"Josephus Problem"

n	$W(n)$
1	1
2	1
3	3
4	1
5	3
6	5
7	7
8	1
9	3
10	5
11	7
12	9
13	11
14	13

NOTE: Winners are always odd.

A power of 2

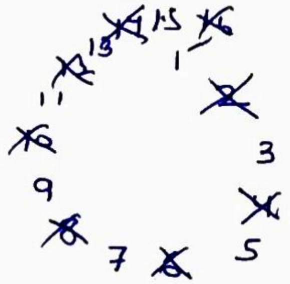
NOTE: If $n = 2^a$ then $W(n) = 1$

total Soldiers

winning position.

Why is it that for powers of two the winning position is always 1

Consider 16 which is 2^4

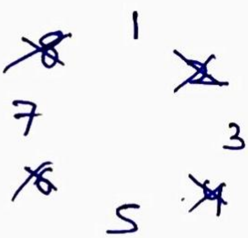


In first pass all the even no. go out.

Now the total no. of positions that are remaining is actually half of the original positions i.e. $\frac{16}{2} = 8$

Step 2

Now, we number the 8 positions starting from 1 using consecutive natural no.



Again repeat step 2

∴ for any powers of 2 w(n) is always 1.

How to justify the pattern(increasing by 2) until ~~resseting~~ ressetting at powers of 2.

$N = \underset{\uparrow}{2^a} + \text{Something}$

biggest power of 2 (that can be subtracted from the numbers).

$77 = 64 + 8 + 4 + 1$

$77 = 2^6 + 2^3 + 2^2 + 2^0$

Binary

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	1	1	0	1

This is binary representation

is a unique way of writing a number in terms of powers of 2 where no power of 2 is repeated.

Now consider

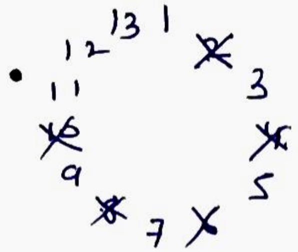
$$77 = \underbrace{64}_{2^a} + \underbrace{13}_{l}$$

$$13 = \underbrace{8}_{2^a} + \underbrace{5}_{l}$$

Consider the case of $n=13$

Now $13 = \underbrace{8}_{2^a} + \underbrace{5}_{l}$

Perform 5 steps



Now notice the total no. of position left after performing 5 moves it is a power of 2 i.e 8 in this case & we know from our previous observation that in case of power of 2 position the starting (first) position is the winning position which is 11 in this case.

If we express n as $2^a + l$ then after performing l steps whose turn it is that position wins.

∴ the winning seat will be $\underbrace{2l+1}$

∵ that will be the turn of the position after performing l steps.

$\boxed{\text{Claim}}$

If $n = 2^a + l$

where $l < 2a$

Then $W(n) = 2l+1$

n soldiers	$2^a + l$	Winner/Survivor $W(n) = 2l + 1$
1	1 + 0	$2^*0+1 = 1$
2	2 + 0	$2^*0+1 = 1$
3	2 + 1	$2^*1+1 = 3$
4	4 + 0	$2^*0+1 = 1$
5	4 + 1	$2^*1+1 = 3$
6	4 + 2	$2^*2+1 = 5$
7	4 + 3	$2^*3+1 = 7$
8	8 + 0	$2^*0+1 = 1$
9	8 + 1	$2^*1+1 = 3$
10	8 + 2	$2^*2+1 = 5$
11	8 + 3	$2^*3+1 = 7$
12	8 + 4	$2^*4+1 = 9$

for $n = 41$

$41 = 32 + 9$

$W(41) = 2\times 9 + 1 = \boxed{19}$

Trick for finding answer to josephus problem.

$41 = 2^5 + 2^3 + 2^0$

41 =	2^5	2^4	2^3	2^2	2^1	2^0
	1	0	1	0	0	1

(*) take the leading digit & put it at the end.

$010011 = 19$

Always work but no explanation/justification.