

14.18 (a) Let $\omega_c = \frac{3}{2} \gamma^3 \frac{c}{\rho}$. Eq. (14.79) reads

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2}{3\pi^2 c} \frac{q\gamma^6}{4} \left(\frac{\omega}{\omega_c}\right)^2 \frac{1}{\gamma^4 (1+\gamma^2 \theta^2)^2} \left[K_{1/3}^2(\xi) + \frac{\gamma^2 \theta^2}{1+\gamma^2 \theta^2} K_{2/3}^2(\xi) \right],$$

where $\xi = \frac{\omega^2}{3c} \left(\frac{1}{\gamma^2} + \theta^2 \right)^{3/2} = \frac{\omega}{\omega_c} (1+\gamma^2 \theta^2)^{3/2}$. We can write the above as

$$\frac{d^2 I}{d\omega d\Omega} = \frac{\sqrt{3} e^2 \gamma}{c} \frac{\sqrt{3} \gamma}{4\pi^2} \left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2 \theta^2)^2 \left[K_{1/3}^2(\xi) + \frac{\gamma^2 \theta^2}{1+\gamma^2 \theta^2} K_{2/3}^2(\xi) \right].$$

Comparing with Eq. (14.91), we know that

$$\int \frac{\sqrt{3} \gamma}{4\pi^2} \left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2 \theta^2)^2 \left[K_{1/3}^2(\xi) + \frac{\gamma^2 \theta^2}{1+\gamma^2 \theta^2} K_{2/3}^2(\xi) \right] d\Omega = \left(\frac{\omega}{\omega_c}\right)^2 \int_{\omega/\omega_c}^{+\infty} K_{5/3}(y) dy.$$

We can apply the above result to prob. 14.17 (b), where

$$\frac{d^2 P}{d\omega d\Omega} = \frac{\sqrt{3} e^2 \gamma}{2\pi c} \frac{\omega_B}{\cos^2 \alpha} \frac{\sqrt{3} \gamma}{4\pi^2} \left(\frac{\omega}{\omega_c}\right)^2 (1+\gamma^2 \psi^2)^2 \left[K_{1/3}^2(\xi) + \frac{\gamma^2 \psi^2}{1+\gamma^2 \psi^2} K_{2/3}^2(\xi) \right], \text{ with } \xi = \frac{\omega}{\omega_c} (1+\gamma^2 \psi^2)^{3/2}.$$

To perform the angular integration, notice that most contribution comes from the region $\psi=0$, which corresponds

to $\theta = \pi/2 - \alpha$. Therefore, $d\Omega = \sin\theta d\theta d\phi = \cos\alpha d\theta d\phi$. Therefore, the integral for the helix motion

should contain an extra $\cos\alpha$ contribution. Finally, we should have

$$\frac{dP}{d\omega} = \frac{\sqrt{3} e^2 \gamma}{2\pi c} \frac{\omega_B}{\cos^2 \alpha} \cdot \cos\alpha \cdot \frac{\omega}{\omega_c} \int_{\omega/\omega_c}^{+\infty} K_{5/3}(y) dy = \frac{\sqrt{3} e^2 \gamma \omega_B}{2\pi c \cos\alpha} G\left(\frac{\omega}{\omega_c}\right).$$

(b) Using the identity $\int_0^{+\infty} y^2 K_{5/3}(y) dy = \frac{16\pi}{9\sqrt{3}}$, we can find the integral as

$$\frac{dP}{d\omega} = \frac{\sqrt{3} e^2 \gamma \omega_B}{2\pi c \cos\alpha} \cdot \omega_c \int_0^{+\infty} G\left(\frac{\omega}{\omega_c}\right) d\left(\frac{\omega}{\omega_c}\right) \quad \text{and}$$

$$\int_0^{+\infty} G(x) dx = \int_0^{+\infty} dx \times \int_x^{+\infty} dy K_{5/3}(y) = \int_0^{+\infty} dy \int_0^y dx \times K_{5/3}(y) = \frac{1}{2} \int_0^{+\infty} y^2 K_{5/3}(y) dy = \frac{8\pi}{9\sqrt{3}}.$$

$$\text{Thus } \frac{dP}{d\omega} = \frac{\sqrt{3} e^2 \gamma \omega_B}{2\pi c \cos\alpha} \cdot \frac{3}{2} \gamma^3 \omega_B \cos\alpha \cdot \frac{8\pi}{9\sqrt{3}} = \frac{2 e^2 \omega_B^2 \gamma^4}{3c}.$$

More mathematical details can be found in *Synchrotron Radiation* by Wiedemann.