

9.16 (a) The current density can be written as $\vec{J}(\vec{r}) = I \sin\left(\frac{2\pi}{d} z\right) \delta(x) \delta(y) \hat{z}$, and

the vector potential can be exactly calculated

$$\begin{aligned} \vec{A}(\vec{r}) &= \hat{z} \frac{\mu_0 I}{4\pi} \frac{e^{ikr}}{r} \int_{-d/2}^{d/2} \sin(kz) e^{-ikz \cos\theta} dz = \hat{z} \frac{\mu_0 I}{4\pi} \frac{e^{ikr}}{kr} \int_{-\pi}^{\pi} \sin(\phi) e^{-i\phi \cos\theta} d\phi \\ &= \hat{z} \frac{-i\mu_0 I}{2\pi} \frac{e^{ikr}}{kr} \frac{\sin(\pi \cos\theta)}{\sin\theta}, \quad \text{with } k = 2\pi/d. \end{aligned}$$

Following the same argument from Eq. (9.55) to Eq. (9.56), we can see that

$$\frac{dP}{d\Omega} = \frac{Z_0 I^2}{8\pi^2} \left[\frac{\sin(\pi \cos\theta)}{\sin\theta} \right]^2.$$

$$\begin{aligned} (b) \quad P &= \int d\Omega \frac{dP}{d\Omega} = \frac{Z_0 I^2}{4\pi} \int_0^\pi \frac{\sin^2(\pi \cos\theta)}{\sin\theta} d\theta = \frac{Z_0 I^2}{4\pi} \times \frac{1}{2} \left[-C_1(4\pi) + \gamma + \log(4\pi) \right] \\ &= \frac{Z_0 I^2}{4\pi} \times 1.55718, \end{aligned}$$

where $C_1(x)$ is the cosine integral function. The radiation resistance is

$$R_{\text{rad}} = \frac{Z_0}{2\pi} \times 1.55718 = 93.366061$$