14.5 (a) For the particle with energy E in a repulsive potential, the shortest distance it can get to the central field with head-on collision is the solution to $E = V(r_{min})$. From the Lamour's formula, the instantaneous power radiated is

$$P\,dt = \frac{2}{3}\frac{z^2e^2}{c^3}|\dot{\vec{v}}|^2\,dt.$$

For nonrelativistic particle, using Newton's second law, we know

$$m\dot{\vec{v}} = -\nabla V = -\frac{\partial V}{\partial r}\hat{e}_r,$$

for head-on collision. Also, $dt = dr/v$, and from energy conservation,

$$E = V(r_{min}) = \frac{1}{2}mv^2 + V(r), \quad \Rightarrow \quad v = \sqrt{\frac{2}{m}}\left(V(r_{min}) - V(r)\right)^{\frac{1}{2}} \qquad (*)$$

Put everything together, we have

$$P\,dt = \frac{2}{3}\frac{z^2e^2}{m^2c^3}\sqrt{\frac{m}{2}}\left|\frac{\partial V}{\partial r}\right|^2 \frac{dr}{\sqrt{V(r_{min})-V(r)}}.$$

Since the particle will be reflected, we finally get

$$\Delta W = \frac{4}{3}\frac{z^2e^2}{m^2c^3}\sqrt{\frac{m}{2}}\int_{r_{min}}^{+\infty}\left|\frac{\partial V}{\partial r}\right|^2 \frac{dr}{\sqrt{V(r_{min})-V(r)}}$$

(b) For $V(r) = \frac{zZe^2}{r}$, the integral can be evaluated as

$$\Delta W = \frac{4}{3}\frac{z^2e^2}{m^2c^3}\sqrt{\frac{m}{2}}(zZe^2)^{3/2}\int_{r_{min}}^{+\infty}\frac{1}{r^4}\frac{dr}{\sqrt{r_{min}^{-1} - r^{-1}}}$$

$$= \frac{4}{3}\frac{z^2e^2}{m^2c^3}\sqrt{\frac{m}{2}}(zZe^2)^{3/2}\, r_{min}^{1/2}\int_{r_{min}}^{+\infty} r^{-7/2}(r - r_{min})^{-1/2}\,dr$$

$$= \frac{4}{3}\frac{z^2e^2}{m^2c^3}\sqrt{\frac{m}{2}}(zZe^2)^{3/2}\, r_{min}^{1/2}\cdot\frac{16}{15\,r_{min}^3} \qquad \left(\int_a^{+\infty}\frac{x^{-7/2}}{\sqrt{x-a}}\,dx = \frac{16}{15a^3}\right)$$

Since $E = \frac{1}{2}mv_0^2 = V(r_{min}) = \frac{zZe^2}{r_{min}}$, $r_{min} = \frac{2zZe^2}{mv_0^2}$. Substitute r_{min} into the above formula, we will get

$$\Delta W = \frac{64}{45}\frac{z^2e^2}{m^2c^3}\sqrt{\frac{m}{2}}(zZe^2)^{3/2}\left(\frac{2zZe^2}{mv_0^2}\right)^{-5/2} = \frac{8}{45}\frac{zmv_0^5}{Zc^3}.$$

The energy loss compared to the total energy is $\frac{\Delta W}{E} \sim \left(\frac{v_0}{c}\right)^3 \ll 1$, which justifies the use of energy conservation (*) to obtain the velocity of the particle.