Local and Robust Self testing using trapped ions

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Abstract

We provide experimental implementation of KCBS self-testing scheme. Theoretical tools are supplemented to render the results in *Physical Review Letters 122 (25), 250403* practical.

1 Introduction

2 Background

2.1 Exclusivity graph approach to contextuality

2.2 Robust self-testing

2.3 Verifying the assumptions experimentally

The self-testing results in reference[1] hinges on three assumptions:

1. The measurements are "sharp". Sharp measurements are outcome-repeatable and minimally disturbing.
2. The measurements follow the compatibility relation according to an odd cycle graph.
3. The black box has no more memory than its information-carrying capacity. Each box is used once, and there is an unlimited supply of them.

In the experiments, we verify sharpness (assumption 1) and compatibility (assumption 2) based on the techniques in reference[2]. To quantify sharpness of an observable A_μ, it is instructive to quantify the probability of getting the same outcome corresponding to two consecutive measurements of the same measurement i.e. A_μ. The appropriate mathematical expression for the same could be given by

$$R_\mu = \frac{\sum_{a_1} N\left(A_\mu = a_1, A_\mu = a_1\right)}{N\left(A_\mu, A_\mu\right)},$$

where $N(.)$ quantifies the number of instances corresponding to the expression inside its following bracket. Coming back to assumption 2, the necessary conditions for compatibility are the absence of context-signalling as well as lack of influence of a precedding compatible measurement on the statistics of an observable. The forward context-signalling is quantified by

$$S^f_{u,v,w;i} = \frac{N_i\left(A_\nu, A_u = a_1\right)}{N_i\left(A_\nu, A_u\right)} - \frac{N_i\left(A_w, A_u = a_1\right)}{N_i\left(A_w, A_u\right)}, \tag{2.1}$$

whereas the backward context-signalling is given by

$$S^b_{u,v,w;i} = \frac{N_i\left(A_u = a_1, A_\nu\right)}{N_i\left(A_\nu, A_u\right)} - \frac{N_i\left(A = a_1, A_w\right)}{N_i\left(A_w, A_u\right)}. \tag{2.2}$$

It is worth mentioning that the expressions for the forward and backward context-signalling are defined for some fixed input state i. In case of no-context signalling, the value of both the expressions 2.2 and 2.1 is zero. Regarding assumption 3, we haven't closed the memory loop-hole. (TODO: Add the compatibility check via contextuality-by-default approach)

3 Experimental results

The experimental implementation corresponding to the work in ref [1] was carried on.

4 Theoretical Analysis

The goal is to provide a lower bound on fidelity for the experimental results in section 2. The following steps are required:

4.1 Regularizing the experimental statistics

In reference [1], authors assume that the measurement operators follow cyclic compatibility structure. However, this need not be true in the real life experimental scenario. Thus, the experimental data may not belong to quantum set due to finite statistics and violation of the cyclic compatibility structure. This demands a regularization of the experimental data for the theoretical analysis. Here, regularization means finding the statistics which is closest to our experimental statistics and belongs to the quantum set. The set of quantum behaviours forms a convex set, known as theta body.

4.2 Providing a lower bound on the fidelity from regularized measurement statistics

4.2.1 Constructing the isometry

A swap circuit between two qudit registers is given by $S = TUVU$ where

$$T = \mathbb{I} \otimes \sum_k |-k\rangle\langle k|$$

$$U = \sum_{k=0}^{d-1} P^k \otimes |k\rangle\langle k|$$

$$V = \sum_{k=0}^{d-1} |k\rangle\langle k| \otimes P^{-k}$$

and

$$P = \sum_{k=0}^{d-1} |k+1\rangle\langle k|.$$

Here, P is a translation operator.
TUVU, Localizing matrix etc....

4.2.2 Deriving the expression for fidelity

Let $\left\{\{\bar{\Pi}_i\}_{i\in V}, |\bar{\psi}\rangle\right\}$ be the ideal configuration and $\{\{\Pi_i\}, |\psi\rangle\}$ be the candidate configuration. Our expression for the fidelity for the aforementioned configurations is given by

$$F(\alpha, \beta) = \alpha \sum_{i\in V} \langle\bar{\psi}|\bar{\Pi}_i\Pi_i|\psi\rangle + \beta\langle\bar{\psi}|\psi\rangle,$$

where α,$\beta \in \mathbb{R}$ and are weights on fidelitiy for states and measured states respectively.

The total fidelity for state and projectors for odd n-cycle scenario in the ideal case is $n+1$.

4.2.3 Providing the lower bound on fidelity

NPA analoge for theta body

5 Conclusions

References

[1] Kishor Bharti, Maharshi Ray, Antonios Varvitsiotis, Naqueeb Ahmad Warsi, AdÃ¡n Cabello, and Leong-Chuan Kwek. Robust self-testing of quantum systems via noncontextuality inequalities.

[2] Florian M Leupold, Maciej Malinowski, Chi Zhang, Vlad Negnevitsky, Adán Cabello, Joseba Alonso, and Jonathan P Home. Sustained state-independent quantum contextual correlations from a single ion. *Physical review letters*, 120(18):180401, 2018.