In[1123]:=
```
Solve[{z == 1 + z (1 - D)^2 + 2 o D (1 - D), o == 1 + o*(1 - C)}, {z, o}]
```

Out[1123]=
$$\left\{\left\{z \to -\frac{C + 2\,D - 2\,D^2}{C\,(-2 + D)\,D},\ o \to \frac{1}{C}\right\}\right\}$$

In[1124]:=
```
Simplify[-(C + 2 D - 2 D^2)/(C (-2 + D) D)]
```

Out[1124]=
$$\frac{C + 2\,D - 2\,D^2}{2\,C\,D - C\,D^2}$$

In[1125]:=
```
F = C + 2 D - 2 D^2
G = 2*C*D - C*D^2
```

Out[1125]= $C + 2\,D - 2\,D^2$

Out[1126]= $2\,C\,D - C\,D^2$

In[1127]:=
```
ReplaceAll[{C -> a + (1 - a) P (P/(P + Q)), D -> a + (1 - a) Q (P/(P + Q))}][{F}]
ReplaceAll[{C -> a + (1 - a) P (P/(P + Q)), D -> a + (1 - a) Q (P/(P + Q))}][{G}]
```

Out[1127]=
$$\left\{a + \frac{(1-a)\,P^2}{P+Q} + 2\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right) - 2\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right)^2\right\}$$

Out[1128]=
$$\left\{2\left(a + \frac{(1-a)\,P^2}{P+Q}\right)\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right) - \left(a + \frac{(1-a)\,P^2}{P+Q}\right)\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right)^2\right\}$$

In[1129]:=
$$\texttt{f[Q\_] :=} \left\{a + \frac{(1-a)\,P^2}{P+Q} + 2\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right) - 2\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right)^2\right\}$$

$$\texttt{g[Q\_] :=} \left\{2\left(a + \frac{(1-a)\,P^2}{P+Q}\right)\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right) - \left(a + \frac{(1-a)\,P^2}{P+Q}\right)\left(a + \frac{(1-a)\,P\,Q}{P+Q}\right)^2\right\}$$

```
DNUM[Q_] := FullSimplify[f'[Q] g[Q] - g'[Q] f[Q]]
DNUM[Q]
```

Out[1132]=
$$\Big\{-\frac{1}{(P+Q)^6}\,2\,(-1+a)\,P^2\,\big(a^4\,(P+Q-P\,Q)^4 + a^2\,(P\,(-1+Q)-Q)\,\big(P^3\,(-1+3\,(-1+P)\,P) + P^2\,(-3+7\,P)\,Q + P\,(-3+(23-18\,P)\,P)\,Q^2 + (-1+P\,(13+3\,P\,(-5+2\,P)))\,Q^3\big) - a^3\,(P\,(-1+Q)-Q)\,\big(P^3\,(-3+(-1+P)\,P) + 9\,(-1+P)\,P^2\,Q - 3\,(-1+P)\,P\,(-3+4\,P)\,Q^2 + (-3+P\,(11+P\,(-11+4\,P)))\,Q^3\big) + P^2\,\big(-P^4 + (-2+P)\,P^3\,Q + P^2\,Q^2 - 4\,(-1+P)\,P\,Q^3 + (-2+P)\,(-1+P)\,Q^4\big) + a\,P\,\big(-3\,P^5\,(-1+Q) + 2\,P\,(5-7\,Q)\,Q^3 + 4\,Q^4 + P^4\,(-2+6\,Q) + P^2\,Q^2\,(6+Q\,(-28+13\,Q)) - P^3\,Q\,(2+Q\,(11+4\,(-4+Q)\,Q))\big)\big)\Big\}$$

In[1133]:=
```
Sgn[Q_] := DNUM[Q] * (P + Q) ^ 6
Sgn[Q]
```

Out[1134]=

$$
\begin{aligned}
&\{-2\,(-1+a)\,P^2\\
&\quad (a^4\,(P+Q-P\,Q)^4 + a^2\,(P\,(-1+Q)-Q)\,(P^3\,(-1+3\,(-1+P)\,P) + P^2\,(-3+7\,P)\,Q + P\,(-3+(23-18\,P)\,P)\,Q^2 +\\
&\qquad (-1+P\,(13+3\,P\,(-5+2\,P)))\,Q^3) - a^3\,(P\,(-1+Q)-Q)\\
&\qquad (P^3\,(-3+(-1+P)\,P) + 9\,(-1+P)\,P^2\,Q - 3\,(-1+P)\,P\,(-3+4\,P)\,Q^2 + (-3+P\,(11+P\,(-11+4\,P)))\,Q^3) +\\
&\quad P^2\,(-P^4 + (-2+P)\,P^3\,Q + P^2\,Q^2 - 4\,(-1+P)\,P\,Q^3 + (-2+P)\,(-1+P)\,Q^4) + a\,P\,(-3\,P^5\,(-1+Q) +\\
&\qquad 2\,P\,(5-7\,Q)\,Q^3 + 4\,Q^4 + P^4\,(-2+6\,Q) + P^2\,Q^2\,(6+Q\,(-28+13\,Q)) - P^3\,Q\,(2+Q\,(11+4\,(-4+Q)\,Q))))\}
\end{aligned}
$$

