8.19 (a). The current ~~can~~ be expressed as $\vec{J} = \delta(x-X)\delta(z)\cdot I_0 \sin\left(\frac{\omega}{c}(h-y)\right)\hat{y}$. To calculate the coefficient for mode λ, it can be seen that only the E_y component will contribute.

For TM and TE modes, we have

$$E_{y\,mn} = \frac{2\pi}{\gamma_{mn}\sqrt{ab}}\begin{Bmatrix} 1/b \\ f/a \end{Bmatrix} \sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right),$$

where $f = 1/\sqrt{2}$, if n is 0, $m>0$. Then,

$$A_{mn} = -\frac{Z_\lambda}{2}\int \vec{J}\cdot\vec{E}_{mn}\,d^3x = -\frac{\pi}{\gamma_{mn}\sqrt{ab}}\begin{Bmatrix} Z_{TM}\,\pi/b \\ Z_{TE}\,f\pi/a \end{Bmatrix}\int_0^h I_0\sin\left(\frac{m\pi X}{a}\right)\sin\left(\frac{\omega}{c}(h-y)\right)\cos\left(\frac{n\pi y}{b}\right)dy$$

$$= \frac{\pi}{2\gamma_{mn}\sqrt{ab}}\begin{Bmatrix} Z_{TM}/b \\ Z_{TE}\,f/a \end{Bmatrix} I_0\sin\left(\frac{m\pi X}{a}\right)\int_0^h\left(\sin\left(\frac{\omega}{c}h+\left(\frac{n\pi}{b}-\frac{\omega}{c}\right)y\right)+\sin\left(\frac{\omega}{c}h-\left(\frac{n\pi}{b}+\frac{\omega}{c}\right)y\right)\right)dy$$

$$= \frac{\pi}{2\gamma_{mn}\sqrt{ab}}\begin{Bmatrix} Z_{TM}/b \\ Z_{TE}\,f/a \end{Bmatrix} I_0\sin\left(\frac{m\pi X}{a}\right)\left[\frac{1}{\frac{n\pi}{b}+\frac{\omega}{c}}-\frac{1}{\frac{n\pi}{b}-\frac{\omega}{c}}\right]\left(\cos\left(\frac{n\pi h}{b}\right)-\cos\left(\frac{\omega}{c}h\right)\right)$$

$$= \frac{\pi}{\gamma_{mn}\sqrt{ab}}\begin{Bmatrix} Z_{TM}/b \\ Z_{TE}\,f/a \end{Bmatrix} I_0\sin\left(\frac{m\pi X}{a}\right)\frac{-\omega/c}{\left(\frac{n\pi}{b}\right)^2-\left(\frac{\omega}{c}\right)^2}\left(\cos\left(\frac{n\pi h}{b}\right)-\cos\left(\frac{\omega}{c}h\right)\right)$$

where $\gamma_{mn} = \pi\left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)^{1/2}$. Therefore, for $m\gg1$ and $n\gg1$, the amplitude decays as $\frac{1}{m}$ and $\frac{1}{n^3}$, respectively.

(b) For TE_{10} mode, $n=0$. $f = 1/\sqrt{2}$, $\gamma_{10} = \pi/a$, $Z_{TE} = \frac{\mu\omega}{k}$

$$A_{10} = \frac{\mu\omega}{\sqrt{2}k\sqrt{ab}}\,\frac{c}{\omega}\,I_0\sin\left(\frac{\pi X}{a}\right)\left(1-\cos\left(\frac{\omega h}{c}\right)\right) = \frac{\sqrt{2}\mu c}{k\sqrt{ab}}\,I_0\sin\left(\frac{\pi X}{a}\right)\sin^2\left(\frac{\omega h}{2c}\right).$$

Therefore, the transmitted power is

$$\frac{1}{2}\int_A (A_{10}\vec{E}_{10}\times A_{10}\vec{H}_{10})\cdot\hat{z}\,da = \frac{A_{10}^2}{2}\int_A(\vec{E}_{10}\times\vec{H}_{10})\cdot\hat{z}\,da = \frac{A_{10}^2}{2Z_{TE}}$$

$$= \frac{\mu^2c^2I_0^2}{k^2ab}\cdot\frac{k}{\mu\omega}\sin^2\left(\frac{\pi X}{a}\right)\sin^4\left(\frac{\omega h}{2c}\right) = \frac{\mu c^2 I_0^2}{k\omega ab}\sin^2\left(\frac{\pi X}{a}\right)\sin^4\left(\frac{\omega h}{2c}\right).$$