2.28 Solution: Similar to Problem 1.10, the potential at the center of the polyhedron can be expressed as

$$\Phi(\mathbf{x}) = -\frac{1}{4\pi}\oint_S \Phi(\mathbf{x}')\frac{\partial}{\partial n'}\left(\frac{1}{R}\right) da' = -\frac{1}{4\pi}\oint_S \Phi(\mathbf{x}')\mathbf{n}'\cdot\nabla'\left(\frac{1}{R}\right) da',$$

where $R = |\mathbf{x} - \mathbf{x}'|$ is the distance from the center to a point on the surface of the polyhedron, and ∇' is the differential operator applied at $\mathbf{x}'$.

We can consider each side separately, *i.e.*, fix only side i at a non-zero constant potential V_i while keeping other sides grounded. After this, the original problem can be solved by simple superposition. Now, the contribution from this side can be written as

$$\Phi_i(\mathbf{x}) = -\frac{V_i}{4\pi}\oint_S \mathbf{n}'\cdot\nabla'\left(\frac{1}{R}\right) da'.$$

Notice that

$$\mathbf{n}'\cdot\nabla'\left(\frac{1}{R}\right) da' = -d\Omega,$$

then

$$\Phi_i(\mathbf{x}) = \frac{\Omega_i}{4\pi}V_i.$$

However, for a regular polyhedron, each side equally subtends a solid angle of $4\pi/n$, and

$$\Phi_i(\mathbf{x}) = \frac{V_i}{n}.$$

Finally, taking into account of the contribution from all sides,

$$\Phi(\mathbf{x}) = \frac{1}{n}\sum_{i=1}^{n} V_i,$$

which is the average of the potentials on the sides of the regular polyhedron.