

# Electromagnetism with magnetic charge and two photons

D. Singleton

Citation: *Am. J. Phys.* **64**, 452 (1996); doi: 10.1119/1.18191

View online: <http://dx.doi.org/10.1119/1.18191>

View Table of Contents: <http://ajp.aapt.org/resource/1/AJPIAS/v64/i4>

Published by the American Association of Physics Teachers

## Related Articles

Causality in Classical Electrodynamics

*Phys. Teach.* **50**, 226 (2012)

Reanalyzing the Ampère-Maxwell Law

*Phys. Teach.* **49**, 343 (2011)

MicroReviews by the Book Review Editor: Maxwell on the Electromagnetic Field: A Guided Study: Thomas K. Simpson

*Phys. Teach.* **48**, 560 (2010)

MicroReviews by the Book Review Editor: The Road to Reality: A Complete Guide to the Laws of the Universe: Roger Penrose

*Phys. Teach.* **48**, 560 (2010)

The relativistic Ampère-Maxwell law in integral form

*Am. J. Phys.* **72**, 1178 (2004)

## Additional information on Am. J. Phys.

Journal Homepage: <http://ajp.aapt.org/>

Journal Information: [http://ajp.aapt.org/about/about\\_the\\_journal](http://ajp.aapt.org/about/about_the_journal)

Top downloads: [http://ajp.aapt.org/most\\_downloaded](http://ajp.aapt.org/most_downloaded)

Information for Authors: <http://ajp.dickinson.edu/Contributors/contGenInfo.html>

## ADVERTISEMENT



**WebAssign®**

**The PREFERRED Online Homework Solution for Physics**

Every textbook publisher agrees! Whichever physics text you're using, we have the proven online homework solution you need. WebAssign supports every major physics textbook from every major publisher.

[webassign.net](http://webassign.net)

# Electromagnetism with magnetic charge and two photons

D. Singleton<sup>a)</sup>

Department of Physics, University of Virginia, Charlottesville, Virginia 22901

(Received 20 April 1995; accepted 18 July 1995)

The Dirac approach to include magnetic charge in Maxwell's equations places the magnetic charge at the end of a string on which the fields of the theory develop a singularity. In this paper an alternative formulation of classical electromagnetism with magnetic and electric charge is given by introducing a second pseudo-four-vector potential,  $C_\mu$ , in addition to the usual four-vector potential,  $A_\mu$ . This avoids the use of singular, nonlocal variables (i.e., Dirac strings) in electrodynamics with magnetic charge, and it makes the treatment of electric and magnetic charge more symmetric, since both charges are now gauge charges. © 1996 American Association of Physics Teachers.

## I. INTRODUCTION

Looking at the three vector form of Maxwell's equations, one immediately notices that they become more symmetric if magnetic, as well as electric, charges are included. This symmetry between electric and magnetic charges is called the dual symmetry of electromagnetism, and it allows one to "rotate" electric and magnetic quantities (i.e., fields and charges) into one another in a manner analogous to a planar rotation.<sup>1</sup> This symmetry does not carry over so well at the level of the gauge potential,  $A_\mu = (\phi_e, \mathbf{A})$ , since the electric and magnetic fields look quite different in terms of these fields. Dirac<sup>2</sup> was able to include magnetic charge in Maxwell's equations, while keeping the usual definitions of the  $\mathbf{E}$  and  $\mathbf{B}$  fields in terms of the gauge potentials, at the expense of having a string singularity in the vector potential,  $\mathbf{A}$ . In order to keep this string from having any physical effect it is necessary to impose a quantization condition on the magnetic and electric charges. Since Dirac's initial work there has been much theoretical work devoted to magnetic monopoles, such as the Wu–Yang fiber bundle formulation of magnetic charge<sup>3</sup> and the 't Hooft–Polyakov monopole<sup>4</sup> where a magnetic charge arises from a non-Abelian gauge theory. In this paper we present an alternative formulation which does not require a Dirac string, but rather involves only local, nonsingular variables. This alternative formulation of Maxwell's equations with magnetic charge requires the introduction of a second four-vector potential,  $C_\mu = (\phi_m, \mathbf{C})$ ,<sup>5</sup> in addition to  $A_\mu$ . The usual definitions of the  $\mathbf{E}$  and  $\mathbf{B}$  fields are expanded so that they contain terms involving this new potential. When these new definitions are inserted into Maxwell's equations one finds that the equations separate into wave-type equations for  $A_\mu$  and  $C_\mu$  whose sources are electric and magnetic charges respectively. This implies that when the theory is quantized there will be two photons—one associated with  $A_\mu$  and the other with  $C_\mu$ .<sup>6</sup> However the two photons must have opposite behavior under the parity or spatial inversion transformation. The advantages of this approach to magnetic charge are the avoidance of the string singularity in the fields, and the expansion of the dual symmetry to the level of the gauge potentials. Two photons arise in this theory, since electric and magnetic charges are now both being treated as gauge charges, which means they must each have an associated gauge boson. The obvious objection to this approach is that there is apparently only one massless photon in nature. Attempts have been made<sup>7</sup> to get around this objection by introducing extra conditions on the two gauge potentials such that the number of degrees of free-

dom is reduced to that of only one photon. In this paper we take the approach that the second gauge potential does represent a real gauge boson, and show what classical electrodynamics would look like in this hypothetical, two-photon world by examining how various equations and quantities, familiar from electromagnetism with only electric charge, change (e.g., Maxwell's equations, the Lorentz force equation, the energy-momentum tensor). We will present most of our results both in three-vector and in covariant four-vector notation. The Hamiltonian formulation of the two-potential theory as well as a discussion of some of the quantum mechanical aspects of the problem has been given by Barker and Graziani.<sup>8</sup> In the last section we will look at some of the peculiar features of magnetic charge theory which persist even in the two-photon model, and we will point out differences and similarities with the Dirac string approach. The fact that this second, parity-odd photon is not seen should not immediately disqualify this theory from further consideration, since one could easily use spontaneous symmetry breaking in the form of the Higgs mechanism<sup>9</sup> to make this second photon massive,<sup>10</sup> thus hiding it, and the gauge symmetry associated with it, until a certain energy scale had been reached (much in the same way that the  $W$  and  $Z$  bosons were not observed directly until a certain accelerator energy had been reached). In this article we are not concerned with creating a realistic model, we simply wish to present an alternative, and largely unknown, formulation of electrodynamics with electric and magnetic charge, whose basic concepts can be grasped with an undergraduate knowledge of electromagnetism. The results derived here are mostly a collection of previous results. However our emphasis that the second potential should be treated as a real, second photon is unique. Throughout this paper we set  $c=1$ .

