathcal{F} = \sin^2 \left[\frac{a}{2} \left(\frac{\omega}{v} - k \cos\theta \right) \right] = \sin^2 \left[v \left(1 + \frac{1}{r^2} + \eta \right) \frac{a}{4D} \right]$