

11.5 In K and K' , we have

$$x_0 = \gamma(x'_0 + \beta x'_{11}) \quad , \quad \text{or} \quad ct = \gamma(ct' + \beta x'_{11}),$$

where $\beta = v/c$, $\gamma = (1 - \beta^2)^{-1/2}$. Then,

$$c \frac{dt}{dt'} = \gamma(c + \beta u'_{11}) \quad \text{or} \quad \frac{dt}{dt'} = \gamma \left(1 + \frac{\vec{u} \cdot \vec{v}}{c^2}\right).$$

Since $\vec{u}_{11} = \frac{\vec{u}'_{11} + \vec{v}}{1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2}}$, we can apply the time derivative to get acceleration \vec{a}_{11} as

$$\vec{a}_{11} = \frac{d\vec{u}_{11}}{dt} = \frac{d}{dt'} \left(\frac{\vec{u}'_{11} + \vec{v}}{1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2}} \right) \frac{dt'}{dt} = \left[\frac{\vec{a}'_{11}}{1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2}} - \frac{\vec{u}'_{11} + \vec{v}}{(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2})^2} \frac{\vec{a}'_{11} \cdot \vec{v}}{c^2} \right] \frac{1}{\gamma \left(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2}\right)}$$

Notice that $(\vec{u}'_{11} + \vec{v}) \vec{a}'_{11} \cdot \vec{v} = \vec{a}'_{11} (\vec{u}'_{11} + \vec{v}) \cdot \vec{v} = \vec{a}'_{11} (\vec{u}'_{11} \cdot \vec{v} + v^2)$, we have

$$\begin{aligned} \vec{a}_{11} &= \vec{a}'_{11} \left[\frac{1}{1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2}} - \frac{(\vec{u}'_{11} \cdot \vec{v} + v^2)/c^2}{(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2})^2} \right] \frac{1}{\gamma \left(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2}\right)} \\ &= \frac{1 - v^2/c^2}{\gamma \left(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2}\right)^3} \vec{a}'_{11} = \frac{(1 - v^2/c^2)^{3/2}}{(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2})^3} \vec{a}'_{11} \end{aligned}$$

Similarly, in the perpendicular direction,

$$\begin{aligned} \vec{a}_\perp &= \frac{d\vec{u}_\perp}{dt} = \frac{d}{dt'} \left(\frac{\vec{u}'_\perp}{\gamma \left(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2}\right)} \right) \frac{dt'}{dt} = \left[\frac{\vec{a}'_\perp}{1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2}} - \frac{\vec{u}'_\perp}{(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2})^2} \frac{\vec{a}'_{11} \cdot \vec{v}}{c^2} \right] \frac{1}{\gamma \left(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2}\right)} \\ &= \frac{(1 - v^2/c^2)}{(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2})^3} \left(\vec{a}'_\perp + \frac{1}{c^2} [\vec{a}'_\perp (\vec{u}'_{11} \cdot \vec{v}) - \vec{u}'_\perp (\vec{a}'_{11} \cdot \vec{v})] \right) \end{aligned}$$

The expression in the square brackets can be written as

$$\vec{a}'_\perp (\vec{u}'_{11} \cdot \vec{v}) - \vec{u}'_\perp (\vec{a}'_{11} \cdot \vec{v}) = (\vec{a}' (\vec{u}'_{11} \cdot \vec{v}) - \vec{u}' (\vec{a}'_{11} \cdot \vec{v})) - (\vec{a}'_{11} (\vec{u}'_{11} \cdot \vec{v}) - \vec{u}'_{11} (\vec{a}'_{11} \cdot \vec{v})) = \vec{v} \times (\vec{a}' \times \vec{u}')$$

Since the last term is in the parallel direction.

$$\text{Therefore,} \quad \vec{a}_\perp = \frac{(1 - v^2/c^2)}{(1 + \frac{\vec{u}'_{11} \cdot \vec{v}}{c^2})^3} \left[\vec{a}'_\perp + \frac{\vec{v}}{c^2} \times (\vec{a}' \times \vec{u}') \right].$$