

13.1 (a) Since $T = Q^2/2m = \frac{2p^2}{m} \sin^2(\frac{\theta}{2}) = \frac{2p^2}{m} \cdot \frac{1}{1 + \cot^2(\theta/2)} = \frac{2p^2}{m} \cdot \frac{1}{1 + (p v b / z e^2)^2}$

$$= \frac{2p^2}{m} \cdot \frac{z^2 e^4 / p^2 v^2}{b^2 + (z e^2 / p v)^2}$$

define $b_{min}^{(e)} = z e^2 / p v$, we have

$$T(b) = \frac{2 z^2 e^4}{m v^2} \frac{1}{b^2 + (b_{min}^{(e)})^2}$$

For $b=0$, $T(0) = 2p^2/m = 2\gamma^2 \beta^2 m c^2$, since $p = \gamma m \beta c$.

(b) Assuming the heavy particle maintains a constant impact parameter, its transverse electric field is given by $E_z = \frac{z e \gamma b}{(b^2 + \gamma^2 v^2 t^2)^{3/2}}$. Then, the total transverse momentum

transferred to the electron is

$$\Delta p = \int_{-\infty}^{+\infty} e E_z dt = 2 z e^2 \gamma b \int_0^{+\infty} \frac{dt}{(b^2 + \gamma^2 v^2 t^2)^{3/2}} = \frac{2 z e^2 \gamma b}{\gamma^3 v^3} \cdot \frac{\gamma^2 v^2}{b^2} \cdot \frac{t}{\sqrt{t^2 + (b/\gamma v)^2}} \Big|_0^{+\infty}$$

$$= \frac{2 z e^2}{b v}$$

and the energy transfer is

$$T = (\Delta p)^2 / 2m = \frac{2 z^2 e^4}{m v^2 b^2}$$

In the result of part (a), for large impact parameter, $b \gg b_{min}^{(e)}$, we will asymptotically get the result here.