

15.7 (a) Similar to Prob. 15.4, the final velocities of the particles are  $\frac{A_2 \beta}{A_1 + A_2}$  and  $\frac{-A_1 \beta}{A_1 + A_2}$ ,

$$\text{and } \vec{E} = z_1 e \frac{A_2 \vec{\beta} / (A_1 + A_2)}{1 - A_2 \beta \cos \theta / (A_1 + A_2)} + z_2 e \frac{-A_1 \vec{\beta} / (A_1 + A_2)}{1 + A_1 \beta \cos \theta / (A_1 + A_2)}.$$

$$= e \vec{\beta} \left( \frac{z_1 A_2 - z_2 A_1}{A_1 + A_2} + \frac{z_1 A_2^2 + z_2 A_1^2}{(A_1 + A_2)^2} \beta \cos \theta + \dots \right),$$

where we have assumed the fission takes place at  $\vec{r} = 0$ , and we have only retained terms to second order of  $\beta$ . Following the same argument as in Prob. 15.6, we have

$$\frac{d^2 I}{d(\hbar \omega) d\Omega} = \frac{e^2 \beta^2 \sin^2 \theta}{4\pi^2 \hbar c} \left( \frac{z_1 A_2 - z_2 A_1}{A_1 + A_2} + \frac{z_1 A_2^2 + z_2 A_1^2}{(A_1 + A_2)^2} \beta \cos \theta \right)^2$$

$$= \frac{2\beta^2 \sin^2 \theta}{4\pi^2} |p + q \beta \cos \theta|^2,$$

With  $2 = \frac{e^2}{\hbar c}$ ,  $p = \frac{z_1 A_2 - z_2 A_1}{A_1 + A_2}$ ,  $q = \frac{z_1 A_2^2 + z_2 A_1^2}{(A_1 + A_2)^2}$ .

Performing the angular integration,

$$\frac{dI}{d(\hbar \omega)} = \int \frac{d^2 I}{d(\hbar \omega) d\Omega} d\Omega = \frac{2\beta^2}{4\pi^2} 2\pi \int_{-1}^1 \sin^2 \theta (p^2 + 2pq \beta \cos \theta + q^2 \beta^2 \cos^2 \theta) d(\cos \theta)$$

$$= \frac{2\beta^2}{2\pi} \left( \frac{4}{3} p^2 + \frac{4}{15} q^2 \beta^2 \right) = \frac{2\beta^2}{3\pi} \left( p^2 + \frac{q^2 \beta^2}{5} \right).$$