## II. THE DUAL PHOTON

The generalized Maxwell equations in the presence of electric and magnetic charges and currents are<sup>1</sup>

$$\begin{aligned}\nabla \cdot \mathbf{E} &= \rho_e, & \nabla \times \mathbf{B} &= \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}_e, \\ \nabla \cdot \mathbf{B} &= \rho_m, & -\nabla \times \mathbf{E} &= \frac{\partial \mathbf{B}}{\partial t} + \mathbf{J}_m.\end{aligned}\tag{1}$$

These equations possess a dual symmetry between electric and magnetic quantities which can be written as

$$\mathbf{E} \rightarrow \mathbf{E} \cos \theta + \mathbf{B} \sin \theta, \quad \mathbf{B} \rightarrow -\mathbf{E} \sin \theta + \mathbf{B} \cos \theta \tag{2}$$

and

$$\begin{aligned}\rho_e &\rightarrow \rho_e \cos \theta + \rho_m \sin \theta, & \rho_m &\rightarrow -\rho_e \sin \theta + \rho_m \cos \theta, \\ \mathbf{J}_e &\rightarrow \mathbf{J}_e \cos \theta + \mathbf{J}_m \sin \theta, & \mathbf{J}_m &\rightarrow -\mathbf{J}_e \sin \theta + \mathbf{J}_m \cos \theta.\end{aligned}\quad (3)$$

The  $\mathbf{E}$ - and  $\mathbf{B}$ -field vectors are distinguished from one another under the spatial inversion or parity transformation ( $\mathbf{r} \rightarrow -\mathbf{r}$ ),  $\mathbf{E}$  being a vector ( $\mathbf{E} \rightarrow -\mathbf{E}$ ) and  $\mathbf{B}$  being a pseudovector ( $\mathbf{B} \rightarrow \mathbf{B}$ ). These definitions and Maxwell's equations then imply that  $\mathbf{J}_e$  ( $\mathbf{J}_m$ ) is a vector (pseudovector) under parity, while  $\rho_e$  ( $\rho_m$ ) is a scalar (pseudoscalar) under parity (i.e.,  $\rho_e \rightarrow \rho_e$  and  $\rho_m \rightarrow -\rho_m$  under the transformation  $\mathbf{r} \rightarrow -\mathbf{r}$ ). In order that the parity properties of all the quantities in the Maxwell equations remain unchanged under the dual rotations of Eqs. (2) and (3), one must require that  $\theta$  be a pseudoscalar. Then  $\cos \theta$  is a scalar since it contains only even powers of  $\theta$ , while  $\sin \theta$  is a pseudoscalar since it contains only odd powers. Our chief requirement will be that this formulation of magnetic charge should obey this dual symmetry at every level. Introducing two four-vector potentials,  $A^\mu = (\phi_e, \mathbf{A})$  and  $C^\mu = (\phi_m, \mathbf{C})$ , the  $\mathbf{E}$  and  $\mathbf{B}$  fields can be written as

$$\begin{aligned}\mathbf{E} &= -\nabla \phi_e - \frac{\partial \mathbf{A}}{\partial t} - \nabla \times \mathbf{C}, \\ \mathbf{B} &= -\nabla \phi_m - \frac{\partial \mathbf{C}}{\partial t} + \nabla \times \mathbf{A}.\end{aligned}\quad (4)$$

In electrodynamics with only electric charge,  $\mathbf{E}$  consists of only the first two terms, while  $\mathbf{B}$  consists of only the last term. Since we still want  $\mathbf{E}$  and  $\mathbf{B}$  to be a vector and a pseudovector under these expanded definitions we need to require that  $\phi_e$  and  $\mathbf{A}$  act as scalars and vectors, and  $\phi_m$  and  $\mathbf{C}$  act as pseudoscalars and pseudovectors under spatial inversion. Inserting these new definitions for the electromagnetic fields into Maxwell's equations they become (after the use of a few standard vector identities such as  $\nabla \cdot [\nabla \times \mathbf{V}] = 0$ ,  $\nabla \times [\nabla \phi] = 0$  and  $\nabla \times [\nabla \times \mathbf{V}] = \nabla[\nabla \cdot \mathbf{V}] - \nabla^2 \mathbf{V}$ )

$$\begin{aligned}\nabla^2 \phi_e + \frac{\partial}{\partial t} (\nabla \cdot \mathbf{A}) &= -\rho_e, & \nabla^2 \phi_m + \frac{\partial}{\partial t} (\nabla \cdot \mathbf{C}) &= -\rho_m, \\ \nabla^2 \mathbf{A} - \frac{\partial^2 \mathbf{A}}{\partial t^2} - \nabla \left( \nabla \cdot \mathbf{A} + \frac{\partial \phi_e}{\partial t} \right) &= -\mathbf{J}_e, \\ \nabla^2 \mathbf{C} - \frac{\partial^2 \mathbf{C}}{\partial t^2} - \nabla \left( \nabla \cdot \mathbf{C} + \frac{\partial \phi_m}{\partial t} \right) &= -\mathbf{J}_m.\end{aligned}\quad (5)$$

In order to simplify this form of Maxwell's equations further, we can use the gauge freedom possessed by the potentials, and impose the Lorentz gauge condition on them:

$$\frac{\partial \phi_e}{\partial t} + \nabla \cdot \mathbf{A} = 0, \quad \frac{\partial \phi_m}{\partial t} + \nabla \cdot \mathbf{C} = 0. \quad (6)$$

Using these gauge conditions in Eq. (5) we arrive at the wave form of Maxwell's equations with electric and magnetic charge

$$\begin{aligned}\nabla^2 \phi_e - \frac{\partial^2 \phi_e}{\partial t^2} &= -\rho_e, & \nabla^2 \phi_m - \frac{\partial^2 \phi_m}{\partial t^2} &= -\rho_m, \\ \nabla^2 \mathbf{A} - \frac{\partial^2 \mathbf{A}}{\partial t^2} &= -\mathbf{J}_e, & \nabla^2 \mathbf{C} - \frac{\partial^2 \mathbf{C}}{\partial t^2} &= -\mathbf{J}_m.\end{aligned}\quad (7)$$

These equations are the inhomogeneous wave equation form of Maxwell's equations. Notice that the parity of the quantities on the left in Eq. (7) agrees with the parity of the quantities on the right. In this approach to magnetic charge one has two four-vector gauge potentials and therefore when these fields are quantized (i.e., second quantization) there are two distinct photons. The difference between the two photons represented by  $A_\mu$  and  $C_\mu$ , is that they transform differently under parity:  $A_\mu$  transforms as a normal four vector while  $C_\mu$  transforms as a pseudo four-vector. The photon associated with  $C_\mu$  will be called the dual or magnetic photon.