In[1135]:=
```
roots =
 Reduce[Sgn[Q] == 0 && Q > 0 && Q < 1 && P > 0 && P < 1 && a > 0 && a < 1, {a, P, Q}, Reals]
```

Out[1135]=

$$
\begin{aligned}
&\Bigg(0 < a \le \frac{1}{2}\left(3-\sqrt{5}\right) \;\&\&\; \frac{-2\,a+a^2}{2\,(-1+a)^2} + \frac{1}{2}\sqrt{\frac{8\,a^2-20\,a^3+17\,a^4-4\,a^5}{(-1+a)^4}} <\\
&\quad P < \text{Root}\big[a^2-3\,a^3+a^4+(4\,a-10\,a^2+2\,a^3)\,\#1+(2-4\,a-5\,a^2+a^3)\,\#1^2+\\
&\qquad (1-9\,a+6\,a^2-2\,a^3)\,\#1^3+(-2-a+3\,a^2-a^3)\,\#1^4+(-2+4\,a-3\,a^2+a^3)\,\#1^5\;\&,\;3\big]\;\&\&\\
&\quad Q == \text{Root}\big[a^2\,P^4-3\,a^3\,P^4+a^4\,P^4-2\,a\,P^5+3\,a^2\,P^5-a^3\,P^5-P^6+3\,a\,P^6-3\,a^2\,P^6+a^3\,P^6+\\
&\qquad (4\,a^2\,P^3-12\,a^3\,P^3+4\,a^4\,P^3-2\,a\,P^4-5\,a^2\,P^4+11\,a^3\,P^4-4\,a^4\,P^4-2\,P^5+6\,a\,P^5-6\,a^2\,P^5+\\
&\qquad\quad 2\,a^3\,P^5+P^6-3\,a\,P^6+3\,a^2\,P^6-a^3\,P^6)\,\#1+(6\,a^2\,P^2-18\,a^3\,P^2+6\,a^4\,P^2+6\,a\,P^3-\\
&\qquad\quad 33\,a^2\,P^3+39\,a^3\,P^3-12\,a^4\,P^3+P^4-11\,a\,P^4+25\,a^2\,P^4-21\,a^3\,P^4+6\,a^4\,P^4)\,\#1^2+\\
&\qquad (4\,a^2\,P-12\,a^3\,P+4\,a^4\,P+10\,a\,P^2-39\,a^2\,P^2+41\,a^3\,P^2-12\,a^4\,P^2+4\,P^3-28\,a\,P^3+\\
&\qquad\quad 56\,a^2\,P^3-44\,a^3\,P^3+12\,a^4\,P^3-4\,P^4+16\,a\,P^4-24\,a^2\,P^4+16\,a^3\,P^4-4\,a^4\,P^4)\,\#1^3+(a^2-3\,a^3+\\
&\qquad\quad a^4+4\,a\,P-14\,a^2\,P+14\,a^3\,P-4\,a^4\,P+2\,P^2-14\,a\,P^2+28\,a^2\,P^2-22\,a^3\,P^2+6\,a^4\,P^2-3\,P^3+\\
&\qquad\quad 13\,a\,P^3-21\,a^2\,P^3+15\,a^3\,P^3-4\,a^4\,P^3+P^4-4\,a\,P^4+6\,a^2\,P^4-4\,a^3\,P^4+a^4\,P^4)\,\#1^4\;\&,\;2\big]\Bigg)\;||\\
