5.32 (a) Using Eq. (5.37),

$$A_\phi(r,\theta) = \frac{\mu_0}{4\pi}\frac{4Ia}{\sqrt{a^2+r^2+2ar\sin\theta}}\frac{(2-k^2)K(k)-2E(k)}{k^2},$$

where

$$k^2 = \frac{4ar\sin\theta}{a^2+r^2+2ar\sin\theta}$$

For point P, $r^2 = a^2+\rho^2+2a\rho\cos\phi$, and $r\sin\theta = a+\rho\cos\phi$, then

$$k^2 = \frac{4a(a+\rho\cos\phi)}{4a^2+4a\rho\cos\phi+\rho^2} = \frac{1+\rho\cos\phi/a}{1+\rho\cos\phi/a+\rho^2/4a^2}$$

For $\rho \ll a$,

$$k^2 = \left(1+\frac{\rho}{a}\cos\phi\right)\left(1-\frac{\rho}{a}\cos\phi-\frac{\rho^2}{4a^2}+\frac{\rho^2}{a^2}\cos^2\phi+\frac{\rho^3}{2a^3}\cos\phi+\frac{\rho^4}{16a^4}\cdots\right)$$

$$= 1-\frac{\rho^2}{4a^2}+o\left(\frac{\rho^2}{a^2}\right)$$

Let $k' = \sqrt{1-k^2} = \rho/2a$, using Gradshtein and Ryzhik, $8^{th}$ ed, 8.113.3 and 8.114.3

$$K(k) = \log\frac{4}{k'},\quad E(k) = 1,$$

we have

$$A_\phi(r,\theta) = \frac{\mu_0}{4\pi}\cdot\frac{4Ia}{2a}\cdot\left[\log\left(\frac{8a}{\rho}\right)-2\right] = \frac{\mu_0 I}{2\pi}\left(\log\left(\frac{8a}{\rho}\right)-2\right)$$

(b). Following the same argument of Prob. 5.26, the vector potential inside the wire has the form of

$$A_\phi = -\frac{\mu_0 I}{4\pi}\frac{\rho^2}{b^2}+C,$$

which must be continuous at the surface of the wire.

$$-\frac{\mu_0 I}{4\pi}+C = \frac{\mu_0 I}{2\pi}\left(\log\left(\frac{8a}{b}\right)-2\right),\qquad C = \frac{\mu_0 I}{4\pi}+\frac{\mu_0 I}{2\pi}\left(\log\left(\frac{8a}{b}\right)-2\right).$$

Therefore, the vector potential is

$$A_\phi = \frac{\mu_0 I}{4\pi}\left(1-\frac{\rho^2}{b^2}\right)+\frac{\mu_0 I}{2\pi}\left(\log\left(\frac{8a}{b}\right)-2\right)$$

(c) The magnetic energy, per unit length, is

$$W = \frac{1}{2}\int\vec{J}\cdot\vec{A}\,d^2x = \frac{I}{2\pi b^2}\cdot 2\pi\int_0^b\left[\frac{\mu_0 I}{4\pi}\left(1-\frac{\rho^2}{b^2}\right)+\frac{\mu_0 I}{2\pi}\left(\log\left(\frac{8a}{b}\right)-2\right)\right]\rho\,d\rho$$

$$= \frac{\mu_0 I^2}{4\pi}\left[\log\left(\frac{8a}{b}\right)-7/4\right],$$

and the total energy is $2\pi a\cdot W = \frac{1}{2}\mu_0 a I^2\left[\log\left(\frac{8a}{b}\right)-7/4\right]$, and $L = \mu_0 a\left[\log\left(\frac{8a}{b}\right)-7/4\right]$