

11.30. Following (11.149), \vec{D} and \vec{H} should transform similarly,

$$\vec{D} = \gamma(\vec{D}' - \vec{\beta} \times \vec{H}') - \frac{\gamma^2}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{D}'), \quad \vec{H} = \gamma(\vec{H}' + \vec{\beta} \times \vec{D}') - \frac{\gamma^2}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{H}').$$

Using the constitutive relation $\vec{D}' = \epsilon \vec{E}'$ and $\vec{H}' = \vec{B}'/\mu$, we have

$$\vec{D} = \gamma\left(\epsilon \vec{E}' - \frac{1}{\mu} \vec{\beta} \times \vec{B}'\right) - \frac{\epsilon \gamma^2}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{E}'), \quad \vec{H} = \gamma\left(\frac{\vec{B}'}{\mu} + \epsilon \vec{\beta} \times \vec{E}'\right) - \frac{\gamma^2}{\mu(\gamma+1)} \vec{\beta}(\vec{\beta} \cdot \vec{B}').$$

Now using (11.149), for \vec{D} , we have

$$\begin{aligned} \vec{D} &= \gamma \left\{ \epsilon \left[\gamma(\vec{E} + \vec{\beta} \times \vec{B}) - \frac{\gamma^2}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{E}) \right] \right. \\ &\quad \left. - \frac{1}{\mu} \vec{\beta} \times \left[\gamma(\vec{B} - \vec{\beta} \times \vec{E}) - \frac{\gamma^2}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{B}) \right] \right\} \\ &\quad - \frac{\epsilon \gamma^2}{\gamma+1} \vec{\beta} \left(\vec{\beta} \cdot \left[\gamma(\vec{E} + \vec{\beta} \times \vec{B}) - \frac{\gamma^2}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{E}) \right] \right) \\ &= \gamma^2 \epsilon \vec{E} + \gamma^2 \epsilon \vec{\beta} \times \vec{B} - \frac{\epsilon \gamma^3}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{E}) \\ &\quad - \frac{\gamma^2}{\mu} \vec{\beta} \times \vec{B} + \frac{\gamma^2}{\mu} \vec{\beta} \times (\vec{\beta} \times \vec{E}) + \frac{\gamma^3}{\mu(\gamma+1)} (\vec{\beta} \times \vec{\beta})(\vec{\beta} \cdot \vec{B}) \\ &\quad - \frac{\epsilon \gamma^3}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{E}) - \frac{\epsilon \gamma^3}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot (\vec{\beta} \times \vec{B})) + \frac{\epsilon \gamma^4}{(\gamma+1)^2} \beta^2 \vec{\beta}(\vec{\beta} \cdot \vec{E}) \\ &= \gamma^2 \epsilon \vec{E} + \gamma^2 \left(\epsilon - \frac{1}{\mu} \right) (\vec{\beta} \times \vec{B}) + \frac{\gamma^2}{\mu} \vec{\beta} \times (\vec{\beta} \times \vec{E}) - \frac{2\epsilon \gamma^3}{\gamma+1} \vec{\beta}(\vec{\beta} \cdot \vec{E}) + \frac{\epsilon \gamma^4}{(\gamma+1)^2} \beta^2 \vec{\beta}(\vec{\beta} \cdot \vec{E}) \end{aligned}$$

Since $\vec{\beta} \times \vec{\beta} = 0$ and $\vec{\beta} \cdot (\vec{\beta} \times \vec{B}) = 0$. Notice that

$$\frac{\gamma^4}{(\gamma+1)^2} \beta^2 - \frac{2\gamma^3}{\gamma+1} = \frac{\gamma^2(\gamma^2-1) - 2\gamma^3}{\gamma+1} = -\gamma^2,$$

$$\begin{aligned} \text{then } \vec{D} &= \gamma^2 \epsilon \vec{E} + \gamma^2 \left(\epsilon - \frac{1}{\mu} \right) (\vec{\beta} \times \vec{B}) + \frac{\gamma^2}{\mu} \vec{\beta} \times (\vec{\beta} \times \vec{E}) - \epsilon \gamma^2 \vec{\beta}(\vec{\beta} \cdot \vec{E}) \\ &= \gamma^2 \epsilon \vec{E} + \gamma^2 \left(\epsilon - \frac{1}{\mu} \right) \vec{\beta} \times \vec{B} + \frac{\gamma^2}{\mu} \vec{\beta} \times (\vec{\beta} \times \vec{E}) - \epsilon \gamma^2 [\vec{\beta} \times (\vec{\beta} \times \vec{E}) + \beta^2 \vec{E}] \\ &= \epsilon \vec{E} \cdot \gamma^2 (1 - \beta^2) + \gamma^2 \left(\epsilon - \frac{1}{\mu} \right) \vec{\beta} \times \vec{B} - \gamma^2 \left(\epsilon - \frac{1}{\mu} \right) [\vec{\beta} \times (\vec{\beta} \times \vec{E})] \\ &= \epsilon \vec{E} + \gamma^2 \left(\epsilon - \frac{1}{\mu} \right) [\beta^2 \vec{E}_\perp + \vec{\beta} \times \vec{B}], \end{aligned}$$

where we have used the fact that $\gamma^2 = (1 - \beta^2)^{-1}$ and $\vec{\beta} \times (\vec{\beta} \times \vec{E}) = \beta^2 \vec{E}_\perp$.

Following the same manipulation, we will have the results for \vec{H} .