5.27 From 5.26, the magnetic field is given by

$$\vec{H} = \frac{I}{2\pi\rho}\hat{\phi}, \qquad b < \rho < a$$

$$\vec{H} = \frac{I\rho}{2\pi b^2}\hat{\phi}, \qquad \rho < b$$

$$\vec{H} = 0, \qquad \rho > a$$

Then, the total magnetic energy per unit length is

$$W = \frac{\mu_0}{2}\int |\vec{H}|^2 d^2x = \pi\mu_0\left(\int_0^b \frac{I^2\rho^2}{4\pi^2 b^4}\rho d\rho + \int_b^a \frac{I^2}{4\pi^2\rho^2}\rho d\rho\right)$$

$$= \frac{\mu_0 I^2}{16\pi} + \frac{\mu_0 I^2}{4\pi}\log\left(\frac{a}{b}\right) = \frac{\mu_0 I^2}{4\pi}\left(\log\left(\frac{a}{b}\right) + \frac{1}{4}\right)$$

By definition, $W = \frac{1}{2}LI^2$ Therefore, $L = \frac{\mu_0}{2\pi}\left(\log\left(\frac{a}{b}\right) + \frac{1}{4}\right)$.

If the inner conductor is a thin hollow tube, then only the magnetic field between the tubes is non-zero. In this case.

$$W = \pi\mu_0\int_b^a \frac{I^2}{4\pi^2\rho^2}\rho d\rho = \frac{\mu_0 I^2}{4\pi}\log\left(\frac{a}{b}\right)$$

and $\quad L = \frac{\mu_0}{2\pi}\log\left(\frac{a}{b}\right)$