9.3 Using Eq. (3.36), the potential inside the sphere is given by

$\Phi_{in}(r,\theta) = V(t)\left[\frac{3}{2}\frac{r}{a}P_1(\cos\theta) - \cdots\right]$, and the induced charge inside the sphere is

$$\sigma_{in} = -\varepsilon_0 \left.\frac{\partial\Phi_{in}}{\partial n}\right|_{r=a} = \varepsilon_0 \left.\frac{\partial\Phi_{in}}{\partial r}\right|_{r=a} = \frac{3\varepsilon_0 V(t)}{2a}P_1(\cos\theta)$$ Similarly,

$\Phi_{out}(r,\theta) = V(t)\left[\frac{3}{2}\left(\frac{a}{r}\right)^2 P_1(\cos\theta) - \cdots\right]$, and the induced charge outside the sphere is

$$\sigma_{out} = -\varepsilon_0 \left.\frac{\partial\Phi_{out}}{\partial n}\right|_{r=a} = \left.\frac{3\varepsilon_0 V a^2}{r^3}\right|_{r=a} = \frac{3\varepsilon_0 V(t)}{a}P_1(\cos\theta).$$ Therefore, the total charge distribution

is $\sigma = \sigma_{in} + \sigma_{out} = \frac{9\varepsilon_0 V(t)}{2a}P_1(\cos\theta)$, and we can calulate the electric dipole as

$\vec{p} = \int \vec{x}\,\rho(\vec{x})\,d^3x$, or in component form,

$$\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = a^2 \int \begin{pmatrix} a\sin\theta\cos\phi \\ a\sin\theta\sin\phi \\ a\cos\theta \end{pmatrix} \frac{9\varepsilon_0 V(t)}{2a} P_1(\cos\theta)\,d\Omega = \begin{pmatrix} 0 \\ 0 \\ \frac{4\pi}{3} \end{pmatrix} \times \frac{9\varepsilon_0 V(t) a^2}{2}$$

$$= 6\pi a^2 \varepsilon_0 V(t)\,\hat{z} = \mathrm{Re}\left[6\pi a^2 \varepsilon_0 V \hat{z}\cdot e^{-i\omega t}\right]$$

At this stage, we can use the results from Eqs. (9.19), (9.23), and (9.24), which give,

with $\vec{p} = 6\pi a^2\varepsilon_0 V\hat{z}$,

$$\vec{H} = \frac{ck^2}{4\pi}(\vec{n}\times\vec{p})\frac{e^{ikr}}{r} = \frac{3\varepsilon_0 c}{2}(ka)^2\frac{e^{ikr}}{r}(\vec{n}\times\hat{z}) = -\frac{3\varepsilon_0 c}{2}(ka^2)\frac{e^{ikr}}{r}\sin\theta\,\hat{\phi}$$

$$\vec{E} = Z_0\vec{H}\times\vec{n} = -\frac{3\varepsilon_0 c Z_0}{2}(ka)^2\frac{e^{ikr}}{r}\sin\theta\,(\hat{\phi}\times\vec{n}) = -\frac{3\varepsilon_0 c Z_0}{2}(ka)^2\frac{e^{ikr}}{r}\sin\theta\,\hat{\theta}.$$

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0}{32\pi^2}k^4|\vec{p}|^2\sin^2\theta = \frac{1}{\mu_0\varepsilon_0}\sqrt{\frac{\mu_0}{\varepsilon_0}}\times\frac{k^2}{32\pi^2}\times 36\pi^2 a^4\varepsilon_0^2 V^2\times\sin^2\theta = \frac{9V^2}{8Z_0}(ka)^4\sin^2\theta$$

$$P = \frac{c^2 Z_0 k^4}{12\pi}|\vec{p}|^2 = \frac{1}{\mu_0\varepsilon_0}\sqrt{\frac{\mu_0}{\varepsilon_0}}\frac{k^4}{12\pi}\times 36\pi^2 a^4\varepsilon_0^2 V^2 = \frac{3\pi(ka)^4 V^2}{Z_0}$$