2.1 Solution: Assume that the infinite plane conductor lies on the xy plane and the point charge is located on the z-axis, then the image charge $-q$ is located at $(0,0,-d)$, which leads to a potential

$$\Phi(x,y,z)=\frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{x^2+y^2+(z-d)^2}}-\frac{1}{\sqrt{x^2+y^2+(z+d)^2}}\right).$$

(a) The surface charge density is given by

$$\begin{aligned}\sigma(x,y) &= -\varepsilon_0\left.\frac{\partial\Phi}{\partial z}\right|_{z=0}=\frac{q}{4\pi}\left.\left(\frac{z-d}{[x^2+y^2+(z-d)^2]^{3/2}}-\frac{z+d}{[x^2+y^2+(z+d)^2]^{3/2}}\right)\right|_{z=0}\\ &= -\frac{q}{2\pi}\frac{d}{[x^2+y^2+d^2]^{3/2}}.\end{aligned}$$

(b) The distance between the point charge and its image charge is $2d$. Using Coulomb's law, the attractive force between them is

$$F=\frac{q^2}{16\pi\varepsilon_0 d^2}.$$

(c) By integrating $\sigma^2/2\varepsilon_0$, we have

$$\begin{aligned}F &= \frac{q^2d^2}{8\pi^2\varepsilon_0}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{dxdy}{[x^2+y^2+d^2]^3}\\ &= \frac{q^2d^2}{4\pi\varepsilon_0}\int_0^{+\infty}\frac{rdr}{[r^2+d^2]^3}\\ &= -\frac{q^2d^2}{16\pi\varepsilon_0}\left.\frac{1}{[r^2+d^2]^2}\right|_0^{+\infty}\\ &= \frac{q^2}{16\pi\varepsilon_0 d^2},\end{aligned}\tag{1}$$

which coincides with the result of (b).

(d) Move the charge along z-axis from $(0,0,d)$ to infinity and notice the force is attractive, the work required is

$$W=\int_d^{+\infty}F(z)dz=\frac{q^2}{16\pi\varepsilon_0 d}.$$

(e) The potential energy between the point charge and its image is

$$-\frac{q^2}{8\pi\varepsilon_0 d},$$

whose absolute value is twice the work required to move the point charge to infinity, result of (d).