&\Bigg(\frac{1}{2}\left(3-\sqrt{5}\right) < a < \boxed{0.401\ldots}\;\&\&\;\text{Root}\big[a^2-3\,a^3+a^4+(4\,a-10\,a^2+2\,a^3)\,\#1+(2-4\,a-5\,a^2+a^3)\,\#1^2+\\
&\qquad (1-9\,a+6\,a^2-2\,a^3)\,\#1^3+(-2-a+3\,a^2-a^3)\,\#1^4+(-2+4\,a-3\,a^2+a^3)\,\#1^5\;\&,\;2\big] <\\
&\quad P < \text{Root}\big[a^2-3\,a^3+a^4+(4\,a-10\,a^2+2\,a^3)\,\#1+(2-4\,a-5\,a^2+a^3)\,\#1^2+\\
&\qquad (1-9\,a+6\,a^2-2\,a^3)\,\#1^3+(-2-a+3\,a^2-a^3)\,\#1^4+(-2+4\,a-3\,a^2+a^3)\,\#1^5\;\&,\;3\big]\;\&\&\\
&\quad Q == \text{Root}\big[a^2\,P^4-3\,a^3\,P^4+a^4\,P^4-2\,a\,P^5+3\,a^2\,P^5-a^3\,P^5-P^6+3\,a\,P^6-3\,a^2\,P^6+a^3\,P^6+\\
&\qquad (4\,a^2\,P^3-12\,a^3\,P^3+4\,a^4\,P^3-2\,a\,P^4-5\,a^2\,P^4+11\,a^3\,P^4-4\,a^4\,P^4-2\,P^5+6\,a\,P^5-6\,a^2\,P^5+\\
&\qquad\quad 2\,a^3\,P^5+P^6-3\,a\,P^6+3\,a^2\,P^6-a^3\,P^6)\,\#1+(6\,a^2\,P^2-18\,a^3\,P^2+6\,a^4\,P^2+6\,a\,P^3-\\
&\qquad\quad 33\,a^2\,P^3+39\,a^3\,P^3-12\,a^4\,P^3+P^4-11\,a\,P^4+25\,a^2\,P^4-21\,a^3\,P^4+6\,a^4\,P^4)\,\#1^2+\\
&\qquad (4\,a^2\,P-12\,a^3\,P+4\,a^4\,P+10\,a\,P^2-39\,a^2\,P^2+41\,a^3\,P^2-12\,a^4\,P^2+4\,P^3-28\,a\,P^3+\\
&\qquad\quad 56\,a^2\,P^3-44\,a^3\,P^3+12\,a^4\,P^3-4\,P^4+16\,a\,P^4-24\,a^2\,P^4+16\,a^3\,P^4-4\,a^4\,P^4)\,\#1^3+\\
&\qquad (a^2-3\,a^3+a^4+4\,a\,P-14\,a^2\,P+14\,a^3\,P-4\,a^4\,P+2\,P^2-14\,a\,P^2+28\,a^2\,P^2-22\,a^3\,P^2+6\,a^4\,P^2-\\
&\qquad\quad 3\,P^3+13\,a\,P^3-21\,a^2\,P^3+15\,a^3\,P^3-4\,a^4\,P^3+P^4-4\,a\,P^4+6\,a^2\,P^4-4\,a^3\,P^4+a^4\,P^4)\,\#1^4\;\&,\;2\big]\Bigg)
\end{aligned}
$$

