5.21 (a) $\int \vec{B}\cdot\vec{H}\, d^3x = \int \vec{H}\cdot(\nabla\times\vec{A})\, d^3x = \int \nabla\cdot(\vec{A}\times\vec{H})\, d^3x + \int \vec{A}\cdot(\nabla\times\vec{H})\, d^3x$

$= \oint (\vec{A}\times\vec{H})\cdot\vec{n}\, da$

The second term vanishes, since $\nabla\times\vec{H} = \vec{J} = 0$, because the magnetic field comes from localized permanent magnetization. The surface integral term also vanishes, as $|\vec{A}| \sim \frac{1}{r^2}$, $|\vec{H}| \sim \frac{1}{r^3}$, both decreasing rapidly enough.

(b) $\delta U = -\vec{m}\cdot\delta\vec{B}$. Since $\vec{B}$ is proportional to m, it is straight forward to show that

$$W = -\frac{1}{2}\int \vec{M}\cdot\vec{B}\, d^3x = -\frac{\mu_0}{2}\int \vec{M}\cdot(\vec{H}+\vec{M})\, d^3x$$

$$= -\frac{\mu_0}{2}\int \vec{M}\cdot\vec{H}\, d^3x + C$$

where $C = \frac{\mu_0}{2}\int |\vec{M}|^2 d^3x$ is a constant. Also, since $\vec{M} = \frac{\vec{B}}{\mu} - \vec{H}$

$$W = -\frac{\mu_0}{2}\int \left(\frac{\vec{B}}{\mu} - \vec{H}\right)\cdot\vec{H}\, d^3x = \frac{\mu_0}{2}\int |\vec{H}|^2 d^3x.$$

using the result from part (a).