

16.1 For the linear potential, $V(r) = \frac{1}{2} m \omega_0^2 r^2$. It is easy to show that for a particle moving in this potential, its kinetic energy coincides with the potential energy. Therefore, $E = m \omega_0^2 r^2$.

Then, $\frac{\partial V}{\partial r} = m \omega_0^2 r$; and $\frac{dE}{dt} = -\frac{\tau}{m} \left(\frac{\partial V}{\partial r} \right)^2 = -\frac{\tau}{m} m^2 \omega_0^2 r^2 = -\omega_0^2 \tau E$,

which leads to the solution $E(t) = E(0) e^{-\tau t}$, with $\tau = \omega_0^2 \tau$.

Also, $\frac{d\vec{L}}{dt} = -\frac{\tau}{m} \left(\frac{1}{r} \frac{\partial V}{\partial r} \right) \vec{L} = -\omega_0^2 \tau \vec{L}$, which leads to $\vec{L}(t) = \vec{L}(0) e^{-\tau t}$.

Therefore, both energy and angular momentum decay exponentially.

16.2. (a) For the electron moving on a circular orbit with radius r , its energy is $E = -ze^2/r$.

Also, $V(r) = -ze^2/r$. Then Eq. (16.13) leads to

$$\frac{dE}{dt} = -\frac{\tau}{m} \left(\frac{\partial V}{\partial r} \right)^2, \quad \frac{ze^2}{2r^2} \frac{dr}{dt} = -\frac{\tau}{m} \frac{z^2 e^4}{r^4}, \quad r^2 \frac{dr}{dt} = -\frac{2ze^2 \tau}{m} = -\frac{4ze^4}{3m^2 c^3}$$

The solution to the above differential equation is $r(t)^3 - r(0)^3 = -\frac{4ze^4}{m^2 c^3} t$,

which can be written as $r(t)^3 = r_0^3 - 9Z(ct)^3 \frac{e^4}{\hbar}$, where $r(0) = r_0$.

(b) Since $r = n^2 a_0 / z$, then $\frac{dr}{dt} = \frac{2na_0}{z} \frac{dn}{dt} = -\frac{4ze^4}{3m^2 c^3 r^2}$, or

$$\begin{aligned} -\frac{dr}{dt} &= \frac{2ze^2 e^4}{3m^2 c^3} \cdot \frac{1}{r^2} \cdot \frac{1}{na_0} = \frac{2ze^2 e^4}{3m^2 c^3} \cdot \frac{z^2}{n^5 a_0^3} = \frac{2ze^2 e^4}{3m^2 c^3} \cdot \frac{z^2}{n^5} \cdot \frac{m^3 e^6}{\hbar^6} \\ &= \frac{2}{3} \frac{z^4 m e^{10}}{n^5 \hbar^6 c^3}, \end{aligned}$$

which agrees with Prob. 14.21(a)