All of the results up to this point can be rewritten in four-vector notation. First we define two field strength tensors and their duals

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \quad G^{\mu\nu} = \partial^\mu C^\nu - \partial^\nu C^\mu, \quad (8)$$

$$\mathcal{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}, \quad \mathcal{G}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}, \quad (9)$$

where  $\epsilon^{\mu\nu\alpha\beta}$  is the rank four Levi-Civita tensor with  $\epsilon^{0123} = +1$ , and having total antisymmetry in its indices. We can write the  $\mathbf{E}$  and  $\mathbf{B}$  fields, of Eq. (4), in terms of these field strength tensors

$$E_i = F^{i0} - \mathcal{F}^{i0}, \quad B_i = G^{i0} + \mathcal{G}^{i0}. \quad (10)$$

The dual symmetry can now be carried over to the potentials

$$\begin{aligned}A_\mu &\rightarrow A_\mu \cos \theta + C_\mu \sin \theta, \\ C_\mu &\rightarrow -A_\mu \sin \theta + C_\mu \cos \theta.\end{aligned}\quad (11)$$

The dual symmetry for the charges and currents can also be written in four-vector language as

$$\begin{aligned}J_e^\mu &\rightarrow J_e^\mu \cos \theta + J_m^\mu \sin \theta, \\ J_m^\mu &\rightarrow -J_e^\mu \sin \theta + J_m^\mu \cos \theta,\end{aligned}\quad (12)$$

where  $J_e^\mu = (\rho_e, \mathbf{J}_e)$  and  $J_m^\mu = (\rho_m, \mathbf{J}_m)$  are the electric and magnetic four currents respectively. Finally, the wave equation form of Maxwell's equations that result from substituting Eq. (4) into Eq. (1) are

$$\begin{aligned}\partial_\mu F^{\mu\nu} &= \partial_\mu \partial^\mu A^\nu = J_e^\nu, \\ \partial_\mu G^{\mu\nu} &= \partial_\mu \partial^\mu C^\nu = J_m^\nu,\end{aligned}\quad (13)$$

where the Lorentz gauge condition ( $\partial_\mu A^\mu = \partial_\mu C^\mu = 0$ ) has been taken for both potentials. Since  $C^\mu$  is a pseudo four-vector, Eq. (13) implies that  $J_m^\mu$  must be a pseudo four-current. One can immediately write down a Lagrange density which yields Eq. (13)

$$\mathcal{L}_M = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - J_e^\mu A_\mu - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} - J_m^\mu C_\mu. \quad (14)$$

One could add terms like  $F_{\mu\nu} \mathcal{G}^{\mu\nu}$ ,  $\mathcal{F}_{\mu\nu} G^{\mu\nu}$ ,  $\mathcal{F}_{\mu\nu} F^{\mu\nu}$ , or  $\mathcal{G}_{\mu\nu} G^{\mu\nu}$  to this Lagrangian and still obtain the Maxwell's equations of Eq. (13) since these extra terms are total divergences due to the antisymmetry of  $\epsilon_{\mu\nu\alpha\beta}$ . In the following section we will discuss the Lorentz force equations for magnetic charge. We will also construct the energy-momentum tensor in terms of  $\mathbf{E}$  and  $\mathbf{B}$  fields, and in terms of  $A_\mu$  and  $C_\mu$ .

It is interesting to observe that one can also think of formulating a theory of magnetic charge with only an additional scalar potential (e.g.,  $\phi_m$ ) in addition to the standard four-vector potential  $A_\mu = (\phi_e, \mathbf{A})$ . The normal four-vector potential,  $A_\mu = (\phi_e, \mathbf{A})$ , takes care of the  $\mathbf{E}$  field (both the curl-free and divergence-free parts) as well as the divergence-free part of the  $\mathbf{B}$  field. The curl-free part of the  $\mathbf{B}$  field can then be

accounted for by adding only an extra scalar,  $\phi_m$ , to the theory. This is in fact the basic idea behind the 't Hooft–Polyakov<sup>4</sup> monopole, where the extra scalar potential is the Higgs field. Looking at 't Hooft's generalized definition of the field strength tensor one sees that the magnetic charge does indeed come from the scalar Higgs field rather than from the gauge fields of the theory.<sup>11</sup> However for the 't Hooft–Polyakov monopole one finds that the magnetic charge is not a gauge charge but a topological charge, which is not explicitly associated with any symmetry. The present formulation of magnetic charge is not as economical as that of 't Hooft and Polyakov, but it is more symmetric since both electric and magnetic charges are treated as gauge charges.

### III. THE LORENTZ FORCE EQUATION AND ENERGY-MOMENTUM TENSOR

The equation for the rate of change of mechanical energy for an electric charge  $e$  moving with velocity  $\mathbf{v}_e$  in external  $\mathbf{E}$  and  $\mathbf{B}$  fields is

$$\frac{dE_e}{dt} = e\mathbf{v}_e \cdot \mathbf{E}. \quad (15)$$

The Lorentz force equation for this particle is then

$$\frac{d\mathbf{p}_e}{dt} = e(\mathbf{E} + \mathbf{v}_e \times \mathbf{B}). \quad (16)$$

These two equations can be combined into one manifestly covariant form as<sup>11</sup>

$$\frac{dp_e^\mu}{d\tau} = m \frac{dU^\mu}{d\tau} = e(F^{\mu\nu} - \mathcal{G}^{\mu\nu})U_\nu, \quad (17)$$

where  $U^\mu = dx^\mu/d\tau$  is the four-velocity of the particle with charge  $e$  and mass  $m$ .  $\tau$  is the proper time of the particle. Usually the covariant expression of the Lorentz force equation involves only the  $F^{\mu\nu}$  term, but because of the expanded definitions of  $\mathbf{E}$  and  $\mathbf{B}$  one has the second term. In electrodynamics with only electric charge it is possible to derive the covariant form of the Lorentz force from a Lagrangian.<sup>1</sup> In the two potential formulation it has been proven<sup>12</sup> that it is impossible, from a single Lagrangian, to derive both the equations for the fields and for the particles. In this paper we will get around this problem by requiring that the generalized Lorentz equations should satisfy the dual symmetry of Eq. (2) or (11). Using this requirement and Eqs. (15)–(17) one arrives at the Lorentz equation for a magnetically charged particle. Looking at the dual rotation where  $\theta = 90^\circ$  so that  $\mathbf{E} \rightarrow \mathbf{B}$ ,  $\mathbf{B} \rightarrow -\mathbf{E}$ , and  $J_e^\mu \rightarrow J_m^\mu$  (i.e.,  $e \rightarrow g$  and  $e\mathbf{v}_e \rightarrow g\mathbf{v}_m$ ) it is found that the rate of energy change equation and the Lorentz force equation become

$$\frac{dE_m}{dt} = g\mathbf{v}_m \cdot \mathbf{B} \quad (18)$$

and

$$\frac{d\mathbf{p}_m}{dt} = g(\mathbf{B} - \mathbf{v}_m \times \mathbf{E}). \quad (19)$$

