14.15 (a) Since the motion is axially symmetric, the result will only depend on the azimuthal angle θ. Without loss of generality, we can choose $\vec{n}$ to be in the x-z plane, i.e., $\vec{n} = (\sin\theta, 0, \cos\theta)$. Then, given $\vec{x}(t) = R(\cos(\omega_0 t), \sin(\omega_0 t), 0)$ and $\vec{v}(t) = \omega_0 R(-\sin(\omega_0 t), \cos(\omega_0 t), 0)$, we know $\vec{n}\cdot\vec{x}(t) = R\sin\theta\cos(\omega_0 t)$ and $\vec{v}(t)\times\vec{n} = \omega_0 R\left(\cos\theta\cos(\omega_0 t), \cos\theta\sin(\omega_0 t), -\sin\theta\cos(\omega_0 t)\right)$. Applying the result of Prob. 14.13,

$$\frac{dP_m}{d\Omega} = \frac{e^2\omega_0^4 m^2}{(2\pi c)^3}\left|\int_0^{2\pi/\omega_0}\begin{pmatrix}\cos\theta\cos(\omega_0 t)\\ \cos\theta\sin(\omega_0 t)\\ -\sin\theta\cos(\omega_0 t)\end{pmatrix}\omega_0 R\exp\left\{im\omega_0\left(t-\frac{R}{c}\sin\theta\cos(\omega_0 t)\right)\right\}dt\right|^2$$

Since $\exp\left\{-im\omega_0\frac{R}{c}\sin\theta\cos(\omega_0 t)\right\} = \sum_{k=-\infty}^{+\infty}(-i)^k J_k\left(m\frac{\omega_0 R}{c}\sin\theta\right)e^{-ik\omega_0 t}$, the integral can be evaluated with the following identities

$$\int_0^{2\pi/\omega_0}\begin{Bmatrix}\cos(\omega_0 t)\\ \sin(\omega_0 t)\end{Bmatrix}\exp\{im\omega_0 t\}\sum_{k=-\infty}^{+\infty}(-i)^k J_k(m\beta\sin\theta)\,e^{-ik\omega_0 t}\,dt$$

$$= \int_0^{2\pi/\omega_0}\begin{Bmatrix}\frac{1}{2}\left(e^{i\omega_0 t}+e^{-i\omega_0 t}\right)\\ \frac{1}{2i}\left(e^{i\omega_0 t}-e^{-i\omega_0 t}\right)\end{Bmatrix}\exp\{im\omega_0 t\}\sum_{k=-\infty}^{+\infty}(-i)^k J_k(m\beta\sin\theta)\,e^{-ik\omega_0 t}\,dt$$

$$= \frac{2\pi}{\omega_0}\begin{Bmatrix}\frac{1}{2}\left((-i)^{m+1}J_{m+1}(m\beta\sin\theta)+(-i)^{m-1}J_{m-1}(m\beta\sin\theta)\right)\\ \frac{1}{2i}\left((-i)^{m+1}J_{m+1}(m\beta\sin\theta)-(-i)^{m-1}J_{m-1}(m\beta\sin\theta)\right)\end{Bmatrix}$$

$$= \frac{2\pi}{\omega_0}\begin{Bmatrix}\frac{1}{2}(-i)^{m+1}\,2\left.\frac{dJ_m(x)}{dx}\right|_{x=m\beta\sin\theta}\\ \frac{1}{2}(-i)^{m+2}\cdot\frac{2m}{m\beta\sin\theta}J_m(m\beta\sin\theta)\end{Bmatrix}$$

Then,

$$\frac{dP_m}{d\Omega} = \frac{e^2\omega_0^4R^2}{2\pi c^3}m^2\left|\begin{pmatrix}\cos\theta\;dJ_m(x)/dx\big|_{x=m\beta\sin\theta}\\ (-i)\cos\theta\cdot J_m(m\beta\sin\theta)/\beta\sin\theta\\ -\sin\theta\;dJ_m(x)/dx\big|_{x=m\beta\sin\theta}\end{pmatrix}\right|^2$$

$$= \frac{e^2\omega_0^4R^2}{2\pi c^3}m^2\left\{\left[\frac{dJ_m(m\beta\sin\theta)}{d(m\beta\sin\theta)}\right]^2+\frac{\cot^2\theta}{\beta^2}J_m^2(m\beta\sin\theta)\right\}.$$

(b) In the non-relativistic limit, $\beta \ll 1$, only $m=1$ component will contribute. Notice that

$J_1(z) \sim z/2$, $\quad dJ_1(z)/dz \sim \frac{1}{2}$, we can arrive at the following asymptotic result.

$$\frac{dP}{d\Omega} \sim \frac{dP_1}{d\Omega} = \frac{e^2 \omega_0^4 R^2}{2\pi c^3}\left(\frac{1}{4} + \frac{1}{4}\cos^2\theta\right) = \frac{e^2 \omega_0^4 R^2}{4\pi c^3}\left(1 - \frac{1}{2}\sin^2\theta\right).$$

Which agrees with Prob. 14.4 (b)

(c) Not sure how to proceed, but it seems to be related to the Bessel function property that

$J_n(x)$ can be approximated by $K_{1/3}$ for $n \sim x$.