14.17. (a) Using the results of section 12.2, the radius of the circular motion, i.e., the projection of the helix motion on the plane perpendicular to the magnetic field, can be determined by

$$cp_\perp = eBr, \quad \text{or } c\gamma m v \cos\alpha = eBr.$$

Therefore, $r = \frac{\gamma mc}{eB} v\cos\alpha = \frac{v\cos\alpha}{\omega_B}$, with $\omega_B = \frac{eB}{\gamma mc}$. We can follow the same argument as in Section 14.4 to show that the pulse length of the radiation burst is $L = r/2\gamma^3$. However, for an observer, due to the helical motion of the particle, its direction is not always pointing to the observer. The observer will feel a component perpendicular to the magnetic field, with an interval of $L\cos\alpha = \frac{v}{2\gamma^3}\frac{\cos^2\alpha}{\omega_B}$. Thus, the fundamental frequency is $\omega_0 = \omega_B/\cos^2\alpha$.

The helix can be parameterized by

$$x(t) = r\cos(\omega_B t), \quad y(t) = r\sin(\omega_B t), \quad z(t) = v\sin\alpha\, t = \frac{v\sin\alpha}{\omega_B}\cdot\omega_B t = c\cdot\omega_B t.$$

The radius of curvature is then

$$\rho = \frac{r^2 + c^2}{r} = \frac{v^2/\omega_B^2}{v\cos\alpha/\omega_B} = \frac{v}{\omega_B\cos\alpha}.$$

By Eq. (14.81), $\omega_c = \frac{3}{2}\gamma^3\omega_B\cos\alpha$.

(b) The power radiated can be obtained from the energy radiated, Eq. (14.79), by

$$\frac{d^2P}{d\omega d\Omega} = \frac{1}{T}\frac{d^2I}{d\omega d\Omega} = \frac{\omega_0}{2\pi}\frac{d^2I}{d\omega d\Omega}$$

$$= \frac{1}{2\pi}\frac{\omega_B}{\cos^2\alpha}\frac{e^2}{3\pi^2 c}\left(\frac{\omega}{\omega_c}\frac{3}{2}\gamma^3\right)^2\left(\frac{1}{\gamma^2}+\psi^2\right)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K_{1/3}^2(\xi)\right]$$

$$= \frac{3e^2\gamma^2}{8\pi^3 c}\left(\frac{\omega}{\omega_c}\right)^2\frac{\omega_B}{\cos^2\alpha}(1+\gamma^2\psi^2)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K_{1/3}^2(\xi)\right],$$

where ψ is the relative angle measured from the helix and $\xi = \frac{\omega}{2\omega_c}(1+\gamma^2\psi^2)^{3/2}$.