In[1]:= `Solve[{z == 1 + z (1 - D)^2 + 2 o D (1 - D), o == 1 + o*(1 - C)}, {z, o}]`

Out[1]= $\left\{\left\{z \rightarrow -\dfrac{C + 2 D - 2 D^2}{C (-2 + D) D}, \; o \rightarrow \dfrac{1}{C}\right\}\right\}$

In[2]:= `Simplify`$\left[-\dfrac{C + 2 D - 2 D^2}{C (-2 + D) D}\right]$

Out[2]= $\dfrac{C + 2 D - 2 D^2}{2 C D - C D^2}$

In[3]:= `F = C + 2 D - 2 D^2`

`G = 2 * C * D - C * D^2`

Out[3]= $C + 2 D - 2 D^2$

Out[4]= $2 C D - C D^2$

In[5]:= `ReplaceAll[{C → a + (1 - a) P (P / (P + Q)), D → a + (1 - a) Q (P / (P + Q))}][{F}]`

`ReplaceAll[{C → a + (1 - a) P (P / (P + Q)), D → a + (1 - a) Q (P / (P + Q))}][{G}]`

Out[5]= $\left\{a + \dfrac{(1 - a) P^2}{P + Q} + 2\left(a + \dfrac{(1 - a) P Q}{P + Q}\right) - 2\left(a + \dfrac{(1 - a) P Q}{P + Q}\right)^2\right\}$

Out[6]= $\left\{2\left(a + \dfrac{(1 - a) P^2}{P + Q}\right)\left(a + \dfrac{(1 - a) P Q}{P + Q}\right) - \left(a + \dfrac{(1 - a) P^2}{P + Q}\right)\left(a + \dfrac{(1 - a) P Q}{P + Q}\right)^2\right\}$

In[7]:= $f[Q\_] := \left\{a + \dfrac{(1 - a) P^2}{P + Q} + 2\left(a + \dfrac{(1 - a) P Q}{P + Q}\right) - 2\left(a + \dfrac{(1 - a) P Q}{P + Q}\right)^2\right\}$

$g[Q\_] := \left\{2\left(a + \dfrac{(1 - a) P^2}{P + Q}\right)\left(a + \dfrac{(1 - a) P Q}{P + Q}\right) - \left(a + \dfrac{(1 - a) P^2}{P + Q}\right)\left(a + \dfrac{(1 - a) P Q}{P + Q}\right)^2\right\}$

`DNUM[Q_] := FullSimplify[f'[Q] g[Q] - g'[Q] f[Q]]`

`DNUM[Q]`

Out[10]= $\left\{-\dfrac{1}{(P + Q)^6} 2 (-1 + a) P^2 \right.$

$(a^4 (P + Q - P Q)^4 + a^2 (P (-1 + Q) - Q) (P^3 (-1 + 3 (-1 + P) P) + P^2 (-3 + 7 P) Q + P (-3 + (23 - 18 P) P) Q^2 +$

$(-1 + P (13 + 3 P (-5 + 2 P))) Q^3) - a^3 (P (-1 + Q) - Q)$

$(P^3 (-3 + (-1 + P) P) + 9 (-1 + P) P^2 Q - 3 (-1 + P) P (-3 + 4 P) Q^2 + (-3 + P (11 + P (-11 + 4 P))) Q^3) +$

$P^2 (-P^4 + (-2 + P) P^3 Q + P^2 Q^2 - 4 (-1 + P) P Q^3 + (-2 + P) (-1 + P) Q^4) + a P (-3 P^5 (-1 + Q) +$

$\left. 2 P (5 - 7 Q) Q^3 + 4 Q^4 + P^4 (-2 + 6 Q) + P^2 Q^2 (6 + Q (-28 + 13 Q)) - P^3 Q (2 + Q (11 + 4 (-4 + Q) Q)))))\right\}$

