10.12 (a) Using the same geometry as in Fig. 10.13, the incident wave's electric field is

$$\vec{E}_i = E_0 \vec{\varepsilon}_2 e^{ik(z\cos\alpha + x\sin\alpha)}$$

At the opening. $(\vec{n} \times \vec{E}_i)_{z=0} = -E_0 \vec{\varepsilon}_1 e^{ikx\sin\alpha}$ Then the scattered electric field becomes

$$\vec{E}(\vec{x}) = \frac{ie^{ikr}}{2\pi r} \vec{k} \times \int_{S_1} \vec{n} \times \vec{E}_i(\vec{x}') e^{-i\vec{k}\cdot\vec{x}'} da'$$

$$= -\frac{ie^{ikr}}{2\pi r} (\vec{k} \times \vec{\varepsilon}_1) \int_0^a \rho d\rho \int_0^{2\pi} d\beta \exp\{ik\rho[\sin\alpha\cos\beta - \sin\theta\cos(\phi-\beta)]\},$$

$$= -\frac{ie^{ikr}}{r} a^2 E_0 (\vec{k} \times \vec{\varepsilon}_1) \frac{J_1(ka\xi)}{ka\xi},$$

where ξ is defined as exactly in the book. The time-averaged diffracted power is given by

$$\frac{dP}{d\Omega} = P_i \frac{(ka)^2}{4\pi} (\cos^2\theta + \sin^2\theta \sin^2\phi) \left| \frac{2J_1(ka\xi)}{ka\xi} \right|^2,$$

With $P_i = \frac{E_0^2}{2Z_0} \pi a^2$. Here, we have used the result $\vec{k} \times \vec{\varepsilon}_1 = k(\hat{j}\cos\theta - \hat{k}\sin\theta\sin\phi)$.

(b) Comparing with Section 10.9, we can see that the polarization of the diffracted wave is different. Also, the electric field is normally incident, we don't have the $\cos\alpha$ term.