In[1]:=
```
Solve [{z == 1 + z (1 – B) ^ 2 + 2 o B (1 – B), o == 1 + o * (1 – H)}, {z, o}]
```

Out[1]=
$$\left\{\left\{z \to -\frac{2\,B - 2\,B^2 + H}{(-2 + B)\,B\,H},\ o \to \frac{1}{H}\right\}\right\}$$

In[2]:=
```
Simplify [-(2 B – 2 B^2 + H)/((–2 + B) B H)]
```

Out[2]=
$$\frac{2\,B - 2\,B^2 + H}{2\,B\,H - B^2\,H}$$

In[3]:=
```
F = H + 2 B – 2 B ^ 2
G = 2 * H * B – H * B ^ 2
```

Out[3]=
$$2\,B - 2\,B^2 + H$$

Out[4]=
$$2\,B\,H - B^2\,H$$

In[5]:=
```
ReplaceAll [{H → a + (1 – a) P (P / (P + Q)), B → a + (1 – a) Q (P / (P + Q))}][{F}]
ReplaceAll [{H → a + (1 – a) P (P / (P + Q)), B → a + (1 – a) Q (P / (P + Q))}][{G}]
```

Out[5]=
$$\left\{a + \frac{(1-a)\,P^2}{P+Q} + 2\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right) - 2\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right)^2\right\}$$

Out[6]=
$$\left\{2\left(a + \frac{(1-a)\,P^2}{P+Q}\right)\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right) - \left(a + \frac{(1-a)\,P^2}{P+Q}\right)\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right)^2\right\}$$

In[7]:=
```
f[Q_] := {a + (1 – a) P^2/(P + Q) + 2 (a + (1 – a) P Q/(P + Q)) – 2 (a + (1 – a) P Q/(P + Q))^2}
g[Q_] := {2 (a + (1 – a) P^2/(P + Q)) (a + (1 – a) P Q/(P + Q)) – (a + (1 – a) P^2/(P + Q)) (a + (1 – a) P Q/(P + Q))^2}
DNUM [Q_] := FullSimplify [f '[Q] g[Q] – g '[Q] f[Q]]
DNUM [Q]
```

Out[10]=
$$\left\{-\frac{1}{(P+Q)^6}2\,(-1+a)\,P^2\right.$$
$$\left(a^4\,(P + Q - P\,Q)^4 + a^2\,(P\,(-1+Q) - Q)\left(P^3\,(-1 + 3\,(-1+P)\,P) + P^2\,(-3+7\,P)\,Q + P\,(-3 + (23 - 18\,P)\,P)\,Q^2 + (-1 + P\,(13 + 3\,P\,(-5 + 2\,P)))\,Q^3\right) - a^3\,(P\,(-1+Q) - Q)\right.$$
$$\left(P^3\,(-3 + (-1+P)\,P) + 9\,(-1+P)\,P^2\,Q - 3\,(-1+P)\,P\,(-3+4\,P)\,Q^2 + (-3 + P\,(11 + P\,(-11 + 4\,P)))\,Q^3\right) +$$
$$P^2\left(-P^4 + (-2+P)\,P^3\,Q + P^2\,Q^2 - 4\,(-1+P)\,P\,Q^3 + (-2+P)\,(-1+P)\,Q^4\right) + a\,P\left(-3\,P^5\,(-1+Q) + 2\,P\,(5-7\,Q)\,Q^3 + 4\,Q^4 + P^4\,(-2+6\,Q) + P^2\,Q^2\,(6 + Q\,(-28 + 13\,Q)) - P^3\,Q\,(2 + Q\,(11 + 4\,(-4+Q)\,Q))\right)\Big)\Big\}$$

In[11]:=
```
Sgn[Q_] := FullSimplify [DNUM[Q] * (P + Q) ^ 6]
Sgn[Q]
```

Out[12]=
$$\{-2\,(-1+a)\,P^2\,(a^4\,(P+Q-P\,Q)^4+a^2\,(P\,(-1+Q)-Q)\,(P^3\,(-1+3\,(-1+P)\,P)+P^2\,(-3+7\,P)\,Q+P\,(-3+(23-18\,P)\,P)\,Q^2+(-1+P\,(13+3\,P\,(-5+2\,P)))\,Q^3)-a^3\,(P\,(-1+Q)-Q)\,(P^3\,(-3+(-1+P)\,P)+9\,(-1+P)\,P^2\,Q-3\,(-1+P)\,P\,(-3+4\,P)\,Q^2+(-3+P\,(11+P\,(-11+4\,P)))\,Q^3)+P^2\,(-P^4+(-2+P)\,P^3\,Q+P^2\,Q^2-4\,(-1+P)\,P\,Q^3+(-2+P)\,(-1+P)\,Q^4)+a\,P\,(-3\,P^5\,(-1+Q)+2\,P\,(5-7\,Q)\,Q^3+4\,Q^4+P^4\,(-2+6\,Q)+P^2\,Q^2\,(6+Q\,(-28+13\,Q))-P^3\,Q\,(2+Q\,(11+4\,(-4+Q)\,Q)))))\}$$

