13.6 (a) In the nonrelativistic limit, $\beta \ll 1$, then $\lambda = \omega/v$, which is real. Equation (13.36) becomes

$$\left(\frac{dE}{dx}\right)_{b>k_0^{-1}} = \frac{2}{\pi}\frac{z^2e^2}{v^2}\int_0^{+\infty} \mathrm{Re}\left(\frac{i\omega}{\epsilon(\omega)}\right)\left(\frac{\omega}{k_0 v}\right)K_1\left(\frac{\omega}{k_0 v}\right)K_0\left(\frac{\omega}{k_0 v}\right)d\omega .$$

Using the small limit approximation of the modified Bessel function, we have

$$x K_1(x) K_0(x) \sim \log\left(\frac{1.123}{x}\right),$$

which leads to the expression of the energy loss as

$$\left(\frac{dE}{dx}\right)_{k_0 b>1} \simeq \frac{2z^2e^2}{\pi v^2}\int_0^{+\infty} \mathrm{Re}\left(\frac{i\omega}{\epsilon(\omega)}\right)\log\left(\frac{1.123\,k_0 v}{\omega}\right)d\omega .$$

(b) Given the form of the dielectric constant,

$$\frac{1}{\epsilon(\omega)} = \frac{\omega^2 + i\omega\Gamma}{\omega^2 - \omega_p^2 + i\omega\Gamma} = \frac{\frac{\omega^2}{\omega_p^2} + \frac{i\omega}{\omega_p}\frac{\Gamma}{\omega_p}}{\left(\frac{\omega^2}{\omega_p^2} - 1\right) + \frac{i\omega}{\omega_p}\frac{\Gamma}{\omega_p}}$$

In the limit of $\Gamma \ll \omega_p$, we can approximately write the above equation as

$$\frac{1}{\epsilon(\omega)} \to \mathrm{P.V.}\left(\frac{\omega^2}{\omega^2-\omega_p^2}\right) - i\pi\frac{\omega^2}{\omega_p^2}\delta\left(\frac{\omega^2}{\omega_p^2}-1\right)$$

Then,

$$\left(\frac{dE}{dx}\right)_{k_0 b>1} \simeq \frac{2z^2e^2}{\pi v^2}\int_0^{+\infty} \omega\cdot\pi\frac{\omega^2}{\omega_p^2}\delta\left(\frac{\omega^2}{\omega_p^2}-1\right)\log\left(\frac{1.123\,k_0 v}{\omega}\right)d\omega$$

$$= \frac{2z^2e^2}{v^2}\int_0^{+\infty} \omega\cdot\frac{\omega^2}{\omega_p^2}\frac{\omega_p^2}{2\omega}\delta(\omega-\omega_p)\log\left(\frac{1.123\,k_0 v}{\omega}\right)d\omega$$

$$= \frac{z^2e^2}{v^2}\omega_p^2\log\left(\frac{1.123\,k_0 v}{\omega_p}\right).$$

Combine with the result from Prob 13.5, the total energy loss becomes

$$\frac{dE}{dx} = \frac{z^2e^2}{v^2}\omega_p^2\log\left(\frac{1}{\Lambda_1 k_0 b_{min}}\right) + \frac{z^2e^2}{v^2}\omega_p^2\log\left(\frac{\Lambda_2 k_0 v}{\omega_p}\right) = \frac{z^2e^2}{v^2}\omega_p^2\log\left(\frac{\Lambda v}{\omega_p b_{min}}\right),$$

where Λ is a constant of order 1.