2.21 Solution: On the two-dimensional complex plane, a point z is located in the circle $|w| = b$, and its image point will fall outside of the circle and on the line connecting the origin and z. The exact location of the image point is given by $z' = b^2/\bar{z}$. Applying the Cauchy's theorem, we have

$$F(z) = \frac{1}{2\pi i} \oint_{|w|=b} \frac{F(w)}{w - z} dw,$$

and

$$\frac{1}{2\pi i} \oint_{|w|=b} \frac{F(w)}{w - z'} dw = 0.$$

Take the difference, we can obtain the following equivalent result,

$$\begin{aligned}
F(z) &= \frac{1}{2\pi i} \oint_{|w|=b} F(w) \left(\frac{1}{w - z} - \frac{1}{w - z'} \right) dw \\
&= \frac{1}{2\pi i} \oint_{|w|=b} F(w) \frac{z - b^2/\bar{z}}{w^2 + b^2 z/\bar{z} - (z + b^2/\bar{z})w} dw \\
&= \frac{1}{2\pi i} \oint_{|w|=b} F(w) \frac{|z|^2 - b^2}{\bar{z}w + b^2 z/w - (|z|^2 + b^2)} \frac{dw}{w}.
\end{aligned}$$

Introduce the parameterization $w = be^{i\phi'}$ and $z = \rho e^{i\phi}$ in polar coordinates, the above result becomes

$$\begin{aligned}
F(\rho, \phi) &= \frac{1}{2\pi} \int_0^{2\pi} F(b, \phi') \frac{\rho^2 - b^2}{\rho b e^{i(\phi' - \phi)} + \rho b e^{-i(\phi' - \phi)} - (\rho^2 + b^2)} d\phi' \\
&= \frac{1}{2\pi} \int_0^{2\pi} F(b, \phi') \frac{b^2 - \rho^2}{b^2 + \rho^2 - 2\rho b \cos(\phi' - \phi)} d\phi'.
\end{aligned}$$