14.19 (a) Using Eq. (14.70), the radiation intensity is given by

$$\frac{d^2I}{d\omega d\Omega} = \frac{\omega^2}{4\pi^2 c}\left|\int dt \int d^3x\, \vec{n}\times\left[\vec{n}\times\left(\nabla\times\left(\vec{\mu}(t)\delta(\vec{x}-\vec{r}(t))\right)\right)\right] e^{i\omega(t-\vec{n}\cdot\vec{x}/c)}\right|^2$$

Notice that $\nabla\times\left(\vec{\mu}(t)\delta(\vec{x}-\vec{r}(t))\, e^{i\omega(t-\vec{n}\cdot\vec{x}/c)}\right) = \left[\nabla\times\left(\vec{\mu}(t)\delta(\vec{x}-\vec{r}(t))\right)\right] e^{i\omega(t-\vec{n}\cdot\vec{x}/c)}$

$$+\left(\nabla e^{i\omega(t-\vec{n}\cdot\vec{x}/c)}\right)\times\vec{\mu}(t)\delta(\vec{x}-\vec{r}(t)),$$

and integrate by parts, we have

$$\int d^3x\, \vec{n}\times\left[\vec{n}\times\left(\nabla\times(\vec{\mu}(t)\delta(\vec{x}-\vec{r}(t)))\right)\right] e^{i\omega(t-\vec{n}\cdot\vec{x}/c)} = -\int d^3x\, \vec{n}\times\left[\vec{n}\times\left(\nabla e^{i\omega(t-\vec{n}\cdot\vec{x}/c)}\times\vec{\mu}(t)\delta(\vec{x}-\vec{r}(t))\right)\right]$$

Since $\nabla e^{i\omega(t-\vec{n}\cdot\vec{x}/c)} = -\frac{i\omega}{c}\nabla(\vec{n}\cdot\vec{x})\, e^{i\omega(t-\vec{n}\cdot\vec{x}/c)} = -\frac{i\omega}{c}\vec{n}\, e^{i\omega(t-\vec{n}\cdot\vec{x}/c)}$, we will get

performing the ~~spatial~~ integration with respect to Dirac delta function, we will get

$$\int d^3x\, \vec{n}\times\left[\vec{n}\times\left(\nabla\times(\vec{\mu}(t)\delta(\vec{x}-\vec{r}(t)))\right)\right] e^{i\omega(t-\vec{n}\cdot\vec{x}/c)} = \frac{i\omega}{c}\vec{n}\times\left[\vec{n}\times\left(\vec{n}\times\vec{\mu}(t)\right)\right] e^{i\omega(t-\vec{n}\cdot\vec{r}(t)/c)}$$

$$= -\frac{i\omega}{c}\left(\vec{n}\times\vec{\mu}(t)\right) e^{i\omega(t-\vec{n}\cdot\vec{r}(t)/c)}$$

Therefore, $\displaystyle\frac{d^2I}{d\omega d\Omega} = \frac{\omega^4}{4\pi^2 c^3}\left|\int dt\left(\vec{n}\times\vec{\mu}(t)\right) e^{i\omega(t-\vec{n}\cdot\vec{r}(t)/c)}\right|^2.$

(b) For $\vec{\mu}(t) = \mu_0\left(\sin(\omega_0 t),\, 0,\, \cos(\omega_0 t)\right)$,

$$\vec{n}\times\vec{\mu}(t) = \mu_0\left(\cos(\omega_0 t)\sin\theta\sin\phi,\ \sin(\omega_0 t)\cos\theta - \cos(\omega_0 t)\sin\theta\cos\phi,\ -\sin(\omega_0 t)\sin\theta\sin\phi\right)$$

The magnetic moment is also fixed at origin, $\vec{r}(t)\equiv 0$. Then,

$$\int dt\left(\vec{n}\times\vec{\mu}(t)\right) e^{i\omega(t-\vec{n}\cdot\vec{r}(t)/c)} = \mu_0\int_{-T/2}^{T/2}\begin{pmatrix}\cos(\omega_0 t)\sin\theta\sin\phi\\ \sin(\omega_0 t)\cos\theta - \cos(\omega_0 t)\sin\theta\cos\phi\\ -\sin(\omega_0 t)\sin\theta\sin\phi\end{pmatrix} e^{i\omega t}\,dt$$

For $T\to\infty$, by Riemann-Lebesgue theorem, only when $\omega=\omega_0$, the integral does not vanish. We can only perform the integral in one period, to obtain

$$\mu_0\frac{\omega_0 T}{2\pi}\cdot\frac{2\pi}{\omega_0}\cdot\frac{1}{2}\begin{pmatrix}\sin\theta\sin\phi\\ -i\cos\theta-\sin\theta\cos\phi\\ i\sin\theta\sin\phi\end{pmatrix} = \mu_0\frac{T}{2}\begin{pmatrix}\sin\theta\sin\phi\\ -i\cos\theta-\sin\theta\cos\phi\\ i\sin\theta\sin\phi\end{pmatrix}$$

The radiation intensity becomes

$$\frac{dI}{d\Omega} = \frac{\omega_0^4}{4\pi^2 c^3} \mu_0^2 \frac{T^2}{4} \left(2\sin^2\theta \sin^2\phi + \cos^2\theta + \sin^2\theta \cos^2\phi \right)$$

$$= \frac{\omega_0^4}{4\pi^2 c^3} \mu_0^2 \frac{T^2}{4} \left(1 + \sin^2\theta \sin^2\phi \right)$$

The total power radiated is

$$I = \frac{\omega_0^4}{4\pi^2 c^3} \mu_0^2 \frac{T^2}{4} \cdot \frac{16\pi}{3} = \frac{\omega_0^4}{3\pi c^3} \mu_0^2 T^2$$

I am not sure what's the meaning of total time averaged power, and will just stop here.

Alternatively, we can express the integral of $\int_{-T/2}^{T/2} \cos(\omega_0 t) e^{i\omega t} dt$ and $\int_{-T/2}^{T/2} \sin(\omega_0 t) e^{i\omega t} dt$

in the form of $\dfrac{\sin[(\omega \pm \omega_0)T/2]}{\omega \pm \omega_0}$, and use the Dirac delta function approximation

$$\lim_{T \to 0} \frac{\sin^2(xT)}{x^2 \pi T} = \delta(x)$$ to obtain the same result.