5.20 (a) $\vec{F} = \int (\nabla\times\vec{M})\times\vec{B}\, d^3x + \oint (\vec{M}\times\vec{n})\times\vec{B}\, da$

Since $\nabla(\vec{M}\cdot\vec{B}) = (\vec{M}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{M} + \vec{M}\times(\nabla\times\vec{B}) + \vec{B}\times(\nabla\times\vec{M})$

and $(\vec{M}\times\vec{n})\times\vec{B} = \vec{n}(\vec{M}\cdot\vec{B}) - \vec{M}(\vec{n}\cdot\vec{B})$.

we have

$$\vec{F} = \int \left[-\nabla(\vec{M}\cdot\vec{B}) + (\vec{M}\cdot\nabla)\vec{B} + (\vec{B}\cdot\nabla)\vec{M} + \vec{M}\times(\nabla\times\vec{B})\right] d^3x$$
$$+ \oint \left[\vec{n}(\vec{M}\cdot\vec{B}) - \vec{M}(\vec{n}\cdot\vec{B})\right] da$$

Using the result that

$$\int \nabla(\vec{M}\cdot\vec{B})\, d^3x = \oint \vec{n}(\vec{M}\cdot\vec{B})\, da,$$
$$\int (\vec{M}\cdot\nabla)\vec{B}\, d^3x = \oint (\vec{M}\cdot\vec{n})\vec{B}\, da - \int (\nabla\cdot\vec{M})\vec{B}\, d^3x$$
$$\int (\vec{B}\cdot\nabla)\vec{M}\, d^3x = \oint (\vec{B}\cdot\vec{n})\vec{M}\, da - \int (\nabla\cdot\vec{B})\vec{M}\, d^3x,$$

and $\nabla\cdot\vec{B} = 0$

$\nabla\times\vec{B} = 0$

we have

$$\vec{F} = -\int (\nabla\cdot\vec{M})\vec{B}\, d^3x + \oint (\vec{M}\cdot\vec{n})\vec{B}\, da$$

(b) The volume integral term is zero, since $\nabla\cdot\vec{M} = 0$. For the surface integral,

$$\vec{F} = M\int \left(\cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\phi-\phi_0)\right)\begin{pmatrix} B_0(1+\beta y) \\ B_0(1+\beta x) \\ 0 \end{pmatrix} da$$

$$= MB_0 \int \left(\cos\theta\cos\theta_0 + \sin\theta\sin\theta_0\cos(\phi-\phi_0)\right)\begin{pmatrix} 1+\beta R\sin\theta\sin\phi \\ 1+\beta R\sin\theta\cos\phi \\ 0 \end{pmatrix} d(\cos\theta)\, d\phi$$

$$= MB_0\cdot\beta R\cdot\begin{pmatrix} \frac{4}{3}\cdot\pi\ \sin\theta_0\sin\phi_0 \\ \frac{4}{3}\cdot\pi\ \sin\theta_0\cos\phi_0 \\ 0 \end{pmatrix} = \frac{4}{3}\pi\beta B_0 MR\left(\sin\theta_0\sin\phi_0,\ \sin\theta_0\cos\phi_0,\ 0\right)$$