

In[55]:= **Solve** [{z == 1 + z (1 - B)^2 + 2 o B (1 - B), o == 1 + o * (1 - H)}, {z, o}]

Out[55]= $\left\{\left\{z \rightarrow -\frac{2 B - 2 B^2 + H}{(-2 + B) B H}, o \rightarrow \frac{1}{H}\right\}\right\}$

In[56]:= **Simplify** $\left[-\frac{2 B - 2 B^2 + H}{(-2 + B) B H}\right]$

Out[56]= $\frac{2 B - 2 B^2 + H}{2 B H - B^2 H}$

In[57]:= **F = H + 2 B - 2 B^2**
G = 2 * H * B - H * B^2

Out[57]= $2 B - 2 B^2 + H$

Out[58]= $2 B H - B^2 H$

In[59]:= **ReplaceAll** [{H → a + (1 - a) P (P / (P + Q)), B → a + (1 - a) Q (P / (P + Q))}][{F}]
ReplaceAll [{H → a + (1 - a) P (P / (P + Q)), B → a + (1 - a) Q (P / (P + Q))}][{G}]

Out[59]= $\left\{a + \frac{(1 - a) P^2}{P + Q} + 2 \left(a + \frac{(1 - a) P Q}{P + Q}\right) - 2 \left(a + \frac{(1 - a) P Q}{P + Q}\right)^2\right\}$

Out[60]= $\left\{2 \left(a + \frac{(1 - a) P^2}{P + Q}\right) \left(a + \frac{(1 - a) P Q}{P + Q}\right) - \left(a + \frac{(1 - a) P^2}{P + Q}\right) \left(a + \frac{(1 - a) P Q}{P + Q}\right)^2\right\}$

In[61]:= **f**[Q_] := $\left\{a + \frac{(1 - a) P^2}{P + Q} + 2 \left(a + \frac{(1 - a) P Q}{P + Q}\right) - 2 \left(a + \frac{(1 - a) P Q}{P + Q}\right)^2\right\}$

g[Q_] := $\left\{2 \left(a + \frac{(1 - a) P^2}{P + Q}\right) \left(a + \frac{(1 - a) P Q}{P + Q}\right) - \left(a + \frac{(1 - a) P^2}{P + Q}\right) \left(a + \frac{(1 - a) P Q}{P + Q}\right)^2\right\}$

DNUM[Q_] := **FullSimplify** [f'[Q] g[Q] - g'[Q] f[Q]]

DNUM[Q]

Out[64]= $\left\{-\frac{1}{(P + Q)^6} 2 (-1 + a) P^2 \right. \\
(a^4 (P + Q - P Q)^4 + a^2 (P (-1 + Q) - Q) (P^3 (-1 + 3 (-1 + P) P) + P^2 (-3 + 7 P) Q + P (-3 + (23 - 18 P) P) Q^2 + \\
(-1 + P (13 + 3 P (-5 + 2 P))) Q^3) - a^3 (P (-1 + Q) - Q) \\
(P^3 (-3 + (-1 + P) P) + 9 (-1 + P) P^2 Q - 3 (-1 + P) P (-3 + 4 P) Q^2 + (-3 + P (11 + P (-11 + 4 P))) Q^3) + \\
P^2 (-P^4 + (-2 + P) P^3 Q + P^2 Q^2 - 4 (-1 + P) P Q^3 + (-2 + P) (-1 + P) Q^4) + a P (-3 P^5 (-1 + Q) + \\
2 P (5 - 7 Q) Q^3 + 4 Q^4 + P^4 (-2 + 6 Q) + P^2 Q^2 (6 + Q (-28 + 13 Q)) - P^3 Q (2 + Q (11 + 4 (-4 + Q) Q)))\left.\right\}$

In[65]:= **Sgn[Q_] := FullSimplify [DNUM [Q] * (P + Q) ^ 6]**
Sgn[Q]

