16.10 (a) The Abraham-Lorentz equation $m(\dot{\vec{v}} - \tau\ddot{\vec{v}}) = \vec{F}$ can be expressed as

$$\frac{d}{dt}\left(e^{-t/\tau}\dot{\vec{v}}\right) = -\frac{1}{m\tau}\vec{F}(t)e^{-t/\tau}.$$

To eliminate the runaway solution, we require that $\dot{\vec{v}}(+\infty)$ to be finite. Integrating from t to $+\infty$,

we have $$e^{-u/\tau}\dot{\vec{v}}(u)\Big|_t^{+\infty} = -\frac{1}{m\tau}\int_t^{+\infty}\vec{F}(u)e^{-u/\tau}du,$$

or $$-e^{-t/\tau}\dot{\vec{v}}(t) = -\frac{1}{m\tau}\int_t^{+\infty}\vec{F}(u)e^{-u/\tau}du \Rightarrow m\dot{\vec{v}}(t) = \frac{1}{\tau}\int_t^{+\infty}\vec{F}(u)e^{(t-u)/\tau}du.$$

Make the substitution $u = t+\tau s$ in the integral, we will obtain

$$m\dot{\vec{v}}(t) = \int_0^{+\infty} e^{-s}\vec{F}(t+\tau s)ds.$$

(b) The force can be Taylor expanded as $F(t+\tau s) = \sum_{n=0}^{\infty}\frac{\tau^n s^n}{n!}\frac{d^n\vec{F}(t)}{dt^n}$. Using the identity,

$\int_0^{+\infty} e^{-s}s^n ds = \Gamma(n+1) = n!$, we have

$$m\dot{\vec{v}}(t) = \sum_{n=0}^{\infty}\tau^n\frac{d^n\vec{F}(t)}{dt^n}.$$

Keeping only the first two terms, $m\dot{\vec{v}}(t) = \vec{F}(t) + \tau\frac{d\vec{F}(t)}{dt}$. We can approximate $\vec{F}(t)$ as $m\dot{\vec{v}}(t)$,

the $d\vec{F}/dt = m\ddot{\vec{v}}(t)$. This leads to the Abraham-Lorentz equation, $m(\dot{\vec{v}}(t) - \tau\ddot{\vec{v}}(t)) = \vec{F}(t)$.

(c) For $t>0$, $F(t+\tau s) = F_0$ for $s>0$. Then $m\dot{v}(t) = F_0\int_0^{+\infty}e^{-s}ds = F_0$, $\dot{v}(t) = F_0/m$.

For $t<0$, $F(t+\tau s) = F_0$, for $s>-t/\tau$. Then, $m\dot{v}(t) = F_0\int_{-t/\tau}^{+\infty}e^{-s}ds = F_0e^{t/\tau}$, $\dot{v}(t) = F_0e^{t/\tau}/m$.

Now, $$v(t) = \int_{-\infty}^{t}\dot{v}(u)du = \begin{cases}\frac{F_0\tau}{m}e^{t/\tau}, & t<0\\ \frac{F_0}{m}(t+\tau), & t>0\end{cases}$$

Without radiation reaction, the particle should stay at rest until the force is turned on at $t=0$.

Instead, now it acquires some velocity even before the force is applied. This breaks the causality of physical laws.