In[11]:= `Sgn[Q_] := FullSimplify [DNUM[Q] * (P + Q) ^ 6]`
`Sgn[Q]`

Out[12]= $\{-2(-1+a)P^2$

$(a^4(P+Q-PQ)^4 + a^2(P(-1+Q)-Q)(P^3(-1+3(-1+P)P)+P^2(-3+7P)Q+P(-3+(23-18P)P)Q^2+$

$(-1+P(13+3P(-5+2P)))Q^3)-a^3(P(-1+Q)-Q)$

$(P^3(-3+(-1+P)P)+9(-1+P)P^2Q-3(-1+P)P(-3+4P)Q^2+(-3+P(11+P(-11+4P)))Q^3)+$

$P^2(-P^4+(-2+P)P^3Q+P^2Q^2-4(-1+P)PQ^3+(-2+P)(-1+P)Q^4)+aP(-3P^5(-1+Q)+$

$2P(5-7Q)Q^3+4Q^4+P^4(-2+6Q)+P^2Q^2(6+Q(-28+13Q))-P^3Q(2+Q(11+4(-4+Q)Q)))))\}$

In[13]:= **roots =**
 Reduce [Sgn[Q] == 0 && Q > 0 && Q < 1 && P > 0 && P < 1 && a > 0 && a < 1, {a, P, Q}, Reals]

Out[13]= $\Big(0 < a \leq \frac{1}{2}\left(3-\sqrt{5}\right) \&\& \frac{-2a+a^2}{2(-1+a)^2} + \frac{1}{2}\sqrt{\frac{8a^2-20a^3+17a^4-4a^5}{(-1+a)^4}} <$

$P < \text{Root}\big[a^2-3a^3+a^4+(4a-10a^2+2a^3)\#1+(2-4a-5a^2+a^3)\#1^2+$

$(1-9a+6a^2-2a^3)\#1^3+(-2-a+3a^2-a^3)\#1^4+(-2+4a-3a^2+a^3)\#1^5 \&, 3\big] \&\&$

$Q == \text{Root}\big[a^2P^4-3a^3P^4+a^4P^4-2aP^5+3a^2P^5-a^3P^5-P^6+3aP^6-3a^2P^6+a^3P^6+$

$(4a^2P^3-12a^3P^3+4a^4P^3-2aP^4-5a^2P^4+11a^3P^4-4a^4P^4-2P^5+6aP^5-6a^2P^5+$

$2a^3P^5+P^6-3aP^6+3a^2P^6-a^3P^6)\#1+(6a^2P^2-18a^3P^2+6a^4P^2+6aP^3-$

$33a^2P^3+39a^3P^3-12a^4P^3+P^4-11aP^4+25a^2P^4-21a^3P^4+6a^4P^4)\#1^2+$

$(4a^2P-12a^3P+4a^4P+10aP^2-39a^2P^2+41a^3P^2-12a^4P^2+4P^3-28aP^3+$

$56a^2P^3-44a^3P^3+12a^4P^3-4P^4+16aP^4-24a^2P^4+16a^3P^4-4a^4P^4)\#1^3+$

$(a^2-3a^3+a^4+4aP-14a^2P+14a^3P-4a^4P+2P^2-14aP^2+28a^2P^2-22a^3P^2+6a^4P^2-$

$3P^3+13aP^3-21a^2P^3+15a^3P^3-4a^4P^3+P^4-4aP^4+6a^2P^4-4a^3P^4+a^4P^4)\#1^4 \&, 2\big]\Big) ||$

$\Big(\frac{1}{2}\left(3-\sqrt{5}\right) < a < \boxed{\circ\ 0.401\ldots} \&\& \text{Root}\big[a^2-3a^3+a^4+(4a-10a^2+2a^3)\#1+(2-4a-5a^2+a^3)\#1^2+$

$(1-9a+6a^2-2a^3)\#1^3+(-2-a+3a^2-a^3)\#1^4+(-2+4a-3a^2+a^3)\#1^5 \&, 2\big] <$

$P < \text{Root}\big[a^2-3a^3+a^4+(4a-10a^2+2a^3)\#1+(2-4a-5a^2+a^3)\#1^2+$

$(1-9a+6a^2-2a^3)\#1^3+(-2-a+3a^2-a^3)\#1^4+(-2+4a-3a^2+a^3)\#1^5 \&, 3\big] \&\&$

