

11.19 (a) In the rest frame of the decaying particle, the total 4-momentum before the decay is $p = (M, \vec{0})$. After the decay, due to momentum conservation, we must have, for the two particles, $p_1 = (E_1, \vec{p}_1)$, $p_2 = (E_2, \vec{p}_2)$, with $\vec{p}_1 = -\vec{p}_2$. Since $p = p_1 + p_2$, we have $(M - E_1, -\vec{p}_1) = (E_2, \vec{p}_2)$. Squaring both sides,

$$(M - E_1)^2 - |\vec{p}_1|^2 = E_2^2 - |\vec{p}_2|^2, \text{ or } M^2 - 2ME_1 + E_1^2 - |\vec{p}_1|^2 = E_2^2 - |\vec{p}_2|^2.$$

Since $m_1^2 = E_1^2 - |\vec{p}_1|^2$, we have $M^2 - 2ME_1 + m_1^2 = m_2^2$, which leads to

$$E_1 = \frac{M^2 + m_1^2 - m_2^2}{2M}$$

Repeat the same procedure for particle 2, we will have

$$E_2 = \frac{M^2 + m_2^2 - m_1^2}{2M}.$$

(b) The kinetic energy for particle 1 is

$$T_1 = E_1 - m_1 = \frac{M^2 + m_1^2 - m_2^2}{2M} - m_1 = \frac{M^2 - 2m_1M + m_1^2 - m_2^2}{2M}$$

$$= \frac{(M - m_1)^2 - m_2^2}{2M} = \frac{(M - m_1 - m_2)(M - m_1 + m_2)}{2M}$$

$$= \Delta M \cdot \frac{2M - 2m_1 - (M - m_1 - m_2)}{2M} = \Delta M \left(1 - \frac{m_1}{M} - \frac{\Delta M}{2M} \right),$$

where $\Delta M = M - m_1 - m_2$.