These can be written in covariant form as<sup>12</sup>

$$\frac{dp_m^\mu}{d\tau} = g(G^{\mu\nu} + \mathcal{F}^{\mu\nu})U_\nu. \quad (20)$$

For a particle carrying both types of charges (i.e., a dyon) the equation of motion is given by the sum of Eqs. (16) and (19), or covariantly by Eqs. (17) and (20).

From these Lorentz equations one can arrive at the proper energy-momentum tensor. The time components of Eqs. (17) and (20) give a statement of energy conservation for a system of particles and fields. In three vector form this yields

$$\frac{dE}{dt} = \int (\mathbf{J}_e \cdot \mathbf{E} + \mathbf{J}_m \cdot \mathbf{B}) d^3x. \quad (21)$$

Since we are now considering a system of an arbitrary number of particles we have made the replacements  $e\mathbf{v}_e \rightarrow \int \mathbf{J}_e d^3x$  and  $g\mathbf{v}_m \rightarrow \int \mathbf{J}_m d^3x$ .  $E$  is the mechanical energy of the particles, and the right-hand side of the equation is the rate at which external  $\mathbf{E}$  and  $\mathbf{B}$  fields do work on the electric and magnetic charges. Using Maxwell's equations (written in terms of the  $\mathbf{E}$  and  $\mathbf{B}$  fields) to replace  $\mathbf{J}_e$  and  $\mathbf{J}_m$ , and applying some standard vector identities it is possible to rewrite Eq. (21) as

$$\frac{dE}{dt} = - \frac{\partial}{\partial t} \int \frac{1}{2} (\mathbf{E}^2 + \mathbf{B}^2) d^3x - \int \nabla \cdot (\mathbf{E} \times \mathbf{B}) d^3x. \quad (22)$$

By making the following definition

$$T^{00} = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2), \quad T^{i0} = (\mathbf{E} \times \mathbf{B})_i, \quad (23)$$

one can rewrite Eq. (22) in the suggestive form

$$\frac{dE}{dt} = - \int \frac{\partial T^{00}}{\partial t} d^3x - \int \partial_i T^{i0} d^3x. \quad (24)$$

This form of the energy conservation equation shows the connection of the particles mechanical energy to the components of the energy-momentum tensor for the fields. By moving  $T^{00}$  to the left-hand side of the equation it can be interpreted as the energy density of the  $\mathbf{E}$  and  $\mathbf{B}$  fields. The quantity  $T^{i0}$  appears as a three divergence in Eq. (24), which can be converted to a surface integral, thus allowing  $T^{i0}$  to be interpreted as an energy flux (the Poynting vector). These are exactly the same expressions for the field energy density and energy flux as in the theory with only electric charge. The difference lies in the definitions of the  $\mathbf{E}$  and  $\mathbf{B}$  fields which have the expanded forms of Eq. (4). This difference will become apparent when we write down the covariant form of the energy-momentum tensor in terms of the gauge potentials. Now, by looking at the spatial components of Eqs. (17) and (20), it is possible to obtain a momentum conservation equation for a system of particles interacting with external fields. Written in three-vector form this becomes

$$\frac{d\mathbf{P}}{dt} = \int (\rho_e \mathbf{E} + \mathbf{J}_e \times \mathbf{B} + \rho_m \mathbf{B} - \mathbf{J}_m \times \mathbf{E}) d^3x, \quad (25)$$

where  $\mathbf{P}$  is the mechanical momentum of the particles. Again by using Maxwell's equations it is possible to replace the charge and current densities ( $\rho_e, \rho_m, \mathbf{J}_e, \mathbf{J}_m$ ) by derivatives of the fields to get

$$\begin{aligned} \frac{d\mathbf{P}}{dt} = & - \frac{d}{dt} \int (\mathbf{E} \times \mathbf{B}) d^3x + \int [\mathbf{E}(\nabla \cdot \mathbf{E}) - \mathbf{E} \times (\nabla \times \mathbf{E}) \\ & + \mathbf{B}(\nabla \cdot \mathbf{B}) - \mathbf{B} \times (\nabla \times \mathbf{B})] d^3x. \end{aligned} \quad (26)$$

By making the following definitions

$$\begin{aligned} T^{0i} & \equiv (\mathbf{E} \times \mathbf{B})_i \\ T^{ij} & \equiv E_i E_j + B_i B_j - \frac{1}{2} \delta_{ij} (\mathbf{E}^2 + \mathbf{B}^2), \end{aligned} \quad (27)$$

the equation for momentum conservation can be written as

$$\frac{dP_i}{dt} + \frac{d}{dt} \int T^{0i} d^3x = \int \frac{\partial T_{ij}}{\partial x_j} d^3x. \quad (28)$$

This shows the connection between the particles mechanical momentum and the components of the energy-momentum tensor for the fields. Again the equation for momentum conservation has exactly the same form as the theory with only electric charge. The  $T^{0i}$  term on the left-hand side of the equation can be interpreted as the momentum carried by the fields. The term on the right-hand side is a three divergence, which can be written as a surface integral:

$$\oint T_{ij} n_j da, \quad (29)$$

where  $n_j$  is the  $j^{\text{th}}$  component of the unit outward normal to the surface. Thus  $T_{ij} n_j$  can be interpreted as  $i^{\text{th}}$  component of the flow per unit area of momentum across the surface. Equation (28) is the statement of momentum conservation of the system of particles and fields. Collecting all the various components of the energy-momentum tensor for the fields, one has

$$T^{00} = \frac{1}{2}(\mathbf{E}^2 + \mathbf{B}^2), \quad T^{i0} = T^{0i} = (\mathbf{E} \times \mathbf{B})_i, \quad (30)$$

$$T^{ij} = E_i E_j + B_i B_j - \frac{1}{2} \delta_{ij} (\mathbf{E}^2 + \mathbf{B}^2).$$