Out[66]=
$$\{-2(-1+a)P^2$$

$$(a^4(P+Q-PQ)^4 + a^2(P(-1+Q)-Q)(P^3(-1+3(-1+P)P) + P^2(-3+7P)Q + P(-3+(23-18P)P)Q^2 +$$

$$(-1+P(13+3P(-5+2P)))Q^3) - a^3(P(-1+Q)-Q)$$

$$(P^3(-3+(-1+P)P) + 9(-1+P)P^2Q - 3(-1+P)P(-3+4P)Q^2 + (-3+P(11+P(-11+4P)))Q^3) +$$

$$P^2(-P^4 + (-2+P)P^3Q + P^2Q^2 - 4(-1+P)PQ^3 + (-2+P)(-1+P)Q^4) + aP(-3P^5(-1+Q) +$$

$$2P(5-7Q)Q^3 + 4Q^4 + P^4(-2+6Q) + P^2Q^2(6+Q(-28+13Q)) - P^3Q(2+Q(11+4(-4+Q)Q)))\}$$

In[67]:= **roots =**
Reduce [Sgn[Q] == 0 && Q > 0 && Q < 1 && P > 0 && P < 1 && a > 0 && a < 1, {a, P, Q}, Reals]

Out[67]=
$$\left(0 < a \leq \frac{1}{2}(3 - \sqrt{5}) \&\& \frac{-2a + a^2}{2(-1+a)^2} + \frac{1}{2} \sqrt{\frac{8a^2 - 20a^3 + 17a^4 - 4a^5}{(-1+a)^4}} < \right.$$

$$P < \text{Root}[a^2 - 3a^3 + a^4 + (4a - 10a^2 + 2a^3)\#1 + (2 - 4a - 5a^2 + a^3)\#1^2 +$$

$$(1 - 9a + 6a^2 - 2a^3)\#1^3 + (-2 - a + 3a^2 - a^3)\#1^4 + (-2 + 4a - 3a^2 + a^3)\#1^5 \&, 3] \&\&$$

$$Q == \text{Root}[a^2P^4 - 3a^3P^4 + a^4P^4 - 2aP^5 + 3a^2P^5 - a^3P^5 - P^6 + 3aP^6 - 3a^2P^6 + a^3P^6 +$$

$$(4a^2P^3 - 12a^3P^3 + 4a^4P^3 - 2aP^4 - 5a^2P^4 + 11a^3P^4 - 4a^4P^4 - 2P^5 + 6aP^5 - 6a^2P^5 +$$

$$2a^3P^5 + P^6 - 3aP^6 + 3a^2P^6 - a^3P^6)\#1 + (6a^2P^2 - 18a^3P^2 + 6a^4P^2 + 6aP^3 -$$

$$33a^2P^3 + 39a^3P^3 - 12a^4P^3 + P^4 - 11aP^4 + 25a^2P^4 - 21a^3P^4 + 6a^4P^4)\#1^2 +$$

$$(4a^2P - 12a^3P + 4a^4P + 10aP^2 - 39a^2P^2 + 41a^3P^2 - 12a^4P^2 + 4P^3 - 28aP^3 +$$

$$56a^2P^3 - 44a^3P^3 + 12a^4P^3 - 4P^4 + 16aP^4 - 24a^2P^4 + 16a^3P^4 - 4a^4P^4)\#1^3 +$$

$$(a^2 - 3a^3 + a^4 + 4aP - 14a^2P + 14a^3P - 4a^4P + 2P^2 - 14aP^2 + 28a^2P^2 - 22a^3P^2 + 6a^4P^2 -$$

$$3P^3 + 13aP^3 - 21a^2P^3 + 15a^3P^3 - 4a^4P^3 + P^4 - 4aP^4 + 6a^2P^4 - 4a^3P^4 + a^4P^4)\#1^4 \&, 2] \parallel$$

$$\left(\frac{1}{2}(3 - \sqrt{5}) < a < \text{Root}[a^2 - 3a^3 + a^4 + (4a - 10a^2 + 2a^3)\#1 + (2 - 4a - 5a^2 + a^3)\#1^2 +$$

$$(1 - 9a + 6a^2 - 2a^3)\#1^3 + (-2 - a + 3a^2 - a^3)\#1^4 + (-2 + 4a - 3a^2 + a^3)\#1^5 \&, 3] < \right.$$

$$P < \text{Root}[a^2 - 3a^3 + a^4 + (4a - 10a^2 + 2a^3)\#1 + (2 - 4a - 5a^2 + a^3)\#1^2 +$$

$$(1 - 9a + 6a^2 - 2a^3)\#1^3 + (-2 - a + 3a^2 - a^3)\#1^4 + (-2 + 4a - 3a^2 + a^3)\#1^5 \&, 3] \&\&$$

$$Q == \text{Root}[a^2P^4 - 3a^3P^4 + a^4P^4 - 2aP^5 + 3a^2P^5 - a^3P^5 - P^6 + 3aP^6 - 3a^2P^6 + a^3P^6 +$$

