12.19 (a) For β and γ as space indices, the tensor becomes $M^{\alpha ij} = \Theta^{\alpha i}x^j - \Theta^{\alpha j}x^i$.

The conservation condition $\partial_\alpha M^{\alpha ij} = 0$ is $\frac{1}{c}\frac{\partial}{\partial t}M^{0ij} + \partial_k M^{kij} = 0$, where k is also a space index. Integrate this condition, we will have $\frac{1}{c}\frac{d}{dt}\int(\Theta^{0i}x^j - \Theta^{0j}x^i)\,d^3x + \int \partial_k M^{kij} d^3x = 0$.

The second integral over the entire space is zero, as it is a boundary term while the EM field is localized. Then, we have $\frac{1}{c}\frac{d}{dt}\int(\Theta^{0i}x^j - \Theta^{0j}x^i)\,d^3x = 0$. However, $\Theta^{0i} = c g^i$ is just the momentum of the EM field. The condition becomes $\frac{d}{dt}\int \vec{x}\times\vec{g}_{em}\,d^3x = 0$, which is the statement of angular momentum conservation

(b) For $\beta = 0$, $\partial_\alpha M^{\alpha\beta\gamma} = \partial_\alpha(\Theta^{\alpha 0}x^\gamma - \Theta^{\alpha\gamma}x^0) = \partial_\alpha \Theta^{\alpha 0}x^\gamma$, where we have used the conservation law to drop the second term. Then, take γ as a space index j.

$$\partial_\alpha \Theta^{\alpha 0}x^j = \frac{1}{c}\frac{\partial}{\partial t}\Theta^{00}x^j + \partial_k \Theta^{k0}x^j = \frac{1}{c}\frac{\partial}{\partial t}\Theta^{00}x^j + \partial_k(\Theta^{k0}x^j) - \Theta^{j0} = 0$$

Integrate over the entire space, the second term will drop out, we have

$$\frac{1}{c}\frac{d}{dt}\int \Theta^{00}x^j\,d^3x = \int \Theta^{j0}\,d^3x$$

Since $\Theta^{00} = u$, $\Theta^{j0} = cp^j$, we can write the above equation as

$$\frac{1}{c}\frac{d}{dt}\int \vec{x}u\,d^3x = c\vec{P}_{em}.$$

Define $\int \vec{x}u\,d^3x = \vec{X}\int u\,d^3x = \vec{X}E_{em}$, then we have

$$\frac{d\vec{X}}{dt} = \frac{c^2\vec{P}_{em}}{E_{em}}$$