14.4 (a) $\vec{v}(t) = \dot{\vec{x}}(t) = -a\omega_0 \sin(\omega_0 t)\,\hat{z}$, $\dot{\vec{v}}(t) = -a\omega_0^2 \cos(\omega_0 t)\,\hat{z}$.

Since $\vec{v}$ and $\dot{\vec{v}}$ are in the same direction, we can use Eq. (14.21),

$$\frac{dP(t)}{d\Omega} = \frac{e^2}{4\pi c^3}|\dot{\vec{v}}|^2 \sin^2\theta = \frac{e^2 a^2 \omega_0^4}{4\pi c^3}\cos^2(\omega_0 t)\sin^2\theta.$$

and the time averaged power radiated per unit solid angle is,

$$\left\langle \frac{dP}{d\Omega} \right\rangle = \frac{e^2 a^2 \omega_0^4}{8\pi c^3}\sin^2\theta, \qquad \text{since} \quad \langle \cos^2(\omega_0 t)\rangle = 1/2.$$

(b) $\vec{x}(t) = R(\cos(\omega_0 t), \sin(\omega_0 t))$, $\dot{\vec{v}}(t) = -\omega_0^2 R(\cos(\omega_0 t), \sin(\omega_0 t))$.

The angle Θ between $\dot{\vec{v}}(t)$ and $\vec{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ can be determined by

$$\cos\Theta = \dot{\vec{v}}(t)\cdot\vec{n} / |\dot{\vec{v}}(t)| = -\sin\theta\cos(\phi - \omega_0 t)$$

Then,
$$\frac{dP(t)}{d\Omega} = \frac{e^2}{4\pi c^3}|\dot{\vec{v}}|^2 \sin^2\Theta = \frac{e^2\omega_0^4 R^2}{4\pi c^3}\left(1 - \sin^2\theta\cos^2(\phi - \omega_0 t)\right)$$

and
$$\left\langle \frac{dP(t)}{d\Omega} \right\rangle = \frac{e^2\omega_0^4 R^2}{4\pi c^3}\left(1 - \frac{1}{2}\sin^2\theta\right).$$