6.22 (a) See Prob 6.21(b) for detailed calculation.

(b) $\vec{B}_{sym} = \nabla \times \vec{A} = \frac{\mu_0}{8\pi} \nabla \times \left[\frac{\vec{p}(\vec{x}\cdot\vec{v}) + \vec{v}(\vec{x}\cdot\vec{p})}{r^3} \right]$.

Let us consider a general case, the curl of $\vec{a}(\vec{x}\cdot\vec{b})/r^3$. It is straightforward to show

$$\nabla \times \left(\frac{\vec{a}(\vec{x}\cdot\vec{b})}{r^3} \right) = \nabla\left(\frac{1}{r^3}\right) \times \vec{a}(\vec{x}\cdot\vec{b}) + \frac{1}{r^3} \nabla \times (\vec{a}(\vec{x}\cdot\vec{b}))$$

$$= -\left(\frac{3\vec{x}}{r^5} \times \vec{a}\right)(\vec{x}\cdot\vec{b}) + \frac{1}{r^3}\left(\nabla(\vec{x}\cdot\vec{b}) \times \vec{a}\right)$$

$$= -\left(\frac{3\vec{x}}{r^5} \times \vec{a}\right)(\vec{x}\cdot\vec{b}) + \frac{1}{r^3}\left[(\vec{b}\cdot\nabla)\vec{x} \right] \times \vec{a}$$

$$= -\left(\frac{3\vec{x}}{r^5} \times \vec{a}\right)(\vec{x}\cdot\vec{b}) + \frac{1}{r^3}(\vec{b}\times\vec{a}).$$

Then, $\vec{B}_{sym} = \frac{\mu_0}{8\pi}\left[\nabla\times \frac{\vec{p}(\vec{x}\cdot\vec{v})}{r^3} + \nabla\times\frac{\vec{v}(\vec{x}\cdot\vec{p})}{r^3} \right] = \frac{\mu_0}{8\pi}\left[-\left(\frac{3\vec{x}}{r^5}\times\vec{p}\right)(\vec{x}\cdot\vec{v}) - \left(\frac{3\vec{x}}{r^5}\times\vec{v}\right)(\vec{x}\cdot\vec{p}) \right]$

$$= -\frac{3\mu_0}{8\pi r^3}\,\vec{n}\times\left[\vec{p}(\vec{n}\cdot\vec{v}) + \vec{v}(\vec{n}\cdot\vec{p}) \right]$$

(d) $\vec{B} = \nabla\times\vec{A} = \frac{\mu_0}{4\pi}\nabla\times\left(\frac{1}{r^2}\vec{v}(\vec{p}\cdot\vec{n}) \right) = \frac{\mu_0}{4\pi}\left[\nabla\left(\frac{1}{r^2}\right)\times\vec{v}(\vec{p}\cdot\vec{n}) + \frac{1}{r^2}\nabla\times(\vec{v}(\vec{p}\cdot\vec{n})) \right]$

We know $\nabla\left(\frac{1}{r^2}\right) = -\frac{2\vec{n}}{r^3}$,

$$\nabla\times[\vec{v}(\vec{p}\cdot\vec{n})] = [\nabla(\vec{p}\cdot\vec{n})]\times\vec{v} = (\vec{p}\cdot\nabla)\vec{n}\times\vec{v} = \frac{1}{r}[\vec{p} - \vec{n}(\vec{p}\cdot\vec{n})]\times\vec{v}.$$

Then, $\vec{B} = \frac{\mu_0}{4\pi}\left[-\frac{2}{r^3}(\vec{n}\times\vec{v})(\vec{p}\cdot\vec{n}) + \frac{1}{r}\left(\vec{p} - \vec{n}(\vec{p}\cdot\vec{n})\right)\times\vec{v} \right]$

$$= \frac{\mu_0}{4\pi}\left[\frac{1}{r}\vec{p}\times\vec{v} - \frac{3}{r^3}(\vec{p}\cdot\vec{n})(\vec{n}\times\vec{v}) \right] = \frac{\mu_0}{4\pi}\vec{v}\times\frac{3\vec{n}(\vec{n}\cdot\vec{p}) - \vec{p}}{r^3}$$

The final result can be written as $\vec{B} = \frac{1}{c^2}\vec{v}\times\vec{E}$, with $\vec{E} = \frac{3\vec{n}(\vec{n}\cdot\vec{p}) - \vec{p}}{4\pi\varepsilon_0 r^3}$ is the dipole electric field. From this we can see that a moving dipole generates a magnetic field.

(c) I don't want to do it now. Maybe later. Maybe never.