In[13]:=
```
roots =
 Reduce [Sgn[Q] == 0 && Q > 0 && Q < 1 && P > 0 && P < 1 && a > 0 && a < 1, {a, P, Q}, Reals ]
```

Out[13]=
$$\left(0<a\le\frac{1}{2}\left(3-\sqrt{5}\right)\&\&\ \frac{-2\,a+a^2}{2\,(-1+a)^2}+\frac{1}{2}\sqrt{\frac{8\,a^2-20\,a^3+17\,a^4-4\,a^5}{(-1+a)^4}}<\right.$$
$$P<\text{Root}\left[a^2-3\,a^3+a^4+(4\,a-10\,a^2+2\,a^3)\,\#1+(2-4\,a-5\,a^2+a^3)\,\#1^2+(1-9\,a+6\,a^2-2\,a^3)\,\#1^3+(-2-a+3\,a^2-a^3)\,\#1^4+(-2+4\,a-3\,a^2+a^3)\,\#1^5\ \&,\ 3\right]\&\&$$
$$Q==\text{Root}\left[a^2\,P^4-3\,a^3\,P^4+a^4\,P^4-2\,a\,P^5+3\,a^2\,P^5-a^3\,P^5-P^6+3\,a\,P^6-3\,a^2\,P^6+a^3\,P^6+(4\,a^2\,P^3-12\,a^3\,P^3+4\,a^4\,P^3-2\,a\,P^4-5\,a^2\,P^4+11\,a^3\,P^4-4\,a^4\,P^4-2\,P^5+6\,a\,P^5-6\,a^2\,P^5+2\,a^3\,P^5+P^6-3\,a\,P^6+3\,a^2\,P^6-a^3\,P^6)\,\#1+(6\,a^2\,P^2-18\,a^3\,P^2+6\,a^4\,P^2+6\,a\,P^3-33\,a^2\,P^3+39\,a^3\,P^3-12\,a^4\,P^3+P^4-11\,a\,P^4+25\,a^2\,P^4-21\,a^3\,P^4+6\,a^4\,P^4)\,\#1^2+(4\,a^2\,P-12\,a^3\,P+4\,a^4\,P+10\,a\,P^2-39\,a^2\,P^2+41\,a^3\,P^2-12\,a^4\,P^2+4\,P^3-28\,a\,P^3+56\,a^2\,P^3-44\,a^3\,P^3+12\,a^4\,P^3-4\,P^4+16\,a\,P^4-24\,a^2\,P^4+16\,a^3\,P^4-4\,a^4\,P^4)\,\#1^3+(a^2-3\,a^3+a^4+4\,a\,P-14\,a^2\,P+14\,a^3\,P-4\,a^4\,P+2\,P^2-14\,a\,P^2+28\,a^2\,P^2-22\,a^3\,P^2+6\,a^4\,P^2-3\,P^3+13\,a\,P^3-21\,a^2\,P^3+15\,a^3\,P^3-4\,a^4\,P^3+P^4-4\,a\,P^4+6\,a^2\,P^4-4\,a^3\,P^4+a^4\,P^4)\,\#1^4\ \&,\ 2\right]\Bigg)\ ||$$

