**11.10** Solution: (a) Let $\boldsymbol{\epsilon} = (\sin\theta\cos\phi, \sin\theta\cos\phi, \cos\theta)$, then

$$\boldsymbol{\epsilon}\cdot\mathbf{S} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\cos\theta & \sin\theta\sin\phi \\ 0 & \cos\theta & 0 & \sin\theta\cos\phi \\ 0 & -\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{pmatrix}. \tag{1}$$

By straightforward calculation, it can be shown that $(\boldsymbol{\epsilon}\cdot\mathbf{S})^3 = -\boldsymbol{\epsilon}\cdot\mathbf{S}$.

Similarly,

$$\boldsymbol{\epsilon}'\cdot\mathbf{K} = \begin{pmatrix} 0 & \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \sin\theta\cos\phi & 0 & 0 & 0 \\ \sin\theta\sin\phi & 0 & 0 & 0 \\ \cos\theta & 0 & 0 & 0 \end{pmatrix}, \tag{2}$$

and $(\boldsymbol{\epsilon}'\cdot\mathbf{K})^3 = \boldsymbol{\epsilon}'\cdot\mathbf{K}$.

(b) Using the result from part (a), we can find that, for $n \geq 1$,

$$(\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^{2n} = (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^{2n-3}(\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^3 = (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^{2n-3}(\hat{\boldsymbol{\beta}}\cdot\mathbf{K}) = (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^{2n-2} = \cdots = (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^2,$$

and, for $n \geq 0$,

$$(\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^{2n+1} = (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^{2n-2}(\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^3 = (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^{2n-2}(\hat{\boldsymbol{\beta}}\cdot\mathbf{K}) = (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^{2n-1} = \cdots = \hat{\boldsymbol{\beta}}\cdot\mathbf{K},$$

Then,

$$\begin{aligned}
\exp(-\zeta\hat{\boldsymbol{\beta}}\cdot\mathbf{K}) &= \sum_{n=0}^{\infty}\frac{(-\zeta)^n}{n!}(\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^n \\
&= I + \sum_{k=0}^{\infty}\frac{(-\zeta)^{2k+1}}{(2k+1)!}(\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^{2k+1} + \sum_{k=1}^{\infty}\frac{(-\zeta)^{2k}}{(2k)!}(\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^{2k} \\
&= I + (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})\sum_{k=0}^{\infty}\frac{(-\zeta)^{2k+1}}{(2k+1)!} + (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^2\sum_{k=1}^{\infty}\frac{(-\zeta)^{2k}}{(2k)!} \\
&= I - (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})\sinh\zeta + (\hat{\boldsymbol{\beta}}\cdot\mathbf{K})^2[\cosh\zeta - 1],
\end{aligned}$$

where we have used

$$\sinh x = \sum_{k=0}^{\infty}\frac{x^{2k+1}}{(2k+1)!}, \qquad \cosh x = \sum_{k=0}^{\infty}\frac{x^{2k}}{(2k)!}.$$