1.13 Solution: To apply the Green's reciprocation theorem from Problem 1.12, consider two separate charge-potential configurations.

(i) As described, there are two infinite grounded parallel conducting planes located at $z = 0$ and $z = d$, with a point charge a located at $z = z_0$ on the z-axis. In this case, we have charge configuration as

$$\rho(\mathbf{x}) = q\delta(x)\delta(y)\delta(z - z_0), \quad \sigma(\mathbf{x}) = \sigma_U(x, y)\delta(z - d) + \sigma_L(x, y)\delta(z),$$

where σ_U and σ_L are unknown surface charge densities on the upper and lower planes. For the potential,

$$\Phi(z_0), \quad \Phi(d) = \Phi(0) = 0,$$

where $\Phi(z_0)$ is unknown.

(ii) For the same two infinite parallel conducting planes located at $z = 0$ and $z = d$, set $\Phi'(0) = 0$ and $\Phi'(d) = V$. Then, it is obvious that $\Phi'(z_0) = (z_0/d)V$. In the meantime, no charge is introduced into this configuration, *i.e.*, $\rho'(\mathbf{x}) = 0$ and $\sigma'(\mathbf{x}) = 0$.

Now, apply the Green's reciprocation theorem,

$$\int_V \rho\Phi' d^3x + \oint_S \sigma\Phi' da = \int_V \rho'\Phi d^3x + \oint_S \sigma'\Phi da,$$

we can see that the left hand side is

$$q\Phi'(z_0) + \int\int \sigma_U(x, y)\Phi'(d)dxdy = q\frac{z_0}{d}V + Q_U V,$$

where $Q_U = \int\int \sigma_U(x, y)(d)dxdy$ is the total charge induced on the upper plane, and the right hand side is simply 0, as there is no charge at all. Therefore,

$$q\frac{z_0}{d}V + Q_U V = 0,$$

which yields

$$Q_U = -q\frac{z_0}{d}.$$