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SYNCHROTRON RADIATION FORMULAE

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In a recent Letter, Epstein and Feldman (1967) pointed out that the usual formulae for synchrotron radiation, as derived by Westfold (1959) and by Le Roux (1961), do not give the correct value for the radiated power received from a highly relativistic electron spiraling around a magnetic field with pitch angle $\psi \neq \pi/2$. They give revised formulae, which differ from the generally accepted ones by a factor of $\sin^2 \psi$. Up to this point, I would agree that they have made a valid statement. Unfortunately, they go on to conclude that most of the theoretical work which has been done with the conventional formulae requires reinterpretation. I believe the latter idea is based on a misunderstanding and offer the following remarks in the hope of forestalling a spate of quite erroneous "corrections."

As I am somewhat more familiar with Le Roux's treatment than with Westfold's, I shall use the former as a model for "conventional" treatments. Le Roux calculates the energy received (per unit frequency interval, per unit area, etc.) by a distant observer for each complete orbit of the electron around the magnetic field. He then obtains a mean power by dividing by the orbital period of the electron. Thus he allows correctly for Doppler shifts and aberration effects, by working in the observer's frame throughout, and his results represent the mean power *emitted* by the electron in a particular direction, frequency range, etc. (P_{emitted}). The energy emitted in a time interval t is received in a shorter time interval $t \sin^2 \psi$, as is evident from Figure 1; thus energy is received at a rate

$$P_{\text{received}} = P_{\text{emitted}}/\sin^2 \psi \,.$$

If we now use one of Le Roux's formulae for P_{emitted}, we obtain just the corresponding Epstein and Feldman result for P_{received}. There is no disagreement between the formulae so long as each is correctly interpreted.

Now suppose we wish to calculate the radiation from some electrons trapped in a radio source. For simplicity, consider just one electron trapped between two magnetic mirrors in an otherwise uniform magnetic field (Fig. 2). (It will be assumed in this and the following examples that the piece of "source" considered is so far from the observer that it subtends a negligible angle; also, the electron velocity is approximated by the speed of light c in "$c \cos \psi$," the component of velocity parallel to the magnetic field.) If the magnetic bottle has length L, the electron takes time $L\,(c \cos \psi)^{-1}$ to go down the bottle from A to B and the same time to go back from B to A again. Radiation will be received by an observer in direction D at a rate P_{received} given by the Epstein-Feldman formulae, but only for a time

$$L(c \cos \psi)^{-1} - (L \cos \psi)/c = L(c \cos \psi)^{-1} \sin^2 \psi \,,$$

that is, $\frac{1}{2} \sin^2 \psi$ of the total time. Thus the *mean* power received by the observer is just the power given by the "conventional" formula multiplied by the mean number of electrons in the source with the right pitch angle ($\frac{1}{2}$ in this case!).

Nothing new is involved in the generalization to many electrons in more complex trapping configurations; the general result is evidently that the mean power from a collection of trapped electrons is correctly given by the "conventional" power per electron times the mean number of electrons in the source with the appropriate pitch angle.

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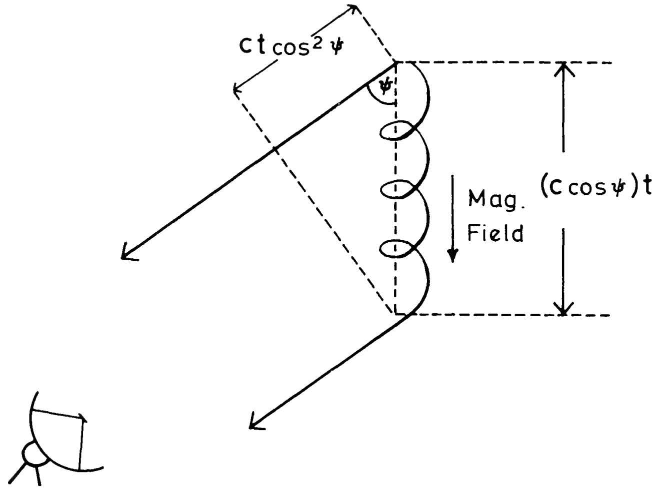

$$P_{received} = \frac{P_{emitted} \cdot t}{t - (ct\cos^2\psi)/c} = \frac{P_{emitted}}{\sin^2\psi}$$

