9.5 (a) From Eq. (9.16), $\vec{A}(\vec{x}) = -i\frac{\mu_0 \omega}{4\pi}\frac{e^{ikr}}{r}\vec{p}$. For the scalar potential,

$$\Phi(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int \rho(\vec{x}')\frac{e^{ik|\vec{x}-\vec{x}'|}}{|\vec{x}-\vec{x}'|}d^3x'$$

Since $|\vec{x}-\vec{x}'| = r - \vec{n}\cdot\vec{x}'$, and $|\vec{x}-\vec{x}'|^{-1} = \frac{1}{r} + \frac{\vec{n}\cdot\vec{x}'}{r^2}$, we have

$$\Phi(\vec{x}) = \frac{e^{ikr}}{4\pi\varepsilon_0}\int \rho(\vec{x}')\left(1 - ik\vec{n}\cdot\vec{x}'\right)\left(\frac{1}{r} + \frac{\vec{n}\cdot\vec{x}'}{r^2}\right)d^3x'$$

$$= \frac{e^{ikr}}{4\pi\varepsilon_0}\int \rho(\vec{x}')\left(\frac{1}{r} - ik\frac{\vec{n}\cdot\vec{x}'}{r} + \frac{\vec{n}\cdot\vec{x}'}{r^2} + \cdots\right)d^3x'$$

$$= \frac{e^{ikr}}{4\pi\varepsilon_0}\left[\frac{q}{r} + \frac{\vec{n}\cdot\vec{p}}{r^2}(1-ikr) + \cdots\right].$$

Where the leading term is the electric monopole contribution and can be dropped for radiation purposes.

Therefore, $\Phi(\vec{x}) = \frac{e^{ikr}}{4\pi\varepsilon_0 r^2}(\vec{n}\cdot\vec{p})(1-ikr)$

(b) Using the relation $\nabla\times(\psi\vec{a}) = \nabla\psi\times\vec{a} + \psi\nabla\times\vec{a}$, the magnetic field is

$$\vec{H} = \frac{1}{\mu_0}\nabla\times\vec{A} = -i\frac{\omega}{4\pi}\nabla\left(\frac{e^{ikr}}{r}\right)\times\vec{p} = -i\frac{\omega}{4\pi}e^{ikr}\left(\frac{ik\vec{n}}{r} - \frac{\vec{n}}{r^2}\right)\times\vec{p}$$

$$= \frac{k\omega}{4\pi}(\vec{n}\times\vec{p})\frac{e^{ikr}}{r}\left(1 - \frac{1}{ikr}\right) = \frac{ck^2}{4\pi}(\vec{n}\times\vec{p})\frac{e^{ikr}}{r}\left(1 - \frac{1}{ikr}\right)$$

For the electric field, $\vec{E} = -\nabla\Phi - \frac{\partial\vec{A}}{\partial t}$. The vector potential has $e^{-i\omega t}$ time dependence, and

$$\frac{\partial\vec{A}}{\partial t} = -\frac{\mu_0\omega^2}{4\pi}\frac{e^{ikr}}{r}\vec{p} = -\frac{k^2}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\vec{p}.$$ For the scalar potential part,

$$\nabla\Phi = \frac{-ike^{ikr}}{4\pi\varepsilon_0 r^2}\vec{n}(\vec{n}\cdot\vec{p}) + \frac{ike^{ikr}}{4\pi\varepsilon_0 r^2}\vec{n}(\vec{n}\cdot\vec{p})(1-ikr) - \frac{e^{ikr}}{2\pi\varepsilon_0 r^3}\vec{n}(\vec{n}\cdot\vec{p})(1-ikr)$$
$$+ \frac{e^{ikr}}{4\pi\varepsilon_0 r^2}(1-ikr)\nabla(\vec{n}\cdot\vec{p}) \qquad \left(\nabla(\vec{n}\cdot\vec{p}) = (\vec{p}\cdot\nabla)\vec{n} = \frac{1}{r}[\vec{p} - \vec{n}(\vec{n}\cdot\vec{p})]\right)$$

$$= \frac{k^2}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\vec{n}(\vec{n}\cdot\vec{p}) - \frac{e^{ikr}}{2\pi\varepsilon_0 r^3}\vec{n}(\vec{n}\cdot\vec{p})(1-ikr) + \frac{e^{ikr}}{4\pi\varepsilon_0 r^3}(1-ikr)[\vec{p} - \vec{n}(\vec{n}\cdot\vec{p})]$$

$$= \frac{k^2}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}\vec{n}(\vec{n}\cdot\vec{p}) + \frac{e^{ikr}}{4\pi\varepsilon_0}\left(\frac{1}{r^3} - \frac{ik}{r^2}\right)[\vec{p} - 3\vec{n}(\vec{n}\cdot\vec{p})]$$

Then, $\vec{E} = -\nabla\Phi - \frac{\partial\vec{A}}{\partial t} = \frac{k^2}{4\pi\varepsilon_0}\frac{e^{ikr}}{r}[\vec{p} - \vec{n}(\vec{n}\cdot\vec{p})] + \frac{e^{ikr}}{4\pi\varepsilon_0}\left(\frac{1}{r^3} - \frac{ik}{r^2}\right)[3\vec{n}(\vec{n}\cdot\vec{p}) - \vec{p}]$

$$= \frac{e^{ikr}}{4\pi\varepsilon_0 r}\left[k^2(\vec{n}\times\vec{p})\times\vec{n} + (3\vec{n}(\vec{n}\cdot\vec{p}) - \vec{p})\left(\frac{1}{r^2} - \frac{ik}{r}\right)\right]$$