Experimental Self-Testing via Noncontextuality Inequalities

Abstract

The steps for experimental self-testing via KCBS inequality is presented and possible challenges are discussed. To read the notes fast, experimentalists can ignore the "non-blue texts". Text in "red" is the major challenge at the moment.

1 Introduction and Steps

We aim to experimentally certify local 3-level systems via non-contextuality inequalities. In particular, we plan to use KCBS inequality and it's generalizations to n-cycle scenario [1]. Based on the measurement statistics for KCBS scenario, it is possible to identify the underlying measurement settings and the state upto an isometry [2]. The aforementioned phenomena is referred to as self-testing [3]. For self-testing to be of practical relevance, one needs to provide a noise-tolerant scheme, often known as robust self-testing scheme. Though the authors in [2] prove that the KCBS inequality and its genaralizations admit robust self-testng, the robustness was obtained only upto a multiplicative constant. Determining the multiplicativative constant was left as an open problem and thus the scheme was not experimental-friendly. In this note, we discuss the steps to devise an experimental friendly self-testing scheme and hence experimentally witness the claims by [2]. We discuss the necessary experimental data required to carry on the whole procedure. While step 1 and 3 are experimental, step 2 is theoretical.

1.1 Step 1 (Experimental Step): Generate the Measurement Statistics

The first step is to generate the experimental statistics for the KCBS scenario and its generalizations. This step has already been done by USTC and the statistics for n upto 51 are available with us. However, we will have to carry on the experiment again for $n = 5, 7$ and 9 (please refer to step 3 for the reason).

1.2 Step 2 (Theoretical Step): Predict Lower Bound on Fidelity

For this step, we provide the technique analogous to Bell-scenario based self-testing tool. There are some missing sub-steps (as of now) which need to be completed. The goal is to provide a lower bound on the fidelity of measurement settings and the state based on the measurement statistics obtained from step 1.

1.2.1 Regularizing the experimental statistics

In reference [2], authors assume that the measurement operators follow cyclic compatibility structure. However, this need not be true in the real life experimental scenario. Thus, the experimental data may not belong to quantum set due to finite statistics and violation of the cyclic compatibility structure. This demands a regularization of the experimental data for the theoretical analysis. Here, regularization means finding the statistics which is closest to our experimental statistics and belongs to the quantum set. We can regularize the experimental data using semi definite programming.

Let P^e be the vector of probabilities obtained for n cycle scenario in the experimental scenario. Here, P^e_i refers to the probability of getting outcome 1 when one measures the quantum state with projector Π_i for $i \in [1, 2, \cdots, n]$. This implies

$P_i^e = Tr[\Pi_i \rho]$ for quantum state ρ and $i \in [1, 2, \cdots, n]$. The cyclic compatibility structure on the projectors demands that $Tr[\Pi_i \Pi_{i+1} \rho] = 0$ for $i \in [1, 2, \cdots, n]$. However, this is not the case based on the data from step 1 and hence we need to regularize it. The regulariazation procedure is an optimization problem which demands finding the statistics from the quantum set which is closest to experimental statistics. Quite specifically, this corresponds to the folowing semi-definite program.

$$min\, |P^e - P| \tag{1.1}$$

such that

$$P \in Q, \tag{1.2}$$

where Q is the quantum set for the corresponding scenario. For a non-contextuality inequality corresponding to graph G with vertices V and edges E, the equation 1.2 can also be written as a series of conditions which correspond to the quantum set:

$$X_{00} = 1$$

$$X_{0i} = X_{ii} = P_i \quad i \in V$$

$$X_{i,j} = 0 \quad i, j \in E$$

and

$$X \geq 0.$$

The above semi-definte program gives us the regularized measurement statistics i.e $P \in Q$. This step can be done for any arbitrary contextuality experiment. Next step is to determine the fidelity of the state and the measurements used in the experiment with the ideal state and the ideal measurements.

1.2.2 Providing a Lower Bound on the Fidelity from Regularized Measurement Statistics

Providing a lower bound on the fidelity of the states and measurement with the ideal configuration corresponds to another SDP. However, the size of the SDP matrix increases with n which suggests that we can carry on the analysis for smaller values of n only, for example $n = 5, 7$ and 9. This step can be broken down into a few sub-steps.

1. Constructing the isometry: The isometry needs to be constructed using the projective measurements used in the scenario. **This step needs to be worked out.**

2. Deriving the expression for the fidelity: Once the isometry in the previous step has been constructed, this step is a trivial calculation. However, it seems the expression for fidelity will contain only expectation value of product of at most two projective measurements for $n = 5$ case. For higher values of n, the expression for fidelity seems to containthe expectation value of product of at most $\frac{(n-1)}{2}$ projective measurements, which also corresponds to the independence number of the n cycle graph. Let us denote the fidelity expression as F.

3. Providing the lower bound on the fidelity : The lower bound on the fidelity can be given by a SDP. Since the expression for the fidelity in the last step for $n = 5$ contains the expecation value of the product of at most two terms, the feasibility region for the SDP is equal to quantum set and hence corresponds to optimization over 6×6 symmmetric positive semidefinite matrices. The SDP is given by

$$min\, F(P)$$

such that

$$P \in Q.$$

However, for $n = 7$onwards, since the SDP doesn't contain expecation value of product of more than two projective measurements, we need to relax our SDP. *The relaxation leads to increase in the size of the SDP matrix and hence problem becomes computationally intractable for large n.*

1.3 Step 3 (Experimental Step): Obtain Fidelities by Experimental Tomography

We need to experimentally find the fidelities of the measurement settings and the state which were employed to generate the statistics for step 1. Thus, it is crucial to tomograph *the physical system corresponding to the experimental statistics* for the the data in step 1. Unless the physical system which corresponded to the measurement statistics is preserved, we need to carry on the experiment for step 1 again and tomograph the underlying projectors and the state. Please carry on this process for small values of n only at the moment, for example $n = 5, 7$ and 9.

2 Conclusion

In conclusion, we should experimentally generate the statistics for $n = 5, 7$ and 9. Moreover, we should experimentally tomograph the underlying state and measurements as well. The theoretical analysis predicts that the size of SDP matrices increase with n and hence optimization may be a difficult problem for large n. This is not surprising as we have similar problem in the Bell scenario based SDP heirarchies [4].

References

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