In[1136]:=
```
a1 := roots[[1]]
a2 := roots[[2]]
a1[[1]]
a1[[2]]
```

Out[1138]=
$$0 < a \leq \frac{1}{2}\left(3 - \sqrt{5}\right)$$

Out[1139]=
$$\frac{-2\,a + a^2}{2\,(-1 + a)^2} + \frac{1}{2}\sqrt{\frac{8\,a^2 - 20\,a^3 + 17\,a^4 - 4\,a^5}{(-1 + a)^4}} < P <$$
$$\text{Root}\left[a^2 - 3\,a^3 + a^4 + \left(4\,a - 10\,a^2 + 2\,a^3\right)\#1 + \left(2 - 4\,a - 5\,a^2 + a^3\right)\#1^2 + \left(1 - 9\,a + 6\,a^2 - 2\,a^3\right)\#1^3 + \left(-2 - a + 3\,a^2 - a^3\right)\#1^4 + \left(-2 + 4\,a - 3\,a^2 + a^3\right)\#1^5\,\&,\ 3\right]$$

In[1140]:=
```
a2[[1]]
a2[[2]]
```

Out[1140]=
$$\frac{1}{2}\left(3 - \sqrt{5}\right) < a < 0.401\ldots$$

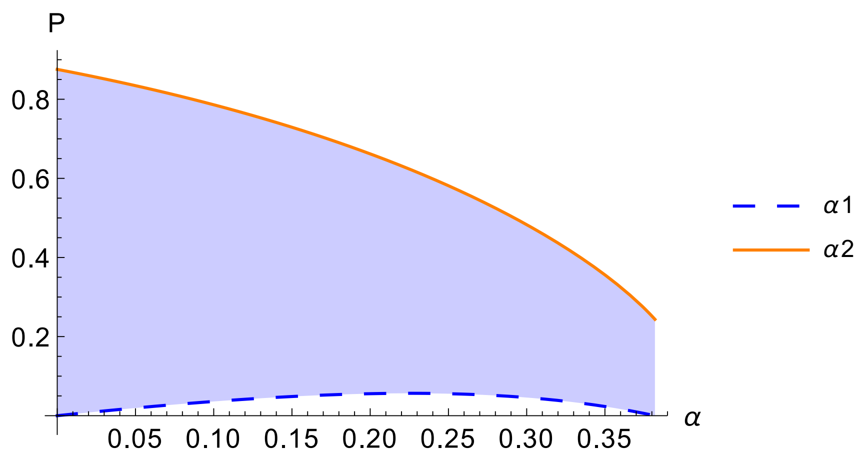
Out[1141]=
$$\text{Root}\left[a^2 - 3\,a^3 + a^4 + \left(4\,a - 10\,a^2 + 2\,a^3\right)\#1 + \left(2 - 4\,a - 5\,a^2 + a^3\right)\#1^2 + \left(1 - 9\,a + 6\,a^2 - 2\,a^3\right)\#1^3 + \left(-2 - a + 3\,a^2 - a^3\right)\#1^4 + \left(-2 + 4\,a - 3\,a^2 + a^3\right)\#1^5\,\&,\ 2\right] <$$
$$P < \text{Root}\left[a^2 - 3\,a^3 + a^4 + \left(4\,a - 10\,a^2 + 2\,a^3\right)\#1 + \left(2 - 4\,a - 5\,a^2 + a^3\right)\#1^2 + \left(1 - 9\,a + 6\,a^2 - 2\,a^3\right)\#1^3 + \left(-2 - a + 3\,a^2 - a^3\right)\#1^4 + \left(-2 + 4\,a - 3\,a^2 + a^3\right)\#1^5\,\&,\ 3\right]$$

In[1142]:=
```
amid = Root[-6784 + 80 832 #1 - 428 644 #1^2 + 1 279 684 #1^3 -
    2 275 131 #1^4 + 2 358 718 #1^5 - 1 175 083 #1^6 - 203 210 #1^7 + 760 072 #1^8 -
    570 650 #1^9 + 217 065 #1^10 - 41 706 #1^11 + 3125 #1^12 &, 2]
```

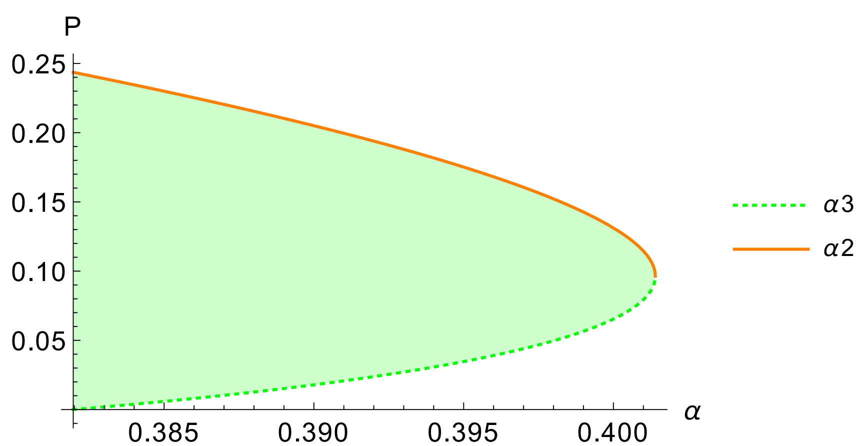
Out[1142]=
0.401…

In[1143]:=
```
graph1 = Plot[{(-2 a + a^2)/(2 (-1 + a)^2) + 1/2 Sqrt[(8 a^2 - 20 a^3 + 17 a^4 - 4 a^5)/(-1 + a)^4],
    Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 + (1 - 9 a + 6 a^2 - 2 a^3) #1^3 +
      (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 3] }, {a, 0, 1/2 (3 - Sqrt[5])},
  Filling -> {1 -> {2}}, PlotStyle -> {{Blue, Dashing[Large]}, Orange},
  AxesLabel -> {Style["α", FontSize -> 15], Style["P", FontSize -> 15]},
  PlotLegends -> {"α1", "α2"}, TicksStyle -> Directive[FontSize -> 15]]
graph2 = Plot[{Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 +
      (1 - 9 a + 6 a^2 - 2 a^3) #1^3 + (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 2],
    Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 + (1 - 9 a + 6 a^2 - 2 a^3) #1^3 +
      (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 3] },
  {a, 1/2 (3 - Sqrt[5]), amid }, Filling -> {1 -> {2}}, PlotStyle -> {{Green, Dotted}, Orange},
  AxesLabel -> {Style["α", FontSize -> 15], Style["P", FontSize -> 15]},
  PlotLegends -> {"α3", "α2"}, TicksStyle -> Directive[FontSize -> 15]]
```