$$(4a^2P^3 - 12a^3P^3 + 4a^4P^3 - 2aP^4 - 5a^2P^4 + 11a^3P^4 - 4a^4P^4 - 2P^5 + 6aP^5 - 6a^2P^5 +$$

$$2a^3P^5 + P^6 - 3aP^6 + 3a^2P^6 - a^3P^6)\#1 + (6a^2P^2 - 18a^3P^2 + 6a^4P^2 + 6aP^3 -$$

$$33a^2P^3 + 39a^3P^3 - 12a^4P^3 + P^4 - 11aP^4 + 25a^2P^4 - 21a^3P^4 + 6a^4P^4)\#1^2 +$$

$$(4a^2P - 12a^3P + 4a^4P + 10aP^2 - 39a^2P^2 + 41a^3P^2 - 12a^4P^2 + 4P^3 - 28aP^3 +$$

$$56a^2P^3 - 44a^3P^3 + 12a^4P^3 - 4P^4 + 16aP^4 - 24a^2P^4 + 16a^3P^4 - 4a^4P^4)\#1^3 +$$

$$(a^2 - 3a^3 + a^4 + 4aP - 14a^2P + 14a^3P - 4a^4P + 2P^2 - 14aP^2 + 28a^2P^2 - 22a^3P^2 + 6a^4P^2 -$$

$$3P^3 + 13aP^3 - 21a^2P^3 + 15a^3P^3 - 4a^4P^3 + P^4 - 4aP^4 + 6a^2P^4 - 4a^3P^4 + a^4P^4)\#1^4 \&, 2] \Big)$$

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In[68]:= a1 := roots [[1]]
a2 := roots [[2]]
a1[[1]]
a1[[2]]
```

Out[70]= $0 < a \leq \frac{1}{2} (3 - \sqrt{5})$

Out[71]= $\frac{-2a + a^2}{2(-1 + a)^2} + \frac{1}{2} \sqrt{\frac{8a^2 - 20a^3 + 17a^4 - 4a^5}{(-1 + a)^4}} < P <$

$$\text{Root}[a^2 - 3a^3 + a^4 + (4a - 10a^2 + 2a^3)\#1 + (2 - 4a - 5a^2 + a^3)\#1^2 + (1 - 9a + 6a^2 - 2a^3)\#1^3 + (-2 - a + 3a^2 - a^3)\#1^4 + (-2 + 4a - 3a^2 + a^3)\#1^5 \&, 3]$$

```
In[72]:= a2[[1]]
a2[[2]]
```

Out[72]= $\frac{1}{2} (3 - \sqrt{5}) < a < 0.401 \dots$

Out[73]= $\text{Root}[a^2 - 3a^3 + a^4 + (4a - 10a^2 + 2a^3)\#1 + (2 - 4a - 5a^2 + a^3)\#1^2 + (1 - 9a + 6a^2 - 2a^3)\#1^3 + (-2 - a + 3a^2 - a^3)\#1^4 + (-2 + 4a - 3a^2 + a^3)\#1^5 \&, 2] <$
 $P < \text{Root}[a^2 - 3a^3 + a^4 + (4a - 10a^2 + 2a^3)\#1 + (2 - 4a - 5a^2 + a^3)\#1^2 + (1 - 9a + 6a^2 - 2a^3)\#1^3 + (-2 - a + 3a^2 - a^3)\#1^4 + (-2 + 4a - 3a^2 + a^3)\#1^5 \&, 3]$

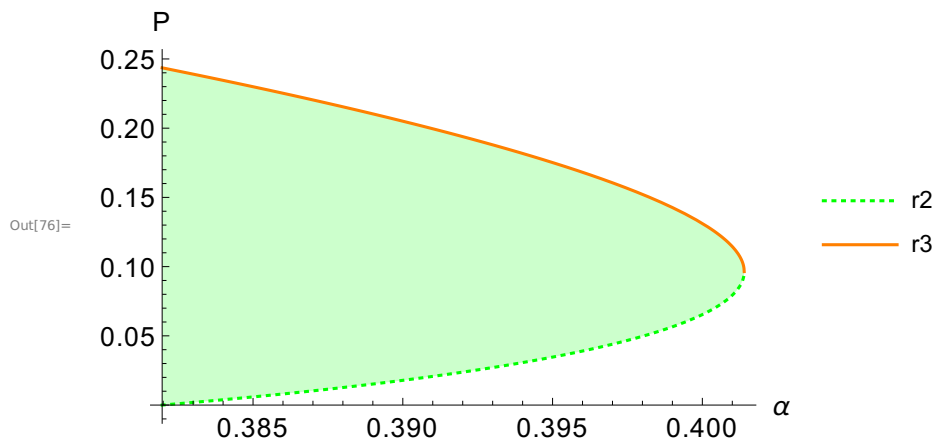
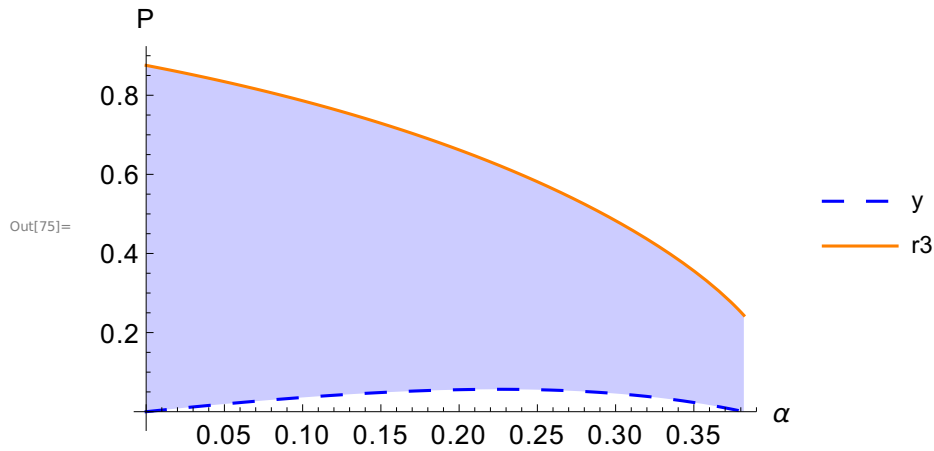
```
In[74]:= amid = Root[-6784 + 80832 #1 - 428644 #1^2 + 1279684 #1^3 -
2275131 #1^4 + 2358718 #1^5 - 1175083 #1^6 - 203210 #1^7 + 760072 #1^8 -
570650 #1^9 + 217065 #1^10 - 41706 #1^11 + 3125 #1^12 &, 2]
```

Out[74]= $0.401 \dots$

