15.3 As in Prob. 15.2, we know

$$\vec{\beta}_i = \beta \vec{e}_z, \quad \vec{\beta}_f = \beta\left(\sin\theta_p \cos\phi_p \vec{e}_x + \sin\theta_p \sin\phi_p \vec{e}_y + \cos\theta_p \vec{e}_z\right),$$

$$\vec{n} = \sin\theta \vec{e}_x + \cos\theta \vec{e}_z, \quad \vec{\varepsilon}_1 = \cos\theta \vec{e}_x - \sin\theta \vec{e}_z, \quad \vec{\varepsilon}_2 = \vec{e}_y.$$

Then,
$$\vec{\beta}_i \cdot \vec{n} = \beta\cos\theta, \quad \vec{\beta}_f \cdot \vec{n} = \beta\left(\sin\theta \sin\theta_p \cos\phi_p + \cos\theta \cos\theta_p\right)$$

$$\vec{\varepsilon}_1^* \cdot \vec{\beta}_i = -\beta\sin\theta, \quad \vec{\varepsilon}_1^* \cdot \vec{\beta}_f = \beta\left(\cos\theta \sin\theta_p \cos\phi_p - \sin\theta \cos\theta_p\right),$$

$$\vec{\varepsilon}_2^* \cdot \vec{\beta}_i = 0, \quad \vec{\varepsilon}_2^* \cdot \vec{\beta}_f = \beta \sin\theta_p \sin\phi_p.$$

Using Eq. (15.2), the photon cross section is

$$\frac{d^3\sigma}{d\Omega_p \, d(\hbar\omega)\, d\Omega} = \frac{k^2}{4} \cdot \frac{e^2}{4\pi^2 c} \frac{\beta^2}{\hbar\omega}\left[\left(\frac{\cos\theta \sin\theta_p \cos\phi_p - \sin\theta \cos\theta_p}{1-\beta\left(\sin\theta \sin\theta_p \cos\phi_p + \cos\theta \cos\theta_p\right)} + \frac{\sin\theta}{1-\beta\cos\theta}\right)^2 + \frac{\sin^2\theta_p \sin^2\phi_p}{\left[1-\beta\left(\sin\theta \sin\theta_p \cos\phi_p + \cos\theta \cos\theta_p\right)\right]^2}\right].$$

All we need to do now is to perform the integration w.r.t. the solid angle Ω_p, with the help from Mathematica. (TODO)