This is exactly the same form as the energy-momentum tensor for electrodynamics with only electric charge. However the covariant expression of the energy-momentum tensor, which is written in terms of the gauge potentials, has extra cross terms between the two potentials,  $A_\mu$  and  $C_\mu$ . Inserting the expanded definitions of  $\mathbf{E}$  and  $\mathbf{B}$  in terms of  $A_\mu$  and  $C_\mu$  into the expressions of Eq. (30) and piecing together the results gives the covariant form of the energy-momentum tensor:

$$T^{\alpha\beta} = F^\alpha_\rho F^{\rho\beta} + \frac{1}{4} g^{\alpha\beta} F_{\mu\nu} F^{\mu\nu} + G^\alpha_\rho G^{\rho\beta} + \frac{1}{4} g^{\alpha\beta} G_{\mu\nu} G^{\mu\nu} \\ + \frac{1}{2} (\mathcal{F}^{\alpha\rho} G^\beta_\rho + \mathcal{F}^{\beta\rho} G^\alpha_\rho) - \frac{1}{2} (\mathcal{G}^{\alpha\rho} F^\beta_\rho + \mathcal{G}^{\beta\rho} F^\alpha_\rho). \quad (31)$$

This is the covariant expression for the energy-momentum tensor of Ref. 12. It is symmetric in its indices  $\alpha$  and  $\beta$  as is required if the angular momentum of the fields is to be conserved. The first four terms of the right-hand side of Eq. (31) are the terms one would expect by a simple generalization of the usual energy-momentum tensor for electrodynamics with only electric charge. The remaining terms are the cross terms between  $A_\mu$  and  $C_\mu$  which occur when one inserts the expressions for  $\mathbf{E}$  and  $\mathbf{B}$  from Eq. (4) into Eq. (30). This form of the energy-momentum tensor ensures that the magnetic and electric charges will interact with one another according to the Lorentz force equations of Eqs. (17) and (20). These cross terms also ensure that the combination of an electric charge and a magnetic charge will carry some angular momentum in their combined  $\mathbf{E}$  and  $\mathbf{B}$  fields.<sup>13</sup> One can reverse the arguments above and starting with the energy-momentum tensor of equation Eq. (31) arrive at the Lorentz equations. Taking the four divergence of  $T_{\mu\nu}$  and using the covariant form of Maxwell's equations gives, after some work,

$$\partial_\mu T^{\mu\nu} = -F^{\nu\rho} J_\rho^e - G^{\nu\rho} J_\rho^m - \mathcal{F}^{\nu\rho} J_\rho^+ + \mathcal{G}^{\nu\rho} J_\rho^-. \quad (32)$$

This shows that in the absence of external sources the energy-momentum tensor is divergenceless, and in the presence of external sources the above equation gives the cova-

riant form of the generalized Lorentz force of Eqs. (17) and (20) if one includes the mechanical energy-momentum tensor for the particles.

#### IV. ANGULAR MOMENTUM AND QUANTIZATION CONDITIONS

In this section we examine some of the peculiar aspects of monopole theory, which occur in this two-potential approach as well as in other formulations. In particular we examine the angular momentum which is carried in the fields produced by two particles—one carrying an electric charge and the other a magnetic charge. We will briefly discuss how the quantization condition between the two types of charges carries over into the two-potential theory, and we will make some comments about symmetry breaking in magnetic charge theories.

To study the angular momentum carried in the electromagnetic fields of some configuration of charges, one looks at the integral of certain components of the rank three tensor:

$$\mathcal{M}^{\alpha\beta\gamma} = T^{\alpha\beta} x^\gamma - T^{\alpha\gamma} x^\beta, \quad (33)$$

where  $T^{\alpha\beta}$  is the energy momentum tensor of Eq. (31). In particular the angular momentum is given by

$$M^{ij} = \int \mathcal{M}^{0ij} d^3x = \int (T^{0i} x^j - T^{0j} x^i) d^3x. \quad (34)$$

The configuration that we will consider is of two particles,  $\mathcal{A}$  and  $\mathcal{B}$ , with particle  $\mathcal{A}$  having an electric charge  $e$  (whose magnitude we will take to be that of an electron or proton) and particle  $\mathcal{B}$  having a magnetic charge  $g$ , which is undetermined at this point. In connection with our introductory comments we will allow the magnetic photon to have a mass  $m$ . Usually gauge bosons are prohibited from having a mass by gauge invariance. A massive gauge boson introduces a term  $\frac{1}{2} m^2 C_\mu C^\mu$  into the Lagrangian, which is then not invariant under the gauge transformation  $C_\mu \rightarrow C_\mu - \partial_\mu \Lambda(x)$ , where  $\Lambda(x)$  is an arbitrary function (note that the field strength tensor,  $G_{\mu\nu}$ , is invariant under this gauge transformation). However it is possible to use the Higgs mechanism<sup>10</sup> to give the magnetic photon a mass while still maintaining a gauge invariant theory. Here we are giving the magnetic photon a mass by hand with the justification that this development can be made consistent with gauge invariance by an application of the Higgs mechanism. Placing the magnetic particle,  $\mathcal{B}$ , at the origin and the electric particle,  $\mathcal{A}$ , a distance  $\mathbf{d}$  from  $\mathcal{B}$  the four-vector potentials produced by these particles are

$$C^\mu_{\mathcal{B}} = \left( \frac{g}{4\pi} \frac{e^{-mr}}{r}, 0, 0, 0 \right), \quad (35)$$

$$A^\mu_{\mathcal{A}} = \left( \frac{e}{4\pi} \frac{1}{r}, 0, 0, 0 \right),$$

where  $r' = |\mathbf{r} - \mathbf{d}|$ . Notice that particle  $\mathcal{A}$  produces a Coulomb potential, while particle  $\mathcal{B}$  produces a Yukawa potential due to the postulated mass of the magnetic photon. Without loss of generality we take particle  $\mathcal{A}$  to be located along the  $+z$ -axis. Then plugging the four-vector potentials of Eq. (35) into the energy-momentum tensor and finally into the expression for the field angular momentum of Eq. (34), we find the angular momentum of this charge configuration to be

$$M^{ij} = L_k = -\frac{egd}{16\pi^2} \int \frac{e^{-mr}}{r'^3} \left( \frac{1}{r} + m \right) [n_z - n_k \cos \theta] d^3x, \quad (36)$$

where  $n_k$  is the unit vector in the  $k$ th direction. Evaluating this in spherical polar coordinates and noticing that only the  $z$  component of  $n_k$  survives the  $\phi$  integration, we arrive at

$$L_z = -\frac{egd}{8\pi} \int_0^\infty r^2 dr \left[ \int_{-1}^1 d(\cos \theta) \times \frac{e^{-mr}}{(r^2 + d^2 - 2rd \cos \theta)^{3/2}} \left( m + \frac{1}{r} \right) (1 - \cos^2 \theta) n_z \right]. \quad (37)$$