FIG. 1.—Geometry of synchrotron emission from spiraling orbits

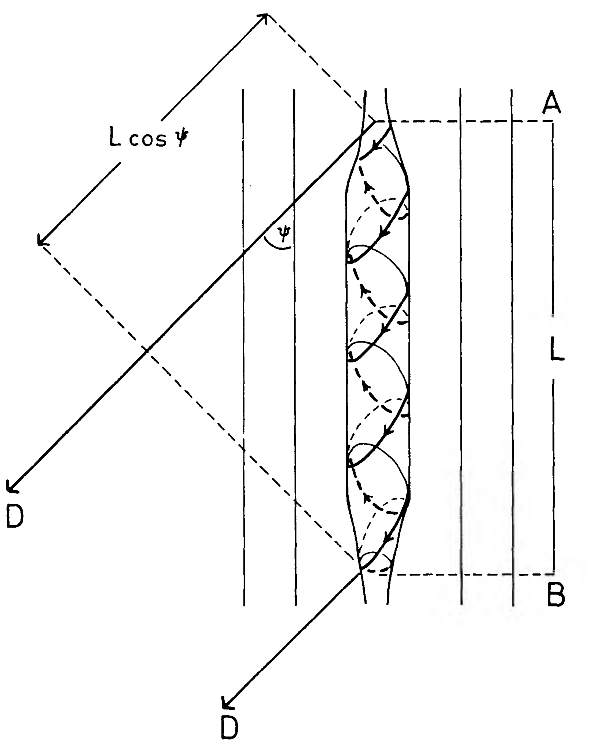

FIG. 2.—Emission of synchrotron radiation from trapped electrons

We might expect, however, that we should have to use the Epstein-Feldman formulae when the radiation comes from an untrapped beam of electrons. To define the situation quite clearly, consider a stream of electrons, all with the same pitch angle ψ, passing across a window of width L (Fig. 3). Let the mean number of electrons in the region behind the window be N, so that $Nc \cos \psi/L$ electrons enter (and leave) the region per unit time. The power from each electron is given by the Epstein-Feldman formulae and is received for a time interval

$$L(c \cos \psi)^{-1} - L \cos \psi/c = L(c \cos \psi)^{-1} \sin^2 \psi \,,$$

as in the previous example. Thus the mean power received is

$$\frac{Nc \cos \psi}{L} \frac{L \sin^2 \psi}{c \cos \psi} \times \text{power from one electron (Epstein-Feldman formula)}$$

$$= N \sin^2 \psi \times \text{power from one electron (Epstein-Feldman formula)}$$

$$= N \times \text{power from one electron ("conventional" formula).}$$

After all, the conventional formula again gives the right answer for the mean power. It might be thought that the window is an artificial restriction, but this is not so. A stream

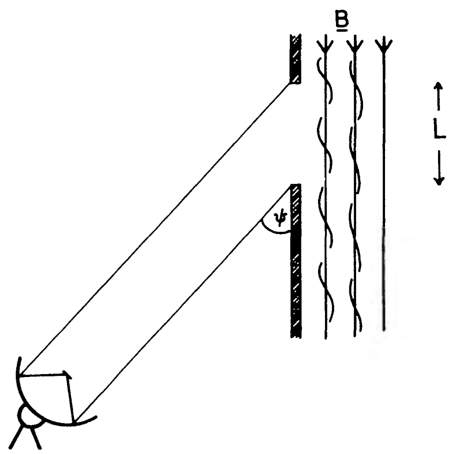

FIG. 3.—Synchrotron emission through a window from electrons with the same pitch angle ψ

of electrons gushing out of a radio source along some jet becomes ineffective beyond a certain distance because, for example, it reaches a region of weaker magnetic field. Woltjer's (1966) model of a quasi-stellar source consists of many such electron streams flowing out radially from the source; it seems the case par excellence for the application of the new formulae; however, the above calculation shows that it is still the conventional formula that gives the correct mean power.

The situation shown in Figure 3 is so close to the situation of a single free electron that one may well ask why different formulae should apply. The correct answer is that there is no significant difference between the two situations; the only difference arises in the ways used to count electrons. It is instructive to consider the situation of Figure 3, but with the "beam" of electrons replaced by a perfectly regular sequence of electrons following each other at distance intervals L. There is then always exactly one electron in the visible region, behind the window. However, radiation is received by the observer for only a fraction $\sin^2 \psi$ of the time (Fig. 4). It is in this question of counting, not in any peculiarity of the source configuration, that the distinction between the two formulae lies.

The conclusion may be stated as follows. The power received from a radio source may be computed correctly in either of two ways: (i) multiply (power per electron, according to conventional formula) by (mean number of electrons *in source* with appropriate pitch angle to radiate toward observer); (ii) multiply (power per electron, according to Epstein and Feldman) by (mean number of electrons contributing to the *radiation observed at a particular instant*).

Theoretical astronomers almost invariably deal with the numbers of electrons present in a source at a given time; thus I conclude that their use of the "conventional" formulae for synchrotron radiation was correct and that no revision of radio-source models will

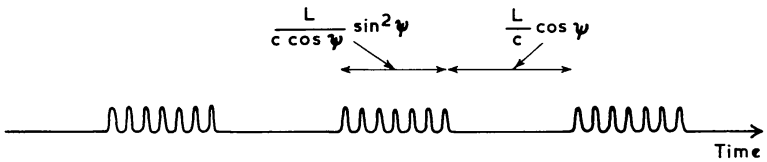

FIG. 4.—Intervals during which radiation is emitted and received

be required in this respect—however wrong they may be in all others! If a source containing trapped electrons is in motion as a whole, then the Doppler and aberration corrections appropriate to the source's velocity must, of course, be applied, as for any other radiation mechanism.

I understand from Professor Ginzburg that the paper (Ginzburg, Sazonov, and Syrovatskii 1968) also cited by Epstein and Feldman reaches conclusions similar to those presented here.

It is a pleasure to thank J. D. Scargle, who introduced me to these questions and discussed them with me about February 1966.

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