$$\left(\frac{1}{2}\left(3-\sqrt{5}\right)<a<\boxed{0.401\ldots}\ \&\&\ \text{Root}\left[a^2-3\,a^3+a^4+(4\,a-10\,a^2+2\,a^3)\,\#1+(2-4\,a-5\,a^2+a^3)\,\#1^2+(1-9\,a+6\,a^2-2\,a^3)\,\#1^3+(-2-a+3\,a^2-a^3)\,\#1^4+(-2+4\,a-3\,a^2+a^3)\,\#1^5\ \&,\ 2\right]<\right.$$
$$P<\text{Root}\left[a^2-3\,a^3+a^4+(4\,a-10\,a^2+2\,a^3)\,\#1+(2-4\,a-5\,a^2+a^3)\,\#1^2+(1-9\,a+6\,a^2-2\,a^3)\,\#1^3+(-2-a+3\,a^2-a^3)\,\#1^4+(-2+4\,a-3\,a^2+a^3)\,\#1^5\ \&,\ 3\right]\&\&$$
$$Q==\text{Root}\left[a^2\,P^4-3\,a^3\,P^4+a^4\,P^4-2\,a\,P^5+3\,a^2\,P^5-a^3\,P^5-P^6+3\,a\,P^6-3\,a^2\,P^6+a^3\,P^6+(4\,a^2\,P^3-12\,a^3\,P^3+4\,a^4\,P^3-2\,a\,P^4-5\,a^2\,P^4+11\,a^3\,P^4-4\,a^4\,P^4-2\,P^5+6\,a\,P^5-6\,a^2\,P^5+2\,a^3\,P^5+P^6-3\,a\,P^6+3\,a^2\,P^6-a^3\,P^6)\,\#1+(6\,a^2\,P^2-18\,a^3\,P^2+6\,a^4\,P^2+6\,a\,P^3-33\,a^2\,P^3+39\,a^3\,P^3-12\,a^4\,P^3+P^4-11\,a\,P^4+25\,a^2\,P^4-21\,a^3\,P^4+6\,a^4\,P^4)\,\#1^2+(4\,a^2\,P-12\,a^3\,P+4\,a^4\,P+10\,a\,P^2-39\,a^2\,P^2+41\,a^3\,P^2-12\,a^4\,P^2+4\,P^3-28\,a\,P^3+56\,a^2\,P^3-44\,a^3\,P^3+12\,a^4\,P^3-4\,P^4+16\,a\,P^4-24\,a^2\,P^4+16\,a^3\,P^4-4\,a^4\,P^4)\,\#1^3+(a^2-3\,a^3+a^4+4\,a\,P-14\,a^2\,P+14\,a^3\,P-4\,a^4\,P+2\,P^2-14\,a\,P^2+28\,a^2\,P^2-22\,a^3\,P^2+6\,a^4\,P^2-3\,P^3+13\,a\,P^3-21\,a^2\,P^3+15\,a^3\,P^3-4\,a^4\,P^3+P^4-4\,a\,P^4+6\,a^2\,P^4-4\,a^3\,P^4+a^4\,P^4)\,\#1^4\ \&,\ 2\right]\Bigg)$$

In[14]:=
```
a1 := roots [[1]]
a2 := roots [[2]]
a1[[1]]
a1[[2]]
```

Out[16]= $0 < a \leq \frac{1}{2}\left(3 - \sqrt{5}\right)$

Out[17]= $\frac{-2\,a + a^2}{2\,(-1+a)^2} + \frac{1}{2}\sqrt{\frac{8\,a^2 - 20\,a^3 + 17\,a^4 - 4\,a^5}{(-1+a)^4}} < P <$
$\text{Root}\left[a^2 - 3\,a^3 + a^4 + (4\,a - 10\,a^2 + 2\,a^3)\,\#1 + (2 - 4\,a - 5\,a^2 + a^3)\,\#1^2 + (1 - 9\,a + 6\,a^2 - 2\,a^3)\,\#1^3 + (-2 - a + 3\,a^2 - a^3)\,\#1^4 + (-2 + 4\,a - 3\,a^2 + a^3)\,\#1^5 \;\&,\; 3\right]$