The remaining  $r$  and  $\theta$  integrals can be performed using some standard integration techniques (see the article by Carter and Cohen<sup>14</sup> for the case when  $m=0$ ). This leads to

$$L_z = -\frac{eg}{2\pi m^2 d^2} [1 - (1 + md)e^{-md}] n_z. \quad (38)$$

Thus, under the assumption that the magnetic photon has a mass  $m$ , we find that the angular momentum in the charge-monopole system depends on the magnitude of the two types of charge,  $e$  and  $g$ , and also on the mass,  $m$ , and the separation,  $d$ , between the two particles. This latter feature (the dependence of the angular momentum on the separation between the charge and the monopole) is a unique feature that arises because we have assumed a mass for the magnetic photon. In the usual analysis of the charge-monopole system<sup>14</sup> the angular momentum is independent of the distance between the two particles. This difference will yield some interesting results when we discuss the quantization conditions between the charges. To make the connection with the usual angular momentum result we let  $m \rightarrow 0$ . Care must be taken since  $m$  occurs in the denominator of Eq. (38). Expanding  $e^{-md}$  out to second order in  $md$  Eq. (38) becomes

$$L_z = \left( -\frac{eg}{4\pi} + \frac{eg}{4\pi} md - \mathcal{O}(m^2 d^2) \right) n_z. \quad (39)$$

On taking the limit  $m \rightarrow 0$ , we recover the standard result which is independent of the distance between the two particles. We have calculated the angular momentum of the fields using four-vector notation. The entire calculation can be done in three-vector form by taking the results for the three-vector form of  $T^{0i}$  and plugging them into Eq. (34) to give

$$\mathbf{L} = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) d^3x, \quad (40)$$

for the angular momentum in the  $\mathbf{E}$  and  $\mathbf{B}$  fields. Expressions for the  $\mathbf{E}$  and  $\mathbf{B}$  fields can be obtained using the potentials of Eq. (35):

$$\begin{aligned} \mathbf{E} &= -\frac{e}{4\pi} \frac{\mathbf{r}'}{r'^3}, \\ \mathbf{B} &= -\frac{g}{4\pi} \frac{e^{-mr}}{r^2} \left( \frac{1}{r} + m \right) \mathbf{r}. \end{aligned} \quad (41)$$

Inserting these expressions into Eq. (40) one can go through the same steps which were taken in the covariant notation to rederive the result for the angular momentum in the  $\mathbf{E}$  and  $\mathbf{B}$  fields of the configuration.

One of the most interesting results of Dirac's string monopole theory is the quantization condition between electric and magnetic charges:

$$\frac{eg}{4\pi} = \frac{n}{2}. \quad (42)$$

[Here, we are only concerned with magnitude, as opposed to Eq. (39) which also gives the direction of  $\mathbf{L}$ ]  $n$  is an integer, and we have set  $\hbar=1$ —it would appear in the numerator of the right-hand side of Eq. (42). This condition arises from the requirement that the wave function of a particle in the presence of the Dirac string be single valued.<sup>2</sup> Thus, even in a first quantized theory (i.e., where the particles are quantized but the fields are treated classically), one has a restriction between electric and magnetic charges. This condition has the physical consequence that the singular string variable has no physical effect on the particles of the theory. It makes the string invisible to the particle. In the two-potential approach we have replaced the nonlocal and singular string with a local and nonsingular gauge field. Since there is no prohibition against having particles interact with the nonsingular gauge field,  $C_\mu$  one does not get a restriction between the electric and magnetic charges as in Eq. (42). However, when the fields,  $A_\mu$  and  $C_\mu$  are quantized (i.e., second quantization), one recovers such a condition, although for the case when the magnetic photon is massive this condition is considerably different from Dirac's condition of Eq. (42).

The basic argument, which is due to Saha and Wilson,<sup>13</sup> is that when the gauge fields are treated as quantum fields, the angular momentum carried in the field configuration of an electric charge and a magnetic charge must be quantized in integer multiples of  $\hbar/2$ . Taking the result of Eq. (38) in conjunction with this quantum restriction on the angular momentum yields the condition

$$\frac{eg}{2\pi m^2 d^2} [1 - (1 + md)e^{-md}] = \frac{n}{2}, \quad (43)$$

where  $n$  is an integer. Only the magnitude of  $L_z$  from Eq. (38) is taken, and as stated previously we are setting  $\hbar=1$ . We shall address most of our comments to the  $n=1$  case. In the limit  $m \rightarrow 0$  we just recover the quantization condition of Eq. (42). When  $m \neq 0$  we get a new quantization condition which involves not only the magnitude of the charges,  $e$  and  $g$ , but the mass of the magnetic photon and the distance between the two particles. Comparing the two conditions from Eq. (42) and Eq. (43) one notices that the magnitude of the magnetic charge,  $g$ , must always be larger for the latter condition. Physically this is easily understood since in the case of the massive magnetic photon the Yukawa field of the magnetically charged particle falls off more rapidly than the equivalent Coulomb case, so in order for the configuration to still have an angular momentum of  $1/2$  we need to have a larger magnetic charge,  $g$ . Thus one can say that as the mass  $m$  increases the magnitude of the magnetic charge must also increase in order to have a minimum angular momentum of  $1/2$ . A caveat to this statement is that one can also change the angular momentum by changing the distance  $d$ . In general one can increase the angular momentum of the charge-monopole system by decreasing the separation between them. Decreasing the distance between the particles allows the electric charge to "see" more of the magnetic charge. It is easy to see that the condition imposed by Eq. (43) is very complicated since it depends on three variables,  $g$ ,  $m$ , and  $d$  ( $e$  is assumed to be fixed to the magnitude of the electron's

charge). The most unusual result however is that if one specifies some mass for the magnetic photon and some magnitude for the magnetic charge, then the condition of Eq. (43) restricts the separation distances between the charge and monopole to be only certain discrete values. This quantization of the separation distance is in a sense a kinematic restriction, since we can think of no dynamical reason that would force the charge-monopole system to have such discrete separations.

