11.15 Suppose the frame, in which $\vec{E}'$ and $\vec{B}'$ are parallel, is moving with $\vec{\beta} = \vec{v}/c$ relative to the original frame, then in this frame,

$$\vec{E}' = \gamma(\vec{E} + \vec{\beta} \times \vec{B}) - \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{E}), \quad \vec{B}' = \gamma(\vec{B} - \vec{\beta}\times\vec{E}) - \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{B})$$

If $\vec{E}'$ and $\vec{B}'$ are parallel to each other, then

$$0 = \vec{E}' \times \vec{B}' = \left[\gamma(\vec{E} + \vec{\beta}\times\vec{B}) - \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{E})\right] \times \left[\gamma(\vec{B} - \vec{\beta}\times\vec{E}) - \frac{\gamma^2}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{B})\right]$$

$$= \gamma^2(\vec{E} + \vec{\beta}\times\vec{B}) \times (\vec{B} - \vec{\beta}\times\vec{E}) - \frac{\gamma^3}{\gamma+1}(\vec{\beta}\cdot\vec{B})(\vec{E} + \vec{\beta}\times\vec{B})\times\vec{\beta}$$

$$- \frac{\gamma^3}{\gamma+1}(\vec{\beta}\cdot\vec{E})(\vec{B} - \vec{\beta}\times\vec{E})\times\vec{\beta}$$

$$= \gamma^2\left[\vec{E}\times\vec{B} + \vec{B}(\vec{\beta}\cdot\vec{B}) - \beta|\vec{B}|^2 - \beta|\vec{E}|^2 + \vec{E}(\vec{\beta}\cdot\vec{E}) - \vec{\beta}\left[(\vec{\beta}\times\vec{B})\cdot\vec{E}\right]\right].$$

$$- \frac{\gamma^3}{\gamma+1}\left[(\vec{\beta}\cdot\vec{B})(\vec{E} + \vec{\beta}\times\vec{B})\times\vec{\beta} + (\vec{\beta}\cdot\vec{E})(\vec{B} - \vec{\beta}\times\vec{E})\times\vec{\beta}\right]$$

$$= \gamma^2\left[\vec{E}\times\vec{B} + \vec{\beta}(\vec{\beta}\cdot(\vec{E}\times\vec{B})) - \beta(|\vec{E}|^2 + |\vec{B}|^2) - \vec{B}(\vec{\beta}\cdot\vec{B}) + \vec{E}(\vec{\beta}\cdot\vec{E})\right]$$

$$- \frac{\gamma^3}{\gamma+1}\left[(\vec{\beta}\cdot\vec{B})(\vec{E} + \vec{\beta}\times\vec{B})\times\vec{\beta} + (\vec{\beta}\cdot\vec{E})(\vec{B} - \vec{\beta}\times\vec{E})\times\vec{\beta}\right]$$

Since $\vec{E}$ and $\vec{B}$ are in the x-y plane, if we choose $\vec{\beta}$ to be in the z-direction, and require $\vec{E}\times\vec{B} + \vec{\beta}(\vec{\beta}\cdot(\vec{E}\times\vec{B})) - \beta(|\vec{E}|^2 + |\vec{B}|^2) = 0$, then we will have $\vec{E}'\times\vec{B}' = 0$. In this case, we have

$$EB\sin\theta(1+\beta^2) - \beta(|\vec{E}|^2 + |\vec{B}|^2) = 0 \implies 2\sin\theta(1+\beta^2) - 5\beta = 0$$

Therefore,
$$\beta = \frac{5 - \sqrt{25 - 16\sin^2\theta}}{4\sin\theta}, \text{ as } \beta < 1.$$

For $\theta \ll 1$, $\beta = \frac{2}{5}\sin\theta \to 0$, as $\vec{E}$ and $\vec{B}$ are almost parallel in the original frame.

For $\theta \to \frac{\pi}{2}$, $\beta \to \frac{1}{2}$. In this case, $\vec{E}$ and $\vec{B}$ are almost perpendicular. Since $\vec{E}\cdot\vec{B}$ is Lorentz invariant, to have $\vec{E}'\times\vec{B}' = 0$, we can only make $\vec{E}'$ disappear as $|\vec{E}| < |\vec{B}|$ in the original frame. Therefore,

$$\vec{\beta} = \frac{\vec{E}\times\vec{B}}{B^2} = \frac{1}{2}\hat{z}$$