12.14 (a) From the Lagrangian, taking the derivative, we have

$$\frac{\partial \mathcal{L}}{\partial(\partial_\alpha A_\beta)} = -\frac{1}{4\pi}\partial^\alpha A^\beta, \qquad \frac{\partial \mathcal{L}}{\partial A_\beta} = -\frac{1}{c}J^\beta.$$

and the Euler-Lagrange equation $\partial_\alpha\left(\frac{\partial \mathcal{L}}{\partial(\partial_\alpha A_\beta)}\right) - \frac{\partial \mathcal{L}}{\partial A_\beta} = 0$ is $\partial_\alpha \partial^\alpha A^\beta = \frac{4\pi}{c}J^\beta$.

Comparing with the Maxwell equation $\partial_\alpha F^{\alpha\beta} = \frac{4\pi}{c}J^\beta$, we can see that the two equations are equivalent if $\partial_\alpha A^\alpha = 0$.

(b) For the EM field part, we have

$$F_{\alpha\beta}F^{\alpha\beta} = (\partial_\alpha A_\beta - \partial_\beta A_\alpha)(\partial^\alpha A^\beta - \partial^\beta A^\alpha) = \partial_\alpha A_\beta \partial^\alpha A^\beta - \partial_\beta A_\alpha \partial^\alpha A^\beta - \partial_\alpha A_\beta \partial^\beta A^\alpha + \partial_\beta A_\alpha \partial^\beta A^\alpha$$

$$= 2\left(\partial_\alpha A_\beta \partial^\alpha A_\beta - \partial_\alpha A_\beta \partial^\beta A^\alpha\right)$$

Under the condition $\partial_\alpha A^\alpha = 0$, the second term can be rewritten as

$$\partial_\alpha A_\beta \partial^\beta A^\alpha = \partial_\alpha\left(A_\beta \partial^\beta A^\alpha\right) - A_\beta \partial^\beta \partial_\alpha A^\alpha = \partial_\alpha\left(A_\beta \partial^\beta A^\alpha\right).$$

Then, the original Lagrangian, (12.85), can be written as

$$\mathcal{L} = -\frac{1}{8\pi}\left(\partial_\alpha A_\beta \partial^\alpha A^\beta - \partial_\alpha\left(A_\beta \partial^\beta A^\alpha\right)\right) - \frac{1}{c}J_\alpha A^\alpha,$$

which differs with the Lagrangian in the problem by a 4-divergence. This contributes to the action an extra boundary term, but has no effect on the equation of motion.