12.17 (a) Using the results of Section 12.2, the radius of curvature is

$$c p_\perp = eB\rho, \quad \text{or} \quad c \cdot \gamma m v \cos\alpha = eB\rho$$

Therefore, $\rho = \frac{\gamma mc}{eB} v\cos\alpha = \frac{v\cos\alpha}{\omega_B}$. We can follow the same argument as in Section 14.4, to show that the pulse length of the radiation burst is $L = \frac{\rho}{2\gamma^3}$. However, for a fixed observer, the radiation has a projection in the observer's plane, and the actual length is $L\cos\alpha$ $= \frac{v\cos^2\alpha}{2\gamma^3\omega_B}$. For relativistic motion, $v \sim c$, and the fundamental frequency is $\omega_0 = \frac{\omega_B}{\cos^2\alpha}$.

If we use the definition of the critical frequency as $\omega_c = \frac{3}{2}\gamma^3 \frac{c}{\rho} = \frac{3}{2}\gamma^3 \omega_B/\cos\alpha$, which would be different from Jackson's result. Don't know why?

(b) From Eq (14.79)

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c}\left(\frac{\omega\rho}{c}\right)^2\left(\frac{1}{\gamma^2}+\psi^2\right)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K_{1/3}^2(\xi)\right]$$

$$= \frac{3e^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2\gamma^6\left(\frac{1}{\gamma^2}+\psi^2\right)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K_{1/3}^2(\xi)\right]$$

$$= \frac{3e^2\gamma^2}{4\pi^2 c}\left(\frac{\omega}{\omega_c}\right)^2\left(1+\gamma^2\psi^2\right)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K_{1/3}^2(\xi)\right]$$

where ψ is measured from the pitch angle, $\psi = \theta - \alpha$, and $\xi = \frac{\omega}{2\omega_c}(1+\gamma^2\psi^2)^{3/2}$.

The helix motion has a period of $T = 2\pi/\omega_0$, then

$$\frac{d^2P}{d\omega d\Omega} = \frac{3e^2\gamma^2}{8\pi^3 c}\left(\frac{\omega}{\omega_c}\right)^2\frac{\omega_B}{\cos^2\alpha}(1+\gamma^2\psi^2)^2\left[K_{2/3}^2(\xi) + \frac{\gamma^2\psi^2}{1+\gamma^2\psi^2}K_{1/3}^2(\xi)\right].$$