6.25 (a) For a single charged particle in the atom, the Lorentz force is

$$\frac{d\vec{p}_i}{dt} = q_i\left(\vec{E}(\vec{r}_i) + \vec{v}_i \times \vec{B}(\vec{r}_i)\right),$$ where $\vec{p}_i$, $\vec{v}_i$, and $\vec{r}_i$ are the particle's momentum, velocity, and location

Then, $\frac{d\vec{P}_{atom}}{dt} = \sum_i \frac{d\vec{p}_i}{dt} = \sum_i q_i\left(\vec{E}(\vec{r}_i) + \vec{v}_i \times \vec{B}(\vec{r}_i)\right)$. Expanding around the location of the atom,

we have $$\frac{d\vec{P}_{atom}}{dt} = \sum_i q_i\left(\vec{E}(\vec{0}) + \vec{r}_i\cdot\nabla\vec{E}(\vec{0}) + \vec{v}_i\times\vec{B}(\vec{0})\right) = \sum_i \left(q_i\vec{r}_i\right)\cdot\nabla\vec{E}(\vec{0}) + \sum_i \frac{d}{dt}\left(q_i\vec{r}_i\right)\times\vec{B}(\vec{0}),$$

where we have use the neutrality of the atom, $\sum_i q_i = 0$. Define the atom dipole moment as

$\vec{d} = \sum_i q_i\vec{r}_i$, then we will have $\frac{d\vec{P}_{atom}}{dt} = (\vec{d}\cdot\nabla)\vec{E} + \dot{\vec{d}}\times\vec{B}$.

(b) By a uniform plane, the gradient should vanish, and we are left with

$$\frac{d\vec{P}_{atom}}{dt} = \dot{\vec{d}}\times\vec{B}.$$ Averaging over volume, and notice that

$\frac{1}{\Delta V}\int_{\Delta V} \vec{P}_{atom}\, d^3x = \vec{g}_{mech}$, $\frac{1}{\Delta V}\int \vec{d}\, d^3x = \vec{P}$, which is the polarization, we have

$$\frac{d\vec{g}_{mech}}{dt} = \dot{\vec{P}}\times\vec{B}.$$ Since $\vec{P} = (\varepsilon-\varepsilon_0)\vec{E}$, the equation now becomes

$$\frac{d\vec{g}_{mech}}{dt} = \frac{\varepsilon-\varepsilon_0}{\varepsilon_0}\dot{\vec{E}}\times\vec{H}\cdot\mu_0\varepsilon_0 = \frac{1}{c^2}(n^2-1)\dot{\vec{E}}\times\vec{H},$$ where $n^2 = \varepsilon/\varepsilon_0$

Finally, $\dot{\vec{E}}\times\vec{H} = \frac{1}{2}\frac{d}{dt}(\vec{E}\times\vec{H})$, and $\vec{g}_{em} = \frac{1}{c^2}\vec{E}\times\vec{H}$, we will arrive at

$$\frac{d\vec{g}_{mech}}{dt} = \frac{1}{2}(n^2-1)\frac{d\vec{g}_{em}}{dt}$$