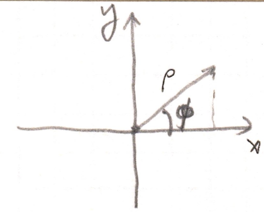
5.15 (a) For a single wire, the magnetic field is given by



$$\vec{H} = \frac{I}{2\pi\rho}\hat{\phi}$$

Therefore, the scalar potential is

$$\Phi_m = -\frac{I}{2\pi}\phi = -\frac{I}{2\pi}\arctan\left(\frac{y}{x}\right).$$

Then, in the two-wire setting, assuming the current at $x = d/2$ along the $+\hat{z}$-direction

$$\Phi_m = \frac{I}{2\pi}\left(\arctan\left(\frac{y}{x+d/2}\right) - \arctan\left(\frac{y}{x-d/2}\right)\right)$$

For $d \ll x, y$,

$$\Phi_m = \frac{Id}{2\pi}\frac{\partial}{\partial x}\left(\arctan\left(\frac{y}{x}\right)\right) = -\frac{Id}{2\pi}\frac{y}{x^2+y^2} = -\frac{Id\sin\phi}{2\pi\rho}.$$

(b) Following the same procedure as 5.14, the potential in the three regions are

$$\Phi_m = -\frac{Id\sin\phi}{2\pi\rho} + \sum_{m=1}^{\infty} a_m\rho^m\sin(m\phi), \qquad \rho < a$$

$$\Phi_m = \sum_{m=1}^{\infty}\left(b_m\rho^m + c_m\rho^{-m}\right)\sin(m\phi). \qquad a < \rho < b$$

$$\Phi_m = \sum_{m=1}^{\infty} d_m\rho^{-m}\sin(m\phi). \qquad \rho > b$$

The continuity conditions

$$\left.\frac{\partial\Phi_m}{\partial\rho}\right|_{\rho=a-} = \mu_r\left.\frac{\partial\Phi_m}{\partial\rho}\right|_{\rho=a+}, \qquad \mu_r\left.\frac{\partial\Phi_m}{\partial\rho}\right|_{\rho=b-} = \left.\frac{\partial\Phi_m}{\partial\rho}\right|_{\rho=b+}$$

$$\left.\frac{\partial\Phi_m}{\partial\phi}\right|_{\rho=a-} = \left.\frac{\partial\Phi_m}{\partial\phi}\right|_{\rho=a+}, \qquad \left.\frac{\partial\Phi_m}{\partial\phi}\right|_{\rho=b-} = \left.\frac{\partial\Phi_m}{\partial\phi}\right|_{\rho=b+}$$

lead to

$$\begin{cases} H_0 + a^2a_1 = \mu_r a^2 b_1 - \mu_r c_1 \\ -H_0 + a^2a_1 = a^2b_1 + c_1 \\ \mu_r b^2 b_1 - \mu_r c_1 = -d_1 \\ b^2 b_1 + c_1 = d_1 \end{cases}$$

Where $H_0 = Id/2\pi$.

These equations can be solved, which leads to

$$d_1 = -\frac{4\mu_r b^2}{(\mu_r+1)^2 b^2 - (\mu_r-1)^2 a^2} H_0$$

$$c_1 = \frac{\mu_r+1}{2\mu_r} d_1$$

$$b_1 = \frac{\mu_r-1}{2\mu_r} \frac{d_1}{b^2}$$

$$a_1 = \frac{\mu_r^2-1}{4\mu_r}\left(\frac{1}{b^2}-\frac{1}{a^2}\right) d_1$$

Thus, the scalar potential outside of the cylinder is reduced in strength

(c) For $\mu_r \gg 1$, and $b = a+t$ with $t \ll b$,

$$F = \frac{4\mu_r b^2}{(\mu_r+1)^2 b^2 - (\mu_r-1)^2 (b-t)^2} \rightarrow \frac{4\mu_r b^2}{\mu_r^2\, 2bt} = \frac{2b}{\mu_r t}$$