```

In[75]:= graph1 = Plot[ $\left\{\frac{-2a + a^2}{2(-1+a)^2} + \frac{1}{2} \sqrt{\frac{8a^2 - 20a^3 + 17a^4 - 4a^5}{(-1+a)^4}}, \right.$ 
      Root[ $a^2 - 3a^3 + a^4 + (4a - 10a^2 + 2a^3)\#1 + (2 - 4a - 5a^2 + a^3)\#1^2 + (1 - 9a + 6a^2 - 2a^3)\#1^3 +$ 
       $(-2 - a + 3a^2 - a^3)\#1^4 + (-2 + 4a - 3a^2 + a^3)\#1^5 \&, 3]$ , {a, 0,  $\frac{1}{2}(3 - \sqrt{5})$ },
      Filling -> {1 -> {2}}, PlotStyle -> {{Blue, Dashing[Large]}, Orange},
      AxesLabel -> {Style["a", FontSize -> 15], Style["P", FontSize -> 15]},
      PlotLegends -> {"y", "r3"}, TicksStyle -> Directive[FontSize -> 15]]
graph2 = Plot[ $\left\{\text{Root}[a^2 - 3a^3 + a^4 + (4a - 10a^2 + 2a^3)\#1 + (2 - 4a - 5a^2 + a^3)\#1^2 + \right.$ 
       $(1 - 9a + 6a^2 - 2a^3)\#1^3 + (-2 - a + 3a^2 - a^3)\#1^4 + (-2 + 4a - 3a^2 + a^3)\#1^5 \&, 2],$ 
      Root[ $a^2 - 3a^3 + a^4 + (4a - 10a^2 + 2a^3)\#1 + (2 - 4a - 5a^2 + a^3)\#1^2 + (1 - 9a + 6a^2 - 2a^3)\#1^3 +$ 
       $(-2 - a + 3a^2 - a^3)\#1^4 + (-2 + 4a - 3a^2 + a^3)\#1^5 \&, 3]$ , {a,  $\frac{1}{2}(3 - \sqrt{5})$ , amid},
      Filling -> {1 -> {2}}, PlotStyle -> {{Green, Dotted}, Orange},
      AxesLabel -> {Style["a", FontSize -> 15], Style["P", FontSize -> 15]},
      PlotLegends -> {"r2", "r3"}, TicksStyle -> Directive[FontSize -> 15]]

