12.13 (a) The Lagrangian is $L = \frac{1}{2}(m_1 v_1^2 + m_2 v_2^2) + \frac{1}{8c^2}(m_1 v_1^4 + m_2 v_2^4) - \frac{q_1 q_2}{r} + \frac{1}{2c^2}\frac{q_1 q_2}{r}\left[\vec{v}_1\cdot\vec{v}_2 + (\vec{v}_1\cdot\hat{r})(\vec{v}_2\cdot\hat{r})\right]$.

Define the total momentum $\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1+m_2)\vec{V}$, where $\vec{V}$ is velocity of the CM frame in the lab frame. Then, $\vec{v}_1 = \vec{V} + \frac{m_2}{m_1+m_2}\vec{v}$, $\vec{v}_2 = \vec{V} - \frac{m_1}{m_1+m_2}\vec{v}$, where $\vec{v} = \vec{v}_1 - \vec{v}_2$ is the relative velocity of the two particles. With these, we can write the Lagrangian as

$$L = \frac{1}{2}\left[m_1\left(\vec{V} + \frac{m_2}{m_1+m_2}\vec{v}\right)^2 + m_2\left(\vec{V} - \frac{m_1}{m_1+m_2}\vec{v}\right)^2\right] + \frac{1}{8c^2}\left[m_1\left(\vec{V} + \frac{m_2}{m_1+m_2}\vec{v}\right)^4 + m_2\left(\vec{V} - \frac{m_1}{m_1+m_2}\vec{v}\right)^4\right]$$

$$- \frac{q_1 q_2}{r} + \frac{1}{2c^2}\frac{q_1 q_2}{r}\left[\left(\vec{V} + \frac{m_2}{m_1+m_2}\vec{v}\right)\cdot\left(\vec{V} - \frac{m_1}{m_1+m_2}\vec{v}\right) + \left[\left(\vec{V} + \frac{m_2}{m_1+m_2}\vec{v}\right)\cdot\hat{r}\right]\left[\left(\vec{V} - \frac{m_1}{m_1+m_2}\vec{v}\right)\cdot\hat{r}\right]\right]$$

This simplifies if $\vec{V} = 0$, and the Lagrangian becomes

$$L = \frac{m_1 m_2}{2(m_1+m_2)}v^2 + \frac{1}{8c^2}\frac{m_1 m_2(m_1^3+m_2^3)}{(m_1+m_2)^4}v^4 - \frac{q_1 q_2}{r} - \frac{1}{2c^2}\frac{q_1 q_2}{r}\frac{m_1 m_2}{(m_1+m_2)^2}\left[v^2 + (\vec{v}\cdot\hat{r})^2\right]$$

The canonical momentum is

$$\vec{p} = \frac{\partial L}{\partial \vec{v}} = \frac{m_1 m_2}{m_1+m_2}\vec{v} + \frac{1}{2c^2}\frac{m_1 m_2(m_1^3+m_2^3)v^2}{(m_1+m_2)^4}\vec{v} - \frac{q_1 q_2}{r}\frac{m_1 m_2}{(m_1+m_2)^2 c^2}\left(\vec{v} + (\vec{v}\cdot\hat{r})\hat{r}\right)$$

(b) From the canonical momentum, we can solve the velocity correct to $1/c^2$,

$$\vec{v} = \frac{m_1+m_2}{m_1 m_2}\vec{p}$$

Then, the Hamiltonian is

$$H = \vec{p}\cdot\vec{v} - L = \frac{m_1+m_2}{m_1 m_2}p^2 - \frac{m_1 m_2}{2(m_1+m_2)}\frac{(m_1+m_2)^2}{m_1^2 m_2^2}p^2 - \frac{1}{8c^2}\frac{m_1 m_2(m_1^3+m_2^3)}{(m_1+m_2)^4}\frac{(m_1+m_2)^4}{m_1^4 m_2^4}p^4 + \frac{q_1 q_2}{r}$$

$$+ \frac{1}{2c^2}\frac{q_1 q_2}{r}\frac{m_1 m_2}{(m_1+m_2)^2}\frac{(m_1+m_2)^2}{m_1^2 m_2^2}\left(p^2 + (\vec{p}\cdot\hat{r})^2\right)$$

$$= \frac{p^2}{2}\left(\frac{1}{m_1} + \frac{1}{m_2}\right) - \frac{p^4}{8c^2}\left(\frac{1}{m_1^3} + \frac{1}{m_2^3}\right) + \frac{q_1 q_2}{r} + \frac{q_1 q_2}{2m_1 m_2 c^2}\frac{p^2 + (\vec{p}\cdot\hat{r})^2}{r}$$