In[18]:=
```
a2[[1]]
a2[[2]]
```

Out[18]= $\frac{1}{2}\left(3 - \sqrt{5}\right) < a <$ 0.401…

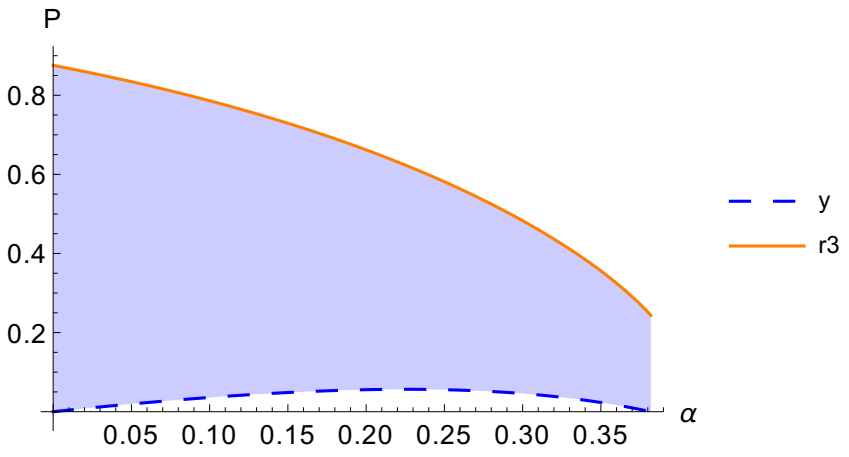
Out[19]= $\text{Root}\left[a^2 - 3\,a^3 + a^4 + (4\,a - 10\,a^2 + 2\,a^3)\,\#1 + (2 - 4\,a - 5\,a^2 + a^3)\,\#1^2 + (1 - 9\,a + 6\,a^2 - 2\,a^3)\,\#1^3 + (-2 - a + 3\,a^2 - a^3)\,\#1^4 + (-2 + 4\,a - 3\,a^2 + a^3)\,\#1^5 \;\&,\; 2\right] <$
$P < \text{Root}\left[a^2 - 3\,a^3 + a^4 + (4\,a - 10\,a^2 + 2\,a^3)\,\#1 + (2 - 4\,a - 5\,a^2 + a^3)\,\#1^2 + (1 - 9\,a + 6\,a^2 - 2\,a^3)\,\#1^3 + (-2 - a + 3\,a^2 - a^3)\,\#1^4 + (-2 + 4\,a - 3\,a^2 + a^3)\,\#1^5 \;\&,\; 3\right]$

In[20]:=
```
amid = Root[-6784 + 80 832 #1 - 428 644 #1^2 + 1 279 684 #1^3 -
    2 275 131 #1^4 + 2 358 718 #1^5 - 1 175 083 #1^6 - 203 210 #1^7 + 760 072 #1^8 -
    570 650 #1^9 + 217 065 #1^10 - 41 706 #1^11 + 3125 #1^12 &, 2]
```

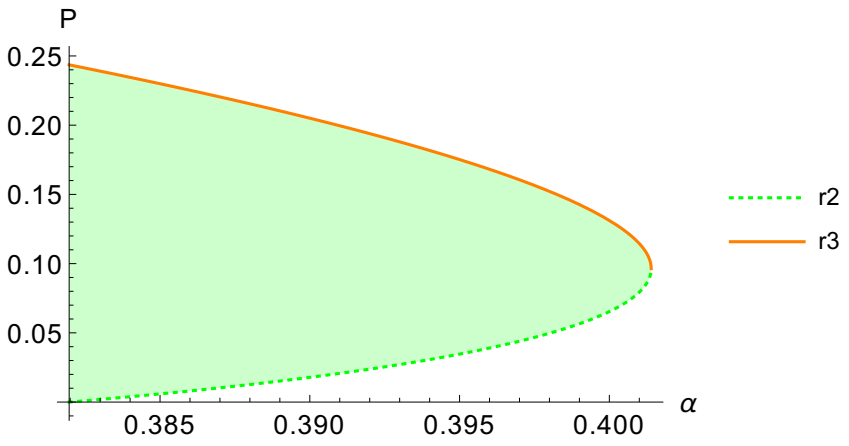
Out[20]= 0.401…

In[21]:=
```
graph1 = Plot[{(-2 a + a^2)/(2 (-1 + a)^2) + 1/2 Sqrt[(8 a^2 - 20 a^3 + 17 a^4 - 4 a^5)/(-1 + a)^4],
   Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 + (1 - 9 a + 6 a^2 - 2 a^3) #1^3 +
     (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 3]}, {a, 0, 1/2 (3 - Sqrt[5])},
  Filling -> {1 -> {2}}, PlotStyle -> {{Blue, Dashing[Large]}, Orange},
  AxesLabel -> {Style["α", FontSize -> 15], Style["P", FontSize -> 15]},
  PlotLegends -> {"y", "r3"}, TicksStyle -> Directive[FontSize -> 15]]
graph2 = Plot[{Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 +
     (1 - 9 a + 6 a^2 - 2 a^3) #1^3 + (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 2],
   Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 + (1 - 9 a + 6 a^2 - 2 a^3) #1^3 +
     (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 3]},
  {a, 1/2 (3 - Sqrt[5]), amid}, Filling -> {1 -> {2}}, PlotStyle -> {{Green, Dotted}, Orange},
  AxesLabel -> {Style["α", FontSize -> 15], Style["P", FontSize -> 15]},
  PlotLegends -> {"r2", "r3"}, TicksStyle -> Directive[FontSize -> 15]]
```