To conclude this section, we will give a speculative argument which suggests that in any monopole theory the photon may acquire a mass. Usually gauge bosons are said to be massless due to the gauge invariance. However, one should add to this statement the requirement that the coupling constant be enough small so that perturbation theory is valid.<sup>15</sup> Some good, but nonrigorous arguments have been given by Wilson<sup>16</sup> and Guth<sup>17</sup> which suggest that there is some critical value of electric charge below which the photon is massless and above which the photon acquires a mass. This critical value is not specified by their analysis, but, due to the smallness of the electric charge of all the known particles, it seems reasonable to assume that this critical value is greater than the known electric charge magnitude. The success of perturbation theory for the electromagnetic interactions of the electron also indicates that this is a good assumption. In quantum electrodynamics in one space and one time dimension Schwinger<sup>18</sup> has shown rigorously that this phenomenon of the photon acquiring a mass does in fact occur. Although Schwinger's 1+1 quantum electrodynamics shows this dynamical mass generation for the photon, it is not clear what relevance it has to theories in 3+1 dimensions. In particular, in Schwinger's model one finds that the critical value of the electric charge is zero,<sup>19</sup> (i.e., the photon will develop a mass if  $e \neq 0$  in 1+1 dimensions). With these motivating statements let us look at the magnetic charge. Since we are trying to argue that the photon in any formulation of a magnetic charge theory will become massive, we will not from the outset assume a mass for the magnetic photon. This means that we have the same quantization condition for both the string and the two-potential theories. Thus, in either case, the magnitude of the magnetic charge is large (i.e., at the minimum  $g = 2\pi/e$  which is large since  $e$  is small), and we are well out of the regime where perturbation theory is valid. Therefore based on the conjectures of Wilson and Guth it is not unreasonable to suggest that the photon may acquire a mass in the presence of magnetic charge with such a large magnitude. This mechanism, which could generate a mass for the photon, can be compared to the technicolor theory of particle physics,<sup>20</sup> which also give a mechanism for dynamically generating masses for the  $W^\pm$  and  $Z$  gauge bosons of the standard model. In technicolor theories one has techniquarks instead of magnetic monopoles, and a super strong "color" force instead of a super strong electromagnetic force.

Coupling Wilson and Guth's conjecture with the duality of electric and magnetic charge (which implies that it should not make a difference whether the large, nonperturbative coupling is electric or magnetic), one is led to the conclusion that in the presence of large magnetic charge the photon may become massive. This could be viewed as a loose argument against a theory of magnetic charge with only a single photon. (The reason for saying that this is only a loose argument, in addition to the fact that Wilson and Guth's conjecture has not been rigorously proven, is that one could always claim

that the mass given to the photon is below the experimental upper limit. However, since the upper bound on the photon's mass is very stringent, this is not a strong counterargument.) The two photon theory of magnetic charge, however, is still viable under this conjectured mechanism for photon mass generation, since one of the photons (i.e., the magnetic photon) can become massive while the other photon (i.e., the electric photon) remains massless. This is in direct analogy with the standard model where the  $Z$  boson acquires a mass while the photon remains massless.

## V. DISCUSSION AND CONCLUSIONS

Based on an old idea of Cabibbo and Ferrari,<sup>5</sup> we have reviewed the covariant Lagrangian formalism of electromagnetism with magnetic charge, which employs two four-vector potentials,  $A_\mu$  and  $C_\mu$ . The Hamiltonian formulation of this approach to magnetic charge, as well as some of the quantum mechanical aspects of this theory can be found in two articles by Barker and Graziani.<sup>8</sup> This strategy has several differences from the usual Dirac formulation of magnetic charge: (a) It allows one to extend the dual symmetry down to the level of the gauge potentials, and it treats magnetic and electric charges in the same way (i.e., both are gauge charges); (b) it deals only with local, nonsingular variables; and (c) since there is no singular string in the theory, there is no quantization condition on the charges in a first quantized theory. If the gauge fields are quantized one recovers a quantization condition which is the same as Dirac's condition only if the magnetic photon is massless. If the magnetic photon is massive one obtains an unusual quantization condition which involves not only the electric and magnetic charge magnitudes, but also the mass of the magnetic photon as well as the distance between the charges. The main disadvantage of this two potential idea is that there is no single Lagrangian from which both the field equations and the Lorentz equations can be derived. The field equations, in terms of the two potentials, can be derived from the Lagrangian of Eq. (14), which is a straightforward extension of the usual field Lagrangian in electromagnetism with only electric charge. According to a result by Rohrlich,<sup>12</sup> it is not possible to use a single Lagrangian to derive both the generalized Lorentz force equations and the field equations when one has both electric and magnetic charges. Here, we side step this problem by not deriving the Lorentz equations via a Lagrangian, but rather from the requirement that the theory respect the dual invariance of Eq. (2). These generalized Lorentz equations in turn allowed one to arrive at an expression for the energy-momentum tensor. A different approach to this problem of the Lorentz force equations can be found in Ref. 21 where the Lagrangian is modified so that it becomes possible to derive the covariant form of the energy-momentum tensor directly from the Lagrangian. Then, since the Lorentz equations imply the form of the energy-momentum tensor and visa versa, it is possible to obtain the Lorentz equations by taking the four divergence of the energy-momentum tensor obtained from the modified Lagrangian. This is similar to the situation in general relativity where the particle equations of motion follow from the field equations rather than being distinct from them.<sup>22</sup> To use this approach, one can add

$$\mathcal{L}_{\text{extra}} = -\frac{1}{4}\mathcal{F}^{\mu\nu}(\partial_\mu C_\nu - \partial_\nu C_\mu) + \frac{1}{4}\mathcal{G}^{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu) \quad (44)$$

to the Lagrangian of Eq. (14). As was mentioned at the end of Sec. II, these added terms will not change Maxwell's



equations since they are total four divergences. Formally, the energy-momentum tensor of Eq. (31) is obtained by treating  $A_\mu$ ,  $C_\mu$ ,  $\mathcal{F}_{\mu\nu}$ , and  $\mathcal{G}_{\mu\nu}$  as independent variables. Then by adding four divergences at the appropriate points, we arrive at the expression from Eq. (31). As already pointed out, taking the four divergence of the energy-momentum tensor then yields the covariant form of the Lorentz equations. In the simple development given in this paper the general Lorentz equations and the energy-momentum tensor are obtained by assuming that the usual form for the Lorentz equations with only electric charge, and then deriving the general form using the dual symmetry. In this way the dual theory does not treat the field equations and the particle equations equivalently since the field equations can be obtained from a Lagrangian, while the particle equations are gotten by requiring that the dual symmetry hold.

