14.20 (a) Applying the result of Prob 14.19 (a), we have

$$\vec{n}\times\vec{\mu}(t) = \left(\mu_z \sin\theta\sin\phi,\ \mu_x\cos\theta - \mu_z \sin\theta\cos\phi,\ -\mu_x \sin\theta\sin\phi\right),$$

and

$$\int dt\ \vec{n}\times\vec{\mu}(t)\, e^{i\omega(t-\vec{n}\cdot\vec{r}(t)/c)} = \mu_0 \int \begin{pmatrix} \tanh(\nu t)\sin\theta\sin\phi \\ \mathrm{sech}(\nu t)\cos\theta - \tanh(\nu t)\sin\theta\cos\phi \\ -\mathrm{sech}(\nu t)\sin\theta\sin\phi \end{pmatrix} e^{i\omega t}\, dt$$

$$= \mu_0 \begin{pmatrix} \frac{i\pi}{\nu}\,\mathrm{cosech}\left(\frac{\omega\pi}{2\nu}\right)\sin\theta\sin\phi \\ \frac{\pi}{\nu}\,\mathrm{sech}\left(\frac{\omega\pi}{2\nu}\right)\cos\theta - \frac{i\pi}{\nu}\,\mathrm{cosech}\left(\frac{\omega\pi}{2\nu}\right)\sin\theta\cos\phi \\ -\frac{\pi}{\nu}\,\mathrm{sech}\left(\frac{\omega\pi}{2\nu}\right)\sin\theta\sin\phi \end{pmatrix},$$

where we have used the identities

$$\int_0^{+\infty} \frac{\cos ax}{\cosh\beta x}\,dx = \frac{\pi}{2\beta}\,\mathrm{sech}\left(\frac{a\pi}{2\beta}\right), \qquad \int_0^{+\infty} \sin ax\,\frac{\sinh(\beta x)}{\cosh(\gamma x)}\,dx = \frac{\pi}{\gamma}\,\frac{\sin\left(\frac{\beta\pi}{2\gamma}\right)\sinh\left(\frac{a\pi}{2\gamma}\right)}{\cosh\left(\frac{a\pi}{\gamma}\right)+\cos\left(\frac{\beta\pi}{\gamma}\right)}$$

and set $\beta=\gamma$ in the second identity. Then

$$\frac{d^2I}{d\omega\, d\Omega} = \frac{\omega^4}{4\pi^2 c^3}\left|\int dt\ \vec{n}\times\vec{\mu}(t)\, e^{i\omega(t-\vec{n}\cdot\vec{r}(t)/c)}\right|^2$$

$$= \frac{\omega^4}{4\pi^2 c^3}\,\mu_0^2\left(\frac{\pi}{\nu}\right)^2\left[\mathrm{cosech}^2\left(\frac{\omega\pi}{2\nu}\right)\sin^2\theta + \mathrm{sech}^2\left(\frac{\omega\pi}{2\nu}\right)\left(\cos^2\theta+\sin^2\theta\sin^2\phi\right)\right]$$

Perform the angular integration,

$$\frac{dI}{d\omega} = \frac{\omega^4}{4\pi^2 c^3}\,\mu_0^2\left(\frac{\pi}{\nu}\right)^2\left[\mathrm{cosech}^2\left(\frac{\omega\pi}{2\nu}\right)\frac{8\pi}{3} + \mathrm{sech}^2\left(\frac{\omega\pi}{2\nu}\right)\left(\frac{4\pi}{3}+\frac{4\pi}{3}\right)\right]$$

$$= \frac{2\pi\omega^4}{3\nu^2 c^3}\,\mu_0^2\left(\mathrm{cosech}^2\left(\frac{\omega\pi}{2\nu}\right) + \mathrm{sech}^2\left(\frac{\omega\pi}{2\nu}\right)\right).$$