Out[21]=

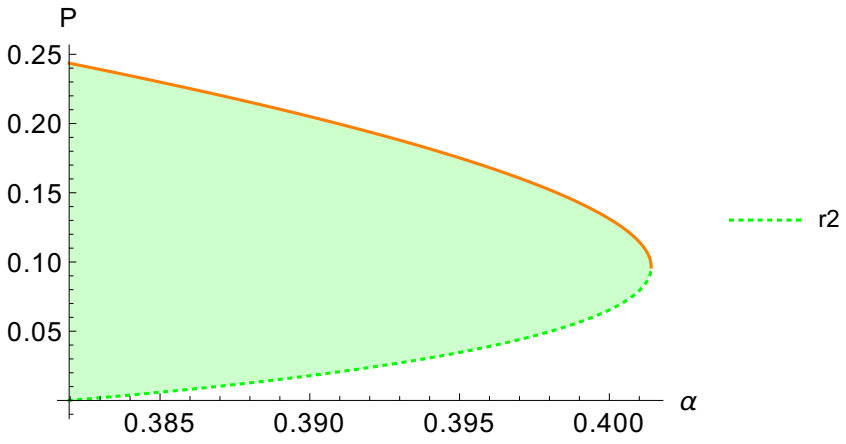


Out[22]=



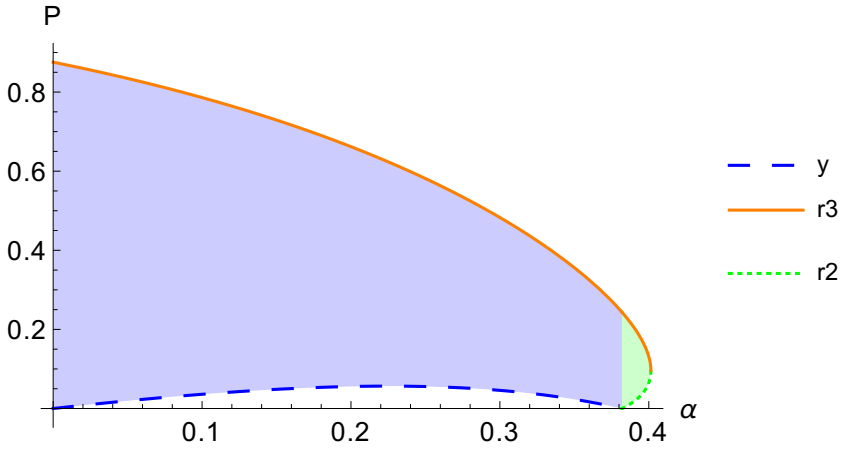
In[23]:=
```
graph2partlegend =
 Plot[{Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 + (1 - 9 a + 6 a^2 - 2 a^3) #1^3 +
      (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 2],
   Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 + (1 - 9 a + 6 a^2 - 2 a^3) #1^3 +
      (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 3] },
  {a, 1/2 (3 - Sqrt[5]), amid }, Filling → {1 → {2}}, PlotStyle → {{Green , Dotted }, Orange },
  AxesLabel → {Style["α", FontSize → 15], Style["P", FontSize → 15]},
  PlotLegends → {"r2"}, TicksStyle → Directive [FontSize → 15]]
```

Out[23]=



In[24]:=
```
graphc = Show[graph1 , graph2partlegend , PlotRange → All]
```

Out[24]=



In[25]:=
```
CloudExport [graph1 , "EPS", "tp-left-side .eps"]
CloudExport [graph2 , "EPS", "tp-right-side .eps"]
CloudExport [graphc , "EPS", "tp.eps"]
CloudExport [graph1 , "PNG", "tp-left-side .png"]
CloudExport [graph2 , "PNG", "tp-right-side .png"]
CloudExport [graphc , "PNG", "tp.png"]
```

Out[25]= CloudObject [https://www.wolframcloud.com/obj/tomer.tuchner/tp-left-side.eps]

Out[26]= CloudObject [https://www.wolframcloud.com/obj/tomer.tuchner/tp-right-side.eps]

Out[27]= CloudObject [https://www.wolframcloud.com/obj/tomer.tuchner/tp.eps]

Out[28]= CloudObject [https://www.wolframcloud.com/obj/tomer.tuchner/tp-left-side.png]

Out[29]= CloudObject [https://www.wolframcloud.com/obj/tomer.tuchner/tp-right-side.png]

Out[30]= CloudObject [https://www.wolframcloud.com/obj/tomer.tuchner/tp.png]