

10.13 (a) We can use Eq. (10.92) to express the diffracted electric field as

$$\vec{E}(\vec{x}) = -\frac{ie^{ikr}}{4\pi r} \vec{k} \times \oint_S \left[\frac{c\vec{k} \times (\vec{n} \times \vec{B}(\vec{x}'))}{k} - \vec{n} \times \vec{E}(\vec{x}') \right] e^{-i\vec{k} \cdot \vec{x}'} da'$$

Since $\vec{E} = E_0 (\vec{E}_1 \cos \alpha - \vec{E}_3 \sin \alpha) e^{ik[z \cos \alpha + x \sin \alpha]}$, $\vec{B} = \frac{E_0}{c} \vec{E}_1 e^{ik[z \cos \alpha + x \sin \alpha]}$, then

$$(\vec{n} \times \vec{E})_{z=0} = E_0 \vec{E}_1 \cos \alpha e^{ikx' \sin \alpha}, \quad (\vec{n} \times \vec{B})_{z=0} = -\frac{E_0}{c} \vec{E}_1 e^{ikx' \sin \alpha}.$$

The diffracted electric field now becomes

$$\begin{aligned} \vec{E}(\vec{x}) &= \frac{ie^{ikr}}{4\pi r} E_0 \vec{k} \times \oint_S e^{-i\vec{k} \cdot \vec{x}'} \left[\frac{\vec{k} \times \vec{E}_1}{k} + \vec{E}_1 \cos \alpha \right] da' \\ &= \frac{ie^{ikr}}{4\pi r} E_0 \vec{k} \times \oint_S e^{-i\vec{k} \cdot \vec{x}'} \left[\vec{E}_1 (\cos \theta + \cos \alpha) - \vec{E}_3 \sin \theta \sin \phi \right] da', \end{aligned}$$

where we have used $\vec{k} \times \vec{E}_1 = k(\vec{E}_1 \cos \alpha - \vec{E}_3 \sin \alpha \sin \phi)$. Compare with Eq. (10.117), we can see that $\cos \alpha$ is replaced by $(\cos \theta + \cos \alpha)/2$, and there is an additional factor proportional to $\vec{k} \times \vec{E}_3$.