Out[1143]=



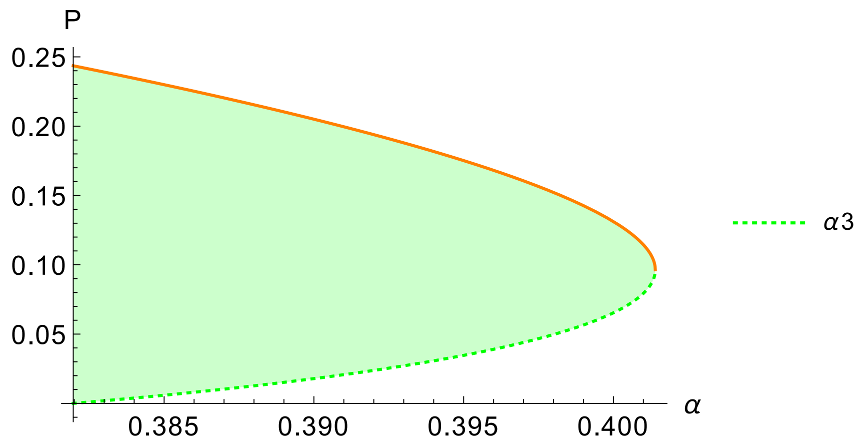
Out[1144]=



In[1145]:=

```
graph2partlegend =
 Plot[{Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 + (1 - 9 a + 6 a^2 - 2 a^3) #1^3 +
      (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 2],
   Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 + (1 - 9 a + 6 a^2 - 2 a^3) #1^3 +
      (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 3] },
  {a, 1/2 (3 - Sqrt[5]), amid }, Filling → {1 → {2}}, PlotStyle → {{Green , Dotted }, Orange },
  AxesLabel → {Style ["α", FontSize → 15], Style ["P", FontSize → 15]},
  PlotLegends → {"α3"}, TicksStyle → Directive [FontSize → 15]]
```

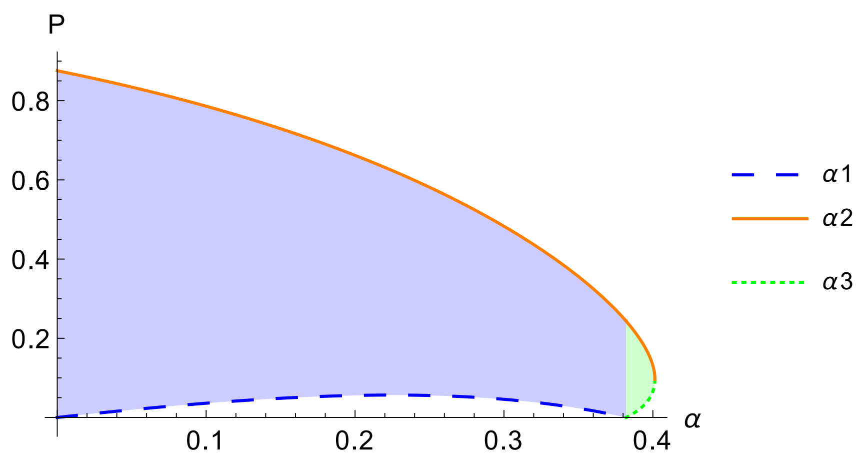
Out[1145]=



In[1146]:=

```
graphc = Show[graph1 , graph2partlegend , PlotRange → All]
```

Out[1146]=



In[1147]:=
```
CloudExport [graph1 , "EPS", "tp-left-size.eps"]
CloudExport [graph2 , "EPS", "tp-right-size.eps"]
CloudExport [graphc , "EPS", "tp.eps"]
```

Out[1147]=
```
CloudObject [https://www.wolframcloud.com/obj/tomer.tuchner/tp-left-size.eps]
```

Out[1148]=
```
CloudObject [https://www.wolframcloud.com/obj/tomer.tuchner/tp-right-size.eps]
```

Out[1149]=
```
CloudObject [https://www.wolframcloud.com/obj/tomer.tuchner/tp.eps]
```