14.14 (a) For the harmonic oscillation, $\vec{r}(t) = a\cos(\omega_0 t)\,\hat{z}$, $\vec{v}(t) = -a\omega_0 \sin(\omega_0 t)\,\hat{z}$. From Prob. 14.13,

$$\left| \int_0^{2\pi/\omega_0} \vec{v}(t)\times\vec{n}\, \exp\left\{ im\omega_0\left(t - \frac{\vec{n}\cdot\vec{r}(t)}{c}\right)\right\} dt \right| = \left| \int_0^{2\pi/\omega_0} \sin\theta \sin(\omega_0 t) \exp\left\{ im\omega_0\left(t - \frac{a}{c}\cos\theta\cos(\omega_0 t)\right)\right\} dt \right| \times a\omega_0$$

The exponential can be expanded by the generating function for Bessel functions,

$$\exp\left\{ -im\omega_0 \frac{a}{c}\cos\theta\cos(\omega_0 t)\right\} = \sum_{n=-\infty}^{+\infty} (-i)^n J_n(m\beta\cos\theta)\, e^{in\omega_0 t} \qquad \text{with } \beta = a\omega_0/c.$$

Then $$\int_0^{2\pi/\omega_0} \sin(\omega_0 t) \exp\left\{ im\omega_0\left(t - \frac{\vec{n}\cdot\vec{r}(t)}{c}\right)\right\} dt = \sum_{n=-\infty}^{+\infty} (-i)^n J_n(m\beta\cos\theta) \int_0^{2\pi/\omega_0} \sin(\omega_0 t)\, e^{i(m-n)\omega_0 t}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} (-i)^n J_n(m\beta\cos\theta)\, \frac{1}{2i}\cdot\frac{2\pi}{\omega_0}\left(\delta_{m+1,n} - \delta_{m-1,n}\right)$$

$$= \frac{\pi}{i\omega_0}\left[(-i)^{m+1} J_{m+1}(m\beta\cos\theta) - (-i)^{m-1} J_{m-1}(m\beta\cos\theta)\right]$$

$$= -(-i)^m \frac{\pi}{\omega_0}\left(J_{m+1}(m\beta\cos\theta) + J_{m-1}(m\beta\cos\theta)\right)$$

$$= -(-i)^m \frac{\pi}{\omega_0}\frac{2m}{m\beta\cos\theta} J_m(m\beta\cos\theta) = -(-i)^m \frac{2\pi}{\omega_0}\frac{J_m(m\beta\cos\theta)}{\beta\cos\theta}$$

Put this into the formula in the result of Prob. 14.13, we will arrive at

$$\frac{dP_m}{d\Omega} = \frac{e^2\omega_0^4 m^2}{(2\pi c)^3}\cdot\frac{(2\pi)^2}{\omega_0^2}\frac{J_m(m\beta\cos\theta)^2}{\beta^2\cos^2\theta}\cdot a^2\omega_0^2\sin^2\theta = \frac{e^2 c\beta^2}{2\pi a^2} m^2 \tan^2\theta\, J_m(m\beta\cos\theta)^2$$

(b) In the non-relativistic limit, $\beta \ll 1$, and only $m=1$ term has significant contribution. Since $J_1(x) \sim x/2$, we then have $\frac{dP}{d\Omega} \simeq \frac{dP_1}{d\Omega} = \frac{e^2 c\beta^4}{8\pi a^2}\sin^2\theta$.

and $$P = \int \frac{dP}{d\Omega}\, d\Omega = \frac{1}{3}\frac{e^2 c\beta^4}{a^2} = \frac{2}{3}\frac{e^2}{c^3}\omega_0^4\, \bar{a^2},$$

where $\bar{a^2} = \frac{1}{2}a^2$ is the mean square amplitude of the oscillation.

This is also in agreement with Prob. 14.12 (a), by setting $\beta \to 0$ in the denominator and performing the angular integration