15.5 Combine Eq. (14.67) and Prob. 14.19(a), we know

$$\frac{d^2 I}{d\omega d\Omega} = \frac{\omega^2}{4\pi^2 c}\left| \sum_i \int_{-\infty}^{0} dt \left\{ e_i \left[\vec{n}\times(\vec{n}\times\vec{\beta}_i) \right] + \frac{\omega}{c}\vec{n}\times\vec{\mu}_i \right\} \exp\{ i\omega(t - \vec{n}\cdot\vec{r}_i(t)/c) \} \right|^2,$$

where $e_1 = -e_2 = e$, $\vec{\beta}_1 = -\vec{\beta}_2 = \beta\hat{z}$, $\vec{r}_1(t) = -\vec{r}_2(t) = c\beta t\hat{z}$, $\vec{S}_1 = -\vec{S}_2 = \vec{S}$, and $\vec{\mu}_1 = \frac{e}{mc}\vec{S} = \frac{-e}{mc}(-\vec{S}) = \vec{\mu}_2 = \vec{\mu}$. Performing the integral, with proper decay factor,

We get

$$\frac{d^2 I}{d\omega d\Omega} = \frac{1}{4\pi^2 c}\left| \frac{e\{\vec{n}\times(\vec{n}\times\vec{\beta})\}}{1-\vec{n}\cdot\vec{\beta}} + \frac{\frac{\omega}{c}\vec{n}\times\vec{\mu}}{1-\vec{n}\cdot\vec{\beta}} + \frac{e\{\vec{n}\times(\vec{n}\times\vec{\beta})\}}{1+\vec{n}\cdot\vec{\beta}} + \frac{\frac{\omega}{c}\vec{n}\times\vec{\mu}}{1+\vec{n}\cdot\vec{\beta}} \right|^2$$

$$= \frac{1}{\pi^2 c}\left| \frac{e\{\vec{n}\times(\vec{n}\times\vec{\beta})\}}{1-(\vec{n}\cdot\vec{\beta})^2} + \frac{\frac{\omega}{c}(\vec{n}\times\vec{\mu})}{1-(\vec{n}\cdot\vec{\beta})^2} \right|^2$$

Following the same calculation procedure in Prob. 15.4 (b) and Section 15.7, we have

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{\pi^2 c}\frac{\beta^2\sin^2\theta}{(1-\beta^2\cos^2\theta)^2} + \frac{1}{\pi^2 c}\left(\frac{\omega}{c}\right)^2 \frac{e^2\hbar^2/4m^2c^2}{(1-\beta^2\cos^2\theta)^2}$$

$$= \frac{e^2}{\pi^2 c}\frac{\beta^2\sin^2\theta + \frac{\hbar^2\omega^2}{4m^2c^4}}{(1-\beta^2\cos^2\theta)^2}$$

Taking into account of the hadron creation cross section,

$$\frac{d^2\sigma}{d\omega d\Omega} = \frac{1}{\hbar\omega}\frac{d^2 I}{d\omega d\Omega}d\sigma_0 = \frac{\alpha}{\pi^2}\frac{d\sigma_0}{\hbar\omega}\frac{\beta^2\sin^2\theta + \frac{\hbar^2\omega^2}{4m^2c^4}}{(1-\beta^2\cos^2\theta)^2}$$

Using the integration identities,

$$\int_{-1}^{1}\frac{1-x^2}{(1-\beta^2x^2)^2}dx = \frac{1}{\beta^2}\left[\frac{1+\beta^2}{2\beta}\log\left(\frac{1+\beta}{1-\beta}\right) - 1\right], \quad \int_{-1}^{1}\frac{dx}{(1-\beta^2x^2)^2} = \frac{1}{2\beta}\log\left(\frac{1+\beta}{1-\beta}\right) + \frac{1}{1-\beta^2}.$$

and performing integration in solid angle, we will obtain

$$\frac{d\sigma}{d(\hbar\omega)} = \frac{2\alpha}{\pi}\frac{d\sigma_0}{\hbar\omega}\left\{\left[\frac{1+\beta^2}{2\beta}\log\left(\frac{1+\beta}{1-\beta}\right) - 1\right] + \frac{\hbar^2\omega^2}{4m^2c^4}\left[\frac{1}{2\beta}\log\left(\frac{1+\beta}{1-\beta}\right) + \frac{1}{1-\beta^2}\right]\right\}$$