$Q == \text{Root}\big[a^2P^4-3a^3P^4+a^4P^4-2aP^5+3a^2P^5-a^3P^5-P^6+3aP^6-3a^2P^6+a^3P^6+$

$(4a^2P^3-12a^3P^3+4a^4P^3-2aP^4-5a^2P^4+11a^3P^4-4a^4P^4-2P^5+6aP^5-6a^2P^5+$

$2a^3P^5+P^6-3aP^6+3a^2P^6-a^3P^6)\#1+(6a^2P^2-18a^3P^2+6a^4P^2+6aP^3-$

$33a^2P^3+39a^3P^3-12a^4P^3+P^4-11aP^4+25a^2P^4-21a^3P^4+6a^4P^4)\#1^2+$

$(4a^2P-12a^3P+4a^4P+10aP^2-39a^2P^2+41a^3P^2-12a^4P^2+4P^3-28aP^3+$

$56a^2P^3-44a^3P^3+12a^4P^3-4P^4+16aP^4-24a^2P^4+16a^3P^4-4a^4P^4)\#1^3+$

$(a^2-3a^3+a^4+4aP-14a^2P+14a^3P-4a^4P+2P^2-14aP^2+28a^2P^2-22a^3P^2+6a^4P^2-$

$3P^3+13aP^3-21a^2P^3+15a^3P^3-4a^4P^3+P^4-4aP^4+6a^2P^4-4a^3P^4+a^4P^4)\#1^4 \&, 2\big]\Big)$

In[14]:= **a1 := roots[[1]]**
a2 := roots[[2]]
a1[[1]]
a1[[2]]

Out[16]= $0 < a \leq \frac{1}{2}\left(3 - \sqrt{5}\right)$

Out[17]= $\frac{-2\,a + a^2}{2\,(-1 + a)^2} + \frac{1}{2}\sqrt{\frac{8\,a^2 - 20\,a^3 + 17\,a^4 - 4\,a^5}{(-1 + a)^4}} < P <$

$\text{Root}\left[a^2 - 3\,a^3 + a^4 + \left(4\,a - 10\,a^2 + 2\,a^3\right)\#1 + \left(2 - 4\,a - 5\,a^2 + a^3\right)\#1^2 + \left(1 - 9\,a + 6\,a^2 - 2\,a^3\right)\#1^3 + \left(-2 - a + 3\,a^2 - a^3\right)\#1^4 + \left(-2 + 4\,a - 3\,a^2 + a^3\right)\#1^5 \,\&,\, 3\right]$

In[18]:= **a2[[1]]**
a2[[2]]

Out[18]= $\frac{1}{2}\left(3 - \sqrt{5}\right) < a < \boxed{\; 0.401\ldots}$

Out[19]= $\text{Root}\left[a^2 - 3\,a^3 + a^4 + \left(4\,a - 10\,a^2 + 2\,a^3\right)\#1 + \left(2 - 4\,a - 5\,a^2 + a^3\right)\#1^2 + \left(1 - 9\,a + 6\,a^2 - 2\,a^3\right)\#1^3 + \left(-2 - a + 3\,a^2 - a^3\right)\#1^4 + \left(-2 + 4\,a - 3\,a^2 + a^3\right)\#1^5 \,\&,\, 2\right] <$

$P < \text{Root}\left[a^2 - 3\,a^3 + a^4 + \left(4\,a - 10\,a^2 + 2\,a^3\right)\#1 + \left(2 - 4\,a - 5\,a^2 + a^3\right)\#1^2 + \left(1 - 9\,a + 6\,a^2 - 2\,a^3\right)\#1^3 + \left(-2 - a + 3\,a^2 - a^3\right)\#1^4 + \left(-2 + 4\,a - 3\,a^2 + a^3\right)\#1^5 \,\&,\, 3\right]$

In[20]:= **amid = Root[-6784 + 80 832 #1 - 428 644 #1^2 + 1 279 684 #1^3 -**
2 275 131 #1^4 + 2 358 718 #1^5 - 1 175 083 #1^6 - 203 210 #1^7 + 760 072 #1^8 -
570 650 #1^9 + 217 065 #1^10 - 41 706 #1^11 + 3125 #1^12 &, 2]

