11.20 The energy-momentum 4-vector before decay is $p = (M, 0)$, while the 4-vector after decay is $p' = (E_1 + E_2, \vec{p}_1 + \vec{p}_2)$. Since the norm of the energy-momentum 4-vector is a Lorentz invariant, we must have

$$M^2 = (E_1 + E_2)^2 - |\vec{p}_1 + \vec{p}_2|^2 = E_1^2 - p_1^2 + E_2^2 - p_2^2 + 2E_1E_2 - 2p_1p_2\cos\theta$$

$$= m_1^2 + m_2^2 + 2E_1E_2 - 2p_1p_2\cos\theta$$

11.21 Following 11.19, we know that

$$T_i = \frac{(M - m_i)^2 - \left[\left(\sum_{j \neq i} E_j \right)^2 - \left| \sum_{j \neq i} \vec{p}_j \right|^2 \right]}{2M}$$

Consider the total energy-momentum 4-vector for all particles other than particle i. In the laboratory frame and the cm frame, we have

$$\left(\sum_{j \neq i} E_j \right)^2 - \left| \sum_{j \neq i} \vec{p}_j \right|^2 = \left(\sum_{j \neq i} W_j \right)^2,$$

where W_j is particle j's energy in the cm frame. Clearly, $W_j \geq m_j$. Then

$$T_i \leq \frac{(M - m_i)^2 - \left(\sum_{j \neq i} m_j \right)^2}{2M} = \Delta M \left(1 - \frac{m_i}{M} - \frac{\Delta M}{2M} \right),$$

following the same procedure as 11.19