

**4.2** Solution: For the effective charge density

$$\rho(\mathbf{x}) = -\mathbf{p} \cdot \nabla \delta(\mathbf{x} - \mathbf{x}_0),$$

the potential from this charge density is

$$\begin{aligned} \Phi(\mathbf{x}) &= \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d^3x' \\ &= -\frac{1}{4\pi\epsilon_0} \int \frac{1}{|\mathbf{x} - \mathbf{x}'|} \mathbf{p} \cdot \nabla' \delta(\mathbf{x}' - \mathbf{x}_0) d^3x' \\ &= \frac{1}{4\pi\epsilon_0} \int \delta(\mathbf{x}' - \mathbf{x}_0) \left( \nabla' \cdot \frac{\mathbf{p}}{|\mathbf{x} - \mathbf{x}'|} \right) d^3x' \\ &= \frac{1}{4\pi\epsilon_0} \int \delta(\mathbf{x}' - \mathbf{x}_0) \frac{\mathbf{p} \cdot (\mathbf{x} - \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|^3} d^3x' \\ &= \frac{1}{4\pi\epsilon_0} \frac{\mathbf{p} \cdot (\mathbf{x} - \mathbf{x}_0)}{|\mathbf{x} - \mathbf{x}_0|^3}, \end{aligned}$$

which is the potential from a dipole moment  $p$  at  $\mathbf{x}_0$ .