5.26 The magnetic induction outside a single wire is

$$\vec{B}_0 = \frac{\mu_0 I}{2\pi\rho}\hat{\phi}.$$

and the corresponding vector potential is

$$\vec{A}_0 = -\hat{z}\frac{\mu_0 I}{2\pi}\log\rho$$

Meanwhile, the magnetic induction inside the wire, with radius a, is

$$\vec{B}_i = \frac{\mu_0 I\rho}{2\pi a^2}\hat{\phi}$$

and the vector potential is

$$\vec{A}_i = \hat{z}\left(-\frac{\mu_0 I\rho^2}{4\pi a^2} + C\right)$$

where C is a constant to satisfy the continuity condition of the vector potential. At $\rho = a$ we must have

$$-\frac{\mu_0 I}{4\pi} + C = -\frac{\mu_0 I}{2\pi}\log a$$

which leads to

$$C = \frac{\mu_0 I}{4\pi} - \frac{\mu_0 I}{2\pi}\log a$$

and the vector potential inside the wire is

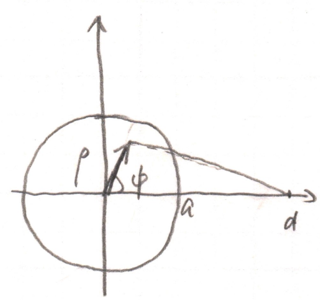
$$\vec{A}_i = \hat{z}\left[\frac{\mu_0 I}{4\pi}\left(1 - \frac{\rho^2}{a^2}\right) - \frac{\mu_0 I}{2\pi}\log a\right]$$

The self magnetic energy for the wire is, per unit length

$$W_{aa} = \frac{1}{2}\int \vec{J}\cdot\vec{A}_i\, d^2x = \frac{I}{2\pi a^2}\int_0^{2\pi} d\phi \int_0^a \rho\, d\rho \left[\frac{\mu_0 I}{4\pi}\left(1 - \frac{\rho^2}{a^2}\right) - \frac{\mu_0 I}{2\pi}\log a\right]$$

$$= \frac{I}{2\pi a^2}\cdot 2\pi\cdot\left[\frac{\mu_0 I}{4\pi}\left(\frac{a^2}{2} - \frac{a^2}{4}\right) - \frac{\mu_0 I}{4\pi}a^2\log a\right] = \frac{\mu_0 I^2}{4\pi}\left(\frac{1}{4} - \log a\right).$$

For the interaction with the other wire



$$W_{ab} = \frac{1}{2}\int \vec{J}\cdot\vec{A}_0\, d^2x$$

$$= \frac{I}{2\pi a^2}\int_0^{2\pi} d\phi \int_0^a \rho\, d\rho \cdot \frac{\mu_0 I}{2\pi}\log\left(\rho^2 + d^2 - 2\rho d\cos\phi\right)^{1/2}$$

$$= \frac{\mu_0 I^2}{8\pi^2 a^2}\int_0^a \rho\, d\rho \int_0^{2\pi} d\phi\left[\log d^2 + \log\left(1 - \frac{2\rho}{d}\cos\phi + \frac{\rho^2}{d^2}\right)\right]$$

$$= \frac{\mu_0 I^2}{2\pi a^2}\int_0^a \rho\log d\, d\rho = \frac{\mu_0 I^2}{4\pi}\log d,$$

Here, we have used the identity

$$\int_0^{\pi} \log\left(1 - 2a\cos\theta + a^2\right) d\theta = \begin{cases} 2\pi \log a, & |a|>1. \\ 0, & \text{otherwise} \end{cases}$$

Then, the magnetic energy for wire a is

$$W_a = W_{aa} + W_{ab} = \frac{\mu_0 I^2}{4\pi}\left(\frac{1}{4} + \log\left(\frac{d}{a}\right)\right)$$

Similarly, for the other wire.

$$W_b = W_{bb} + W_{ba} = \frac{\mu_0 I^2}{4\pi}\left(\frac{1}{4} + \log\left(\frac{d}{b}\right)\right)$$

The total energy is

$$W = W_a + W_b = \frac{\mu_0 I^2}{8\pi}\left(1 + 2\log\left(\frac{d^2}{ab}\right)\right)$$

By the definition of self induction,

$$W = \frac{1}{2} L I^2$$

then

$$L = \frac{\mu_0}{4\pi}\left(1 + 2\log\left(\frac{d^2}{ab}\right)\right)$$