

**1.7** Solution: Choose the origin to be the center of the conductor with radius  $a_1$  and the center of the other conductor is located on the  $x$ -axis at  $d$ . Assume the conductors are parallel to the  $z$ -axis, with linear charge density  $\lambda$  and  $-\lambda$ , respectively. The electric field along the  $x$ -axis is pointing in the positive  $x$  direction is approximately,

$$E(x) = \frac{\lambda}{2\pi\epsilon_0} \left( \frac{1}{x} + \frac{1}{d-x} \right).$$

Then, the potential difference between the two conductors is

$$V = \int_{a_1}^{d-a_2} E(x)dx = \frac{\lambda}{2\pi\epsilon_0} \int_{a_1}^{d-a_2} \left( \frac{1}{x} + \frac{1}{d-x} \right) dx = \frac{\lambda}{2\pi\epsilon_0} \log \left( \frac{(d-a_1)(d-a_2)}{a_1a_2} \right).$$

Since  $d \ll a_1, a_2$ , and let  $a^2 = a_1a_2$ , we can write the potential difference as

$$V \simeq \frac{\lambda}{2\pi\epsilon_0} \log \left( \frac{d^2}{a^2} \right) = \frac{\lambda}{\pi\epsilon_0} \log \left( \frac{d}{a} \right).$$

Therefore, the capacitance is

$$C = \frac{\lambda}{V} = \pi\epsilon_0 \left[ \log \left( \frac{d}{a} \right) \right]^{-1}.$$