8.16 (a) For the TE mode, the eigen equation is

$$4ka\sin\theta - 4\arctan\sqrt{\frac{2\Delta}{\sin^2\theta}-1} = 2p\pi,$$

where $\cos\theta = k_z/k$, $k = n_1\omega/c$. Define

$$f = 4ka\sqrt{1-k_z^2/k^2} - 4\arctan\sqrt{\frac{2\Delta}{1-k_z^2/k^2}-1},$$

then

$$\frac{\partial f}{\partial k_z} = -\frac{4ak_z}{k}\frac{1}{\sqrt{1-k_z^2/k^2}} - \frac{4k_z}{k^2}\frac{1}{1-k_z^2/k^2}\frac{1}{\sqrt{\frac{2\Delta}{1-k_z^2/k^2}-1}}$$

$$= -\frac{4a\cos\theta}{\sin\theta} - \frac{4\cos\theta}{\sin^2\theta}\cdot\frac{\sin\theta}{\beta} = -\frac{4\cos\theta}{\sin\theta}\left(a+\frac{1}{\beta}\right),$$

$$\frac{\partial f}{\partial k} = \frac{4a}{\sqrt{1-k_z^2/k^2}} + \frac{4k_z^2}{k^3}\frac{1}{1-k_z^2/k^2}\frac{1}{\sqrt{\frac{2\Delta}{1-k_z^2/k^2}-1}}$$

$$= \frac{4a}{\sin\theta} + \frac{4\cos^2\theta}{\sin^2\theta}\frac{\sin\theta}{\beta} = \frac{4}{\sin\theta}\left(a+\frac{\cos^2\theta}{\beta}\right).$$

Since $\frac{\partial f}{\partial k_z} + \frac{\partial f}{\partial k}\frac{dk}{dk_z} = 0$, we have

$$\frac{d\omega}{dk_z} = \frac{c}{n_1}\frac{dk}{dk_z} = -\frac{c}{n_1}\frac{\partial f/\partial k_z}{\partial f/\partial k} = \frac{c\cos\theta}{n_1}\frac{a+1/\beta}{a+\cos^2\theta/\beta} = \frac{c\cos\theta}{n_1}\frac{1+\beta a}{\cos^2\theta+\beta a}$$