11.6 (a) From Problem 11.5, the acceleration in the earth frame is given by

$a = \frac{dv}{dt} = \left(1 - \frac{v^2}{c^2}\right)^{3/2} g$. Then, we can determine the velocity of the rocket ship in the earth

by solving the differential equation,

$$\frac{dv}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} = g\,dt \quad \text{or} \quad gt = \int_0^v \frac{du}{\left(1-\frac{u^2}{c^2}\right)^{3/2}} = \frac{v/c}{\sqrt{1-v^2/c^2}}$$

Therefore, the velocity of the rocket ship in the earth frame is $v(t) = \frac{gt}{\sqrt{1+(gt/c)^2}}$

The proper time is related to the earth time by

$$\tau = \int_0^\tau d\tau' = \int_0^t \sqrt{1-v(s)^2/c^2}\,ds = \int_0^t \frac{ds}{\left(1+(gs/c)^2\right)^{1/2}} = \frac{c}{g}\log\left(\frac{gt}{c} + \sqrt{1+\left(\frac{gt}{c}\right)^2}\right) = \frac{c}{g}\sinh^{-1}\left(\frac{gt}{c}\right)$$

or $t = \frac{c}{g}\sinh(g\tau/c)$. When the rocket ship is done with acceleration after 5 years in the ship time, $\tau = 5$, we have $t = 84$. From symmetry, we know that, after the rocket ship returns, the number of year that has passed on earth is $4t = 336$

(b) The distance travelled during the acceleration stage is

$$\int_0^t v(s)ds = \int_0^t \frac{gs}{\sqrt{1+(gs/c)^2}}\,ds = \frac{c^2}{g}\left(\sqrt{1+(gt/c)^2} - 1\right) = 7.85 \times 10^{17}\,m$$

The farthest distance from the earth is twice of this, which is approximately 164 light years.