6.1. (a) $$\Psi(\vec{x},t) = \int \frac{[f(\vec{x}',t')]_{ret}}{|\vec{x}-\vec{x}'|} d^3x' = \int \frac{\delta(x')\delta(y')\delta(t-|\vec{x}-\vec{x}'|/c)}{|\vec{x}-\vec{x}'|} dx'dy'dz'$$

$$= \int \frac{\delta(t-\sqrt{x^2+y^2+(z-z')^2}/c)}{[x^2+y^2+(z-z')^2]^{1/2}} dz'$$

Let $f(z') = t - \sqrt{\rho^2+(z-z')^2}/c$, with $\rho^2 = x^2+y^2$, the roots of $f(z')=0$ are $z'_\pm = z \pm \sqrt{c^2t^2-\rho^2}$, when $ct \geq \rho$

Also $f'(z') = \dfrac{z-z'}{c\sqrt{\rho^2+(z-z')^2}}$, and

$$f'(z'_\pm) = \frac{\mp\sqrt{c^2t^2-\rho^2}}{c^2t}$$

Then, $$\delta(f(z')) = \sum_\pm \delta(z'-z'_\pm)/|f'(z'_\pm)|$$

$$= \frac{ct}{\sqrt{c^2t^2-\rho^2}} \sum_\pm \delta(z'-z'_\pm)$$

Finally, $$\Psi(\vec{x},t) = \int_{-\infty}^{+\infty} \frac{c^2t}{\sqrt{c^2t^2-\rho^2}} \sum_\pm \frac{\delta(z'-z'_\pm)}{[\rho^2+(z-z')^2]^{1/2}} dz'$$

$$= 2\times \frac{c^2t}{\sqrt{c^2t^2-\rho^2}} \times \frac{1}{ct} = \frac{2c}{\sqrt{c^2t^2-\rho^2}}$$

Since the result is only non-zero when $ct > \rho$, we can express the final result as

$$\Psi(\vec{x},t) = \frac{2c\,\Theta(ct-\rho)}{\sqrt{c^2t^2-\rho^2}}$$

(b) $$\Psi(\vec{x},t) = \int \frac{[f(\vec{x}',t')]_{ret}}{|\vec{x}-\vec{x}'|} d^3x' = \int \frac{\delta(x')\delta(t-|\vec{x}-\vec{x}'|/c)}{|\vec{x}-\vec{x}'|} dx'dy'dz'$$

For $\vec{x} = (x,0,0)$,

$$\Psi(x,t) = \int \frac{\delta(t-\sqrt{x^2+y'^2+z'^2}/c)}{\sqrt{x^2+y'^2+z'^2}} dy'dz' = 2\pi \int_0^{+\infty} \frac{\delta(t-\sqrt{x^2+\rho'^2}/c)}{\sqrt{x^2+\rho'^2}} \rho' d\rho'$$

Let $f(\rho') = t - \sqrt{x^2+\rho'^2}/c$, then the root is given by

$$\rho'_0 = \sqrt{c^2t^2-x^2}, \quad \text{when } ct \geq |x|.$$

Also, $f'(\rho') = -\dfrac{\rho'}{c\sqrt{x^2+\rho'^2}}$, $\quad f'(\rho_0') = -\dfrac{\sqrt{c^2t^2-x^2}}{c^2t}$

Then

$$\Psi(x,t) = 2\pi \int_0^{+\infty} \frac{\delta(\rho'-\rho_0')}{\sqrt{x^2+\rho'^2}} \cdot \frac{1}{|f'(\rho_0')|} \cdot \rho' d\rho'$$

$$= 2\pi \cdot \frac{1}{\sqrt{x^2+\rho_0'^2}} \cdot \frac{c^2t}{\sqrt{c^2t^2-x^2}} \cdot \sqrt{c^2t^2-x^2}$$

$$= 2\pi c$$

Again, this is only non-zero when $ct \geq |x|$. so the final result becomes.

$$\Psi(x,t) = 2\pi c\, \Theta(ct-|x|).$$