9.11 The charge density is given by $\rho(\vec{x},t) = q\left[2\delta(z) - \delta(z - a\cos\omega t) - \delta(z + a\cos\omega t)\right]\delta(x)\delta(y)$

We first calculate the dipole moment.

$$\vec{p} = \int \vec{x}\,\rho(\vec{x},t)\,d^3x = \int \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} \rho(\vec{x},t)\,d^3x .$$

The x- and y-components are clearly zero. The z-component is also zero, as can be easily verified.

Now, the quadruple moments. Again, the off diagonal moments must be zero due to the $\delta(x)$ and $\delta(y)$ terms. For the diagonal ones,

$$\begin{Bmatrix} Q_{xx} \\ Q_{yy} \\ Q_{zz} \end{Bmatrix} = \int \begin{Bmatrix} 3x^2 - r^2 \\ 3y^2 - r^2 \\ 3z^2 - r^2 \end{Bmatrix} \rho(\vec{x},t)\,d^3x = q\int \begin{Bmatrix} -z^2 \\ -z^2 \\ 2z^2 \end{Bmatrix} \left[2\delta(z) - \delta(z - a\cos\omega t) - \delta(z + a\cos\omega t)\right] dz$$

$$= q \begin{Bmatrix} -2a^2\cos^2(\omega t) \\ -2a^2\cos^2(\omega t) \\ 4a^2\cos^2(\omega t) \end{Bmatrix} ,$$

or $Q_{33} = Q_0 = 2qa^2\left(1 + \cos(2\omega t)\right) = \mathrm{Re}\left[2qa^2\left(1 + e^{-i2\omega t}\right)\right]$, $Q_{xx} = Q_{yy} = -\frac{1}{2}Q_0$.

Using Eq. (9.51) and (9.52),

$$\frac{dP}{d\Omega} = \frac{c^2 Z_0 k^6}{512\pi^2} \cdot 4q^2a^4 \cdot \sin^2\theta\cos^2\theta = \frac{c^2 Z_0 k^6}{128\pi^2} q^2a^4\sin^2\theta\cos^2\theta$$

$$P = \frac{c^2 Z_0 k^6}{960\pi} 4q^2a^4 = \frac{c^2 Z_0 k^6}{240\pi} q^2a^4 ,$$

where $k = 2\omega$.