14.23 (a) From Prob. 14.13, we know

$$\frac{dP_m(N)}{d\Omega} = \frac{\omega_0^4 m^2}{(2\pi c)^3}\left|\sum_{j=1}^{N} q_j \int_0^{2\pi/\omega_0} \vec{v}_j(t)\times\vec{n}\,\exp\left\{im\omega_0\left(t-\frac{\vec{n}\cdot\vec{x}_j(t)}{c}\right)\right\}dt\right|^2$$

Also, for the particles, $\vec{x}_j(t) = R(\cos(\omega_0 t+\phi_j), \sin(\omega_0 t+\phi_j), 0)$, $\vec{v}_j(t) = \omega_0 R(-\sin(\omega_0 t+\phi_j), \cos(\omega_0 t+\phi_j), 0)$,

then we can the result from Prob. 14.15 (a),

$$\frac{dP_m(N)}{d\Omega} = \frac{\omega_0^4 m^2}{(2\pi c)^3}\left|\sum_{j=1}^{N} q_j \int_0^{2\pi/\omega_0} \begin{pmatrix}\cos\theta\cos(\omega_0 t+\phi_j)\\ \cos\theta\sin(\omega_0 t+\phi_j)\\ -\sin\theta\cos(\omega_0 t+\phi_j)\end{pmatrix} \omega_0 R\,\exp\{im\omega_0 t\}\sum_{k=-\infty}^{+\infty}(-i)^k J_k(m\beta\sin\theta)\,e^{-ik(\omega_0 t+\phi_j)}\,dt\right|^2$$

It is straightforward to verify that

$$\int_0^{2\pi/\omega_0}\begin{Bmatrix}\cos(\omega_0 t+\phi_j)\\ \sin(\omega_0 t+\phi_j)\end{Bmatrix} e^{im\omega_0 t}\sum_{k=-\infty}^{+\infty}(-i)^k J_k(m\beta\sin\theta)\,e^{-ik(\omega_0 t+\phi_j)}\,dt$$

$$= e^{-im\phi_j}\int_0^{2\pi/\omega_0}\begin{Bmatrix}\cos(\omega_0 t)\\ \sin(\omega_0 t)\end{Bmatrix} e^{im\omega_0 t}\sum_{k=-\infty}^{+\infty}(-i)^k J_k(m\beta\sin\theta)\,e^{-ik\omega_0 t}\,dt,$$

Then,

$$\frac{dP_m(N)}{d\Omega} = \frac{\omega_0^4 m^2}{(2\pi c)^3}\left|\sum_{j=1}^{N} q_j e^{-im\phi_j}\cdot\int_0^{2\pi/\omega_0}\begin{pmatrix}\cos\theta\cos(\omega_0 t)\\ \cos\theta\sin(\omega_0 t)\\ -\sin\theta\cos(\omega_0 t)\end{pmatrix}\omega_0 R\,\exp\{im\omega_0 t\}\sum_{k=-\infty}^{+\infty}(-i)^k J_k(m\beta\sin\theta)\,e^{-ik\omega_0 t}\,dt\right|^2$$

$$= \frac{dP_m(1)}{d\Omega}\cdot F_m(N),$$

where $F_m(N) = \left|\sum_{j=1}^{N} q_j e^{-im\phi_j}\right|^2$

(b) If all q_j's are the same, $q_j = q$, and the particles are evenly spaced $\phi_j = 2\pi j/N$, then

$F_m(N) = q^2\left|\sum_{j=1}^{N}\exp\{-im\cdot 2\pi j/N\}\right|^2$ It is easy to verify that the sum is only non-zero when m is a multiple of N, and the sum is N in this case. Therefore, the radiation frequency must be a multiple of $N\omega_0$ and the intensity is N^2 of one particle case. The radiation becomes the coherent sum of individual particle radiation.

(c) Since the radiation frequency must be a multipole of $N\omega_0$, the lowest m in the expression of Prob. 14.15 (a) must be N. For non-relativistic motion, $\beta \ll 1$, $J_N(z) \sim z^N$, $dJ_N(z)/dz \sim z^{N-1}$, both term in the braces are of the order β^{2N-2}. Together with pre-factor, we can see the leading order contribution is of the order β^{2N}. Therefore, the radiation tends to 0 as $N \to \infty$.

(d) Not sure how to do this. Leave it for later.

(e) From parts (c) and (d), it is clear that when the particle number tends to infinity and their positions are symmetrically arranged, there will be no radiation. The configuration is closely related to that of a steady current, which according to our study of the static property of magnetic field, should have no radiation at all.