

$$5.33 \quad (a) \quad \vec{F}_{12} = - \frac{\mu_0}{4\pi} I_1 I_2 \oint \oint \frac{(d\vec{l}_1 \cdot d\vec{l}_2) \vec{r}_{12}}{|\vec{r}_{12}|^3}$$

Since $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 + \vec{r}$, and

$$\nabla_{\vec{r}} \left(\frac{1}{|\vec{r}_1 - \vec{r}_2 + \vec{r}|} \right) = - \frac{\vec{r}_1 - \vec{r}_2 + \vec{r}}{|\vec{r}_1 - \vec{r}_2 + \vec{r}|^3} = - \frac{\vec{r}_{12}}{|\vec{r}_{12}|^3}$$

We have $\vec{F}_{12} = I_1 I_2 \nabla_{\vec{r}} \left(\frac{\mu_0}{4\pi} \oint \oint \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_1 - \vec{r}_2 + \vec{r}|} \right) = I_1 I_2 \nabla_{\vec{r}} M_{12}(\vec{r})$

$$(b) \quad \nabla_{\vec{r}}^2 \left(\frac{1}{|\vec{r}_1 - \vec{r}_2 + \vec{r}|} \right) = -4\pi \delta(\vec{r}_1 - \vec{r}_2 + \vec{r})$$

Since $\vec{r} \neq \vec{r}_2 - \vec{r}_1$, unless two loops touch. Therefore

$$\nabla_{\vec{r}}^2 \left(\frac{1}{|\vec{r}_1 - \vec{r}_2 + \vec{r}|} \right) = 0, \quad \text{and} \quad \nabla_{\vec{r}}^2 M(\vec{r}) = 0$$