14.6 (a) Similar to Prob. 14.5, we can express the energy conservation condition as

$$E = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + V(r) = \text{const}$$

in polar coordinates. Due to the conservation of angular momentum, we know $L = mr^2\dot{\theta}$ is also a constant, then energy conservation can be written as

$$E = \frac{1}{2}m\dot{r}^2 + \frac{L^2}{2mr^2} + V(r),$$

or

$$\frac{dr}{dt} = \sqrt{\frac{2}{m}}\left[E - V(r) - \frac{L^2}{2mr^2}\right]^{1/2}$$

Then, as in Prob 14.5 (a), the total energy loss becomes

$$\Delta W = \frac{4z^2e^2}{3m^2c^3}\sqrt{\frac{m}{2}}\int_{r_{min}}^{+\infty}\left|\frac{\partial V}{\partial r}\right|^2\left[E - V(r) - \frac{L^2}{2mr^2}\right]^{-1/2} dr,$$

where r_{min} is the solution to $E = V(r) + \frac{L^2}{2mr^2}$.

(b) We can determine r_{min} from energy conservation, $E = \frac{zZe^2}{r} + \frac{L^2}{2mr^2}$. Notice that $E = \frac{1}{2}mv_0^2$, and $L = mbv_0$, we have $r^2 - 2sr - b^2 = 0$, where $s = zZe^2/mv_0^2$. The physical solution is $r_{min} = s + \sqrt{s^2+b^2}$. Denote $r^* = s - \sqrt{s^2+b^2}$, and also notice that $|\partial V/\partial r| = zZe^2/r^2$, the total energy radiated is

$$\Delta W = \frac{4z^2e^2}{3m^2c^3}\sqrt{\frac{m}{2}}\frac{(zZe^2)^2}{(mv_0^2/2)^{1/2}}\int_{r_{min}}^{+\infty}\frac{1}{r^3}\frac{dr}{\sqrt{(r-r_{min})(r-r^*)}} = \dots$$

Still need to figure out how to simplify the integral in Mathematica in order to reduce to the form presented in the problem.

For $t \gg 1$, dropping terms of t^{-4} and t^{-5}, we have

$$\Delta W = \frac{2 z m v_0^5}{Z c^3} \cdot \frac{1}{3t^3} \cdot \frac{\pi}{2} = \frac{\pi z^4 Z^2 e^6}{3 m^2 c^3 v_0} \frac{1}{b^3},$$

which reduces to the result of Prob. 14.7 (a).

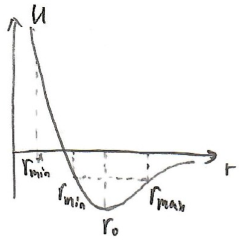
(c) For $t = \cot \theta/2$, $\arctan t = \frac{\pi}{2} - \frac{\theta}{2}$, and

$$\Delta W = \frac{2 z m v_0^5}{Z c^3} \left[- \tan^4\left(\frac{\theta}{2}\right) + \tan^3\left(\frac{\theta}{2}\right) \left(\tan^2\left(\frac{\theta}{2}\right) + \frac{1}{3} \right) \cdot \frac{1}{2}(\pi - \theta) \right]$$

$$= \frac{2 z m v_0^5}{Z c^3} \tan^3\left(\frac{\theta}{2}\right) \left[\frac{1}{6}(\pi - \theta) \left(1 + 3\tan^2\left(\frac{\theta}{2}\right) \right) - \tan\left(\frac{\theta}{2}\right) \right]$$

(d) The effective potential (see Landau and Lifshitz, Mechanics, Fig. 10) is given by

$$U(r) = -\frac{z Z e^2}{r} + \frac{L^2}{2 m r^2}.$$



When the total energy $E = \frac{1}{2} m \dot{r}^2 + U(r)$ is positive, the particle's trajectory is unbounded, $r > r^*_{min}$. Then, the total radiation energy loss can be similarly calculated. However, if the total energy of the particle is $U(r_0) < E < 0$, then the particle is bounded to the central potential. During its periodic motion between r_{min} and r_{max}, it will constantly lose energy due to radiation. Eventually, it will lose all of its energy to radiation and fall to the center.