Out[20]= $\boxed{\; 0.401\ldots}$

In[21]:= $\texttt{graph1 = Plot}\Big[\Big\{\dfrac{-2\,a + a^2}{2\,(-1+a)^2} + \dfrac{1}{2}\,\sqrt{\dfrac{8\,a^2 - 20\,a^3 + 17\,a^4 - 4\,a^5}{(-1+a)^4}}\,,$

$\qquad \texttt{Root}\big[a^2 - 3\,a^3 + a^4 + (4\,a - 10\,a^2 + 2\,a^3)\,\#1 + (2 - 4\,a - 5\,a^2 + a^3)\,\#1^2 + (1 - 9\,a + 6\,a^2 - 2\,a^3)\,\#1^3 +$

$\qquad\quad (-2 - a + 3\,a^2 - a^3)\,\#1^4 + (-2 + 4\,a - 3\,a^2 + a^3)\,\#1^5\,\&,\ 3\big]\Big\},\ \Big\{a,\ 0,\ \dfrac{1}{2}\big(3 - \sqrt{5}\,\big)\Big\},$

$\qquad \texttt{Filling} \to \{1 \to \{2\}\},\ \texttt{PlotStyle} \to \{\{\texttt{Blue},\ \texttt{Dashing[Large]}\},\ \texttt{Orange}\},$

$\qquad \texttt{AxesLabel} \to \{\texttt{Style["}\alpha\texttt{"},\ \texttt{FontSize} \to 15],\ \texttt{Style["P"},\ \texttt{FontSize} \to 15]\},$

$\qquad \texttt{PlotLegends} \to \{\texttt{"y"},\ \texttt{"r3"}\},\ \texttt{TicksStyle} \to \texttt{Directive[FontSize} \to 15]\Big]$

$\texttt{graph2 = Plot}\Big[\Big\{\texttt{Root}\big[a^2 - 3\,a^3 + a^4 + (4\,a - 10\,a^2 + 2\,a^3)\,\#1 + (2 - 4\,a - 5\,a^2 + a^3)\,\#1^2 +$

$\qquad\quad (1 - 9\,a + 6\,a^2 - 2\,a^3)\,\#1^3 + (-2 - a + 3\,a^2 - a^3)\,\#1^4 + (-2 + 4\,a - 3\,a^2 + a^3)\,\#1^5\,\&,\ 2\big],$

$\qquad \texttt{Root}\big[a^2 - 3\,a^3 + a^4 + (4\,a - 10\,a^2 + 2\,a^3)\,\#1 + (2 - 4\,a - 5\,a^2 + a^3)\,\#1^2 + (1 - 9\,a + 6\,a^2 - 2\,a^3)\,\#1^3 +$

$\qquad\quad (-2 - a + 3\,a^2 - a^3)\,\#1^4 + (-2 + 4\,a - 3\,a^2 + a^3)\,\#1^5\,\&,\ 3\big]\Big\},$

$\qquad \Big\{a,\ \dfrac{1}{2}\big(3 - \sqrt{5}\,\big),\ \texttt{amid}\,\Big\},\ \texttt{Filling} \to \{1 \to \{2\}\},\ \texttt{PlotStyle} \to \{\{\texttt{Green},\ \texttt{Dotted}\},\ \texttt{Orange}\},$

$\qquad \texttt{AxesLabel} \to \{\texttt{Style["}\alpha\texttt{"},\ \texttt{FontSize} \to 15],\ \texttt{Style["P"},\ \texttt{FontSize} \to 15]\},$

$\qquad \texttt{PlotLegends} \to \{\texttt{"r2"},\ \texttt{"r3"}\},\ \texttt{TicksStyle} \to \texttt{Directive[FontSize} \to 15]\Big]$

