11.14 (a) $F^{\alpha\beta}F_{\alpha\beta} = -2(|\vec{E}|^2 - |\vec{B}|^2)$, $\mathcal{F}^{\alpha\beta}F_{\alpha\beta} = -4\vec{E}\cdot\vec{B}$, $\mathcal{F}^{\alpha\beta}\mathcal{F}_{\alpha\beta} = 2(|\vec{E}|^2 - |\vec{B}|^2)$.

No further invariants quadratic in $\vec{E}$ and $\vec{B}$.

(b) It is not possible to have only $\vec{E}$ in one frame while only $\vec{B}$ in other frame.

From the Lorentz invariant $F^{\alpha\beta}F_{\alpha\beta}$, it can be seen that, if we can make $\vec{B}$ disappear in one frame, then $F^{\alpha\beta}F_{\alpha\beta} < 0$. If we can make $\vec{E}$ disappear in another frame, we will have $F^{\alpha\beta}F_{\alpha\beta} > 0$, a contradiction. To have $\vec{E} = 0$ in one frame, we must have $|\vec{E}| < |\vec{B}|$ in all frames. Also, if $\vec{E} = 0$, then $\vec{E}\cdot\vec{B} = 0$, which means $\vec{E}$ and $\vec{B}$ are perpendicular in all frames.

(c) Similar to $F^{\alpha\beta}$, we have

$$G^{\alpha\beta} = \begin{pmatrix} 0 & -D_x & -D_y & -D_z \\ D_x & 0 & -H_z & H_y \\ D_y & H_z & 0 & -H_x \\ D_z & -H_y & H_x & 0 \end{pmatrix}, \quad \mathcal{G}^{\alpha\beta} = \begin{pmatrix} 0 & -H_x & -H_y & -H_z \\ H_x & 0 & D_z & -D_y \\ H_y & -D_z & 0 & D_x \\ H_z & D_y & -D_x & 0 \end{pmatrix}$$

The additional invariants are

$$G^{\alpha\beta}G_{\alpha\beta} = -2(|\vec{D}|^2 - |\vec{H}|)^2, \quad G^{\alpha\beta}\mathcal{G}_{\alpha\beta} = -4\vec{D}\cdot\vec{H}, \quad \mathcal{G}^{\alpha\beta}\mathcal{G}_{\alpha\beta} = 2(|\vec{D}|^2 - |\vec{H}|^2)$$

$$F^{\alpha\beta}G_{\alpha\beta} = -2(\vec{E}\cdot\vec{D} - \vec{B}\cdot\vec{H}), \quad F^{\alpha\beta}\mathcal{G}_{\alpha\beta} = -2(\vec{E}\cdot\vec{H} + \vec{B}\cdot\vec{D})$$

$$\mathcal{F}^{\alpha\beta}G_{\alpha\beta} = -2(\vec{E}\cdot\vec{H} + \vec{B}\cdot\vec{D}), \quad \mathcal{F}^{\alpha\beta}\mathcal{G}_{\alpha\beta} = 2(\vec{E}\cdot\vec{D} - \vec{B}\cdot\vec{H})$$