

11.10 (b) It is easy to show, that  $(\hat{\beta} \cdot \vec{k})^{2k} = (\hat{\beta} \cdot \vec{k})^2$ ,  $(\hat{\beta} \cdot \vec{k})^{2k+1} = (\hat{\beta} \cdot \vec{k})$ ,  $k \geq 1$ .

$$\begin{aligned}
 \text{then, } \exp\{-\zeta \hat{\beta} \cdot \vec{k}\} &= \sum_{n=0}^{\infty} \frac{(-\zeta)^n}{n!} (\hat{\beta} \cdot \vec{k})^n \\
 &= 1 + \sum_{k=0}^{\infty} \frac{(-\zeta)^{2k+1}}{(2k+1)!} (\hat{\beta} \cdot \vec{k})^{2k+1} + \sum_{k=1}^{\infty} \frac{(-\zeta)^{2k}}{(2k)!} (\hat{\beta} \cdot \vec{k})^{2k} \\
 &= 1 + (\hat{\beta} \cdot \vec{k}) \sum_{k=0}^{\infty} \frac{(-\zeta)^{2k+1}}{(2k+1)!} + (\hat{\beta} \cdot \vec{k})^2 \sum_{k=1}^{\infty} \frac{(-\zeta)^{2k}}{(2k)!} \\
 &= 1 + (\hat{\beta} \cdot \vec{k}) \sinh(-\zeta) + (\hat{\beta} \cdot \vec{k})^2 [\cosh(-\zeta) - 1] \\
 &= 1 - (\hat{\beta} \cdot \vec{k}) \sinh \zeta + (\hat{\beta} \cdot \vec{k})^2 [\cosh \zeta - 1]
 \end{aligned}$$

where we have use the expansion

$$\sinh \zeta = \sum_{k=0}^{\infty} \frac{\zeta^{2k+1}}{(2k+1)!}, \quad \cosh \zeta = \sum_{k=0}^{\infty} \frac{\zeta^{2k}}{(2k)!}$$