Interval-Based Global Sensitivity Analysis for Epistemic Uncertainty

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The objective of sensitivity analysis is to understand how the input uncertainty of a mathematical model contributes to its output uncertainty. In the context of a digital twin, sensitivity analysis is paramount for the automatic verification and validation of physical models, and can also be used as a decision support tool to determine on which parameter to invest more empirical effort. Yet, sensitivity analysis often requires making assumptions about the inputs such as probability distribution functions, as it is the case in variance-based methods, or relies on surrogate models that also introduce more assumptions, as in kriging or polynomial chaos. It can be the case that one cannot reliably assign probability distribution functions if the model is dominated by epistemic uncertainties, or the complexity of the model is such that surrogate models cannot accurately capture its behaviour.
We present a non-probabilistic sensitivity analysis method which requires no assumptions about the input probability distributions: the uncertainty in the input is expressed in the form of intervals, and employs the width of the output interval as the only measure, in the same way that interval analysis. As a positive by-product, the method also returns all the information that could be obtained with the reduction of the input interval to a single value, also called pinching. We use the Ishigami function as test case to show the performance of the proposed method, and compare it with Sobol indices.

Keywords: epistemic uncertainty, uncertainty quantification, sensitivity analysis, interval arithmetic, sobol indices, digital twin.

1. Introduction

Prediction is inherent in science, because prediction is essential to test theories and their consequences. Thanks to the power of modern computation, the prediction of mathematical models (i.e. their output) representing natural phenomena can now be tested at unprecedented scales. Digital twins attempt to exploit this advantage by modelling physical systems and allowing their evaluation via simulation, as these systems are often technically and economically prohibitive to operate in the physical world (Wagg et al. (2020)). However, digital twins are often so complex that it is not possible to infer their prediction from experience and judgement alone. Therefore, it is desirable to verify and validate the prediction of these models without having to rely on subjectivity (I et al. (2020)). Sensitivity analysis can help with this task, by indicating what model parameters are responsible for the prediction of the model, and how that prediction depends on them (Saltelli et al. (2004)).

Generally, sensitivity analysis methods fall within three categories: derivative-based, distribution-based, and regression-based Razavi et al. (2021). Derivative-based approaches attempt

to compute the derivative of the model functions, either analytically or numerically, and measure the change in the output when the inputs are perturbed around a base point ([reference]). Distribution-based methods, such as Sobol' indices, decompose the output variance and assigns the partitions to the input variances, indicating how much of the output variance is caused by each input variance (Saltelli et al. (2010)). Lastly, regression-based approaches employ correlation coefficients, regression coefficients, or other machine learning methods (Sudret (2008)).

However, these approaches present some limitations that can make them unsuitable in certain cases. For instance, the analytical description of the functions in the model are not always available, since it is not uncommon to deal with black-box models or models with too many functions that make unpractical or difficult their analytical derivation. Also, derivative-based methods require defining a base point for each input parameter, and a perturbation size. It is not rare to find a situation where there is not consensus about those elements. A similar argument can be made for distribution-based methods, which require a precise definition of the probability distribution functions of the model input parameters. Lastly, to successfully apply regression-based methods, it is necessary to know the behavior of the model under investigation, which is not always known. For example, partial correlation coefficients assume model linearity, or monotonicity in the case of partial rank coefficients (Saltelli and Marivoet (1990)). For these reasons, it is desirable to find a sensitivity analysis method that:

(1) Does not require knowing the analytical description of the model functions.
(2) Does not require defining base values for any input parameter.
(3) Does not require to assume the input parameters follow a precise probability distribution function.
(4) It is independent of the model behavior.

This paper attempts to present an interval-based global sensitivity analysis method that fulfills these requirements. Section 2 introduces some

basic concepts in interval uncertainty propagation which are required to perform the analysis. In Section 3 it is explained how the sensitivity indices are calculated in the interval approach, and the pinching measure that can be calculated additionally. Lastly, Section 4 compares the performance of the interval-based method against Sobol' indices in two test cases (Ishigami function and a basic physics problem), followed by the conclusion in section 5.

(1) Explain sensitivity analysis. [DONE]
(2) Explain main approaches and their limitation. [DONE]
(3) Explain epistemic uncertainty in engineering -slides- (we cannot afford those limitations) [DONE]
(4) Explain structure of the paper [DONE]

2. Interval Uncertainty Propagation

Minimum unit of assumption (or knowledge).

(1) Explain epistemic uncertainty can be modeled as intervals.
(2) Explain interval uncertainty can be propagated with: arithmetic, sampling
(3) In te case the analytical function(s) are available, interval arithmetic gets the rigorous calculation. If the function(s) are not available, the propagation can be performed with sampling.

3. Interval-Based Sensitivity Index

(1) Explain the area stuff
(2) Explain extreme cases (e.g. perfect dependence = index of 1 = no area, independence = index of 0 = all area)
(3) Explain pinching.

4. Application

(1) Explain simple function (or physics based example?).
(2) Explain comparison with Sobol indices.
(3) Show results.

(1) Explain Ishigami function
(2) Explain comparison with sobol indices (arguably the most common SA method)
(3) Show results.

5. Conclusion

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