

8.2

(a) Given the peak value H_0 of the magnetic field, inside the wire, the azimuthal magnetic field is then given by $H = H_0 a/r$, and using the relation $\vec{B} = \pm \sqrt{\mu\epsilon} \hat{z} \times \vec{E}$, the radial electric field is $E = \sqrt{\frac{\mu}{\epsilon}} H = \sqrt{\frac{\mu}{\epsilon}} H_0 \frac{a}{r}$. Then, the transmitted power becomes $\frac{1}{2} EH$, and

$$P = \frac{1}{2} \int EH da = \frac{1}{2} \sqrt{\frac{\mu}{\epsilon}} H_0^2 \int_0^{2\pi} d\phi \int_a^b \frac{a^2}{r^2} r dr = \sqrt{\frac{\mu}{\epsilon}} \pi a^2 H_0^2 \log\left(\frac{b}{a}\right).$$

(b) Using Eq. (8.58),

$$-\frac{dP}{dz} = \frac{1}{2\sigma\delta} \oint |\vec{n} \times \vec{H}_t|^2 d\ell = \frac{1}{2\sigma\delta} \left(\int_0^{2\pi} H_0^2 a d\phi + \int_0^{2\pi} H_0^2 \frac{a^2}{b^2} b d\phi \right)$$

$$= \frac{\pi a^2}{\sigma\delta} H_0^2 \left(\frac{1}{a} + \frac{1}{b} \right),$$

and $\gamma = -\frac{1}{2P} \frac{dP}{dz} = \frac{1}{2\sigma\delta} \sqrt{\frac{\epsilon}{\mu}} \frac{\left(\frac{1}{a} + \frac{1}{b}\right)}{\log\left(\frac{b}{a}\right)}$.

(c) The voltage between the cylinder is $V = \int_a^b E dr = \sqrt{\frac{\mu}{\epsilon}} H_0 a \log\left(\frac{b}{a}\right)$, and the current in one of the cylinders is $I = \oint \vec{K} \cdot d\vec{\ell} = 2\pi a H_0$. Therefore, $Z_0 = \frac{V}{I} = \frac{1}{2\pi} \sqrt{\frac{\mu}{\epsilon}} \log\left(\frac{b}{a}\right)$.

(d) The Joule heating is given by $|dP/dz| = \frac{1}{2} I^2 R$, which leads to

$$R = \frac{2}{I^2} \left| \frac{dP}{dz} \right| = \frac{1}{2\pi\sigma\delta} \left(\frac{1}{a} + \frac{1}{b} \right).$$

For the inductance, the energy in the wire is

$$W_{\text{wire}} = \frac{\mu}{2} \int |\vec{H}|^2 da = \frac{\mu}{2} \cdot 2\pi \int_a^b H_0^2 \frac{a^2}{r^2} r dr = \pi a^2 \mu H_0^2 \log\left(\frac{b}{a}\right)$$

In the cylinders,

$$W_{\text{cyl}} = \frac{\mu_c \delta}{2} \left(2\pi a \int_0^{+\infty} H_0^2 e^{-2\xi/\delta} d\xi + 2\pi b \int_0^{+\infty} H_0^2 \frac{a^2}{b^2} e^{-2\xi/\delta} d\xi \right)$$

$$= \frac{\mu_c \delta}{2} \pi a^2 H_0^2 \left(\frac{1}{a} + \frac{1}{b} \right).$$

Then $L = \frac{2}{I^2} (W_{\text{wire}} + W_{\text{cyl}}) = \frac{\mu}{2\pi} \log\left(\frac{b}{a}\right) + \frac{\mu_c \delta}{4\pi} \left(\frac{1}{a} + \frac{1}{b} \right).$

Here, the integration is performed as if we have a surface current.