Let $x = \omega\pi/2\nu$, and $\frac{dI}{dx} = \frac{dI}{d\omega}\frac{d\omega}{dx} = \frac{2\nu}{\pi}\frac{dI}{d\omega} = \frac{4\omega^4}{3\nu c^3}\,\mu_0^2\left(\mathrm{cosech}^2 x + \mathrm{sech}^2 x\right)$

Finally, $\frac{dI}{dx} = \frac{4}{3}\frac{\mu_0^2}{\nu c^3}\left(\frac{2\nu}{\pi}x\right)^4\left(\mathrm{cosech}^2 x + \mathrm{sech}^2 x\right) = \frac{4}{3}\left(\frac{\nu}{c}\right)^3\mu_0^2\left\{16\left(\frac{x}{\pi}\right)^4\left(\mathrm{cosech}^2 x + \mathrm{sech}^2 x\right)\right\}$

We can find the mean for $x$ as

$$\langle x\rangle = \int_0^{+\infty} x\frac{dI}{dx}\,dx \Big/ \int_0^{+\infty}\frac{dI}{dx}\,dx = \frac{465\,\zeta(5)}{32}\Big/\frac{\pi^4}{16} = 2.47498$$

(b) In Gauss unit, $Z_0 = 4\pi/c$. Then, the result of Prob. 9.7(a) reads

$$\frac{dP(t)}{d\Omega} = \frac{1}{4\pi c^3}\left|\ddot{\vec{\mu}}(t')\times\vec{n}\right|^2.$$

Here, $t' = t - r/c$. Since the magnetic moment is fixed at origin, $r$ is also fixed and no extra angular dependence is introduced due to retardation. The second order derivatives lead to

$$\ddot{\vec{\mu}}(t') = \mu_0 v^2\left(\operatorname{sech}(vt')\left(\tanh^2(vt') - \operatorname{sech}^2(vt')\right),\ 0,\ -2\tanh(vt')\operatorname{sech}^2(vt')\right).$$

and

$$\ddot{\vec{\mu}}(t')\times\vec{n} = \mu_0 v^2\Big(-2\tanh(vt')\operatorname{sech}^2(vt')\sin\theta\sin\phi\,\hat{i}$$
$$+\left(\operatorname{sech}(vt')\left(\tanh^2(vt') - \operatorname{sech}^2(vt')\right)\cos\theta + 2\tanh(vt')\operatorname{sech}^2(vt')\sin\theta\cos\phi\right)\hat{j}$$
$$-\operatorname{sech}(vt')\left(\tanh^2(vt') - \operatorname{sech}^2(vt')\right)\sin\theta\sin\phi\,\hat{k}\Big).$$

After squaring, 

$$\left|\ddot{\vec{\mu}}(t')\times\vec{n}\right|^2 = \mu_0^2 v^4\Big(4\tanh^2(vt')\operatorname{sech}^4(vt')\sin^2\theta$$
$$+\operatorname{sech}^2(vt')\left(\tanh^2(vt') - \operatorname{sech}^2(vt')\right)^2\left(\cos^2\theta + \sin^2\theta\sin^2\phi\right)\Big)$$

Performing the angular integration,

$$P(t) = \frac{2v^4}{3c^3}\mu_0^2\left(4\tanh^2(vt')\operatorname{sech}^4(vt') + \operatorname{sech}^2(vt')\left(\tanh^2(vt') - \operatorname{sech}^2(vt')\right)^2\right)$$

The total power radiated is

$$I = \int_{-\infty}^{+\infty} P(t)\,dt = \frac{2v^4}{3c^3}\mu_0^2\int_{-\infty}^{+\infty}\left(4\tanh^2(vt')\operatorname{sech}^4(vt') + \operatorname{sech}^2(vt')\left(\tanh^2(vt') - \operatorname{sech}^2(vt')\right)^2\right)dt',$$

Since $dt = dt'$. After the integration is done, we are left with

$$I = \frac{4}{3}\left(\frac{v}{c}\right)^3\mu_0^2,$$

which agrees with part (a)