14.12 (a) Given the harmonic motion of the charge, $z(t') = a\cos(\omega_0 t')$, its velocity and acceleration are $\vec{v}(t') = \dot{\vec{z}}(t') = -a\omega_0 \sin(\omega_0 t')\,\hat{z}$, $\dot{\vec{v}}(t') = \ddot{\vec{z}}(t') = -a\omega_0^2\cos(\omega_0 t')\,\hat{z}$. In this case, the velocity and the acceleration are in the same direction, we can then apply Eq. (14.38) as

$$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c^3}\,\frac{|\vec{n}\times(\vec{n}\times\dot{\vec{v}})|^2}{(1-\vec{v}\cdot\vec{n}/c)^5}.$$

Since $\vec{v}\cdot\vec{n} = -a\omega_0\cos\theta\sin(\omega_0 t')$, $|\vec{n}\times(\vec{n}\times\dot{\vec{v}})| = a\omega_0^2\sin\theta\cos(\omega t')$, the final result becomes

$$\frac{dP(t')}{d\Omega} = \frac{e^2}{4\pi c^3}\,\frac{(a\omega_0^2\sin\theta\cos(\omega_0 t'))^2}{\left(1+\frac{a\omega_0}{c}\cos\theta\sin(\omega_0 t')\right)^5} = \frac{e^2 c\beta^4}{4\pi a^2}\,\frac{\sin^2\theta\cos^2(\omega_0 t')}{(1+\beta\cos\theta\sin(\omega_0 t'))^5}$$

(b) The average power per unit solid angle is

$$\frac{dP}{d\Omega} = \frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}\frac{dP(t')}{d\Omega}\,dt' = \frac{e^2 c\beta^4}{4\pi a^2}\,\frac{\omega_0}{2\pi}\int_0^{2\pi/\omega_0}\frac{\sin^2\theta\cos^2(\omega_0 t')}{(1+\beta\cos\theta\sin(\omega_0 t'))^5}\,dt'$$

The integral can be transformed to

$$\sin^2\theta\int_{-\pi}^{\pi}\frac{\cos^2 t'}{(1+\beta\cos\theta\sin t')^5}\,dt' \overset{t'=2\arctan u}{=\!=\!=} \sin^2\theta\int_{-\infty}^{+\infty}\frac{\left(\frac{1-u^2}{1+u^2}\right)^2}{\left(1+\beta\cos\theta\frac{2u}{1+u^2}\right)^5}\,\frac{2du}{1+u^2}$$

$$= \sin^2\theta\int_{-\infty}^{+\infty}\frac{2(1-t^4)^2}{(t^2+2t\beta\cos\theta+1)^5}\,du = \frac{\pi(4+\beta^2\cos^2\theta)}{4(1-\beta^2\cos^2\theta)^{7/2}}\sin^2\theta,$$

where the last step is obtained with the help of Mathematica. Then,

$$\frac{dP}{d\Omega} = \frac{e^2 c\beta^4}{32\pi a^2}\,\frac{4+\beta^2\cos^2\theta}{(1-\beta^2\cos^2\theta)^{7/2}}\sin^2\theta$$