7.2? (a) Expanding $\vec{E}(\vec{x}, t-\tau)$ as a Taylor series in τ,

$$\vec{E}(\vec{x}, t-\tau) = \vec{E}(\vec{x}, t) - \tau \frac{\partial \vec{E}(\vec{x},t)}{\partial t} + \frac{\tau^2}{2} \frac{\partial^2 \vec{E}(\vec{x},t)}{\partial t^2} + \cdots$$

We have $$\vec{D}(\vec{x}, t) = \varepsilon_0 \left\{ \vec{E}(\vec{x},t) + \int d\tau\, G(\tau) \left(\vec{E}(\vec{x},t) - \tau \frac{\partial \vec{E}(\vec{x},t)}{\partial t} + \frac{\tau^2}{2} \frac{\partial^2 \vec{E}(\vec{x},t)}{\partial t^2} \cdots \right) \right\}$$

Since $G(\tau) = \omega_p^2 e^{-\gamma\tau/2} \frac{\sin(\nu_0 \tau)}{\nu_0} \theta(\tau)$, with $\nu_0 = \sqrt{\omega_0^2 - \gamma^2/4}$, we can perform the integral up to second order.

$$\int_{-\infty}^{+\infty} G(\tau) d\tau = \frac{\omega_p^2}{\nu_0} \int_0^{+\infty} e^{-\gamma\tau/2} \sin(\nu_0 \tau) d\tau = \frac{\omega_p^2}{\nu_0} \frac{\nu_0}{\nu_0^2 + \gamma^2/4} = \frac{\omega_p^2}{\omega_0^2},$$

$$\int_{-\infty}^{+\infty} \tau G(\tau) d\tau = \frac{\omega_p^2}{\nu_0} \int_0^{+\infty} \tau e^{-\gamma\tau/2} \sin(\nu_0 \tau) d\tau = \frac{\omega_p^2}{\nu_0} \frac{2\nu_0 \cdot \gamma/2}{(\nu_0^2 + \gamma^2/4)^2} = \frac{\gamma \omega_p^2}{\omega_0^4},$$

$$\int_{-\infty}^{+\infty} \tau^2 G(\tau) d\tau = \frac{\omega_p^2}{\nu_0} \int_0^{+\infty} \tau^2 e^{-\gamma\tau/2} \sin(\nu_0 \tau) d\tau = -\frac{\omega_p^2}{\nu_0} \frac{2\nu_0(\nu_0^2 - 3\gamma^2/4)}{(\nu_0^2 + \gamma^2/4)^3} = -\frac{2\omega_p^2(\omega_0^2 - \gamma^2)}{\omega_0^6}$$

Then, $$\vec{D}(\vec{x}, t) = \varepsilon_0 \left\{ \vec{E}(\vec{x},t) + \frac{\omega_p^2}{\omega_0^2} \vec{E}(\vec{x},t) - \frac{\gamma\omega_p^2}{\omega_0^4} \frac{\partial \vec{E}(\vec{x},t)}{\partial t} - \frac{\omega_p^2(\omega_0^2 - \gamma^2)}{\omega_0^6} \frac{\partial^2 \vec{E}(\vec{x},t)}{\partial t^2} \right\}$$

$$= \varepsilon_0 \left\{ \left(1 + \frac{\omega_p^2}{\omega_0^2}\right) \vec{E}(\vec{x},t) - \frac{\gamma\omega_p^2}{\omega_0^4} \frac{\partial \vec{E}(\vec{x},t)}{\partial t} - \frac{\omega_p^2(\omega_0^2 - \gamma^2)}{\omega_0^6} \frac{\partial^2 \vec{E}(\vec{x},t)}{\partial t^2} \right\}$$

(b) $$\vec{D}(\vec{x}, t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \vec{D}(\vec{x}, \omega) e^{-i\omega t} d\omega = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \varepsilon(\omega) \vec{E}(\vec{x}, \omega) e^{-i\omega t} d\omega$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \vec{E}(\vec{x}, \omega)\, \varepsilon\left(i\frac{\partial}{\partial t}\right) e^{-i\omega t} d\omega = \frac{1}{\sqrt{2\pi}} \varepsilon\left(i\frac{\partial}{\partial t}\right) \int_{-\infty}^{+\infty} \vec{E}(\vec{x}, \omega) e^{-i\omega t} d\omega$$

$$= \varepsilon\left(i\frac{\partial}{\partial t}\right) \vec{E}(\vec{x}, t)$$

With the substitution $\omega \to i\partial/\partial t$,

$$\varepsilon\left(i\frac{\partial}{\partial t}\right) = 1 + \frac{\omega_p^2}{\omega_0^2 + \partial_t^2 + \gamma\partial_t} = 1 + \frac{\omega_p^2}{\omega_0^2}\left(1 - \frac{\gamma\partial_t + \partial_t^2}{\omega_0^2} + \frac{\gamma^2 \partial_t^2}{\omega_0^4}\right) = 1 + \frac{\omega_p^2}{\omega_0^2} - \frac{\gamma\omega_p^2}{\omega_0^4}\frac{\partial}{\partial t} - \frac{\omega_p^2(\omega_0^2 - \gamma^2)}{\omega_0^6}\frac{\partial^2}{\partial t^2}$$

The coefficients of the MacLaurin series matches those obtained in part (a) up to second order.