Out[21]=

Out[22]=
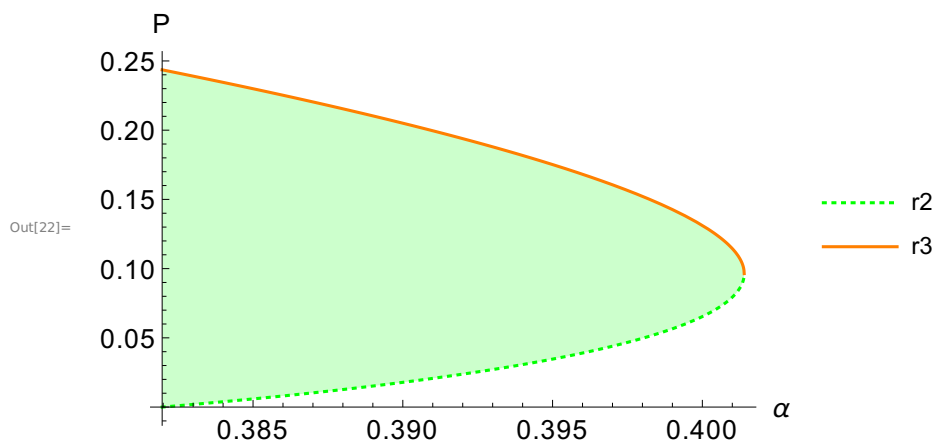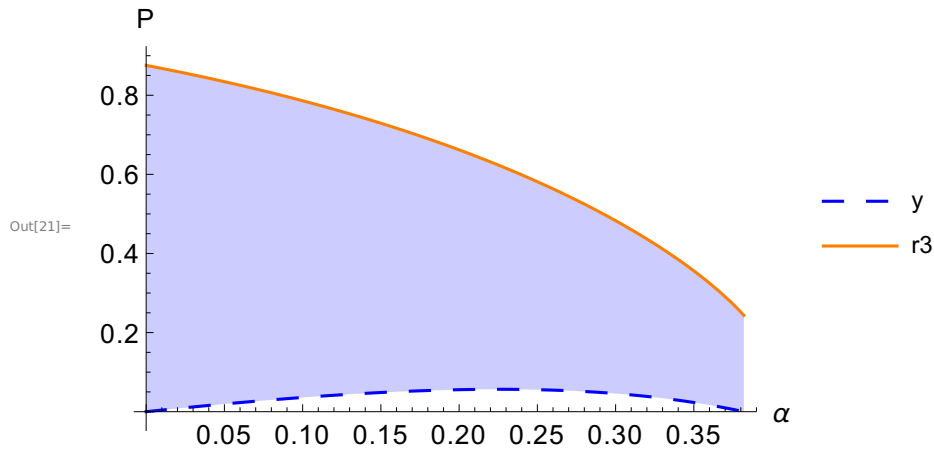
In[23]:= **graph2partlegend** =

$\mathsf{Plot}\big[\{\mathsf{Root}\big[a^2 - 3\,a^3 + a^4 + (4\,a - 10\,a^2 + 2\,a^3)\,\#1 + (2 - 4\,a - 5\,a^2 + a^3)\,\#1^2 + (1 - 9\,a + 6\,a^2 - 2\,a^3)\,\#1^3 +$
$(-2 - a + 3\,a^2 - a^3)\,\#1^4 + (-2 + 4\,a - 3\,a^2 + a^3)\,\#1^5\,\&,\,2],$
$\mathsf{Root}\big[a^2 - 3\,a^3 + a^4 + (4\,a - 10\,a^2 + 2\,a^3)\,\#1 + (2 - 4\,a - 5\,a^2 + a^3)\,\#1^2 + (1 - 9\,a + 6\,a^2 - 2\,a^3)\,\#1^3 +$
$(-2 - a + 3\,a^2 - a^3)\,\#1^4 + (-2 + 4\,a - 3\,a^2 + a^3)\,\#1^5\,\&,\,3]\,\},$
$\{a,\,\frac{1}{2}\,(3 - \sqrt{5}),\,\mathsf{amid}\,\},\,\mathsf{Filling} \to \{1 \to \{2\}\},\,\mathsf{PlotStyle} \to \{\{\mathsf{Green},\,\mathsf{Dotted}\},\,\mathsf{Orange}\,\},$
$\mathsf{AxesLabel} \to \{\mathsf{Style}["\alpha",\,\mathsf{FontSize} \to 15],\,\mathsf{Style}["P",\,\mathsf{FontSize} \to 15]\},$
$\mathsf{PlotLegends} \to \{"r2"\},\,\mathsf{TicksStyle} \to \mathsf{Directive}\,[\mathsf{FontSize} \to 15]\big]$

