8.11 From 8.9, calculate

$$\nabla\times(\nabla\times\vec{E}) = \nabla(\nabla\cdot\vec{E}) - \nabla^2\vec{E} = -\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}E_z\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial\phi^2}E_z\right]\hat{z}$$

$$= \hat{z}\,E_0\left[\left((1+\nu)^2\left(\frac{\rho}{R}\right)^{\nu-1} - \nu^2\left(\frac{\rho}{R}\right)^{\nu-2}\right)\frac{\sin 2\phi}{R^2} + \frac{4\sin 2\phi}{R^2}\left(\frac{\rho}{R}\right)^{\nu-2}\left(1-\frac{\rho}{R}\right)\right]$$

$$= \hat{z}\,\frac{E_0\sin 2\phi}{R^2}\left[(\nu^2+2\nu-3)\left(\frac{\rho}{R}\right)^{\nu-1} - (\nu^2-4)\left(\frac{\rho}{R}\right)^{\nu-2}\right].$$

then

$$\int_V \vec{E}^*\cdot(\nabla\times(\nabla\times\vec{E}))\,d^3x = d\cdot\int_0^{\pi/2}d\phi\int_0^R \rho\,d\rho$$

$$\times\ \frac{E_0^2\sin^2(2\phi)}{R^2}\left(\frac{\rho}{R}\right)^{\nu}\left(1-\frac{\rho}{R}\right)\left[(\nu+3)(\nu-1)\left(\frac{\rho}{R}\right)^{\nu-1} - (\nu+2)(\nu-2)\left(\frac{\rho}{R}\right)^{\nu-2}\right]$$

$$= \frac{\pi d}{2}\cdot\frac{E_0^2}{R^2}\int_0^R\left(\frac{\rho}{R}\right)^{\nu+1}\left(1-\frac{\rho}{R}\right)\left[(\nu+3)(\nu-1)\left(\frac{\rho}{R}\right)^{\nu-1} - (\nu+2)(\nu-2)\left(\frac{\rho}{R}\right)^{\nu-2}\right]d\rho$$

$$= \frac{\pi d}{2}E_0^2\,\frac{\nu^2+\nu+4}{2\nu(\nu+1)(2\nu+1)}$$

For the denominator,

$$\int_V \vec{E}^*\cdot\vec{E}\,d^3x = d\cdot E_0^2\int_0^{\pi/2}\sin^2(2\phi)\,d\phi\int_0^R\left(\frac{\rho}{R}\right)^{2\nu}\left(1-\frac{\rho}{R}\right)^2\rho\,d\rho$$

$$= \frac{\pi d}{2}E_0^2\,\frac{R^2}{4\nu^3+18\nu^2+26\nu+12} = \frac{\pi d}{2}E_0^2\,\frac{R^2}{2(2\nu+3)(\nu+1)(\nu+2)}$$

Therefore.
$$k^2R^2 = \frac{(\nu+2)(2\nu+3)(\nu^2+\nu+4)}{\nu(2\nu+1)}$$

The minimum value can be achieved at $\nu = 1.56758$, with the minimum as 27.0967,

which leads to $(kR)_{min} = 5.20545$