By introducing a second, pseudo four-vector potential,  $C_\mu$ , it is possible to obtain a formulation of electrodynamics with both electric and magnetic charges, which is an alternative to Dirac's string approach. The basic difference between the two-potential approach and the string approach is that one replaces a nonlocal, singular variable (i.e., the Dirac string) with a local, nonsingular variable (i.e., the pseudo four-potential  $C_\mu$ ). This two-potential approach is not new, but usually it is argued that the second potential does not represent a second photon. This is done by introducing extra conditions on the gauge potentials<sup>7</sup> so that the number of degrees of freedom reduce to those of only one photon. Here, we take the view point that this second potential does represent a second, magnetic photon, which has the opposite behavior under parity from that of the usual photon, and we develop what electrodynamics would look like in a hypothetical world that did have two massless photons. To make the theory more realistic, one could easily apply the Higgs mechanism to give the magnetic photon a mass, taking it out of the observable particle spectrum until a certain energy scale had been reached. Finally, a speculative argument is given which implies that the large magnitude of the magnetic charge in either the string approach or the two potential approach leads to a dynamical generation of mass for the photon. In a theory with only one photon, this is bad since we know that there is one massless photon. In a theory of magnetic charge with two photons, however, it can be arranged so that one photon becomes massive while the other remains massless.

## ACKNOWLEDGMENTS

The author would like to thank David New and Marla Cantor for help and encouragement during the writing of this paper. Additional thanks goes to A. Yoshida who has discussed many aspects of this paper with the author.

- <sup>a)</sup>Present address: D. Singleton, Dept. of Physics, Virginia Commonwealth University, Richmond, VA 23284-2000; E-mail: dasingle@hibbs.vcu.edu.
- <sup>1</sup>J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), 2nd ed. p. 251.
- <sup>2</sup>P. A. M. Dirac, "Quantised Singularities in the Electromagnetic Field," *Proc. R. Soc. A* **133**, 60–72 (1931); P. A. M. Dirac, "The Theory of Magnetic Poles," *Phys. Rev.* **74**, 817–830 (1948).
- <sup>3</sup>T. T. Wu and C. N. Yang, "Concept of Nonintegrable Phase Factor and Global Formulation of Gauge Fields," *Phys. Rev. D* **12**, 3845–3857 (1975).
- <sup>4</sup>G. 't Hooft, "Magnetic Monopoles in Unified Gauge Theories," *Nucl. Phys. B* **79**, 276–284 (1974); A. M. Polyakov, "Particle Spectrum in Quantum Field Theory," *JETP Lett.* **20**, 194–195 (1974).
- <sup>5</sup>N. Cabibbo and E. Ferrari, "Quantum Electrodynamics with Dirac Monopoles," *Nuovo Cimento* **23**, 1147–1154 (1962).
- <sup>6</sup>C. R. Hagen, "Noncovariance of Dirac Monopole," *Phys. Rev.* **140**, B 804–B 810 (1965).
- <sup>7</sup>D. Zwanzinger, "Local-Lagrangian Field Theory of Electric and Magnetic Charges," *Phys. Rev. D* **3**, 880–891 (1971).
- <sup>8</sup>W. Barker and F. Graziani, "Quantum Mechanical Formulation of Electron-Monopole Interaction without Dirac Strings," *Phys. Rev. D* **18**, 3849–3857 (1978); "A Heuristic Potential Theory of Electric and Magnetic Monopoles without Strings," *Am. J. Phys.* **46**, 1111–1115 (1978).
- <sup>9</sup>P. W. Higgs, "Broken Symmetries, Massless Particles and Gauge Fields," *Phys. Lett.* **12**, 132–133 (1964); "Broken Symmetries and the Masses of Gauge Bosons," *Phys. Rev. Lett.* **13**, 508–509 (1964); "Spontaneous Symmetry Breaking without Massless Bosons," *Phys. Rev.* **145**, 1156–1163 (1966).
- <sup>10</sup>D. Singleton, "Magnetic Charge as a Hidden Gauge Symmetry," *Int. J. Theor. Phys.* **34**, 37–46 (1995).
- <sup>11</sup>J. Arafune, P. G. O. Freund, and C. J. Goebel, "Topology of Higgs Field," *J. Math. Phys.* **16**, 433–437 (1975).
- <sup>12</sup>F. Rohlich, "Classical Theory of Magnetic Monopoles," *Phys. Rev.* **150**, 1104–1111 (1966).
- <sup>13</sup>M. N. Saha, "On the Origin of Mass in Neutrons and Protons," *Ind. J. Phys.* **10**, 145–151 (1936); M. N. Saha, "Note on Dirac's Theory of Magnetic Poles," *Phys. Rev.* **75**, 1968 (1949); H. A. Wilson, "Note on Dirac's Theory of Magnetic Poles," *ibid. Rev.* **75**, 309 (1949).
- <sup>14</sup>E. F. Carter and H. A. Cohen, "Classical Problem of Charge and Pole," *Am. J. Phys.* **41**, 994–1005 (1974).
- <sup>15</sup>Kerson Huang, *Quarks, Leptons and Gauge Fields* (World Scientific, Singapore, 1982), p. 50.
- <sup>16</sup>K. Wilson, "Confinement of Quarks," *Phys. Rev. D* **10**, 2445–2459 (1974).
- <sup>17</sup>A. Guth, "Existence Proof of a Nonconfining Phase in Four Dimensional  $U(1)$  Lattice Gauge Theory," *Phys. Rev. D* **21**, 2291–2307 (1980).
- <sup>18</sup>J. Schwinger, "Gauge Invariance and Mass. II," *Phys. Rev.* **128**, 2425–2429 (1962).
- <sup>19</sup>B. Holstein, "Anomalies for Pedestrians," *Am. J. Phys.* **61**, 142–147 (1993).
- <sup>20</sup>Ta-Pei Cheng and Ling-Fong-Li, *Gauge Theory of Elementary Particle Physics* (Oxford U. P., New York, 1984), p. 402–406.
- <sup>21</sup>W. A. Rodrigues *et al.*, "The Classical Problem of the Charge and Pole Motion. A Satisfactory Formalism by Clifford Algebras," *Phys. Lett. B* **220**, 195–199 (1989).
- <sup>22</sup>A. Einstein, L. Infeld, and B. Hoffmann, "The Gravitational Equations and the Problem of Motion," *Ann. Math.* **39**, 65–100 (1938); A. Einstein, and L. Infeld, "On the Motion of Particles in General Relativity Theory," *Can. J. Math.* **1**, 209–241 (1949).

## COMPLEMENTARITY

Bohr was not satisfied to limit the idea of complementarity to physics. He saw it everywhere: instinct and reason, free will, love and justice, and on and on. Pais remarks that he finds this way of viewing things "liberating." I have always found it to be a dead end.

Jeremy Bernstein, *Cranks, Quarks, and the Cosmos* (BasicBooks, New York, 1993), p. 44.