Out[23]=
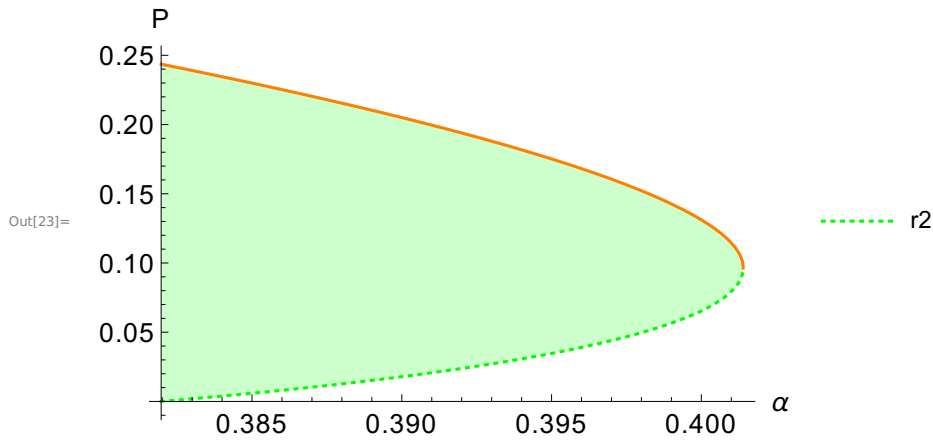
In[24]:= **graphc = Show[graph1 , graph2partlegend , PlotRange → All]**

Out[24]=
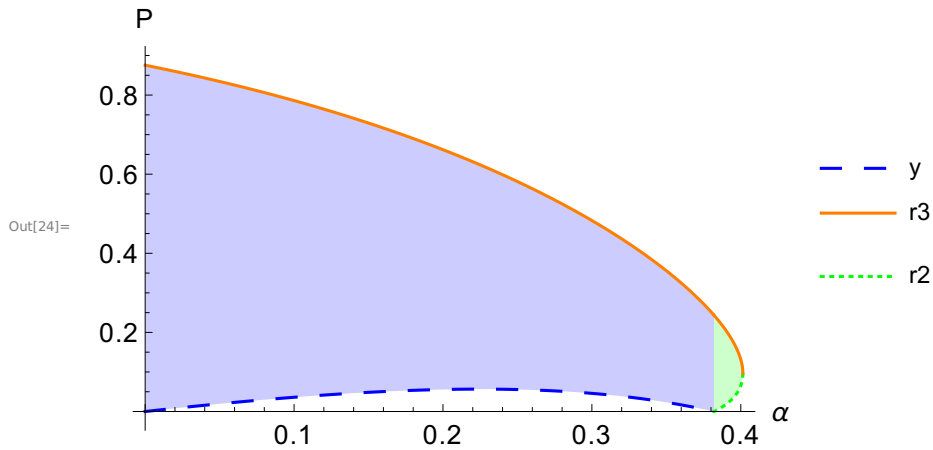
In[25]:= **CloudExport [graph1 , "EPS", "tp-left-size .eps"]**
CloudExport [graph2 , "EPS", "tp-right -size .eps"]
CloudExport [graphc , "EPS", "tp.eps"]

Out[25]= CloudObject $\left[\text{https ://www .wolframcloud .com/obj/tomer .tuchner /tp-left-size .eps}\right]$

Out[26]= CloudObject $\left[\text{https ://www .wolframcloud .com/obj/tomer .tuchner /tp-right -size .eps}\right]$

Out[27]= CloudObject $\left[\text{https ://www .wolframcloud .com/obj/tomer .tuchner /tp.eps}\right]$