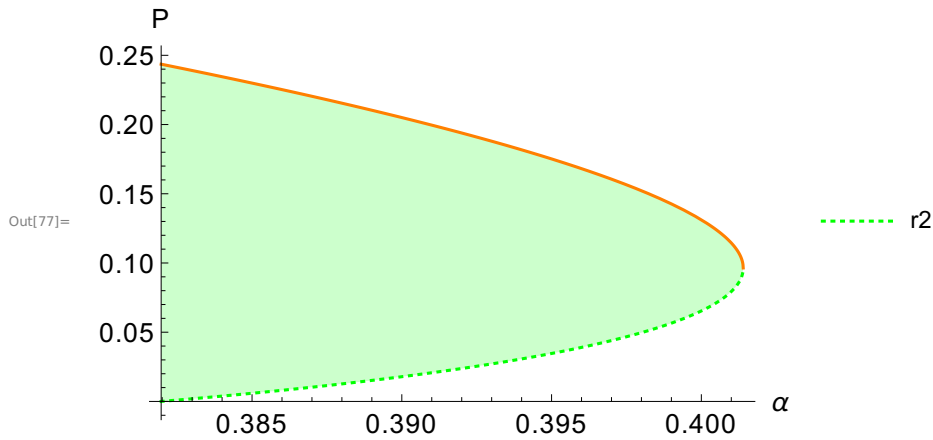
```



```

In[77]:= graph2partlegend =
  Plot[{Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 + (1 - 9 a + 6 a^2 - 2 a^3) #1^3 +
    (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 2],
    Root[a^2 - 3 a^3 + a^4 + (4 a - 10 a^2 + 2 a^3) #1 + (2 - 4 a - 5 a^2 + a^3) #1^2 + (1 - 9 a + 6 a^2 - 2 a^3) #1^3 +
    (-2 - a + 3 a^2 - a^3) #1^4 + (-2 + 4 a - 3 a^2 + a^3) #1^5 &, 3] },
  {a,  $\frac{1}{2}(3 - \sqrt{5})$ , amid}, Filling -> {1 -> {2}}, PlotStyle -> {{Green, Dotted}, Orange},
  AxesLabel -> {Style["α", FontSize -> 15], Style["P", FontSize -> 15]},
  PlotLegends -> {"r2"}, TicksStyle -> Directive[FontSize -> 15]]

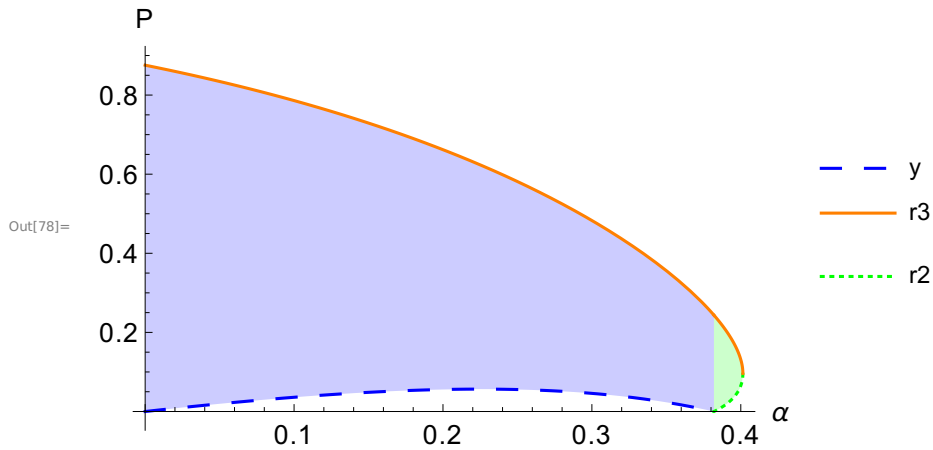
```



```

In[78]:= graphc = Show[graph1, graph2partlegend, PlotRange -> All]

```



```
In[79]:= CloudExport [graph1 , "EPS", "tp-left-size.eps"]  
          CloudExport [graph2 , "EPS", "tp-right-size.eps"]  
          CloudExport [graphc , "EPS", "tp.eps"]  
  
Out[79]= CloudObject [https://www.wolframcloud .com/obj/tomer.tuchner /tp-left-size.eps]  
  
Out[80]= CloudObject [https://www.wolframcloud .com/obj/tomer.tuchner /tp-right-size.eps]  
  
Out[81]= CloudObject [https://www.wolframcloud .com/obj/tomer.tuchner /tp.eps]
```