

The vector potential from the loop with radius a is given by

$$A_\phi(r, \theta) = \frac{\mu}{4\pi} \frac{4\pi a}{\sqrt{r^2 + a^2 + 2ars\sin\theta}} \left[\frac{(2-k^2)K(k) - 2E(k)}{k^2} \right]$$

In the configuration, $r^2 = b^2 + d^2$, $r\sin\theta = b$. Therefore, the vector potential on the loop with radius b is

$$A_\phi(b, d) = \frac{\mu}{4\pi} \frac{4\pi a}{\sqrt{(a+b)^2 + d^2}} \left[\frac{(2-k^2)K(k) - 2E(k)}{k^2} \right]$$

with $k^2 = \frac{4ab}{(a+b)^2 + d^2}$

The vector potential has the same direction as the current, and the energy is

$$\begin{aligned} W_{ab} &= \int \vec{J}_b \cdot \vec{A}_a d^3x = \mu I^2 \frac{2ab}{\sqrt{(a+b)^2 + d^2}} \left[\frac{(2-k^2)K(k) - 2E(k)}{k^2} \right] \\ &= \mu I^2 \sqrt{ab} \left[\left(\frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right] \end{aligned}$$

and the mutual inductance is

$$M_{ab} = \frac{W_{ab}}{I^2} = \mu \sqrt{ab} \left[\left(\frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right]$$

When $d \ll a, b$ and $a \approx b$, $k \rightarrow 1$. The mutual inductance diverges.