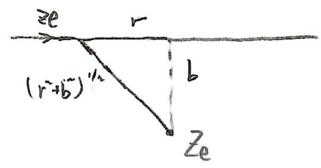
14.7. (a) Since the particle is assumed to move in a straight line with constant speed, we can write $dt = dr/v_0$. Then,



$$P\,dt = \frac{2}{3}\frac{z^2e^2}{m^2c^3}|\nabla V|^2\frac{dr}{v_0}$$

$$= \frac{2}{3}\frac{z^2e^2}{m^2c^3}\frac{z^2Z^2e^4}{(r^2+b^2)^2}\frac{dr}{v_0}$$

So, the total energy radiated is

$$\Delta W = \frac{2}{3}\frac{z^4Z^2e^6}{m^2c^3v_0}\int_{-\infty}^{+\infty}\frac{dr}{(r^2+b^2)^2} = \frac{2}{3}\frac{z^6Z^2e^6}{m^2c^3v_0}\frac{\pi}{2b^3} = \frac{\pi z^6Z^2e^6}{3m^2c^3v_0}\frac{1}{b^3}$$

(b) For $b = r_c = 2zZe^2/mv_0^2$,

$$\Delta W = \frac{\pi}{24}\frac{z m v_0^5}{Zc^3},$$

comparable to Prob. 14.5 (b)

(c) $$\chi = 2\pi\int_{b_{min}}^{+\infty}\Delta W(b)\,b\,db = \frac{2\pi^2 z^4Z^2e^6}{3m^2c^3v_0}\int_{b_{min}}^{+\infty}\frac{db}{b^2} = \frac{2\pi^2 z^4Z^2e^6}{3m^2c^3v_0 b_{min}}$$

Following Eq. (13.16), we can set $b_{min} = \hbar/mv_0$, then

$$\chi = \frac{2\pi^2}{3}Z\left(\frac{Ze^2}{\hbar c}\right)\frac{z^4e^4}{mc^2}$$

Which is close to the Heitler-Bethe result. ($2\pi^2 \approx 19.739$)